Measuring malapportionment, gerrymander, and turnout effects in multi-party systems*

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Abstract

We extend the measurement and breakdown of partisan bias—i.e., undue advantage conferred to some party in the conversion of votes into legislative seats—to multi-party systems. Extant methods to estimate the gerrymander, malapportionment, and turnout sources of partisan bias were limited to two-party competition. We apply the empirical procedure developed to the study of recent lower chamber elections in Mexico. Analysis reveals advantageous, albeit modest partisan bias for Mexico's former hegemonic ruling party, and especially for the left, relative to the right. The method uncovers systematic and large turnout-based bias in favor of the PRI that has been offset by district boundaries substantively helping one or both other major parties.

This paper describes an empirical method for the analysis of the difference between the vote share that a party receives in the electorate and the seat share it subsequently wins in a plurality election. This difference is at the heart of debates about electoral reform and has received much scholarly attention in two-party competition from political scientists, economists, sociologists, geographers, mathematicians, and statisticians.[1] The concern has

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[1] See Altman and McDonald (2011), Balinski and Young (2001), Brady and Grofman (1991), Cain (1985), Cox and Katz (2002), Dahl (1956), Engstrom (2006), Erikson (1972), Gelman and King (1994), Grofman (1983), Grofman, Koetzle and Brunell (1997), Gudgin and Taylor (1980), Johnston (2002), Kendall and Stuart (1950), King and Browning (1987), Niemi and Fett (1986), Rae (1967), Rossiter, Johnston and Pattie (1997), Taagepera (1973), Trelles and Martínez (2012), Tufte (1973).

been the derivation of votes–seats curves assessing the electoral system's most fundamental function: the conversion of electoral support into assembly representation (Lijphart 1994).

The standard approach measures two distinct votes–seats curve characteristics, responsiveness and partisan bias (King and Browning 1987, Tufte 1973). The system's *responsiveness* is a measure of the seat bonus granted to large parties in detriment of the small. That plurality systems grant a substantial, if variable majority seat bonus, and that bonus size drops markedly when the formula is proportional representation, is well established (Linzer 2012, Taagepera and Shugart 1989). Responsiveness is a symmetric distortion, in the sense that substituting the top vote-getting party shifts the seat bonus earner accordingly. In contrast, *partisan bias* introduces an asymmetry in the seats–votes relationship. Partisan bias grants the beneficiary undue advantage in the ability to win assembly seats. Parties favored by systematic bias buy seats with fewer votes than others, countering a central tenet of democracy.

Theory highlights three sources of partisan bias. One is *malapportionment*—differences in district populations. A party with stronger voting bases in smaller districts receives a seat bonus nationwide (Jackman 1994, Johnston 2002). Another is *gerrymandering*—the practice of drawing district boundaries to achieve partisan advantage. Packing adversaries in few districts is a classic way to force them into vote wasting (Cox and Katz 2002, Engstrom 2006). The other is *turnout differences* across districts. A party enjoying stronger support in high- than low-turnout districts pays a seat penalty (Campbell 1996, Rosenstone and Hansen 1993). Our method measures the independent contribution to partisan bias of the three sources.

The procedure builds upon work by Grofman, Koetzle and Brunell (1997). Our contribution in three-fold. First, unlike Grofman, Koetzle and Brunell (and unlike every work cited in footnote 1) our approach drops the restrictive assumption of two-party competition. National two-party systems remain exceptional even among plurality systems (Cox 1997), so extending measurement to multi-party competition clears the way to empirical verification of theoretical propositions in numerous systems that were beyond reach. Second, we take "creeping malapportionment" (Johnston 2002), often ignored, into account. District size differentials arise by commission when cartography is adopted that deliberately underrepresents some citizens, but also by omission when failure to redistrict perpetuates secular demographic imbalance. Lastly, we inspect the case of Cámara de Diputados elections in Mexico to assess the proposed measuring method in a multi-party setting. Analysis uncovers small but systematic partisan bias against the right relative to the country's former hegemonic ruling party, but especially relative to the left. Decomposition of bias into additive components reveals that parts are often greater than the whole, contributing in opposing directions and therefore offsetting one another to a large extent.

The paper proceeds thus. Replicas of three models on which we build our argument are described in sections 1, 2, and 3. Each model removes obstacles: King (1990) the estimation of partisan bias in multi-party systems; Grofman, Koetzle and Brunell (1997) the challenge of measuring the size and polarity of three addtitive sources of partisan bias independently; and Linzer (2012) how to estimate quantities of interest with few observation points given that general elections are not really comparable. Our method stands at the intersection of this model trio. The rest of the paper applies the procedure to the case of recent Mexican Cámara de Diputados elections. While the main purpose is an assessment of the method, analysis is quite instructive in itself. Section 4 describes the mixed-member electoral system, isolating the plurality tier for analysis. Data for four consecutive elections in the 2003–2012 period, sources, and caveats are elaborated. Section 5 opens a digression inspecting malapportionment, which is shown to be substantial in the period. Population projections are discussed in order to control for creeping malapportionment. Section 6 reports results of the analysis, and section 7 concludes.

1 Partisan bias in the multi-party context

Formalizing partisan bias and responsiveness is the first step towards measurement. The two-party case is simpler (King and Browning 1987, Taagepera 1973, Tufte 1973) and extends to multi-party competition. It is a generalization of the cube law stipulating that

$$\frac{s}{1-s} = e^{\lambda}\left(\frac{v}{1-v}\right)^{\rho} \iff \text{logit}(s) = \lambda + \rho\,\text{logit}(v) \tag{1}$$

where s is the seat share won by the left party with vote share v; λ is the left party's bias relative to the right party (positive values favor the left, negative values favor the right); and ρ is responsiveness. Set $\lambda = 0$ and a system without partisan bias ensues. Figure 1 shows how parameters affect the votes-seats translation function.

Black lines illustrate variable responsiveness without partisan bias. A system with $\rho = 1$ is perfect PR, the ideal type against which real districts are often contrasted. It appears as the dotted diagonal line: every party winning $x\%$ of the vote gets, precisely, $x\%$ of seats. $\rho = 3$ characterizes the classic cube law, the dotted curve over-representing the winner (points above the diagonal). Here a party with 55% of the vote wins two-thirds of the seats, but with 33% it wins only one-tenth of the seats. As responsivity heightens, the curve grows steeper, until barely crossing the majority threshold suffices to win all the seats available.

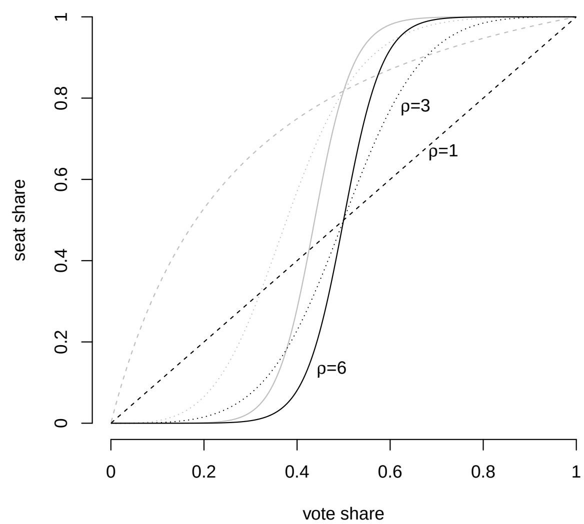

Figure 1: Illustration of model parameters. Partisan bias is set to $\lambda = 0$ in black lines. Gray lines replicate the black one-by-one with $\lambda = +1.5$.

