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A method for measuring and decomposing electoral bias for the three-party case, illustrated by the British case

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ABSTRACT

The measurement of bias in election results, whereby one or more parties are advantaged in the translation of votes into seats at the expense of others, is attracting increasing attention. So far, almost all of the analytical work – aimed at both identifying the extent of bias in an election result and establishing its causes – has focused on either two-party systems or on the largest two parties in multi-party systems. Building on the firm foundations of one such approach, this paper introduces an original procedure for analysing bias in three-party systems using a readily-appreciated metric for both evaluating the degree of bias and decomposing it into the various causal factors. This is illustrated using the example of the 2005 British general election and a comparison of the results from two-party and three-party analyses of six recent elections there.

Disproportional election results, where a party's share of the votes cast is incommensurate with its share of the seats allocated, are more likely to occur under some types of electoral system than others. The greatest levels of disproportionality are mostly generated in elections determined on a plurality basis in single-member legislative districts. That disproportionality almost invariably involves a 'winner's bonus', whereby the party with the largest share of the votes cast obtains a greater share of the seats. However, the second-placed party may not be disadvantaged; its vote and seat shares may be such as to give it a seats' bonus also. Third and smaller parties generally suffer most in the allocation of seats from votes, leading to the formation of a strong two-party system as anticipated by Duverger's Law (Duverger, 1954).

A further feature of such electoral systems and their outcomes, less frequently noted, is that in many cases the disproportionality does not favour each of the two dominant parties to the same extent. For example, one party may get a larger winner's bonus than the other would with the same vote share. According to Grofman and King (2007, 6) this indicates an absence of *partisan symmetry*, a requirement that '… the electoral system treat similarly-situated parties equally, so that each receives the same fraction of legislative seats for a particular vote percentage as the other party would have received if it had the same percentage'.

Where this requirement is not met there is partisan asymmetry – or *partisan bias* (Grofman and King, 2007, 32). Its measurement has been the subject of debate (see King, 1990; Gelman and King, 1994; Grofman et al., 1997; Gelman et al., 2004), leading King et al. (2005, 9) to conclude that;

> A consensus exists about using the symmetry standard to evaluate partisan bias in electoral systems. But such a consensus does not answer the subsidiary question: how to measure symmetry itself in order to determine whether partisan bias exists.

Grofman and King (2007, 32), on the other hand, claim that 'the degree of deviation from symmetry of treatment … is easily quantified, and made specific as to direction'.

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Quantifying the reasons for such deviation has not proved straightforward, however, hence the literature to which this paper is a significant extension.

Much of the debate about the quantification of partisan bias – leading to Grofman and King's conclusion – has occurred in the context of USA elections, especially to the House of Representatives, where two parties predominate.[1] Little attention has been paid to countries which use the same electoral system but where the two-party predominance expected by Duverger's Law is absent, as in Great Britain where, since the 1970s, a third party and other smaller parties have presented a substantial challenge to Conservative and Labour, which predominated during the preceding three decades.[2] Nevertheless, analyses of partisan bias there have continued to focus on those two parties. The results, although very illuminating with regard to the situation as it affects the two major participants, present a partial view only of the extent and nature of the partisan asymmetry involved in the translation of votes into seats for all three parties.

The British case therefore indicates the need to extend discussion of partisan bias and its measurement into *three-party* situations generally. A recent paper (Borisyuk et al., 2008) laid the foundations for such an extension, but empirical applications of the proposed algebra for analysing and decomposing three-party bias produced some unresolved issues (as noted there, p. 225). That paper adapted an algebra developed for the analysis of two-party situations, and the empirical applications did not all produce sensible results. Rather than adapt it further, however, this paper returns to the first principles of the original formulation and, using the same underlying approach, develops a new algebra for identifying and decomposing bias in three-party situations. It thus represents a departure from what has become a standard approach to bias estimation in the British context over recent decades, offering an alternative new procedure built on the same foundations but specifically designed for the three-party situation, with potential wide application to other electoral systems where three parties predominate.

This paper is thus an original contribution to the analysis of bias in plurality electoral systems where three parties all win substantial shares of both the votes cast and the legislative seats into which those totals are translated. It returns to the first principles established in a seminal study of electoral bias, and derives a new algebra (set out in full in the Appendix to this paper) for the identification and decomposition of bias in three-party systems. Its application is illustrated with data for the 2005 British general election,[3] and a final section presents summary findings for the preceding five contests, all of which can readily be characterised as three-party in nature. This provides, for the first time, an evaluation of not only the degree to which each of those three parties was advantaged or disadvantaged in the procedure for translating votes into seats, and how, but also the extent to which that advantage/disadvantage involved both, or only one, of the other two parties.

Table 1
Percentage shares of the votes and seats at British general elections, 1983–2005.

		Conservative	Labour	LD	Other
1983	Votes	43.5	28.3	26.0	2.2
	Seats	62.7	33.0	3.6	0.6
	Seats-Votes	19.2	4.7	−22.4	−1.6
1987	Votes	43.3	31.5	23.1	2.1
	Seats	59.4	36.1	3.5	0.9
	Seats-Votes	16.1	4.6	−19.6	−1.2
1992	Votes	42.8	35.2	18.3	3.7
	Seats	52.9	42.7	3.2	1.1
	Seats-Votes	10.1	7.5	−15.1	−2.6
1997	Votes	31.5	44.4	17.2	6.9
	Seats	25.7	65.2	7.2	1.9
	Seats-Votes	−5.8	20.8	−10.0	−5.0
2001	Votes	32.7	42.0	18.9	6.4
	Seats	25.9	64.4	8.1	1.6
	Seats-Votes	−6.8	22.4	−10.8	−4.8
2005	Votes	33.2	36.2	22.7	7.9
	Seats	31.5	56.7	9.9	1.9
	Seats-Votes	−1.7	20.5	−12.8	−6.0

Source: C. Rallings and Thrasher, 2007.

2. Disproportionality and bias at recent British general elections: the two-party case

A method for measuring and decomposing bias developed by Ralph Brookes (1953, 1959, 1960) for use in New Zealand (he termed the outcome 'distorted representation' rather than bias) has been widely applied in recent analyses of British election results (e.g. Johnston et al., 2001, 2002, 2006).[4] His approach has the major benefits of using a readily-appreciated metric and being decomposable into the various bias sources that he identified.[5] Its major drawback, however, is that whereas it is excellent for analysing a system where two parties predominate its application to recent British elections is constrained by the growth of three-party politics. Despite modifications by Mortimore (1992: see Johnston et al., 1999), it focuses on the two largest parties only, treating the third as a source of bias affecting the other two rather than as also potentially either suffering or benefiting from bias – where its share of the seats allocated is incommensurate with its share of the votes cast.

[1] Indeed, the software designed to measure partisan bias, JudgeIt, originally developed by King and others, depends upon a two-party system and is therefore inappropriate for the British case, and other systems with strong third parties.

[2] Throughout this paper we deal with the situation in Great Britain only, rather than the United Kingdom. Northern Ireland is excluded because it has a separate party system and the dominant British parties do not compete for votes there.

[3] Data for the 2010 general election only became available after this paper was almost completed but a minor reference to it is contained in Fig. 7 towards the end of this paper. A more detailed examination of the application of the three-party bias method to the 2010 general election may be found in, G. Borisyuk et al. "*Electoral bias in 2010: Evaluating its extent in a three-party system*" paper presented to the EPOP annual conference, Essex, September 2010.

[4] The only other attempts to measure and account for bias in Great Britain have been those by Curtice (2001; see also Curtice and Steed, 1986), which although it identified the various sources of bias did not quantify their relative importance, and Blau's (2001) important critique of the Brookes' method.

[5] An alternative approach, developed almost contemporaneously with Brookes', identifying the same basic bias components, is Soper and Rydon (1958), who developed some early ideas of Brookes (1953).

