Measuring malapportionment, gerrymander, and turnout effects in multi-party systems

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Casa matemática Oaxaca
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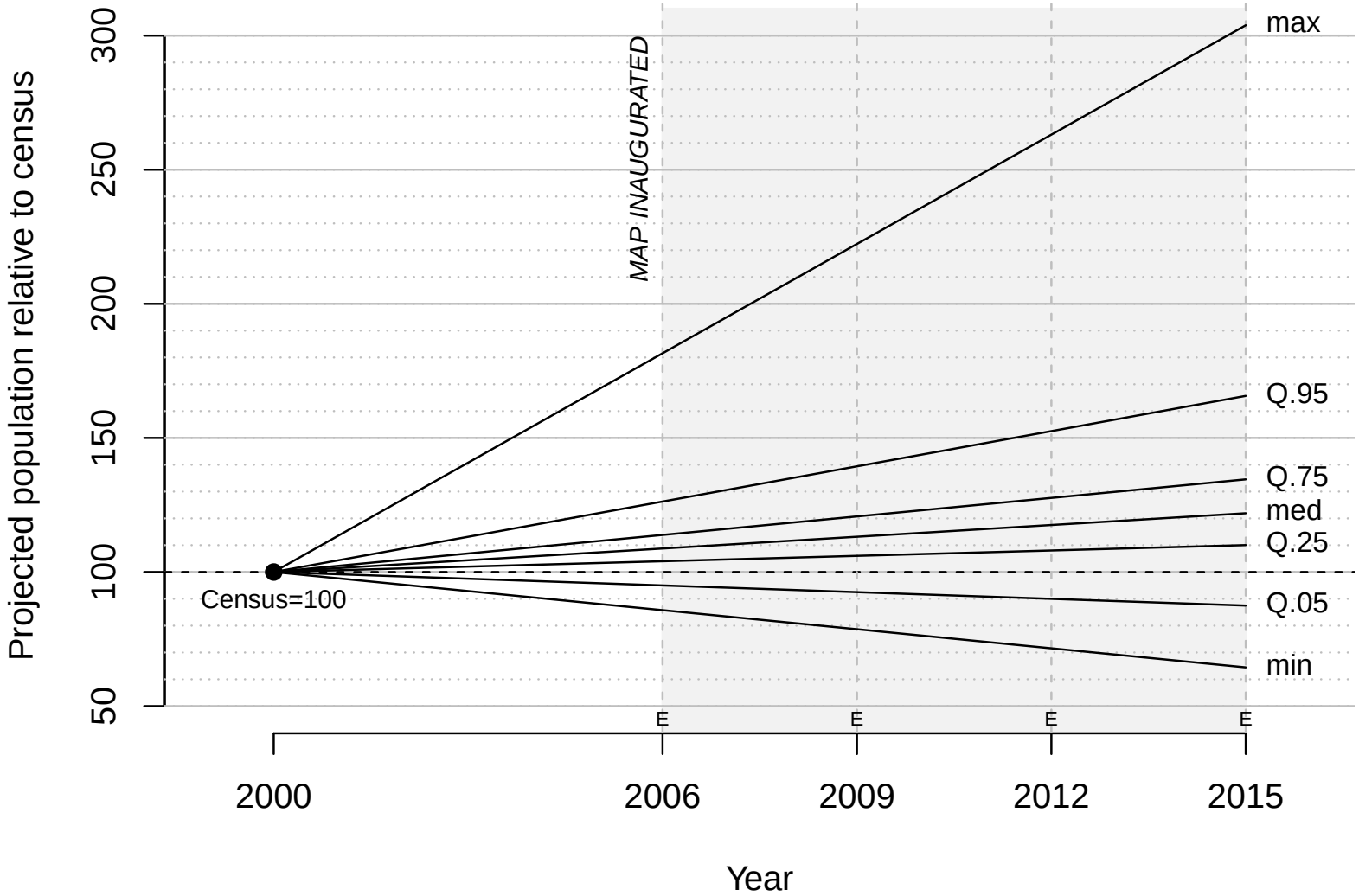
District populations: linear projection



Background on Mexico

- Hegemonic party 1929–1997
- Three major parties: PRD (left) PRI PAN (right) and minors
- Lower chamber of Congress elected every 3 years, concurrent w presidential race every 6 years
- Mixed system: 300 SMD + 200 PR seats
- Single-term limits removed in 2018
- Autonomous regulator (IFE) organizes elections and redistricting

Questions

Did 1997 reform remove **party bias** in representation?

