Malapportionment and party bias in Mexico's mixed-member system*

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Abstract

Automated redistricting in consultation with parties since 1997 rid redistricting in Mexico of the most flagrant distorsions on representation. But more subtle ones remain. Rules and practice introduce some degree of malapportionment, small in comparative perspective (Samuels and Snyder 2001), but nonetheless substantial. We inspect district maps, votes, and seats in Mexico to evaluate two forms of political distortion that may arise from malapportionment: party bias and responsiveness (otherwise known as plurality bias). State-level data reveal no evidence of systematic party bias. What bias there is is of another kind, a big bonus for larger parties (or vote responsiveness) typical of plurality rule in single-member districts. Since most states distribute few seats each, the large party bonus of states' federal districts doubles the estimate with national data. Since the PRI is the largest party in most states, it receives a substantial seat bonus.

A classic approach to assess electoral systems is the derivation of vote–seat curves.[1] Such curves capture how votes in general elections are converted into parliamentary seats and therefore evaluate the most fundamental task of the electoral system (Lijphart 1994). Particular attention has been given to plurality systems and the ways in which the votes–seats curve depart from proportional representation.

Mexico's lower chamber of Congress has been elected with a mixed-member electoral system for decades. Such systems give voters a direct role in the election of representatives

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[1]E.G., Dahl (1956), Erikson (1972), Gelman and King (1994), Gudgin and Taylor (1979), King (1990), King and Browning (1987), Rae (1967), Taagepera (1973), Taagepera and Shugart (1989), Tufte (1973).

from single-member districts (SMDs), while also using some form proportional representation (PR) to improve the odds that parties will receive about as many seats as they are worth in votes (Shugart and Wattenberg 2001*a*). Underneath the PR-tier lies a standard plurality system. We suspect strong distorsions in the form of party bias may be arising in the SMD-tier due to substantial malapportionment. These could be avoided.

Proving this requires methods to measure party bias and to assess how much of it is due to malapportionment. We rely on a model by King (1990) generalizing the fit of a vote–seat curve with parameters for party bias and large-party premium to multiparty competition. We then rely on a procedure set forth by Grofman, Koetzle and Brunell (1997) for two-party competition, extending it to the study of Mexico's multi-party elections.

Section 1 investigates malapportionment in Mexican elections, showing that it is substantial. District boundaries in Mexico are not drawn by politicians but by experts under the aegis of a collegiate electoral regulator where all three major parties share power (Estévez, Magar and Rosas 2008). This removes suspiscions of the flagrant partisan manipulation such as gerrymandering. But, the section shows, other distrosions remain, most notoriously malapportionment.

Section 2 discusses partisan bias and three separable sources behind it. Distributive-, turnout-, and malapportionment-based party bias can not only be distinguished analytically, their independent contributions can be estimated empirically by operationalizing parties' national vote shares in slightly different ways.

Section 3 elaborates a vote–seat curve capturing party bias and responsiveness, and a way to estimate it with MCMC.

Section 4 offers results of the estimation. Analysis takes advantage of a redistricting plan that was finalized for the 2015 election (but never adopted). The map removed a good deal of "creeping" malapportionment, so comparing hypothetical 2006, 2009, and 2012 elections with the new map to the actual elections offers perspective to evaluate the three sources of party bias.

1 One person, one vote?

> The second condition [of majority rule] is that each individual be
> treated the same as far as his influence on the outcome is concerned
> —May's Theorem (1952)

Malapportionment arises when sparsely populated regions get the same representation as more densely populated ones. It can be found whenever multiple districts are drawn for the purpose of seat allocation. District size differentials may be actively designed, adopting cartography that deliberately underrepresents some citizens. Senates often grant states equal representation, regardless of population. But it more commonly germinates passively by failure to redraw district boundaries that compensate for secular demographic imbalance. “Creeping malapportionment” (Johnston 2002) ensues.

1.1 Malapportionment in a mixed-member system

The lower chamber elects 300 members in SMDs and 200 members by PR (Weldon 2001). It could be argued that malapportionment in mixed-member systems is of little, if any consequence. The PR tier is, after all, specifically designed to compensate for imbalances ensuing from SMDs. We debunk such claim.

Compensating *parties* bears relation to, but is not the same as compensating *citizens* of more densely populated districts. Keeping these compensations distinct is important. Much of the evidence presented here, as in the scholarly literature, deals with party vote:seat ratios. From the normative standpoint, however, it is the ‘one person, one vote’ principle—one of Dahl’s (1972) preconditions of democratic government—that malapportionment antagonizes, and no degree of party compensation will redress this imbalance. The exception would be a system with perfectly district-based parties, where measures to achieve party proportionality are, in fact, compensations for the district citizenry only. Shift away from fully local parties, towards party nationalization, and party compensation stops accruing to citizens of the underrepresented districts only.

A related reason is reelection. Mexico recently removed single-term limits for legislators. Ambitious deputies elected in 2018 will, again, be allowed to seek reelection, restoring the electoral connection that was severed in the 1930s (Dworak 2003). The reform should reinvigorate the relation between citizens and their representatives in the SMD tier. The persistence of substantial variations in district size, however, acts against realizing the full potential of the electoral connection (Mayhew 1974)—widespread disatisfaction and even protests in past years underscore how bad this is needed for the consolidation of Mexico’s young democracy.

Malapportionment distorts representation in mixed-member systems, and this matters. We next show its prevalence.

1.2 Apportionment to the states

Redistricting takes two steps. The three-hundred national legislative seats are distributed between the states, then district boundary maps are drawn for each of the thirty-two states.

