Measuring malapportionment, gerrymander, and turnout effects in multi-party systems

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Motivation

Empirical procedure to measure and analyze the difference between the **votes** and **seats** that parties win in general elections

Central concern of electoral reform debates

Balinski&Young 2001, Cain 1985, Cox&Katz 2002, Engstrom 2006, Erikson 1972, Gelman&King 1994, Grofman 1983, Grofman et al. 1997, Gudgin&Taylor 1980, Johnston 2002, Kendall&Stuart 1950, King&Browning 1987, Niemi&Fett 1986, Rae 1967, Rossiter et al. 1997, Taagepera 1973, Tufte 1973

Procedures exist, but limited to (exceptional) two-party competition

Contributions

1. Extention to multi-party competition
2. Overcome small-N obstacle
3. Apply to recent Mexican elections

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Overview

Key quantity of interest is the system's **partisan bias** = undue advantage coferred to some party in the conversion of votes into seats

- Potential distortion wherever districts are drawn to allocate seats (Tufte 1973, Johnston 2002)

Theory highlights three sources of partisan bias

1. Gerrymanders (Cox&Katz 2002)
2. Turnout differentials (Rosenstone&Hansen 1993)
3. Malapportionment and demographic shifts (Jackman 1994)

Procedure to break down three sources in two-party competition (cf. Grofman et al. 1987)

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UK general election 2015

	v	s	$s - v$
Conservative	.369	.509	$+.140$
Labour	.305	.357	$+.052$
UK Independence Party	.126	.002	$-.125$
Liberal Democrat	.079	.012	$-.066$
Scottish National Party	.047	.086	$+.039$
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	small	large
too concentrated	enjoys	suffers
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Overview: results

Three models

- Grofman, Koetzle&Brunell (1997) $\rightarrow$ partisan bias breakdown
- King (1990) $\rightarrow$ multi-party partisan bias
- Linzer (2012) $\rightarrow$ data scarcity

Findings

1. Persistent bias against the right
2. Components of bias often larger than the whole
3. Gerrymanders have offset PRI's large turnout advantage

Road map

1. Partisan bias: sources, measurement
2. Mexican Cámara de Diputados elections
3. Malapportionment
4. Results

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What is partisan bias

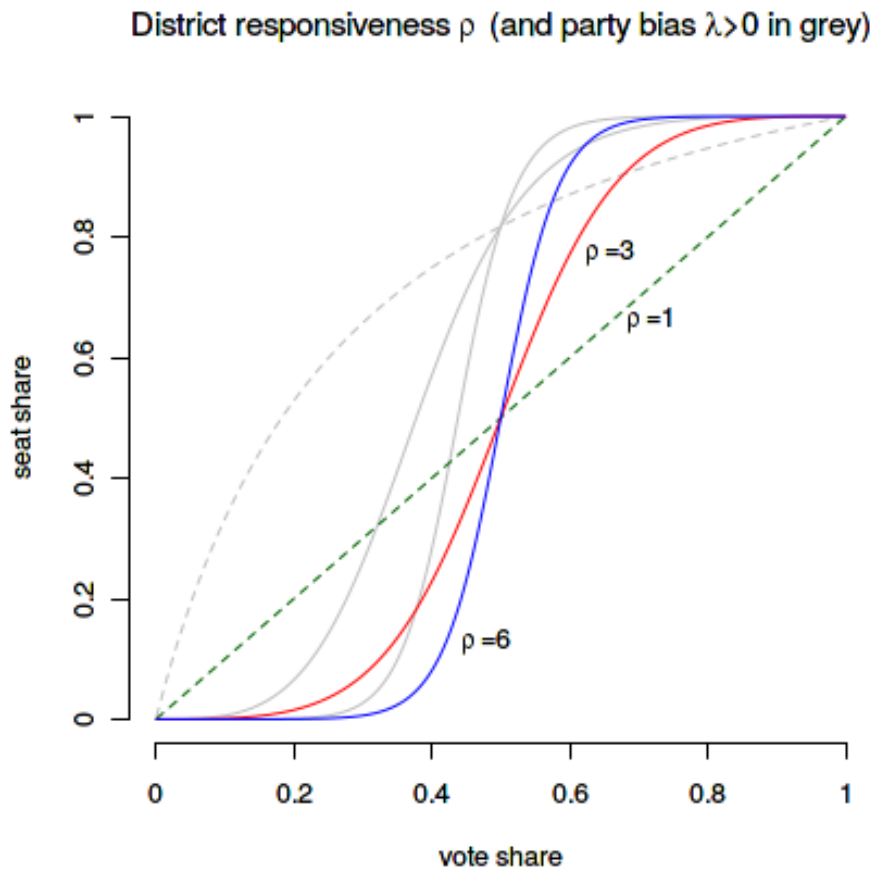
It is the excess/defect seat share that a party with half of the votes wins:

