Cooling down a coke can

Experiment study

Fluid Mechanics and Transport Processes

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“Cooling down a coke can”
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Chapitre 1

Introduction

This report outlines the physics behind an experiment produced during the MECA-H3001 “Fluid Mechanics and Transport Processes” course of the ULB (Université Libre de Bruxelles) on Friday, the sixteenth of October 2015.

The experiment can be found by following this link [1] and was described as following :

- Three coke cans were available as well as a bucket of ice and water (at 0°C) and a drill to spin the can inside the bucket.
- One of the cans was used to determine the initial temperature (16°C) of the fluid (essentially water) inside the can.
- One can was left inside the bucket for 60 seconds and a final temperature of 11.9°C was measured as a result of the conductive heat transfer with the surrounding fluid.
- One can was spun inside the bucket for 60 seconds at about 1000 rounds per minute, and a final temperature of 11°C was measured as a result of the convective and conductive heat transfer.

In the first chapter, convective and conductive processes that are related to the experiment will be described. Those phenomena will then be applied to the problem via a mathematical model, including a discussion of simplifications brought to the problem in order to simplify calculations. This model will be compared to the experimental results, after what a discussion on it's reliability followed by a conclusion on mathematical models in general will be presented.

1. https://www.youtube.com/watch?v=MSwc_IAPh3E

Chapitre 2

Heat transfer processes

They are three different ways of transferring heat : conduction, convection and radiation. Due to its nature, radiation can be neglected for this experiment and will thus not be presented here.

2.1 Conduction

Thermal conduction is a heat transfer process without macroscopic movement of matter. It is initiated by a difference of temperature between contiguous bodies (or inside a body). This difference of temperature implies a difference of internal energy : the energy is higher in the warmer area than in the cooler. By diffusion and collisions between the particles which can be molecules in a fluid or conduction electrons in a solid, particles in the warmer area transfer kinetic energy to the other particles, making them moving or vibrating faster. This creates a heat flow from the warmer area to the cooler until the system reaches thermal equilibrium. Furthermore, conduction is an irreversible process.

Conduction is described by the following general equation, which is demonstrated in Professor Jean-Marie Buchlin's course[2], Chapter 13.

$$\frac{\delta T}{\delta t} = \bigtriangledown.(\alpha \bigtriangledown T) + \dot{Q}_v \tag{2.1}$$

This equation can not be used by itself because of it's nature (second degree partial derivative equation). It thus needs conditions linked to properties of the system. Those can be geometrical, physical, temporal or border conditions.

Conditions used and simplifications of the general equation above will be discussed in chapter 3

2.2 Convection

Convection is a heat transfer in fluid. Convection occurs when some fluid is in movement. The movement lead to an advection (heat is transported by matters when it's moving).

Convection is described as following :

$$Convection = Conduction + Advection \tag{2.2}$$

Seeing this, it is easily to understand that convection is superior than conduction in fluids in a flux situation. Flow properties have a major impact in heat transfers.

As convection depends on the flow (laminar, turbulent,...), we will discuss the equation to use in the next chapter(Mathematical model,chapter 3).

Chapitre 3

Mathematical model

The idea behind mathematical models is to create a simplified version of a problem, that is accurate enough to predict the behaviour of a system, but simple enough to be resolved with few calculations.

This means that some simplifications of the equations seen before can be made, using the properties of the studied system.

We will first describe general simplifying assumptions for our experiment and explain why we can use them, after what we will go ahead and create two simplified models : one for the non-spinning can and one for the spinning can.

3.1 General simplifying assumptions

The first assumption we will consider is that the fluid contained in the can has properties similar to water. Coke is indeed an aqueous solution containing sugar and other ingredients, but at relatively low concentrations. Properties of water can be found in annex A and are extracted from *"Perry's Chemical Engineers' Handbook"[1]*.

Another assumption is that the can is a perfect cylinder with an height of $h = 116mm$ and a diameter of $d = 66mm$ [1]. In the reality, the shape of a can is a bit different to support pressure but difference should not be significant in our calculations. The material used for cans is Aluminium and since it is extremely thin(less than $1mm$) and has an excellent thermal conductivity ($> 200\frac{W}{K.m}$), we will consider it has no influence on heat transfers. We will thus consider a cylinder of water inside water maintained at another temperature and without any matter exchange, as shown in figure 3.1.

Furthermore, we will consider that all heat exchanges between the can and surrounding ice-cold water takes place on de sides of the can and not on it's top or bottom. The total surface of the can

1. dimensions are standard ones for European Aluminium $330ml$ cans, those can be found on webpackaging.com

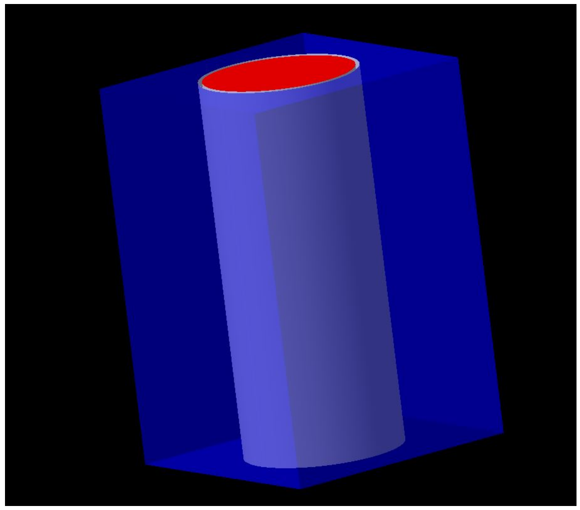


FIGURE 3.1 – The red cylinder is hotter water, the blue box is colder water. No matter exchanges, only heat.

is given by :

$$\begin{aligned} Surface &= 2.Surface_{circle} + Surface_{rectangularside} \\ \Leftrightarrow S &= 2(\pi(\frac{d}{2})^2) + \pi dh \\ \Leftrightarrow S &= 30.8944cm^3 \end{aligned} \tag{3.1}$$

Bottom and top circular surfaces have a total surface of $2xSurface_{circle} = 2(\pi(\frac{d}{2})^2) = 6.8424cm^3$, this is thus about 22.14% of total surface. Reason why we decided to ignore such a large portion of the can's area is because half of it is not even in contact with ice during the experiment (the system holding the can covers it's top and non-spinning can is not totally in ice), reducing the area to consider at about 11% of total exchange area. This can still be seen as a large percentage, but because it is on the bottom of the can and because we are trying to cool down a liquid and cold fluids tend to be more dense than hot fluid, the liquid in contact with the bottom part will be cooler, making heat transfer far less efficient there.