- Potential problem wherever districts are drawn to allocate seats (Tufte 1973, Johnston 2002)

If party bias remains, what **factors** drive it?

- Do parties use redistricting in their advantage?
- How do demographic shifts over time affect parties?
- Turnout differentials?

Answers

1. Persistent bias against the right
2. Components of bias often cancel each other

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1. Persistent bias against the right
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What is party bias

It is the excess/defect seat share that a party with half of the votes gets:

$$(s \mid v{=}.5) - .5$$

- Two-party system
- Constant-sum game
- Vote wasting: too-concentrated large party or too-dispersed small party suffer bias (Calvo&Rodden 2015)

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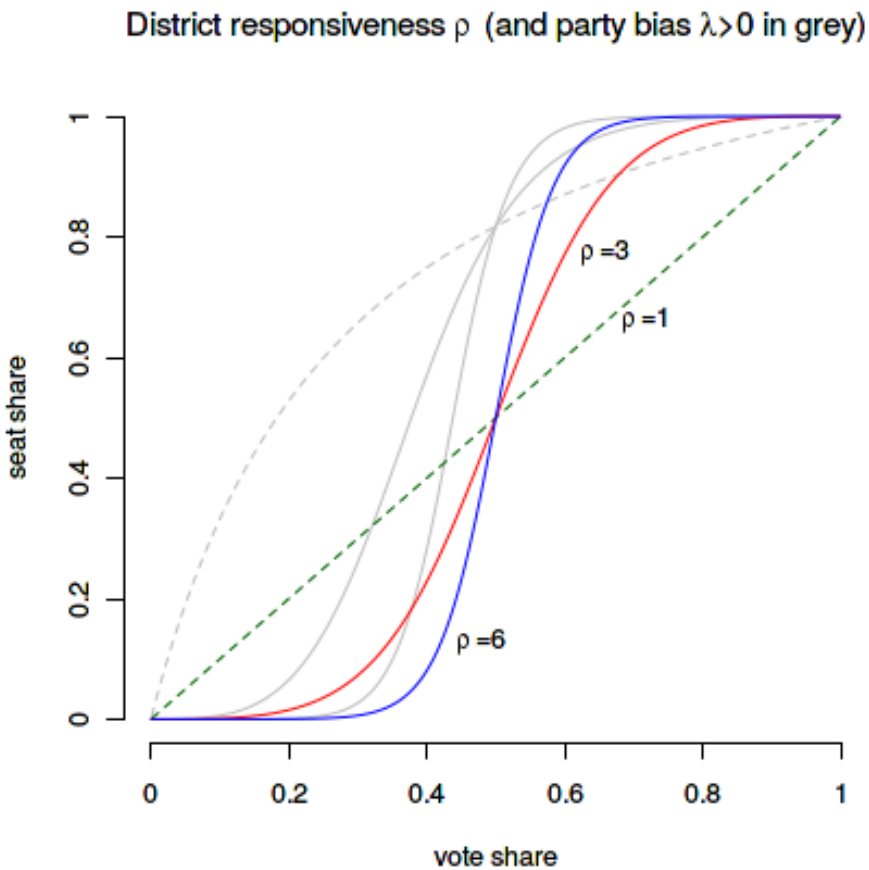
Obstacle 1: measure party bias

Fitting votes–seats curves: $s = f(v)$
(Rae 1967, Tufte 1973, King&Browning 1987)

$$\frac{s}{1-s} = \lambda \left(\frac{v}{1-v} \right)^{\rho}$$



Three sources of party bias

			Raw votes			Vote shares		Seat shares	
Districts	Pop.	Turnout	left	right	total	left	right	left	right
Gerrymandering									
1 and 2	420	.5	147	63	210	**.7**	**.3**	1	0
3, 4 and 5	420	.5	84	126	210	**.4**	**.6**	0	1
nationwide	2100	.5	546	504	1050	.52	.48	.4	.6
Turnout									
1 and 2	420	**.70**	200	100	300	.67	.33	1	0
3, 4 and 5	420	**.35**	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6
Malapportionment									
1 and 2	**600**	.5	200	100	300	.67	.33	1	0
3, 4 and 5	**300**	.5	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6

