Measuring malapportionment, gerrymander, and turnout effects in multi-party systems

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Empirical procedure to measure and analyze the difference between the **votes** and **seats** that parties win in general elections

Central concern of electoral reform debates
Kendall&Stuart 1950, Rae 1967, Erikson 1972, Tufte 1973, Taagepera 1973,
Johnston 1979, Gudgin&Taylor 1980, Grofman 1983, Cain 1985,
Niemi&Fett 1986, King&Browning 1987, Gelman&King 1994,
Balinski&Young 2001, Cox&Katz 2002, Engstrom 2006...

Methods exist, but limited to (exceptional) two-party competition

Contributions

1. Extension to multi-party competition

2. Overcome small-N obstacle

3. Apply to recent Mexican elections

Motivation

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Key quantity of interest is the system's **partisan bias** = undue advantage conferred to some party in the conversion of votes into legislative seats

$\rightarrow$ Potential distortion wherever districts are drawn to allocate seats

Theory highlights three sources of partisan bias

1. Gerrymanders (e.g., Cox&Katz 2002)
2. Turnout differentials (Rosenstone&Hansen 1993)
3. Malapportionment and demographic shifts (Johnston 1979)

Procedure measures three sources

	v	s	s − v
Conservative	.369	.509	+.140
Labour	.305	.357	+.052
UK Independence Party	.126	.002	−.125
Liberal Democrat	.079	.012	−.066
Scottish National Party	.047	.086	+.039
Green	.038	.002	−.036

	small	large
too concentrated	helps	hurts
too dispersed	hurts	helps

Three models

- Grofman, Koetzle & Brunell (1997) $\rightarrow$ partisan bias breakdown
- King (1990) $\rightarrow$ multi-party partisan bias
- Linzer (2012) $\rightarrow$ data scarcity

Findings

1. Persistent bias against Mexican right
2. Components of bias often larger than the whole
3. Gerrymanders have offset PRI's large turnout advantage

Road map

Road map

What is partisan bias

> It is the excess/defect seat share that a
> party with half of the votes wins:
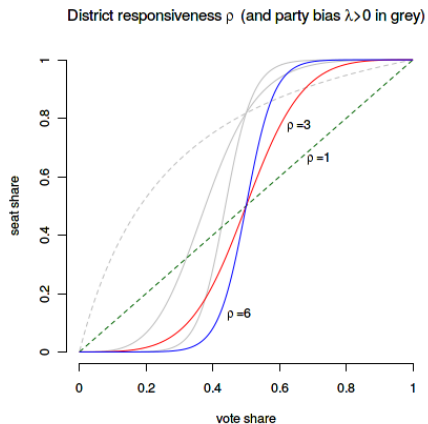
$$\lambda = (s \mid v = .5) - .5$$

- Two-party, balanced system assumed
- .5 threshold inappropriate for multi-party/imbalanced competition
- Alternative threshold not evident
- Relative quantity preferable

Fitting votes–seats curves: $s = f(v)$

District responsiveness ρ (and party bias λ>0 in grey)

$$\frac{s}{1-s} = \left(\frac{v}{1-v}\right)^3$$

$$\frac{s}{1-s} = \lambda \left(\frac{v}{1-v}\right)^{\rho} \iff$$

$$\text{logit}(s) = \ln \lambda + \rho \, \text{logit}(v)$$

Three sources of partisan bias

Districts	Pop.	Turnout	Raw votes			Vote shares		Seat shares	
			left	right	total	left	right	left	right
Gerrymander									
1 and 2	420	.5	147	63	210	**.7**	**.3**	1	0
3, 4 and 5	420	.5	84	126	210	**.4**	**.6**	0	1
nationwide	2100	.5	546	504	1050	.52	.48	.4	.6
Turnout									
1 and 2	420	**.70**	200	100	300	.67	.33	1	0
3, 4 and 5	420	**.35**	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6
Malapportionment									
1 and 2	**600**	.5	200	100	300	.67	.33	1	0
3, 4 and 5	**300**	.5	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6