Partisan bias is asymmetric party treatment in the votes–seats conversion. Gray lines replicate the values of ρ just discussed but with $\lambda = +1.5$ added. Bias in favor of the left party achieves a leftward pull of lines. In other words, a bias-favored party will require less effort to reach the threshold for large-party over-representation, cooking artificial parliamentary majorities with substantially less than a vote majority—as routinely occurred in the United Kingdom since the 1970s. (The gray dotted convex line shows how, due to logit links in Equation 1, partisan bias also reshapes the function's trace.)

A multi-party, estimable version of equation 1 is King (1990) (another is Calvo and Micozzi 2005). A transformation akin to a multinomial logit's departure from the dichotomous kind is used, formulating that party p's ($p = 1, 2, \ldots, P$) expected seat share is

$$E(s_p) = \frac{e^{\lambda_p} \times v_p^{\rho}}{\sum_{q=1}^{P} e^{\lambda_q} \times v_q^{\rho}} \tag{2}$$

with data and parameters now indexed for party identification. Setting $\lambda_1 = 0$ restricts the remainder $\lambda_{p \neq 1}$ to express partisan bias in relation to party $p = 1$ without loss of generality. This is convenient. Partisan bias in two-party competition is asymmetry in a votes–seats curve centered on $v = .5$, as in Figure 1. While the partisan bias shift operates likewise, there is no reason to expect a .5-centered curve in multi-party competition. Nor is it a priori evident what vote share serves as center point, a difficulty towards expressing a partisan bias estimate $\hat{\lambda}_p$ as a percentage points advantage or handicap for party p in the votes-to-seats conversion—as routinely done in analysis of two-party systems (e.g., Cox and Katz 2002). Finding $\lambda_{p\neq 1} < 0$ is evidence of bias against party $p \neq 1$ relative to party $p = 1$.

2 Three sources of partisan bias

At the root of partisan bias in systems with multiple districts are differences in the geographic concentration of parties' vote strength. One party with 20 percent of the vote nationwide evenly spread across districts may fail to win a single seat, but another with much less support that is concentrated in few districts will, in fact, get seats. In general, vote concentration helps smaller parties while hurting larger ones through vote (and therefore seat) wasting (Calvo and Rodden 2015). In the end, several forces add up and interact to yield partisan bias (Gudgin and Taylor 1980).

Grofman, Koetzle and Brunell (1997, henceforth GKB) demonstrate that what we shall now call *raw partisan bias* (λ) has three clear and distinct sources and offer a procedure to separate empirically the independent, additive contribution of each.[2]

- *Gerrymanders* (GKB call this source of partisan bias 'distributional') correspond to different party distributions of vote wasting across districts. Vote wasting may be deliberate (e.g., the strategy of packing opponent in few districts mentioned above), but it may also arise by the accidents of geography (e.g., when districts cannot cross state boundaries and the state is a party stronghold).

- *Turnout* differentials across districts. Those who abstain from voting are passively lowering the bar to win a district's seat. Parties stronger in the lower turnout districts will achieve victories with fewer votes than others, improving their votes:seats ratio. Turnout differentials arise when correlates of participation, such as socio-economic status, vary systematically across districts, or when parties mobilize more effectively in some areas than in others.

- *Malapportionment* arises when sparsely populated regions get the same representation as more densely populated ones. It can be found whenever multiple districts are drawn for the purpose of seat allocation. District size differentials may be designed,

[2]Other elements highlighted by Gudgin and Taylor (1980) that our analysis of raw partisan bias ignores are the cube-law's bonus, large third-party votes, and possible interactions between all the elements. The bonus is, in fact, captured by the system's responsiveness parameter and therefore distinct from partisan bias in our framework (more on this in section 3). Calvo (2009) models departures from bipartism explicitly. Interactions remain interesting venues for future research.

			Raw votes			Vote shares		Seat shares	
Districts	Pop.	Turnout	left	right	total	left	right	left	right
Gerrymander-based partisan bias only									
1 and 2	420	.5	147	63	210	**.7**	**.3**	1	0
3, 4 and 5	420	.5	84	126	210	**.4**	**.6**	0	1
nationwide	2100	.5	546	504	1050	.52	.48	.4	.6
Turnout-based partisan bias only									
1 and 2	420	**.70**	200	100	300	.67	.33	1	0
3, 4 and 5	420	**.35**	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6
Malapportionment-based partisan bias only									
1 and 2	**600**	.5	200	100	300	.67	.33	1	0
3, 4 and 5	**300**	.5	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6

Table 1: Illustrative five-district system scenarios

> adopting cartography that deliberately under-represents some citizens. Senates often grant states equal representation, regardless of population. But it inevitably germinates passively by failure to redraw district boundaries that compensate for secular demographic imbalance. Creeping malapportionment ensues.

The scenarios in Table 1, which draw heavily from examples in GKB, illustrate the sources operating in isolation from one another. The division of vote and seat shares nationwide and the degree of partisan bias remain constant in all scenarios: the left party suffers a 12 percentage point deficit in representation, with 52% of votes but just 40% of seats (it won two of five districts); and the right party enjoys 12 percent overrepresentation, winning 60% of seats with just 48% of votes. Other traits change, one at a time. The first scenario has equal-sized and constant-turnout districts[3] that nonetheless manifest partisan differences in votes wasted, the left party winning seats by wider margins (+.4) than the right (+.2). The sole source of partisan bias is gerrymanders. Shifting district boundaries might re-allocate wasted votes in such way that another district tips towards the left.

The next scenario has equal-sized districts and winning margins uncorrelated with the vote distribution, but varying turnout that is not orthogonal to vote shares. Right and left are winning seats with the exact same margins, but the right wins in lower-turnout districts—half, in fact, the turnout of districts won by the left. As a consequence, right seats come quite cheaper than the left's. In this case, partisan bias is the product of turnout differentials alone, against the left. Extend the reach of the left's mobilization effort beyond districts 1

[3]A less restrictive scenario allows size and turnout differences across districts with distributions that are independent of the distribution of partisan support.

and 2, and other seats might be won.

The other scenario has equal-turnout districts and winning margins uncorrelated with party vote strength, but different district sizes that do correlate with the latter. Again, both parties are winning with equal margins, but the right is doing it in districts half as populous as those won by the left. The consequence is a more efficient conversion of quite similar total votes into seats for the right than the left. This is partisan bias attributable to malapportionment by itself.