Obstacle 2: measure the sources of party bias

Grofman, Koetzle & Brunell 1997:

Three additive components

raw party bias = gerrymandering (distributional)
+ malapportionment
+ turnout

1. Fitting votes–seats curve with v yields **raw** party bias
2. with $\bar{v}$ yields the **gerrymandering**-based
3. with $\bar{w}$ yields gerrymandering + malapportionment

→ Subtract (3) – (2) = **malapportionment**-based
→ Subtract (1) – (3) = **turnout**-based

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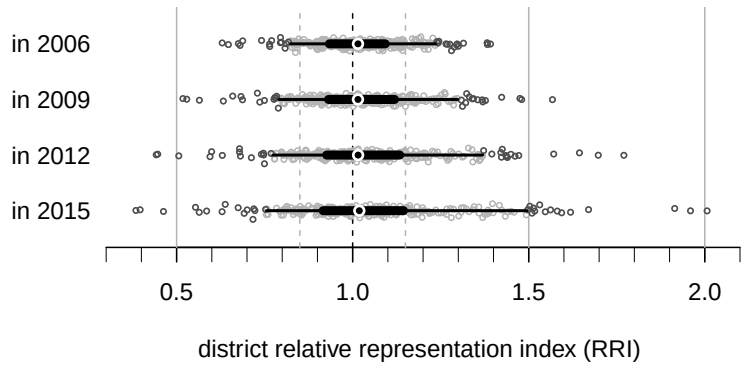
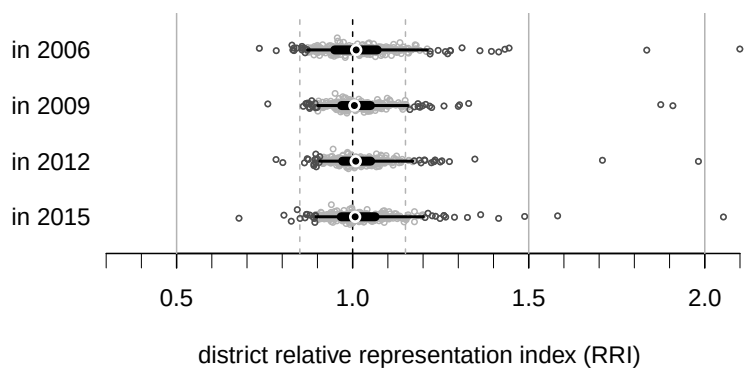
Malapportionment is substantial

$$RRI = \frac{Q}{\text{district size}}$$

2006 map (drawn with 2000 census)



2015 map (drawn with 2010 census)