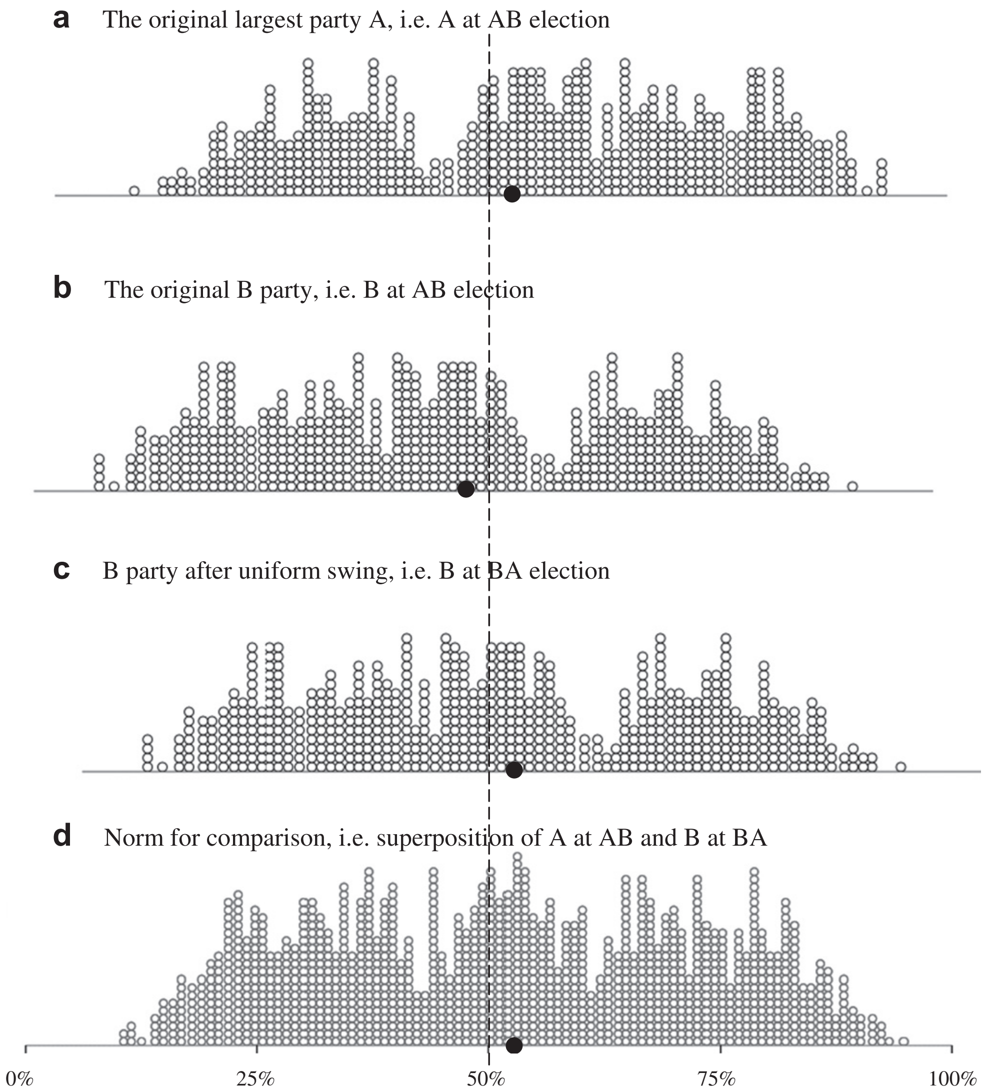

Fig. 1. Brookes' two-party method: Calculation of bias.

That Great Britain currently has a three-party system is clearly demonstrated by the vote shares data in Table 1 for the six general elections held during the period 1983–2005. In none of those contests did the winning party obtain a majority of the votes cast and, with the exception of the 2005 election, the winning party's vote share varied by only 2.4 percentage points around a mean of 43.3 per cent. There was slightly more variation in the shares obtained by the second- and third-placed parties, however, with the former obtaining on average 32.0 per cent and the latter 21.0. That final figure is crucial to the argument developed here; with a third party (in each case the Liberal Democrats[6]) gaining on average more than one-fifth of the votes cast and winning an increasing proportion of seats it is clearly not sensible to try and measure partisan bias as if it were a predominantly two-party system comparable to the United States.

The disproportionality generated by those results is shown in Table 1, which gives each party's shares of the votes cast and seats allocated. The winner's bonus (the percentage seat share less the percentage vote share) was very substantial at each contest: 20, 16 and 10 percentage points for the Conservatives at the first three elections respectively, and 21, 22 and 20 points for Labour at the last three. There is also a hint of partisan bias as the elections differ in the second-placed party's performance. In the first three elections second-placed Labour enjoyed an advantage in seat over vote share but that is not true for the Conservative party in the final set of three where it finished second. (For example, in 1987 Labour received 31.5 per cent of the votes cast but won 36.1

[6] Throughout this paper, we refer to the third party as the Liberal Democrats. In 1983 and 1987, two parties – Liberals and Social Democrat – contested the general elections as the Alliance, with an agreed single candidate in each constituency. The two parties merged in 1988 (see Crewe and King, 1995) and adopted their current name in 1989.

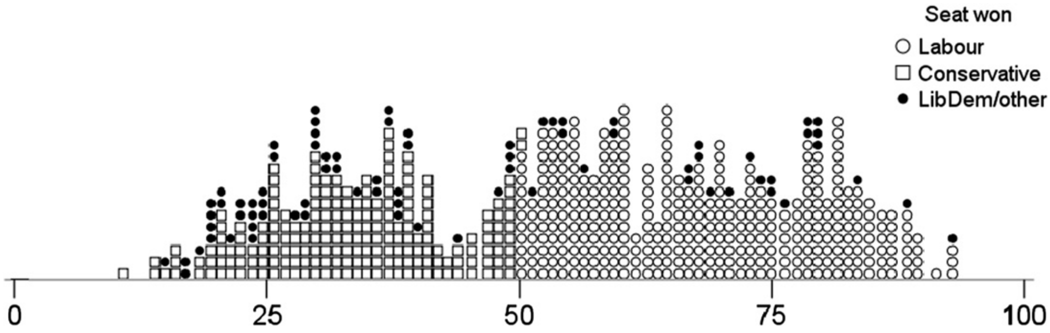

Fig. 2. Three players at 2005 British General Election.

per cent of the seats; in 2001 the Conservatives won the same share of the votes, but only 25.7 per cent of the seats.) In addition, Labour's winner's bonus was generally larger than the Conservatives'; in 2001 with 42.0 per cent of the votes Labour gained 64.4 per cent of the seats, whereas in 1992 with 42.8 per cent of the votes the Conservatives were allocated only 52.9 per cent of the seats. These figures suggest bias (or partisan asymmetry, or distorted representation) favouring Labour at each of the six contests. Substantial under-representation in the House of Commons was the experience of the Liberal Democrats at all six elections, however.

How might this asymmetry affecting all three parties be evaluated quantitatively? In answering that question, this paper presents a major extension of Brookes' method – rather than a further modification – to incorporate three-party situations. Brookes' method defines partisan bias as the difference between two parties in the number of seats they would win if they had a particular share of the votes cast. It then decomposes that bias into separate components including efficiency of vote distribution as well as effects separately produced by electorate size, turnout and minor parties' votes. Brookes (1960, 166) operationalises this decomposition by assuming 'a uniform percentage shift in support between the major parties'. We maintain that there are four important elements to the Brookes method. The first two are completely independent of any operationalising procedure: a quantitative measure of bias that can be partitioned into separate components; and formulae for the decomposition themselves. The other two elements – deriving a norm for comparison and estimating the expected number of seats for each party under certain scenarios (these are described in more detail in Borisyuk et al., 2008 and below) – are clearly integral to the way in which the method is operationalised. Brookes believed that uniform swing was the simplest assumption for the first of the latter pair, but our interpretation of his original procedure is that it merely operates to construct a '*norm for comparison*', which allows us to compare data from the actual election with this benchmark. It is not unusual, of course, for students of electoral system effects to want to derive some basis for considering the seat/vote curve. (See, for example, discussion in Blau (2001, 2004) about the use of the Edgeworth expansion of the normal distribution that retains the same mean, standard deviation and kurtosis as the actual frequency distribution of vote shares.) It is important to note that our interpretation does not offer a justification for using uniform swing as the basis for a counterfactual – 'what would happen if the election had been won by another party?'; it simply uses it as the means of establishing a reasonable norm, derived on rational grounds, against which the actual result can be compared. It is important, therefore, that our 'notionals' should be considered as technical steps that help in the necessary construction of the symmetrical multidimensional distribution that retains features of the actual electoral outcome and is independent from the size of electoral area (constituency). In short, we do not actually argue, or expect the reader to believe, that Labour gets 18.9% of the vote while the Liberal Democrats receive 42% at a general election but rather that this is merely part of a *technical construction* of the multidimensional distribution and permits us to compare the actual outcome with a norm for comparison.