The formalization of the votes-seats curve in section 1 assumed that votes in Equations 1 and 2 are the party's share of the national vote v_p—the party's vote aggregated across districts divided by the total raw vote nationwide. This standard mode of national aggregation of district-level vote returns measures raw partisan bias. Noting that party p's raw vote in district d is the product of its district vote share v_{dp} and the district's raw vote, the party's vote share nationwide can be expressed as

$$v_p = \sum_d v_{dp} \times \frac{\text{raw vote}_d}{\text{total raw vote}}. \tag{3}$$

This algebraic transformation eases consideration of two alternative national aggregations of district returns in GKB's separation argument. One is party p's mean district vote share, defined as

$$\bar{v}_p = \sum_d v_{dp} \times \frac{1}{\text{total districts}}. \tag{4}$$

The other is party p's population-weighted mean district vote share, defined as

$$\bar{w}_p = \sum_d v_{dp} \times \frac{\text{population}_d}{\text{total population}}. \tag{5}$$

Following the insight of Tufte's (1973) seminal work (further elaborated in Gelman and King 1994), fitting the votes–seats curve using $\bar{v}_p$ instead of v_p yields gerrymander-based partisan bias. This is so because $\bar{v}_p$ aggregates district vote shares with disregard to district size and voter turnout. In the same spirit, GKB show that relying on $\bar{w}_p$ (an aggregate compounding district vote shares and relative district populations) yields estimates conflating gerrymander- and malapportionment-based partisan bias. So subtracting partisan bias estimated with $\bar{v}_p$ from partisan bias estimated with $\bar{w}_p$ yields pure malapportionment-based partisan bias. And, because raw partisan bias conflates all three sources, subtracting partisan bias estimated with $\bar{w}_p$ from partisan bias estimated with v_p yields pure turnout-based partisan bias.[4]

[4]The notation (subscripts dropped) that GKB use for v, $\bar{v}$, and $\bar{w}$ is R, P, and M, respectively.

In sum, the GKB procedure consists of repeatedly fitting equation 2, alternatively using v_p, then $\bar{v}_p$, and $\bar{w}_p$. Denoting λ_p^v, $\lambda_p^{\bar{v}}$, and $\lambda_p^{\bar{w}}$ party p's partisan bias parameter from each fitting, the following subtractions bring forth the quantities of interest:

a. raw partisan bias $= \lambda_p^v$,

b. gerrymander-based partisan bias $= \lambda_p^{\bar{v}}$,

c. malapportionment-based partisan bias $= \lambda_p^{\bar{w}} - \lambda_p^{\bar{v}}$, and

d. turnout-based partisan bias $= \lambda_p^v - \lambda_p^{\bar{w}}$.

It is easy to verify that raw partisan bias is the sum of the three components in the GKB framework ($\mathrm{a} = \mathrm{b} + \mathrm{c} + \mathrm{d}$).

3 Measurement via Monte Carlo simulation

The final obstacle is fitting the votes–seats curve to data of interest. The general problem is one of scarcity of data points to achieve this. Each party fielding candidates in a general election offers one point in a votes–seats coordinate system, and relatively few parties do in each election. One common approach to overcome this limitation is pooling several elections in the historical record for estimation (e.g., Márquez 2014*a*). Multi-election studies inevitably sacrifice comparability, especially over the longer haul (Jackman 1994). Single-election studies are therefore preferable (Niemi and Fett 1986), with alternative procedures to multiply data points desirable. Dis-aggregation in a federal case is one strategy: reliance on state-level congressional election votes and seats, instead of standard national-level aggregates. But by treating state party systems as the national, this has pitfalls similar to longitudinal multiplication.

The multiplication approach adopted here, inspired by Linzer (2012) and explored by Márquez (2014*b*), relies on Monte Carlo simulation instead.[5] Towards this goal, a probability density of national party vote returns is approximated from observed district outcomes with a finite mixture model. The FMM works with district-level data, assuming sub-populations with known distributions are present (e.g., some districts where party 1 vote grows at the expense of party 2's, others where they grow jointly) but information

[5]We did not pursue Linzer's swing ratios. The relation of that quantity with the notion of partisan bias adopted here is straightforward in balanced two-party competition (see Linzer 2012:410), but not when multiple parties compete. We therefore followed his method part of the way, borrowing his code to generate hypothetical national party vote and seat pairs, then fitting a standard votes–seats curve on those.

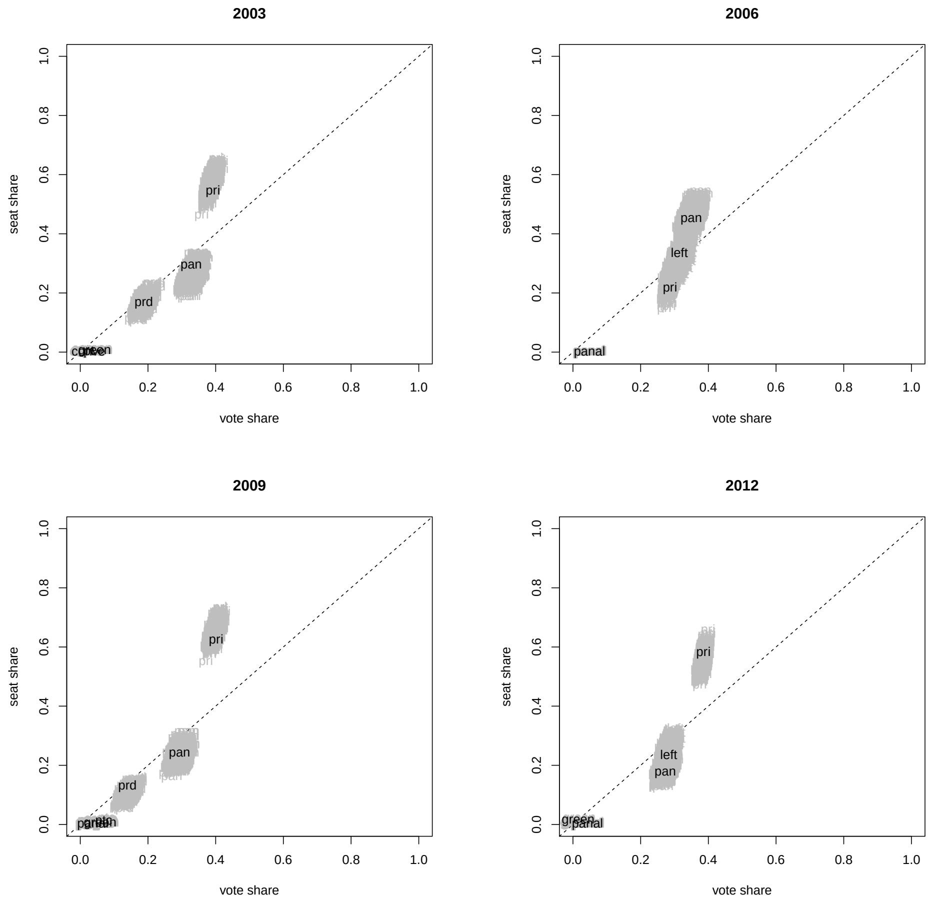

Figure 2: Votes and plurality seats in four elections. Actual data in black, simulated elections in gray. Source: prepared with data from `www.ine.org.mx`.

to match districts to sub-populations is unavailable. A mix of the known distributions describes the unknown distribution. Repeated draws of hypothetical district outcomes from the mix reflect variation in the sources of partisan bias: in district size, in turnout, and in vote choice (information fed to the FMM); and aggregating them nationwide yields a large sample of vote–seat simulations that are supported by the data.[6]

Figure 2 describes the output of the simulation process for the case study that section 4 presents in detail. Observed national votes received and seats won appear as black labels for three consecutive elections. Simulated elections are surrounding clouds of gray labels. The mixture offers reliable counter-factual predictions about party seats shares for vote shares near observed points only (about ± 10 percent, Linzer 2012:fn. 8). The single-election approach is not suited for extreme counter-factual prediction (Gelman and King 1994). We

[6]An online appendix describing the multiplication procedure in detail, and with commented code extending Linzer's procedure will be posted upon publication.

show below that the approach is nonetheless very attractive.