$$\lambda = (s \mid v = .5) - .5$$

- Two-party system assumed
- .5 threshold unappropriate for multi-party (or imbalanced) competition
- Alternative threshold not evident: relative quantity better
- Constant-sum game

Fitting votes–seats curves: $s = f(v)$
(Rae 1967, Tufte 1973, King&Browning 1987)

$$\frac{s}{1-s} = \lambda \left(\frac{v}{1-v} \right)^{\rho}$$
$$\text{logit}(s) = \ln \lambda + \rho \text{logit}(v)$$



Three sources of partisan bias

			Raw votes			Vote shares		Seat shares	
Districts	Pop.	Turnout	left	right	total	left	right	left	right
Gerrymander									
1 and 2	420	.5	147	63	210	**.7**	**.3**	1	0
3, 4 and 5	420	.5	84	126	210	**.4**	**.6**	0	1
nationwide	2100	.5	546	504	1050	.52	.48	.4	.6
Turnout									
1 and 2	420	**.70**	200	100	300	.67	.33	1	0
3, 4 and 5	420	**.35**	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6
Malapportionment									
1 and 2	**600**	.5	200	100	300	.67	.33	1	0
3, 4 and 5	**300**	.5	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6

Obstacle 1: partisan bias breakdown

Grofman, Koetzle & Brunell (1997): Three ways to aggregate district returns nationwide

$$v = \sum_d v_d \times \frac{\text{raw vote}_d}{\text{total raw vote}} \tag{1}$$

$$\bar{v} = \sum_d v_d \times \frac{1}{\text{total districts}} \tag{2}$$

$$\bar{w} = \sum_d v_d \times \frac{\text{population}_d}{\text{total population}} \tag{3}$$

Fitting votes–seats curve with (1), (2), or (3) yields components
$\rightarrow$ with v you get **raw** partisan bias
$\rightarrow$ with $\bar{v}$ you get **gerrymander**-based
$\rightarrow$ with $\bar{w}$ you get **gerrymander** + **malapportionment**-based

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Obstacle 1: partisan bias breakdown

Trick is to estimate λ with each national vote measure in turn

Formulas:

a. raw partisan bias $= \lambda^{v}$

b. gerrymander-based $= \lambda^{\bar{v}}$

c. malapportionment-based $= \lambda^{\bar{w}} - \lambda^{\bar{v}}$

d. turnout-based $= \lambda^{v} - \lambda^{\bar{w}}$

e. $a = b + c + d$

Obstacle 2: multi-party competition

King (1990) is a votes-seats curve for a P-party system ($P \geq 2$)

$$E(s_p) = \frac{e^{\lambda_p} v_p^{\rho}}{\sum_{q=1}^{P} e^{\lambda_q} v_q^{\rho}}$$

- Akin to switch from dichotomous to multinomial logit regression
- Restricting $\lambda_1 = 0$ expresses bias relative to party $p = 1$

