Formula sheet MECA-H-300

Transport Phenomena

October 2015

The definition of the dimensionless numbers as well as the physical understanding of the terms of the equations must be known.

Part I: Momentum

Local Equations

- **Newton's Law:**

$$\tau_{xy} = -\mu \frac{\partial u}{\partial y}$$

- **Mass conservation:**

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho \vec{V}\right) = 0$$

- **Navier-Stokes equations:**
  - Laminar flow, incompressible and Newtonian fluid:

$$\frac{D\vec{V}}{Dt} = -\frac{1}{\rho}\nabla P + \frac{\mu}{\rho}\nabla^2 \vec{V} + \vec{F}$$

- **Euler equation:**

$$\rho \frac{D\vec{V}}{Dt} = -\nabla P + \rho \vec{F}$$

- **Bernoulli equation**

$$\rho \frac{V^2}{2} + P + \rho g z = C^{te} = P_{tot}$$

- **Laminar boundary layer equation for a flat plate:**

$$\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = \frac{\mu}{\rho} \cdot \frac{\partial^2 u}{\partial y^2}$$

  - Boundary layer thickness: $\delta = \frac{5x}{\sqrt{Re_x}}$
  - Friction coefficient: $C_f = \frac{0,664}{\sqrt{Re_x}}$

Integral method :

- **Mass conservation:**

$$\frac{d[m]_{tot}}{dt} = <\rho V_n>_1 S_1 - <\rho V_n>_2 S_2$$

- **Momentum:**

$$\frac{d\vec{\mathcal{P}}_{tot}}{dt} = -\Delta_1^2 \left[<\rho V^2> + P\right]\vec{S} - \vec{F}_{paroi} + m_{tot}\vec{g}$$

- **Boundary layer :**
  - Boundary layer thickness: $\delta^{**} = \delta \int_0^1 \left(\frac{u}{U_{ext}} - \frac{u^2}{U_{ext}^2} \right) \cdot d\left(\frac{y}{\delta} \right)$
  - Friction coefficient: $C_f = 2 \frac{d\delta^{**}}{dx}$

- **Pressure drops**

☛ **Distributed pressure drops:** $\Delta P_{tot} = \lambda \frac{L}{D} \frac{\rho U^2}{2}$

- Laminar regime: $\lambda == \frac{64}{Re_{D_h}}$
- Turbulent regime, smooth walls: **Blasius** $\lambda = \frac{0,316}{Re_{D_h}^{0,25}}$
- Turbulent regime, smooth/rough transition: **Colebrook** $\frac{1}{\sqrt{\lambda}} = 1,14 - 0,87 \ln\left(\Lambda\% + \frac{9,34}{Re_{D_h} \sqrt{\lambda}} \right)$
- Turbulent regime, rough walls: **Nikuradse** $\frac{1}{\sqrt{\lambda}} = 1,14 - 0,87 \ln\left(\Lambda\% \right)$

☛ **Concentrated pressure drops:** $\Delta P_{tot} = K \frac{\rho U_2^2}{2}$

- Values of K for different configurations:

Elbow

r_c/D	K_c
1	**0,35**
2	**0,19**
4	**0,16**
6	**0,21**
8	**0,28**
10	**0,32**

Contraction

D_2/D_1	K_e
0	**0,5**
0,2	**0,49**
0,4	**0,42**
0,6	**0,27**
0,8	**0,20**
0,9	**0,1**

Diffuser

D_1/D_2	K_E
0	**1**
0,2	**0,87**
0,4	**0,70**
0,6	**0,41**
0,8	**0,15**

Local equations

- **Fourier's Law:**

$$q_y = -k \cdot \frac{dT}{dy}$$

- **Heat transfer with heat source ($k=C^{te}$) :**

$$\rho\, C \frac{\partial T}{\partial t} = k\nabla^2 T + \dot{Q}_v$$

  - Laplacian in Cartesian coordinates $\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$
  - Laplacian in cylindrical coordinates: $\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right)$
  - Laplacian in spherical symmetry: $\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r} \right)$

    ☛ Boundary conditions:
    Diriclet: $T_{paroi} = T_o(\vec{s}_{paroi}, t)$

    Neuman: $q_{paroi} = -k \left.\frac{\partial T}{\partial n}\right|_{paroi} = q_o(\vec{s}_{paroi}, t)$

    Mixed: $q_p = -k \left.\frac{\partial T}{\partial n}\right|_{p} = h\ \left(T_p - T_f\right)$

  - h = convective heat transfer coefficient

- **Semi-infinite Solid**
  - Step of temperature: $\Theta = \frac{T - T_f}{T_o - T_f} = \frac{2}{\sqrt{\pi}} \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}} \right)$

    ☛ α = thermal diffusivity

  - Periodic perturbation (ω) with amplitude A_o : $\Theta = \frac{T - T_o}{A_o} = e^{-x\sqrt{\frac{\omega}{2\alpha}}} \cos\left(\omega t - x\sqrt{\frac{\omega}{2\alpha}} \right)$

