Introduction to Graphics Pipeline

2017 Fall

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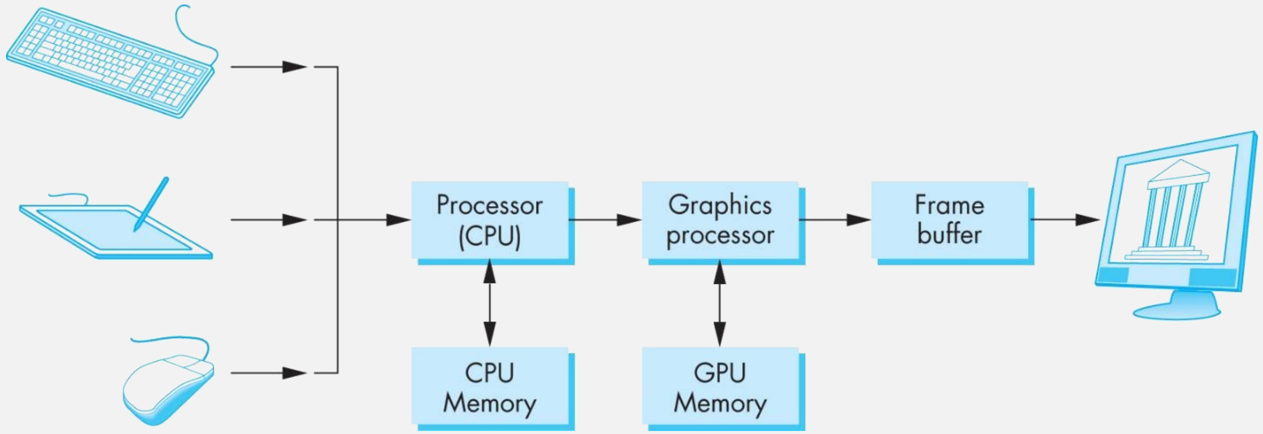
Applications of Computer Graphics

- Display of information
  - Architectural/mechanical drafting systems
  - Cartography
  - Plotting packages to visualize multiple large data sets
  - Medical imaging (CT/MRI)
- Design
  - Computer-aided design (CAD)
  - Very-large-scale integrated (VLSI) circuits design
- Simulation and animation
  - Training of pilots
  - 2D/3D motion-pictures in TV/advertising industries
  - Virtual Reality (VR)
- User interface
  - Windowing systems
  - Browser interface

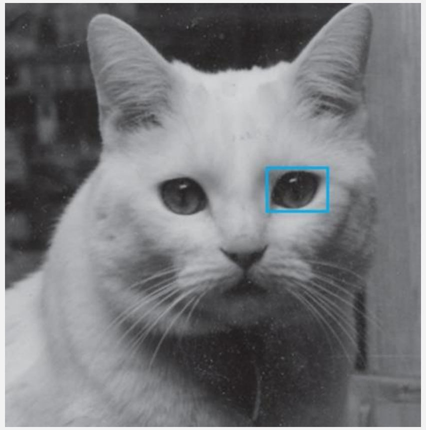
A Graphics System

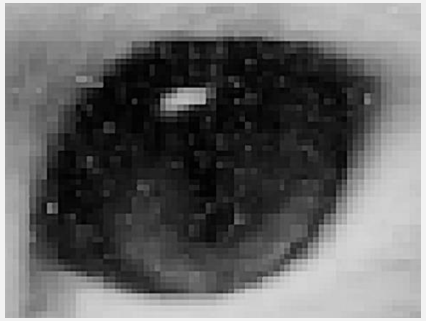
- Components of a general-purpose computer system:
  - Input devices
  - Central Processing Unit
  - Graphics Processing Unit
  - Memory
  - Frame buffer
  - Output devices



Pixels & Frame Buffer

- The image we see on the output device is an array (the raster) of picture elements (pixels) produced by the graphics system.
- Pixels are stored in a part of memory called the frame buffer.
- Resolution: the number of pixels in the frame buffer.
- Depth/Precision: the number of bits used for each pixel.
  - 1-bit-deep frame buffer: only two colors
  - 8-bit-deep frame buffer: 256 colors
  - Full-color/True-color/RGB-color system: 24 (or more) bits per pixel
  - HDR systems: 12 (or more) bits per color component



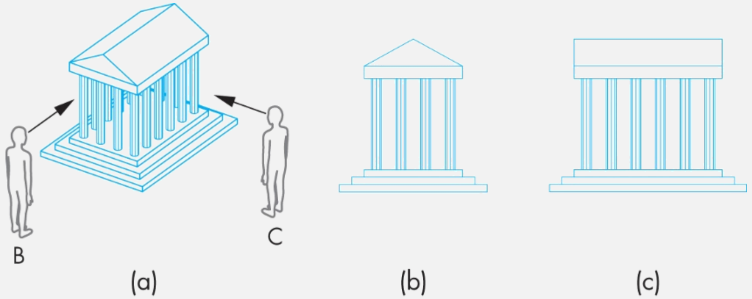


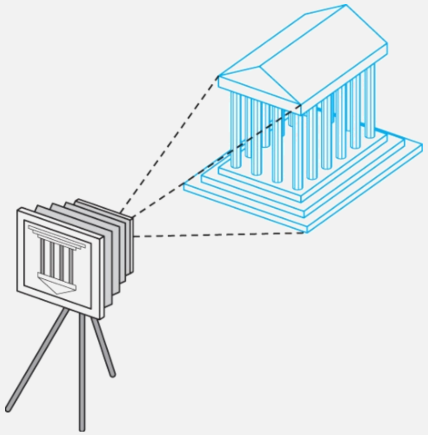
CPU & GPU

- Main graphical function of the processor:
  - Rasterization/Scan conversion: Conversion of geometric entities (such as lines, circles, polygons) to pixel colors and locations in the frame buffer.
- Frame buffer was part of the standard memory that could be directly addressed by the CPU in early graphics system.
- Today, graphical systems are characterized by special-purpose graphical processing units (GPUs) that can perform graphical operations with high degree of parallelism.
- GPU can be either on the mother board of the system or on a graphics card.

Objects & Viewers

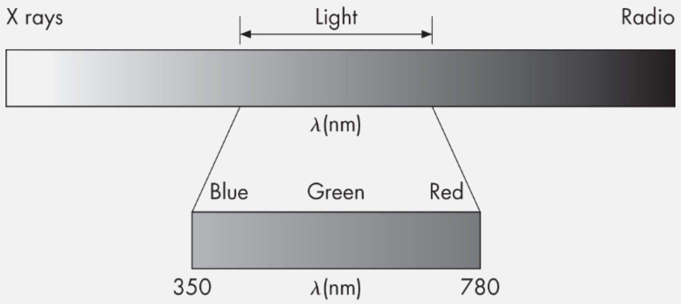
- Objects:
  - Can be defined/approximated by a set of locations in space, i.e. vertices.
- Viewers:
  - Who form the image of the objects.
- Both objects and viewers exist in a 3D world. However, the formed image is 2D.

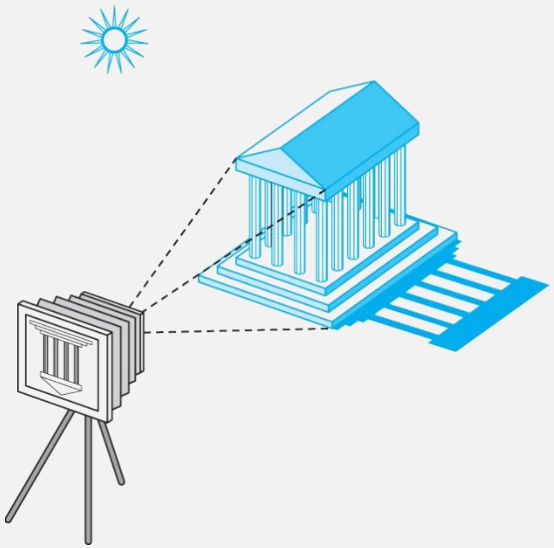




Light & Images

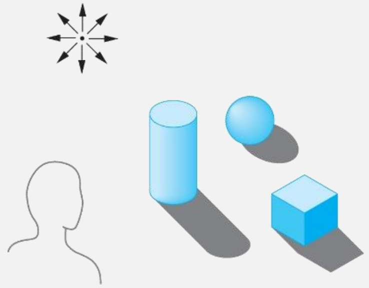
- Visible spectrum: 350~780 nm
- Point source:
  - Emits energy from a single location at one or more frequencies equally in all directions.
- Light bulb:
  - Emits light over an area and emitting more light in one direction than another.
- Complex light sources can be approximated by a number if carefully placed point sources.

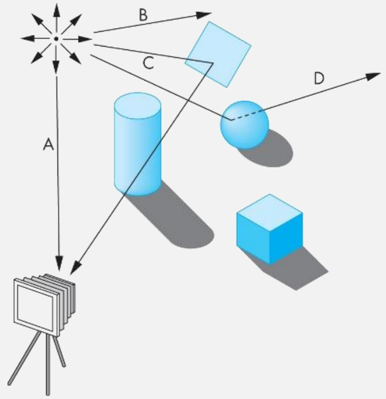




Imaging Models

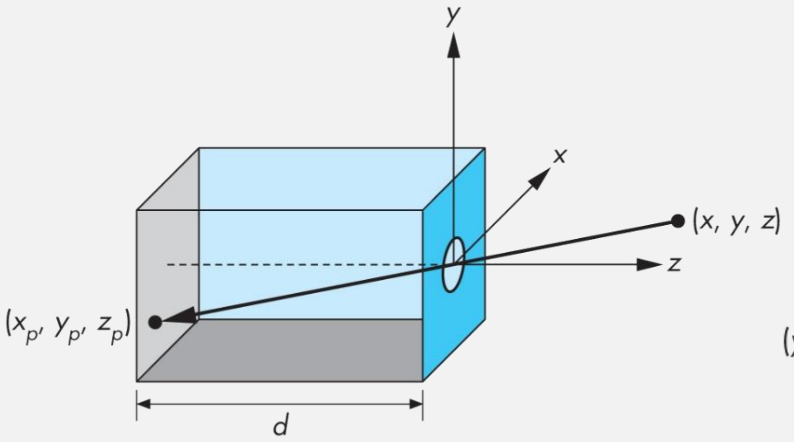
- How we can form images from a set of objects with different light-reflecting properties and different light sources?
  - Building the imaging model by following light from a source.
    - E.g. Raytracing and photon mapping
    - Can provide a close approximation to the physical world, but is not well suited for real-time computation.
  - Conservation of energy.
    - E.g. Radiosity
    - Works best for surfaces that scatter the incoming light equally in all directions.

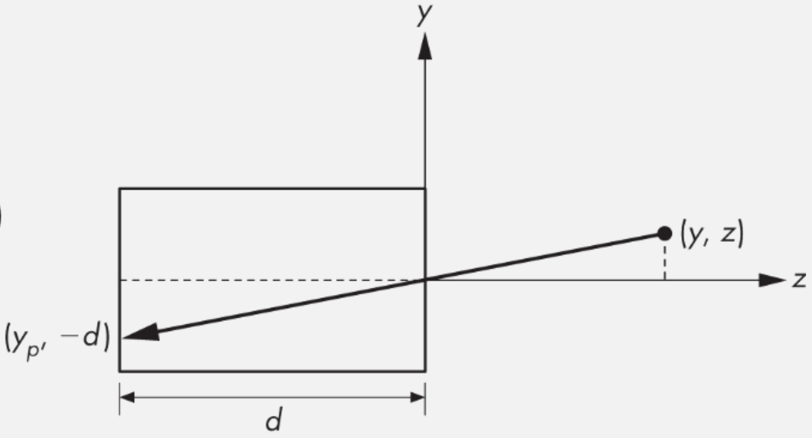




Imaging Systems: Pinhole Camera

- The pinhole camera:
  - Suppose that the camera is oriented along the z-axis, with the pinhole at the origin of our coordinate system.
  - Assume that the hole is so small that only a single ray of light from a point can enter it.



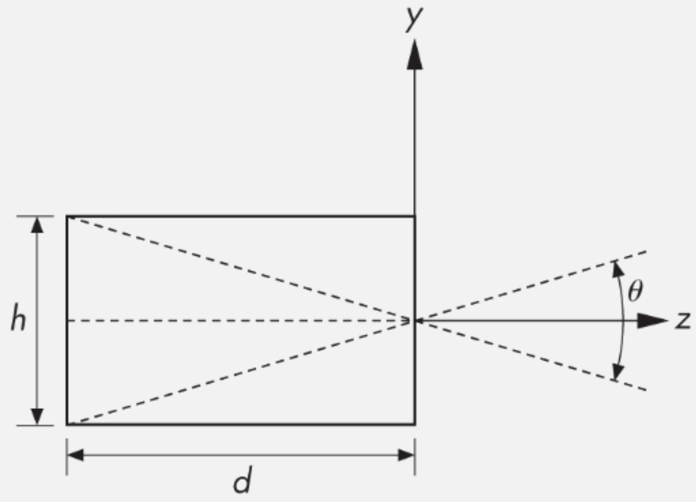


$$y_p = -\frac{y}{z/d}$$

$$x_p = -\frac{x}{z/d}$$

Imaging Systems: Pinhole Camera (Cont.)

- The field/angle of view:
  - The angle made by the largest object that our camera can image on its film plane.
  - The ideal pinhole camera has an infinite depth of filed. (Every point in its filed of view is in focus)
- Disadvantages:
  - Admits only a single ray from a point source, and therefore almost no light enters the camera.
  - The camera cannot be adjusted to have a different angle of view.

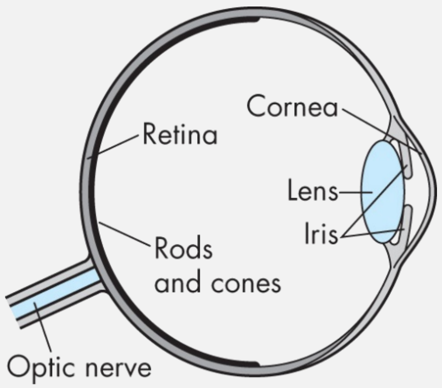


$$\tan\frac{\theta}{2} = \frac{h/2}{d}$$

$$\theta = 2\tan^{-1}\frac{h}{2d}$$

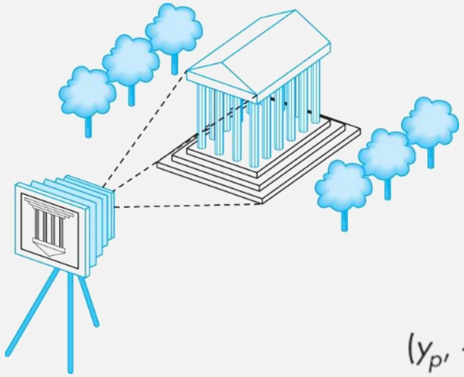
Imaging Systems: Human Visual System

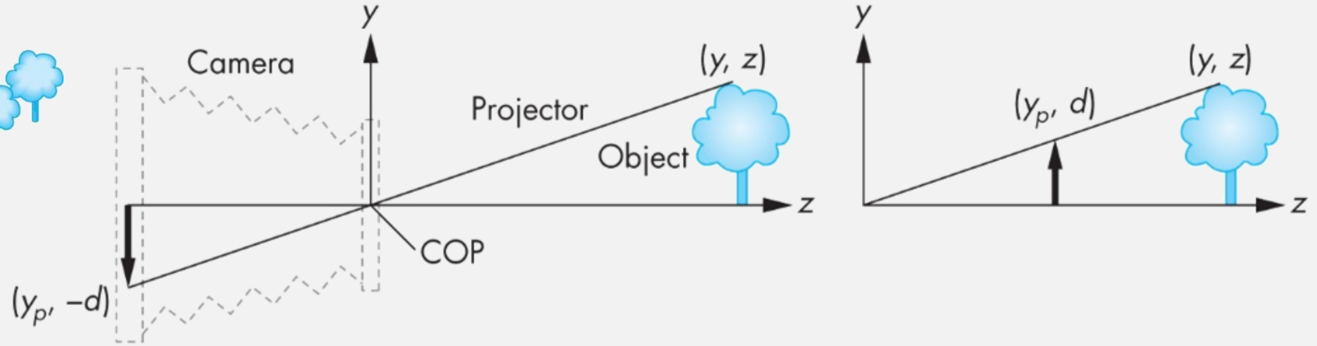
- Sensors in the human eye do not react uniformly to light energy at different wavelength.
  - Most sensitive to green light and least sensitive to red and blue.
- There are three types of cones and therefore we can use three standard primaries to approximate any color that we can perceive.
  - Intensity is a physical measure of light energy.
  - Brightness is a measure of how intense we perceive the light from an object.