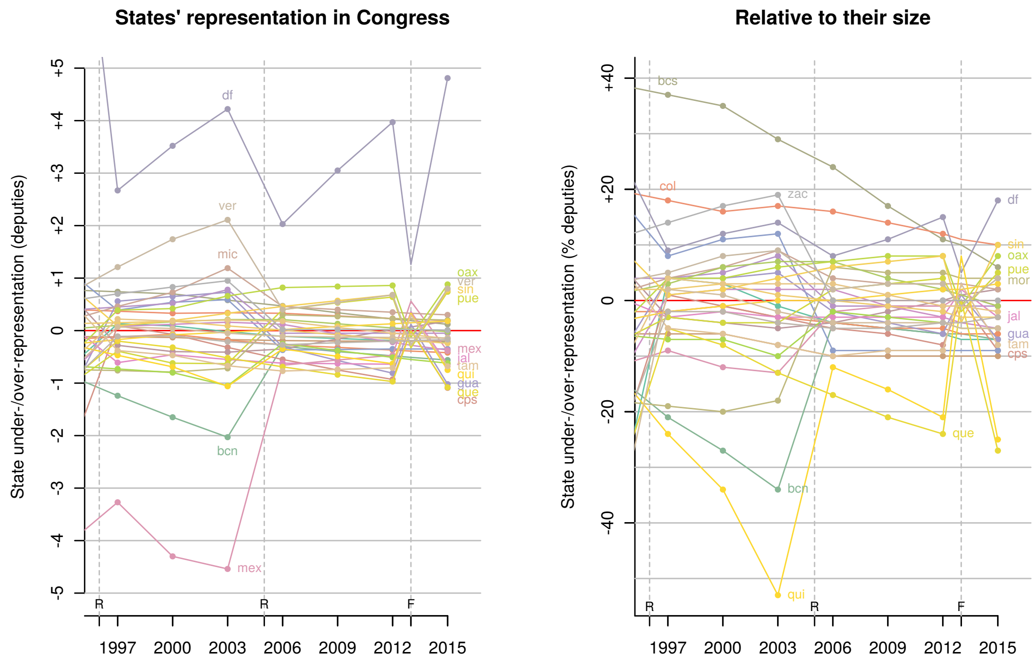


Figure 1: Demography and state apportionment in Congress (lower chamber). Lines connect the seats that each state had relative to those it should have had on a population basis. Population projections from censuses used for this computation. Letters R in the horizontal axis indicate redistricting, letter F a redistricting failure (the effect that the proposed map would have had is reported on the dotted line).

Malapportionment can be introduced in each step (see Snyder and Samuels 2004).

Unlike the U.S., little, if any public debate about methods of apportionment has been undertaken (Balinski and Ramírez González 1996, Szpiro 2010). The Hamilton method of largest remainders is used (Balinski and Young 2001:10). The quota Q (or price of a seat) is the nation's population divided by 300, the number of seats. A first allocation is made by dividing each state's population in the last general census by Q, rounded down. States with populations $< 2Q$ nonetheless get two seats (with Colima and Baja California Sur in this group in earlier apportionment rounds, but no more). Unallocated seats, if any, are awarded to states with largest fractional remainders.

Figure 1 summarizes states' congressional representation. In order to take creeping malapportionment into account, we projected each state's population at the time of a federal election between 1997 and 2015.[2] With moving population estimates for election years, a

[2]Projections up to 2010, inclusive, was by linear interpolation of census populations, and by linear extrapolation for 2012 and 2015. The years 1995 and 2005 had population counts, 2000 and 2010 had general

time-series of states' fair share of congressional seats was then computed (i.e., time-varying Qs). Subtracting the actual number of seats apportioned to the state yields the state's over- or under-representation in election years. A state with perfect apportionment appears at zero in Figure 1. The slope in the line indicates deviations, downward for states losing relative weight in the lower chamber, positive for winners. The left panel reports this measure in absolute terms (so that $+3$ indicates a state with three deputies more than its fair share), the right panel relative to seats apportioned to the state ($+20$ indicates twenty percent more deputies than its fair share). Vertical dashed lines mark redistricting events (the "F" towards 2015 indicates a failed proposal to redistrict).

Creeping malapportionment is manifest, points drifting between redistricting events. Since the first term of the subtraction is a real number and the second an integer, a fraction is bound to remain. Yet, upon redistricting, fractions in a well-apportioned systems should all be less than 1 quota in absolute value. The left panel shows that this is mostly the case, but exceptions are, and have always been present. The 2006 redistricting corrected the most egregious deviations from the ideal. The states of Baja North and of Mexico, with deficits of 2 and 4.5 deputies relative to population in 2003, respectively (deficits of 30 and less than 10 percent of the deputies they actually held), achieved near-parity in 2006. Over-represented Veracruz behaved near symmetrically to Baja in absolute terms (not relative). And a distortion strongly favoring the Federal District ($+4$ deputies) was somewhat attenuated in 2006, but never removed.

The notorious case of the DF reveals that relying on obsolete population data is problematic. The constitution mandates the use of the census for redistricting, but has no obligation to redistrict as soon as it becomes available. As a consequence, drawing a new map for 2006 with data from the year 2000 injected a fair amount of creeping malapportionment into the new plan right at its inception. Compared to the 2000 census, our projected state population are off by 9.7 percent on average, with a standard deviation of 6.9 percent. Most states remained withing the $[-1, 1]$ range, but not the DF. With one extra year away from the reference census, the 1997 map had even less success achieving proper apportionment. One year closer to census makes a huge difference, as the abandoned map for 2015 reveals (although this is probably compounded with a less dynamic demography): the dotted vertical line at year 2013 reports how the new plan might have looked, have it been adopted. With this in mind, it is puzzling that the redistributive nature of apportionment has not pushed for the adoption of alternative methods. The seventeen states that were under-represented in 2006 jointly controlled a majority (162) of single-member districts;

censuses. The former collect much fewer respondent attributes than the latter, but report a population figure that we used.

fourteen of them have always been below the red line indicating fair representation.

1.3 Representation in district boundaries

Despite automation and straightforward, formal redistricting criteria, unequally-sized districts are common practice in Mexico. In fact, Mexican parties in general, and IFE in particular, have been remarkably tolerant of this practice. This form of malapportionment is related, in some degree, to states' imbalanced apportionment in Congress, which perforce creates size differences across states' mean district sizes. But size inequality within states is also prevalent and substantial.