In Brookes' original presentation bias is defined as the difference between the number of seats won by the leading party – *A* – at an election and the number that would be won by its main opponent – *B* – if *B* had obtained the same share of the votes. As demonstrated by the full algebra set out in the Appendix, therefore, if *A* obtains a larger share of the seats than *B* from the same share of the votes, then the positive bias favouring *A* is the inverse of the negative bias suffered by *B*.

This method is illustrated by considering the two-party percentage shares of votes cast at the 2005 British general election (i.e. [Conservative + Labour] = 100). Fig. 1a shows the vote share across 627 parliamentary constituencies[7] for the largest party *A* (Labour) and Fig. 1b presents party *B*'s share (Conservative). These distributions are necessarily the mirror image of each other.[8] Also indicated on both distributions is the overall vote share for each party; 52 per cent for Labour and 48 for the Conservatives.[9] (In both Figs. 1 and 2, each constituency is shown as a separate symbol.)

Brookes' method begins by asking what would happen to the allocation of seats if instead of coming second at the actual election (*AB* – the parties are listed in order according to their share of the votes won, with the largest first), party *B*

[7] There were 628 constituencies in Britain in 2005 but neither of the two major parties contested the Speaker's constituency.

[8] In effect the two-dimensional problem can be reduced to a single dimension. Correspondingly, for the three-party case we can present the results in two-dimensional space.

[9] These values differ from the mean values of the distribution (54% and 46% respectively) because of the unequal size of constituencies.

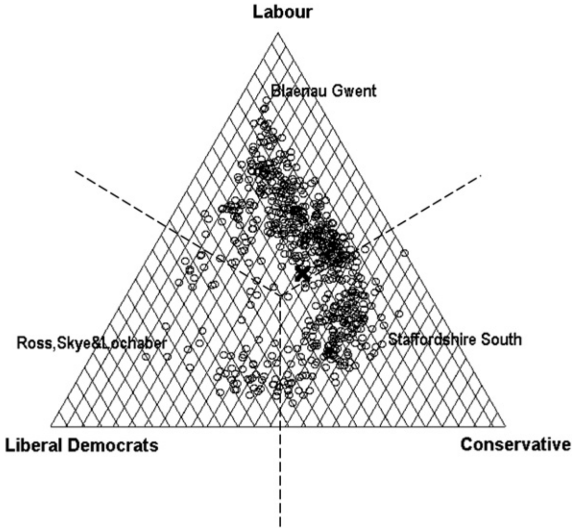

Fig. 3. Distribution of the three-party vote shares.

came first, receiving *A*'s vote share at the actual contest. Using the principle of reverse vote shares the method applies a uniform swing to each constituency to create a notional election result (the notional *BA* election) such that party *B* now wins 52 per cent of the two-party vote total. Following the application of uniform swing to the vote share in each of the 627 constituencies, party *B*'s distribution slides to the right such that its overall percentage of the votes becomes 52 per cent in the notional *BA* election (Fig. 1c).

These three figures represent the conventional understanding of the Brookes' method. However, another interpretation is possible. Fig. 1d shows the superposition of the distribution for party A at the actual election and the distribution for party B at the notional reverse vote share election (literally, a combination of Fig. 1a,c). This superposition that is critical to our interpretation of the Brookes method because it in effect constitutes the *norm for comparison* and retains many important features of the actual data.

Because Brookes' method focuses on the two largest parties only, all constituencies lying to the right of 50 per cent are 'won' by the respective party. Thus Fig. 1a shows the number of seats won by party *A* at the actual election (i.e. '*x*' in previous notation) and Fig. 1c shows the number of seats ('*y*') 'won' by party *B* at the notional election, *BA*.

In Brookes' original formulation – to which we are returning here – partisan bias towards party *A* is measured as the difference between the number of points to the right of 50 per cent in Fig. 1a and an *average* of the number of points to the right of 50 per cent in Fig. 1d. It is clear from the graphs that, in effect, Brookes' method compares the distribution of seats at the actual *AB* election (Fig. 1a) with the norm for comparison (Fig. 1d).[10]

[10] For superposition AB and BA, vote shares now has zero correlation with size of constituency and has symmetrical shape of distribution (a norm distribution). Because of zero correlation with constituency size, the mean of the distribution equals overall vote share.

Having set out the principles underpinning Brookes' original formulation, we use these as the foundation for the extension to the three-party situation.

Applying his algebra to the six general elections shown in Table 1 produces interesting results (Johnston et al., 1999, 2006). At the first two contests (1983 and 1987), which the Conservatives won with comfortable margins in votes cast over Labour (Table 1), there was a small total bias of below 15 seats favouring the Conservatives. Over the next four contests, however, the amount of partisan bias increased substantially, reaching a level of more than 100 seats and benefiting Labour rather than the Conservatives (Johnston et al., 2006); bias both increased substantially and changed direction.

However, these results treat the Liberal Democrats as a minor party only (its vote share in each constituency is unchanged in Brookes' procedure) and take no account of their substantial vote-winning capacity which was unmatched by the allocation of seats – although the party won 18.3 per cent of the votes it gained only 62 seats in 2005, just under 10 per cent of the total. At that election some constituencies had a clear three-party battle, whereas in others there was a two-party contest between the Liberal Democrats and one of the other two parties – with the other (i.e. either Conservative or Labour) being, in effect, a minor party.

The importance of the Liberal Democrats in redefining the nature of party competition in Britain is further illustrated from the distribution of Labour's share of the *two-party* (i.e. Labour + Conservative) vote in 2005 (Fig. 2), on which is also indicated which party won each seat. Some seats in the centre of the distribution – where Labour and the Conservatives got very similar shares of the two-party vote – were won by either the Liberal Democrats or one of the 'other' parties. Many more Liberal Democrat victories are towards the distribution's extremes, however, especially at the left-hand end, indicating constituencies where Labour got only a small share of the (Labour + Conservative) two-party vote, so that the 'real' contest was between the Conservatives and the Liberal Democrats.

This pattern in Fig. 2 complements that in Table 1; although the Liberal Democrats were the third-largest party, their share of the vote (averaging about 20 per cent across the six elections) was not evenly distributed across the constituencies. But, as Table 1 makes clear, they rarely gained sufficient votes in a constituency to win it, so that their seat share was incommensurate with their vote share. This was especially the case in 1983, when the Liberal Democrats were only two points behind Labour in the vote share but Labour won nine times more seats. The implication is that not only was the 1983 outcome slightly biased towards the Conservatives relative to Labour but also probably substantially more biased towards Labour relative to the Liberal Democrats. Similarly in 2005, whereas the Conservatives' vote share was 1.46 times that of the Liberal Democrats, the ratio for seats share was 3.18:1. These elements of the disproportionality clearly indicate the need for a bias measure which takes into account the vote and seat shares of all three parties rather than just the first two.

