Components of Partisan Bias Originating from Single-Member Districts in Multi-Party Systems: The Case of Mexico*

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Abstract

We extend the measurement of the components of partisan bias—i.e., undue advantage conferred to some party in the conversion of votes into legislative seats—to single-member district systems in the presence of multiple parties. Methods to estimate the contributions to partisan bias from malapportionment, boundary delimitations, and turnout are limited to two-party competition. In order to assess the spatial dimension of multi-party elections, we propose an empirical procedure combining three existing approaches. We apply the proposed method to the study of recent national lower chamber elections in Mexico. Analysis reveals advantageous, albeit modest, partisan bias in favor of Mexico's former hegemonic ruling party, and especially for the left, relative to the right. The method uncovers systematic and large turnout-based bias in favor of the PRI that has been offset by district geography substantively helping one or both other major parties.

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A fundamental function of representative democracy is the conversion of parties' electoral support into legislative representation (Lijphart 1994). Often, scholars measure the quality of representation by examining the difference between the vote share that a party receives in the electorate and the seat share it subsequently wins in elections to the legislature. The congruence of vote and seat shares is at the heart of electoral reform debates and has received much attention from political scientists, economists, sociologists, geographers, mathematicians, and statisticians studying electoral systems utilizing single-member, plurality-win districts operating within party systems that have competition among two major political parties.[1]

The standard approach to study votes-seats curves focuses on two characteristics: responsiveness and partisan bias (King and Browning 1987, Tufte 1973). *Responsiveness* measures a change in seats for a change in votes, or the slope of the votes-seats curve. In a perfect proportional representation (PR) system, a party receives a seat share equal to its vote share and responsiveness equals one Linzer (2012), Taagepera and Shugart (1989). For many reasons, responsiveness is rarely equal to one; even in PR systems, thresholds to win a seat preclude a smooth translation of votes into seats. In district systems, responsiveness deviates further from PR because of how voters are assigned to geographical units. In the extreme, when every district is perfectly competitive between the parties, a small change

[1] See Altman and McDonald (2011), Balinski and Young (2001), Brady and Grofman (1991), Cain (1985), Cox and Katz (2002), Engstrom (2006), Erikson (1972), Gelman and King (1994), Grofman (1983), Grofman, Koetzle and Brunell (1997), Gudgin and Taylor (1980), Johnston (2002), Kendall and Stuart (1950), King and Browning (1987), Niemi and Fett (1986), Rae (1967), Rossiter, Johnston and Pattie (1997), Taagepera (1973), Trelles and Martínez (2012), Tufte (1973).

in votes yields a large change in seats, or high responsiveness. If every district is perfectly uncompetitive, seat shares are largely unaffected by vote shares, and responsiveness is near zero. In two-party systems, responsiveness can be described as a symmetric distortion of the seats to votes curve, in the sense that a party wins seats at the expense of their opposition (Grofman and King 2008). In contrast, *partisan bias* introduces an asymmetry in the votes-seats relationship. The term "partisan bias" describes an undue advantage in the ability to win legislative seats. A party favored by systematic bias win seats with fewer votes than their opposition, which can lead to counter-majoritarian outcomes when the party winning the most votes fails to win a legislative majority.

Theory highlights three sources of partisan bias. One is *malapportionment*—differences in district populations. A party with stronger voting bases in smaller-population districts receives a seat bonus nationwide (Jackman 1994, Johnston 2002). Another is *distributional*, and is often associated with partisan gerrymandering—the practice of strategically drawing district boundaries to achieve partisan bias. Partisan gerrymandering strategies involve wasting an opposition party's votes by either packing their supporters into a few districts they win by overwhelming majorities or spreading them thin across several districts that they cannot win (Cox and Katz 2002, Engstrom 2006, Owen and Grofman 1988). Distributional distortions may occur through the intentional practice of gerrymandering, or unintentionally through the confluence of geography and the rules governing the drawing of district boundaries. The third source is *turnout differences* across districts. A party enjoying stronger support in high-turnout districts pays a seat penalty relative to opposition parties that do well in low-turnout districts; the latter parties win seats with fewer votes (Campbell 1996, Rosenstone and Hansen 1993).

We explore the independent contribution of these three sources of partisan bias in multi-party systems. Our method to achieve this builds upon work by Grofman, Koetzle and Brunell (1997). Our contribution is three-fold. First, unlike Grofman, Koetzle and Brunell (and unlike previous works—see footnote 1), our approach drops the restrictive assumption of two-party competition. National two-party systems remain exceptional even among plurality systems (Cox 1997), so extending measurement to multi-party competition clears the way to test theoretical propositions using empirical data from numerous systems previously beyond reach. Second, we take often-ignored "creeping malapportionment" (Johnston 2002) into account. Malapportionment is most-often described as a deliberate choice to over-represent citizens residing in small-population districts and under-represent those in large population districts. Creeping malapportionment—notably prevalent in the United States prior to Supreme Court decisions in the 1960s—arises by the failure to redistrict using the most current population counts from a government census. Third, we apply these advancements to examine Mexican lower-chamber federal legislative elections to assess our method in a multi-party setting. Since democratizing in the second half of the 1990s, three major parties routinely win most votes, but up to 11 parties have fielded candidates for the Cámara de Diputados. We uncover small, but systematic, partisan bias against the right relative to the country's former hegemonic ruling party, but especially relative to the left. Decomposition of bias into the three additive components reveals that the parts are often greater than the whole, contributing in opposing directions and, therefore, offsetting one another to a large extent.

We proceed as follows. We describe the three models upon which we build our approach in sections 1, 2, and 3. Each model removes obstacles: King (1990) measures partisan bias in multi-party systems; Grofman, Koetzle and Brunell (1997) breaks down the size and polarity of three independent sources of partisan bias; and Linzer (2012) estimates quantities of interest with a limited number of observation points. Our method stands at the intersection of this trio. The remainder of the paper applies our proposed procedure to a case of substantive interest to students of elections and political geography, in general, and Latin America, in particular. Section 4 describes Mexico's mixed-member electoral system, isolating the plurality tier for analysis. We describe the sources and limits of the data we analyze for five consecutive elections between 2003 and 2015. Section 5 is an examination of substantial creeping malapportionment in these elections. Section 6 reports results. Section 7 concludes with a discussion of the importance to the method for future scholars and practical applications.

1 Partisan bias in the multi-party context

We begin by formalizing partisan bias and responsiveness. The two-party case (King and Browning 1987, Taagepera 1973, Tufte 1973) extends in a straight-forward manner to multi-party competition. In the two-party case, partisan bias and responsiveness are typically conceptualized as a generalization of the cube law stipulating that:

$$\frac{s}{1-s} = e^{\lambda}\left(\frac{v}{1-v}\right)^{\rho} \iff \text{logit}(s) = \lambda + \rho\,\text{logit}(v) \tag{1}$$

where s is the seat share won by a party with vote share v; λ is the party's bias relative to the opposition party (positive values favor the party, negative values favor the opposition);

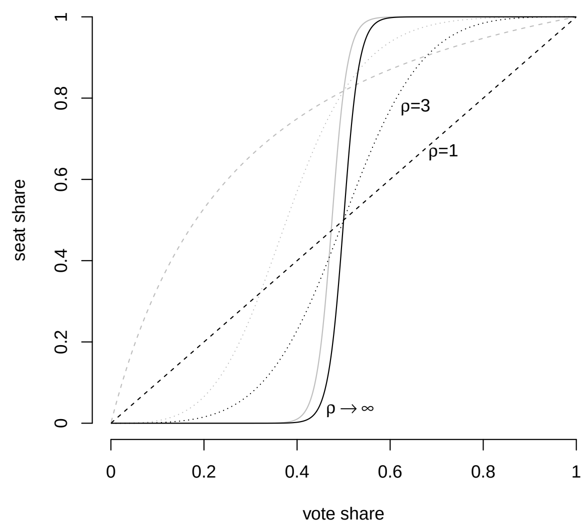

Figure 1: Illustration of model parameters. Partisan bias is set to $\lambda = 0$ in black lines. Gray lines replicate the black one-by-one with $\lambda = +1.5$.

and ρ is responsiveness. When $\lambda = 0$, a system has no partisan bias. The expression on the right is an algebraic transformation, convenient for estimation. Figure 1 shows how the parameters affect the votes-seats translation function.

The three centered lines, which intersect at fifty percent of the vote and fifty percent of the seats, illustrate variable responsiveness without partisan bias. A system with $\rho = 1$ is perfect proportional representation, the ideal type against which electoral systems are often contrasted. PR appears as the dotted diagonal line: every party winning v% of the vote gets, precisely, $s = v$% of seats. As responsiveness grows, the curve becomes steeper, over-representing the winner (points above the diagonal). At the limit, when ρ tends to infinity, every district is a microcosm of the national electorate, such that the party receiving 51% of the vote wins all districts and receives 100% delegates. $\rho = 3$, the dotted line, characterizes the classic cube law that many have associated with plurality rule in single-member districts

(Taagepera 1973). With cube responsiveness, a party with 55% of the vote wins two-thirds of the seats, but with 33% it wins only one-tenth of the seats.

Responsiveness is a symmetric property of the electoral system: *any* party coming first in votes will accrue the large party bonus. Partisan bias, in contrast, can be defined as *asymmetric* party treatment within the votes-seats function. Gray lines crossing fifty percent seats to the left of fifty percent of the votes in Figure 1 replicate the values of ρ just discussed, but with $\lambda = +1.5$ added. A bias-favored party requires fewer votes to reach the threshold for large-party over-representation, thereby generating manufactured parliamentary majorities with less than a vote majority (Lijphart 1994). (The dotted convex line shows how, due to logit links in Equation 1, partisan bias also reshapes the function's trace.) When bias is present, parties winning identical vote shares nationwide earn different shares of seats.

A multi-party, estimable version of equation 1 is King (1990; another is provided by Calvo and Micozzi 2005). A transformation—akin to multinomial logit's departure from the dichotomous version—formulates party p's ($p = 1, 2, \ldots, P$) expected seat share as:

$$E(s_p) = \frac{e^{\lambda_p} \times v_p^{\rho}}{\sum_{q=1}^{P} e^{\lambda_q} \times v_q^{\rho}} \tag{2}$$

with parameters indexed to identify the parties. Setting $\lambda_1 = 0$ restricts the remainder $\lambda_{p \neq 1}$ to express partisan bias in relation to party $p = 1$ without loss of generality. This is convenient for multipartism. Partisan bias in two-party competition is the shift away from $s = .5$ when the votes-seats curve is evaluated or "centered" at $v = .5$ (in Figure 1, it is the gap between the black lines and gray lines crossing points). While partisan bias shifts operate similarly, there is no reason to expect a curve centered at $v = .5$ in multiparty competition. Nor is it evident *a priori* what vote share serves as a center point[2]—which poses a difficulty when expressing a partisan bias estimate $\hat{\lambda}_p$ as a percentage points advantage or handicap for party p, as is commonly done in analysis of two-party systems (e.g., Cox and Katz 2002). With the $\lambda_1 = 0$ restriction, estimating $\lambda_{p \neq 1} < 0$ is evidence of bias against party $p \neq 1$ relative to party $p = 1$.

2 Three sources of partisan bias

At the root of partisan bias in systems with multiple districts are differences in the geographic concentration of parties' supporters. A party with 20% of the vote that is evenly spread nationwide across districts may fail to win a single seat; while another, with much less support, may win multiple seats because its support is geographically concentrated. In general, vote concentration helps smaller parties while hurting larger ones through vote (and therefore seat) wasting (Calvo and Rodden 2015). In the end, several forces add up and interact to yield partisan bias (Gudgin and Taylor 1980).

Grofman, Koetzle and Brunell (1997, henceforth GKB) demonstrate that what we call *raw partisan bias* (λ) has three clear and distinct sources, and offer a procedure to separate empirically the independent, additive contribution of each.[3]

[2]From Calvo and Micozzi (2005:Fig. 1), we can expect that center to shift progressively to the left as party competition increases. We note that partisan bias achieves precisely such a leftward pull—but for a favored party only: a more efficient votes to seats conversion. For unfavored parties, the pull is rightward. So whereas the effect of multipartism remains symmetric, that of partisan bias does not. We are grateful to an anonymous referee for pointing this out.