Obstacle 3: a multiparty system

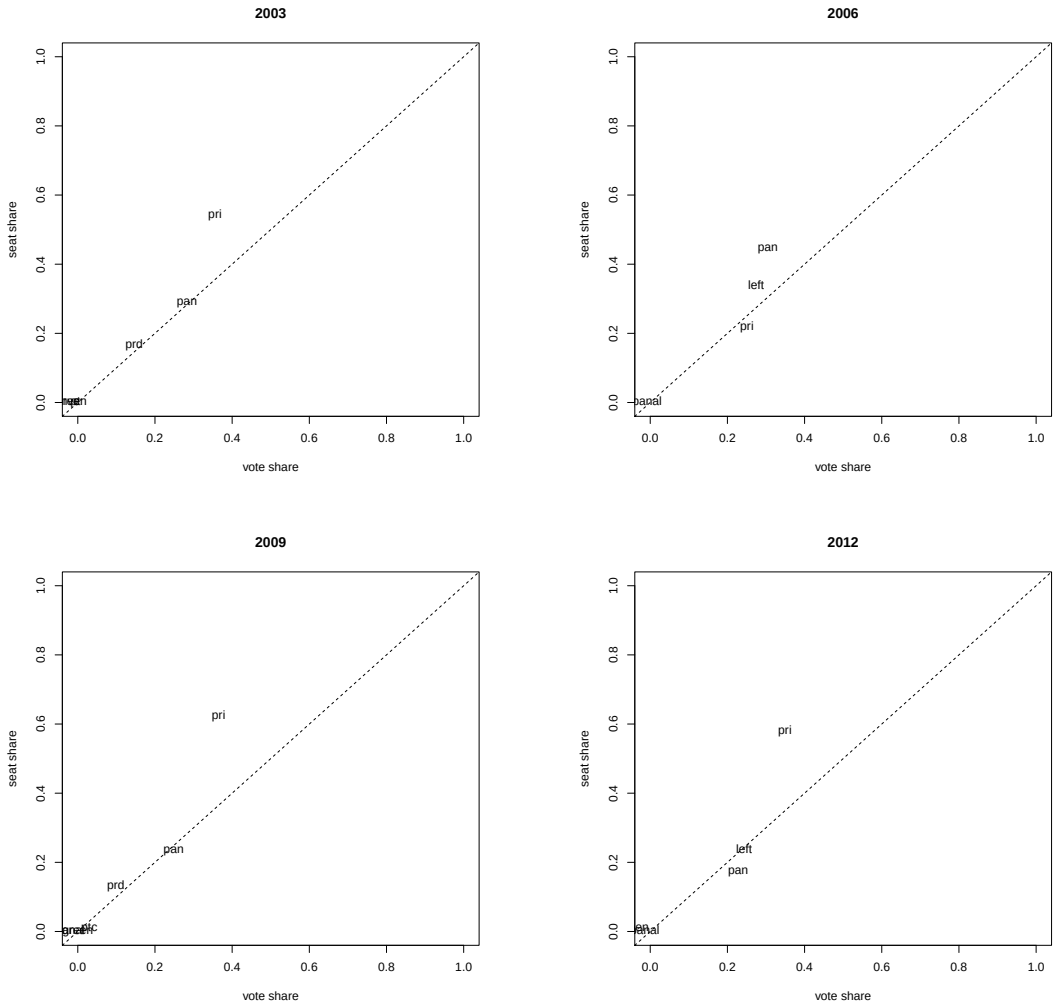
King 1990:

$$E(s_p) = \frac{e^{\lambda_p} v_p^{\rho}}{\sum_{q=1}^{P} e^{\lambda_q} v_q^{\rho}}$$

Bias is expressed relative to a
baseline party (PRI in our case)

Obstacle 4: small-N

- Linzer 2012: approximates prob. distribution of national party vote returns from observed district outcomes (FMM)
- Use to simulate many elections w Monte Carlo draws