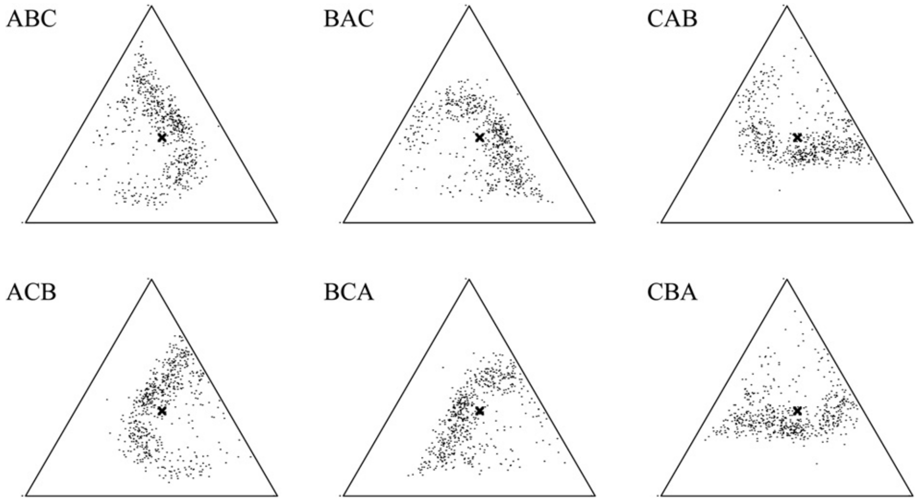

Fig. 4. Distribution of the three-party vote shares: ABC (actual), ACB, BAC, BCA, CAB, and CBA (notional elections).

3. Towards a partisan bias measure for the three-party case

In Brookes' procedure for evaluating partisan bias in the two-party case the establishment of a norm of comparison for the actual distribution of seats is central to the measurement of bias. This also applies with our proposed extension to the three-party case in which the bias towards a party is measured as the difference between the actual number of seats gained and a norm of comparison which is the expected unbiased number of seats that, on average, the three parties could win under similar conditions at a notional election; this is formally defined in the second section of the Appendix.

In Borisyuk et al. (2008) the expected norm was based on the actual election result plus just two notional elections. The first notional election saw the actual second-placed party awarded the same vote share as the actual first-placed party (i.e. the order of the election result was changed from *ABC* to *BAC*) whereas in the second the original third-placed party was given the vote share captured by the first-placed party at the actual election (i.e. *ABC* was converted to *CBA*). The actual number of seats won was thus compared with a norm comprising the mean number of seats gained by the leading party under three scenarios – the actual election and two notional elections. This ignored three other scenarios, and was the reason for the unresolved problems with the empirical applications noted above. This paper thus extends the approach to incorporate the *entire* set of possible outcomes (i.e. including the three other potential notional elections: *ACB*, *BCA* and *CAB*). This extension to the three-party case does that whilst retaining many of the basic principles underpinning the Brookes' original formulation.

In the two-party case the distribution of vote share can be depicted one-dimensionally (e.g. Figs. 1 and 2). Three-party vote share can be best captured by triangular graphs (for early proponents of this technique see Upton, 1976; Miller, 1977; Gudgin and Taylor, 1979; for a recent example that employs this method see Curtice and Firth, 2008). Fig. 3 shows the actual 2005 election result. The point for the national three-party vote share (39, 36, 25) is represented by a cross. The area inside the triangle is divided into three, each of which shows where the respective parties won seats. Where the lines intersect at the centre of the triangle the vote share for each of the three parties is 33.3 per cent. Points towards the peak of the triangle are constituencies where the largest party (in this case Labour) performed well. For example, in Blaenau Gwent, the Labour candidate received 86 per cent of the three-party vote, leaving just 6 per cent for the Conservative and 11 per cent for the Liberal Democrats.[11] The constituency of Staffordshire South occupies an entirely different part of the triangle, located towards the bottom right corner. Here, the Conservatives dominated the three-party vote, winning a 62 per cent share comfortably ahead of Labour on 21 per cent and Liberal Democrats on 17 per cent.[12] Finally, the Liberal Democrats performed exceptionally well (with 70 per cent) in their party leader's Scottish constituency of Ross, Skye and Lochaber.

In constructing the norm for comparison for this extended procedure we have three parties, *A*, *B* and *C* with overall vote shares, α, β, and γ respectively, where $(\alpha + \beta + \gamma = 100)$. The principle is to consider all six possible combinations assigning those three values to parties *A*, *B* and *C* – viz. *ABC* (actual election), *ACB*, *BAC*, *BCA*, *CAB*, and

[11] Blaenau Gwent was won by a 'fourth-party' candidate (an independent) with 58 per cent of the votes cast so that the three figures quoted here refer to the remaining 38.9 per cent of the votes won by the three main parties. Blaenau Gwent is thus an excellent example of a seat where the impact of minor parties is felt – as discussed in the section below on decomposing bias.

[12] South Staffordshire is included in our data although the election in that particular constituency was held some six weeks after the general election owing to the death of one of the original candidates.

CBA. This is shown on six triangular graphs (Fig. 4). The first (*ABC*) repeats what was shown in Fig. 3 while the triangle *ACB* below it shows the notional election where the positions of the second- and third-placed parties, *B* and *C*, have been reversed but that for the first party, *A*, is unchanged. It is important to note that the top of each triangle always shows the largest party, the right-hand side shows the second-placed party while the third-placed party is shown on the left-hand side.

Fig. 5 shows the superposition of these six configurations to create what will be used as the 'norm of comparison'. This procedure is a precise extension of what was done in the two-party case, where Fig. 1d represented the superposition of Fig. 1a and c. Once again, the area inside the triangle is divided into three sections The top section, for example, shows the total number of seats that would be won by the largest party (with vote share of α) across the six elections (the actual plus the five notional ones). Likewise, the section on the right shows seats won by whichever party came second (vote share β) while that on the left represents seats won by the third party with national vote share γ. The next stage of the process compares the actual number of seats won by each party with the expected unbiased number of seats derived from construction of the norm of comparison.

The distribution of points in Fig. 5 clearly differs from a Gaussian/normal distribution because the points are not at their densest around the overall national vote share and instead form their own distinctive shape. This means that we cannot calculate expected values for the bias equations by simple reference to a normal distribution. Instead, our approach considers all points in that scatter plot and identifies the number in each patterned area.[13] The estimate for the unbiased number of seats is 1/6th of the number of dots within each corresponding patterned area (equations (10)–(12) in the Appendix). We use this fraction because these dots represent the superposition of *six* scenarios and altogether there are *six times as many* dots as seats contested at the actual election (i.e. each constituency appears six times). Technically, we get the same outcome by separately six scenarios considering, calculating the values for each of them, and averaging the results.

In the two-party case, total bias (as defined by Johnston et al., 1999 and Blau, 2001) may be either negative or positive dependent upon the direction of bias towards or against the leading party. For three-party competition, however, there is no simple dichotomy and theoretically it may be in one of six possible directions. Three of these directions

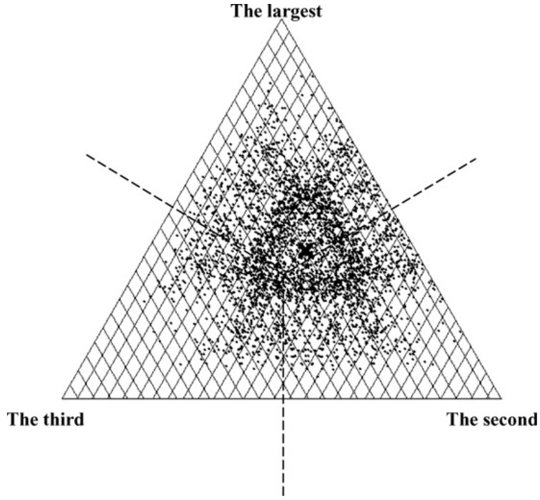

Fig. 5. The superposition ABC + ACB + BAC + BCA + CAB + CBA.

depict the situation when just one party has a positive bias while the remaining two parties have a negative bias. Three others are when two parties show a positive bias of seats while a single-party experiences negative bias. The resulting equation for total bias is in equation (13).

Applying equations (10)–(13) produces our estimates of bias at the 2005 British general election using this new procedure (Table 2). The six blocks give the results of the actual election (*ABC*) and those for the five notional outcomes of that election, showing for each party its share of the votes and the number of seats it would win under that scenario. Thus, for example, the second block shows that if the positions of the Conservatives and the Liberal Democrats were reversed (i.e. notional election *ACB*), the latter would just overtake the former in seats won – compared to a threefold difference between the two at the actual election; the third block shows that if the Labour and Conservative positions were reversed (i.e. *BAC*), nevertheless Labour would still have 59 more seats than the notional victor.