[3]Other elements highlighted by Gudgin and Taylor (1980) that our analysis of raw partisan bias ignores are the cube-law's bonus, large third-party votes, and possible interactions between all the elements. The bonus is, in fact, captured by the system's responsiveness parameter and therefore distinct from partisan bias in our framework (more on this in section 3). Calvo (2009) models departures from bipartism explicitly.

- *Boundary-delimitation* (GKB call this source of partisan bias 'distributional') corresponds to different party distributions of vote-wasting across districts. Vote-wasting may be deliberate (e.g., the gerrymandering strategy of wasting opponents votes), but may also arise incidentally through geography (e.g., when districts cannot cross state boundaries and a state is a party stronghold) or because of legal constraints applied to the redistricting process (e.g., creating districts to provide ethnic or minority representation).

- *Turnout* differentials across districts. Those who abstain from voting—either due to voting qualifications or by choice—lower the bar to win a district's seat. Parties stronger in lower-turnout districts achieve victories with fewer votes than other parties, improving their relative votes:seats ratio. Turnout differentials arise when correlates of participation, such as socio-economic status or voting-eligibility, vary systematically across districts, or when parties mobilize more voters in some districts than in others, such as those predicted to be competitive.

- *Malapportionment* arises when sparsely populated regions get the same representation as more densely populated ones. It may be found whenever multiple districts are drawn for the purpose of seat allocation. District size differentials may be designed by adopting cartography that deliberately under-represents some persons. For example, upper legislative chambers in federal systems often grant states equal representation, regardless of population. Even when districts were drawn precisely equal in population immediately following a census, malapportionment inevitably accumu-

Interactions remain interesting avenues for future research.

			Raw votes			Vote shares		Seat shares	
Districts	Pop.	Turnout	left	right	total	left	right	left	right
Distributional-based partisan bias only									
1 and 2	420	.5	147	63	210	**.7**	**.3**	1	0
3, 4 and 5	420	.5	84	126	210	**.4**	**.6**	0	1
nationwide	2100	.5	546	504	1050	.52	.48	.4	.6
Turnout-based partisan bias only									
1 and 2	420	**.70**	200	100	300	.67	.33	1	0
3, 4 and 5	420	**.35**	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6
Malapportionment-based partisan bias only									
1 and 2	**600**	.5	200	100	300	.67	.33	1	0
3, 4 and 5	**300**	.5	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6

Table 1: Illustrative five-district system scenarios

lates over time, as secular demographic population changes at different rates across regions, this leads to what is known as creeping malapportionment.

The scenarios in Table 1, which draw heavily from examples in GKB, illustrate the sources operating in isolation from one another. The division of vote and seat shares nationwide and the degree of partisan bias remain constant in all scenarios: the left party suffers a 12 percentage point deficit in representation, with 52% of votes but just 40% of seats (it won two of five districts); and the right party enjoys 12 percent overrepresentation, winning 60% of seats with just 48% of votes. The components of partisan bias are changed, one at a time. The first scenario has equal-sized and constant-turnout districts[4] that nonetheless manifest partisan differences in votes wasted, the left party winning seats by wider margins (+.4) than the right (+.2). The sole source of partisan bias is boundary-delimitation. Shifting district boundaries might re-allocate wasted votes in such way that another district tips towards the left.

[4]A less restrictive scenario allows size and turnout differences across districts with distributions that are independent of the distribution of partisan support.

The second scenario has equal-sized districts and winning margins uncorrelated with the vote distribution, but turnout is varied in a manner that is not orthogonal to vote shares. Right and left are winning seats with the exact same margins, but the right wins in lower-turnout districts—half, in fact, of the turnout in districts won by the left. As a consequence, the right wins seats with fewer votes than the left. In this case, partisan bias is the product of turnout differentials alone, against the left.

The third scenario has equal-turnout districts and winning margins uncorrelated with party vote strength, but different district population sizes that do correlate with the latter. Again, both parties win with equal margins, but the right wins districts half as populous as those won by the left. The consequence is a more efficient conversion of voted for the right—a similar total vote total yields a very different number of seats. This is partisan bias attributable to malapportionment by itself.

The formalization of the votes-seats curve in section 1 assumed that the votes in Equations 1 and 2 are the party's share of the national vote v_p, i.e., the party's vote aggregated across districts divided by the total raw vote nationwide. This standard mode of national aggregation of district-level vote returns measures raw partisan bias. Noting that party p's raw vote in district d is the product of its district vote share v_{dp} and the district's total raw vote, the party's vote share nationwide can be expressed as

$$v_p = \sum_d v_{dp} \times \frac{\text{total raw vote}_d}{\text{total raw vote}}. \quad (3)$$

GKB use this algebraic transformation to ease consideration of two alternative national aggregations of district returns, which then provide means of separating the partisan bias components. One formulation is party p's mean district vote share, defined as:

$$\bar{v}_p = \sum_d v_{dp} \times \frac{1}{\text{total districts}}. \tag{4}$$

The other is party p's population-weighted mean district vote share, defined as:

$$\bar{w}_p = \sum_d v_{dp} \times \frac{\text{population}_d}{\text{total population}}. \tag{5}$$

Following the insight of Tufte's (1973) seminal work (further elaborated in Gelman and King 1994), fitting the votes-seats curve using $\bar{v}_p$ instead of v_p yields distributional-based partisan bias. This is because $\bar{v}_p$ aggregates district vote shares without regard to district size and voter turnout. In the same spirit, GKB show that relying on $\bar{w}_p$ (an aggregate compounding district vote shares and relative district populations) yields estimates conflating boundary- and malapportionment-based partisan bias. Cleverly, subtracting partisan bias estimated with $\bar{v}_p$ from partisan bias estimated with $\bar{w}_p$ yields pure malapportionment-based partisan bias. Furthermore, because raw partisan bias conflates all three sources, subtracting partisan bias estimated with $\bar{w}_p$ from partisan bias estimated with v_p yields pure turnout-based partisan bias.[5]

In sum, the GKB procedure consists of repeatedly fitting equation 2, alternatively using v_p, then $\bar{v}_p$, and $\bar{w}_p$. Denoting λ_p^v, $\lambda_p^{\bar{v}}$, and $\lambda_p^{\bar{w}}$ party p's partisan bias parameter from each fitting, the following subtractions bring forth the quantities of interest:

a. raw partisan bias $= \lambda_p^v$,

[5] The notation (subscripts dropped) that GKB use for v, $\bar{v}$, and $\bar{w}$ is R, P, and M, respectively.

b. distributional-based partisan bias $= \lambda_p^{\bar{v}}$,

c. malapportionment-based partisan bias $= \lambda_p^{\bar{w}} - \lambda_p^{\bar{v}}$, and

d. turnout-based partisan bias $= \lambda_p^{v} - \lambda_p^{\bar{w}}$.

It is easy to verify that raw partisan bias is the sum of the three components in the GKB framework ($\text{a} = \text{b} + \text{c} + \text{d}$).

3 Estimation via Monte Carlo simulation

The final obstacle we face is fitting the votes-seats curve to data of interest, with the general problem being a scarcity of observations. Each party fielding candidates in a general election corresponds to one point in a votes-seats coordinate system, and relatively few parties do so in each election. A common approach to overcome this limitation is to pool data across several elections (e.g., Márquez 2014). However, such multi-election studies are not capable of revealing election-specific dynamics (Jackman 1994). Since some of these dynamics, such as turnout and creeping malapportionment, are of central interest, analysis of each election is therefore preferable (Niemi and Fett 1986), but requires a procedure to increase the number of observations.

We use a multiplication approach inspired by Linzer (2012), relying on Monte Carlo simulation.[6] Towards this goal, a probability density of national party vote returns is approximated from observed district outcomes with a finite mixture model (FMM). The FMM

[6]We did not pursue Linzer's swing ratios. The relation of that quantity with the notion of partisan bias adopted here is straightforward in balanced, two-party competition (see Linzer 2012:410), but not when multiple parties compete. We therefore partially follow his method, borrowing his code to simulate national party vote and seat pairs, then fitting a standard votes-seats curve on those. Commented code extending Linzer's procedure will be posted upon publication.

works up from district-level data, assuming sub-populations with known distributions are present—e.g., some districts where party 1's vote grows at the expense of party 2's vote, others where they grow jointly at the expense of party 3—but information to match districts to sub-populations is unavailable. A mix of known distributions describes the unknown distribution. The on-line appendix elaborates on the approximation of the unknown probability distribution governing the fundamentals of district-level party competition. Repeated draws of hypothetical district outcomes from the mix reflect variation in the sources of partisan bias: in district size, in turnout, and in vote choice (information fed to the FMM). Aggregating the draws nationwide yields a large sample of vote-seat simulations that are supported by the data.

Figure 2 presents the output of the simulation process for the Mexico case study that section 4 presents in detail. Observed national votes received and seats won appear as black labels for five consecutive elections. Simulated elections are surrounding clouds of gray labels. These counter-factual predictions are most reliable near observed points (about ± 5 percent, Linzer 2012:fn. 8). The single-election approach is not suited for extreme counter-factual prediction (something generally true for any approach, Gelman and King 1994). However, given the challenges with longitudinal studies, this is the best available approach.

Another technical problem is that parties may not field candidates in all legislative districts. The mixture method handles this issue by considering patterns of district contestation separately. This method does require adjustment when parties form partial coalitions—e.g., when parties A and B field joint candidates in some districts but run against each other in others. This issue occurred in recent Mexican elections, and we address it in greater detail

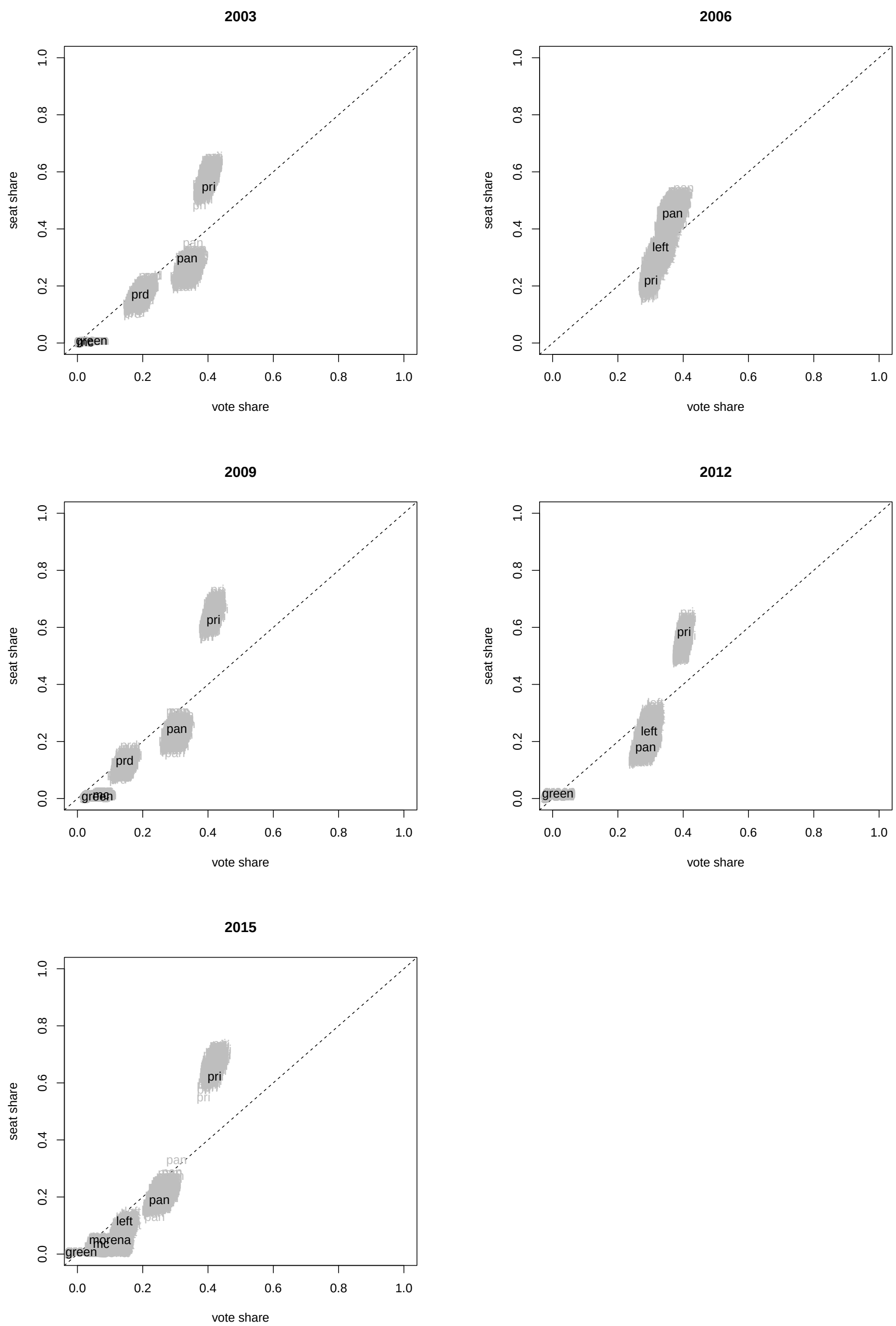

Figure 2: Votes and plurality seats in five elections. Actual data in black, simulated elections in gray. Source: prepared with data from `www.ine.org.mx`.

in the next section.