The last general simplification we will make is to consider the mix of ice and water surrounding the cans as a continuous layer of water maintained at 0℃. This should be correct enough considering that we had a large quantity of melting ice in an isolated box and because liquid water is more flexible than ice, the can has a larger surface in contact with molten ice (and thus water) than with ice itself.

3.2 Simplified models

With these four simplifications in mind, we will now build two models, one for the non-spinning can and one for the spinning can.

3.2.1 Non-spinning can

Heat transfer processes in presence are conduction and natural convection. Natural convection appears because of the density change of fluid related to temperature : the cooler the fluid, the denser it is. There is no forced convection because there is no flux in the fluid.

The convective heat transfer coefficient for natural convection (in the can) is given by :

$$h_x = Ra^{\alpha} = cGr^{\alpha}Pr^{\alpha} \tag{3.2}$$

Where Gr is the Grashof Number and Pr the Prandtl number. Prandtl is found in annex A : water being at 16°C(about $290K$), we have $Pr = 7.56$.
Grashof number is calculated using the following formula (taken from annex B) :

$$Gr = \frac{\beta g \Delta T_{ref} d^3}{\nu^2} \tag{3.3}$$

Where :

- β is the volume expansion coefficient, given by $\frac{1}{\rho}\frac{\delta\rho}{\delta T}$. Using the tables in annex A, we can approximate $\beta = 1.001 \times 10^{-3} \frac{\frac{1}{1.000\times10^{-3}} - \frac{1}{1.001\times10^{-3}}}{5} = 1.998 \times 10^{-4} K^{-1}$.
- $g = 9.81 \frac{m^2}{s}$ is gravity.
- $\Delta T_{ref} = 16 - 0 = 16$°C is the difference of temperature between the water inside the can and the ice-cold water outside of it.
- $d = 33 \times 10^{-3} m$ is the diameter of the cylinder.
- $\nu = \frac{\mu}{\rho} = \frac{1080\times10^{-6}}{\frac{1}{1.001\times10^{-3}}} = 1.081 \times 10^{-6} \frac{m^2}{s}$ is water's kinematic viscosity.

All values used are from annex A.

Rayleigh is thus :

$$Ra = GrPr = 7.56 \times \frac{1.998 \times 10^{-4} \times 9.81 \times 16 \times (66 \times 10^{-3})^3}{(1.081 \times 10^{-6})^2} = 5.8329 \times 10^7 \tag{3.4}$$

Since $Ra < 10^9$, we are in a laminar case. We consider that the ice-cold water around the can is at a constant temperature. We now need to determine which equation to use for Nusselt number.

The easiest approximation is to describe the cylinder as a rectangular enclosure, with H (the height of the enclosure) equal to $h = 116mm$ and L (the characteristic length of the enclosure) equal to $\frac{d}{2} = 33mm$. This gives us an H on L ratio of $\frac{H}{L} = 3.352$. We will thus use equation 9-53 at page 555 of “Heat and Mass Transfer : Fundamentals and Applications” [3], chapter 9-5 :

$$\begin{gathered} Nu = 0.22(\frac{Pr}{0.2 + Pr}Ra)^{0.28}(\frac{H}{L})^{-1/4} \\ \Leftrightarrow Nu = 2.3835 \times 10^1 \end{gathered} \tag{3.5}$$

and :

$$\begin{gathered} Nu = \frac{hL}{k} \\ \Leftrightarrow h = \frac{Nuk}{L} = \frac{2Nuk}{d} \\ \Leftrightarrow h = 4.3191 \times 10^2 \frac{W}{m^2K} \end{gathered} \tag{3.6}$$

Biot number is given by $Bi = \frac{hL}{k} = \frac{hd}{2k} = 23.8347 > 1$! This indicates that *we should not considerate our system as a Lumped system*, the temperature inside the can is not homogeneous ! But, as our temperature is measured manually by putting a thermometer inside the can, and because the size of the thermometer is not negligible compared to the small radius of the can ; we will simplify the system by using the equations for a Lumped system in the case of sensible heat transfer :

$$\begin{aligned}
\rho C V \frac{dT}{dt} &= -h S_{tot}(T - T_{ext}) \\
\Leftrightarrow \frac{h S_{tot}}{\rho C V} dt &= \frac{1}{(T_{ext} - T)} dT \\
\Leftrightarrow \frac{4h}{\rho C d} dt &= \frac{1}{(T_{ext} - T)} dT \\
\Leftrightarrow \frac{4h}{\rho C d} t &= ln(\frac{T_0 - T_{ext}}{T_f - T_{ext}}) \\
\Leftrightarrow e^{\frac{4h}{\rho C d} t} &= \frac{T_0 - T_{ext}}{T_f - T_{ext}} \\
\Leftrightarrow T_f &= T_{ext} + \frac{T_0 - T_{ext}}{e^{\frac{4h}{\rho C d} t}}
\end{aligned} \tag{3.7}$$

This, when we use the parameters of our experiment, would have given $T_f = 284.14K = 10.99$°C after $t = 60s$.