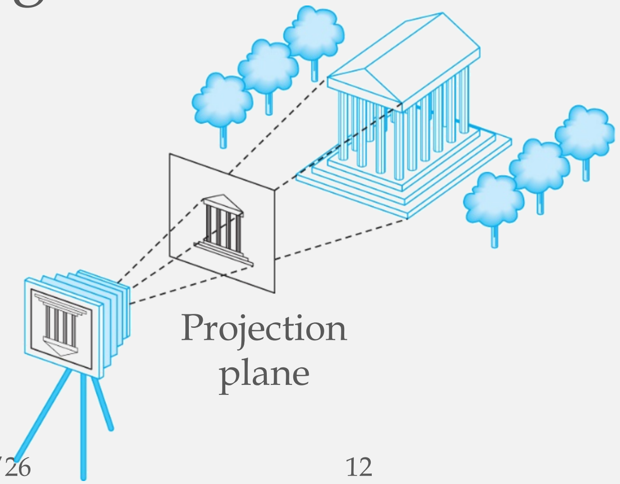


The Synthetic-Camera Model

- The specification of the objects is independent of the specification of the viewer.
  - Within a graphics library, there will be separate functions for specifying the objects and the viewer.
- We can compute the image using simple geometric calculation
- Draw another plane in front of the lens to avoid flipping

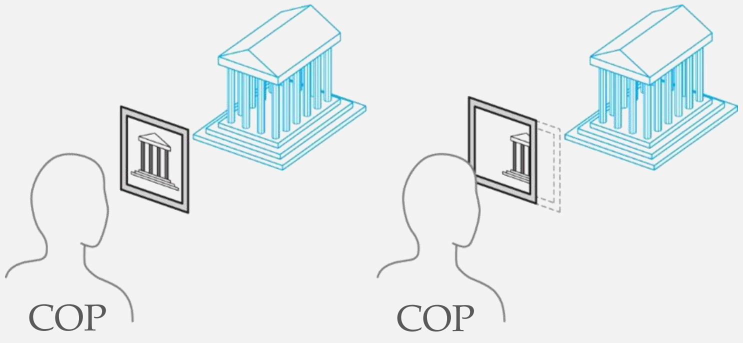






The Synthetic-Camera Model (Cont.)

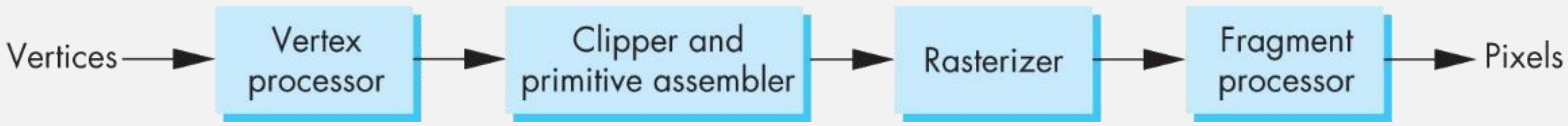
- Not all objects can be imaged onto the pinhole camera's film plane, and the synthetic camera move the limitation to the front by placing a clipping rectangle/window in the projection plane.
- What determines which object will appear in the image?
  - The location of the center of projection (COP)
  - The location and orientation of the projection plane
  - The size of the clipping rectangle



LookAt(COP, at, up);

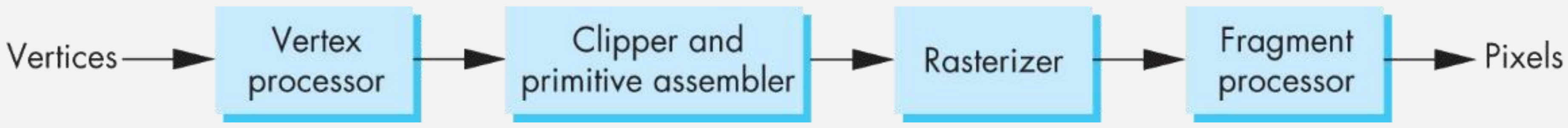
Perspective(field_of_view, aspect_ratio, near, far);

The Graphics Pipeline



- The graphics pipeline or rendering pipeline refers to the sequence of steps used to create a 2D raster representation of a 3D scene/model.
- Vertex processing
  - Each vertex is processed independently.
  - To carry out coordinate transformations.
    - Each change of the camera coordinate can be represented by a matrix.
  - To compute a color for each vertex.
- Clipper and Primitive Assembly
  - Efficient clipping must be done on a primitive-by-primitive basis rather than on a vertex-by-vertex basis.

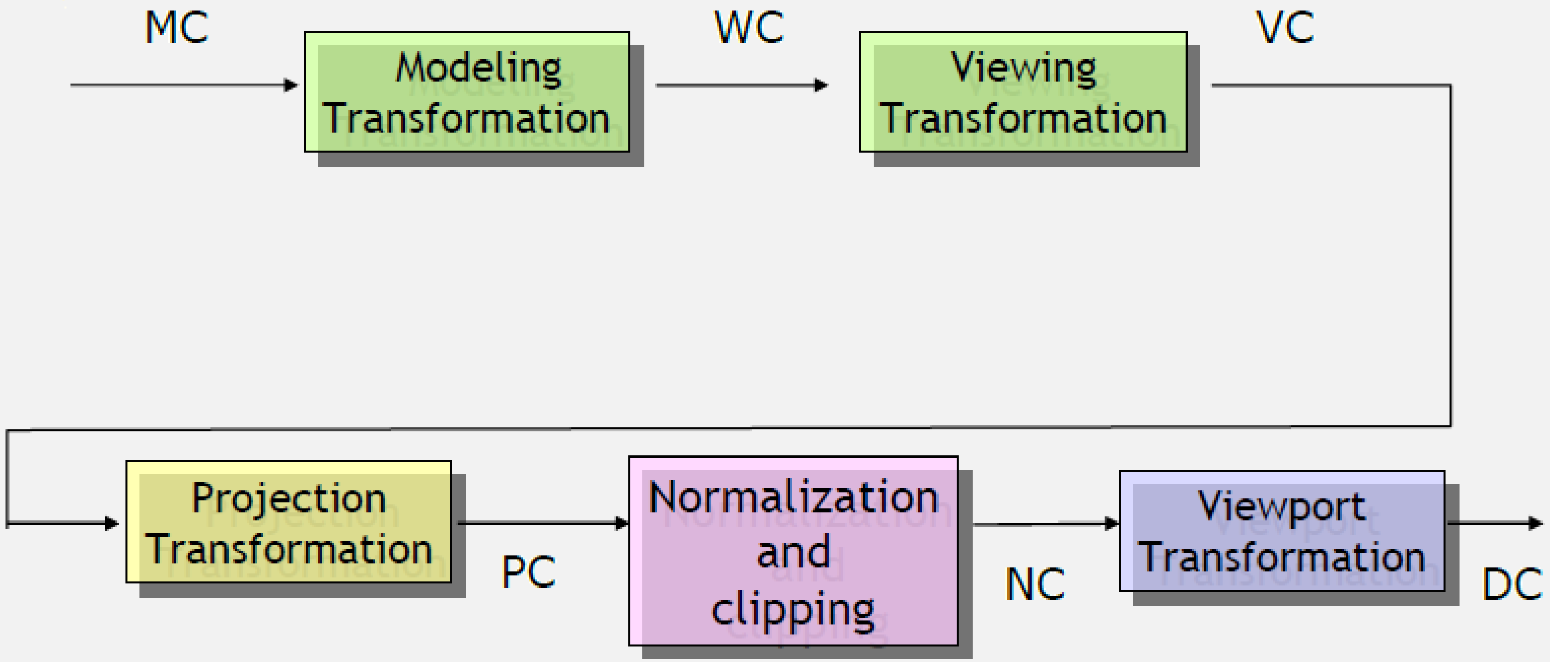
The Graphics Pipeline (Cont.)



- Rasterization (Scan conversion)
  - Primitives emerging from the clipper are still represented in terms of their vertices and must be converted to pixels in the frame buffer.
  - Determine which pixels in the frame buffer are inside the polygon.
  - Output of rasterization is a set of fragments (potential pixels with color, location, and depth information) for each primitive.
- Fragment Processing
  - Update the pixels in the frame buffer according to the processed fragments. (Some surfaces may not be visible because of occlusion)
  - The color of pixels in each fragment can be altered by texture mapping or bump mapping.

Viewing with A Computer

- Pipeline View



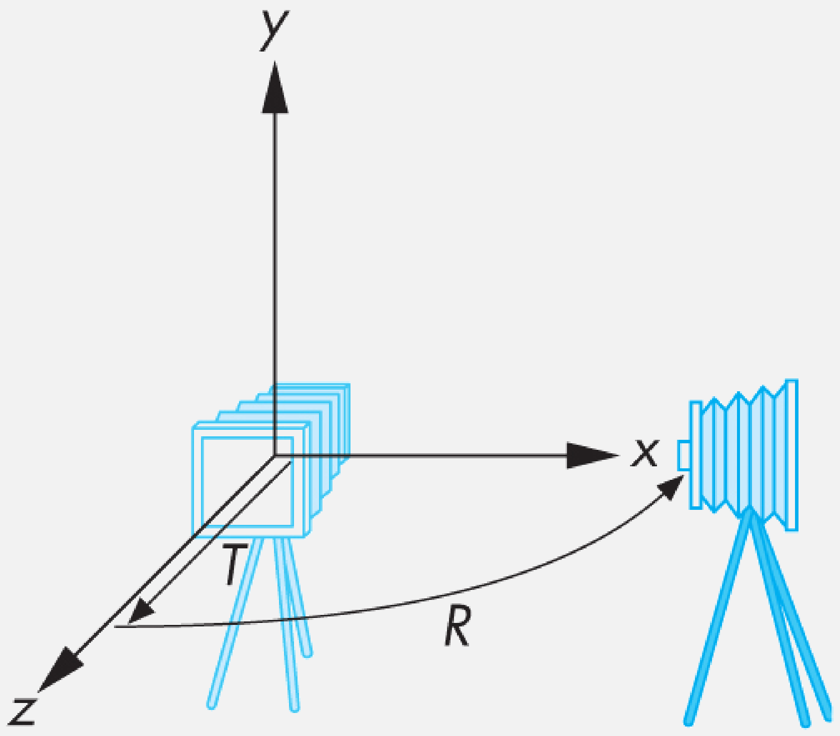
Viewing with A Computer (Cont.)

- Three aspects of the viewing process implemented in the pipeline:
  - Positioning the camera
    - Setting the model-view matrix
  - Selecting a lens
    - Setting the projection matrix: orthogonal or perspective
  - Normalization & Clipping
    - Setting the view volume

Moving the Camera

- We can move the camera to any desired position by a sequence of rotations and translations

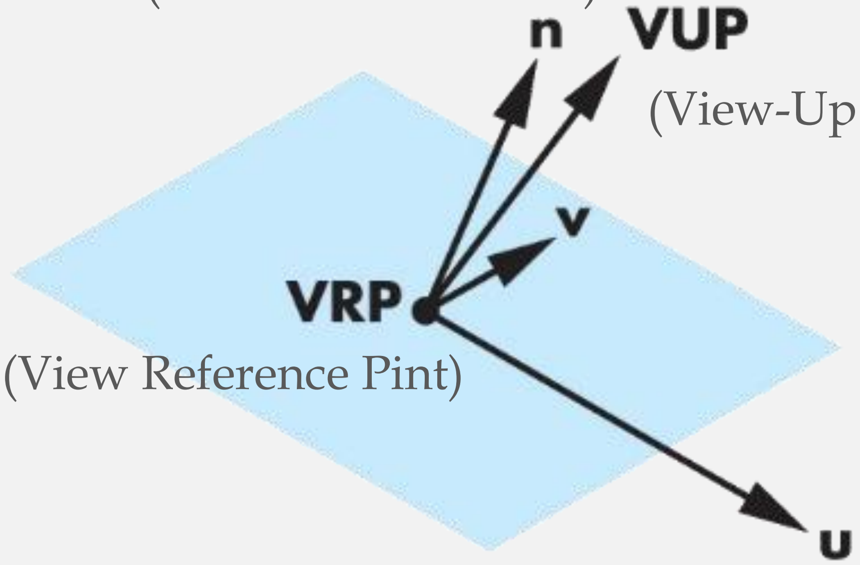
- Example: side view
  - Rotate the camera
  - Move it away from origin
  - View matrix C = TR



How to Obtain the View Matrix ?

VPN (View-Plane Normal)

(View-Up Vector)



(View Reference Pint)

Given $\mathbf{VRP} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$, $\mathbf{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 1 \end{bmatrix}$, $\mathbf{v_{up}} = \begin{bmatrix} \mathbf{v_{up}}_x \\ \mathbf{v_{up}}_y \\ \mathbf{v_{up}}_z \\ 1 \end{bmatrix}$

$$\mathbf{v} = \alpha\mathbf{n} + \beta\mathbf{v_{up}}$$

To simplify, set $\beta = 1$ and $\alpha = -\frac{\mathbf{v_{up}} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}}$

$$\Rightarrow \mathbf{v} = \mathbf{v_{up}} - \frac{\mathbf{v_{up}} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}}\mathbf{n}$$

$$\mathbf{u} = \mathbf{v} \times \mathbf{n}$$

How to Obtain the View Matrix ? (Cont.)

Normalize **u**, **v**, and **n**, and set the rotation matrix as:

$$\mathbf{A} = \begin{bmatrix} u'_x & v'_x & n'_x & 0 \\ u'_y & v'_y & n'_y & 0 \\ u'_z & v'_z & n'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

What we want is the opposite direction ($\mathbf{A}^{-1}$)that represent the vectors in the original system in the $\mathbf{u}'\mathbf{v}'\mathbf{n}'$ coordinate system. Hence, the rotation matrix of the model-view matrix is:

$$\mathbf{R}' = \mathbf{A}^{-1} = \mathbf{A}^T = \begin{bmatrix} u'_x & u'_y & u'_z & 0 \\ v'_x & v'_y & v'_z & 0 \\ n'_x & n'_y & n'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally, by multiplying the translation matrix **T**, we have:

$$\mathrm{C} = \mathrm{R'T'} = \begin{bmatrix} u'_x & u'_y & u'_z & 0 \\ v'_x & v'_y & v'_z & 0 \\ n'_x & n'_y & n'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u'_x & u'_y & u'_z & -xu'_x - yu'_y - zu'_z \\ v'_x & v'_y & v'_z & -xv'_x - yv'_y - zv'_z \\ n'_x & n'_y & n'_z & -xn_x - yn'_y - zn'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

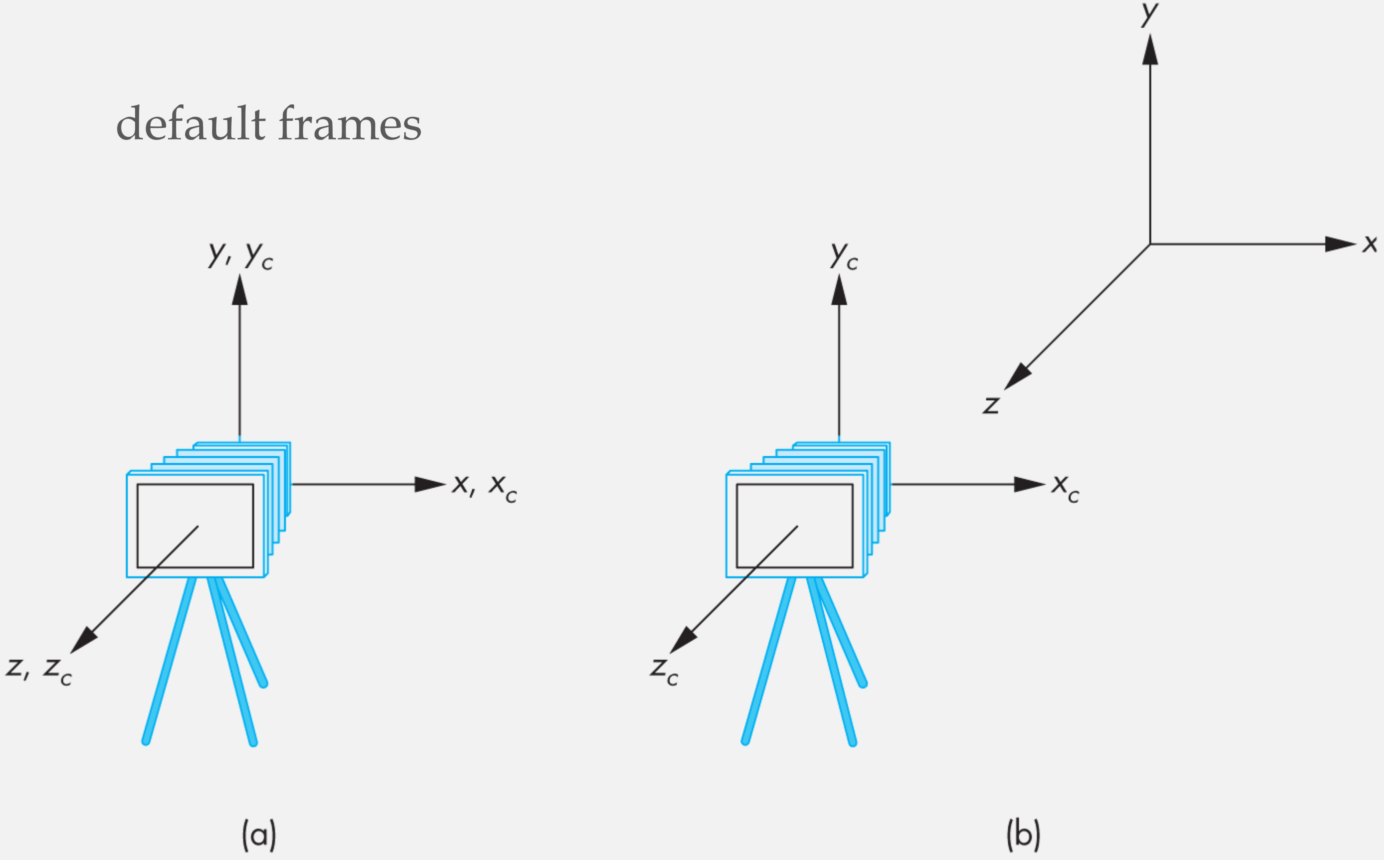
Moving the Camera Frame

- If we want to visualize object with both positive and negative z values we can either
  - Move the camera in the positive z direction
    - Translate the camera frame
  - Move the objects in the negative z direction
    - Translate the world frame
- Both of these views are equivalent and are determined by the model-view matrix
  - Want a translation (`glTranslatef(0.0,0.0,-d);`)
  - `d` > 0