4 Mexican Cámara de Diputados elections

We demonstrate our procedure by analysis of recent elections of Mexico's lower chamber of Congress. The Cámara de Diputados has been elected with a mixed-member electoral system for decades. Systems of this nature give voters a direct role in the election of representatives from single-member districts, while additionally using some form of PR to reduce votes-seats distortions often magnified in plurality systems (Shugart and Wattenberg 2001).[7] We examine, in isolation, the elections held in the single-member plurality-win districts. We thus single out, for analysis, the tier where the potential for distortion of the votes-to-seats conversion is at its fullest—fertile ground to see our procedure at work.

Even if the mixed system, in the end, will attenuate votes-seats distorsions in the overall composition of the Cámara, the plurality tier bias we analyze retains potential to mess up representation. Some inter-party distorsion may carry to the PR tier. Cox (1997) shows how in SMDs, election contests tend to devolve into competition between the two leading party's candidates in each district. If systematic partisan bias further depresses seat-winning opportunities of trailing parties, these will find themselves disadvantaged in the PR tier as their voters abandon their candidates to select one of the top two candidates most likely to win the district election. And compensation is not free of intra-party distributive consequences. A party with a plurality seat deficit vis-à-vis its votes will receive extra Diputados

[7] Each voter casts a unique, non-exclusive, pooling vote to choose among candidates in 300 single-member districts with seats allocated by plurality. Votes then transfer to the party to which the candidate originally voted belongs, in order to allocate seats in five second-tier districts of magnitude 40, by closed-list Hare PR, using a 2% threshold (3% since 2015). Compensation tops over-representation at 8% or less. See Weldon (2001).

from the national lists. Unlike district candidates, who are often nominated by the local party, PR lists are overwhelmingly decided by the national party leadership (Poiré 2002). Evidence of systematic differences in how plurality and list Deputies behave in the Cámara (Kerevell 2015) suggests they do indeed represent different sets of interests. Plurality bias therefore deprives local parties of some influence.[8]

Mexico held its first free and fair congressional election in 1997. Electoral rules have remained fundamentally unchanged since, but district maps have been redrawn, using machine-assisted mapping, by and independent electoral board (Lujambio and Vives 2008, Trelles and Martínez 2012). The "1997 map" was used up to 2003, the other elections were conducted using the "2006 map". Another was proposed in 2015, but the board rejected implementation of this plan when redistricting became conflated with a broader package of electoral reforms. We examine Diputado midterm elections of 2003, 2009, and 2015, and elections concurrent with the presidential races of 2006 and 2012. We take advantage of redistricting in the period to add perspective to the analysis, with a hypothetical map approach. (We ask how the 2003 votes would have converted into seats if the 2006 map had been used? If the 2015 map had been implemented for that year's election?)

Analysis relies on district- and sección-level vote returns, the former to simulate national vote and seat shares by party, the latter to re-aggregate votes into hypothetical district maps.[9] Party vote shares are defined as the number of votes won divided by the effective

[8]Plurality bias has normative consequences too. Compensating *parties* bears relation to, but is not the same as, compensating *citizens* of oversized, electorally uncompetitive, low turnout districts. Keeping these compensations distinct is important. Much of the evidence presented here, as in the scholarly literature, deals with party votes to seats ratios. From the normative standpoint, however, it is the 'one person, one vote' principle—one of Dahl's (1972) preconditions of democratic government—that bias antagonizes, and party compensation is not designed to redress this imbalance.

[9]Data is from IFE's official election returns, available at `www.ine.mx`. *Secciones electorales* are analogous to U.S. census tracts (median sección population in the 2010 census was 1,280, with a maximum at

vote, which is a district's vote total minus voided ballots, votes for write-in candidates, and votes for small parties dropped from analysis (cf. Linzer 2012:fn. 4). To gauge malapportionment, district-level populations (and, for hypothetical map analysis, sección-level populations) were compiled from years 2000 and 2010 census data, and the 2005 population count, prepared by Mexico's census bureau (INEGI). Linear 2000–05 and 2005–10 projections provide point estimates of between-census election-year populations.

Six parties are included. Three are major parties, with vote shares above 15% (albeit volatile, as seen in Figure 2 and reported in detail in the on-line appendix): the right-of-center National Action Party (PAN) that controlled the presidency in the observed period up to 2012; the formerly hegemonic Institutional Revolutionary Party (PRI) in the presidency since 2012; and the left-of-center Democratic Revolution Party (PRD). Minor parties had vote shares between 2 and 9%. Major parties contested every district systematically, but often in pre-election coalitions. The PRI routinely coalesced with the Green party, but other than in 2012, when they nominated join candidates in every plurality district, the deal was partial—the partners fielded joint candidates in some districts, but competed against each other in the remainder. Partial PRI-Green coalitions complicate national votes and seats aggregation. The option of computing separate aggregates for PRI, for Green, and for PRI-Green is attractive for describing the situation faithfully, but we decided against it. Notably, the PRI would wrongly appear as not contesting numerous districts, thereby artificially underestimating its true electoral strength. We opted instead to exploit the coali-

79,232; median tract population in the 2010 census was 3,995, with a maximum at 37,452). Secciones are the basic units for district cartography. The 1997, 2006, and 2015 maps (kindly shared by IFE's cartography department) relate more than sixty-six thousand secciones to 300 congressional districts. This made reconstitution of hypothetical election outcomes in the period possible. Upon publication, all data and code for our analysis will be distributed through a durable publicly accessible archive *which?*.

tion partners' size asymmetry by considering PRI-Green votes won in tandem as if the PRI had won them solo—thus contributing returns in every district for the national aggregate. While this approach is far from satisfactory (the Green is the largest and most successful of minor parties, it may soon qualify as major) the solution is practical, and credits the fact that the partners never failed to team electorally to some degree throughout the period. Section 6 elaborates how results change when partial coalitions are differently specified.[10]

Previous research gives reason to suspect partisan bias. Márquez (2014), using a multi-election approach to analyze votes and seats won over two decades, uncovers a degree of responsivity characteristic of plurality systems and substantive partisan bias against the right. Our proposed procedure offers a way to answer questions of theoretical interest by showing the contributions of malapportionment, distributional, and turnout to bias in each election.

5 One Mexican, one vote?

Prevalent malapportionment adds to the attractiveness of the application case. Districts of unequal size are common practice in Mexico despite a set of clear quantitative redistricting criteria that includes population equality. One way malapportionment arises around the world is when apportionment formulas assign districts to geographic units, such as states, according to their populations that are not neatly divided by the number of seats (Balin-

[10]Minor parties included—all personalistic—are the Green, MC, and, in 2015 only, Morena, an important splinter from the PRD. Ten smaller/ephemeral parties were dropped from analysis (among them are the PT and the PANAL, which had presence throughout the period). Electoral coalitions occurred thus: the PRD fielded common candidates with MC and the PT nationwide in 2006 and in 2012 (they are labeled 'left' in plots); MC and PT ran together nationwide in 2009; and the PRI-Green covered one-third of districts jointly in 2003, all districts in 2006, one-fifth in 2009, and two-thirds in 2012.

ski and Young 2001). This is compounded in Mexico by a time lag between the conduct of the national census and redistricting, what Johnston (2002) calls "creeping malapportionment." Ironically, the constitution mandates the use of the census for redistricting, but the government has no obligation to redistrict as soon as population counts become available. In practive, six or more years have routinely passed between a new census and the time new district boundaries are inaugurated. If districts experienced similar population growths, this would be inconsequential. With growth differentials, however, substantial malapportionment exists even as a new map is implemented.[11]

Another cause of malapportionment is bureaucratic discretion. Small deviations around mean district population are usually unavoidable simply because populations cannot be sliced so finely to create perfectly balanced districts. Courts have struck down new U.S. House district maps bearing less than 1% differences within states without proper justification (Tucker 1985). U.S. redistricting authorities generally view *de minimus* population deviations of as little as one or zero persons between congressional districts as desirable to inoculate against litigation—although larger differences are permitted for elections at other levels of government. In stark contrast, Mexico's electoral board has permitted deviations between 10% (in 2006) and 15% (in 1997 and 2015) above or below mean state district size (Lujambio and Vives 2008, Trelles and Martínez 2012). The greater size deviation is designed to give deference to competing redistricting criteria, such as avoiding district lines that bisect municipalities or keeping units with large indigenous populations within the same district.

[11]The comparative survey by Snyder and Samuels (2004) ranked Mexico among well-apportioned cases. The measure reported is for the 1997 map, but no guidance is offered about the population figures used in denominators. We suspect reliance—as the board did then and still does now—on raw 1990 census data, severely underestimated Mexico's malapportionment.

We follow Ansolabehere, Gerber and Snyder (2002) to examine how malapportionment distorts representation. We measure a district's relative representation index as $RRI = \frac{1/\text{district size}}{300/\text{national population}}$, where the numerator is the number of seats per person in the district and the denominator is the average number of seats per person nationwide (300 is the number plurality seats). A district with unity index value has representation matching the 'one person, one vote' ideal perfectly. Values above one indicate over-representation, values below one under-representation, and the measure is continuous. An example shows how the index is interpreted. The 3rd district of Aguascalientes in 2012 had about 306,000 inhabitants, and 300 divided by Mexico's population was about 387,000. So the district in question had 26% more representation than the national average, for an index value of 1.26. We project inter-census populations linearly to estimate yearly district populations when computing *RRI*s.[12]

The percentiles corresponding to *RRI*s at .85 and 1.15 (the bounds of the board's ±15% tolerance range) in 2006 were 10 and 87, respectively, implying that $10+100-87=23\%$ of districts exceeded IFE's discretionary malapportionment range in the map's inaugural year, as caused by the census lag. By 2012, more than one-third districts were outside the tolerance range, and by 2015 just shy of two-fifths. As the U.S. Supreme Court found in the 1960s, using antiquated population data impairs drawing equal-sized district boundaries and substantially distorts representation.

Figure 3 summarizes observed malapportionment. Vertical dashed lines in gray mark

[12]More precisely, the 2000–2005 rate of growth was used before year 2006, and the 2005–2010 rate afterwards. Population projections for different maps were done after sección census populations had been aggregated into actual or hypothetical districts. Performing linear projection on secciones before any aggregation might have been preferable (because they are much smaller geographic units), but a fair amount of over-populated secciones are routinely split into new ones between elections, complicating the projection exercise.