Small deviations around a state's mean district population are unavoidable, especially as a map ages. But what constitutes a small deviation to begin with is hard to define. Courts in the U.S. have struck down new district maps bearing less than 1% differences without proper justification (Tucker 1985). Redistricting authorities generally view a *de minimus* population deviations of as little as one or zero persons between congressional districts as desirable to inoculate against litigation. In stark contrast, IFE has considered deviations between 10% (in 2006) and 15% (in 1997 and 2015) above or below mean state district size perfectly normal (Lujambio and Vives Segl 2008, Trelles and Martínez 2012). If a pair of districts had populations exactly at the bounds of the ± 15 spread, the citizens at the bottom end would be worth one-third more in Congress than those at the top end. Surprisingly, no party has ever challenged this practice in Court.

Following Ansolabehere, Gerber and Snyder (2002), we measure a district's relative representation index as $RRI = \frac{1/\text{district size}}{300/\text{national population}} = \frac{Q}{\text{district size}}$, where the numerator is the number of congressional seats per person in the district and the denominator is the average number of congressional seats per person nationwide (300 is the number of congressional seats).[3] Since the denominator represents the inverse of the apportionment quota Q, the second equality is a convenient algebraic transformation. A district with unity index value has representation matching the 'one person, one vote' ideal perfectly. Values above one indicate over-representation, values below one under-representation, and the measure is continous. An example shows how the index is interpreted. The size of the 3rd district of Aguascalientes in 2012 was about 306,000 inhabitants, and the (projected) quota about 387,000, so the district in question had 26 percent more representation than the national average, for an index value of 1.26. As before, we used population projections to compute *RRI*s.[4]

[3] Ansolabehere et al.'s *RRI*s were devised to measure how well are represented counties (some of which have several districts) in U.S. state assemblies. In our case, all units are a whole districts.

[4] Census data reported at the electoral *sección* level (used to aggregate district populations in two maps)

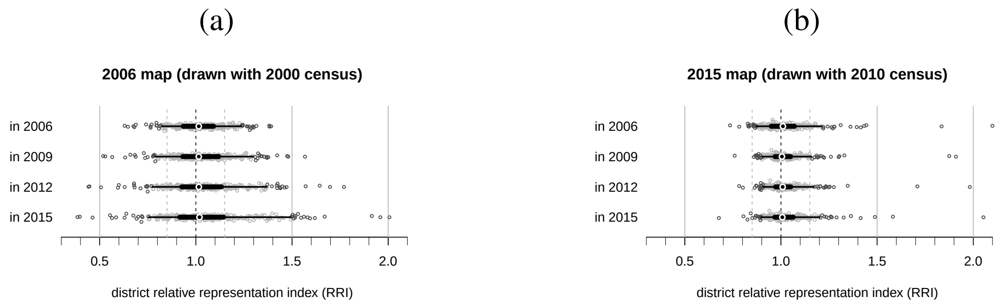


Figure 2: Representation in four elections and two maps. Panel (a) portrays the 2006 map (status quo), panel (b) the hypothetical 2015 map. Points are districts, finer horizontal lines connect the 5th and 95th percentiles, thicker lines the quartiles, and white circles indicate the median.

Redistricting automation should make districts within states tend towards zero malapportionment. Population balance is, after all, the criterion that weighs more in the cost function (Altman, Magar, McDonald and Trelles 2014). But this criterion weighs less than the others combined in the redistricting algorithm, and appears to be trumped rather easily.[5] *RRI*s can also be expressed relative to state populations by replacing the denominator with the average number of congressional seats per person in the district's parent state (i.e., the number of seats in the state over the state's population). With three seats apportioned to the state of Aguascalientes and a total population of about 1,200,000, the 3rd district had 33 percent more representation than the state average, for an index value of 1.33. This is instructive to inspect within-state representation. The percentiles corresponding to state *RRI*s equal to .85 and 1.15 (the bounds of IFE's ± 15 percent tolerance range) in 2006 were 6 and 89, respectively, implying that $6 + 100 - 89 = 17$ percent of districts were outside the range relative to the average state population. That was the map's inaugural year. By 2012, as many as 27 percent districts off the population tolerance range, and by 2015 one third will be. Drawing equal-sized district boudaries with old population data is no easy task, and distorts representation within states over and above between-state imbalances.

Figure 2 returns to *RRI*s computed with the national average. Nationwide, the ± 15 band has been substantively surpassed. Plots summarize the distribution of citizen representation as population has fluctuated. Consider the top plot first. Each point represents one district. The fine horizontal line connects the *RRI* values corresponding to the 5th and 95th percentiles—both well outside the tolerance band, represented by dashed gray lines, since

are available since 2005 only. We projected the 2005–2010 changes to 2006 and 2015 for this exercise. Since many districts are made of full municipalities, we could use that unit to interpolate all the way to 1997.

[5]The other criteria are the preservation of municipal boundaries, travel times within the district, and district compactness. All are achieved by exploiting tolerated population leeway.

the map's inauguration. The thick horizontal line is the inter-quartile range, nearly covering the full tolerance band by 2015—implying that soon half the districts will be off-range! For the 2015 midterm congressional election, citizens in the plot's right-most districts (in central Monterrey and two in battered Juárez) will be worth *four times more* in Congress than citizens in the left-most districts (one each in suburban Monterrey and Mexico City, the other in Cancún). Politically, citizens at one quartile will be nearly worth twice as much as those at the other quartile.

The bottom plot offers a counterfactual, adopting the hypothetical map that was drawn towards the 2015 election but abandoned. It is quite clear that, even if outliers remain in its inauguration, the hypothetical map would have represented Mexicans much better than the status quo (note the narrower horizontal lines). Taking the mental experiment further, it is possible to aggregate section-level population projections for 2009 into the hypothetical map to assess it performance in the year closest to the census on which the map was prepared. Like fish, demographic information must be fresh: all but a handful of hypothetical districts in that year are within the ±15 band.