Moving Camera back from Origin

frames after translation by –d
d > 0

default frames

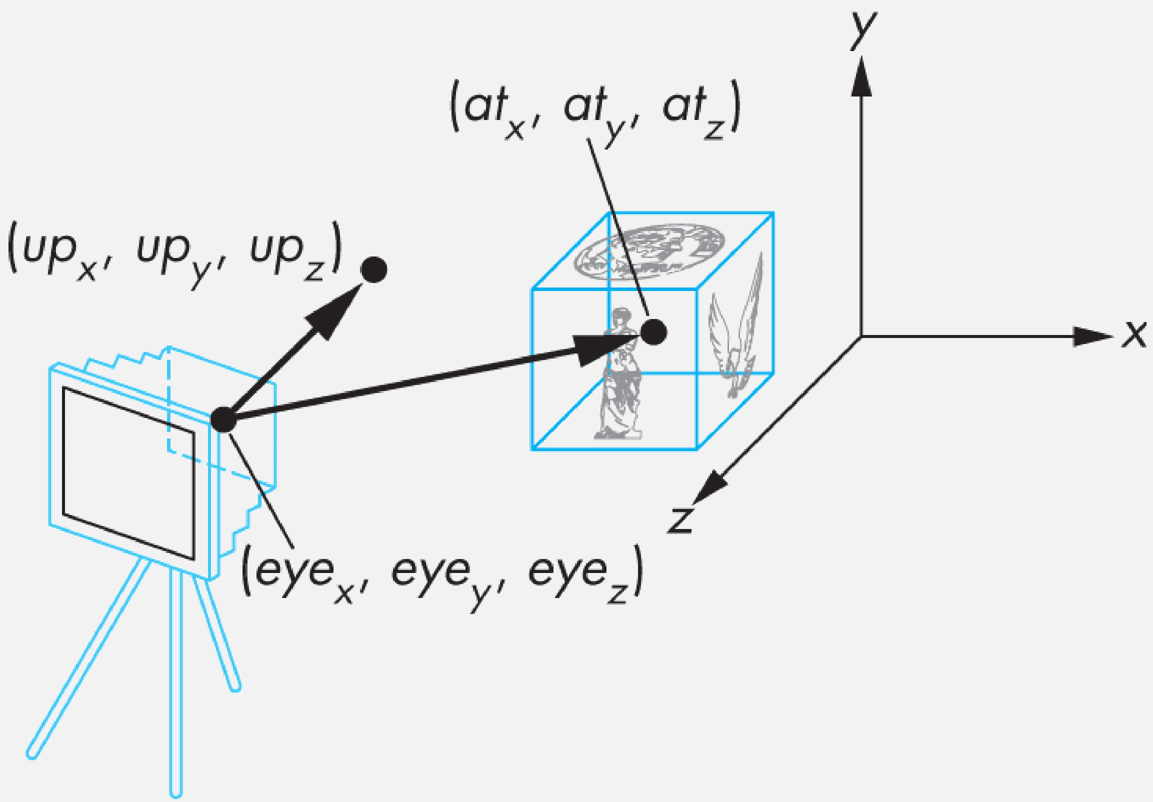


The OpenGL Camera

- In OpenGL, initially the object and camera frames are the same
  - Default model-view matrix is an identity
- The camera is located at origin and points in the negative z direction
- OpenGL also specifies a default view volume that is a cube with sides of length 2 centered at the origin
  - Default projection matrix is an identity

How to Set the Camera Position/Orientation?

- OpenGL: gluLookAt(eye_x, eye_y, eye_z, at_x, at_y, at_z, up_x, up_y, up_z)



$$\mathbf{vpn} = \boldsymbol{a} - \boldsymbol{e}$$

$$\mathbf{n} = \frac{\mathbf{vpn}}{|\mathbf{vpn}|}$$

$$\mathbf{u} = \frac{\mathbf{v_{up}} \times \mathbf{n}}{|\mathbf{v_{up}} \times \mathbf{n}|}$$

$$\mathbf{v} = \frac{\mathbf{n} \times \mathbf{u}}{|\mathbf{n} \times \mathbf{u}|}$$

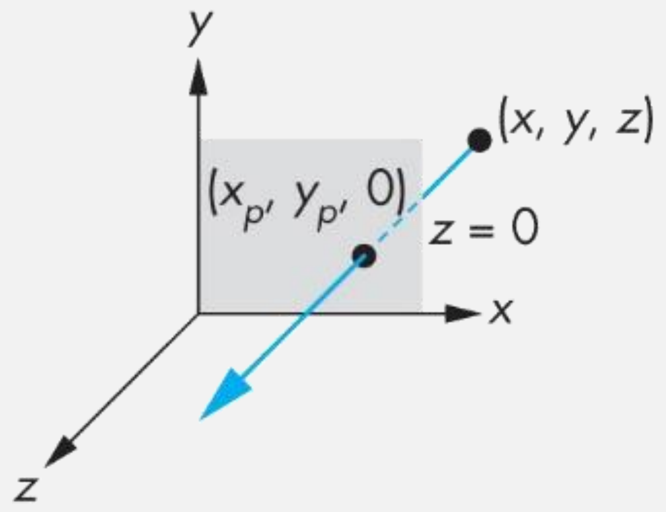
Viewing with A Computer

- Three aspects of the viewing process implemented in the pipeline:
  - Positioning the camera
    - Setting the model-view matrix
  - Selecting a lens
    - Setting the projection matrix: orthogonal or perspective
  - Normalization & Clipping
    - Setting the view volume

Projections and Normalization

- The default projection in the eye (camera) frame is orthogonal

- For points within the default view volume
  - $x_p = x$
  - $y_p = y$
  - $z_p = 0$



Homogeneous Coordinate Representation

- $x_p = x$
- $y_p = y$
- $z_p = 0$
- $w_p = 1$

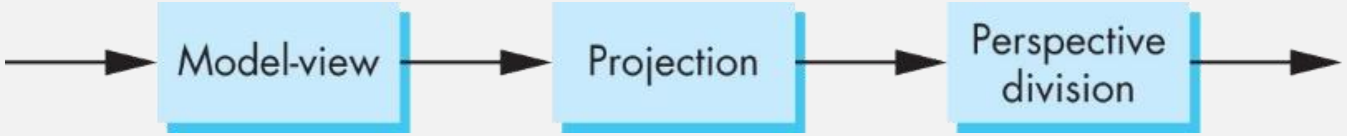
default orthographic projection

$$\mathbf{q} = \mathbf{M}\mathbf{p}$$

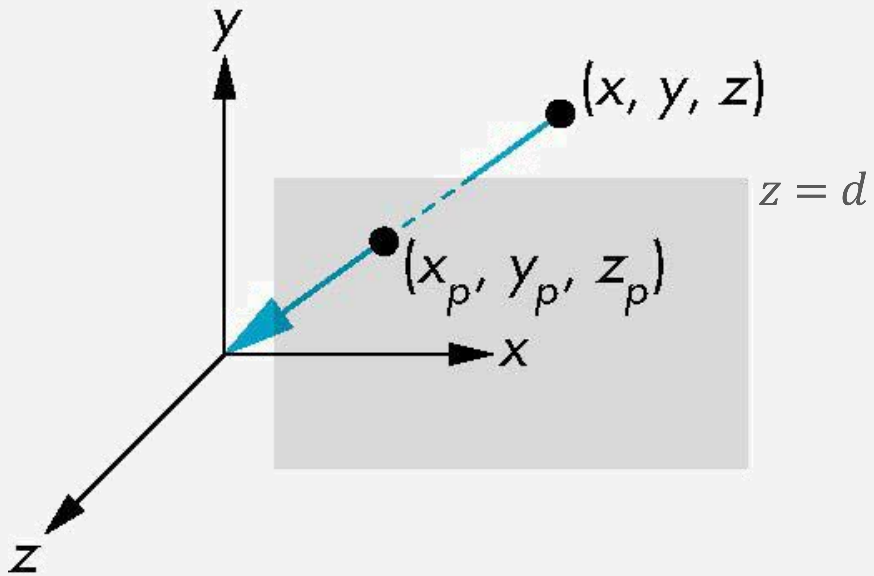
$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In practice, we can let **M= I** and set the *z* term to zero later

Simple Perspective Projections

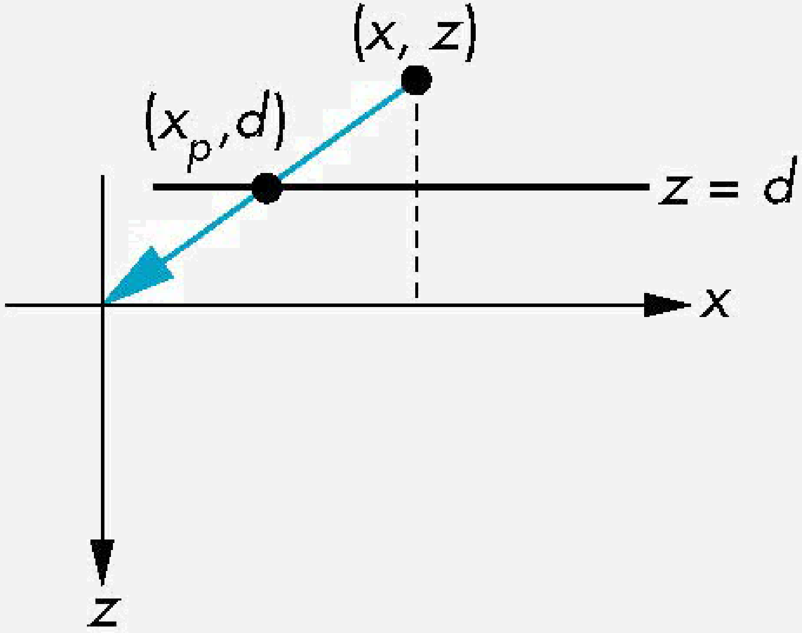


- Center of projection : at the origin
- Projection plane $z = d, d < 0$



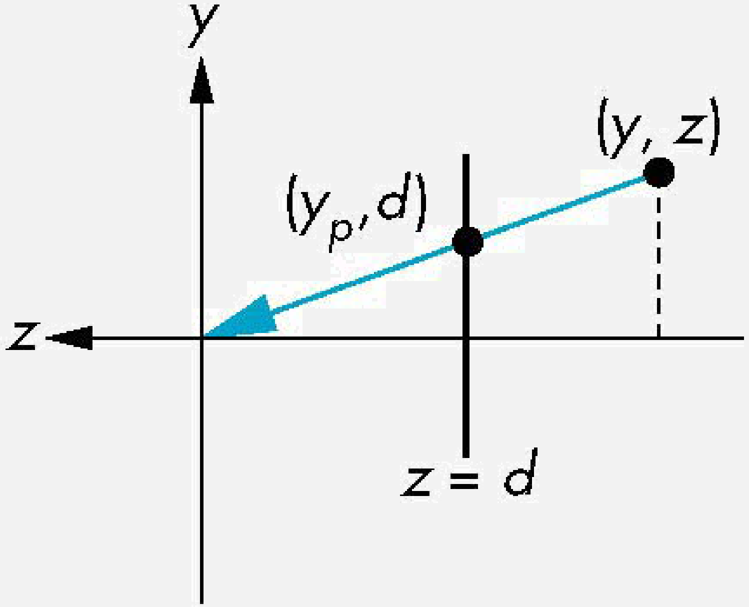
Perspective Equations

Top view



$\frac{x}{z/d}$

Side view



$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d = \frac{z}{z/d}$$

Homogeneous Coordinate Representation

Consider **q= Mp** where

$$x_p = \frac{x}{z/d}$$

$$y_p = \frac{y}{z/d}$$

$$z_p = d = \frac{z}{z/d}$$

$$\not\Leftrightarrow$$

$$\mathrm{q} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \qquad \mathrm{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \qquad \mathrm{p} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective Division

The desired perspective equations:

$$x_p = \frac{x}{z/d}$$

$$y_p = \frac{y}{z/d}$$

$$z_p = d$$

$$q = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

- However $w \neq 1$, so we must divide by w to return from homogeneous coordinates. This perspective *division* yields

$$q' = \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \\ 1 \end{bmatrix} = \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix}$$

Viewing with A Computer (Cont.)

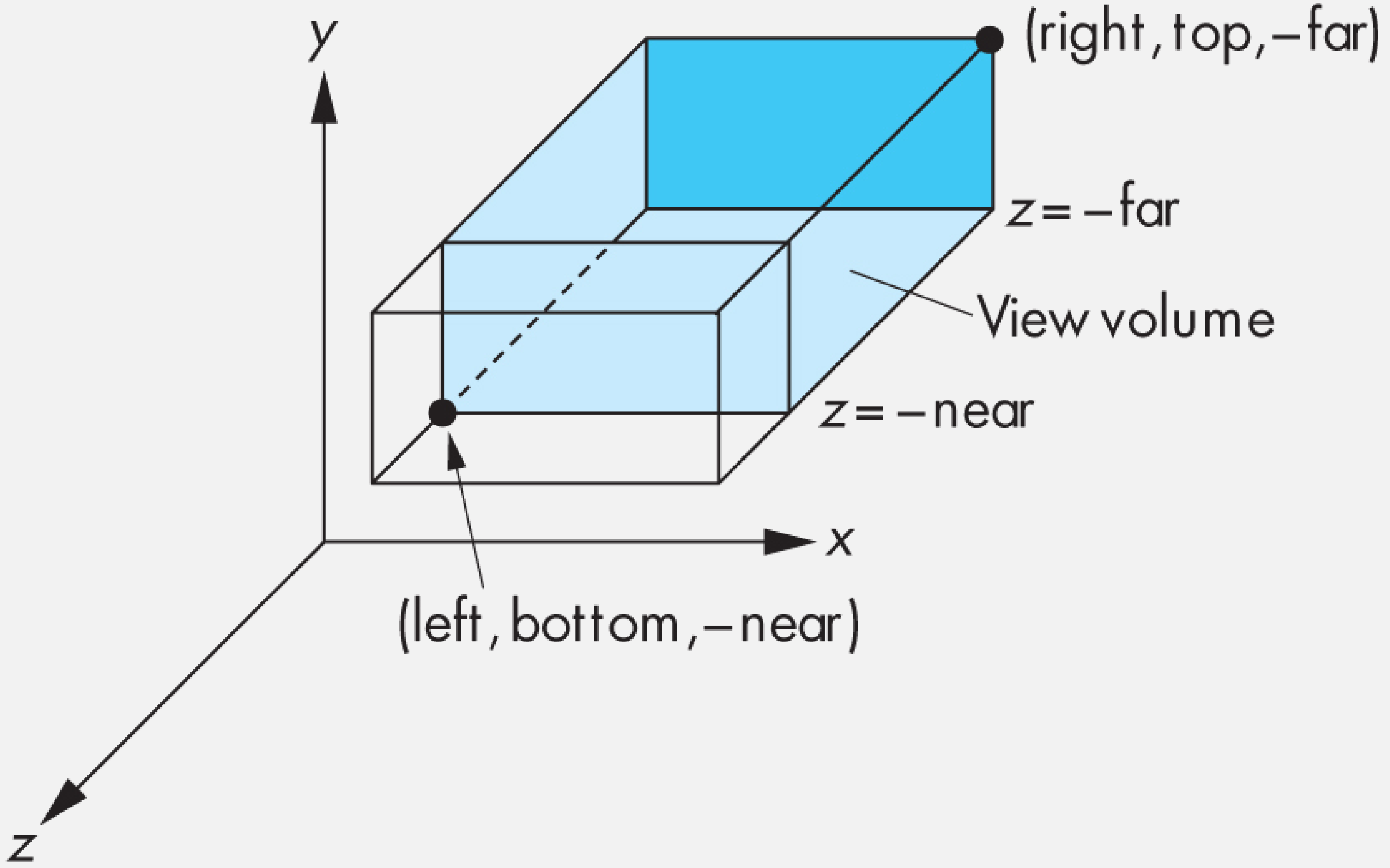
- Three aspects of the viewing process implemented in the pipeline:
  - Positioning the camera
    - Setting the model-view matrix
  - Selecting a lens
    - Setting the projection matrix: orthogonal or perspective
  - Normalization & Clipping
    - Setting the view volume

Taking Clipping into Account

- After the view transformation, a simple projection and viewport transformation can generate screen coordinate.
- However, projecting all vertices are usually unnecessary.
- Clipping with 3D volume.
- Associating projection with clipping and normalization.