Obstacle 1: partisan bias breakdown

Grofman, Koetzle & Brunell (1997): Three ways to aggregate
district returns nationwide

$$v_p = \sum_d v_{dp} \times \frac{\text{raw vote}_d}{\text{total raw vote}} \qquad (1)$$

$$\bar{v}_p = \sum_d v_{dp} \times \frac{1}{\text{total districts}} \qquad (2)$$

$$\bar{w}_p = \sum_d v_{dp} \times \frac{\text{population}_d}{\text{total population}} \qquad (3)$$

Fitting votes–seats curve with (1), (2), or (3) yields components
$\rightarrow$ with v you get **raw** partisan bias
$\rightarrow$ with $\bar{v}$ you get **gerrymander**-based
$\rightarrow$ with $\bar{w}$ you get **gerrymander** + **malapportionment**-based

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Obstacle 1: partisan bias breakdown

Trick is to estimate λ with each national vote measure in turn

Formulas:

a raw partisan bias $= \lambda^v$

b gerrymander-based $= \lambda^{\bar{v}}$

c malapportionment-based $= \lambda^{\bar{w}} - \lambda^{\bar{v}}$

d turnout-based $= \lambda^v - \lambda^{\bar{w}}$

e $a = b + c + d$

Obstacle 2: multi-party competition

Calvo&Micozzi (2005) pool opposition in denominator with control for effective num. parties

King (1990) is a votes-seats curve for a P-party system ($P \geq 2$)

$$E(s_p) = \frac{e^{\lambda_p} v_p^\rho}{\sum_{q=1}^{P} e^{\lambda_q} v_q^\rho}$$

- Akin to switch from dichotomous to multinomial logit regression
- Restricting $\lambda_1 = 0$ expresses bias relative to party $p = 1$

In sum:
Fit equation above using v, then $\bar{v}$, then $\bar{w}$;
rely on subtraction formulas to get measures of raw partisan bias and its components

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Road map

- Hegemonic party 1929–1997
- Three major parties: PRD (left), PRI, PAN (right) and minors
- Lower chamber of Congress elected every 3 years, concurrent w presidential race every 6 years
- Mixed system: 300 SMD + 200 PR seats
- Single-term limits removed in 2018
- Independent board (IFE) manages elections and redistricting

Obstacle 3: small-N

- A general election with P parties offers P data points to fit a votes-seats curve
- P typically small
- Multi-year approach: pool historic record... may compare apples/oranges in long-haul (cf. Márquez 2014a)
- Single-election approach preferable... but requires data multiplication strategy

Monte Carlo simulation

- Linzer (2012): approximates prob. distribution of national party vote returns from observed district outcomes (FMM)
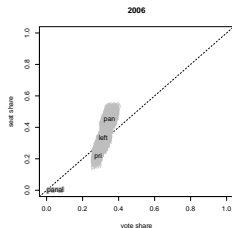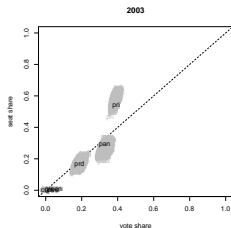- Use to simulate 1,000 elections (cf. Márquez 2014b)

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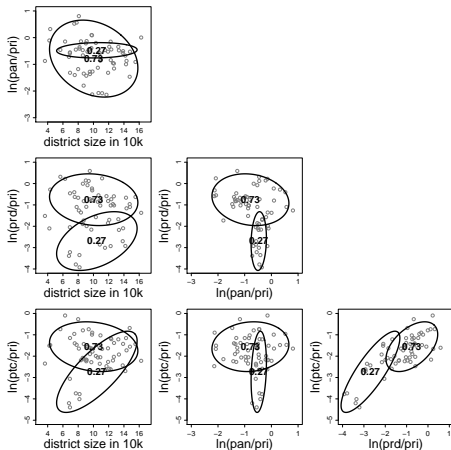- Use to simulate 1,000 elections (cf. Márquez 2014b)