The evidence is unambiguous: malapportionment in Mexico is systematic and substantial.[6] Whether it translates into party bias is the subject to which we now turn attention.

2 Party Bias and Responsiveness

Consider now the relation between congressional party votes (x-axis) and seats (y-axis) portrayed in Figure 3. Panel (a) relates shares at the state level (i.e., votes/seats won by the party in each of the state's federal districts, summed and divided by the total votes/seats in the state) and panel (b) at the national level (i.e., 300-district aggregates), analyzed separately. Each point reports one party's vote-seat relation at the unit of aggregation in one election between 2006 and 2012. For instance, the dot floating to the left of the cloud and above the dashed diagonal line in panel (a) is the Green party in the state of Chiapas in 2012, where it won 3 out of 12 seats. The chart shows that 9 percent of the vote statewide awarded the party 25 percent of the seats, an outstanding achievement for any party. Both clouds manifest a steep upwards slope characteristic of plurality systems (Taagepera 1973).[7] Points below the diagonal indicate under-representation, those above

[6]The comparative survey by Snyder and Samuels (2004) ranked Mexico among well-apportioned cases. The measure reported is for the 1997 map, but no guidance is offered about the population figures used in denominators. We suspect reliance—as IFE did then and still does now—on raw 1990 census data, severely underestimating malapportionment.

[7]Adding the excluded PR seats would level the slope considerably. Doing this would be easy with national aggregates. It is not evident how to carry it with state aggregates, since PR seats are awarded in five second-

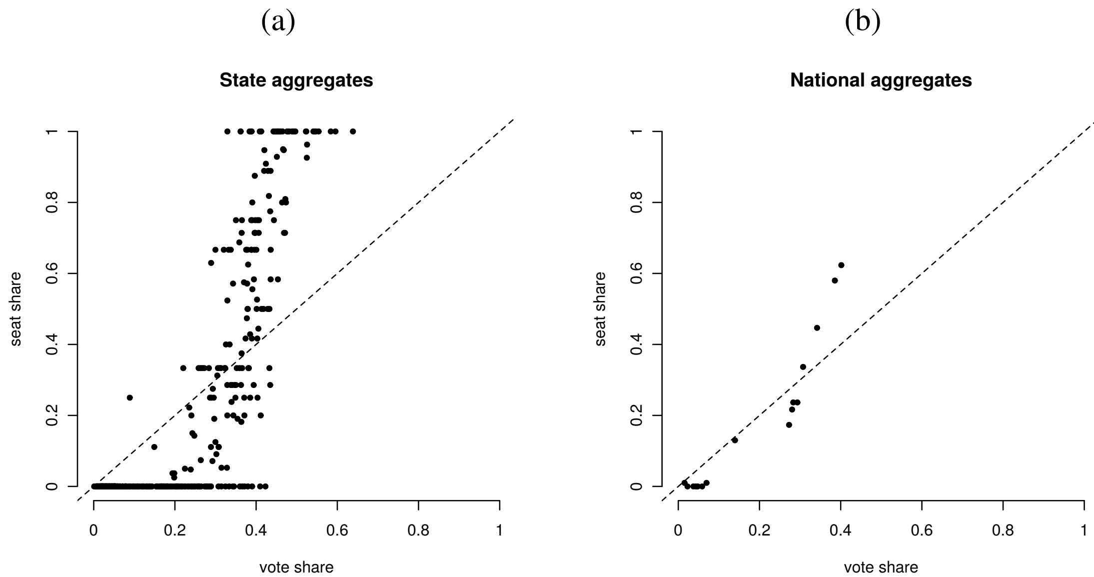


Figure 3: Votes and plurality seats in the 2006, 2009, and 2012 federal deputy elections. Panel (a) reports state aggregates and each point is a party-state-year. Panel (b) reports standard national aggregates and each point is a party-year. Source: prepared with data from `www.ine.org.mx`.

over-representation. Differences are notable among major parties: the PRI achieved over-representation in three-fifths of election-states in the period, the PAN in two-fifths, and the PRD in one-fourth only. Might the system confer undue advantage to the PRI?

2.1 Two classes of distortion

Undue advantage is known, in the specialized literature, as partisan bias, and is one goal that strategic redistricters pursue. It is not, however, alone: scholarship highlights district responsiveness, also known as majority premium, as another goal. These ought to be distinguished (this paragraph draws heavily on Cox and Katz 2002, ch. 3). Partisan bias helps the beneficiary buy seats with fewer votes than others. Because seat distribution is a constant sum game, bias in favor of someone always implies bias against someone else. One way of introducing party bias in district lines is with the conventional redistricting strategy known as packing: group your adversary's voters in few districts, wasting votes to win unnecessarily safe seats, thus raising the price of victory. Responsiveness, on the other hand, is the feature granting a seat bonus to large parties. Maximal responsiveness occurs within each SMD in isolation: the winner takes all, the rest nothing. The same could be achieved in a

tier districts grouping several states each.

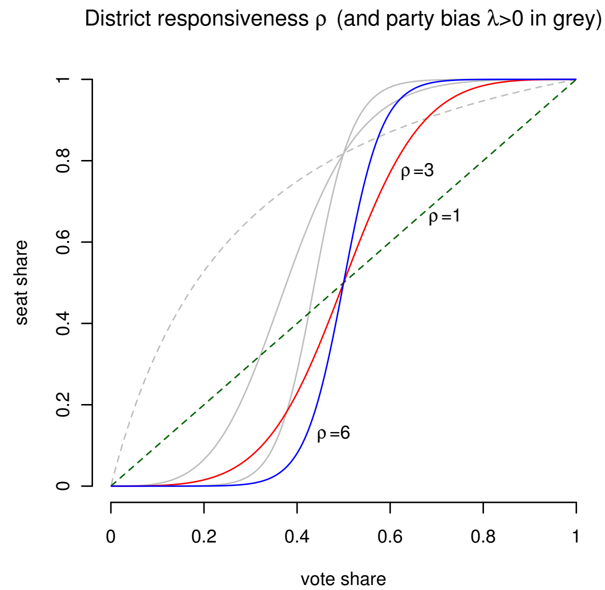


Figure 4: Illustration of model parameters. Party bias is set to $\lambda = 0$ in non-grey lines. Grey lines replicate the colored ones with $\lambda = +1.5$.

whole state by drawing lines so that every district is representative of the state's electorate (Cox and Katz's microcosm strategy). The party with most votes statewide wins every seat, maximizing the vote responsiveness of the proposal.