Another problem is parties fielding candidates in less than all legislative districts. Strategic parties often withdraw in anticipation of a hopeless race. The mix can handle this challenge by considering patterns of district contestation separately. More problematic are partial coalitions between parties—when parties A and B field joint candidates in some districts but run against each other in the rest, complicating national votes and seats aggregation. We now turn attention to a description the case study. We return to elaborate on the solution adopted for partial coalitions after this. A better grip of the minutiae of Mexican congressional elections eases exposition of the problem at hand.

4 Mexican Cámara de Diputados elections

We demonstrate the procedure's usefulness by examining recent elections of Mexico's lower chamber of Congress. The Cámara de Diputados has been elected with a mixed-member electoral system for decades. Systems of this nature give voters a direct role in the election of representatives from single-member districts, while also using some form proportional representation to improve the odds that parties will receive about as many seats as they are worth in votes (Shugart and Wattenberg 2001*a*). Above the PR-tier, however, lies a standard plurality system that we examine in isolation. This is justified because diputado campaigns and voting take place in the plurality tier.[7]

Like other plurality systems, we suspect strong distortions in the form of partisan bias to be arising. Substantial malapportionment is one cause for concern. The procedure offers a way to contrast malapportionment to gerrymanders and turnout effects, and answer theoretically interesting questions.

A new election arbiter was inaugurated, and the district map redrawn, towards the first free and fair congressional election in 1997. Election management was delegated to a newly-appointed Federal Electoral Institute (IFE), an independent board (Lujambio and Vives Segl 2008). Closer inspection of the of the IFE's structure and process reveals how, in practice, it is a power-sharing agent of the major congressional parties (Estévez, Magar and Rosas 2008). Systematic partisan segmentation of the board is evidence that major-party influence in electoral regulation is unremitting. This will guide expectations in the analysis.

[7]Each voter casts a unique, non-exclusive, pooling vote to choose among candidates in 300 single-member districts with seats allocated by plurality. Votes then transfer to the party to which the candidate originally voted belongs, in order to allocate seats in five second-tier districts of magnitude 40, by closed-list Hare proportional representation, using a 2 percent threshold (Weldon 2001).

The board redrew district boundaries, using machine-assisted mapping, in 1997 and 2006. Redistricting was ready for adoption, but postponed, in 2015. We take advantage of this failure in the analysis.

Redistricting is done in two phases: seats are apportioned to states, based on population; then a machine-assisted map is drawn (Altman, Magar, McDonald and Trelles 2014, Trelles and Martínez 2012 describe and analyze the automated process). Section 5 reveals structural features of the process that introduce substantial malapportionment.

We examine diputado elections concurrent with the presidential races of 2006 and 2012, and the midterm elections of 2003 and 2009. All elections took place under near-identical rules but different maps. The earliest in the series used the 1997 map, others the 2006 map. Márquez (2014*a*) used a multi-election approach of votes and seats won over two decades, uncovering a degree of responsivity characteristic of plurality systems and substantive partisan bias against the PAN. We follow the single-election route instead, examining each election in the period in isolation.

Data was compiled from IFE's official election returns, available at `www.ine.mx`. Analysis requires district-level data to simulate national vote and seat shares by party. We chose to rely on sección-level returns in order to aggregate vote returns in the three races into districts corresponding to the 2015 map that was never adopted.[8] Contrast of actual elections (using the 2006 map) to hypothetical ones (using the 2015 map) is illustrative. Party district-level vote shares are the number of votes won divided by the district's effective vote. The effective vote is the district's vote total minus voided ballots, votes for write-in candidates, and votes for parties failing to clear the 2 percent threshold nationwide.[9] Analysis also required sección-level population figures. Years 2000 and 2010 census data, and 2005 population count were compiled from data prepared by the census bureau (INEGI) for the purpose of redistricting. Linear interpolation gave us a point estimate of election year sección-level populations.

Seven parties remained for analysis. Three are major, with vote shares above 15 percent (albeit volatile, as seen in Figure 2): the right-of-center National Action Party (PAN) that controlled the presidency throughout the period; the formerly hegemonic Institutional Revolutionary Party (PRI); and the left-of-center Democratic Revolution Party (PRD). Minor parties with vote shares not much above the 2-percent threshold complete the slate:

[8] *Secciones electorales* are analogous to U.S. census tracts, but bigger (median population in the 2010 census was 1,280, with a maximum at 79,232). The 1997, 2006, and 2015 maps (kindly shared by IFE's cartography department) relate more than 66 thousand *secciones*, the basic units for district cartography, to 300 congressional districts. This makes reconstitution of hypothetical election outcomes in the period possible.

[9] Linzer (2012:fn. 4) recommends likewise. Five parties were removed thus in 2003 (PAS, PSN, FC, MP, and PLM) and one in 2006 (ASDC).

the Green (personalistic), the PANAL (close to the teachers' union), Convergencia (personalistic), and PT (personalistic of Maoist bend). Major parties contested every district systematically, but often in pre-election coalitions. The PRD fielded common candidates with Convergencia and the PT nationwide in 2006 and in 2012 (they are labeled 'left' in plots); the PRI did it with the Green in 2003 and 2006; and Convergencia and PT ran together in 2009. Partial PRI–Green coalitions took place in 2009 and 2012—they fielded joint candidates in some districts but ran against each other in the rest. Partial coalitions complicate national votes and seats aggregation. The option of computing separate aggregates for PRI, for Green, and for PRI–Green was attractive for describing the situation faithfully, but was abandoned due to result sensitivity. The PRI would wrongly appear as fielding no candidates in large numbers of districts (one-fifth of all in 2009, two-thirds in 2012), artificially diluting its true electoral strength. We opted instead to exploit the partners' size asymmetry by considering PRI–Green votes won in tandem as if votes the PRI won solo—thus contributing returns in all districts for the national aggregate (it appears as 'pri' in plots). While far from satisfactory—the Green is the largest and most successful of minor parties (it may soon qualify as major), overestimating the PRI's strength—the solution is practical while crediting the fact that Greens never failed to join the PRI electorally in the period. Alternative solutions are nonetheless welcome.