This value is quite far from what is observed in our experiment. This is due to some simplifications we made, like, for example, using the Lumped system equations, or some of our parameters choice.

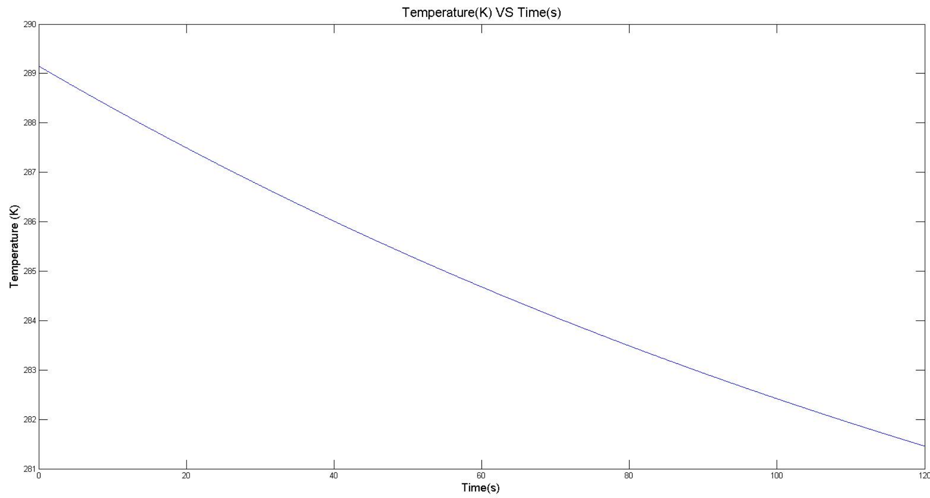
Looking at our parameters, the approximation made for β can be refined. If, instead of using the value approximated over the temperature interval, we use the value for β at 15°C given in Appendix A-1, table A-9 of "Heat and Mass Transfer : Fundamentals and Applications" [3] : $\beta = 1.38 \times 10^{-4} K^{-1}$. Injecting this in our model already adds some accuracy, with a temperature of 11.40°C ($284.55K$).

Moreover, the values of β and Pr are not fixed, they depend on temperature. To get even more accurate results, we will now take "mean values" of those two numbers. In order to do so, since their evolution is small in our interval of temperatures (1.38×10^{-4} to 0.733×10^{-4} for β and 8.09 to 9.45×10^{-4} for Prandtl [2]), we make the assumption that their variation is linear and that we can use the linear mean value on the interval. In the end, we have $\beta = 1.056 \times 10^{-4} K^{-1}$ and $Pr = 8.77$.

Those corrections, when injected in our model, give us $T_f = 284.68K = 11.53$°C after $t = 60s$. This value seems acceptable, considering the simplifications made on the system. The error compared to the experimental results is about 3.11%, and we have to take in account the fact that the temperature measurement made during the experiment where not immediate and that the thermometer itself was warm.

Graph 3.2.1 gives the evolution of temperature(K) with time(s) for our last model with enhanced approximations of Prandtl and β.

2. Be warned that the values for Prandtl are different than the one used before : this is due to the fact we decided to use values from the same source ("Heat and Mass Transfer : Fundamentals and Applications" [3]) here to ensure they are calculated the same way. This also shows the differences we can have for the same number, depending of your source and the temperatures it uses as standards



3.2.2 Spinning can

Chapitre 4

Comparison between reality and the mathematical model

Chapitre 5

Conclusion

Annexe A

Water Thermo-Physical Properties

These chart can be are extracted from “Perry’s Chemical Engineers’ Handbook”, Chapter 2[1].

TABLE 2-352 Saturated Water Substance—Temperature (SI units)