- **Heat equation for a moving fluid 2D:**
  - Laminar flow, incompressible and Newtonian fluid, constant thermal conductivity:

$$\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y} \right)^2$$

Global equations

- **Thermal Resistance :**
  - Series: $\Re_{th,tot} = \frac{T_1 - T_2}{\dot{Q}} = \sum_{i=1}^{N} \Re_{th,i} = \sum_{i=1}^{N} \frac{\delta_i}{S k_i}$

- Parallel: $\frac{\dot{Q}}{T_1 - T_2} = \frac{1}{\Re_{th,tot}} = \sum_{i=1}^{N} \frac{1}{\Re_{th,i}}$
- Global exchange coefficient: $\frac{1}{h_{tot}} = S\Re_{th,tot} = \sum_{i=1}^{N} \frac{\delta_i}{k_i} + \sum_{i=1}^{N} \frac{1}{h_i}$

- **Lumped systems**

$$\frac{dU}{dt} = hS_{tot}(T_{ext} - T)$$

1. Sensible heat transfer (heating up or cooling down):

$$\rho_s C_s V \frac{dT}{dt} = -hS_{tot}(T - T_{ext})$$

2. Latent heat transfer (fusion or evaporation):

$$\rho \mathcal{L}_{SL} S \frac{de}{dt} = hS_{tot}(T - T_{ext})$$

- Contact temperature

$$T_c = \frac{b_1 T_1 + b_2 T_2}{b_1 + b_2} \neq f(t) \qquad \text{Effusivity} \quad b = \sqrt{k\rho C}$$

- **Forced convection correlation (gas and liquids)**
  - Flat plate, laminar regime $Re_x < 3\ 10^5$ $\quad Nu_x = \frac{h_x x}{k_f} = 0,332\, Pr^{1/3} \sqrt{Re_x}$
  - Flat plate, turbulent regime $\quad Nu_x = \frac{h_x x}{k_f} = 0,029\, Pr^{0,43}\, Re_x^{0,8}$
  - Mean coefficient over L $\quad \overline{Nu}_L = \frac{1}{n} \cdot Nu_x (\text{en } x = L)$
    - n exponent of Re_x
  - Laminar boundary layer $\quad \overline{Nu}_L = \frac{\bar{h} \cdot L}{k_f} = 0,664 \cdot Pr^{1/3} \cdot \sqrt{Re_L}$
  - Laminar then turbulent boundary layer

    $\overline{Nu}_L = 0,036 \cdot Pr^{0,43} \left(Re_L^{0,8} - Re_c^{0,8} \right) + 0,664 \cdot Pr^{1/3} \cdot Re_c^{0,5}$
    - $Re_c = 2\ x10^5$
  - Around a cylinder $40 \leq Re_D \leq 10^5$: $\quad \overline{Nu}_D = \left(0,4 Re_D^{0,5} + 0,06 Re_D^{2/3} \right) Pr^{0,4}$
  - Around a sphere $3,5 \leq Re_D \leq 8.10^4$: $\quad \overline{Nu}_{sph} = 2 + \overline{Nu}_{cyl}$
  - In a tube
    - Laminar with $T_{wall} = C^{te}$: $\quad Nu_D = 3,66$
    - Laminar with $q_{wall} = C^{te}$: $\quad Nu_D = 4,36$
    - Turbulent: $\quad Nu_D = 0,023 Re_D^{0,8}\, Pr^{0,4}$

- **Natural convection correlations (gas and liquids)**
  - Isothermal vertical flat plate

- Laminar $10^4 < Ra < 10^9$: $Nu_L = 0,59 Ra_L^{1/4}$
- Turbulent $10^9 .< Ra < 10^{13}$: $Nu_L = 0,10 Ra_L^{1/3}$

- Constant heat flux vertical flat plate
  - Laminar $10^5 < Ra < 10^{11}$: $Nu_L = 0,750 \left[Ra_L^* \right]^{0,20}$
  - Turbulent $2\ 10^{13} < Ra < 10^{16}$: $Nu_L = 0,645 \left[Ra_L^* \right]^{0,22}$

- Around a vertical cylinder $Nu_D = C Ra_D^n$

Ra_D	C	n
10^{-10} - 10^{-2}	0,675	0,058
10^{-2} - 10^{2}	1	0,148
10^{2} - 10^{4}	0.85	0,188
10^{4} - 10^{7}	0,48	1/4
10^{7} - 10^{12}	0,125	1/3

- Around a sphere $10^5 \leq Ra_D \leq 10^9$ $Nu_D = 2 + 0,5 Ra_D^{1/4}$

- Vertical plate in confined space $Nu_e = \frac{k_e}{k_f} = C \cdot Ra_e^n \cdot \left[\frac{L}{e} \right]^{-m}$

	Ra_e	C	n	m
Gas				
	$2000\text{-}2.10^5$	0,197	1/4	1/9
	$2.10^5\text{-}10^7$	0,073	1/3	1/9
Liquid	<2000	1	0	0
	$10^4\text{-}10^7$	0,45	1/4	0,3
	$10^6\text{-}10^9$	0,046	1/3	0