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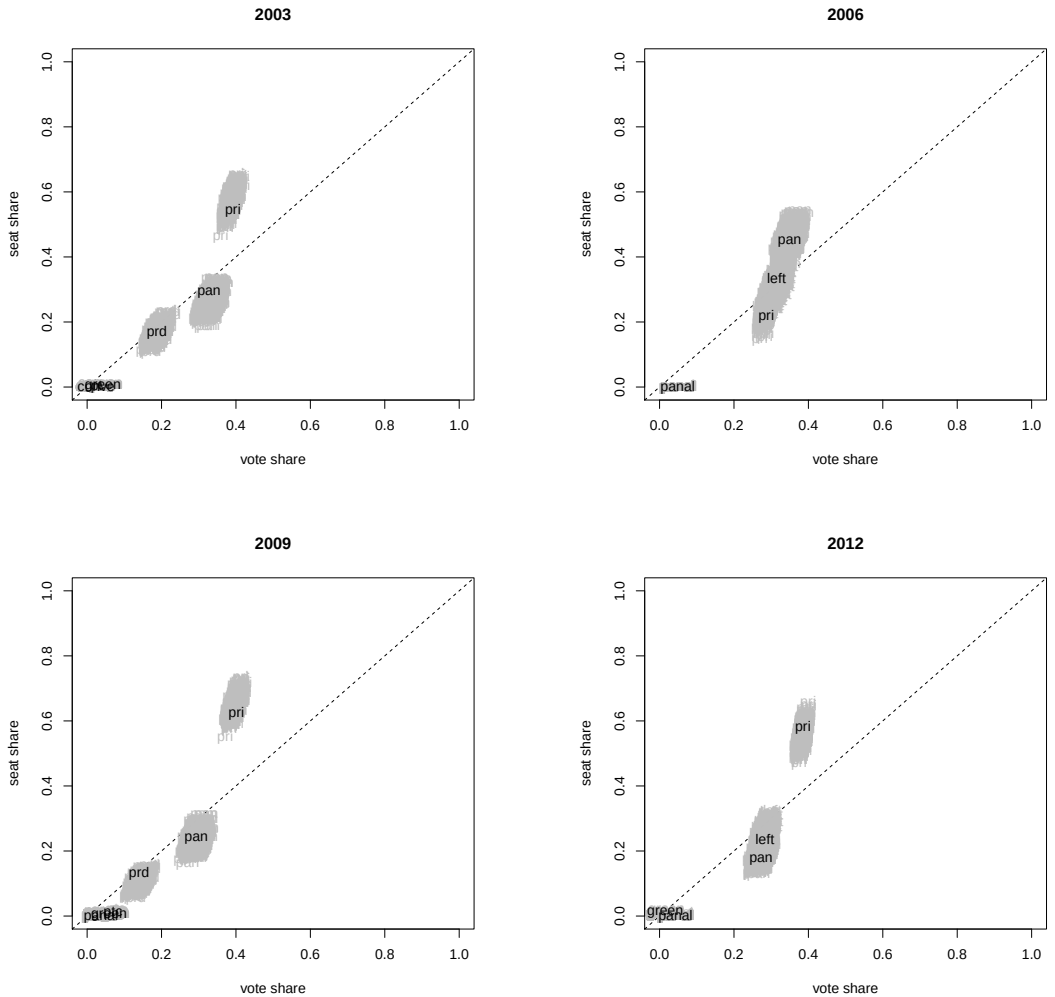