Temp., K	Pressure, bar*	Volume, m³/kg		Enthalpy, kJ/kg		Entropy, kJ/(kg·K)		Specific heat, C_p, kJ/(kg·K)		Viscosity, Ns/m²		Thermal conductivity, W/(m·K)		Prandtl no.		Surface tension, N/m	Temp., K
		Condensed†	Vapor	Condensed†	Vapor	Condensed†	Vapor	Condensed†	Vapor	Condensed†	Vapor	Condensed†	Vapor	Condensed†	Vapor	Condensed†	
150	6.30.–11	1.073.–3	9.55.+9	–539.6	2273	–2.187	16.54	1.155				3.73					150
160	7.72.–10	1.074.–3	9.62.+8	–525.7	2291	–2.106	15.49	1.233				3.52					160
170	7.29.–9	1.076.–3	1.08.+8	–511.7	2310	–2.026	14.57	1.311				3.34					170
180	5.38.–8	1.077.–3	1.55.+7	–497.8	2328	–1.947	13.76	1.389				3.18					180
190	3.23.–7	1.078.–3	2.72.+6	–483.8	2347	–1.868	13.03	1.467				3.04					190
200	1.62.–6	1.079.–3	5.69.+5	–467.5	2366	–1.789	12.38	1.545				2.91					200
210	7.01.–6	1.081.–3	1.39.+5	–451.2	2384	–1.711	11.79	1.623				2.79					210
220	2.65.–5	1.082.–3	3.83.+4	–435.0	2403	–1.633	11.20	1.701				2.69					220
230	8.91.–5	1.084.–3	1.18.+4	–416.3	2421	–1.555	10.79	1.779				2.59					230
240	3.72.–4	1.085.–3	4.07.+3	–400.1	2440	–1.478	10.35	1.857				2.50					240
250	7.59.–4	1.087.–3	1.52.+3	–381.5	2459	–1.400	9.954	1.935				2.42					250
255	1.23.–3	1.087.–3	956.4	–369.8	2468	–1.361	9.768	1.974				2.38					255
260	1.96.–3	1.088.–3	612.2	–360.5	2477	–1.323	9.590	2.013				2.35					260
265	3.06.–3	1.089.–3	400.4	–351.2	2486	–1.281	9.461	2.052				2.31					265
270	4.69.–3	1.090.–3	265.4	–339.6	2496	–1.296	9.255	2.091				2.27					270
273.15	6.11.–3	1.091.–3	206.3	–333.5	2502	–1.221	9.158	2.116				2.26					273.15
273.15	0.00611	1.000.–3	206.3	0.0	2502	0.000	9.158	4.217	1.854	1750.–6	8.02.–6	0.569	0.0182	12.99	0.815	0.0755	273.15
275	0.00697	1.000.–3	181.7	7.8	2505	0.028	9.109	4.211	1.855	1652.–6	8.09.–6	0.574	0.0183	12.22	0.817	0.0753	275
280	0.00990	1.000.–3	130.4	28.8	2514	0.104	8.980	4.198	1.858	1422.–6	8.29.–6	0.582	0.0186	10.26	0.825	0.0748	280
285	0.01387	1.000.–3	99.4	49.8	2523	0.178	8.857	4.189	1.861	1225.–6	8.49.–6	0.590	0.0189	8.81	0.833	0.0743	285
290	0.01917	1.001.–3	69.7	70.7	2532	0.251	8.740	4.184	1.864	1080.–6	8.69.–6	0.598	0.0193	7.56	0.841	0.0737	290
295	0.02617	1.002.–3	51.94	91.6	2541	0.323	8.627	4.181	1.868	959.–6	8.89.–6	0.606	0.0195	6.62	0.849	0.0727	295
300	0.03531	1.003.–3	39.13	112.5	2550	0.393	8.520	4.179	1.872	855.–6	9.09.–6	0.613	0.0196	5.83	0.857	0.0717	300
305	0.04712	1.005.–3	27.90	133.4	2559	0.462	8.417	4.178	1.877	769.–6	9.29.–6	0.620	0.0201	5.20	0.865	0.0709	305
310	0.06221	1.007.–3	22.93	154.3	2568	0.530	8.318	4.178	1.882	695.–6	9.49.–6	0.628	0.0204	4.62	0.873	0.0700	310
315	0.08132	1.009.–3	17.82	175.2	2577	0.597	8.224	4.179	1.888	631.–6	9.69.–6	0.634	0.0207	4.16	0.883	0.0692	315
320	0.1053	1.011.–3	13.98	196.1	2586	0.649	8.151	4.180	1.895	577.–6	9.89.–6	0.640	0.0210	3.77	0.894	0.0683	320
325	0.1351	1.013.–3	11.06	217.0	2595	0.727	8.046	4.182	1.903	528.–6	10.09.–6	0.645	0.0213	3.42	0.901	0.0675	325
330	0.1719	1.016.–3	8.82	237.9	2604	0.791	7.962	4.184	1.911	489.–6	10.29.–6	0.650	0.0217	3.15	0.908	0.0666	330
335	0.2167	1.018.–3	7.09	258.8	2613	0.854	7.881	4.186	1.920	453.–6	10.49.–6	0.655	0.0220	2.88	0.916	0.0658	335
340	0.2713	1.021.–3	5.74	279.8	2622	0.916	7.804	4.188	1.930	420.–6	10.69.–6	0.660	0.0223	2.66	0.925	0.0649	340
345	0.3372	1.024.–3	4.683	300.7	2630	0.977	7.729	4.191	1.941	389.–6	10.89.–6	0.665	0.0226	2.45	0.933	0.0641	345
350	0.4163	1.027.–3	3.846	321.7	2639	1.038	7.657	4.195	1.954	365.–6	11.09.–6	0.668	0.0230	2.29	0.942	0.0632	350
355	0.5100	1.030.–3	3.180	342.7	2647	1.097	7.588	4.199	1.968	343.–6	11.29.–6	0.671	0.0233	2.14	0.951	0.0623	355
360	0.6209	1.034.–3	2.645	363.7	2655	1.156	7.521	4.203	1.983	324.–6	11.49.–6	0.674	0.0237	2.02	0.960	0.0614	360
365	0.7514	1.038.–3	2.212	384.7	2663	1.214	7.456	4.209	1.999	306.–6	11.69.–6	0.677	0.0241	1.91	0.969	0.0605	365
370	0.9040	1.041.–3	1.861	405.8	2671	1.271	7.394	4.214	2.017	289.–6	11.89.–6	0.679	0.0245	1.80	0.978	0.0595	370
373.15	1.0133	1.044.–3	1.679	419.1	2676	1.307	7.356	4.217	2.029	279.–6	12.02.–6	0.680	0.0248	1.76	0.984	0.0589	373.15
375	1.0815	1.045.–3	1.574	426.8	2679	1.328	7.333	4.220	2.036	274.–6	12.09.–6	0.681	0.0249	1.70	0.987	0.0586	375
380	1.2869	1.049.–3	1.337	448.0	2687	1.384	7.275	4.226	2.057	260.–6	12.29.–6	0.683	0.0254	1.61	0.995	0.0576	380
385	1.5233	1.053.–3	1.142	469.2	2694	1.439	7.218	4.232	2.080	248.–6	12.49.–6	0.685	0.0258	1.53	1.004	0.0566	385
390	1.794	1.058.–3	0.980	490.4	2702	1.494	7.163	4.239	2.104	237.–6	12.69.–6	0.686	0.0263	1.47	1.013	0.0556	390
400	2.455	1.067.–3	0.731	532.9	2716	1.605	7.058	4.256	2.158	217.–6	13.05.–6	0.688	0.0272	1.34	1.033	0.0536	400
410	3.302	1.077.–3	0.553	575.6	2729	1.708	6.959	4.278	2.221	200.–6	13.42.–6	0.688	0.0282	1.24	1.054	0.0515	410
420	4.370	1.088.–3	0.425	618.6	2742	1.810	6.865	4.302	2.291	185.–6	13.79.–6	0.688	0.0293	1.16	1.075	0.0494	420
430	5.699	1.099.–3	0.331	661.8	2753	1.911	6.775	4.331	2.369	173.–6	14.14.–6	0.685	0.0304	1.09	1.10	0.0472	430