Why do we use normalization ?

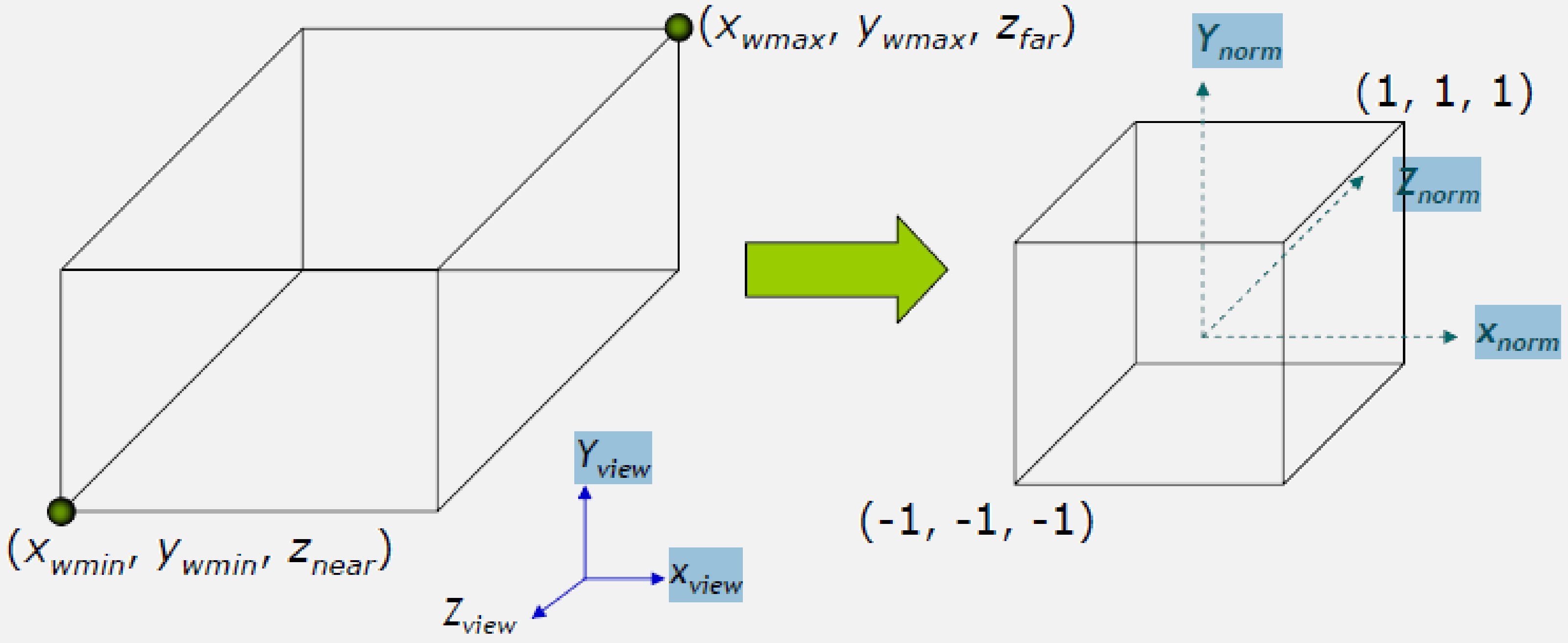
Orthogonal Viewing Volume



Orthogonal Normalization

glOrtho(left,right,bottom,top,near,far)

normalization ⇒ find transformation to convert specified clipping volume to default



Orthogonal Normalization Matrix

- Two steps
  - T: Move center to origin
  - S: Scale to have sides of length 2

$$\mathbf{T} = \mathbf{T}\left(-\frac{(right+left)}{2}, -\frac{(top+bottom)}{2}, -\frac{(far+near)}{2}\right)$$

$$\mathbf{S} = \mathbf{S}\left(\frac{2}{(right-left)}, \frac{2}{(top-bottom)}, \frac{2}{(near-far)}\right)$$

$$\mathbf{P} = \mathbf{ST} = \begin{bmatrix} \frac{2}{xw_{\max} - xw_{\min}} & 0 & 0 & -\frac{xw_{\max} + xw_{\min}}{xw_{\max} - xw_{\min}} \\ 0 & \frac{2}{yw_{\max} - yw_{\min}} & 0 & -\frac{yw_{\max} + yw_{\min}}{yw_{\max} - yw_{\min}} \\ 0 & 0 & \frac{2}{z_{near} - z_{far}} & \frac{z_{near} + z_{far}}{z_{near} - z_{far}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Orthogonal Projection

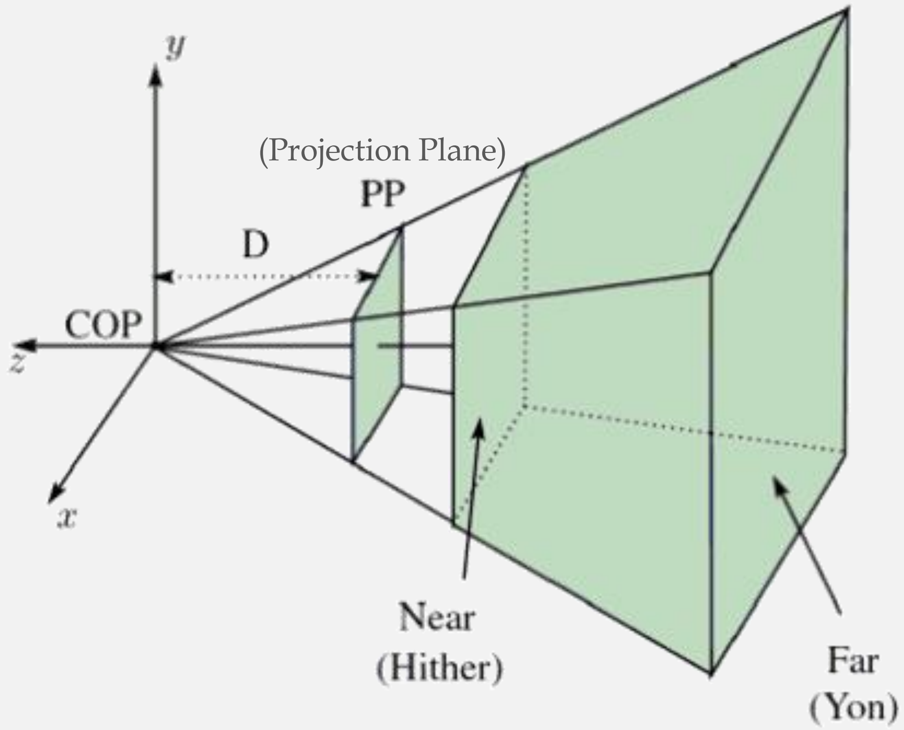
- Set z = 0
- Equivalent to the homogeneous coordinate transformation

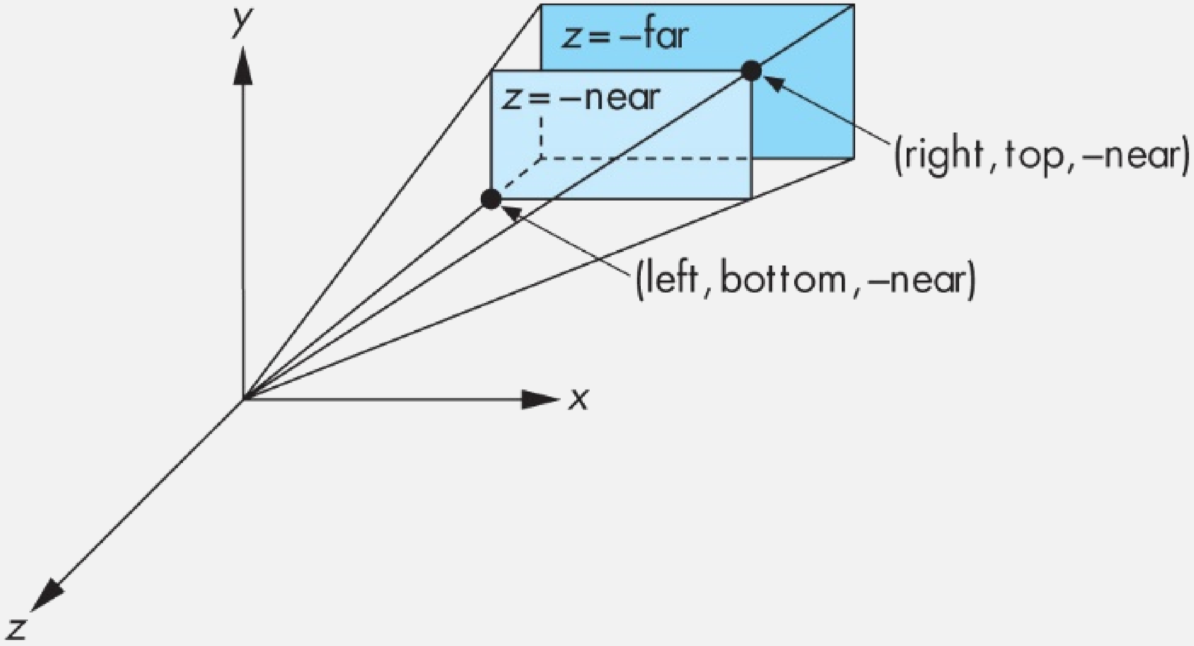
$$\mathbf{M}_{\mathrm{orth}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Hence, general orthogonal projection in 4D is

$$\mathbf{P} = \mathbf{M}_{\mathrm{orth}}\mathbf{ST}$$

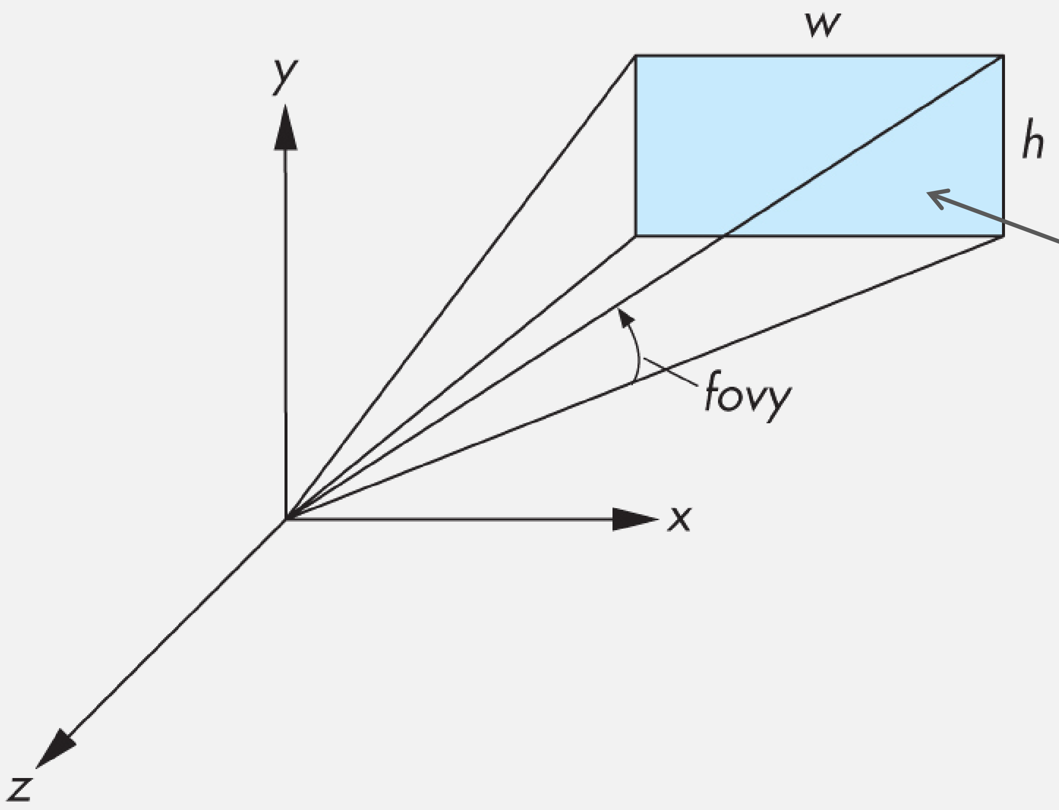
Perspective Viewing Volume





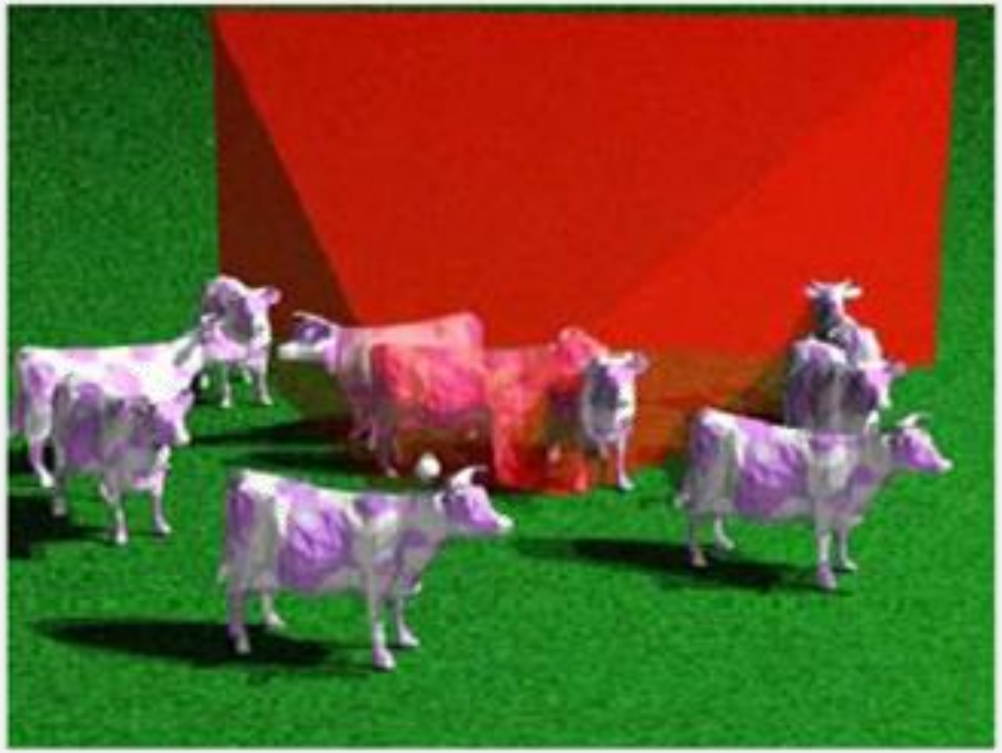
Using Field/Angle of View

- In addition to directly assigning the viewing frustum, assigning field of view may be more user-friendly.