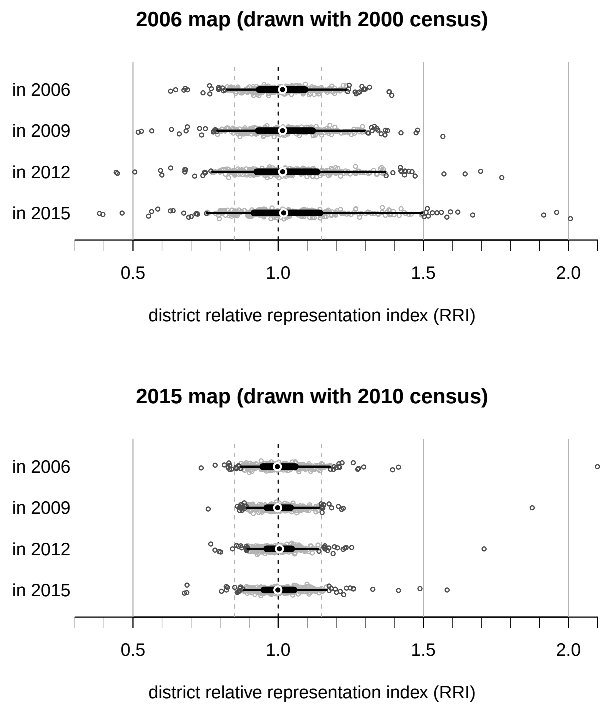

Figure 3: Representation in four elections and two maps. Points are districts. Panels portray (top) the status quo and (bottom) the hypothetical maps. Finer horizontal lines connect the 5th and 95th percentiles, thicker lines the quartiles, and white circles indicate the median.

the 15% tolerance band, which has been amply and systematically surpassed. Consider the top plot, portraying the status quo map, first. Each point represents one district. The fine horizontal line connects the *RRI* values corresponding to the 5th and 95th percentiles—both well outside the tolerance range, since the map's inception. The thick horizontal line is the inter-quartile range, not far from covering the upper bound of the tolerance range by 2012, and towards coverinf the full tolerance range by year 2015 (if it did, half the districts would be off-range). In the Cámara elected in the 2015 midterm, citizens in the plot's right-most districts (in central Monterrey and two in battered Juárez) will be worth *four times more* in Congress than citizens in the left-most districts (one each in suburban Monterrey and Mexico City, the other in Cancún). In matters politic, citizens at one quartile will be worth nearly twice as much as those at the other quartile.

The bottom plot is hypothetical, analyzing the map proposed for 2015, but not adopted. It is quite clear that, using more up to date reference census counts improves representation substantially (note the narrower horizontal lines). Applying our method on the same election in both the actual and the hypothetical maps offers perspective to gauge the effect that a drop in malapportionment has on partisan bias and its components.

6 Results

We now turn to estimating overall bias and its components. We fit equation 2 using MCMC estimation to generate overall bias and responsiveness estimates. The on-line appendix details model implentation and links to data and computer code for replication.[13] The

[13]Gelman and Hill (2007) is a comprehensive introduction to MCMC estimation. For each vote operationalization (v, $\bar{v}$, and $\bar{w}$), three chains were iterated 50 thousand times, taking every 50th observation of the

responsiveness parameter is of secondary interest here, but useful for assessment of model fit. Judging the 90% Bayesian confidence intervals (i.e., the 5th to 95th percentile range of $\hat{\rho}$'s posterior sample) reveals sizable shifts in the estimate between congressional elections: from a low of $[2, 2.3]$ in 2015 to a high of $[2.6, 3.0]$ in 2006. The large-party premium of recent Mexican plurality congressional races is about one-sixth smaller than the power of the putative cube law of plurality elections (Taagepera 1973).

The raw partisan bias estimates (i.e., the λ_p^v parameters) are of direct interest to our investigation. Figure 4 summarizes posterior samples for different parties. We choose the PRI as the reference category and therefore express partisan bias measures relative to the reference (it is for this reason the PRI is absent from the figure). Recall that a negative estimate for a given party is evidence of bias against that party relative to the PRI. Several patterns are noteworthy. Estimate precision (i.e., how concentrated the posterior cloud appears) is consistently higher for major parties than for minor parties (this is true for Convergencia as well, excluded from the plot to save space). Among major parties, the PAN's estimates are the most precise with variation around the median posterior value (taken as the point estimate) nearly indistinguishable at the chosen scale every year. The PRD's estimates are slightly less precise in midterm elections (2003 and 2009) than in presidential election years.

The size and polarity of the bias estimates reveal important party differences. The PAN experienced negative, albeit small partisan bias vis-à-vis the PRI in every year observed. Leaving aside the question of how meaningful the estimated quantities are—we analyze

final 25 thousand to sample the posterior distribution. The Gelman-Hill $\hat{R} \approx 1$, evidence that the chains had reached a steady state. Convergence also inspected visually in chain trace plots of each model parameter, as reported in the on-line appendix. Estimation performed with open source software `Jags` (Plummer 2003), implemented in `R` (R Dev. Core Team 2011) with library `R2jags` (Su and Yajima 2012).

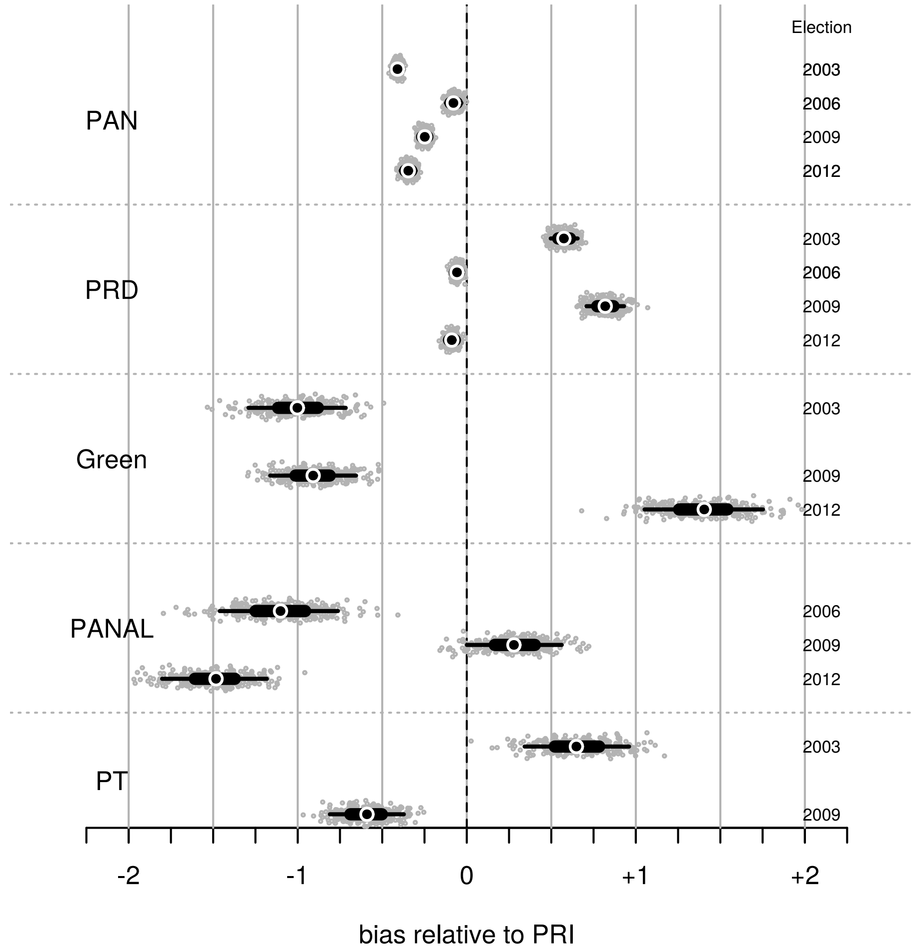

Figure 4: Raw partisan bias in four elections. The plot describes the posterior samples (small gray points) of estimated parameters $\hat{\lambda}_p$ for five parties. Some parties did not run in some years. Finer horizontal black lines connect the 5th and 95th percentile values of the posterior sample, thicker lines the quartiles, and white circles indicate the median value.

swing ratios below—the near exception was 2006, when the tiny estimate's cloud slightly overlaps with the vertical line at zero. In contrast, the PRD experienced favorable and substantive bias relative to the PRI in some years but not in others. Paradoxically, partisan bias in favor of the leftist PRD is a mirror image of its electoral fortunes: bias vanished when its candidates for Congress rode López Obrador's presidential campaign coattails (the party's national congressional vote was 30 percent on average in 2006 and 2012), but emerged in midterm elections (when its vote averaged 16 percent). In spite of losing about half of its support from presidential to midterm elections, the PRD converted votes into seats much more efficiently than either the PRI or PAN in midterm election years. How can a party experience less partisan bias when it fares worse at the polls? Decomposing the components of partisan bias can reveal whether or not this dynamic is due to PRD winning smaller or lower-turnout districts.

Table 2 reports the estimated total and additive components of partisan bias. Bias estimates for the PAN, the PRD, and one minor party (the green) relative to PRI's for the elections as they were conducted in the implemented maps are presented in the first three columns. Numbers in parentheses report the share of the posterior sample with a sign opposite to that reported in the table, serving as a test of the estimate's statistical significance (the probability that the estimate's sign is wrong).

Turnout played favorably for the PRI relative to other major parties in every election in the period, as indicated by systematic negative signs. Years 2009, when sluggish economic performance and civil warfare hurt the incumbent PAN and deep internal divisions the PRD, and 2012, when favorable presidential coattails aided the PRI's congressional candidates, experienced more modest turnout effects. In all others, winning lower-turnout districts gave

partisan bias	Actual map PAN–PRI	Actual map PRD–PRI	Actual map Green–PRI	Hypothetical map PAN–PRI	Hypothetical map PRD–PRI	Hypothetical map Green–PRI
2003 election					(with 2006 map)	
total	−.19 (0)	+.52 (0)	−1.23 (0)	−.41 (0)	+.57 (0)	−1.06 (0)
distrib.	+.04 (.03)	+.50 (0)	−1.11 (0)	−.13 (0)	+.63 (0)	−.96 (0)
turnout	−.23 (0)	−.10 (0)	−.07 (0)	−.27 (0)	−.11 (0)	−.07 (0)
malapp.	+.00 (.45)	+.12 (0)	−.05 (0)	−.01 (.33)	+.04 (0)	−.02 (0)
2006 election						
total	+.22 (0)	+.09 (0)				
distrib.	+.32 (0)	+.29 (0)				
turnout	−.09 (.07)	−.25 (0)				
malapp.	−.01 (.36)	+.05 (.25)				
2009 election						
total	−.25 (0)	+1.01 (0)	−.85 (0)			
distrib.	−.20 (0)	+1.01 (0)	−.87 (0)			
turnout	−.06 (.19)	−.04 (0)	+.04 (0)			
malapp.	+.00 (.46)	+.04 (0)	−.02 (0)			
2012 election						
total	−.52 (0)	−.27 (0)	+.52 (.01)			
distrib.	−.48 (0)	−.25 (0)	+.53 (.01)			
turnout	−.06 (.09)	−.07 (0)	+.01 (0)			
malapp.	+.01 (.40)	+.05 (0)	−.02 (0)			
2015 election					(with 2015 map)	
total	−.17 (0)	+.26 (0)	+.01 (.47)	−.27 (0)	+.26 (0)	+.12 (.27)
distrib.	+.02 (.35)	+.40 (0)	+.10 (.32)	−.15 (0)	+.43 (.06)	+.14 (.24)
turnout	−.19 (0)	−.19 (0)	−.04 (.46)	−.12 (0)	−.19 (0)	−.02 (.11)
malapp.	+.00 (.38)	+.05 (0)	−.05 (.42)	+.01 (.42)	+.01 (0)	−.01 (.11)

Table 2: Relative partisan bias and its additive components. Entries report the median of the posterior sample of parameters estimated with the single-election models. Numbers in parentheses are the share of the posterior sample with sign opposite to the reported point estimate's. The right columns report bias estimates using election data re-arranged according to new district boundaries (i.e., a hypothetical 2003 election with the 2006 map and a hypothetical 2015 election with the map not implemented).

the PRI a springboard to more efficient votes-to-seats conversion against one (2003), the other (2006), or both (2015) other major parties.