In sum:
Fit equation above using v, then $\bar{v}$, then $\bar{w}$;
rely on subtraction formulas to get measures of partisan bias and its components

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1. Partisan bias: sources, measurement
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Background on Mexico

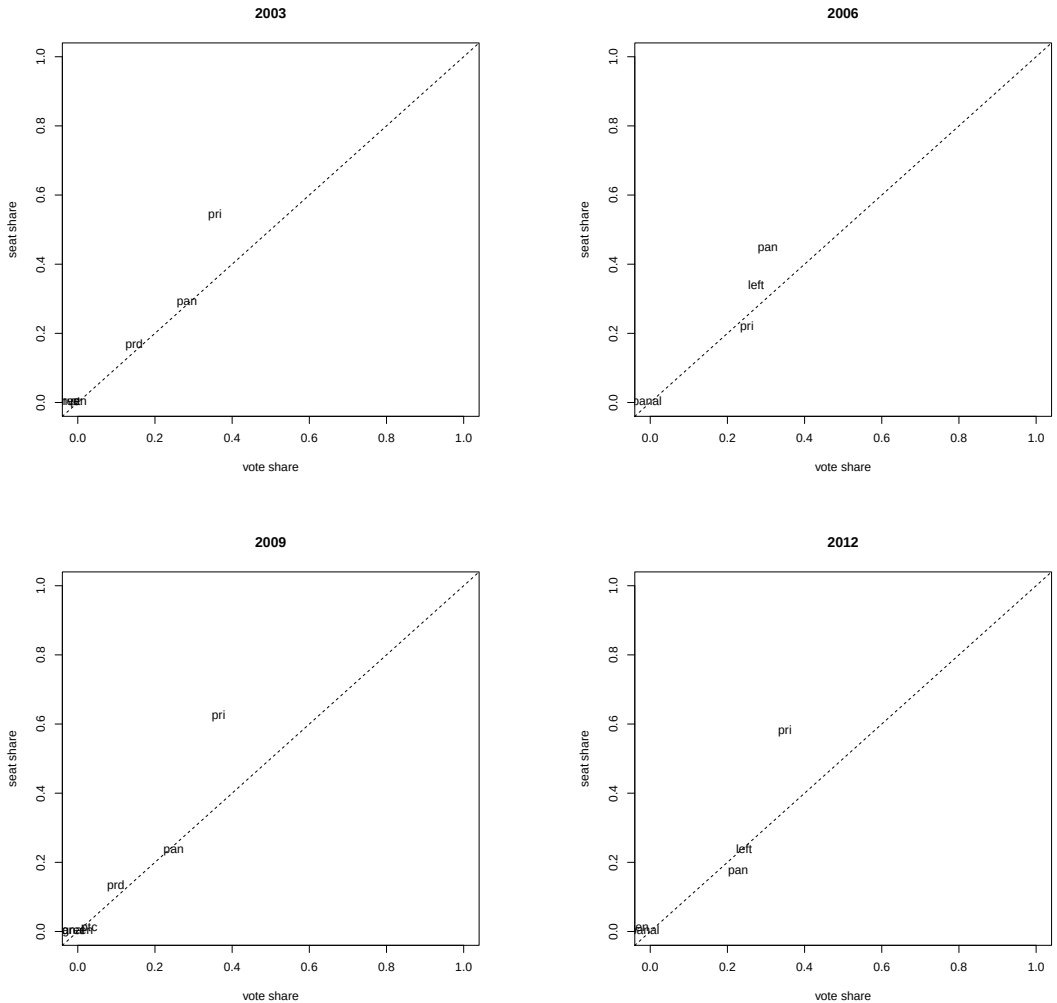
- Hegemonic party 1929–1997
- Three major parties: PRD (left) PRI PAN (right) and minors
- Lower chamber of Congress elected every 3 years, concurrent w presidential race every 6 years
- Mixed system: 300 SMD + 200 PR seats
- Single-term limits removed in 2018
- Independent board (IFE) manages elections and redistricting