Mixture model
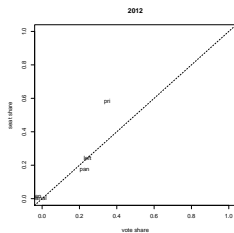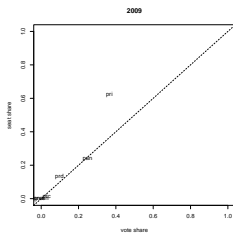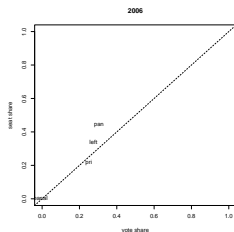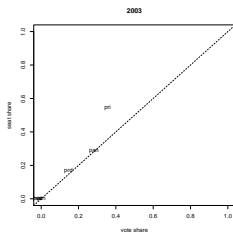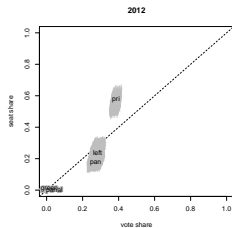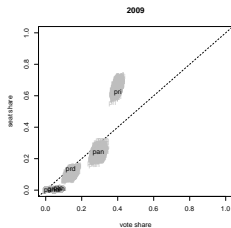
- Combines the properties of two or more prob. density functions: can approximate any arbitrary distribution
- Seek components (multivariate normals) and weights of log-ratio vote shares

Road map

Malapportionment occurs when sparsely populated areas get the same representation as the densely populated

Can arise by commission or by omission (or by accident)

Census lag: most recent decennial census must be used

- ... but no obligation to redistrict as soon as available
- 6-year lag on average: 199**7**, 200**6**, 201**5**
- 2015 map abandoned
- board considers $\pm 15\,\%$ imbalance normal (!)

District populations: linear projection
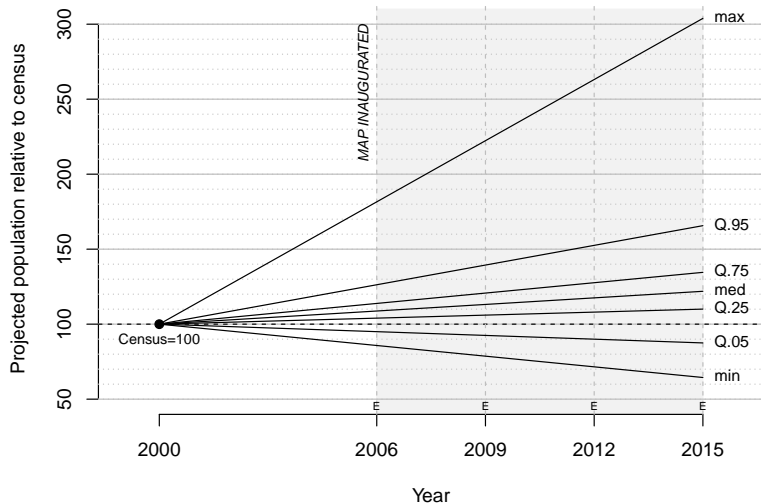
Plus: bureaucratic leeway in new district sizes

$$RRI = \frac{nat.pop./300}{\text{district size}}$$

2006 map (drawn with 2000 census)

district relative representation index (RRI)
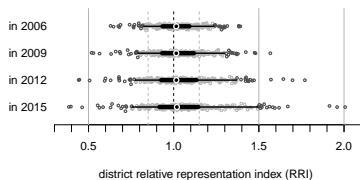
2015 map (drawn with 2010 census)
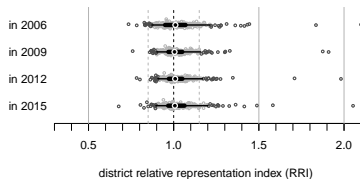
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Road map

Votes–seats curve fitted with MCMC (Jags via R)

100k iterations of 3 chains, every 500th obs. of last 50k used to sample posterior density

Visual inspection of model parameters to verify chain convergence

bias relative to PRI

partisan bias	Actual map			Hypothetical map		
	pan–pri	prd–pri	min–pri	pan–pri	prd–pri	min–pri
2003 election				(with 2006 map)		
raw	−.37 (0)	+.72 (0)	−1.01 (0)	−.41 (0)	+.57 (0)	−1.00 (0)
gerrym.	−.09 (0)	+.69 (0)	−.88 (0)	−.13 (0)	+.62 (0)	−.90 (0)
turnout	−.26 (0)	−.11 (0)	−.08 (0)	−.26 (0)	−.09 (0)	−.09 (0)
malapp.	−.01 (.11)	+.14 (0)	−.05 (0)	−.02 (.12)	+.05 (0)	−.02 (0)
2006 election						
raw	−.08 (0)	−.06 (0)	-1.10 (0)			
gerrym.	+.28 (0)	+.30 (0)	−.62 (0)			
turnout	−.36 (0)	−.41 (0)	−.43 (0)			
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malapp.	+.01 (.42)	+.06 (0)	−.02 (0)	−.00 (.38)	+.01 (0)	+.00 (0)