- Horizontal plate in confined space $Nu_e = \frac{k_e}{k_f} = C \cdot Ra_e^n$

	Ra_e	C	n
Gas	< 1700	1	0
	1700 - 7000	0,059	0,4
	$7000 - 3,2\ 10^5$	0,212	1/4
	$> 3,2\ 10^5$	0,061	1/3
Liquid	< 1700	1	0
	1700 - 6000	0,012	0,6
	$6000 - 3,7\ 10^4$	0,375	0,2
	$3,7\ 10^4 - 10^8$	0,13	0,3
	$> 10^8$	0,057	1/3

- **Energy balance by radiation** $\alpha_\lambda + \tau_\lambda + r_\lambda = 1$
- **Stefan-Boltzmann Law's** $M^o = \int_0^\infty M^o_\lambda d\lambda = \sigma T^4$ avec $\sigma=5{,}67\ 10^{-8}\ W/m^2.K^4$
- **Radiosity of a grey body** $J = \varepsilon M^o + rE$
- **Heat flux density lost by a grey body** $q_{perdue} = \varepsilon M^o - \alpha E = J - E$
- **Radiative heat flux between two grey bodies**

$$\dot{Q}_{ray} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 S_1} + \dfrac{1}{S_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 S_2}}$$

- **Radiative heat transfer coefficient** $h_r = G(\varepsilon, F)\cdot\sigma\cdot\left(T_1^2 + T_2^2\right)\left(T_1 + T_2\right)$

- **Total energy equation**

$$\frac{d\left[E_c + E_i + E_p\right]_{tot}}{dt} = -\Delta_1^2\left[<\rho\left(\frac{V^2}{2} + e_i + \frac{P}{\rho} + gz\right)V_n > S\right] + \dot{Q} + \dot{\overline{W}}$$

- **Mechanical energy equation for an incompressible fluid**

$$\frac{d\left[E_c + E_p\right]_{tot}}{dt} = -\Delta_1^2\left[<\rho\left(\frac{V^2}{2} + \frac{p}{\rho} + gz\right)V_n > S\right] + \dot{\overline{W}} - \dot{E}_v$$

- **Generalized Bernoulli equation for a flow in a pipe**

$$-\Delta_1^2\left[<\rho\left(\frac{V^2}{2} + \frac{p}{\rho} + e_p\right)V_n > S\right] = \dot{E}_v$$

- **Internal energy equation for an incompressible fluid**

$$\frac{d\left[E_i\right]_{tot}}{dt} = -\Delta_1^2\left[<\rho e_i V_n > S\right] + \dot{Q} + \dot{E}_v$$

Part III : Mass

- **Fick's Law**

$$j_A(x) = -\rho \mathcal{D}_{AB} \cdot \frac{d\omega_A}{dx} \qquad \text{with} \quad \mathcal{D}_{AB} = \mathcal{D}_{BA} = \mathcal{D}$$

- **Diffusion into a moving fluid**

$$\vec{n}_A - \omega_A\left(\vec{n}_A + \vec{n}_B\right) = -\rho \mathcal{D} \nabla \omega_A \qquad \text{with} \quad \omega_A = \frac{\rho_A}{\rho}$$

  - Absolute flux density $\vec{n}_A = \rho_A \vec{u}_A = \vec{j}_A + \rho_A \vec{U}$
  - Barycentric velocity $\vec{U} = \dfrac{\rho_A \vec{u}_A + \rho_B \vec{u}_B}{\rho}$

- **Molecular diffusion evaporation of a liquid in a column L**

  - Mass fraction distribution $\dfrac{1-\omega_A(z)}{1-\omega_{A,0}} = \left[\dfrac{1-\omega_{A,L}}{1-\omega_{A,0}}\right]^{\frac{z}{L}}$
  - Evaporation mass flow $\dot{m}_A \approx \left[\mathcal{D}\dfrac{M_A}{\mathcal{R}T} \cdot \dfrac{S}{L}\right]\left(P_{sat}(T) - P_{A,L}\right)$
  - Saturation pressure (T in °C) : $p_{sat}(T) = 10^{7,625\frac{T}{241+T}+2,787}$
  - Mass Diffusivity (T in °C et P in Pa) : $\mathcal{D} = \dfrac{2,26}{P}\left(\dfrac{T+273,15}{273,15}\right)^{1,81}$

- **Mass boundary layer**

$$u\frac{\partial \rho_A}{\partial x} + v\frac{\partial \rho_A}{\partial y} = \mathcal{D}\frac{\partial^2 \rho_A}{\partial y^2}$$

- **Correlation of forced convection mass transfer**

$$j_{A,p} = -\mathcal{D}_{AB} \cdot \left.\frac{d\rho_A}{dy}\right|_{y=0} = h_m\left(\rho_{A,p} - \rho_{A,ext}\right)$$

Same expressions of the heat transfer, replacing Nu with Sh and Pr with Sc.

- **Perfect gas equation:**

$$p_v = \rho_v \frac{\mathcal{R}}{M_v} T_v \qquad \text{with} \quad \mathcal{R} = 8314 \text{ J/kmol.K}$$