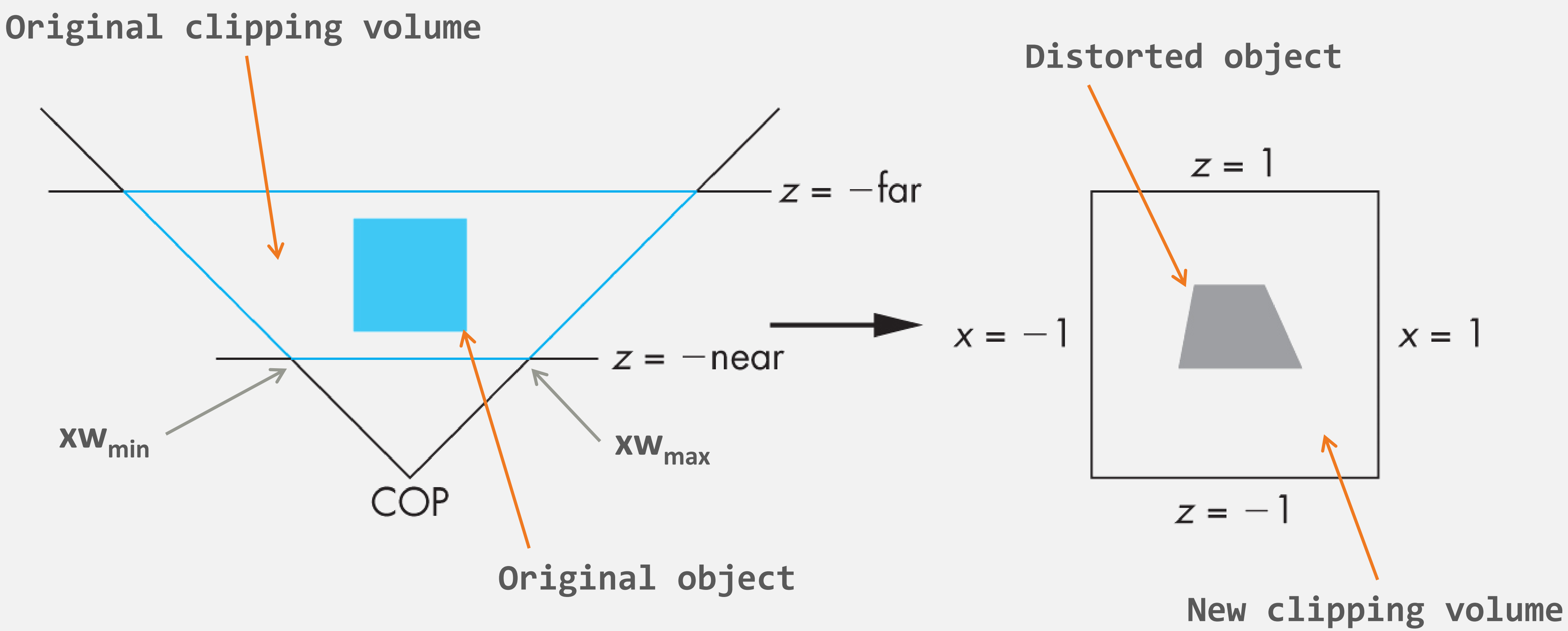


aspect =**w/h**

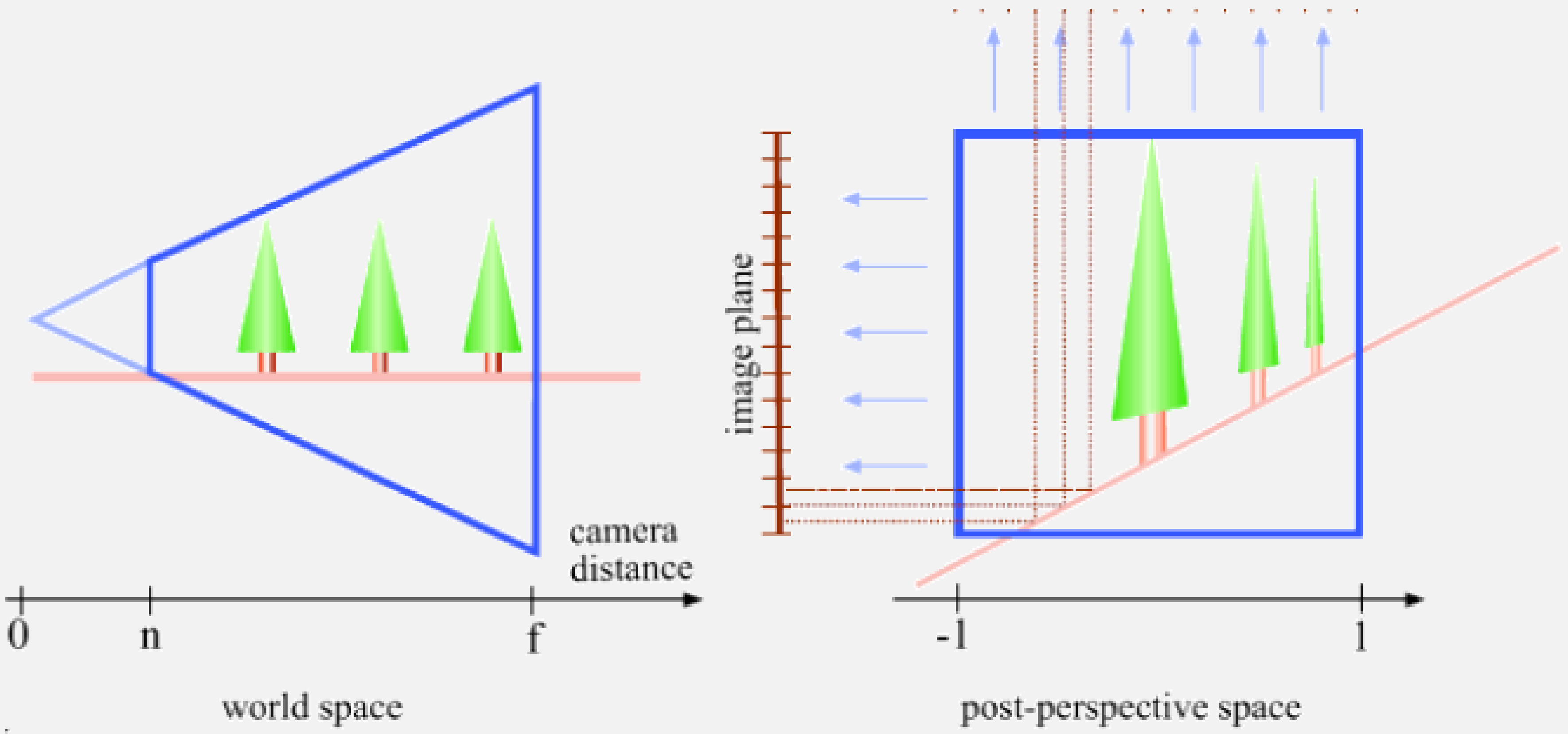
Clipping for Perspective Views



Perspective Normalization

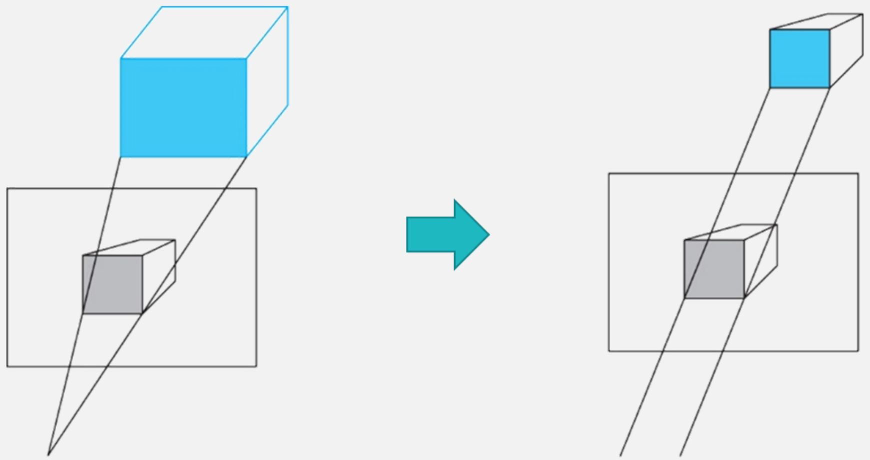


Perspective Normalization (Cont.)



Normalization

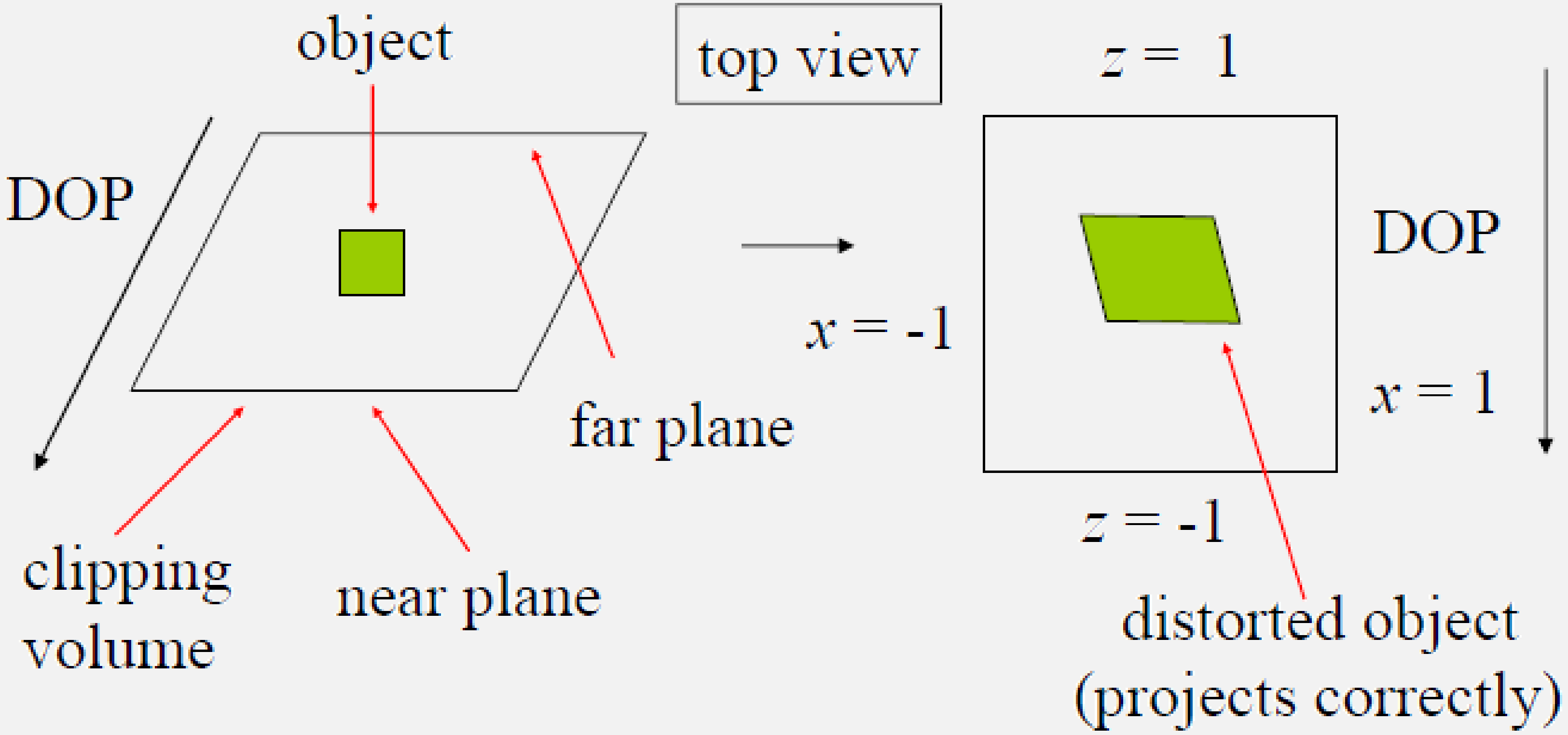
- Rather than derive a different projection matrix for each type of projection, we can **convert all projections to orthogonal projections** with the default view volume



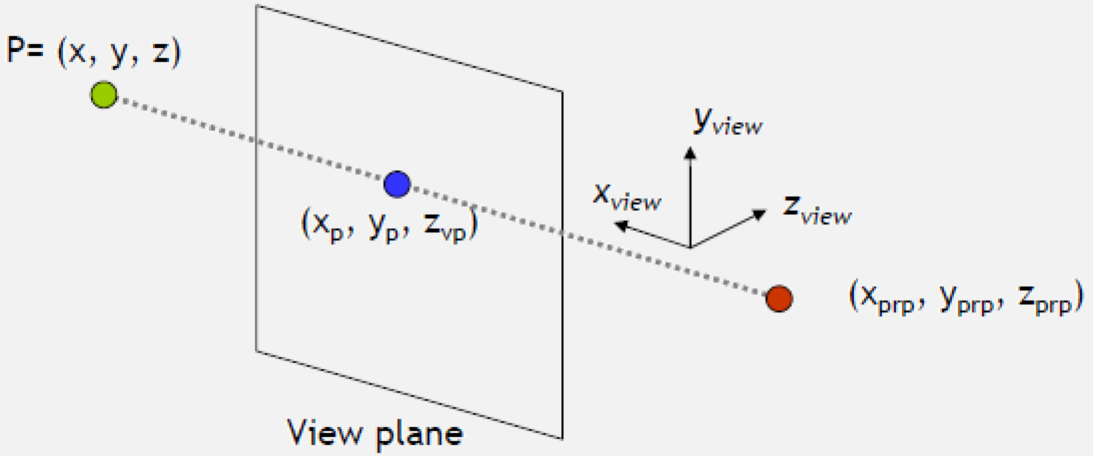
- This strategy allows us to use **standard transformations** in the pipeline and makes for **efficient clipping**

Effect on Clipping

- The projection matrix **P= STH** transforms the original clipping volume to the default clipping volume



Perspective-Projection Transformation



$$x_p = (1-u)x + ux_{prp}$$

$$y_p = (1-u)y + uy_{prp}$$

$$u = 0 \sim 1$$

$$\boxed{\begin{aligned} x_p &= x\left(\frac{z_{prp}-z_{vp}}{z_{prp}-z}\right) + x_{prp}\left(\frac{z_{vp}-z}{z_{prp}-z}\right) \\ y_p &= y\left(\frac{z_{prp}-z_{vp}}{z_{prp}-z}\right) + y_{prp}\left(\frac{z_{vp}-z}{z_{prp}-z}\right) \end{aligned}}$$

Given $x_{prp}=y_{prp}=z_{prp}=0$, $z_{vp} = z_{near}$

$$\boxed{\begin{aligned} x_p &= x\left(\frac{-z_{near}}{-z}\right) \\ y_p &= y\left(\frac{-z_{near}}{-z}\right) \end{aligned}}$$

Perspective-Projection Transformation (Cont.)

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

$$\boxed{M_{pers} = \begin{bmatrix} -z_{near} & 0 & 0 & 0 \\ 0 & -z_{near} & 0 & 0 \\ 0 & 0 & s_z & t_z \\ 0 & 0 & -1 & 0 \end{bmatrix}}$$

After perspective division, the point (*x,y,z*,1) goes to

$$x_p = x\left(\frac{-Z_{near}}{-Z}\right)$$

$$y_p = y\left(\frac{-Z_{near}}{-Z}\right)$$

$$z_p = \frac{s_z z + t_z}{-z} = -\left(s_z + \frac{t_z}{z}\right)$$

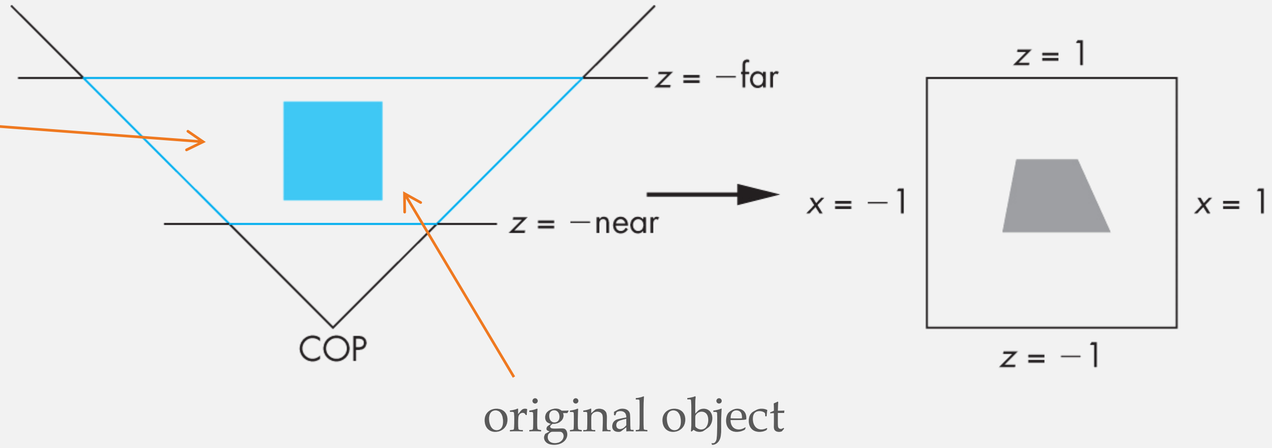
To make $-1 \le z_p \le 1$

$$S_z = \frac{Z_{near} + Z_{far}}{Z_{near} - Z_{far}}$$

$$t_z = \frac{-2Z_{near}Z_{far}}{Z_{near} - Z_{far}}$$

Further Normalization

$$M_{pers} = \begin{bmatrix} -z_{near} & 0 & 0 & 0 \\ 0 & -z_{near} & 0 & 0 \\ 0 & 0 & \dfrac{z_{near}+z_{far}}{z_{near}-z_{far}} & \dfrac{-2z_{near}z_{far}}{z_{near}-z_{far}} \\ 0 & 0 & -a & 0 \end{bmatrix}$$



$$M_{normpers} = \begin{bmatrix} -z_{near}\dfrac{2}{xw_{max}-xw_{min}} & 0 & 0 & 0 \\ 0 & -z_{near}\dfrac{2}{yw_{max}-yw_{min}} & 0 & 0 \\ 0 & 0 & \dfrac{z_{near}+z_{far}}{z_{near}-z_{far}} & \dfrac{-2z_{near}z_{far}}{z_{near}-z_{far}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Notes