Overall, the results of the six separate outcomes suggest that Labour was the major beneficiary from partisan bias; not only was it the outright winner in terms of seats allocated in the two contests when it was placed first (*ABC*, *ACB*), but it also gained most seats in the two where it was placed second (*BAC*, *CAB*); indeed, in the latter it had an absolute majority over all other parties.[14] The Liberal Democrats, on the other hand, experienced substantial negative partisan bias: in the two contests in which they are placed first they come a poor second in the allocation of seats in one (*CAB*) and third in the other (*CBA*).

The figures at the foot of Table 2 confirm this; total bias is calculated for each party as the number of seats it obtained in the actual election minus one-sixth of the sum

[13] A possible alternative would be to use Monte Carlo simulation such that points/values should be randomly drawn with a sample size equal to the number of seats at the actual election. The number of points located within each of the three patterned areas may be used as an estimate for the unbiased number of seats won. Samples of the same size could be drawn randomly from the distribution repeatedly and points/values calculated for each re-sample. Taking an average of these sample results would give an approximation for the unbiased number of seats for each party (the first-, second- and third-parties). This approach has the possible additional advantage that we could calculate not only expected values but also errors and confidence intervals.

[14] Which would remain the case even if the 18 Northern Ireland MPs were included.

Table 2
Measuring three-party bias: 2005 General Election.

	Three-party Vote Share, %	Number of Seats
Election: ABC (actual)		
A (Labour)	39	355
B (Conservative)	36	198
C (Liberal Democrat)	25	62
Other		12
Election: ACB (notional)		
A (Labour)	39	374
B (Conservative)	25	112
C (Liberal Democrat)	36	129
Other		12
Election: BAC (notional)		
A (Labour)	36	308
B (Conservative)	39	249
C (Liberal Democrat)	25	57
Other	13	
Election: BCA (notional)		
A (Labour)	25	178
B (Conservative)	39	304
C (Liberal Democrat)	36	130
Other		15
Election: CAB (notional)		
A (Labour)	36	349
B (Conservative)	25	93
C (Liberal Democrat)	39	172
Other		13
Election: CBA (notional)		
A (Labour)	25	180
B (Conservative)	36	255
C (Liberal Democrat)	39	178
Other	14	
Bias towards each party		
A: $355 - [(355 + 374 + 249 + 304 + 172 + 178)/6] = 83.0$		
B: $198 - [(198 + 129 + 308 + 130 + 349 + 255)/6] = -30.2$		
C: $62 - [(62 + 112 + 57 + 178 + 93 + 180)/6] = -51.7$		
Total bias: $abs(83.0) + abs(-30.2) + abs(-51.7) = 164.9$,		

where '*abs*(.)' is the absolute value, i.e. the magnitude of a number irrespective of its sign.

of the seats it would win across all six scenarios. A positive partisan bias of 83.0 seats favours Labour, with negative biases of 30.2 and 51.7 respectively for the Conservatives and Liberal Democrats. The overall bias (i.e. the sum of those three values, irrespective of sign) is thus just under 165 seats – in a House of Commons with 627 British members (i.e. excluding the Speaker). Removing that bias would involve on average changing the result in one-seventh of the country's constituencies (83 of 627). This estimate of the pro-Labour bias is commensurate with analyses using the two-party measure (e.g. Johnston et al., 2006) but what that application could not show amongst other things was that the largest proportion of this was delivered to Labour at the Liberal Democrats' expense.

4. Decomposing the three-party bias estimates

A great strength of Brookes' method is that it not only estimates total bias in a readily-appreciated metric but also decomposes that bias into one of four categories. The first of these has been labelled differently (gerrymander, vote distribution, efficiency) but we prefer the term 'geography' (denoted by 'G' in the Appendix equations). It shows the degree of asymmetry in the distribution of partisan voting strength across constituencies for the parties being considered (Gudgin and Taylor, 1979 address the two-party situation). In a 'first-past-the-post' voting system a party performs well in the translation of votes into seats (in terms of the geography of its vote across the constituencies) by winning small and losing big; it should avoid accumulating surplus votes in constituencies it wins (i.e. those additional to the number required to win a constituency) and if it cannot win a constituency then it is best to attract as few as votes as possible there since these are literally 'wasted' (see Johnston et al., 2001).

The second component within electoral bias stems from malapportionment, i.e. differences in electorate size across constituencies (denoted by component 'E'). A party that is stronger in constituencies with relatively small electorates will tend to perform better (as shown by comparing its percentage of all votes cast and percentage of seats) than one which performs better in the larger constituencies. The level of abstention ('A') is the third component and becomes relevant when one party wins its seats where electoral turnout is low compared with its rivals whose victories are achieved in constituencies with on average higher turnouts. Finally, there is the minor party effect, component 'M'; here it is restricted to those parties outside the main three.

Brookes' algebra enables the contribution of each of these four components (G, E, A and M) to be calculated, in the same metric as the total bias. In addition, there are also interactions across each combination of bias components – the combined impact of abstentions and electorate size is an example of two-way interactions; there are also three- and four-way interactions. These are not separately calculated here; instead we just report the net interaction, which is the difference between the total bias for each party (equations (10)–(12)) and the sum of four components identified here (equations (15) and (16)).

The results of these calculations for the 2005 election are in Table 3 and allow an evaluation of the sources of the bias either favouring or disadvantaging each of the parties.[15] Thus, for example, of the total bias of 83.0 seats towards Labour the four blocks below that figure show that just under half (40.6) resulted from differences in the geography of the three parties' support across the constituencies. A further 10.5 and 16.2 seats (13 and 20 per cent of the total respectively) came from the electorate size (E) and abstentions (A) components, and a small amount (2.5 seats) from the impact of minor party (M) votes. There was also a substantial interaction effect, undoubtedly reflecting Labour's greater electoral strength in both the smaller constituencies and those with the lower turnouts (see Johnston et al., 2006).

Labour's positive bias resulting from the G component was complemented by the negative bias of −45.8 for this component for the Liberal Democrats, which accounted for

15 The SPSS code for performing these calculations may be obtained from the authors.

Table 3
2005 British General Election: components of three-party bias.

Party	Labour	Conservative	LD
Overall result			
Three-party vote share (%)	39	36	25
Seats won	355	198	62
Expected unbiased number of seats	272.0	228.2	113.7
Bias (seats won – expected)	83.0	−30.2	−51.7
Decomposition of bias			
Vote distribution (G)	40.6	−4.5	−45.8
From: non-symmetry between other parties	−4.2	−8.6	1.2
… bigger rival*	2.5	−24.3	−32.5
… smaller rival	42.3	28.4	−14.4
Electorate size (E)	10.5	−11.5	−2.6
From: non-symmetry between other parties	0.3	−0.5	−0.1
.bigger rival	7.1	−9.4	−3.9
.smaller rival	3.0	−1.6	1.5
Abstention (A)	16.2	−13.9	−9.5
From: non-symmetry between other parties	0.6	−0.6	−0.1
.bigger rival	10.4	−12.4	−9.9
.smaller rival	5.4	−0.9	−0.4
Minor party votes(M)	2.5	−2.7	−0.5
From: non-symmetry between other parties	0.1	0.0	0.0
.bigger rival	1.8	−2.2	−0.9
.smaller rival	2.4	−0.4	0.4
Net interactions	13.2	2.5	6.7

* Note: The bigger rival for Labour is Conservative and the smaller rival is Liberal Democrats. For the Conservatives, the bigger and the smaller rival parties are Labour and the Liberal Democrats respectively. For the Liberal Democrats, they are Labour and the Conservatives.