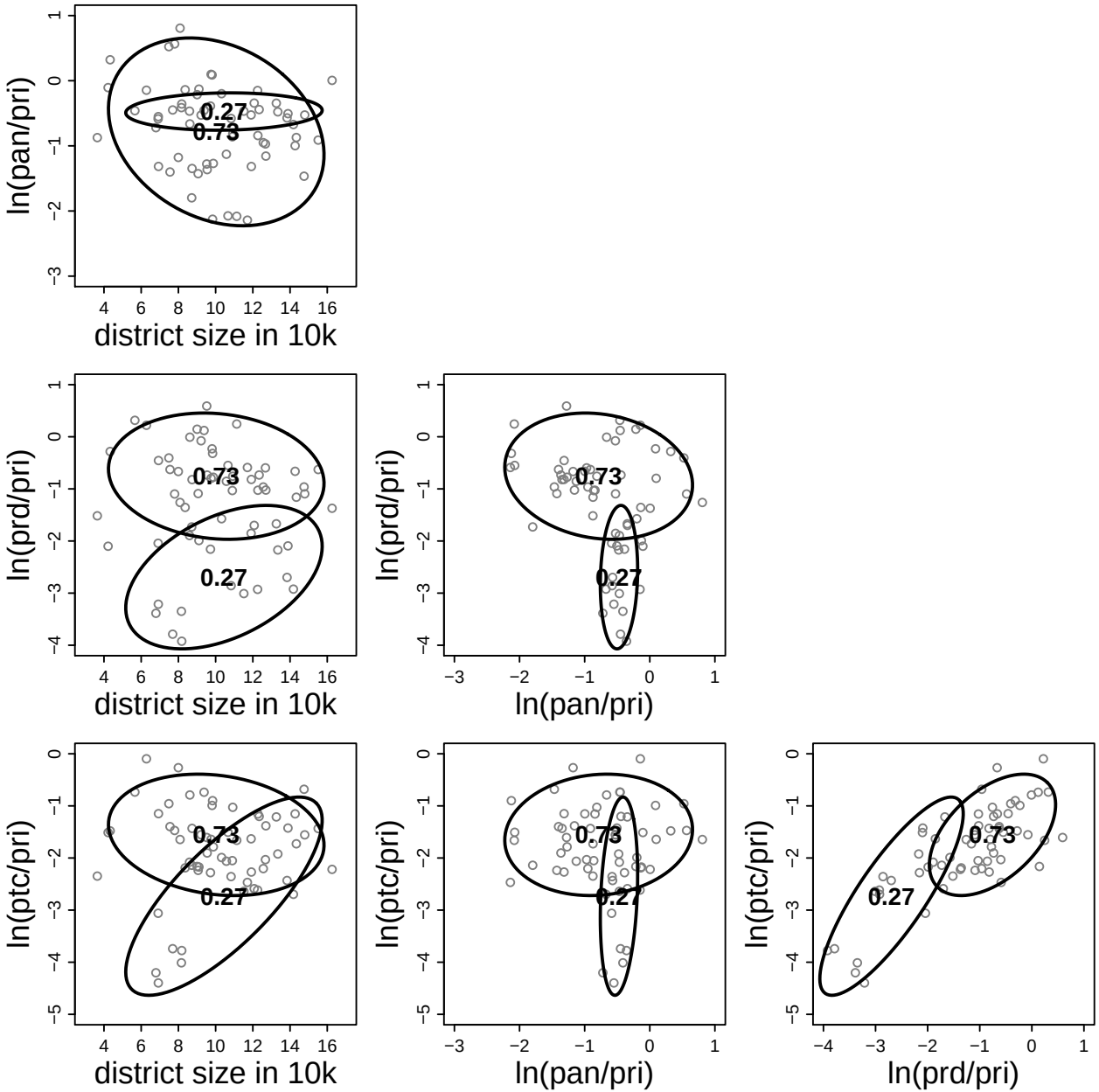
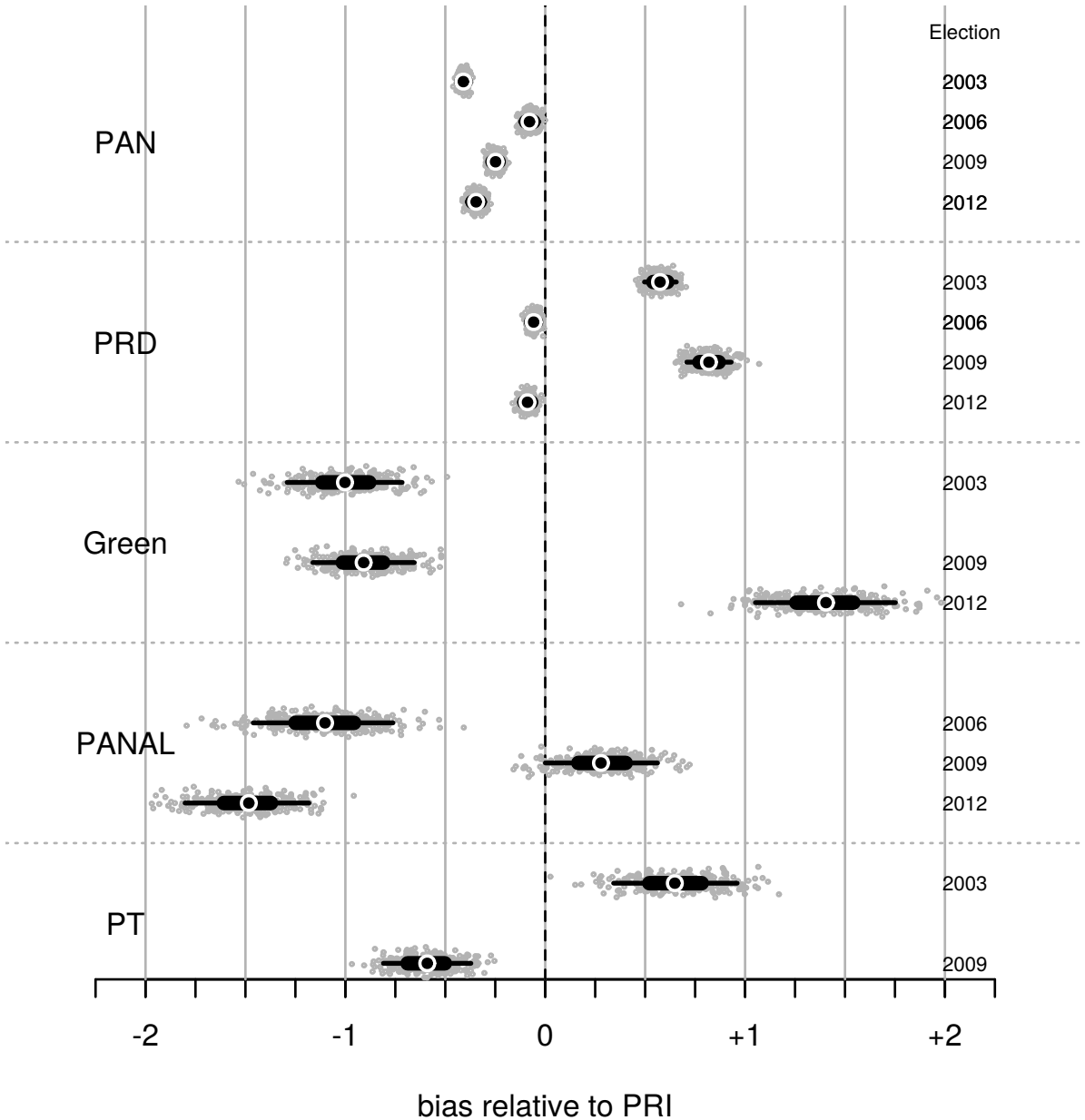