Before showing the procedure in action, the next section turns attention to a potentially important source of partisan bias.

5 One Mexican, one vote?

It could be argued that malapportionment in mixed-member systems is of little, if any consequence. The PR tier is, after all, specifically designed to compensate for imbalances ensuing from plurality races. We debunk this claim and show the prevalence of malapportionment in the case study.

Compensating *parties* bears relation to, but is not the same as compensating *citizens* of over-populated districts. Keeping these compensations distinct is important. Much of the evidence presented here, as in the scholarly literature, deals with party votes:seats ratios. From the normative standpoint, however, it is the 'one person, one vote' principle—one of Dahl's (1972) preconditions of democratic government—that malapportionment antagonizes, and party compensation is not designed to redress this imbalance.[10] Malappor-

[10]The exception would be a system with perfectly district-based parties, where measures to achieve party proportionality are, in fact, compensations for the district citizenry only. Shift away from fully local parties, towards party nationalization, and party compensation stops accruing to citizens of the underrepresented

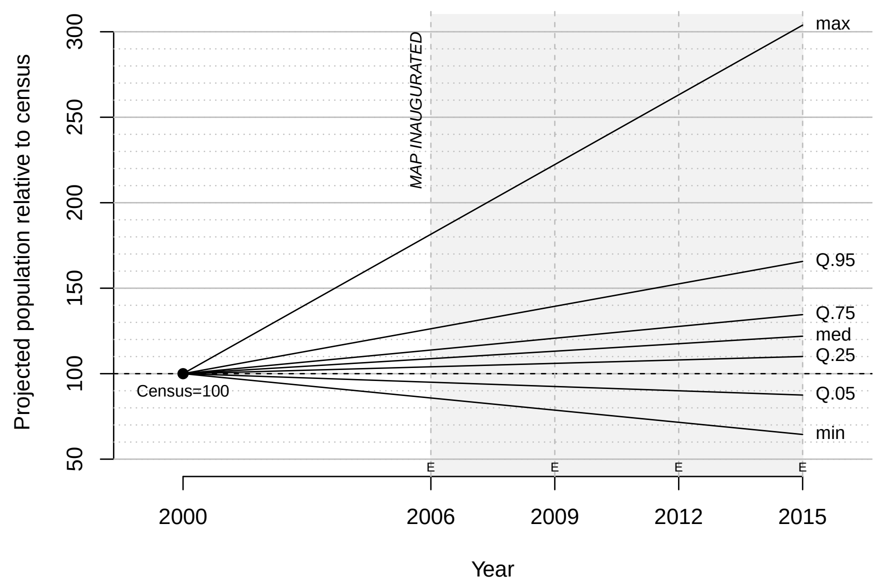

Figure 3: The 2006 map and demographic change. Population projections relied on the 2000–2010 censuses rate of change. Letters E in the horizontal axis indicate elections using this map. Source: prepared with data from `www.inegi.org.mx` and `www.ine.mx`.

tionment has the potential to substantially distort representation even in mixed-member systems.

Unequally-sized districts are common practice in Mexico in spite of the automated application of clear quantitative redistricting criteria since 1997. In fact, Mexican parties in general, and IFE in particular, have been remarkably tolerant of this practice. The very structure of the redistricting process involves a census lag making substantial malapportionment almost inevitable—even when a map is inaugurated! Grasping this anomaly requires a move away from the static census figures used by redistricters in the production of legal plans, and towards an attempt to embrace the constantly changing nature of demography. Demography, after all, is at the heart of the need to redistrict periodically.

The census lag is a major source of malapportionment. The constitution mandates the use of the census for redistricting, but has no obligation to redistrict as soon as data become available. As a consequence, six or more years had passed by the time newly-drawn district boundaries were inaugurated since democratization, injecting a fair amount obsolescence into the new maps. Malapportionment had crept in even before the map had aged. Figure 3 illustrates our point. We rely on linear projection of inter-census populations to estimate

districts only.

yearly district growth.[11] Leaving the question of original size disparities for later, we focus attention on the demography of boundary drawing.

Compared to the 2000 census, projected district populations in 2006 are off by 9.7 percent in absolute value on average, with a standard deviation of 10.6 percent. With one additional year away from the reference census, the 1997 map had even less success achieving proper apportionment at inception. Indexing 2000 census district populations at 100, as the figure does, reveals how different the most demographically dynamic units were in paper and in reality. The fastest-growing district was 88 percent larger in 2006 than what census data otherwise suggested (the line labeled 'max'). The district shedding most population was 16 percent smaller (the 'min' line). These are outliers, but central tendencies reflect sizable magnitude lags as well. The inter-quartile range (lines Q.25 and Q.75) covered 1–13 percent above census in the 2006 election, and expanded to 4–20, 7–27, and 10–35 in the three subsequent diputado elections using that map.

The census lag is not alone in breeding malapportionment. Small deviations around mean district population are unavoidable worldwide, especially as a map ages. But what constitutes significant deviation depends on the political context. Courts in the U.S. have struck down new district maps bearing less than 1% differences without proper justification (Tucker 1985). Redistricting authorities generally view a *de minimus* population deviations of as little as one or zero persons between congressional districts as desirable to inoculate against litigation. In stark contrast, the election board has considered deviations between 10% (in 2006) and 15% (in 1997 and 2015) above or below mean state district size perfectly normal (Lujambio and Vives Segl 2008, Trelles and Martínez 2012). The spread is designed to give leeway in the explicit attempt to minimize the number of municipalities that must be partitioned into two (or more) federal districts. It also accommodates the constitutional requirement to keep municipalities with large indigenous populations within the same district.[12] The cost of leeway is that a pair of districts with populations exactly at the bounds of the larger spread make citizens at the bottom end worth one-third more in Congress than those at the top end. Surprisingly, no party has ever challenged this spread in court.

[11]More precisely, the 2000–2005 rate of growth was used before year 2006, and the 2005–2010 rate afterwards. Population projections for different maps were done after sección census populations had been aggregated into districts. Performing linear projection on secciones before any aggregation might have been preferable (because based on much smaller units), but a fair amount of over-populated secciones are routinely broken into new ones between elections, complicating the projection exercise.

[12]The rationale is minority protection. In practice, however, there is no consideration of primordial differences between contiguous, and possibly antagonistic indigenous communities—all fall in the 'indigenous' category, and grouped in one congressional district. The case of Arizona's Hopi and Navajo tribes comes to mind.

The census lag and bureaucratic leeway compound to produce actual malapportionment in congressional districts. We inspect how it distorts representation as Ansolabehere, Gerber and Snyder (2002). We measure a district's relative representation index as $RRI = \frac{1/\text{district size}}{300/\text{national population}}$, where the numerator is the number of congressional seats per person in the district and the denominator is the average number of congressional seats per person nationwide (300 is the number of congressional seats). A district with unity index value has representation matching the 'one person, one vote' ideal perfectly. Values above one indicate over-representation, values below one under-representation, and the measure is continuous. An example shows how the index is interpreted. The size of the 3rd district of Aguascalientes in 2012 was about 306,000 inhabitants, and the (projected) quota about 387,000, so the district in question had 26 percent more representation than the national average, for an index value of 1.26. As before, we used population projections to compute *RRI*s.