- Turnout always pro-PRI
- Malapp. always pro-left
- Redistricting abates malapp.
- Possibly cancelling effects

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Bias decomposition instructive

For 2006, Márquez (2014b) interprets point below the 45° line as anti-PRI partisan bias

Curva local de curules–votos
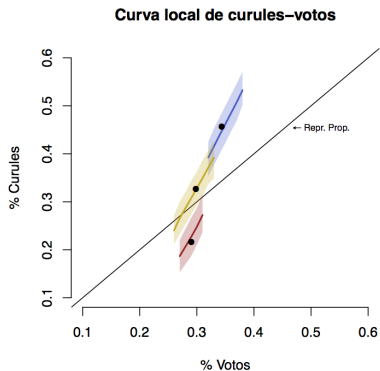
- Turnout still played for the PRI in 2006
- District lines did not
- Below 45° line conflates ρ and λ

Road map

Draw Mexico project $=$ offspring of *Public Mapping Project in U.S.*

Remove opaqueness from redistricting process

`DistrictBuilder` is open-source, web-based software

- enables widespread DIY redistricting thru cloud computing
- internet lets anyone draw/inspect maps: crowdsourcing
- redistricting contests in 6 US states $\rightarrow$ hundreds of legal plans

Application to Mexico (▸ Link: MexDemo) (Donations anyone?)

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2. Surprisingly small malapportionent effects
3. Pro-PRI turnout-based bias
4. Gerrymander effects large and volatile
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6. To-do: v-s swing ratios instead of λ's parametric assumptions
7. To-do: PR-tier compensates *national* party

Thank you!

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7. To-do: PR-tier compensates *national* party

Thank you!
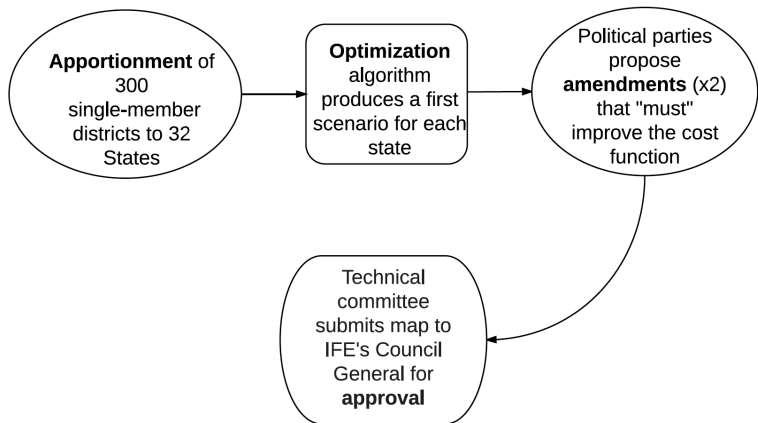
$$\texttt{Score} = .4 \times \texttt{PopBalance} + .3 \times \texttt{MunicBoundaries}$$
$$+ .2 \times \texttt{TravelTime} + .1 \times \texttt{Compactness}$$

Plus: minority representation (40 % indigenous pop.)

Board tolerates ± 15 % imbalance to accommodate this

Optimization algorithm

Simulated annealing = probabilistic meta-heuristic for optimization
locates a good approximation to the global optimum of the cost
function in a large search space

Thousands of iterations using electoral *secciones*

Combinatorial optimization algorithm used to generate the first
scenario in each state

Simulated Annealing
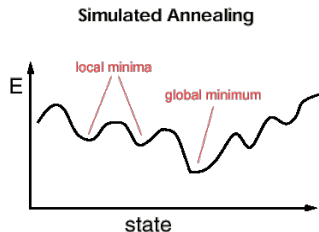
IFE claims that this is a public process, but the
operation and procedures are done behind closed doors

Proposals and counterproposals