Formalizing party bias and responsiveness reveals how these district characteristics relate and opens the way towards their estimation. The two-party case is simpler (King and Browning 1987, Taagepera 1973, Tufte 1973) and extends to multiparty competition. It is a generalization of the cube law stipulating that

$$\frac{s}{1-s} = e^{\lambda} * \left(\frac{v}{1-v}\right)^{\rho} \iff \texttt{logit}(s) = \lambda + \rho * \texttt{logit}(v) \tag{1}$$

where s is the seat share that the left party won with vote share v; λ is the left party's bias relative to the right party (positive values favor the left, negative values favor the right); and ρ is the districts' responsiveness. With $\lambda = 0$ a system with no party bias ensues. Figure 4 shows how the parameters affect the vote-to-seats conversion.

Non-gray lines lack party bias to illustrate variable responsiveness. A system with $\rho = 1$ is perfect PR, the ideal type against which real districts are often contrasted. It appears as the dotted green diagonal: every party winning x% of the vote gets, precisely, x% of seats. $\rho = 3$ characterizes the classic cube law, the red curve over-representing the winner (points above the diagonal). Here a party with 55% of the vote earns two-thirds of the seats, but with 33% it earns only one-tenth of the seats. As responsivity heightens, the curve grows steeper, until barely crossing the majority threshold suffices to win all the seats available.

2.2 Sources of party bias

Party bias is asymmetric treatment of parties in the vote–seat conversion. Grey lines replicate the values of ρ just discussed but with $\lambda = +1.5$ added. Bias in favor of the left party achieves a leftward pull of lines. In other words, a bias-favored party will require less effort to reach the threshold for large-party over-representation, cooking artificial parliamentary majorities with substantially less than a vote majority—as routinely occurred in the United Kingdom since the 1970s. (The grey dotted line demonstrates how, due to logit links in Equation 1, party bias also reshapes the function's trace.)

At the root of party bias in systems with multiple districts are differences in the geographic concentration of parties' vote strength. One party with 20 percent of the vote nationwide evenly spread across districts may fail to win a single seat, yet another with much less support but concentrated in few districts will, in fact, get seats. At the same time, vote concentration by larger parties leads to vote (and seat) wasting (Calvo and Rodden 2015). In the end, several forces add up and interact to yield party bias (Gudgin and Taylor 1980). Analysis of the components of party bias

Grofman, Koetzle and Brunell (1997) demonstrate that what we shall now call *raw party bias* (λ) has three clear and distinct sources and offer a procedure to separate empirically the independent contribution of each. Our analysis applies the GKB method to recent Mexican elections.[8] The first source of party bias is *distributional* (or vote wasting). It corresponds to partisan support differently spread across districts mentioned above. Vote wasting may be caused intentionally through gerrymandering (e.g., packing your opponent's support in few districts), but it may also arise by the accidents of geography (e.g., when districts do not cross state boundaries and the state is a party stronghold). The second source of party bias is differential *turnout* across districts. Those who abstain from voting are passively lowering the bar to win a district's seat. Parties stronger in the lower turnout districts will achieve victories with fewer votes than adversaries, improving their votes:seats ratio. Turnout differentials arise when correlates of participation, such as socioeconomic status, vary systematically across districts, or when parties mobilize more effec-

[8]Other elements highlighted by Gudgin and Taylor (1980) that our analysis of raw party bias ignores are the cube-law's bonus, large third-party votes, and possible interactions between all the elements. The bonus is, in fact, captured by the system's responsiveness parameter and therefore distinct from party bias in our framework (more on this below). Calvo (2009) models departures from bipartism explicitly. Interactions remain interesting venues for future research.

	Raw votes			Vote shares			
Districts	left	right	total	left	right	Pop.	Turnout
Distributional-based party bias only							
1 and 2	7	3	10	**.7**	**.3**	20	.5
3, 4 and 5	4	6	10	**.4**	**.6**	20	.5
nationwide	26	24	50	.52	.48	100	.5
Turnout-based party bias only							
1 and 2	12	6	18	.67	.33	30	**.6**
3, 4 and 5	3	6	9	.33	.67	30	**.3**
nationwide	33	30	63	.52	.48	150	.42
Malapportionment-based party bias only							
1 and 2	12	6	18	.67	.33	**36**	.5
3, 4 and 5	3	6	9	.33	.67	**18**	.5
nationwide	33	30	63	.52	.48	126	.5

Table 1: Illustrative five-district system scenarios

tively in some areas than others (Rosenstone and Hansen 1993). The third source of party bias is *malapportionment*.

The scenarios in Table 1, which draw heavily from examples in GKB, illustrate the sources operate in isolation from the others. The division of vote and seat shares nationwide and the degree of party bias remain constant in all scenarios: the left party suffers a 12 percentage point deficit in representation, with 52% of votes but just 40% of seats (it won two of five districts); and the right party enjoys 12 percent overrepresentation, winning 60% of seats with just 48% of votes. Other traits change, one at a time. The first scenario has equal-sized and constant-turnout districts[9] that nonetheless manifest partisan differences in votes wasted, the left party winning seats by larger margins (.4) than the right (.2). The source of party bias is distributional only. A different map could re-allocate wasted votes in such way that another district tips towards the left.