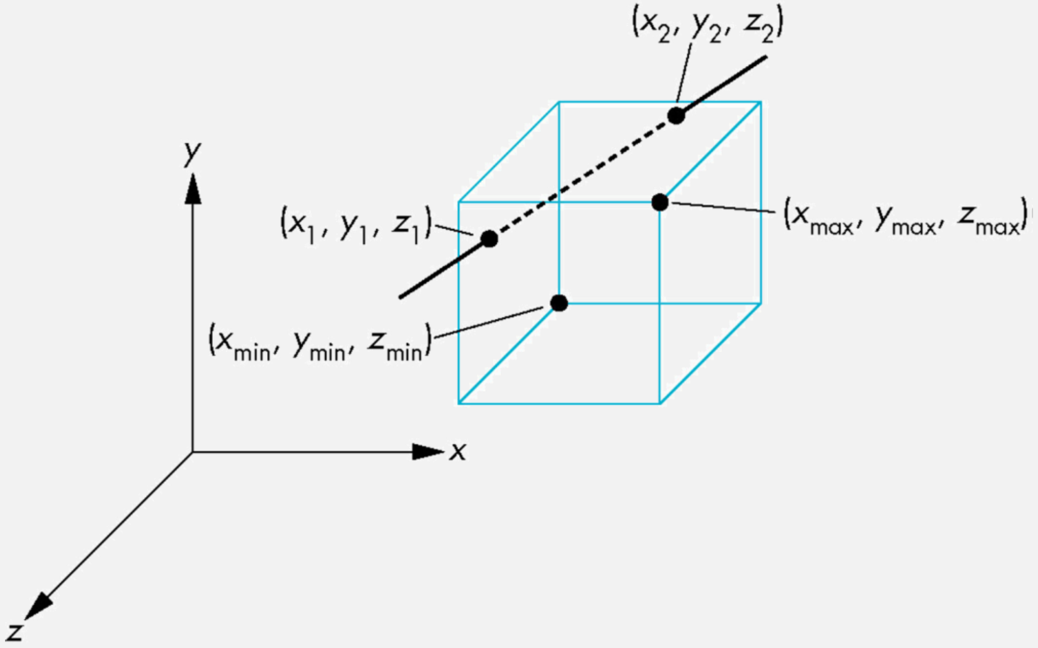
- Normalization let us clip against a simple cube regardless of type of projection
- Delay final "projection" until end
  - Important for ***hidden-surface removal*** to retain depth information as long as possible

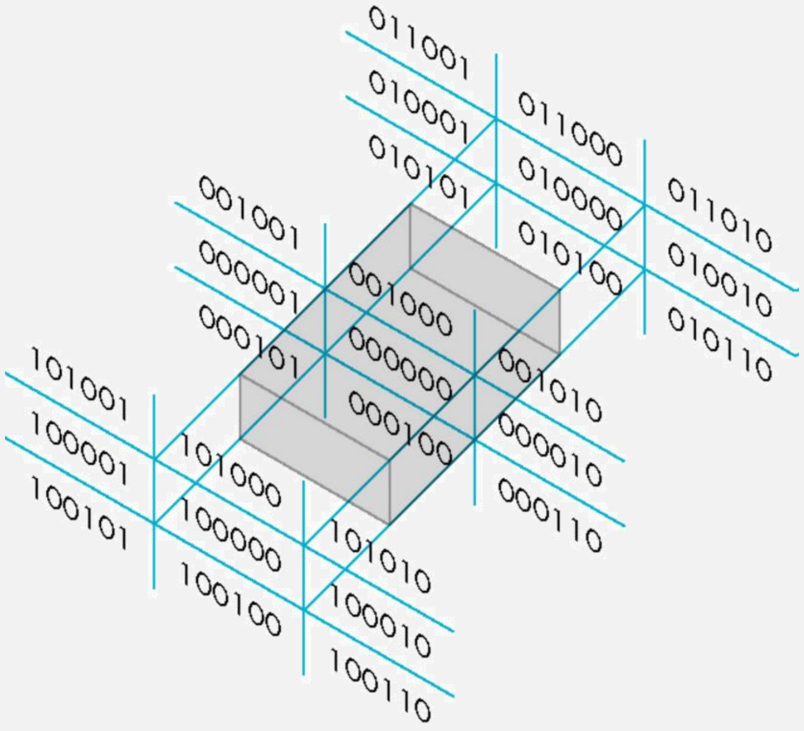
Why do we do it this way?

- Normalization allows for ***a single pipeline*** for both perspective and orthogonal viewing
- We stay in four dimensional homogeneous coordinates as long as possible to retain three-dimensional information needed for hidden-surface removal and shading
- ***Clipping is now "easier".***

Cohen-Sutherland Method in 3D

- Use 6-bit outcodes
  - When needed, clip line segment against planes





Cohen-Sutherland Method in 3D (Cont.)

Check for outcodes:

$$-1 \le x_p \le 1,\ -1 \le y_p \le 1,\ -1 \le z_p \le 1$$

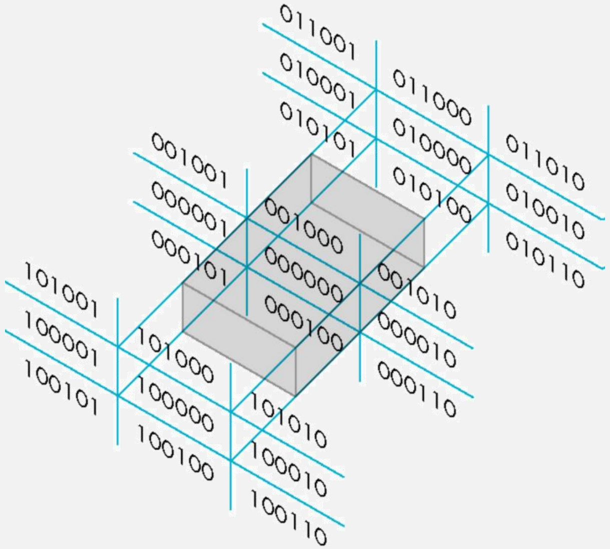
Since

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \xRightarrow{\text{SRT}\ldots} \ldots\ldots \Rightarrow \begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} \xRightarrow{\text{Divide } h} \ldots\ldots \Rightarrow \begin{bmatrix} x_h/h \\ y_h/h \\ z_h/h \\ 1 \end{bmatrix} = \begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix}$$

To avoid unnecessary float division, We can check

$$-h \le x_h \le h,\ -h \le y_h \le h,\ -h \le z_h \le h$$

Cohen-Sutherland Method in 3D (Cont.)



- If outcode(A)==outcode(B)==0
  - Accept the whole line segment.
- If(outcode(A) and outcode(B))!=0
  - Reject the line segment.
- Other cases
  - Calculate an intersection (according to outcode bits)
  - Then check outcode again
- Note: use parametric forms

$$x_h = x_{ha} + (x_{hb} - x_{ha})u$$

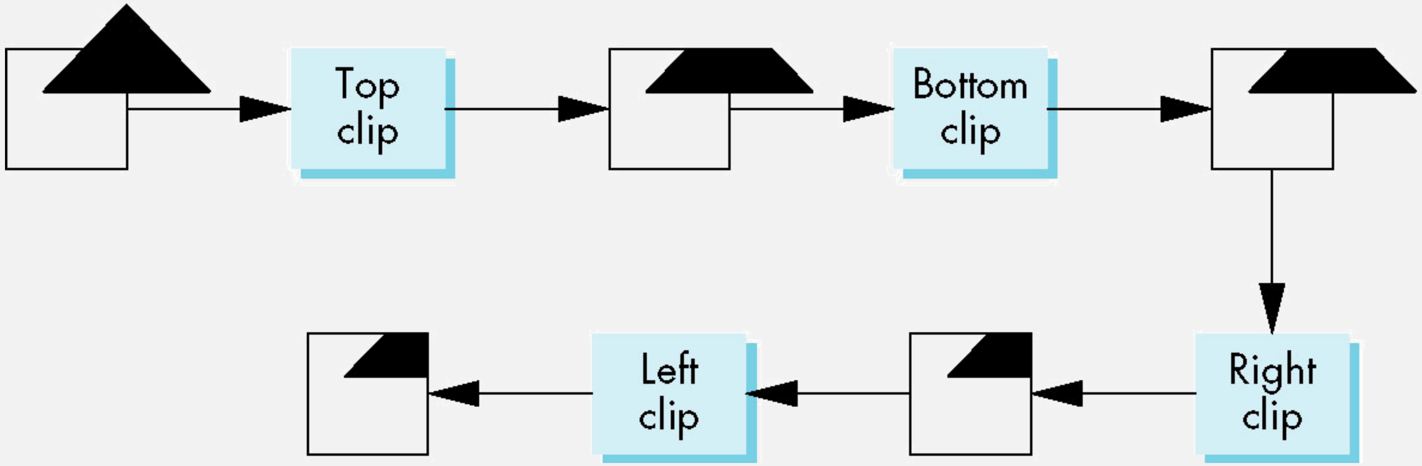
$$y_h = y_{ha} + (y_{hb} - y_{ha})u$$

$$z_h = z_{ha} + (z_{hb} - z_{ha})u$$

$$h = h_a + (h_b - h_a)u$$

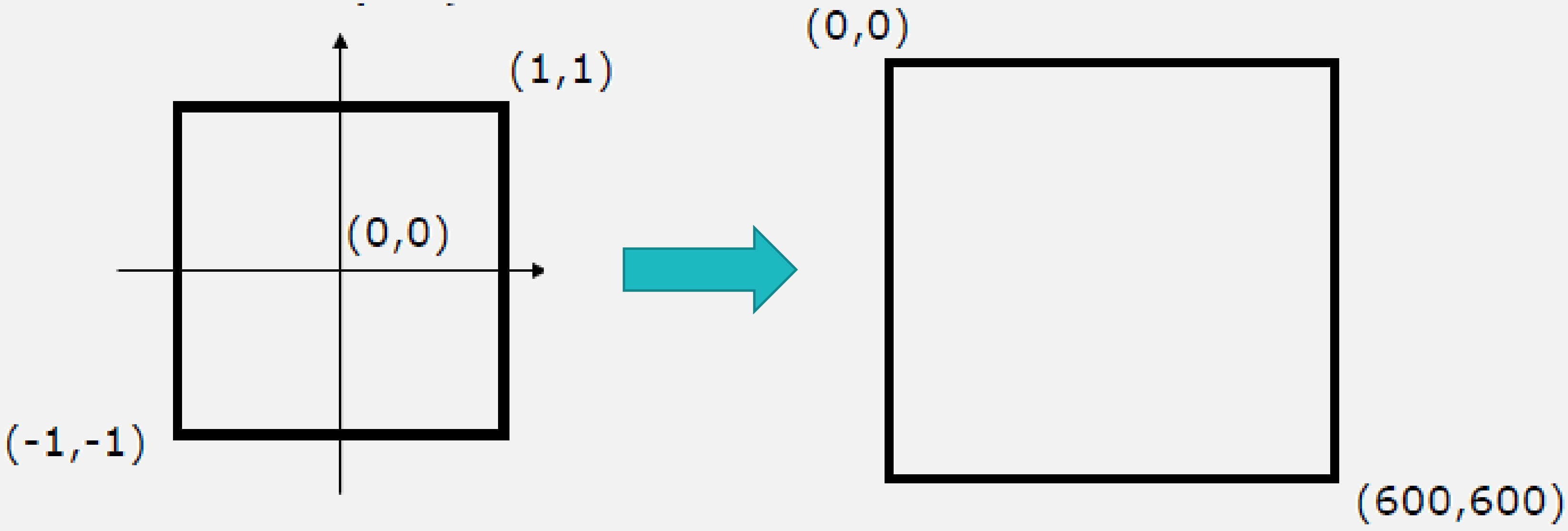
Polygon Clipping in 3D

- Similar to 2D clipping
  - Bounding box
  - Clipping with each clipping plane
  - Etc…..



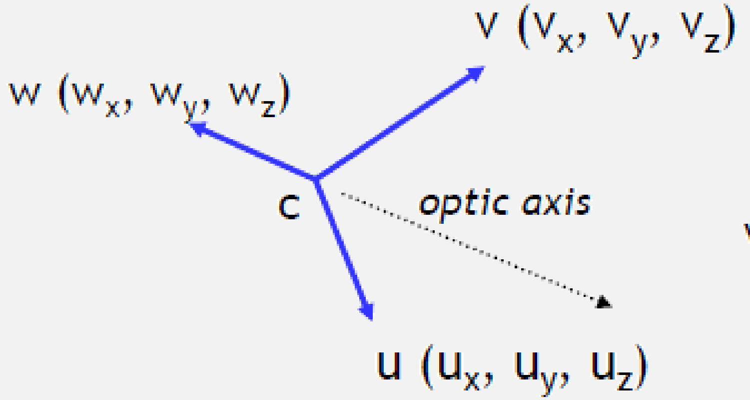
Viewport Transformation

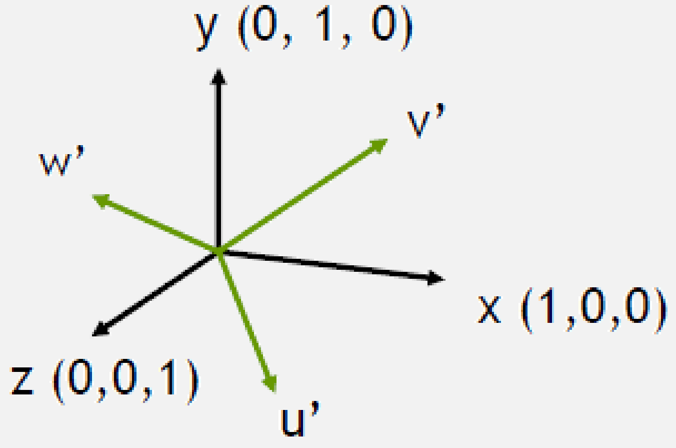
- From the working coordinate to the coordinate of display device.



