

# Measuring malapportionment, gerrymander, and turnout effects in multi-party systems

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Empirical procedure to measure and analyze the difference between the **votes** and **seats** that parties win in general elections

Central concern of electoral reform debates

Kendall&Stuart 1950, Rae 1967, Erikson 1972, Tufte 1973, Taagepera 1973, Johnston 1979, Gudgin&Taylor 1980, Grofman 1983, Cain 1985, Niemi&Fett 1986, King&Browning 1987, Gelman&King 1994, Balinski&Young 2001, Cox&Katz 2002, Engstrom 2006...

Methods exist, but limited to (exceptional) two-party competition

## Contributions

- 1 Extension to multi-party competition
- 2 Overcome small- $N$  obstacle
- 3 Apply to recent Mexican elections

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Key quantity of interest is the system's **partisan bias** = undue advantage conferred to some party in the conversion of votes into legislative seats

→ Potential distortion wherever districts are drawn to allocate seats

Theory highlights three sources of partisan bias

- 1 Gerrymanders (e.g., Cox&Katz 2002)
- 2 Turnout differentials (Rosenstone&Hansen 1993)
- 3 Malapportionment and demographic shifts (Johnston 1979)

Procedure measures three sources

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Procedure measures three sources

	$v$	$s$	$s - v$
Conservative	.369	.509	+.140
Labour	.305	.357	+.052
UK Independence Party	.126	.002	-.125
Liberal Democrat	.079	.012	-.066
Scottish National Party	.047	.086	+.039
Green	.038	.002	-.036

	small	large
too concentrated	helps	hurts
too dispersed	hurts	helps

## Three models

- Grofman, Koetzle & Brunell (1997) → partisan bias breakdown
- King (1990) → multi-party partisan bias
- Linzer (2012) → data scarcity

## Findings

- 1 Persistent bias against the right
- 2 Components of bias often larger than the whole
- 3 Gerrymanders have offset PRI's large turnout advantage



- 1 Partisan bias: sources, measurement
- 2 Mexican Cámara de Diputados elections
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- 4 Results
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# What is partisan bias

It is the excess/defect seat share that a party with half of the votes wins:

$$\lambda = (s \mid v = .5) - .5$$

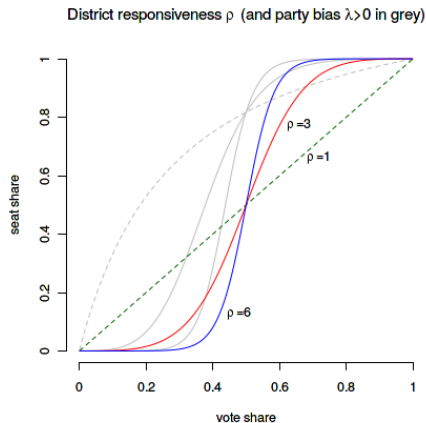
- Two-party, balanced system assumed
- .5 threshold inappropriate for multi-party/imbalanced competition
- Alternative threshold not evident
- Relative quantity preferable

Fitting votes–seats curves:  $s = f(v)$

$$\frac{s}{1-s} = \left( \frac{v}{1-v} \right)^3$$

$$\frac{s}{1-s} = \lambda \left( \frac{v}{1-v} \right)^\rho \iff$$

$$\text{logit}(s) = \ln \lambda + \rho \text{logit}(v)$$



# Three sources of partisan bias

Districts	Pop.	Turnout	Raw votes			Vote shares		Seat shares	
			left	right	total	left	right	left	right
Gerrymander									
1 and 2	420	.5	147	63	210	.7	.3	1	0
3, 4 and 5	420	.5	84	126	210	.4	.6	0	1
nationwide	2100	.5	546	504	1050	.52	.48	.4	.6
Turnout									
1 and 2	420	.70	200	100	300	.67	.33	1	0
3, 4 and 5	420	.35	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6
Malapportionment									
1 and 2	600	.5	200	100	300	.67	.33	1	0
3, 4 and 5	300	.5	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6

# Obstacle 1: partisan bias breakdown

Grofman, Koetzle & Brunell (1997): Three ways to aggregate district returns nationwide

$$v_p = \sum_d v_{dp} \times \frac{\text{raw vote}_d}{\text{total raw vote}} \quad (1)$$

$$\bar{v}_p = \sum_d v_{dp} \times \frac{1}{\text{total districts}} \quad (2)$$

$$\bar{w}_p = \sum_d v_{dp} \times \frac{\text{population}_d}{\text{total population}} \quad (3)$$