The second scenario has equal-sized districts and winning margins uncorrelated with the vote distribution, but varying turnout that is not orthogonal to vote shares. Right and left are winning seats with the exact same margins, but the right winning in lower turnout districts—half in fact the turnout of districts won by the left. As a consequence, right seats com quite cheaper than the left's. In this case, party bias is the product of turnout differentials alone playing against the left. If the fruits of the left's mobilization effort extended to beyond districts 1 and 2, other seats might have been won.

The final scenario has equal-turnout districts and winning margins uncorrelated with

[9]A less restrictive scenario allows size and turnout differences across districts with distributions that are independent of the distribution of partisan support.

party vote strength, but district size differences that do correlate with the latter. Again, both parties are winning with the same margins, but the right is doing this in districts half as populous as those won by the left. The consequence is a more efficient conversion of quite similar total votes into seats for the right than the left. This is party bias attributable to malapportionment by itself.

How do GKB separate the three sources of party bias empirically? The discussion of vote-seat curves has so far assumed that votes in Equation 1 are one party's share of the aggregate (national or state) total vote.[10] Using this standard mode of aggregation of district-level vote returns yields a raw party bias estimate. But, as pioneered by Tufte (1973; also Gelman and King 1994), fitting the vote–seat curve using party p's mean district vote share $\bar{v}_p$ instead yields distributional-based party bias. This is so because $\bar{v}_p$ aggregates district vote shares but disregards district size and voter turnout. In the same spirit, relying on party p's population-weighted mean district vote share $\bar{w}_p$ (an aggregate compounding district vote shares and relative district populations) yields estimates conflating distributive- and malapportionment-based party bias. So subtracting party bias estimated with $\bar{v}_p$ from party bias estimated with $\bar{w}_p$ yields pure malapportionment-based party bias. And, because raw party bias conflates all three sources, subtracting party bias estimated with $\bar{w}_p$ from party bias estimated with v_p yields pure turnout-based party bias.

We apply this separation method to recent Mexican congressional elections in search of malapportionment-based distorsions in the votes-to-seats conversion.

3 Expectations

On paper, IFE is an autonomous, non-partisan election regulator. Members of its Council-General are thoroughly vetted and recruited from among professionals without party affiliation and admitted after winning consensual endorsement in the Chamber of Deputies. Once in office, councilors enjoy secure tenure. Their budget, which includes generous public financing for political parties and their electoral campaigns, is subject to few political whims. Yet closer inspection of the structure and process reveals how the Council-General is, in practice, a power-sharing agent of the major congressional parties (Estévez, Magar and Rosas 2008). The systematic partisan segmentation of the Council-General is quite

[10]Noting that party p's raw vote in district d is the product of its district vote share v_{dp} and the district's raw vote, the party's aggregate vote share can be expressed as $v_p = \sum_d (v_{dp} \times \frac{\text{raw vote}_d}{\text{total raw vote}})$. This algebraic transformation eases comparation to the other aggregate vote measures in GKB's formalization of the separation argument: party p's mean district vote share is $\bar{v}_p = \sum_d (v_{dp} \times \frac{1}{\text{total districts}})$; and party p's population-weighted mean district vote share is $\bar{w}_p = \sum_d (v_{dp} \times \frac{\text{population}_d}{\text{total population}})$. The notation GKB use for these expressions is R, P, and M, respectively.

conclusive evidence that major-party influence in electoral regulation is unremitting.

Two general expectations on party bias follow from this view of electoral regulation.

1. When a map is adopted, districts have no bias favoring one major party relative to another. If a new map has bias, it must be against minor parties relative to major ones.

2. As maps age and district populations drift away from census relative levels, party bias (malapportionment-based) can creep in. Party bias should therefore grow with each failure to redistrict.

4 Data and results

A multiparty and estimable version of equation 1 (King 1990) establishes that party p's ($p = 1, 2, \ldots, P$) expected seat share is

$$E(s_p) = \frac{e^{\lambda_p} * v_p^{\rho}}{\sum_{q=1}^{P} e^{\lambda_q} * v_q^{\rho}} \tag{2}$$

with data and parameters now indexed to identify the parties (another is Calvo and Micozzi 2005). Setting $\lambda_2 = 0$ restricts the remainder $\lambda_{p \neq 2}$ to express party bias with relation to the PRI's ($p = 2$ in the dataset). This is convenient. Party bias in two-party competition is asymmetry in a vote–seat curve centered on $v = .5$ (see Figure 4). There is no reason to expect a .5-centered curve in multiparty competition, nor is it a priori evident what vote should serve as center point, a difficulty towards expressing the party bias estimate $\hat{\lambda}_p$ as a percentage points advantage or handicap for party p in the votes-to-seats conversion. Finding $\lambda_{p \neq 2} < 0$ would be evidence of PRI-favoring bias.

A common estimation strategy relies on a time-series of election returns. Márquez (2014) does this using votes and seats won over two decades, uncovering a degree of responsivity characteristic of plurality systems and substantive party bias against the PAN. Multi-election studies increase the number of data points for estimation, but inevitably sacrifice comparability, especially over the longer haul (Jackman 1994). It is district-level margins, after all, that determine seats won, and two district margin distributions could conceivably yield very similar national vote aggregates but quite different seat shares—a classical problem of over-determination. Single-election studies are therefore preferable (Niemi and Fett 1986), and procedures to multiply data points desirable. Linzer (2012, discussed below—and maybe used in future incarnations of this paper!) approximates a mixture density of district outcomes with which to perform Monte Carlo simulation towards this end.

The estimation strategy adopted here relies on state-level congressional election aggregates, instead of standard national-level aggregates. One disadvantage of the approach is that states have few districts (9.4 on average), and this will amplify the system's responsiveness (Taagepera 1973). It should not, however, affect estimates of party bias, our substantive interest. And the approach offers a 32-fold hike in data points (patent between panels in Figure 3), clearing the way for single-election analyses. The actual district structure remained constant throughout the 2006, 2009, and 2012 federal deputies elections studied here.