The distributional component often predominates among the components of bias: always for the PRD and the green, in 2006–2012 for the PAN. Owing to formidable entry barriers in the election law, no minor party is regional-based. Meager nationwide support provides few opportunities for minor parties to win plurality seats—hence the negative sign most get most of the time. Years 2012 and 2015 are exceptions for the greens, when they nominated and won the coalition's candidate in a concurrent gubernatorial race whose coattails returned three congressional seats (Magar 2012). The distributional component's volatility for major parties, in size and in polarity, is consistent with the absence of partisan gerrymandering—as we might expect from the major-party power-sharing arrangements on the election board that draws districts. Also notable is how the total bias sum can hide large components that contribute in opposite directions and therefore cancel out. The PRD's extraordinary performance in 2006 led them to the lowest major-party measure of total partisan bias (in absolute value) in the period. Decomposition of raw partisan bias makes it possible to note that the left's distributional advantage was key to compensate for even larger turnout disadvantages.

Also distinctive—and surprising given the presence of substantial malapportionment—is how generally small the malapportionment component of partisan bias is compared to other sources. The PAN experienced no bias relative to the PRI attributable to district size differentials over the period—these are most of the estimates, by the way, with sizable probabilities of being the wrong sign. The party's success was therefore not likelier in districts confined at one end of the RRI distribution. The PRD was slightly advantaged relative to the PRI in every year observed. This is very likely due to overrepresentation of Mexico City's Federal District—a PRD stronghold—but the effect is easily eclipsed by the other components of partisan bias. (The drop from +.12 to +.05 between 2003 and 2006 actually coincides with reapportionment and the accessory reduction—not removal—of the capital's overrepresentation in Congress, see Altman, Magar, McDonald and Trelles 2014.) Malapportionment-driven bias is not much larger for the minor party, whose perennial small vote shares locate at the wrong end of the system's responsiveness to size.

For further perspective, we repeated the 2003 and 2015 estimations with hypothetical outcomes using the district boundaries of the 2006 and 2015 maps, respectively (reported in the right three columns of Table 2). As expected, redistricting mitigated significant malapportionment-based partisan bias systematically: under hypothetical, more balanced districts, statistically insignificant bias observed against the PAN remains thus (the probability that the estimate reported has wrong sign is always 11 ore more); and the pro-PRD's discernible bias relative to the PRI shrinks to about one third its original size.

We close with an assessment of how meaningful partisan bias is in recent congressional elections. We discussed how translating the bias estimates into a percentage point advantage or handicap for each party in the votes-to-seats conversion is not straightforward in multi-party settings. We therefore gauge this with an alternative quantity of substantive interest: vote-seat swing ratios (Niemi and Fett 1986, Tufte 1973). Swing ratios measure the sensitivity of individual parties' seat shares to marginal changes in voter preferences, and are computed by the percentage change in seats associated with a one-percent change in the party's national congressional vote. A party with unity swing ratio can expect to receive its fair share of additional seats. Larger values indicate that parties can expect to

		PAN		PRI		PRD	
Year	Variable	β	(SE)	β	(SE)	β	(SE)
2003	v	1.84	(.06)	2.44	(.07)	1.75	(.05)
	$v \times$ reMap	$+.06$	(.08)	$+.08$	(.10)	$-.12$	(.06)
2006	v	2.22	(.06)	1.97	(.10)	1.69	(.05)
	$v \times$ reMap	$+.01$	(.08)	$-.15$	(.14)	$+.01$	(.08)
2009	v	1.77	(.08)	2.22	(.08)	1.60	(.06)
	$v \times$ reMap	$+.19$	(.11)	$+.15$	(.12)	$+.05$	(.08)
2012	v	2.10	(.07)	3.88	(.12)	2.10	(.06)
	$v \times$ reMap	$-.16$	(.09)	$-.18$	(.17)	$+.02$	(.09)
2015	v	1.57	(.05)	1.84	(.06)	1.26	(.04)
	$v \times$ reMap	$-.04$	(.08)	$+.08$	(.10)	$-.01$	(.06)

Table 3: Vote-seat swing ratios. Also in the right side, but not reported, were a dummy indicating data simulated with the hypothetical map (reMap), and a constant. Method of estimation: OLS.

win more (> 1) and lower values indicate parties can expect to win less (< 1) than one percent of seats for a unit percentage change in vote share. (We rule out negative swing ratios corresponding to a party losing seats as it wins votes; for violations of the monotonicity principle of representation, see Balinski and Young 2001).

We derive swing ratios by regressing a party's seat shares in simulated elections on the party's simulated vote shares (Linzer 2012). To also gauge the effects of redistricting, we pool the latter with hypothetical elections using the map that supplanted the actual one (i.e., the 2006 map for the 2003 election and the 2015 map for the rest). Interacting this with a dummy reMap (equal 1 for hypothetical simulated elections, 0 otherwise) yields the fitted equation: $s_p = \beta_0 + \beta_1 v + \beta_2 \text{reMap} + \beta_3 v \times \text{reMap} + \text{error}$. Coefficient β_1 is the swing ratio, coefficient β_3 is the swing ratio change attributable to redistricting.

Table 3 reports results. In general, major parties enjoyed quite favorable swing ratios in the period—2.02 on average, indicating a 2 percentage points hike in seats for an extra percentage point in votes. But a good deal of change, both between parties and between elections, is evident. The PRD enjoyed the smallest four-election average swing ratios (1.7), the PRI the largest (2.3), the PAN somewhere in between (1.9). Given 300 seats, the PRI at its most elastic (in 2012) would have earned nearly 12 more plurality seats with just one extra percent votes nationwide. Underscoring the importance of bias in the single-member district tier, a dozen seats would have amply sufficed to give the coalition majority status that it failed to achieve in the Cámara.[14] Contrast this with the 2.5 and 3.2 additional percentage points in votes, respectively, that it would have taken the PAN and the PRD at their least elastic (in 2015) in order to earn the same dozen extra plurality seats.

7 Conclusion

We develop a generalized procedure to estimate the components of partisan bias—from malapportionment, boundary-delimitation, and turnout—in national electoral systems utilizing single-member, plurality-win districts. A method to estimate these bias components has been available for some time, but for two-party competition only. Our innovation is to intersect three extant empirical models to extend the procedure to multi-party systems. We then use the procedure to study recent Mexican Cámara de Diputados elections.

Our analysis reveals how the plurality component of Mexico's mixed electoral system gives persistent advantage to some parties in recent congressional elections. Relative to the PAN, there is evidence of small, but systematic partisan bias in favor of the PRI in the votes-

[14]Like the analysis, this statement obviates the compensatory PR tier and the 8 percent ceiling on over-representation. When the full mixed system is considered—as implied by the slice of seats needed for majority status—swing ratios will approximate unity. See Weldon (2001) and Márquez and Aparicio (2010).

to-seats conversion, and of a larger, if more volatile, bias favorable to the PRD throughout the period. These findings, derived from simulated data to overcome methodological complications, are in contrast with evidence of substantive anti-PRI bias in a multi-election study (Márquez 2014).

The analysis of the components of partisan bias adds further depth to our findings. Partisan bias sources vary in importance and, to a fair extent, run counter each other. The prevalence of substantial malapportionment in Mexico has not, as a matter of fact, translated into systematic partisan bias. Malapportionment has helped the leftist PRD relative to other major parties, growing in strength as maps aged and further malapportionment crept in. However, the contribution is much smaller than, and easily offset by, parties' turnout differences and boundary-delimitation biases. The PRI of the democratic era retains an edge in low-turnout districts, increasing its capacity to turn votes into seats in every election studied. And in spite of a nominally neutral redistricting system, the PRD in most years, and the PAN in 2006 were able to overcome a large turnout disadvantage through more favorable line drawing.

That these components mostly work against each other to yield modest total partisan bias is fortunate, but they may not act in such a manner in future elections. Furthermore, our analysis demonstrates that mitigating one source of bias through reform may unintentionally yield greater overall bias, when a counterweight against other bias sources is lifted. We expect that the magnitude of the bias components will change in light of electoral reform that will allow members elected in 2018 to run for consecutive reelection, which could introduce new turnout distortions. A new national census and a new map may also introduce a new mix of bias components. Counter-factual analysis—our inspection of elections that preceded a redistricting by reconstituting returns according to the new map—demonstrates a method informing future redistricting decision-making in Mexico and other, similar countries. Proposed maps are inherently counterfactual; if malapportionment and turnout biases persist, the fortunate outcome of modest overall bias in future redistricting may be achieved, ironically, through manipulation of districts, that is, through gerrymandering.

Online appendix

We offer here a step-by-step description of the procedure's implementation for analysis of recent Mexican elections. Code to replicate the analysis is available *here*.

Step 1: Aggregate sección-level into district-level votes

Table 4 summarizes five Diputado elections analyzed. Part A reports official returns almost straight from the source (we have aggregated a residual others category for convenience). Data includes the number of plurality districts that parties/coalitions contested, the national vote share v, and the share of plurality seats, out of 300, won s. Statistics on partial coalitions (e.g., between the PRI and Green in all but one year) appear here: the number of districts where it happened or not, and the share of votes and seats won in tandem.

Part B aggregates the partial coalitions by allocating their votes in those districts to the major party involved, as discussed in the text. It reports two metrics for national vote shares: v_1 has effective votes in the denominator (total votes minus voided ballots, votes

Part A: before partial coalitions allocated to major party

	2003			2006			2009			2012			2015		
	dis.	v	s	dis.	v	s	dis.	v	s	dis.	v	s	dis.	v	s
PAN	300	.32	.27	300	.34	.45	300	.29	.24	300	.27	.17	300	.22	.18
PRI	203	.24	.40	—	—	—	237	.28	.46	101	.11	.17	50	.04	.08
PRI coal.	97	.14	.15	300	.28	.22	63	.12	.17	199	.27	.41	250	.33	.53
PRD	300	.18	.18	—	—	—	300	.14	.13	—	—	—	200	.05	.02
PRD coal.	—	—	—	300	.31	.34	—	—	—	300	.28	.24	100	.08	.10
Green	203	.04	—	—	—	—	237	.06	—	101	.02	.01	50	.01	—
MC	300	.02	—	—	—	—	—	—	—	—	—	—	300	.06	.03
MC coal.	—	—	—	—	—	—	300	.07	.01	—	—	—	—	—	—
Morena	—	—	—	—	—	—	—	—	—	—	—	—	300	.09	.05
Other	300	.06	—	300	.07	—	300	.04	—	300	.05	—	300	.12	.01

Part B: partial coalitions given to major party, shares of effective vote and district population

Party	2003			2006			2009			2012			2015		
	v_1	v_2	s	v_1	v_2	s	v_1	v_2	s	v_1	v_2	s	v_1	v_2	s
PAN	.32	.08	.27	.34	.11	.45	.29	.07	.24	.27	.11	.17	.22	.07	.18
PRI	.38	.10	.55	.28	.09	.22	.40	.09	.63	.38	.16	.58	.38	.11	.62
PRD	.18	.05	.18	.31	.10	.34	.14	.03	.13	.28	.12	.24	.13	.04	.12
Green	.04	.01	—	—	—	—	.06	.01	—	.02	.01	.01	.01	.00	—
MC	.02	.01	—	—	—	—	.07	.02	.01	—	—	—	.06	.02	.03
Morena	—	—	—	—	—	—	—	—	—	—	—	—	.09	.03	.05
Abtention	—	.75	—	—	.71	—	—	.77	—	—	.60	—	—	.72	—

Table 4: Summary of five elections. Entries in part A indicate the number of plurality districts contested by each party/coalition, the national vote share won (v), and the share of plurality seats won (s). In part B, the national vote share relative to effective votes with coalitions allocated to the corresponding major party (v_1) and the same including non-voters compared to total population (v_2). Prepared with data from `www.ine.mx` and `www.inegi.org.mx`

for write-in candidates, and votes for small parties dropped from analysis[15]); v_2 has total district population in the denominator (so that abstention is the share of the total population not voting for one of the parties/coalitions in the analysis). The separation method relies upon relative district populations and turnout statistics to compute partisan bias.