440	7.333	1.110.–3	0.261	705.3	2764	2.011	6.689	4.36	2.46	162.–6	14.50.–6	0.682	0.0317	1.04	1.12	0.0451	440
450	9.319	1.123.–3	0.208	749.2	2773	2.109	6.607	4.40	2.56	152.–6	14.85.–6	0.678	0.0331	0.99	1.14	0.0429	450
460	11.71	1.137.–3	0.167	793.5	2782	2.205	6.528	4.44	2.68	143.–6	15.19.–6	0.673	0.0346	0.95	1.17	0.0407	460
470	14.55	1.152.–3	0.136	838.2	2789	2.301	6.451	4.48	2.79	136.–6	15.54.–6	0.667	0.0363	0.92	1.20	0.0385	470
480	17.90	1.167.–3	0.111	883.4	2795	2.395	6.377	4.53	2.94	129.–6	15.88.–6	0.660	0.0381	0.89	1.23	0.0362	480
490	21.83	1.184.–3	0.0922	929.1	2799	2.479	6.312	4.59	3.10	124.–6	16.23.–6	0.651	0.0401	0.87	1.25	0.0339	490
500	26.40	1.203.–3	0.0766	975.6	2801	2.581	6.233	4.66	3.27	118.–6	16.59.–6	0.642	0.0423	0.86	1.28	0.0316	500
510	31.66	1.222.–3	0.0631	1023	2802	2.673	6.163	4.74	3.47	113.–6	16.95.–6	0.631	0.0447	0.85	1.31	0.0293	510
520	37.70	1.244.–3	0.0525	1071	2801	2.765	6.093	4.84	3.70	108.–6	17.33.–6	0.621	0.0475	0.84	1.35	0.0269	520
530	44.58	1.268.–3	0.0445	1119	2798	2.856	6.023	4.95	3.96	104.–6	17.72.–6	0.608	0.0506	0.85	1.39	0.0245	530
540	52.38	1.294.–3	0.0375	1170	2792	2.948	5.953	5.08	4.27	101.–6	18.1.–6	0.594	0.0540	0.86	1.43	0.0221	540
550	61.19	1.323.–3	0.0317	1220	2784	3.039	5.882	5.24	4.64	97.–6	18.6.–6	0.580	0.0583	0.87	1.47	0.0197	550
560	71.08	1.355.–3	0.0269	1273	2772	3.132	5.808	5.43	5.09	94.–6	19.1.–6	0.563	0.0637	0.90	1.52	0.0173	560
570	82.16	1.392.–3	0.0228	1328	2757	3.225	5.733	5.68	5.67	91.–6	19.7.–6	0.548	0.0698	0.94	1.59	0.0150	570
580	94.51	1.433.–3	0.0193	1384	2737	3.321	5.654	6.00	6.40	88.–6	20.4.–6	0.528	0.0767	0.99	1.68	0.0128	580
590	108.3	1.482.–3	0.0163	1443	2717	3.419	5.569	6.41	7.35	84.–6	21.5.–6	0.513	0.0841	1.05	1.84	0.0105	590
600	123.5	1.541.–3	0.0137	1506	2682	3.520	5.480	7.00	8.75	81.–6	22.7.–6	0.497	0.0929	1.14	2.15	0.0084	600
610	137.3	1.612.–3	0.0115	1573	2641	3.627	5.318	7.85	11.1	77.–6	24.1.–6	0.467	0.103	1.30	2.60	0.0063	610
620	159.1	1.705.–3	0.0094	1647	2588	3.741	5.259	9.35	15.4	72.–6	25.9.–6	0.444	0.114	1.52	3.46	0.0045	620
625	169.1	1.778.–3	0.0085	1697	2555	3.805	5.191	10.6	18.3	70.–6	27.0.–6	0.430	0.121	1.65	4.20	0.0035	625
630	179.7	1.856.–3	0.0075	1734	2515	3.875	5.115	12.6	22.1	67.–6	28.0.–6	0.412	0.130	2.0	4.8	0.0026	630
635	190.9	1.935.–3	0.0066	1783	2466	3.950	5.025	16.4	27.6	64.–6	30.0.–6	0.392	0.141	2.7	6.0	0.0015	635
640	202.7	2.075.–3	0.0057	1841	2401	4.037	4.912	26	42	59.–6	32.0.–6	0.367	0.155	4.2	9.6	0.0008	640
645	215.2	2.351.–3	0.0045	1931	2292	4.223	4.732	90		54.–6	37.0.–6	0.331	0.178	12	26	0.0001	645
647.3‡	221.2	3.170.–3	0.0032	2107	2107	4.443	4.443	∞	∞	45.–6	45.0.–6	0.238	0.238	∞	∞	0.0000	647.3‡

*1 bar = 10^5 N/m^2.

†Above the solid line, the condensed phase is solid; below it, liquid.

‡Critical temperature.

NOTE: The notations 6.30.–11, 1.073.–3, 9.55.+9, etc. signify 6.30×10^{-11}, 1.073×10^{-3}, 955×10^{9}, etc.