The method of estimation is MCMC (Jackman 2000).[11] To obtain the GKB separate party bias sources, equation 2 was estimated as it appears, then replacing v_p by $\bar{v}_p$, then by $\bar{w}_p$. State-aggregate data was used to produce single-election estimates of the 2006, 2009, and 2012 elections. Three-election national aggregates were also fitted for comparison. And, for further perspective on malapportionment-based party bias, we finally repeated all estimation with hypothetical outcomes of the redistricting proposal that was not adopted in 2015.[12]

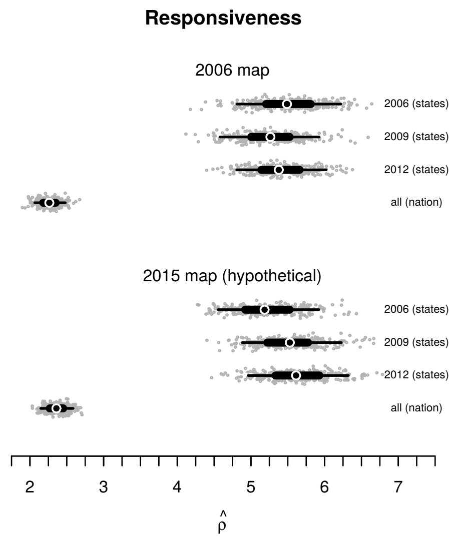
Figure 5 reports summaries of parameter estimates (using v_p). The district responsiveness parameter (panel a) is of secondary interest, yet offers ground to evaluate model performance. The plot reports point estimates (white circles are the median of ρ's posterior sample) for each election separately with state aggregates, for all years pooled together with national aggregates, and comparing actual districts (2006 map) to the abandoned redistricting proposal (2015 map). Owing to data points with few congressional seats (between 2 and 40), $\hat{rho}$ is consistently very high—between 5 and 6—in state-aggregates estimations, as expected. The large party premium resulting from this approach is twice the power of the putative cube law of plurality elections (Taagepera 1973). It drops sharply, to around 2.5, in national-aggregates estimation with 300 seats.

Panels (b) and (c) report raw party bias. It is noteworthy that, unlike ρ, the national-

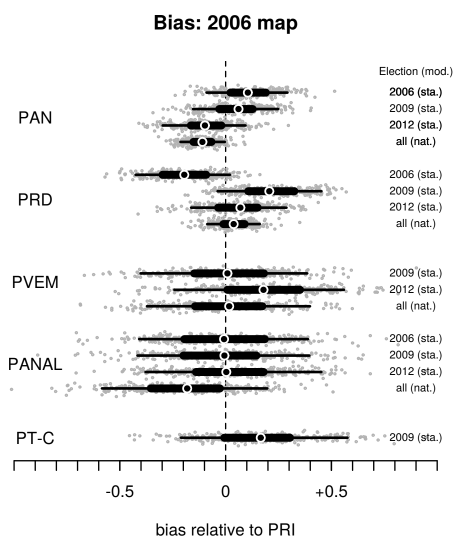
[11]Three chains were iterated 10 thousand times, taking every fiftieth observation of the last 5 thousand to sample the posterior distribution. The Gelman–Hill $\hat{R} \approx 1$ (Gelman and Hill 2007), evidence that the chains had reached a steady state. Convergence was also inspected visually with chain traceplots for each of the model's parameters. Estimation performed with open-source software: JAGS (Plummer 2003), implemented in R (R Dev. Core Team 2011) with library R2jags (Su and Yajima 2012). Data and commented code to replicate the analysis will be posted on-line upon publication.

[12]Hypothetical outcomes were generated with *sección*-level vote returns. *Secciones electorales* are analogous to U.S. census tracts, but bigger (median population in the 2010 census was 1,280, with a maximum at 79,232). The 2006 map (actual districts) and 2015 map (abandoned) relate more than 66 thousand *secciones*, the basic units for district cartography, to congressional districts. This makes reconstitution of hypothetical election outcomes in the period possible. Altman et al. (2014) describe the redistricting process in detail.

(a)



(b)



(c)

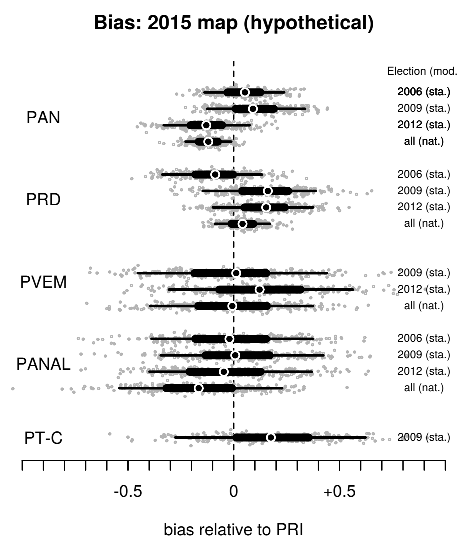


Figure 5: Redistricting, responsiveness, and party bias. Plots describe the posterior sample of estimated parameters $\hat{\rho}$ (panel a) and $\hat{\lambda_p}$ for five parties (panels b and c). The state aggregates model was used for single-election estimation, the national aggregates model for multi-elections. Finer lines connect the 5th and 95th percentile values, thicker lines the quartiles, and white circles indicate the median value.

aggregate estimates of the λ parameters are not markedly different, other than being twice more precise for major parties, than the state-aggregate estimates. (Estimate precision can be assessed with the 50% and 90% confidence intervals, the thicker and thinner horizontal lines, respectively; the cloud of gray points is the full posterior sample.) Even more remarkable is how *little evidence of raw party bias there is*. With two exceptions—relative to the PRI, the PRD experienced negative bias in 2006, the PAN negative bias in the pooled dataset—value zero belongs in all 90% confidence intervals, and in most 50% ones as well. This is even true for small parties: noise kills even the few signals that are not weak.