Two components 2009



Results: raw party bias



Results: components

	Actual map			Hypothetical map		
party bias	pan–pri	prd–pri	min–pri	pan–pri	prd–pri	min–pri
2003 election				(with 2006 map)		
raw	−.37 (0)	+.72 (0)	−1.01 (0)	−.41 (0)	+.57 (0)	−1.00 (0)
gerrym.	−.09 (0)	+.69 (0)	−.88 (0)	−.13 (0)	+.62 (0)	−.90 (0)
turnout	−.26 (0)	−.11 (0)	−.08 (0)	−.26 (0)	−.09 (0)	−.09 (0)
malapp.	−.01 (.11)	+.14 (0)	−.05 (0)	−.02 (.12)	+.05 (0)	−.02 (0)
2006 election						
raw	−.08 (0)	−.06 (0)	-1.10 (0)			
gerrym.	+.28 (0)	+.30 (0)	−.62 (0)			
turnout	−.36 (0)	−.41 (0)	−.43 (0)			
malapp.	−.00 (.42)	+.05 (0)	−.05 (0)			
2009 election						
raw	−.25 (0)	+.82 (0)	−.91 (0)			
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gerrym.	−.28 (0)	−.07 (0)	+1.41 (0)	−.24 (0)	−.05 (.06)	+1.02 (0)
turnout	−.07 (.02)	−.08 (0)	+.02 (0)	−.08 (.26)	−.09 (0)	+.01 (0)
malapp.	+.01 (.42)	+.06 (0)	−.02 (0)	−.00 (.38)	+.01 (0)	+.00 (0)

- Turnout always pro-PRI
- Malapp. always pro-left
- Redistricting abates malapp.
- Possibly cancelling effects

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3. Gerrymandering effects large and volatile
4. Pro-PRI turnout-based bias
5. District lines can compensate for turnout disadvantage
6. To-do: add PR-tier to analysis
7. To-do: study inter-election volatility