Fitting votes–seats curve with (1), (2), or (3) yields components

→ with  $v$  you get **raw** partisan bias

→ with  $\bar{v}$  you get **gerrymander**-based

→ with  $\bar{w}$  you get **gerrymander** + **malapportionment**-based

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# Obstacle 1: partisan bias breakdown

Trick is to estimate  $\lambda$  with each national vote measure in turn

Formulas:

- a** raw partisan bias  $= \lambda^v$
- b** gerrymander-based  $= \lambda^{\bar{v}}$
- c** malapportionment-based  $= \lambda^{\bar{w}} - \lambda^{\bar{v}}$
- d** turnout-based  $= \lambda^v - \lambda^{\bar{w}}$
- e**  $a = b + c + d$



## Obstacle 2: multi-party competition

King (1990) is a votes-seats curve for a  $P$ -party system ( $P \geq 2$ )

$$E(s_p) = \frac{e^{\lambda_p} v_p^\rho}{\sum_{q=1}^P e^{\lambda_q} v_q^\rho}$$

- Akin to switch from dichotomous to multinomial logit regression
- Restricting  $\lambda_1 = 0$  expresses bias relative to party  $p = 1$

In sum:

Fit equation above using  $v$ , then  $\bar{v}$ , then  $\bar{w}$ ;

rely on subtraction formulas to get measures of raw partisan bias and its components

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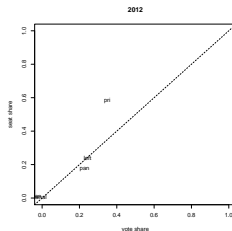
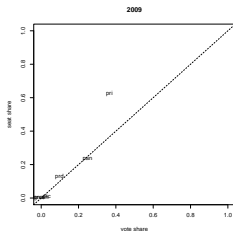
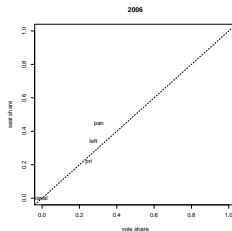
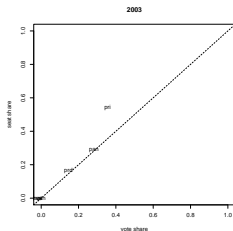
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- Hegemonic party 1929–1997
- Three major parties: PRD left PRI PAN right and minors
- Lower chamber of Congress elected every 3 years, concurrent w presidential race every 6 years
- Mixed system: 300 SMD + 200 PR seats
- Single-term limits removed in 2018
- Independent board (IFE) manages elections and redistricting

- A general election with  $P$  parties offers  $P$  data points to fit a votes-seats curve
- $P$  typically small
- Multi-year approach: pool historic record... may compare apples/oranges in long-haul
- Single-election approach preferable... but requires data multiplication strategy

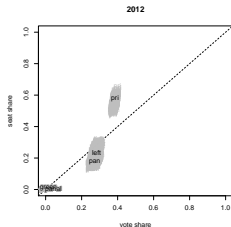
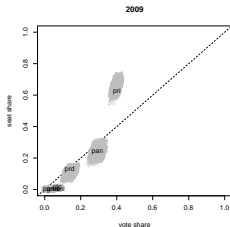
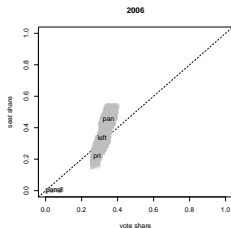
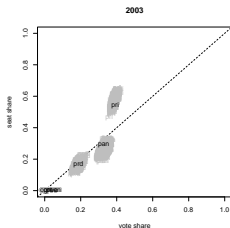
# Monte Carlo simulation

- Linzer (2012): approximates prob. distribution of national party vote returns from observed district outcomes (FMM)
- Use to simulate 1,000 elections



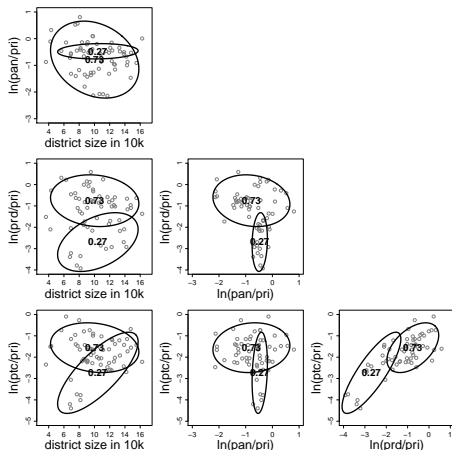
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# Mixture model