The percentiles corresponding to *RRI*s at .85 and 1.15 (the bounds of the board's ± 15 percent tolerance range) in 2006 were 10 and 87, respectively, implying that $10 + 100 - 87 = 23$ percent of districts were off-range.[13] That was the map's inaugural year. By 2012, as many as 35 percent districts off the tolerance range, and by 2015 just shy of two-fifths. Drawing equal-sized district boundaries with old population data is no easy task, and distorts representation within states over and above between-state imbalances.

Figure 4 summarizes the effects of creeping (and original) malapportionment in representation. Vertical dashed lines in gray mark the board's tolerance band. It has been amply surpassed, systematically. Consider the top plot, portraying the status quo map, first. Each point represents one district. The fine horizontal line connects the *RRI* values corresponding to the 5th and 95th percentiles—both well outside the tolerance band, since the map's inception. The thick horizontal line is the inter-quartile range, not far from covering the full tolerance band since 2012—when it does, half the districts will be off-range! For the 2015 midterm congressional election, citizens in the plot's right-most districts (in central Monterrey and two in battered Juárez) will be worth *four times more* in Congress than citizens in the left-most districts (one each in suburban Monterrey and Mexico City, the other in Cancún). In matters politic, citizens at one quartile will be nearly worth twice as much as those at the other quartile.

The bottom plot offers a counter-factual, adopting the hypothetical map that was drawn towards the 2015 election but abandoned. It is quite clear that, even if outliers remain

[13]This is not entirely accurate, as the ± 15 range is in reference to average state population (not average national population, analyzed here, see Altman et al. 2014). The percentiles using state population *RRI*s are .06 and .89, or 17 percent out-of-legal-range districts in 2006.

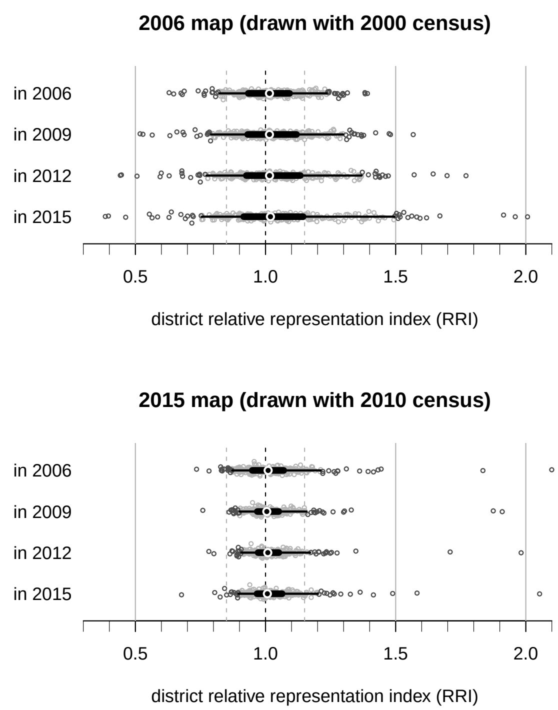

Figure 4: Representation in four elections and two maps. Panels portray (top) the status quo and (bottom) the hypothetical maps. Finer horizontal lines connect the 5th and 95th percentiles, thicker lines the quartiles, and white circles indicate the median. Points are districts.

upon inauguration, the hypothetical map would have represented Mexicans much better than the status quo (note the narrower horizontal lines). Taking the mental experiment further, it is possible to aggregate section-level population projections for 2009 into the hypothetical map to assess it performance in the year closest to the census on which the map was prepared. Like fish, fresh demographic information is a must: all but a handful of hypothetical districts in that year are within the tolerance band.

The evidence is unambiguous: malapportionment in Mexico is systematic and substantial.[14] We next inspect how much of it translates into partisan bias.

6 Results

We relied on MCMC estimation to fit equation 2.[15] The responsiveness parameter is of secondary interest here, but useful to assess general model fit. Judging the 90-percent Bayesian confidence intervals (i.e., the 5th to 95th percentile range of $\hat{\rho}$'s posterior sample) reveals sizable shifts in the estimate between congressional elections: from a low of $[2.2, 2.4]$ in 2012 to a high of $[2.6, 3]$ in 2006. The large-party premium of recent Mexican plurality congressional races is about one-sixth smaller than the power of the putative cube law of plurality elections (Taagepera 1973).

Raw partisan bias results (i.e., the λ_p^v parameters) bear direct interest. Figure 5 summarizes posterior samples for different parties. The PRI's absence is due to its choice are reference to express partisan bias measures. Several patterns are noteworthy. Estimate precision (i.e., how concentrated the posterior cloud appears) is consistently lower for minor than for major parties (this is true for Convergencia as well, excluded from the plot to save space). Among major parties, the PAN's estimates excel in precision, variation around the median posterior value (taken as the point estimate) nearly indistinguishable at the chosen scale every year. The PRD's generally precise estimates are slightly less so in midterm elections (2003 and 2009) than in presidential election years. And the size and polarity of estimates reveal important party differences. The PAN experienced negative, albeit small

[14]The comparative survey by Snyder and Samuels (2004) ranked Mexico among well-apportioned cases. The measure reported is for the 1997 map, but no guidance is offered about the population figures used in denominators. We suspect reliance—as IFE did then and still does now—on raw 1990 census data, severely underestimating malapportionment.

[15]Gelman and Hill (2007) is a comprehensive introduction to MCMC estimation. For each vote opeartionalization (v, $\bar{v}$, and $\bar{w}$), three chains were iterated 10 thousand times, taking every fiftieth observation of the last 5 thousand to sample the posterior distribution. The Gelman–Hill $\hat{R} \approx 1$, evidence that the chains had reached a steady state. Convergence was also inspected visually in chain traceplots of each of the model's parameters. Estimation performed with open-source software Jags (Plummer 2003), implemented in R (R Dev. Core Team 2011) with library R2jags (Su and Yajima 2012). Data and commented code to replicate the analysis will be posted on-line upon publication.