By 2D scaling and translation

By Coordinate Transformations

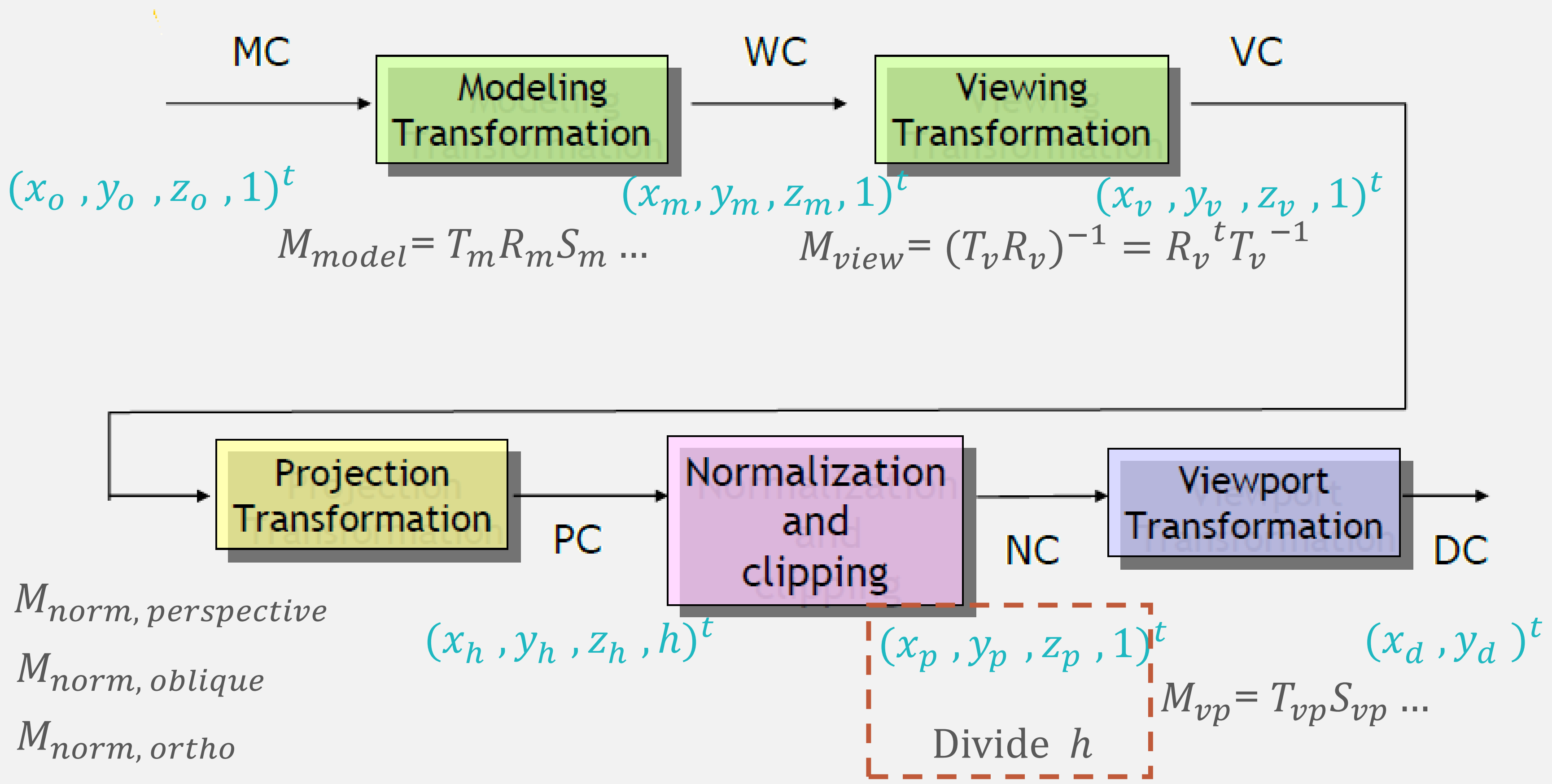




$$\begin{bmatrix} x_{wc} \\ y_{wc} \\ z_{wc} \\ 1 \end{bmatrix} = \begin{bmatrix} u'_x & v'_x & w'_x & 0 \\ u'_y & v'_y & w'_y & 0 \\ u'_z & v'_z & w'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x'_{vc} \\ y'_{vc} \\ z'_{vc} \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x'_{vc} \\ y'_{vc} \\ z'_{vc} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -c_x \\ 0 & 1 & 0 & -c_y \\ 0 & 0 & 1 & -c_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{vc} \\ y_{vc} \\ z_{vc} \\ 1 \end{bmatrix}$$

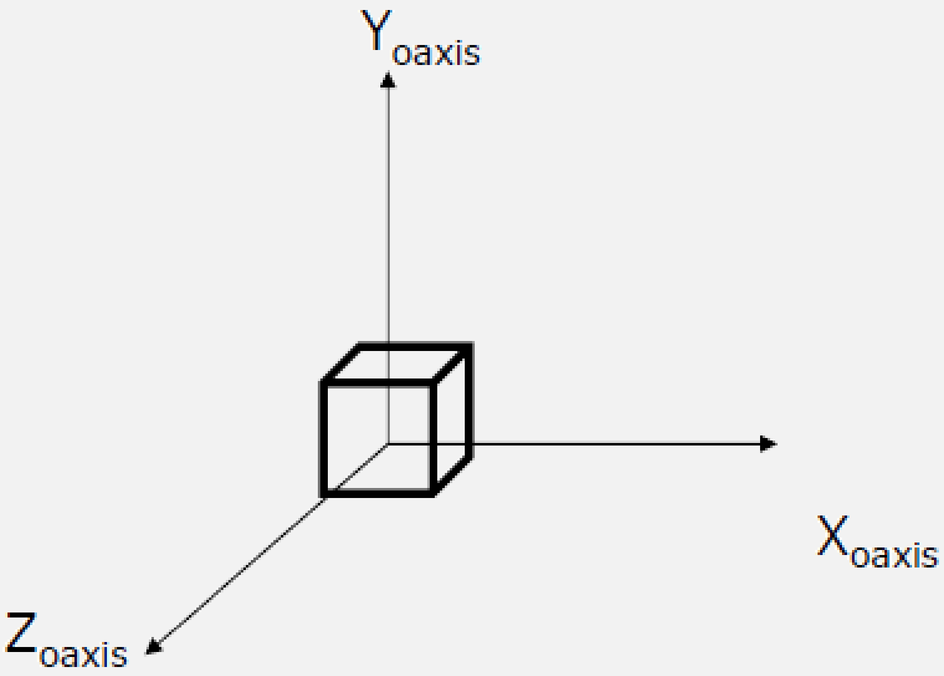
Example

Pipeline View



Loading an Object

- $(x_o\ , y_o\ , z_o\ , 1)^t$

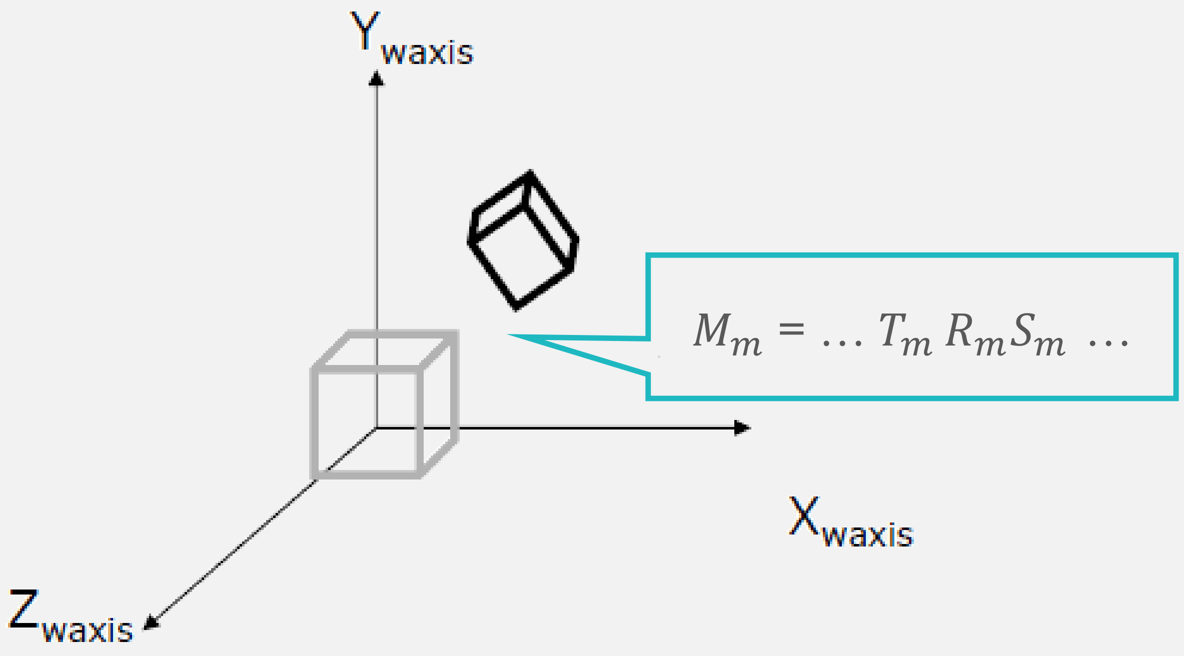


object coordinates

Modeling Transformation

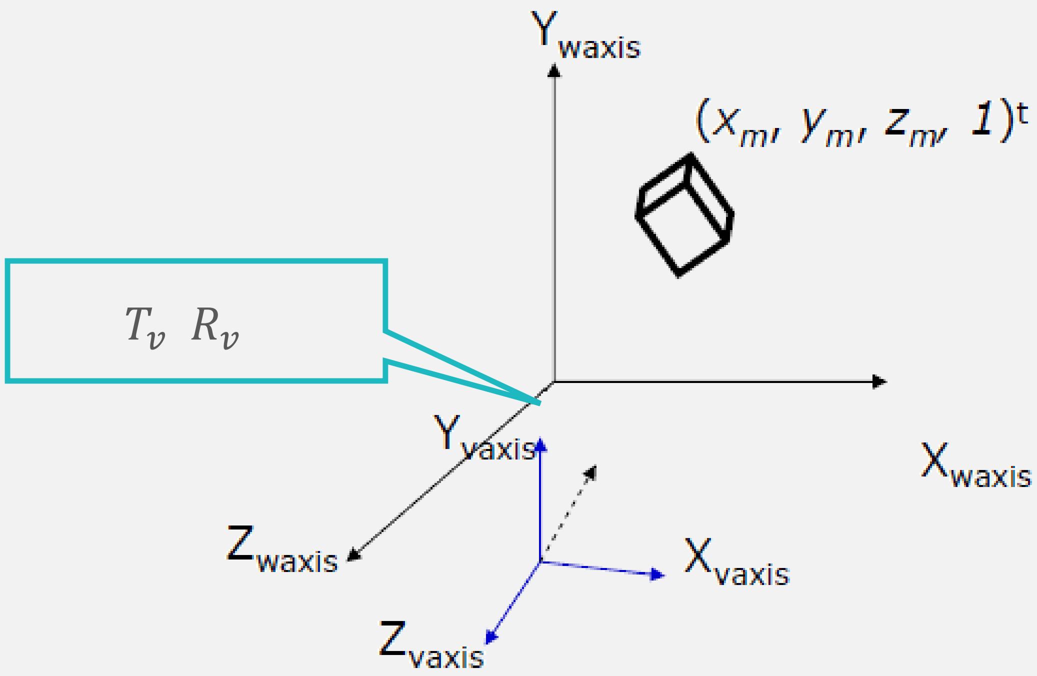
- $(x_m, y_m, z_m, 1)^t = M_m(x_o, y_o, z_o, 1)^t$

  where $M_m = \ldots T_m R_m S_m \ldots$



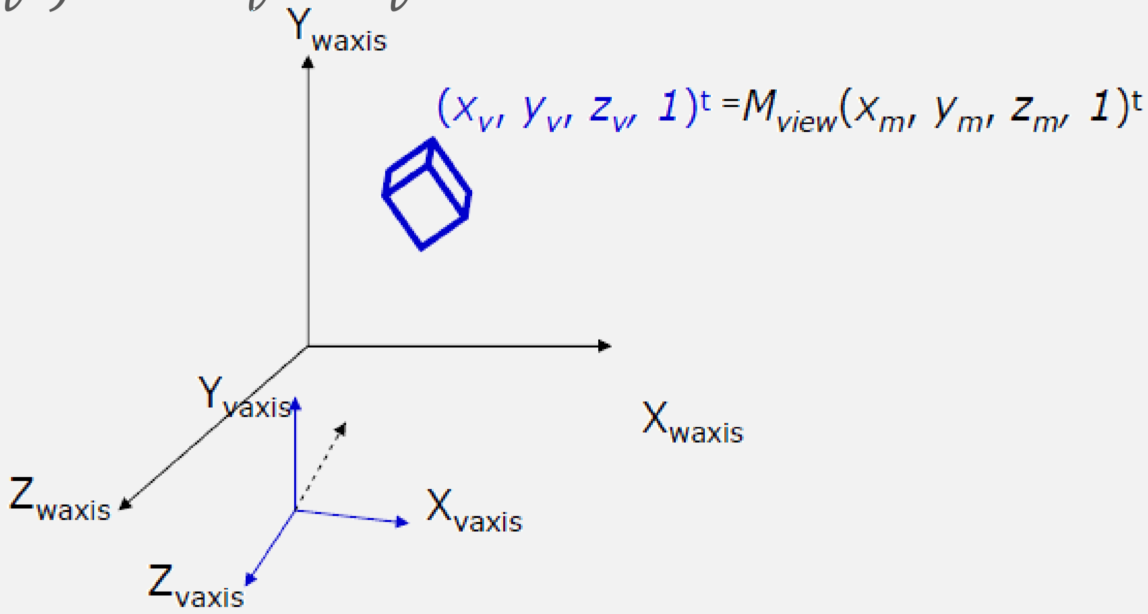
Put a Virtual Camera

- Move a camera from the origin (by $T_v R_v$)

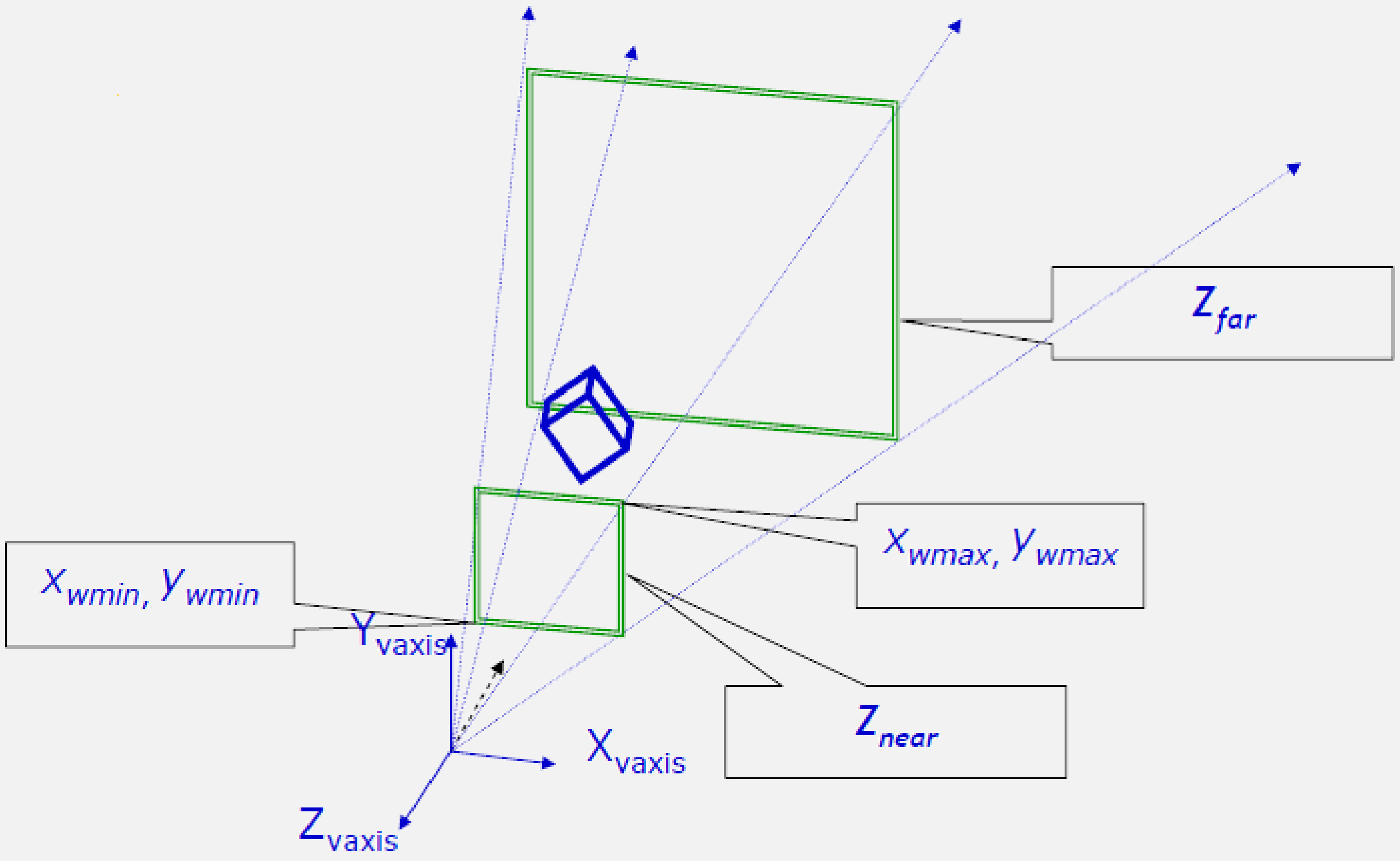


Virtual Camera's Coordinate

- Change the object's coordinate
- $(x_v, y_v, z_v, 1)^t = M_{view}(x_m, y_m, z_m, 1)^t$
- $M_{view} = (T_v, R_v)^{-1} = R_v^{-1} T_v^{-1}$