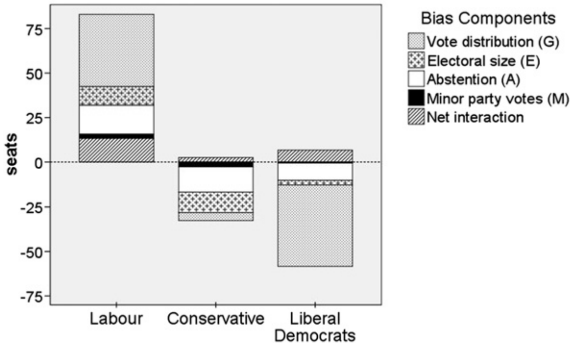

Fig. 6. 2005 British General Election: components of bias.

some 89 per cent of the total negative bias which that party experienced: the geography of its support operated very much to its disadvantage (largely, too many wasted votes in seats that it lost). For the Conservatives, on the other hand, its negative overall bias stemmed largely from the E and A components: compared to Labour especially, its support was over-concentrated in the constituencies with larger electorates and higher turnouts. Fig. 6 clearly shows the overall distribution of electoral bias at the 2005 general election and the relative contributions to that bias provided by the different components.

As well as identifying the various sources of each party's bias at the 2005 general election – which are dominated by the geography of support for Labour (positively) and Liberal Democrats (negatively) – it is also possible to identify whether each component operated at a similar scale in all cross-pair comparisons. This is shown by the three sets of data in each block of Table 3. Regarding the abstentions (A) component, for example, Labour's positive bias of 16.2 seats derived from this source, 10.4 came from the contrast with its larger rival (the Conservatives) and 5.4 from its smaller rival (Liberal Democrats): there were greater differences between Labour and the Conservatives in the average number of abstentions in the constituencies that they would win across the six contests than between Labour and the Liberal Democrats. The same is true with regards to electorate size: there is a greater difference between Labour and the Conservatives in the average electorate of seats won than between Labour and the Liberal Democrats.

Most of the results of this decomposition show how the source of each party's bias is split between its two rivals. Thus, for example, most of Labour's positive outcome from the G bias is to the detriment of the Liberal Democrats. Indeed, the 42.3 seat advantage shown there is larger than the overall benefit to Labour of 40.6 because of the asymmetry in the geography of support for its two rivals. Liberal Democrats, on the other hand, are substantially disadvantaged in the geography of their support not only relative to Labour (a negative bias of 32.5 seats) but also to the Conservatives (14.4 seats). This produces the asymmetry just noted because the Conservatives have a positive G bias relative to the Liberal Democrats (28.4 seats) which more than counters the negative bias (24.3 seats) it suffers relative to Labour.

The geography of Labour's support in 2005 was more efficient than that of either of the other two parties'; the Conservatives' was less efficient than Labour's but more efficient than the Liberal Democrats'; and the Liberal Democrats' was less efficient than that of either of the other two – they wasted too many votes won in the 'wrong places' where their chances of victory were slight. The last point is probably typical of third parties that contest constituencies everywhere; in this case, by winning only one in five votes the Liberal Democrats were almost bound to suffer from a poor vote distribution unless much of their vote-winning was highly targeted on relatively few seats.[16] It is also worth noting that the bias components regarding the votes for minor parties are small. This is to be expected given that the procedure is specifically designed for the three-party case and 'others' captured just 12 of the remaining seats at the 2005 election with 7.9 per cent of the votes. Plaid Cymru contested only the 40 Welsh constituencies, winning three of them (its share of the votes cast in Wales only was 12.5 per cent, which yielded 7.5 per cent of the 40 seats); the SNP contested all 59 Scottish constituencies, obtaining 17.6 per cent of the votes there and winning 6 (10.2 per cent) of the seats; the Respect party contested 26 constituencies in Britain, winning one; and two independents were elected.

[16] Which would be counter-productive. Unless it builds a base in a wider range of constituencies the party can never expect to be a potential party of (single-party) government.

5. Comparing the two-party and three-party analyses: Great Britain 1983–2005

This search for a new method to measure and decompose electoral bias was stimulated by the results of recent British general elections, a country which now has a party system quite different to that studied by Brookes when he first developed his procedure. (New Zealand's party system in the 1950s was even more dominated by two parties than was Britain's prior to 1970.) Although others subsequently modified his procedure to take account of the growing impact of a third party this did not fully incorporate it into the analysis and a radical redesign, based on Brookes' principles and procedures, was clearly needed. Having completed such a redesign, therefore, this section compares the two methods applied to British general elections from 1983 to 2005.

The relevant comparisons are shown in Table 4, where the two-party bias is calculated according to Mortimore's (1992) modification of Brookes' method which compares the two parties as if they each achieved the vote share obtained by the winning party *at* the actual election rather than the comparison as if they had each won half of the two-party vote total, as used by Johnston et al. (2001, 2006). There are some obvious substantial differences relating to specific elections. In 1983, for example, whereas the two-party method produces an estimated total bias of only 11 seats the three-party method comes out at 176. The small positive bias towards the Conservative (six seats) now becomes a negative bias of nine seats but the main difference lies in the large pro-Labour bias of 89 seats and the large negative disadvantage of 78 seats for the Liberal Democrats. Once the third party is incorporated into the analysis, the narrow Labour lead over the Liberal Democrats in vote share (Table 1) but the large disparity in the seat allocation between the two indicates a very substantial bias favouring the former. It is unsurprising to find rather large difference in bias estimates comparing the two- and three-party cases alongside one another. The two-party method, adapted to meet the demands of greater three-party competition, could only assign bias within the system as a whole to one or other of the two main parties whereas the three-party method is free to incorporate the third party explicitly.

Using the three-party method the least biased election of the seven is 1997 (Fig. 7; the 2010 general election is included here), when Labour won an electoral landslide.

Table 4
British General Elections 1983–2005: Comparing two- and three-party methods.

Party Method	Conservative		Labour		Liberal Democrat		Total Bias	
	2prt	3prt	2prt	3prt	2prt	3prt	2prt	3prt
1983	5.5	−8.7	−5.5	89.3	–	−78.0	11.0	176.0
1987	6.5	4.5	−6.5	60.8	–	−64.2	13.0	129.5
1992	−16.5	−11.3	16.5	55.8	–	−41.7	33.0	108.8
1997	−31.0	−5.0	31.0	14.5	–	−7.8	62.0	27.3
2001	−48.0	−35.2	48.0	56.5	–	−19.8	96.0	111.5
2005	−50.0	−30.2	50.0	83.0	–	−51.7	100.0	164.8

2prt – two-party method; 3prt – three-party method.

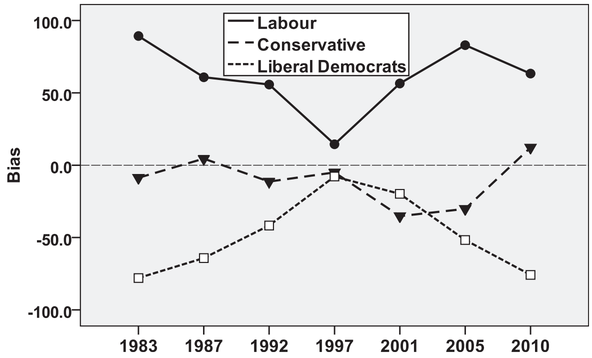

Fig. 7. Three-Party Bias at British General Elections, 1983–2010.

The Brookes' two-party method for this election shows total bias as 62 seats with a pro-Labour bias of 31 seats. By contrast, the three-party method calculates total bias at less than half that figure, with only a modest pro-Labour bias and rather small negative biases for the Conservatives and Liberal Democrats. This confirms our interpretation of the three-party method's superiority. The 1997 election result was certainly disproportional – Labour won more than two-thirds of the seats (418 of 641) with just 44 per cent of the vote – but the decomposition suggests that it was not particularly biased. The Conservative party won just 31 per cent of the votes and 26 per cent of the seats while the Liberal Democrats demonstrated the success of their seat-targeting strategy then: their overall vote share fell slightly (from 18 per cent of the three-party vote in 1992 to 17 in 1997) but they more than doubled their share of seats (20 of 634 in 1992 and 46 of 641 in 1997: Table 1). At subsequent elections, the Liberal Democrats' share of the seats increased by less than the increase in their share of the votes, thus increasing the partisan bias operating against them after reducing it between 1992 and 1997.