Data discussed in the last two paragraphs are national-level aggregates. Analysis in fact proceeds from much smaller aggregates: district- and sección-level returns. Sección-level votes are used to reproduce district returns using hypothetical maps (e.g., the 2003 votes had it been held with the 2006 district map)—secciones are the basic building blocks of redistricting. District-level returns are then used as input for the simulation of national elections, as detailed in step 3.

Step 2: Produce inter-census population estimates

Population figures are from census counts in 2000, 2005, and 2010, from data prepared by Mexico's census bureau (INEGI) for the purpose of redistricting. They are available at the sección-level, and district populations can therefore be obtained. With elections taking place between census years, and after, we proceeded to estimate inter-census populations by linear projection of 5-year rates of growth. So the 2000–2005 rate of growth was used to estimate year 2003 district populations, and the 2005–2010 rate for years 2006, 2009, 2012, and 2015. Population projections for different maps were done after sección census populations had been aggregated into actual or hypothetical districts. Performing linear projection on secciones before any aggregation might have been preferable (because they are much smaller geographic units), but a fair amount of over-populated secciones are

[15] All will be listed in this footnote.

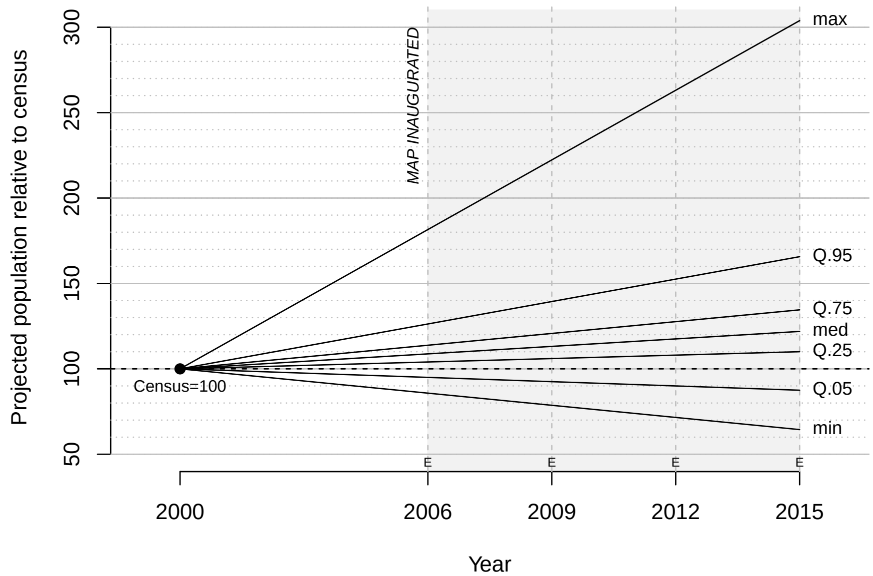

Figure 5: The 2006 map and demographic change. Plotted population projections relied on the 2000–2010 censuses rate of change (see text for details). Letters E in the horizontal axis indicate elections using this map. Source: prepared with data from `www.inegi.org.mx` and `www.ine.mx`.

routinely split into new ones between elections, complicating the projection exercise.

Figure 5 illustrates the projection exercise, and the grip we get on how creeping malapportionment affects representation in Mexico. If every district had experienced the same rate of change, the redistricting census lag would be inconsequential. With variable rates of population growth, malapportionment creeps in. Compared to the 2000 census, projected district populations in 2006, when the map was inaugurated, are off by 9.7% in absolute value on average, with a standard deviation of 10.6%. Indexing 2000 census district populations at 100, as the figure does, reveals how different the most demographically dynamic units were on paper and in reality. The fastest-growing district was 88% larger in 2006 than what census data otherwise suggested (the line labeled 'max'). The district shedding most population was 16% smaller (the 'min' line). These are outliers, but central tendencies reflect sizable lags as well. The inter-quartile range (lines Q.25 and Q.75) covered 1%–13% above census in the 2006 election, and expanded to 4%–20%, 7%–27%, and 10%–35% in the three subsequent Diputado elections using the same map.

Step 3: Simulate national-level elections

In order to simulate national elections (i.e., a vote-seat share for each party), the Linzer method infers the distribution of district-level returns from every pair of parties' correlations. Some parties do not field candidates in every district, so finding patterns of contestation starts the process. For example, two patterns were observed in 2009: candidates from the PAN, PRI, PRD, and MC competed in 63 districts, while candidates from the same plus the Green party competed in the remainder 237 districts. Subsets of districts in each pattern are analyzed separately. To also take district population and turnout into account, the patterns of contestation also include the abstention rate, as discussed in step 1.

The density of district returns is unlikely to conform to a textbook statistical distribution. But a mix of multi-variate normals will capture much of the observed variance. Exploration of district outcomes by plotting the relative performance of party pairs (including abstention) relative to the PRI in 2009 (first pattern of contestation) reveals two apparent clusters of districts. In one, including 21 percent of districts in the pattern, MC won/lost votes relative to the PRI ($ln(\frac{mc}{pri}$ increases) without hurting PAN's performance relative to the PRI ($ln(\frac{pan}{pri})$ constant); in the other cluster, accounting for 79 percent of districts, PAN won/lost votes relative to the PRI without much affecting how MC performed relative to the PRI. The same is generally true of the PRD and the PAN (as seen in the plot of $ln(\frac{prd}{pri})$

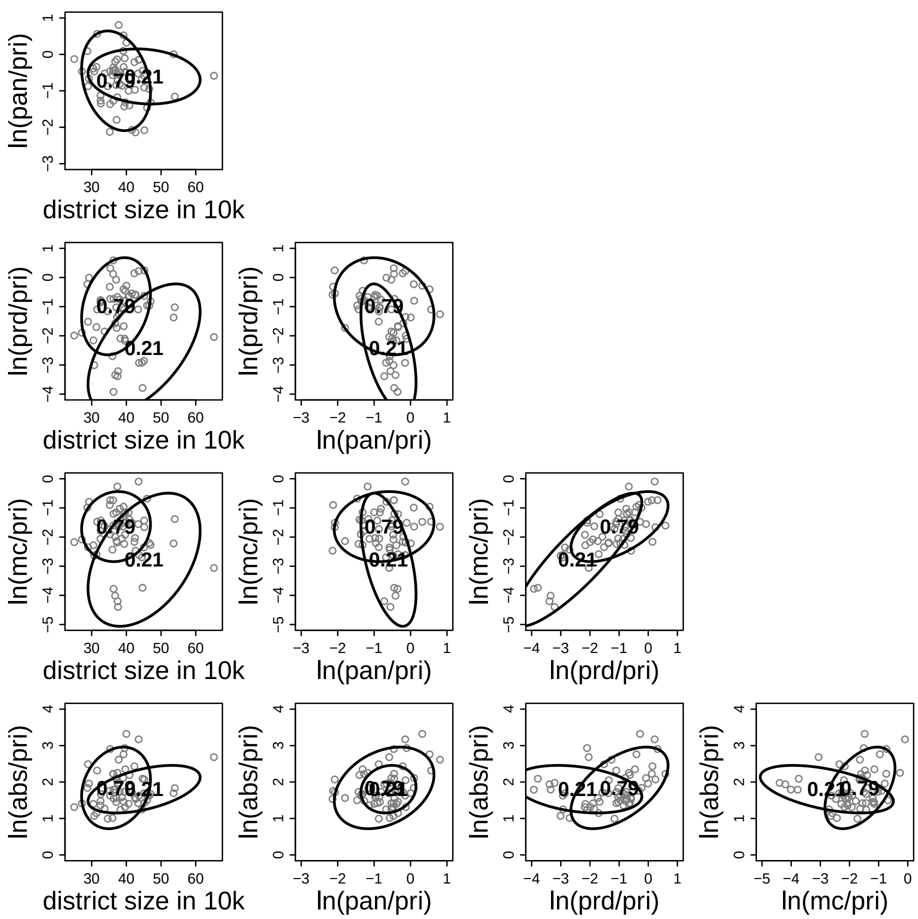

Figure 6: How pairs of parties' votes correlated (vis-à-vis the PRI) in 2009: 63 districts that the Green party did not contest

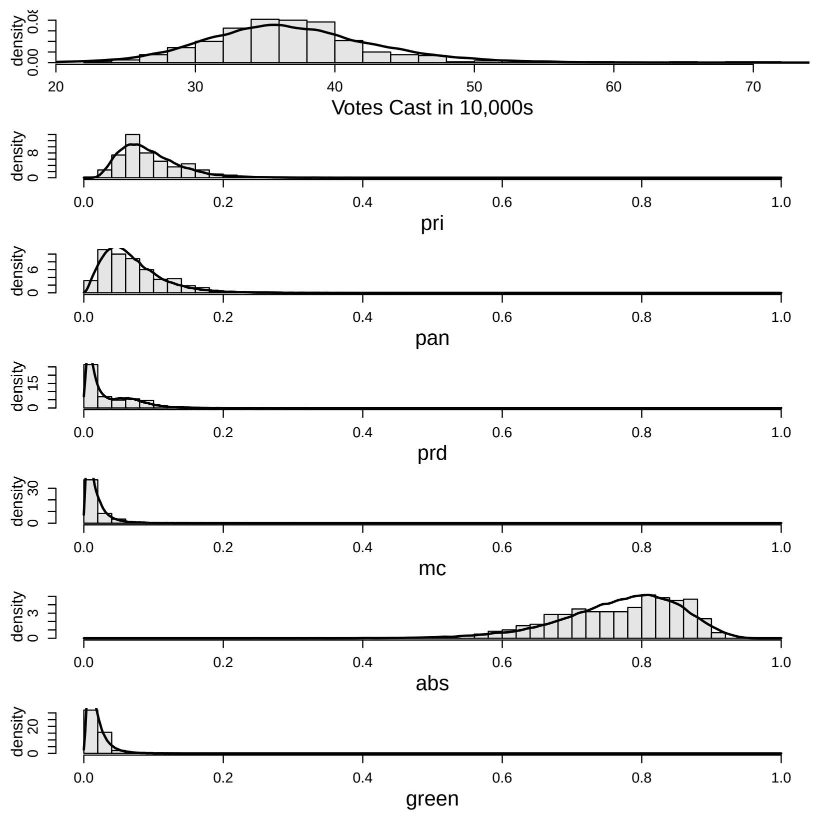

Figure 7: Mixture model's marginal densities in 2009

against $ln(\frac{pan}{pri})$), but not of the PRD and MC (districts where one grew relative to the PRI also had the other growing relative to the PRI). Choosing the number of components to use is impressionistic (Linzer 2012:405): we proceeded with a single component for the first pattern of 2003 (hypothetical map), for both patterns of 2015 (status quo map), and for the second pattern of 2015 (hypothetical map); two components for the second pattern of 2003 (hypothetical map), for the first pattern of 2015 (hypothetical map), and for both patterns of 2003 (status quo map), 2009, and 2012; and three components for the sole pattern of 2006.

Figure 7 compares observed 2009 frequencies with the mixture model's marginal density to verify that there is a good match between the two. Other than a slight bimodality in the abstention rate, marginals follow the contour neatly. Monte Carlo sampling with a weighted mix of the bi-variate normals generated one thousand simulated elections for 2009 in the analysis. The procedure is identical for all other years/maps.