Tables 2-351 and 2-352 are provided for general use. Tables to higher precision are available over certain ranges and for various properties. The most current internationally accepted tables are found in Haar, L., J. S. Gallagher, and G. S. Kell, *NBS/NRC Steam Tables,* Hemisphere, Washington, DC, 1984 (320 pp.). These do not tabulate certain properties at saturation states. A revised release on the IAPWS Skeleton Tables 1985 for the thermodynamic properties of ordinary water substance, Sept. 1993 (15 pp), is apparently the latest international publication. In *J. Phys. Chem. Ref. Data* **17,** 4 (1988): 1439–1540, H. Sato, M. Uematsu, and others review existing steam tables and present the 1985 formulation of skeleton tables. Property codes and programs include Cheng, S. C. and C. Nguyen, *Modeling and Simulation on Microcomputers 1989* (R. W. Allen, ed.), S.C.S. Intl., San Diego, 1989 (pp. 138–141); Garland, W. J. and B. J. Hand, *Nucl. Engng. & Des.,* **113,** (1989): 21–34; Dickey, D. S., *Chem. Eng.* **98,** 9 (1991): 207–8 and **98,** 11: 235–6; Muneer, T. and S. M. Scott, *Proc. Inst. Mech. Eng.,* **205,** (1991): 25–29; and *Energy Convsn. Mgmt.,* **31,** 4 (1991): 315–325. Useful pictorial representations of 20 properties as a function of both temperature (to 800°C) and pressure (to 1000 bar) are given by Grigull, U., J. Bach, et al., *Warme- u. Stoff.,* **1** (1968): 202–213. Property equations for the saturated liquid for the range 0–300°C are given by Charters, W. W. S. and H. A. Sadafi, *Rev. Int. Froid,* **10,** (Mar. 1987): 105–6. Gordon, S., NASA Tech. Paper 1906, 1982 gives detailed tables for ice from 0 K. Ice and snow properties are reviewed by Fukusako, S., *Int. J. Thermophys.,* **11,** 2 (1990): 353–372. See also Wagner, W., A. Saul, et al., *J. Phys. Chem. Ref. Data,* **23,** 3 (1994): 515–525, and Table 2-358.

Annexe B

Formula Sheet

The following formula sheet was given and demonstrated at Pr. Parente's course "MECA-H3001 : Fluid mechanics and transfer processes".

Formula sheet MECA-H-300

Transport Phenomena

October 2015

The definition of the dimensionless numbers as well as the physical understanding of the terms of the equations must be known.

Part I: Momentum

Local Equations

- **Newton's Law:**

$$\tau_{xy} = -\mu \frac{\partial u}{\partial y}$$

- **Mass conservation:**

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho \vec{V}\right) = 0$$

- **Navier-Stokes equations:**
  - Laminar flow, incompressible and Newtonian fluid:

$$\frac{D\vec{V}}{Dt} = -\frac{1}{\rho}\nabla P + \frac{\mu}{\rho}\nabla^2 \vec{V} + \vec{F}$$

- **Euler equation:**

$$\rho \frac{D\vec{V}}{Dt} = -\nabla P + \rho \vec{F}$$

- **Bernoulli equation**

$$\rho \frac{V^2}{2} + P + \rho g z = C^{te} = P_{tot}$$

- **Laminar boundary layer equation for a flat plate:**

$$\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = \frac{\mu}{\rho} \cdot \frac{\partial^2 u}{\partial y^2}$$

  - Boundary layer thickness: $\delta = \dfrac{5x}{\sqrt{Re_x}}$
  - Friction coefficient: $C_f = \dfrac{0,664}{\sqrt{Re_x}}$

Integral method :

- **Mass conservation:**

$$\frac{d[m]_{tot}}{dt} = <\rho V_n>_1 S_1 - <\rho V_n>_2 S_2$$

- **Momentum:**

$$\frac{d\vec{\mathcal{P}}_{tot}}{dt} = -\Delta_1^2\left[<\rho V^2> + P\right]\vec{S} - \vec{F}_{paroi} + m_{tot}\vec{g}$$

- **Boundary layer :**

  - Boundary layer thickness: $\delta^{**} = \delta \int_0^1 \left(\frac{u}{U_{ext}} - \frac{u^2}{U_{ext}^2} \right) \cdot d\left(\frac{y}{\delta} \right)$

  - Friction coefficient: $C_f = 2 \frac{d\delta^{**}}{dx}$

- **Pressure drops**

  ☛ **Distributed pressure drops:** $\Delta P_{tot} = \lambda \frac{L}{D} \frac{\rho U^2}{2}$

  - Laminar regime: $\lambda == \frac{64}{Re_{D_h}}$

  - Turbulent regime, smooth walls: **Blasius** $\lambda = \frac{0,316}{Re_{D_h}^{0,25}}$

  - Turbulent regime, smooth/rough transition: **Colebrook** $\frac{1}{\sqrt{\lambda}} = 1,14 - 0,87 \ln\left(\Lambda\% + \frac{9,34}{Re_{D_h} \sqrt{\lambda}} \right)$

  - Turbulent regime, rough walls: **Nikuradse** $\frac{1}{\sqrt{\lambda}} = 1,14 - 0,87 \ln\left(\Lambda\% \right)$

  ☛ **Concentrated pressure drops:** $\Delta P_{tot} = K \frac{\rho U_2^2}{2}$

  - Values of K for different configurations:

Elbow

r_c/D	K_c
1	0,35
2	0,19
4	0,16
6	0,21
8	0,28
10	0,32

Contraction

D_2/D_1	K_e
0	0,5
0,2	0,49
0,4	0,42
0,6	0,27
0,8	0,20
0,9	0,1

Diffuser

D_1/D_2	K_E
0	1
0,2	0,87
0,4	0,70
0,6	0,41
0,8	0,15

Part II : Energy

Local equations

- **Fourier's Law:**

$$q_y = -k \cdot \frac{dT}{dy}$$

- **Heat transfer with heat source (k=C^te^) :**

$$\rho\, C \frac{\partial T}{\partial t} = k\nabla^2 T + \dot{Q}_v$$

  - Laplacian in Cartesian coordinates $\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$
  - Laplacian in cylindrical coordinates: $\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right)$
  - Laplacian in spherical symmetry: $\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r} \right)$