Furthermore, at the 1997 election – much more so than at either 1992 or 2001 – both Labour and the Liberal Democrats reduced their wasted vote totals by their connivance at tactical voting practices, whereby, for example, in seats where the Labour candidate was likely to come third many putative Labour voters transferred their support to the Liberal Democrat candidate to try and unseat an incumbent Conservative; complementing this, many Liberal Democrat supporters in seats where they were likely to come third voted Labour instead. As Curtice and Steed (1997, 309) noted, 'voters exhibited a striking tendency to opt for whichever of the two opposition parties appeared best placed to defeat the Conservatives locally'. The 1997 general election outcome was certainly highly disproportional but because both Labour and Liberal Democrats each had an effective geographical distribution of their vote, the extent of electoral bias was relatively low.

6. Conclusions

The presence of partisan bias in single-member district, plurality electoral systems is a consequence of several aspects of a country's electoral geography (Johnston and Pattie, 2006) – and can be stimulated by cartographic practices such as malapportionment and gerrymandering. Even where such practices are absent, however, and districting is undertaken by non-partisan bodies (as in the UK), nevertheless bias may be produced because of the geographies underpinning that neutral boundary-drawing process (Gudgin and Taylor, 1979).

Of the various approaches available, that developed by Brookes and subsequently modified in a number of ways, has proved valuable in understanding how the translation of votes into seats operates in the British electoral system. There has been one very important drawback, however: the method, despite the subsequent modifications, treats the British electoral situation as a two-party system, and the role of the third party – which has won on average one-fifth of all votes cast at the last six general elections – is only marginal to the bias calculation. As such, the illumination provided by the Brookes' method has been only partial – indicative of the size and direction of the bias, but not conclusive.

To address that deficiency, this paper has redesigned Brookes' approach to make it fit for the analysis of a three-party situation. Its foundational principles have been retained, but the algebra rewritten to represent the presence of three major parties. Application of that redesigned procedure to the 2005 British general election has adequately illustrated the substantial increase in understanding the nature of partisan bias in that outcome as it differentially impacted upon the three parties, as has a brief evaluation of its comparative performance to the data for a six-election sequence compared to the original Brookes' formulation.[17] As such, this paper has presented a methodology with wide potential applicability in analyses of the translation of votes into seats in situations where three parties each receive a sizeable percentage of votes.

Appendix; The algebra

The basic equations for a two-party system

We prefer to use a different notation to Brookes because of the added complexity introduced by dealing with three rather than just two parties. For all subsequent formulae the subscript relates to the party under consideration while superscripts describe the finishing order for the parties. Thus, in the two-party situation $\boldsymbol{seat_A^{AB}}(\alpha)$ identifies seats won by party A at the election where A is the leading party with α share of the two-party vote and $\boldsymbol{seat_B^{BA}}(\alpha)$ identifies seats won by party B at the notional election where B is the leading party with α share of the two-party vote.

Bias towards the leading party is set out by Brookes as:

Bias to party A is

$$bias_{\mathbf{A}}(a) = x - (x+y)/2 = (x-y)/2, \tag{1}$$

which is simply the negative of bias towards its rival, B:

$$bias_{\mathbf{B}}(a) = y - (x+y)/2 = (y-x)/2. \tag{2}$$

In our revised notation

$$\begin{aligned} bias_{\mathbf{A}}(\boldsymbol{\alpha}) &= seat_{\boldsymbol{A}}^{\boldsymbol{AB}}(\boldsymbol{\alpha}) - \left[seat_{\boldsymbol{A}}^{\boldsymbol{AB}}(\boldsymbol{\alpha}) + seat_{\boldsymbol{B}}^{\boldsymbol{BA}}(\boldsymbol{\alpha})\right]/2 \\ &= \left[seat_{\boldsymbol{A}}^{\boldsymbol{AB}}(\boldsymbol{\alpha}) - seat_{\boldsymbol{B}}^{\boldsymbol{BA}}(\boldsymbol{\alpha})\right]/2 \end{aligned} \tag{3}$$

Bias to the second-placed party (B) is then

$$\begin{aligned} bias_{\boldsymbol{B}}(1-\boldsymbol{\alpha}) &= seat_{\boldsymbol{B}}^{\boldsymbol{AB}}(1-\boldsymbol{\alpha}) - \left[seat_{\boldsymbol{B}}^{\boldsymbol{AB}}(1-\boldsymbol{\alpha}) \right. \\ &\quad \left. + seat_{\boldsymbol{A}}^{\boldsymbol{BA}}(1-\boldsymbol{\alpha})\right]/2 \\ &= \left[seat_{\boldsymbol{B}}^{\boldsymbol{AB}}(1-\boldsymbol{\alpha}) - seat_{\boldsymbol{A}}^{\boldsymbol{BA}}(1-\boldsymbol{\alpha})\right]/2 \end{aligned} \tag{4}$$

If we assume that minor parties win no seats and N equals the total number of seats then we can re-write formula 4 as

$$\begin{aligned} bias_{\boldsymbol{B}}(1-\boldsymbol{\alpha}) &= \left\{\left[N - seat_{\boldsymbol{A}}^{\boldsymbol{AB}}(\boldsymbol{\alpha})\right] - \left[N - seat_{\boldsymbol{B}}^{\boldsymbol{BA}}(\boldsymbol{\alpha})\right]\right\}/2 \\ &= \left[seat_{\boldsymbol{B}}^{\boldsymbol{BA}}(\boldsymbol{\alpha}) - seat_{\boldsymbol{A}}^{\boldsymbol{AB}}(\boldsymbol{\alpha})\right]/2 \end{aligned} \tag{5}$$

which is simply the negative of bias towards its rival, A.

Moreover, we can specify total electoral bias for the two-party case as:

$$\begin{aligned} total\_bias^{AB}(\alpha, 1-\alpha) &= |bias_A(\alpha)| + |bias_B(1-\alpha)| \\ &= \left|seat_A^{AB}(\alpha) - seat_B^{BA}(\alpha)\right|. \end{aligned} \tag{6}$$

This figure is the sum of the absolute values generated by (3) and (5) and indicates the total amount of bias generated by the system given the actual election outcome and its translation into the notional election.

Extending the equations to the three-party case

For the three-party (A, B, C with vote shares α, β, and γ respectively) situation, with the expected norm established, bias for each of the parties is stated as:

$$bias_A(\alpha) = seat_A^{ABC}(\alpha) - expected\_norm\left(\alpha|_{given\_distribution}\right) \tag{7}$$

$$bias_B(\beta) = seat_B^{ABC}(\beta) - expected\_norm\left(\beta|_{given\_distribution}\right) \tag{8}$$

$$bias_C(\gamma) = seat_C^{ABC}(\gamma) - expected\_norm\left(\gamma|_{given\_distribution}\right) \tag{9}$$

The procedure adopted elaborates further on equations (7)–(9). When calculating bias towards party A at the actual election we take the actual number of seats won by A at the election and subtract the average of the number of seats that would be won by the party with vote share α across the six contests. In similar fashion the bias towards party B is calculated with reference to the seats won with vote share β, and bias affecting party C is calculated by using vote share γ. Formally:

Decomposing the three-party bias

Total electoral bias is defined here as a sum of the absolute bias values for the three parties:

[17] Prior to the 2010 UK general election it was argued by some that the review of parliamentary constituency boundaries had apparently failed to address the extent of electoral bias that favoured Labour. Interested readers may wish to consult Borisyuk et al. (2010) for a discussion of this issue.

$$
\begin{aligned}
bias_A(\alpha) &= actual\_seats_A^{ABC} - norm\_seats\left(\alpha|_{given\_distribution}\right) \\
&= seat_A^{ABC} - \frac{seat_A^{ABC} + seat_A^{ACB} + seat_B^{BAC} + seat_B^{BCA} + seat_C^{CAB} + seat_C^{CBA}}{6};
\end{aligned} \tag{10}
$$

$$
bias_B(\beta) = seat_B^{ABC} - \frac{seat_B^{ABC} + seat_B^{CBA} + seat_A^{BAC} + seat_A^{CAB} + seat_C^{ACB} + seat_C^{BCA}}{6}; \tag{11}
$$

$$
bias_C(\gamma) = seat_C^{ABC} - \frac{seat_C^{ABC} + seat_C^{BAC} + seat_A^{BCA} + seat_A^{CBA} + seat_B^{ACB} + seat_B^{CAB}}{6}. \tag{12}
$$

$$
total\_bias^{ABC}(\alpha,\beta,\gamma) = |bias_A(\alpha)| + |bias_B(\beta)| + |bias_C(\gamma)|. \tag{13}
$$