Step 4: Estimate responsiveness and partisan bias

We use MCMC to estimate equation 2, implemented with Jags (Plummer 2003), called from R. Preparation involves generating data objects first: i indexes $I = 100$ simulated observations for a given year; p indexes $P = 6$ parties in the analysis; and $D = 300$ is the number of plurality districts; votes and seats $I \times P$ matrices `v` (where $\mathtt{v}_{i,p}$ is party p's vote share in the ith simulated election) and `S` (where $\mathtt{S}_{i,p}$ are the seats that party p won in the ith simulation); and a P-length vector `dummy` (where $\mathtt{dummy}_p$ equals 1 if party p contested the election in the given year, 0 otherwise). All data objects are bundled in a list

```
lrdata <- list ("S", "v", "I", "J", "D", "dummy").
```

We next randomize a set of initial values for the model parameters (λ needs only $J - 1 = 5$ initial values because we restrict $\lambda_{\mathrm{PRI}} = 0$)

```
lrinits <-function(){ list (lambda=rnorm(J-1), rho=rexp(1)) }
```

and define objects to store parameters' posterior distribution samples

```
lrparameters <-c("lambda", "rho").
```

The formal model estimated appears in Table 5. Reliance on `dummy` is convenient as is allows to proceed with six-party votes ans seats matrices regardless of the year selected—otherwise the code would need to be adapted each year for variable parties. Parties not

```
lambda.rho <- function() {
  for (i in 1:I){   # loop over observations (simulations)
    for (p in 1:P){ # loop over parties (dummy selects who ran)
      S[i,p] ~ dbin(pi[i,p], D[i])  # D is the number of SMD seats
    }
    numerator[i,1] <- dummy[i,1] * exp( lambda[1] ) * v[i,1]^rho
    numerator[i,2] <- dummy[i,2]                    * v[i,2]^rho
    for (p in 3:P){
      numerator[i,p] <- dummy[i,p] * exp( lambda[p-1] ) * v[i,p]^rho
    }
    for (p in 1:P){ # loop over parties
      d1[i,p] <- dummy[i,1] * exp( lambda[1] ) * v[i,1]^rho
      d2[i,p] <- dummy[i,2]                    * v[i,2]^rho # reference
      d3[i,p] <- dummy[i,3] * exp( lambda[2] ) * v[i,3]^rho
      d4[i,p] <- dummy[i,4] * exp( lambda[3] ) * v[i,4]^rho
      d5[i,p] <- dummy[i,5] * exp( lambda[4] ) * v[i,5]^rho
      d6[i,p] <- dummy[i,6] * exp( lambda[5] ) * v[i,6]^rho
      denominator[i,p] <- d1[i,p] + d2[i,p] + d3[i,p]
                         + d4[i,p] + d5[i,p] + d6[i,p]
      pi[i,p] <- numerator[i,p] / denominator[i,p]
    }
  }
  ### priors
  for (q in 1:5){ # P=6 party labels minus reference party is 5
    lambda[q] ~ dnorm( 0, tau.lambda )
  }
  tau.lambda <- pow(.25, -2)
  rho ~ dexp(.75) # has positive range, med about 1, mean 1.25, max 4.5
}
```

Table 5: Code for Bugs model

contesting an election (Morena in 2003–12, Green in 2006, MC in 2006 and 2012), with zero votes and seats columns, need not be dropped from matrices. Instead, multiplying the corresponding $\lambda * v^{\rho}$ term by contestation `dummy` annuls them from equation 2.

Package `r2jags` (Su and Yajima 2012) is an interface to send all this for Jags estimation:

```
results <- jags (  data = lrdata,
                   inits = lrinits,
                   parameters.to.save = lrparameters,
                   model.file = lambda.rho,
                   n.chains = 3,
                   n.iter = 50000,
                   n.burnin = 25000,
                   n.thin = 50  ).
```

We stopped the algorithm after 50 thousand iterations had been completed. The first 25 thousand were discarded to let the model adapt by updating the prior distributions of parameters to the data. Non-informative priors were used across the board: $\lambda_q \sim \mathscr{N}(0,4),\ q = 1,\ldots 5)$ and $\rho \sim \mathtt{exp}(\frac{3}{4})$ (the exponential distribution has positive range and, with this parameterization, a median of about 1, a mean of 1.25, and a maximum value of 4.5). Every fiftieth iteration of the final 25 thousand was saved as sample of parameters' posterior distributions (of size 3 chains $\times\, 25000/50 = 1500$).

To verify convergence, Tables 6–12 plot Gibbs sampler iterations (x-axis) and sampled values of model parameters (y-axis). One election/map is reported per Table, and one parameter per plot, with columns selecting three ways to measure votes: raw vote shares ($\mathtt{v}$), mean district vote shares ($\bar{\mathtt{v}}$), and population-weighted mean district votes ($\bar{\mathtt{v}}$). With three chains utilized in each Jags call, plots allow verification of two things: that chains had reached a steady state; and that all chains (one in green, one in red, one in blue in the

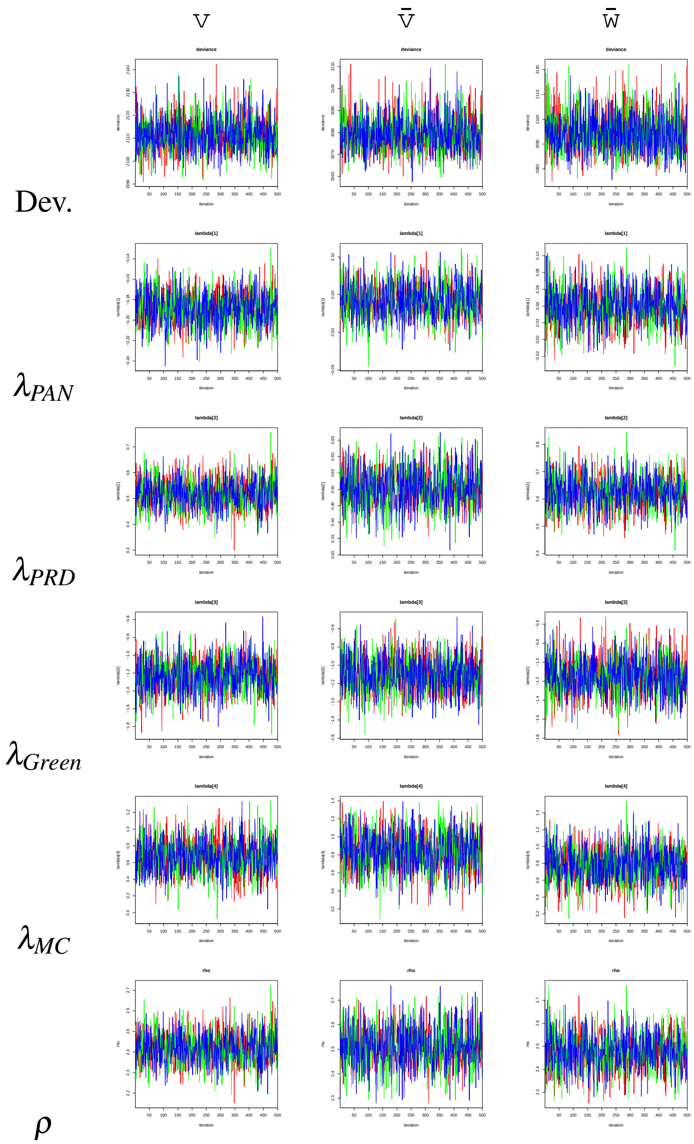

Table 6: Three-chain traceplots for 2003 with status quo map

plots) had converged. With no exception, the parameter samples we report in the text had all converged.

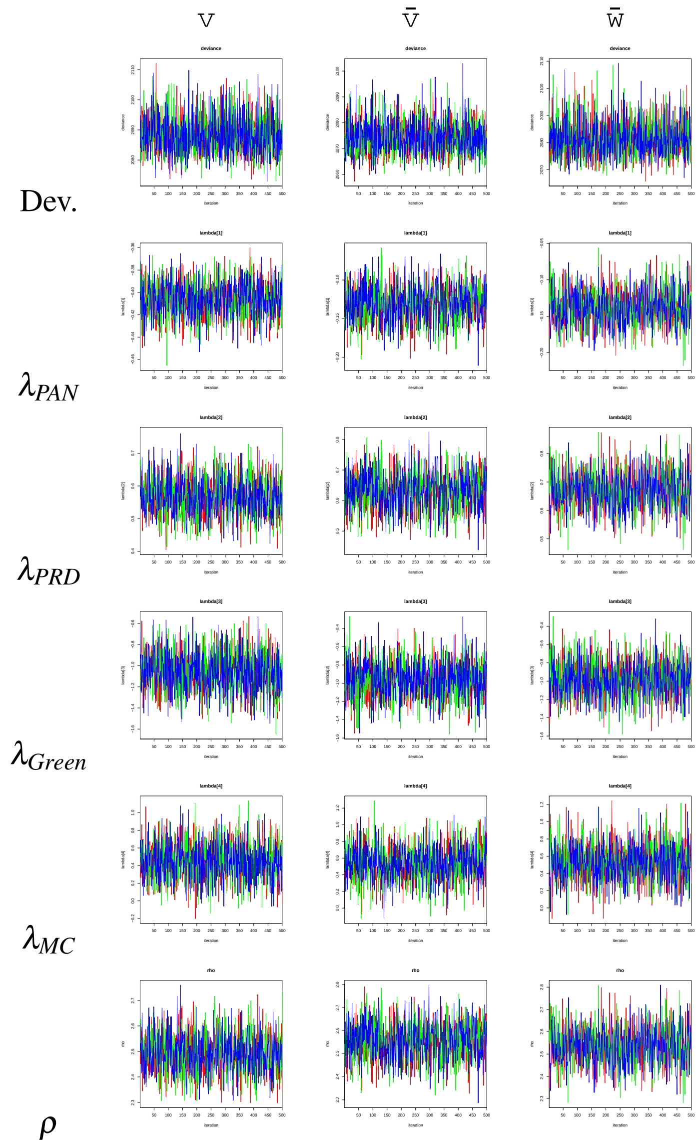

Table 7: Three-chain traceplots for 2003 with hypothetical map

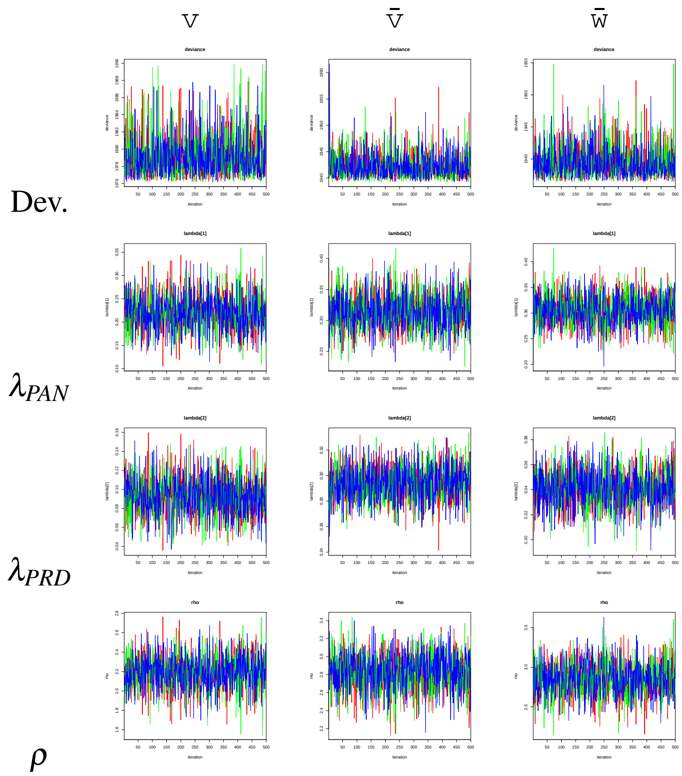

Table 8: Three-chain traceplots for 2006 with status quo map

How the PR tier mitigates under-representation

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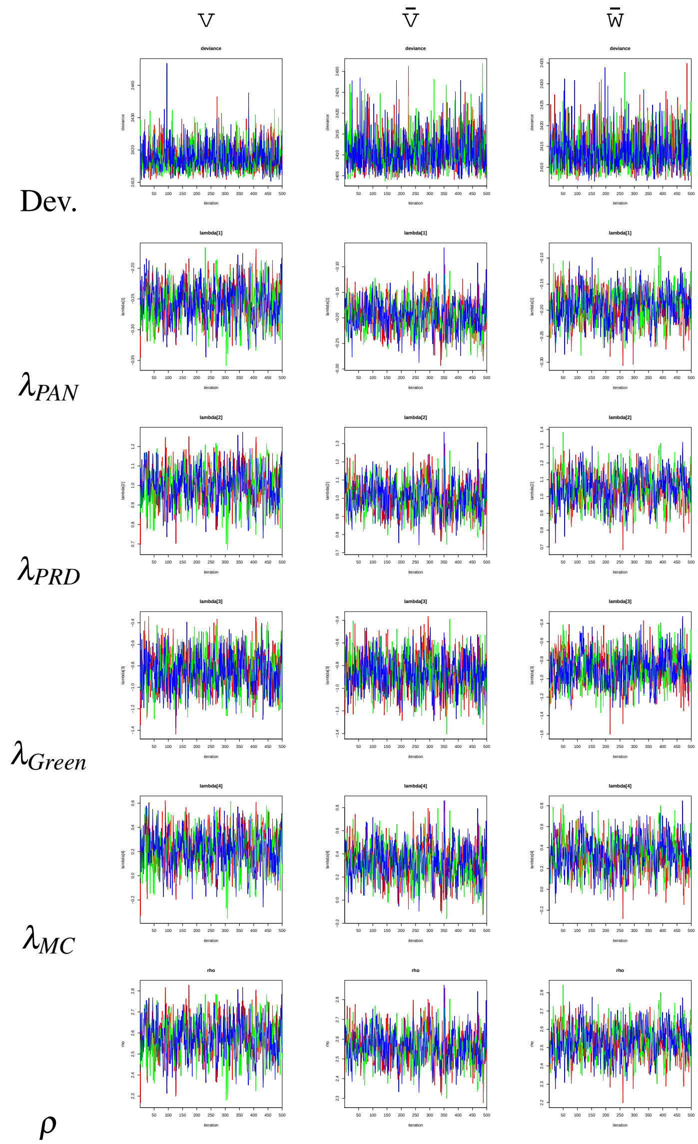