- Combines the properties of two or more prob. density functions: can approximate any arbitrary distribution
- Seek components (multivariate normals) and weights of log-ratio votes shares





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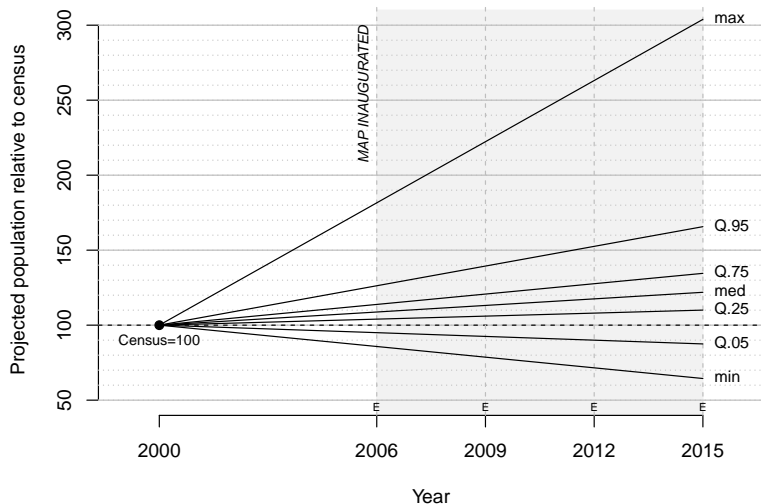
Malapportionment occurs when sparsely populated areas get the same representation as the densely populated

Can arise by commission or by omission (or by accident)

Census lag: most recent decennial census must be used

- ... but no obligation to redistrict as soon as available
- 6-year lag on average: 199**7**, 200**6**, 201**5**
- 2015 map abandoned
- board considers  $\pm 15\%$  imbalance normal (!)

# District populations: linear projection

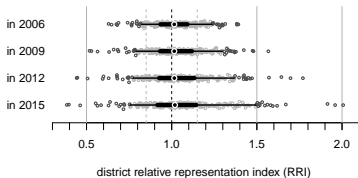


Plus: bureaucratic leeway in new district sizes

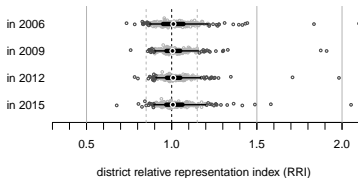
# Malapportionment is substantial

$$RRI = \frac{nat.pop./300}{\text{district size}}$$

2006 map (drawn with 2000 census)



2015 map (drawn with 2010 census)



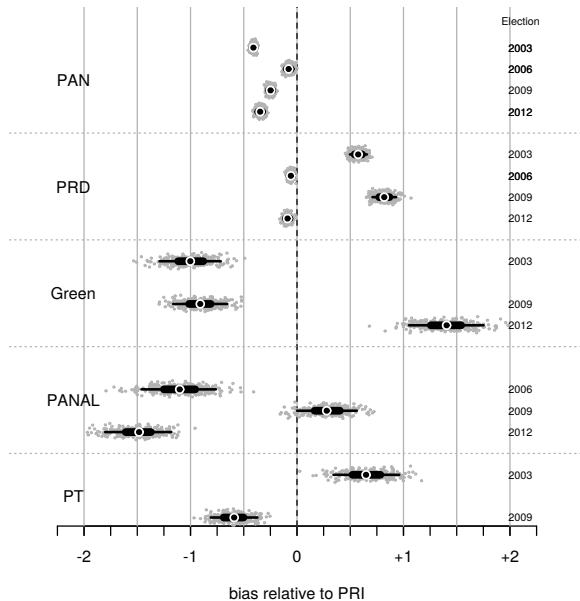
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Votes–seats curve fitted with MCMC (Jags via R)

100k iterations of 3 chains, every 500th obs. of last 50k used to sample posterior density