    ☛ Boundary conditions:
    Diriclet: $T_{paroi} = T_o(\vec{s}_{paroi}, t)$

    Neuman: $q_{paroi} = -k \left.\frac{\partial T}{\partial n}\right|_{paroi} = q_o(\vec{s}_{paroi}, t)$

    Mixed: $q_p = -k \left.\frac{\partial T}{\partial n}\right|_p = h\ \left(T_p - T_f\right)$

  - h = convective heat transfer coefficient

- **Semi-infinite Solid**
  - Step of temperature: $\Theta = \frac{T - T_f}{T_o - T_f} = \frac{2}{\sqrt{\pi}} \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}} \right)$

    ☛ α = thermal diffusivity

  - Periodic perturbation (ω) with amplitude A_o : $\Theta = \frac{T - T_o}{A_o} = e^{-x\sqrt{\frac{\omega}{2\alpha}}} \cos\left(\omega t - x\sqrt{\frac{\omega}{2\alpha}} \right)$

- **Heat equation for a moving fluid 2D:**
  - Laminar flow, incompressible and Newtonian fluid, constant thermal conductivity:

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2$$

Global equations

- **Thermal Resistance :**
  - Series: $\Re_{th,tot} = \frac{T_1 - T_2}{\dot{Q}} = \sum_{i=1}^{N} \Re_{th,i} = \sum_{i=1}^{N} \frac{\delta_i}{S k_i}$

- Parallel: $\dfrac{\dot{Q}}{T_1 - T_2} = \dfrac{1}{\Re_{th,tot}} = \sum_{i=1}^{N} \dfrac{1}{\Re_{th,i}}$
- Global exchange coefficient: $\dfrac{1}{h_{tot}} = S\Re_{th,tot} = \sum_{i=1}^{N} \dfrac{\delta_i}{k_i} + \sum_{i=1}^{N} \dfrac{1}{h_i}$

- **Lumped systems**

$$\frac{dU}{dt} = hS_{tot}(T_{ext} - T)$$

1. Sensible heat transfer (heating up or cooling down):

$$\rho_s C_s V \frac{dT}{dt} = -hS_{tot}(T - T_{ext})$$

2. Latent heat transfer (fusion or evaporation):

$$\rho \mathcal{L}_{SL} S \frac{de}{dt} = hS_{tot}(T - T_{ext})$$

- Contact temperature

$$T_c = \frac{b_1 T_1 + b_2 T_2}{b_1 + b_2} \neq f(t) \qquad \text{Effusivity} \quad b = \sqrt{k\rho C}$$

- **Forced convection correlation (gas and liquids)**
  - Flat plate, laminar regime $Re_x < 3\ 10^5$ $\quad Nu_x = \dfrac{h_x x}{k_f} = 0{,}332\,Pr^{1/3}\sqrt{Re_x}$
  - Flat plate, turbulent regime $\quad Nu_x = \dfrac{h_x x}{k_f} = 0{,}029\,Pr^{0{,}43}\,Re_x^{0{,}8}$
  - Mean coefficient over L $\quad \overline{Nu}_L = \dfrac{1}{n} \cdot Nu_x (\text{en } x = L)$
    - n exponent of Re_x
  - Laminar boundary layer $\quad \overline{Nu}_L = \dfrac{\overline{h} \cdot L}{k_f} = 0{,}664 \cdot Pr^{1/3} \cdot \sqrt{Re_L}$
  - Laminar then turbulent boundary layer

    $$\overline{Nu}_L = 0{,}036 \cdot Pr^{0{,}43}\left(Re_L^{0{,}8} - Re_c^{0{,}8}\right) + 0{,}664 \cdot Pr^{1/3} \cdot Re_c^{0{,}5}$$

    - $Re_c = 2\ x10^5$
  - Around a cylinder $40 \leq Re_D \leq 10^5$: $\quad \overline{Nu}_D = \left(0{,}4\,Re_D^{0{,}5} + 0{,}06\,Re_D^{2/3}\right)Pr^{0{,}4}$
  - Around a sphere $3{,}5 \leq Re_D \leq 8.10^4$: $\quad \overline{Nu}_{sph} = 2 + \overline{Nu}_{cyl}$
  - In a tube
    - Laminar with $T_{wall} = C^{te}$: $\quad Nu_D = 3{,}66$
    - Laminar with $q_{wall} = C^{te}$: $\quad Nu_D = 4{,}36$
    - Turbulent: $\quad Nu_D = 0{,}023\,Re_D^{0{,}8}\,Pr^{0{,}4}$

- **Natural convection correlations (gas and liquids)**
  - Isothermal vertical flat plate

- Laminar $10^4 < Ra < 10^9$: $Nu_L = 0,59 Ra_L^{1/4}$
- Turbulent $10^9 < Ra < 10^{13}$: $Nu_L = 0,10 Ra_L^{1/3}$

- Constant heat flux vertical flat plate
  - Laminar $10^5 < Ra < 10^{11}$: $Nu_L = 0,750 \left[Ra_L^* \right]^{0,20}$
  - Turbulent $2\ 10^{13} < Ra < 10^{16}$: $Nu_L = 0,645 \left[Ra_L^* \right]^{0,22}$

- Around a vertical cylinder $Nu_D = C Ra_D^n$

Ra_D	C	n
$10^{-10} - 10^{-2}$	0,675	0,058
$10^{-2} - 10^{2}$	1	0,148
$10^{2} - 10^{4}$	0.85	0,188
$10^{4} - 10^{7}$	0,48	1/4
$10^{7} - 10^{12}$	0,125	1/3