Brookes' algebra enables the contribution of each of these four components (geography – G; electorate size – E; abstentions – A; and minor parties – M) to be calculated, in the same metric as the total bias. In order to achieve that for the three-party case being developed here, we rearrange formulae (10)–(12) such that bias towards party A, for example, is:

$$
\begin{aligned}
bias_A(\alpha) = &\frac{seat_A^{ABC} - seat_A^{ACB}}{6} + \frac{seat_A^{ABC} - seat_B^{BAC}}{6} \\
&+ \frac{seat_A^{ABC} - seat_B^{BCA}}{6} + \frac{seat_A^{ABC} - seat_C^{CAB}}{6} \\
&+ \frac{seat_A^{ABC} - seat_C^{CBA}}{6}
\end{aligned} \tag{14}
$$

The notation used here replicates that used for the two-party method, as set out in Brookes' (1960) original formulation with the addition of the use of subscripts and superscripts as described earlier. We also omit here for the sake of simplicity references to vote shares, α, β and γ. Hence:

$seat_A^{ABC}$ – number of seats won by party A at actual election;
$seat_A^{ACB}$ – number of seats won by party A under ACB scenario;
$seat_B^{BAC}, seat_B^{BCA}$ – number of seats won by party B under BAC and BCA scenarios respectively;
$seat_C^{CAB}, seat_C^{CBA}$ – number of seats won by party C under CAB and CBA scenarios respectively;
$P_A^{ABC}, P_A^{ACB}, P_B^{BAC}, P_B^{BCA}, P_C^{CAB}, P_C^{CBA}$ – combined vote totals for three major parties where corresponding party won seats under particular scenarios;
$R_A^{ABC}, R_A^{ACB}, R_B^{BAC}, R_B^{BCA}, R_C^{CAB}, R_C^{CBA}$ – average electorate;
$D_A^{ABC}, D_A^{ACB}, D_B^{BAC}, D_B^{BCA}, D_C^{CAB}, D_C^{CBA}$ – average number of abstentions;
$U_A^{ABC}, U_A^{ACB}, U_B^{BAC}, U_B^{BCA}, U_C^{CAB}, U_C^{CBA}$ – average number of minor party votes.

We can now specify the formulae for the four components of bias, in this case towards party A.

Decomposition of bias towards party B and party C yields similar formulae; for example, the decomposition of the

$$
\begin{aligned}
G_{toward\_A} = &\frac{seat_A^{ACB}}{6}\left(\frac{P_A^{ABC}}{P_A^{ACB}} - 1\right) + \frac{seat_B^{BAC}}{6}\left(\frac{P_A^{ABC}}{P_B^{BAC}} - 1\right) + \frac{seat_B^{BCA}}{6}\left(\frac{P_A^{ABC}}{P_B^{BCA}} - 1\right) + \frac{seat_C^{CAB}}{6}\left(\frac{P_A^{ABC}}{P_C^{CAB}} - 1\right) + \frac{seat_C^{CBA}}{6}\left(\frac{P_A^{ABC}}{P_C^{CBA}} - 1\right) \\
E_{toward\_A} = &\frac{seat_A^{ACB}}{6}\left(\frac{R_A^{ACB}}{R_A^{ABC}} - 1\right) + \frac{seat_B^{BAC}}{6}\left(\frac{R_B^{BAC}}{R_A^{ABC}} - 1\right) + \frac{seat_B^{BCA}}{6}\left(\frac{R_B^{BCA}}{R_A^{ABC}} - 1\right) + \frac{seat_C^{CAB}}{6}\left(\frac{R_C^{CAB}}{R_A^{ABC}} - 1\right) + \frac{seat_C^{CBA}}{6}\left(\frac{R_C^{CBA}}{R_A^{ABC}} - 1\right) \\
A_{toward\_A} = &\frac{seat_A^{ACB}}{6}\left(\frac{R_A^{ABC}}{R_A^{ABC} - D_A^{ABC}}\left(\frac{D_A^{ABC}}{R_A^{ABC}} - \frac{D_A^{ACB}}{R_A^{ACB}}\right)\right) + \frac{seat_B^{BAC}}{6}\left(\frac{R_A^{ABC}}{R_A^{ABC} - D_A^{ABC}}\left(\frac{D_A^{ABC}}{R_A^{ABC}} - \frac{D_B^{BAC}}{R_B^{BAC}}\right)\right) \\
&+ \frac{seat_B^{BCA}}{6}\left[\frac{R_A^{ABC}}{R_A^{ABC} - D_A^{ABC}}\left(\frac{D_A^{ABC}}{R_A^{ABC}} - \frac{D_B^{BCA}}{R_B^{BCA}}\right)\right] + \frac{seat_C^{CAB}}{6}\left[\frac{R_A^{ABC}}{R_A^{ABC} - D_A^{ABC}}\left(\frac{D_A^{ABC}}{R_A^{ABC}} - \frac{D_C^{CAB}}{R_C^{CAB}}\right)\right] + \frac{seat_C^{CBA}}{6}\left[\frac{R_A^{ABC}}{R_A^{ABC} - D_A^{ABC}}\left(\frac{D_A^{ABC}}{R_A^{ABC}} - \frac{D_C^{CBA}}{R_C^{CBA}}\right)\right] \\
M_{toward\_A} = &\frac{seat_A^{ACB}}{6}\left[\frac{R_A^{ABC}}{R_A^{ABC} - U_A^{ABC}}\left(\frac{U_A^{ABC}}{R_A^{ABC}} - \frac{U_A^{ACB}}{R_A^{ACB}}\right)\right] + \frac{seat_B^{BAC}}{6}\left[\frac{R_A^{ABC}}{R_A^{ABC} - U_A^{ABC}}\left(\frac{U_A^{ABC}}{R_A^{ABC}} - \frac{U_B^{BAC}}{R_B^{BAC}}\right)\right] \\
&+ \frac{seat_B^{BCA}}{6}\left[\frac{R_A^{ABC}}{R_A^{ABC} - U_A^{ABC}}\left(\frac{U_A^{ABC}}{R_A^{ABC}} - \frac{U_B^{BCA}}{R_B^{BCA}}\right)\right] + \frac{seat_C^{CAB}}{6}\left[\frac{R_A^{ABC}}{R_A^{ABC} - U_A^{ABC}}\left(\frac{U_A^{ABC}}{R_A^{ABC}} - \frac{U_C^{CAB}}{R_C^{CAB}}\right)\right] + \frac{seat_C^{CBA}}{6}\left[\frac{R_A^{ABC}}{R_A^{ABC} - U_A^{ABC}}\left(\frac{U_A^{ABC}}{R_A^{ABC}} - \frac{U_C^{CBA}}{R_C^{CBA}}\right)\right]
\end{aligned} \tag{15}
$$

component relating to the electorate size effect relevant to party *C* would read:

$$E_{toward\_C} = \frac{seat_C^{BAC}}{6}\left(\frac{R_C^{BAC}}{R_C^{ABC}} - 1\right) + \frac{seat_A^{BCA}}{6}\left(\frac{R_A^{BCA}}{R_C^{ABC}} - 1\right) + \frac{seat_A^{CBA}}{6}\left(\frac{R_A^{CBA}}{R_C^{ABC}} - 1\right) + \frac{seat_B^{ACB}}{6}\left(\frac{R_B^{ACB}}{R_C^{ABC}} - 1\right) + \frac{seat_B^{CAB}}{6}\left(\frac{R_B^{CAB}}{R_C^{ABC}} - 1\right) \quad (16)$$

This compares the actual position of the third party *C* with that of the third (in terms of overall vote share) party under each of the five notional election scenarios.

There are also interactions between these four components – the combined impact of both abstentions and electorate size, for example. These (which can be two-way, three-way, and four-way) are not separately calculated here; the net interaction is simply reported, calculated as the difference between the total bias figure for each party (Table 2) and the sum of the four components calculated using the formulae above.

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