Table 9: Three-chain traceplots for 2009 with status quo map

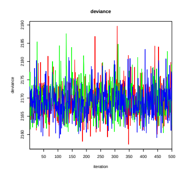
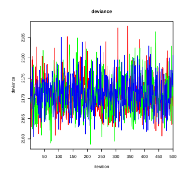
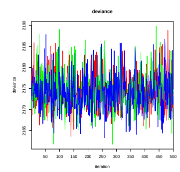
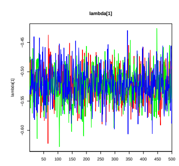
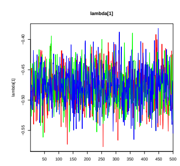
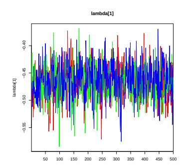
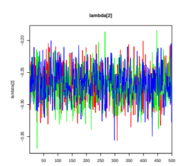
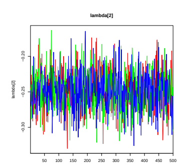
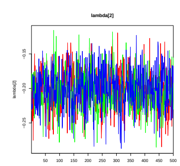
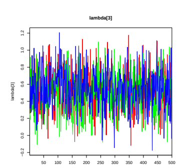
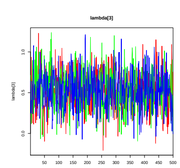
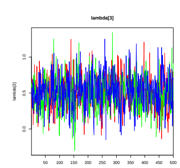
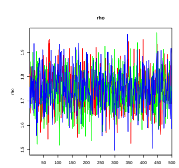
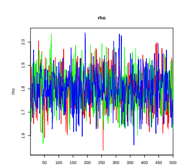
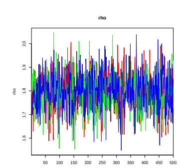
	$\mathtt{V}$	$\bar{\mathtt{V}}$	$\bar{\mathtt{W}}$
Dev.			
λ_{PAN}			
λ_{PRD}			
λ_{Green}			
ρ			

Table 10: Three-chain traceplots for 2012 with status quo map

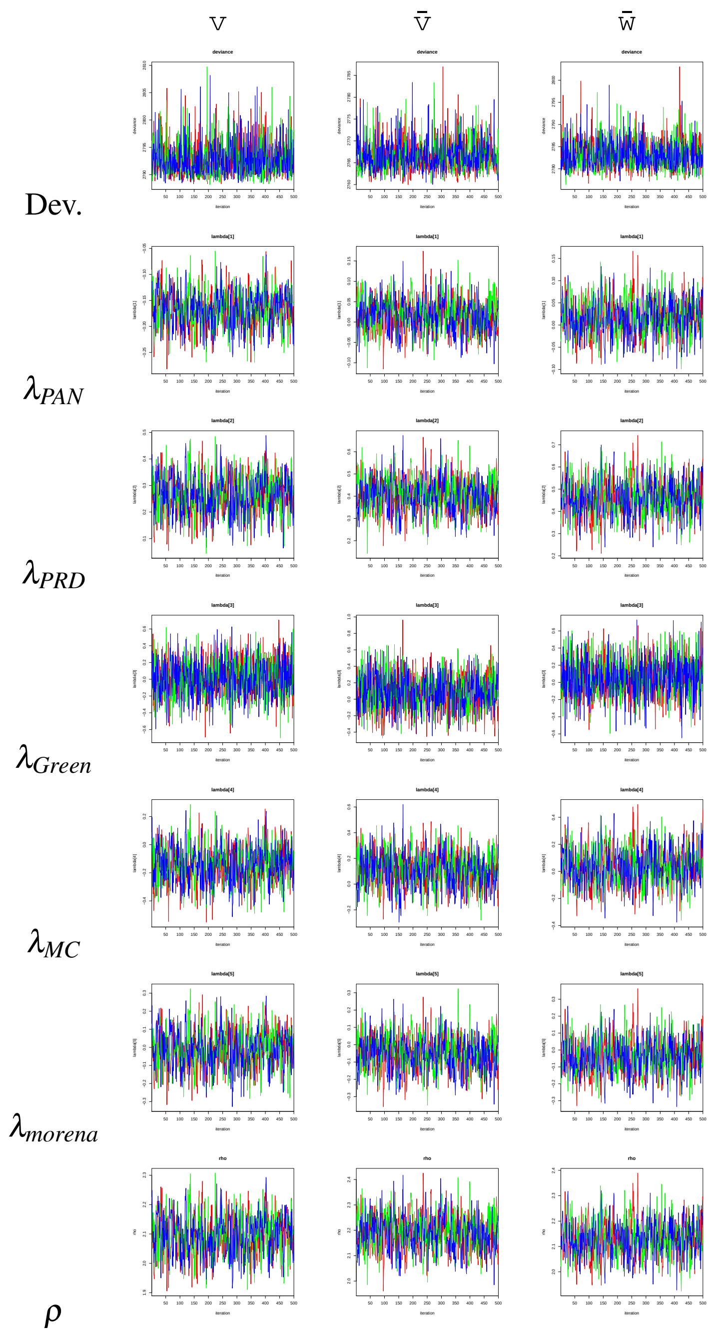


Table 11: Three-chain traceplots for 2015 with status quo map

	V	$\bar{V}$	$\bar{W}$
Dev.			
λ_{PAN}			
λ_{PRD}			
λ_{Green}			
λ_{MC}			
λ_{morena}			
ρ			

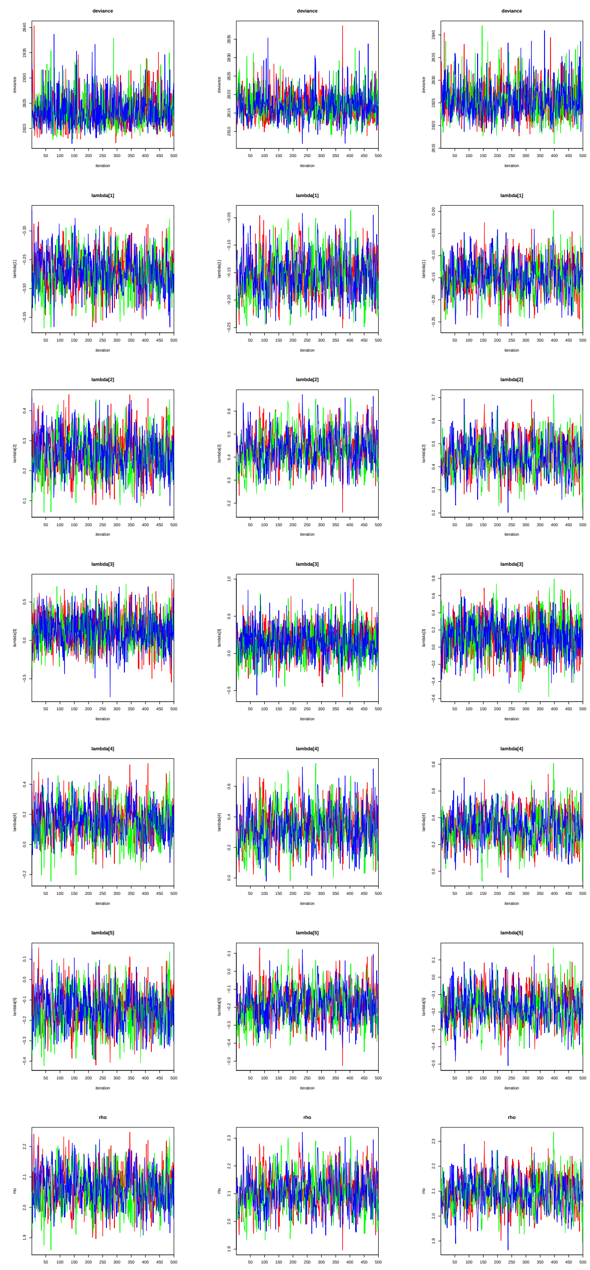
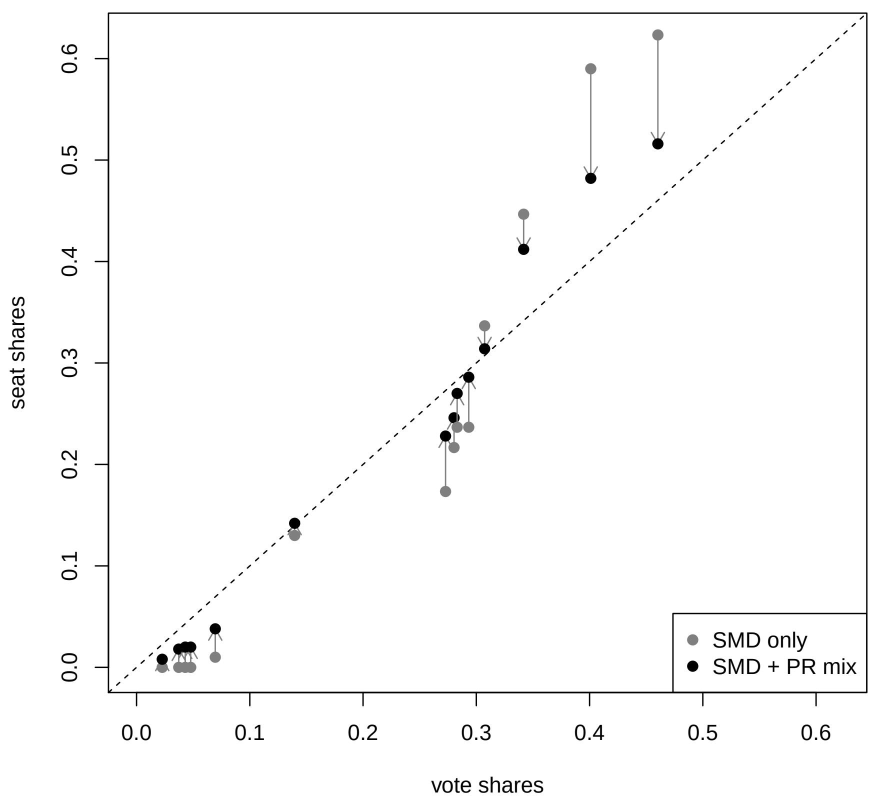


Table 12: Three-chain traceplots for 2015 with hypothetical map


seat shares
0.0
0.1
0.2
0.3
0.4
0.5
0.6
vote shares
0.0
0.1
0.2
0.3
0.4
0.5
0.6
SMD only
SMD + PR mix

Presented at the 2014 Annual Meeting of the American Political Science Association, Washington DC.

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