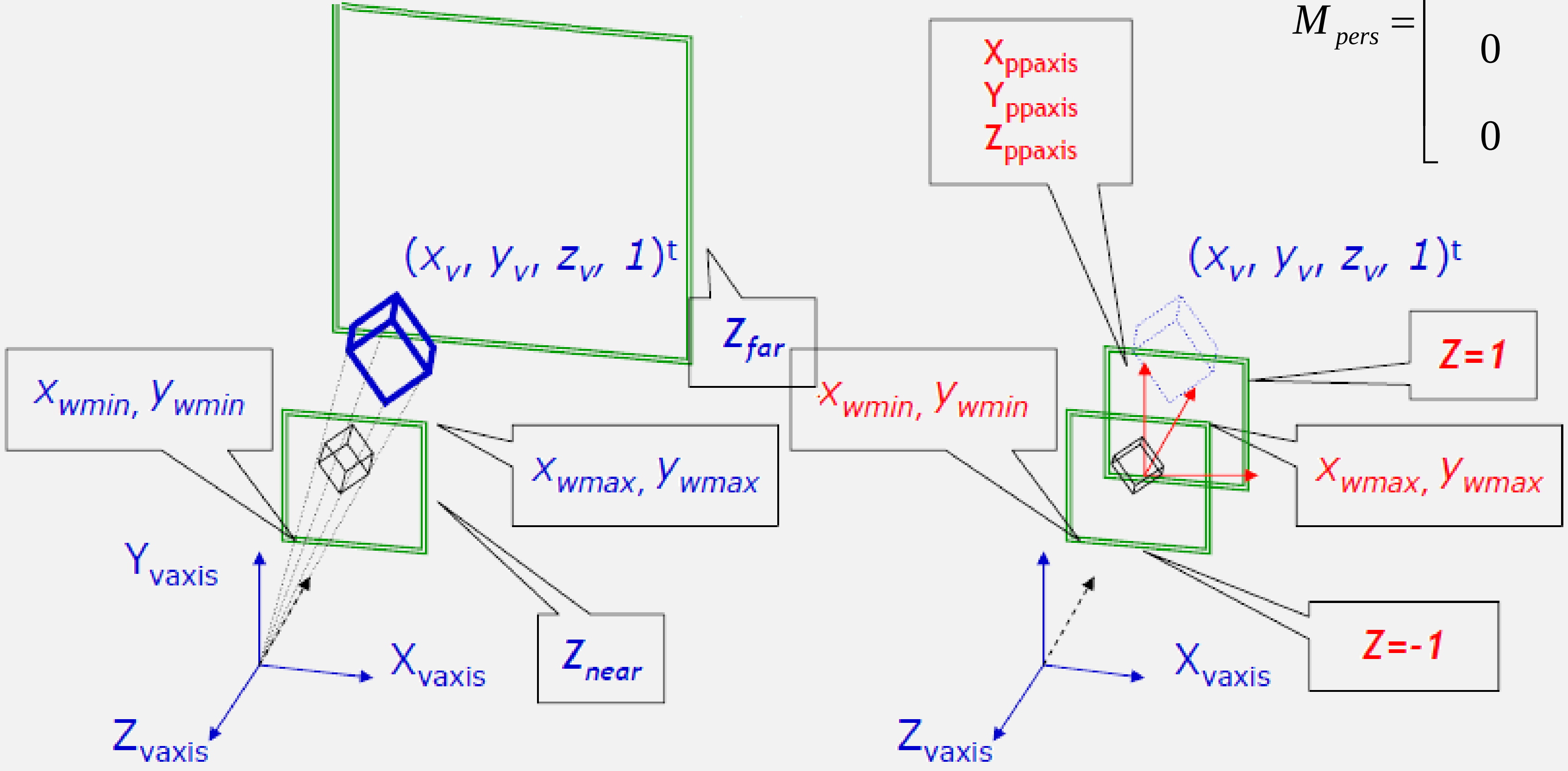
Virtual Camera's Coordinate (Cont.)



Perspective Projection

This matrix is usually combined with the normalization matrix.

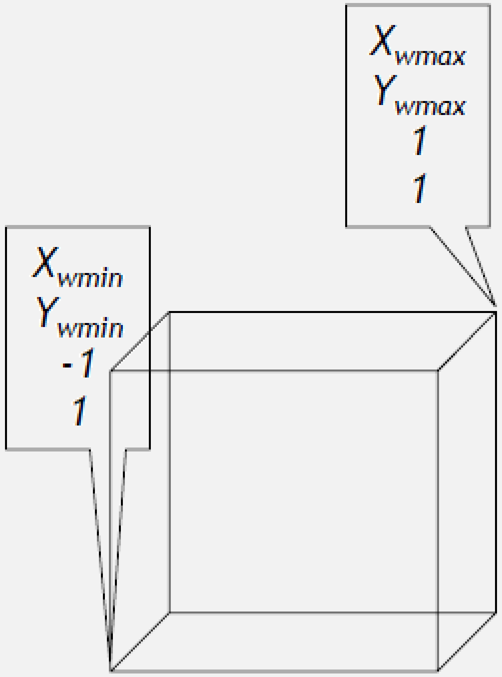
$$M_{pers} = \begin{bmatrix} -z_{near} & 0 & 0 & 0 \\ 0 & -z_{near} & 0 & 0 \\ 0 & 0 & \dfrac{z_{near}+z_{far}}{z_{near}-z_{far}} & \dfrac{-2z_{near}z_{far}}{z_{near}-z_{far}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

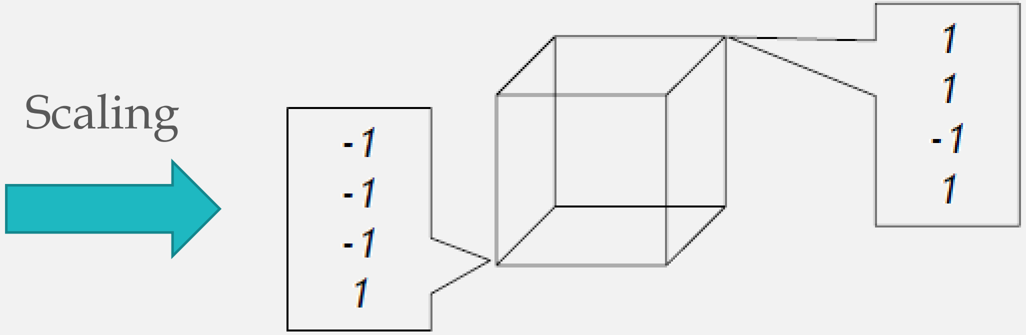


Projection + Normalization

$$M_{normpers} = \begin{bmatrix} -z_{near}\dfrac{2}{xw_{\max} - xw_{\min}} & 0 & 0 & 0 \\ 0 & -z_{near}\dfrac{2}{yw_{\max} - yw_{\min}} & 0 & 0 \\ 0 & 0 & \dfrac{z_{near} + z_{far}}{z_{near} - z_{far}} & \dfrac{-2z_{near}z_{far}}{z_{near} - z_{far}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{2}{xw_{\max} - xw_{\min}} & 0 & 0 & 0 \\ 0 & \dfrac{2}{yw_{\max} - yw_{\min}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} M_{pers}$$

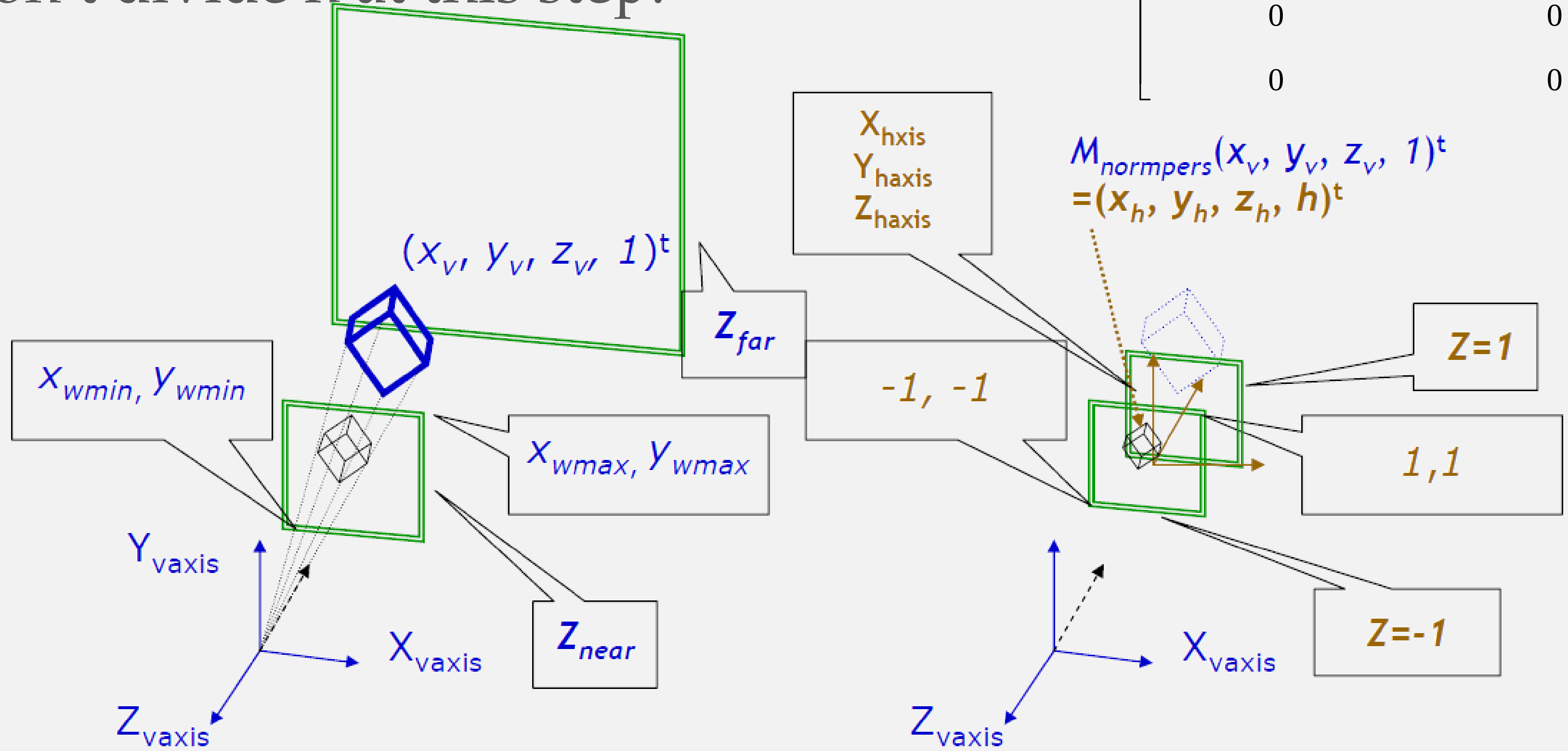




Projection + Normalization (Cont.)

- $(x_h, y_h, z_h, h)^t = M_{normpers}(x_v, y_v, z_v, 1)^t$
- Don't divide h at this step.

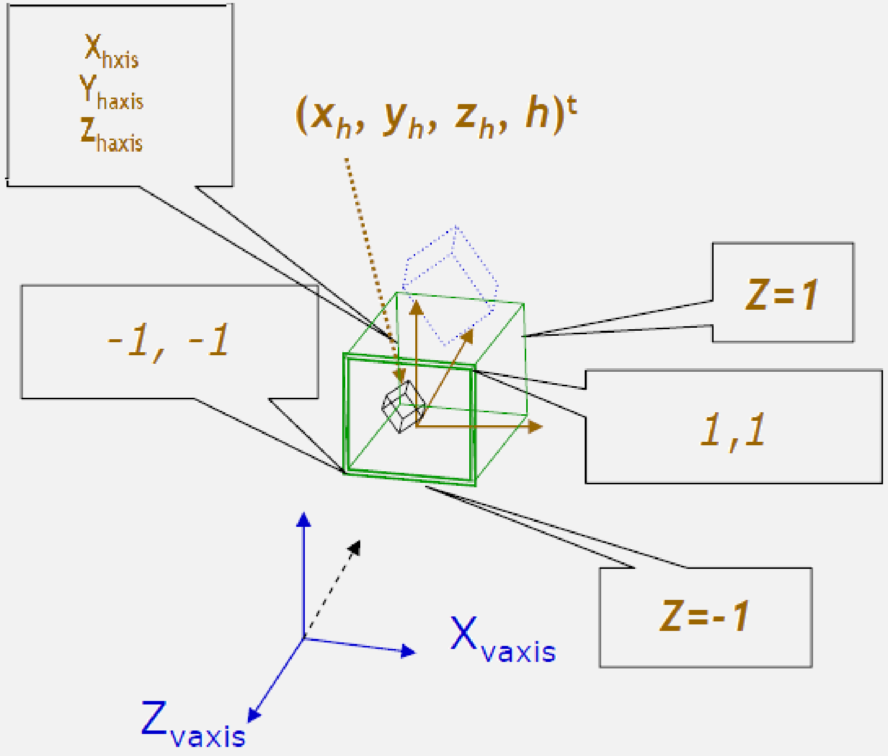
$$M_{normpers} = \begin{bmatrix} -z_{near}\dfrac{2}{xw_{\max}-xw_{\min}} & 0 & 0 & 0 \\ 0 & -z_{near}\dfrac{2}{yw_{\max}-yw_{\min}} & 0 & 0 \\ 0 & 0 & \dfrac{z_{near}+z_{far}}{z_{near}-z_{far}} & \dfrac{-2z_{near}z_{far}}{z_{near}-z_{far}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



Clipping

- Perform clipping with $(x_h, y_h, z_h, h)^t$
- Avoid unnecessary division $-h \leq x_h \leq h,\ -h \leq y_h \leq h,\ -h \leq z_h \leq h$
- Use parametric forms for intersection

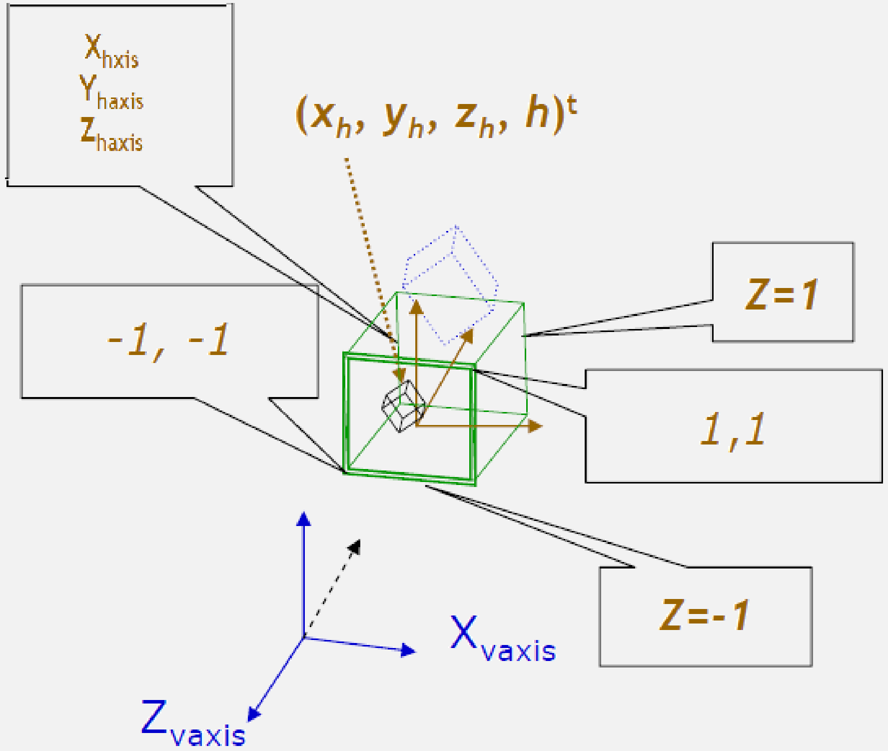
$$x_h = x_{ha} + (x_{hb} - x_{ha})u$$
$$y_h = y_{ha} + (y_{hb} - y_{ha})u$$
$$z_h = z_{ha} + (z_{hb} - z_{ha})u$$
$$h = h_a + (h_b - h_a)u$$

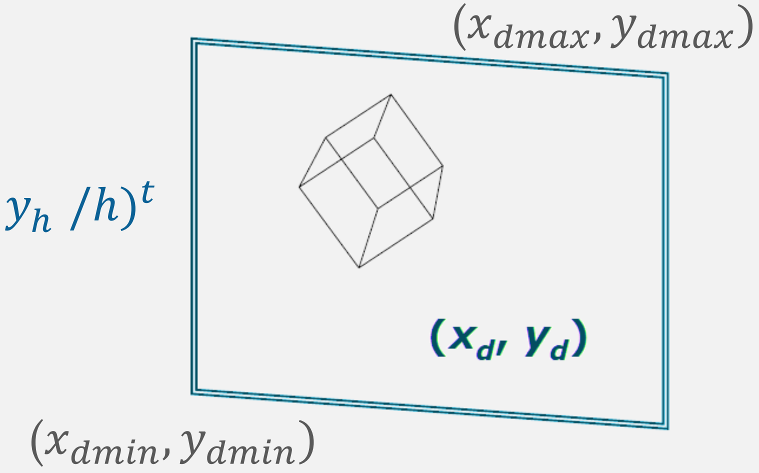


Viewport Transformation

- $(x_d, y_d, z_d, 1)^t = M_{viewport}(x_h, y_h, z_h, h)^t$

  or $(x_d, y_d)^t = {}_{SUB}M_{viewport}(x_p, y_p)^t$, $(x_p, y_p)^t = (x_h/h, y_h/h)^t$





$$M_{viewport} = \begin{bmatrix} \frac{x_{d\max} - x_{d\min}}{2} & 0 & 0 & \frac{x_{d\max} + x_{d\min}}{2} \\ 0 & \frac{y_{d\max} - y_{d\min}}{2} & 0 & \frac{y_{d\max} + y_{d\min}}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rasterization

- Line drawing or polygon filling with

  $(x_d\ , y_d\ , z_d\ , 1)^t$ or $(x_d\ , y_d\ )^t$ and z_h