- Around a sphere $10^5 \le Ra_D \le 10^9$ $Nu_D = 2 + 0,5 Ra_D^{1/4}$

- Vertical plate in confined space $Nu_e = \dfrac{k_e}{k_f} = C \cdot Ra_e^n \cdot \left[\dfrac{L}{e} \right]^{-m}$

	Ra_e	C	n	m
Gas				
	$2000 - 2.10^5$	0,197	1/4	1/9
	$2.10^5 - 10^7$	0,073	1/3	1/9
Liquid	<2000	1	0	0
	$10^4 - 10^7$	0,45	1/4	0,3
	$10^6 - 10^9$	0,046	1/3	0

- Horizontal plate in confined space $Nu_e = \dfrac{k_e}{k_f} = C \cdot Ra_e^n$

	Ra_e	C	n
Gas	< 1700	1	0
	1700 - 7000	0,059	0,4
	$7000 - 3,2\ 10^5$	0,212	1/4
	$> 3,2\ 10^5$	0,061	1/3
Liquid	< 1700	1	0
	1700 - 6000	0,012	0,6
	$6000 - 3,7\ 10^4$	0,375	0,2
	$3,7\ 10^4 - 10^8$	0,13	0,3
	$> 10^8$	0,057	1/3

- **Energy balance by radiation** $\alpha_\lambda + \tau_\lambda + r_\lambda = 1$
- **Stefan-Boltzmann Law's** $M^o = \int_0^\infty M^o_\lambda d\lambda = \sigma T^4$ avec σ=5,67 10^{-8} W/m^2.K^4
- **Radiosity of a grey body** $J = \varepsilon M^o + rE$
- **Heat flux density lost by a grey body** $q_{perdue} = \varepsilon M^o - \alpha E = J - E$
- **Radiative heat flux between two grey bodies**

$$\dot{Q}_{ray} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 S_1} + \frac{1}{S_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 S_2}}$$

- **Radiative heat transfer coefficient** $h_r = G(\varepsilon, F)\cdot\sigma\cdot\left(T_1^2 + T_2^2\right)\left(T_1 + T_2\right)$

- **Total energy equation**

$$\frac{d\left[E_c + E_i + E_p\right]_{tot}}{dt} = -\Delta_1^2\left[<\rho\left(\frac{V^2}{2} + e_i + \frac{P}{\rho} + gz\right)V_n > S\right] + \dot{Q} + \dot{\overline{W}}$$

- **Mechanical energy equation for an incompressible fluid**

$$\frac{d\left[E_c + E_p\right]_{tot}}{dt} = -\Delta_1^2\left[<\rho\left(\frac{V^2}{2} + \frac{p}{\rho} + gz\right)V_n > S\right] + \dot{\overline{W}} - \dot{E}_v$$

- **Generalized Bernoulli equation for a flow in a pipe**

$$-\Delta_1^2\left[<\rho\left(\frac{V^2}{2} + \frac{p}{\rho} + e_p\right)V_n > S\right] = \dot{E}_v$$

- **Internal energy equation for an incompressible fluid**

$$\frac{d\left[E_i\right]_{tot}}{dt} = -\Delta_1^2\left[<\rho e_i V_n > S\right] + \dot{Q} + \dot{E}_v$$

Part III : Mass

- **Fick's Law**

$$j_A(x) = -\rho \mathcal{D}_{AB} \cdot \frac{d\omega_A}{dx} \qquad \text{with} \quad \mathcal{D}_{AB} = \mathcal{D}_{BA} = \mathcal{D}$$

- **Diffusion into a moving fluid**

$$\vec{n}_A - \omega_A\left(\vec{n}_A + \vec{n}_B\right) = -\rho \mathcal{D} \nabla \omega_A \qquad \text{with} \quad \omega_A = \frac{\rho_A}{\rho}$$

  - Absolute flux density $\vec{n}_A = \rho_A \vec{u}_A = \vec{j}_A + \rho_A \vec{U}$
  - Barycentric velocity $\vec{U} = \dfrac{\rho_A \vec{u}_A + \rho_B \vec{u}_B}{\rho}$

- **Molecular diffusion evaporation of a liquid in a column L**

  - Mass fraction distribution $\dfrac{1-\omega_A(z)}{1-\omega_{A,0}} = \left[\dfrac{1-\omega_{A,L}}{1-\omega_{A,0}}\right]^{\frac{z}{L}}$
  - Evaporation mass flow $\dot{m}_A \approx \left[\mathcal{D} \dfrac{M_A}{\mathcal{R} T} \cdot \dfrac{S}{L}\right]\left(P_{sat}(T) - P_{A,L}\right)$
  - Saturation pressure (T in °C) : $p_{sat}(T) = 10^{7,625\frac{T}{241+T}+2,787}$
  - Mass Diffusivity (T in °C et P in Pa) : $\mathcal{D} = \dfrac{2,26}{P}\left(\dfrac{T+273,15}{273,15}\right)^{1,81}$

- **Mass boundary layer**

$$u\frac{\partial \rho_A}{\partial x} + v\frac{\partial \rho_A}{\partial y} = \mathcal{D}\frac{\partial^2 \rho_A}{\partial y^2}$$

- **Correlation of forced convection mass transfer**

$$j_{A,p} = -\mathcal{D}_{AB} \cdot \left.\frac{d\rho_A}{dy}\right|_{y=0} = h_m\left(\rho_{A,p} - \rho_{A,ext}\right)$$

Same expressions of the heat transfer, replacing Nu with Sh and Pr with Sc.

- **Perfect gas equation:**

$$p_v = \rho_v \frac{\mathcal{R}}{M_v} T_v \qquad \text{with} \quad \mathcal{R} = 8314 \text{ J/kmol.K}$$

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