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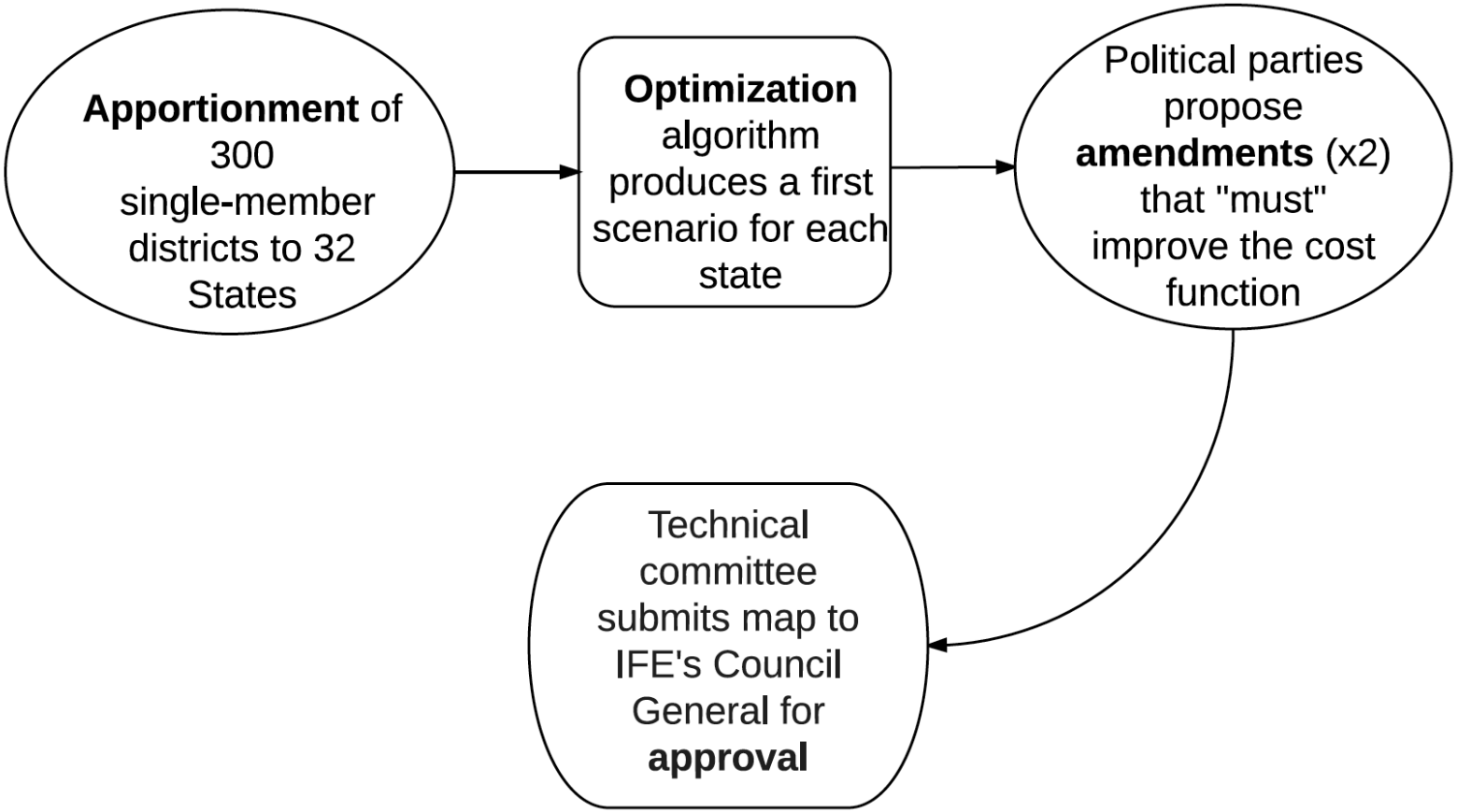
The redistricting process



The redistricting process

Redistricting by experts in 1997, 2006, 2015 (abandoned), and now 2018

1. apportionment of 300 seats to 32 states
2. optimization algorithm $\rightarrow$ proposal
3. parties propose amendments ("must" improve score)
4. new map

$$\texttt{Score} = .4 \times \texttt{PopBalance} + .3 \times \texttt{MunicBoundaries} + .2 \times \texttt{TravelTime} + .1 \times \texttt{Compactness}$$

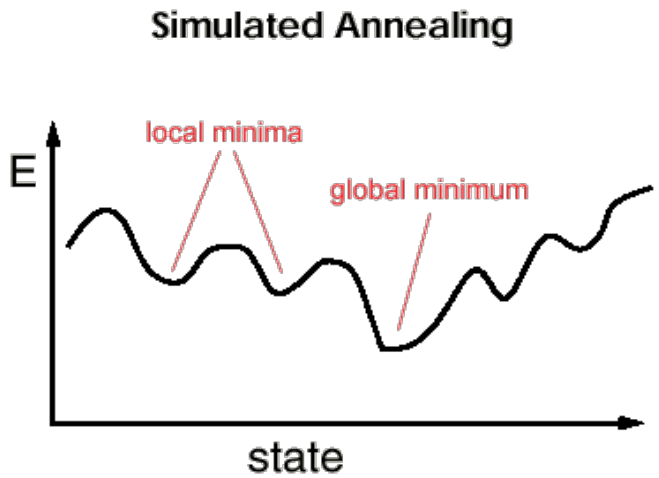
IFE considers $\pm 15\,\%$ imbalance normal (!)

Optimization algorithm

Simulated annealing = probabilistic meta-heuristic for optimization locates a good approximation to the global optimum of the cost function in a large search space

Thousands of iterations using electoral *secciones*

Combinatorial optimization algorithm used to generate the first scenario in each state



IFE claims that this is a public process, but the operation and procedures are done behind closed doors

Party amendments