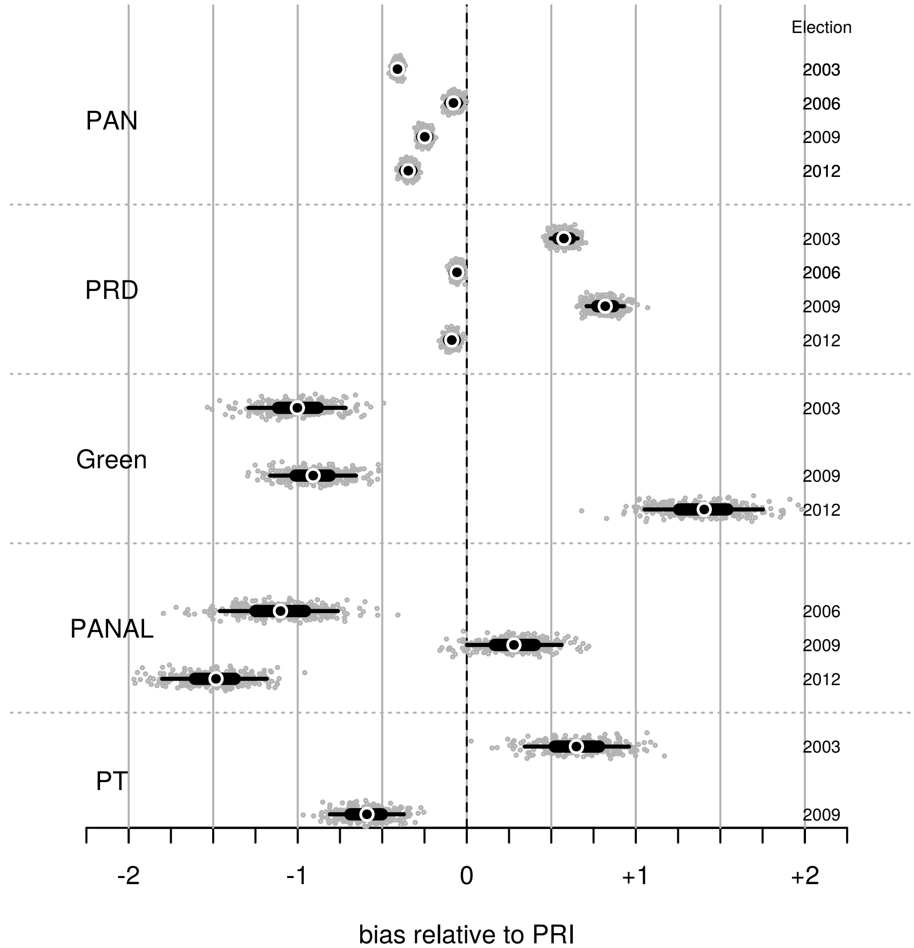

Figure 5: Raw partisan bias in four elections. The plot describes the posterior samples (small gray points) of estimated parameters $\hat{\lambda}_p$ for five parties. Some parties did not run in some years. Finer horizontal black lines connect the 5th and 95th percentile values of the posterior sample, thicker lines the quartiles, and white circles indicate the median value.

partisan bias vis-à-vis the PRI in every year observed. Leaving aside the question of how meaningful the estimated quantities are—we analyze swing ratios below with this object—the near exception was 2006, when the tiny estimate's cloud slightly overlaps with the vertical line at zero.

In stark contrast, the left experienced favorable and substantive bias relative to the PRI in some years but not in others. In this respect, partisan bias for the left is a mirror image of its electoral fortunes: it vanished when its candidates for Congress rode López Obrador's presidential campaign coattails (the party's national congressional vote was 30 percent on average in 2006 and 2012), but materialized otherwise (when vote averaged 16 percent).[16] In spite of losing about half of its support, the PRD converted votes into seats much more efficiently than either other major party in midterm election years. Whether this was due to winning smaller or lower-turnout districts can be answered with our procedure.

Table 2 breaks down the components of partisan bias. Ignore the right column for a moment. Bias estimates for the PAN, the PRD, and a selected minor party relative to PRI's are included. The minor party selected for 2006, 2009, and 2012 is the Green, and PANAL for 2006 (when Greens fielded no candidates outside the coalition with the PRI). Numbers in parentheses report the share of the posterior sample with sign opposite to that reported, serving as a test of the estimate's statistical significance (the probability that the estimate's sign is wrong).

Turnout played favorably for PRI relative to other major parties in every election in the period, as indicated by systematic negative signs. An edge in lower-turnout districts gave the PRI a springboard to more efficient votes-to-seats conversion. The apex was the 2006 election, concurrent with a presidential race where the PRI finished in distant third place. By retracting to its core districts, the low-turnout effect grew in size quite remarkably as negative presidential coattails depressed the party's congressional vote in swing districts nationwide. The opposite happened in 2012, when favorable presidential coattails to some extent helped the party's congressional candidates.

That the gerrymander element often predominates in the mix is quite notable, as was the PRD's case in midterm elections, the PAN's in 2012, and systematically for the selected minor parties. Since, owing to formidable entry barriers in the election law, no minor party is regional-based, the pattern is quite expected for them: spreading meager support across the board returns few, if any seats—hence the negative sign. Year 2012 is the exception,

[16] Parties' national vote shares in the period were:

Year	PAN	PRI	PRD	Green	PANAL
2003	.32	.38	.18	.04	—
2006	.34	.28	.31	—	.05
2009	.29	.40	.14	.06	.04
2012	.27	.38	.28	.02	.04

partisan bias	Actual map PAN–PRI	Actual map PRD–PRI	Actual map min–PRI	Hypothetical map PAN–PRI	Hypothetical map PRD–PRI	Hypothetical map min–PRI
2003 election				(with 2006 map)		
raw	−.37 (0)	+.72 (0)	−1.01 (0)	−.41 (0)	+.57 (0)	−1.00 (0)
gerrym.	−.09 (0)	+.69 (0)	−.88 (0)	−.13 (0)	+.62 (0)	−.90 (0)
turnout	−.26 (0)	−.11 (0)	−.08 (0)	−.26 (0)	−.09 (0)	−.09 (0)
malapp.	−.01 (.11)	+.14 (0)	−.05 (0)	−.02 (.12)	+.05 (0)	−.02 (0)
2006 election						
raw	−.08 (0)	−.06 (0)	−1.10 (0)			
gerrym.	+.28 (0)	+.30 (0)	−.62 (0)			
turnout	−.36 (0)	−.41 (0)	−.43 (0)			
malapp.	−.00 (.42)	+.05 (0)	−.05 (0)			
2009 election						
raw	−.25 (0)	+.82 (0)	−.91 (0)			
gerrym.	−.11 (0)	+1.01 (0)	−.79 (0)			
turnout	−.14 (0)	−.24 (0)	−.12 (0)			
malapp.	−.00 (.36)	+.05 (0)	−.00 (0)			
2012 election				(with 2015 map)		
raw	−.35 (0)	−.09 (0)	+1.40 (0)	−.32 (0)	−.13 (0)	+1.03 (0)
gerrym.	−.28 (0)	−.07 (0)	+1.41 (0)	−.24 (0)	−.05 (.06)	+1.02 (0)
turnout	−.07 (.02)	−.08 (0)	+.02 (0)	−.08 (.26)	−.09 (0)	+.01 (0)
malapp.	+.01 (.42)	+.06 (0)	−.02 (0)	−.00 (.38)	+.01 (0)	+.00 (0)

Table 2: Relative major-party raw bias and its additive components. Entries report the median of the posterior sample of parameters estimated with the single-election models. Numbers in parentheses are the share of the posterior sample with sign opposite to the reported point estimate's. The right column reports bias estimates using election data re-arranged according to district boundaries adopted after an election (i.e., a hypothetical 2003 election with the 2006 map and a hypothetical 2012 election with the 2015 map).

when Greens nominated the coalition's candidate in a concurrent gubernatorial race whose coattails returned three congressional seats (Magar 2012). And the gerrymander component's volatility for major parties, in size and in polarity, is consistent with the absence of partisan gerrymandering—as expected from a major-party power-sharing of the election board. Also notable is how the raw bias sum hides large components that contribute in opposite directions and therefore cancel out. The PAN and PRD's extraordinary performances in 2006 led them to the lowest measures of raw partisan bias (in absolute value) in the period. Separation makes it possible to note that a gerrymander advantage was key to compensate for even larger turnout disadvantages. District boundaries played to their advantage. The PRD experienced something similar, but less balanced, in 2009, when a superb gerrymander advantage was only partially offset by a more modest turnout disadvantage.