Visual inspection of model parameters to verify chain convergence

# Results: raw partisan bias



# Results: components

	Actual map			Hypothetical map		
partisan bias	pan-pri	prd-pri	min-pri	pan-pri	prd-pri	min-pri
<b>2003 election</b>				(with 2006 map)		
raw	-.37 (0)	+.72 (0)	-1.01 (0)	-.41 (0)	+.57 (0)	-1.00 (0)
gerrym.	-.09 (0)	+.69 (0)	-.88 (0)	-.13 (0)	+.62 (0)	-.90 (0)
turnout	-.26 (0)	-.11 (0)	-.08 (0)	-.26 (0)	-.09 (0)	-.09 (0)
malapp.	-.01 (.11)	+.14 (0)	-.05 (0)	-.02 (.12)	+.05 (0)	-.02 (0)
<b>2006 election</b>						
raw	-.08 (0)	-.06 (0)	-1.10 (0)			
gerrym.	+.28 (0)	+.30 (0)	-.62 (0)			
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raw	-.25 (0)	+.82 (0)	-.91 (0)			
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raw	-.35 (0)	-.09 (0)	+1.40 (0)	-.32 (0)	-.13 (0)	+1.03 (0)
gerrym.	-.28 (0)	-.07 (0)	+1.41 (0)	-.24 (0)	-.05 (.06)	+1.02 (0)
turnout	-.07 (.02)	-.08 (0)	+.02 (0)	-.08 (.26)	-.09 (0)	+.01 (0)
malapp.	+.01 (.42)	+.06 (0)	-.02 (0)	-.00 (.38)	+.01 (0)	+.00 (0)

- Turnout always pro-PRI
- Malapp. always pro-left
- Redistricting abates malapp.
- Possibly cancelling effects



# Results: components

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*Draw Mexico* project = offspring of *Public Mapping Project in U.S.*

Remove opaqueness from redistricting process

DistrictBuilder is open-source, web-based software

- enables widespread DIY redistricting thru cloud computing
- internet lets anyone draw/inspect maps: crowdsourcing
- redistricting contests in 6 US states → hundreds of legal plans

Application to **Mexico** [▶ Link: MexDemo](#) (Donations anyone?)

- 1 Rel. to the right, persistent pro-PRI, and esp. pro-left raw bias
- 2 Malapportionment effects are (surprisingly) small
- 3 Pro-PRI turnout-based bias
- 4 Gerrymander effects large and volatile
- 5 District lines can compensate for turnout disadvantage
- 6 To-do: add PR-tier to analysis
- 7 To-do: inspect inter-election volatility

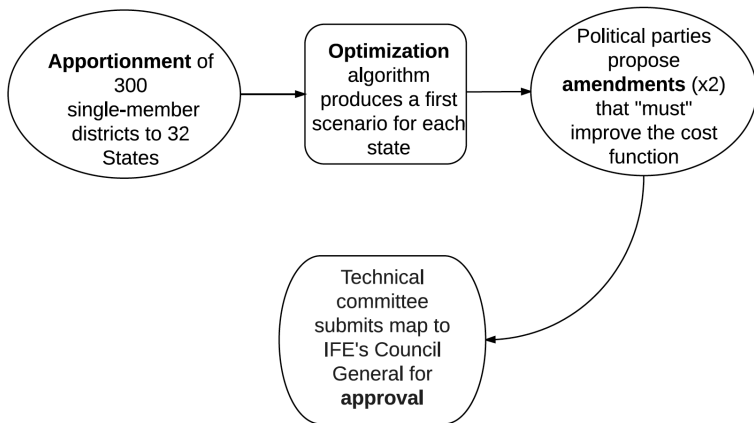
Thank you!

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**Thank you!**



# The redistricting process



# The redistricting process

$$\begin{aligned}\text{Score} = & .4 \times \text{PopBalance} + .3 \times \text{MunicBoundaries} \\ & + .2 \times \text{TravelTime} + .1 \times \text{Compactness}\end{aligned}$$

Plus: minority representation (40 % indigenous pop.)

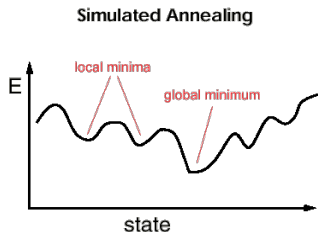
Board tolerates  $\pm 15\%$  imbalance to accommodate this

# Optimization algorithm

Simulated annealing = probabilistic meta-heuristic for optimization  
locates a good approximation to the global optimum of the cost  
function in a large search space

Thousands of iterations using electoral *secciones*

Combinatorial optimization algorithm used to generate the first  
scenario in each state



IFE claims that this is a public process, but the  
operation and procedures are done **behind closed doors**

## Proposals and counterproposals