Major-party bias is expected: small parties lack representation in IFE.

Moreover, the redistricting proposal made little difference in parameters.

Owing to few districts per state, the estimated responsiveness at the state level ($\hat{\rho} \approx 5.5$) is twice Márquez's nationwide ($\hat{\rho} = 2.6$). All three parties experience situations of large party bonus and small party penalty that, to a good extent, cancel each other in the national statistic.

Regarding party bias, signals that are not weak all tend to be accompanied by a good deal of noise, with few exceptions. At the national level, and over a longer haul, Márquez discovers bias in favor of the PRI, but mostly in favor of the PRD, and against the PAN, that seems not the product of chance alone. Analysis at the state level reveals no such biases. As said, Figures 5.b–d express bias relative to the PRI. Although PAN experienced weak signal in its favor in the whole 2006–12 period, a fair density of the blue cloud is, in fact, negative. PRD vs. PRI bias is clearly centered at zero in the full period. The left did experience significant bias in isolated years: against in 2006, in favor in 2009. Perhaps many voters who strategically abandoned the hopeless PRI presidential candidate in 2006 to vote for López Obrador did so in districts where PRI's congressional candidates still won. No other year for no other party reveals any bias unaccompanied by much noise.

5 Conclusion

** To be written **

** Policy recommendations: When redistricting plans are drawn, malapportionment does not exist *de jure*. It exists *de facto*. Either an obligation to redistrict "as soon as reasonably possible" (use the language of Supreme Court and British practice) after a census is produced should be added to the Constitution. Or room should be given to attempts the use of less obsolete population data, such as official lustrum counts, or even population projections. Will remove institutionally-driven creeping malapportionment built into maps at their inception. **

party bias	2006 map PAN–PRI	PRD–PRI	PAN–PRD	2015 map (hypothetical) PAN–PRI	PRD–PRI	PAN–PRD
2006 election (states)						
raw	0.10	−0.20	0.30	0.05	−0.09	0.14
distrib.	0.18	−0.19	0.37	0.14	−0.09	0.23
turnout	−0.10	−0.01	−0.09	−0.06	0.00	−0.06
malapp.	0.02	0.00	0.02	−0.03	0.00	−0.03
2009 election (states)						
raw	0.06	0.21	−0.15	0.09	0.16	−0.07
distrib.	0.09	0.16	−0.07	0.13	0.12	0.01
turnout	−0.04	0.05	−0.09	−0.03	0.02	−0.05
malapp.	0.02	0.00	0.02	−0.01	0.02	−0.03
2012 election (states)						
raw	−0.10	0.07	−0.17	−0.13	0.15	−0.28
distrib.	−0.09	0.02	−0.11	−0.07	0.14	−0.21
turnout	−0.04	0.03	−0.07	−0.05	0.00	−0.05
malapp.	0.04	0.01	0.03	−0.01	0.02	−0.03
All elections (nation)						
raw	−0.11	0.04	−0.15	−0.12	0.04	−0.16
distrib.	−0.04	0.13	−0.17	−0.04	0.17	−0.21
turnout	−0.07	−0.13	0.06	−0.08	−0.13	0.05
malapp.	0.00	0.05	−0.05	0.00	0.00	0.00

Table 2: Relative major-party bias. Single-election entries report raw bias and its additive components from parameters estimated with the state aggregates model. Multi-election bias estimated with the national aggregates model.

Mexico's federal districts exhibit no party bias when analyzed at the state level. Big system responsiveness, typical of first-past-the-post system in units with few federal districts, is associated with the districts. So PRI's tendency to be over-represented more than PAN and PRD, and the PRI's winning of a couple extra seats with the redistricting proposal is not the product of partisan bias. If the PRI wins more this is due to its status as the largest party in more states that the other two major parties combined.

Appendix 1: BUGS code

```
bugsModel <- function() {
    for (i in 1:I){     # loop over state-years
        for (p in 1:P){ # loop over parties (dummy selects those who ran that year)
            S[i,p] ~ dbin(pi[i,p], D[i])  # D is number SMD seats in obs. i's state
        }
        numerator[i,1] <- dummy[i,1] * exp( lambda[1] + rho * log(v[i,1]) )
        numerator[i,2] <- dummy[i,2] * exp(             rho * log(v[i,2]) )
        for (p in 3:P){
            numerator[i,p] <- dummy[i,p] * exp( lambda[p-1] ) * v[i,p]^rho
        }
        for (p in 1:P){ # loop over parties (dummy=1 selects those who ran that year)
            d1[i,p] <- dummy[i,1] * exp( lambda[1] ) * v[i,1]^rho
            d2[i,p] <- dummy[i,2]                    * v[i,2]^rho
            d3[i,p] <- dummy[i,3] * exp( lambda[2] ) * v[i,3]^rho
            d4[i,p] <- dummy[i,4] * exp( lambda[3] ) * v[i,4]^rho
            d5[i,p] <- dummy[i,5] * exp( lambda[4] ) * v[i,5]^rho
            d6[i,p] <- dummy[i,6] * exp( lambda[5] ) * v[i,6]^rho
            d7[i,p] <- dummy[i,7] * exp( lambda[6] ) * v[i,7]^rho
            denominator[i,p] <- d1[i,p]+d2[i,p]+d3[i,p]+d4[i,p]+d5[i,p]+d6[i,p]+d7[i,p]
            pi[i,p] <- numerator[i,p] / denominator[i,p]
        }
    }
    ### priors
    for (q in 1:6){ # there are 7 party labels in the 3-election data, PRI is reference
        lambda[q] ~ dnorm( 0, tau.lambda )
    }
    tau.lambda <- pow(.25, -2)
    rho ~ dexp(.75) # this has positive range, median close to 1, mean 1.25, max 4.5
}
```

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