Also distinctive, and frankly surprising is how generally small the malapportionment component of partisan bias is compared to other sources. The PAN experienced no bias attributable to district size differentials over the period—these are the only estimates, by the way, with sizable probabilities that the estimate is of wrong sign. The party's success was therefore not likelier in districts confined at one end of the RRI distribution, as the left was to some extend. The PRD was slightly advantaged relative to the PRI in every year observed. This is very likely due to overrepresentation of the Federal District—a PRD stronghold—but the effect is easily eclipsed by the other components of partisan bias (The drop from +.14 to +.05 between 2003 and 2006 actually coincides with reapportionment and the accessory reduction—not removal—of the capital's overrepresentation in Congress, see Altman et al. 2014) Malapportionment-driven bias is not much larger for minor parties, whose perennial small vote shares locate at the wrong end of the system's responsiveness to size. For further perspective on malapportionment, we repeated the 2003 and 2012 estimations with hypothetical outcomes using the district boundaries of the map that was re-drawn immediately after those elections (reported in the right column of Table 2). As expected, redistricting mitigated malapportionment-based partisan bias systematically: under hypothetical, more balanced districts, the statistically nil quantity against the PAN actually observed (the probability that the estimate reported has wrong sign is .11) remained so; and the pro-PRD's discernible bias relative to the PRI shrank to about one third its original size. The same is true inspecting the 2012 election in light of districts re-drawn with updated population figures.

We close with an assessment of how meaningful partisan bias has been in recent congressional elections. As said, translating the bias estimates into a percentage point advantage or handicap for each party in the votes-to-seat conversion is not straightforward in

Year	Variable	PAN β	PAN (SE)	PRI β	PRI (SE)	PRD β	PRD (SE)
2003	v	1.84	(.06)	2.44	(.07)	1.75	(.05)
	$v \times$ reMap	$+.06$	(.08)	$+.08$	(.10)	$-.12$	(.06)
2006	v	2.18	(.07)	2.17	(.10)	1.73	(.05)
	$v \times$ reMap	$+.13$	(.10)	$-.36$	(.14)	$-.07$	(.08)
2009	v	1.67	(.07)	2.18	(.07)	1.42	(.04)
	$v \times$ reMap	$+.44$	(.10)	$+.26$	(.11)	$+.18$	(.07)
2012	v	2.31	(.07)	3.86	(.13)	2.35	(.06)
	$v \times$ reMap	$-.24$	(.09)	$+.03$	(.17)	$-.11$	(.09)

Table 3: Vote–seat swing ratios. Also in the right side, but not reported, were a dummy indicating data simulated with the hypothetical map, and a constant. Method of estimation: OLS.

multi-party settings. We therefore gauge this with an alternative quantity of substantive interest: vote–seat swing ratios (Niemi and Fett 1986, Tufte 1973). Swing ratios (or the vote elasticity of seats) are the percentage change in seats associated with a one-percent change in the party's national congressional vote. If the previous section aimed to measure this type of effect system-wide (the responsiveness parameter), swing ratios measure the sensitivity of individual parties' seat shares to changes in voter preferences. A party with unity swing ratio can expect to receive its fair share of seats. Anything else indicates that parties can expect to win more (> 1) or less (< 1) than one percent of seats for a unit percentage change in vote share.(We rule out negative swing ratios corresponding to a party losing seats as it wins votes; for violations of the monotonicity principle of representation, see Balinski and Young 2001).

We derive swing ratios by regressing a party's seat shares in simulated elections on the party's simulated vote shares (Linzer 2012). To also gauge the effects of redistricting, we pool the latter with hypothetical elections using the map that supplanted the actual one (i.e., the 2006 map for the 2003 election and the 2015 map for the 2006–12 elections). Interacting this with dummy reMap (equal 1 for hypothetical simulated elections, 0 otherwise) yields the fitted equation: $s_p = \beta_0 + \beta_1 v + \beta_2 \text{reMap} + \beta_3 v \times \text{reMap} + \text{error}$. Coefficient β_1 is the swing ratio, coefficient β_3 the swing ratio change attributable to redistricting.

Table 3 reports major party results. In general, major parties enjoyed quite favorable swing ratios in the period—2.14 on average, indicating a hike in seats surpassing 2 percentage points for an extra percentage point in votes. But a good deal of change, both between parties and between elections, is palpable. The left enjoyed the smallest four-election average swing ratios (1.8), the PRI the largest (2.7), the PAN somewhere in between (2.0).

Given 300 plurality seats, the PRI at its most elastic (in 2012) would have earned nearly 12 more seats with just one extra percent votes nationwide. A dozen seats would have amply sufficed to give the coalition majority status that it failed to achieve in the Cámara de Diputados. Contrast this with the nearly 2.5 and 3 additional percentage points in votes, respectively, that it would have taken the PAN and the PRD at their least elastic (in 2009) in order to earn the same dozen extra seats.

7 Conclusion

We develop a procedure to estimate the components of partisan bias in national elections separately: the gerrymander, the malapportionment, and the turnout components. A method to achieve this has been available for some time, but for two-party competition only. By intersecting three extant empirical models, we extend the procedure to multi-party systems. We demonstrate the procedure's usefulness with an application to the study of recent Cámara de Diputados elections in Mexico.

Analysis revealed how the plurality component of the mixed electoral system gave persistent advantage to some parties in recent congressional elections. Relative to the PAN, there is evidence of small, but systematic partisan bias in favor of the PRI in the votes-to-seats conversion, and of a larger, but more volatile bias favorable to the PRD throughout the period. These findings, derived from simulated data to overcome methodological complications, are in contrast with evidence of substantive anti-PRI bias in a multi-election study (Márquez 2014*a*).

The breakdown of partisan bias adds further depth to our findings. Partisan bias sources vary in importance and, to a fair extent, run counter each other. The prevalence of substantial malapportionment in Mexico has not, as a matter of fact, translated into systematic partisan bias. It helped the left relative to other major parties, and with growing strength as maps aged and further malapportionment crept in, but the contribution is much smaller than, and easily canceled by those of turnout differences and the geographic distribution of party strength. The PRI of the democratic retains an edge in low-turnout districts, increasing its capacity to turn votes into seats in every election studied. And in spite of a nominally neutral redistricting system, the left in most years, and the PAN in 2006 were able to overcome a large turnout disadvantage through more favorable line drawing. Gerrymandering merits closer inspection in future research.

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