Obstacle 3: small-N

- A general election with P parties offers P data points to fit a votes-seats curve
- P typically small
- Multi-year approach: pool historic record... compares apples/oranges in long-haul
- Single-election apprach preferable... but requires data multiplication strategy

Monte Carlo simulation

- Linzer (2012): approximates prob. distribution of national party vote returns from observed district outcomes (FMM)
- Use to simulate many elections w Monte Carlo draws



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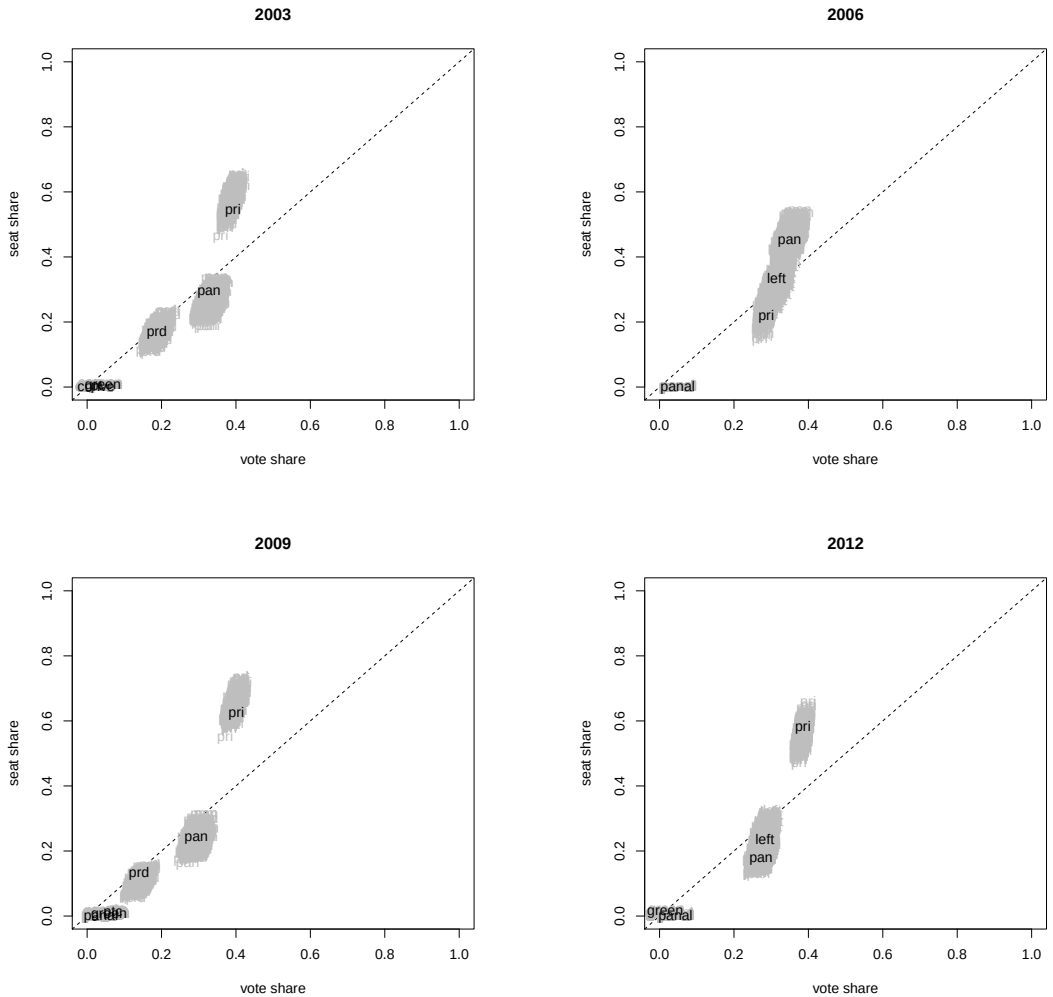
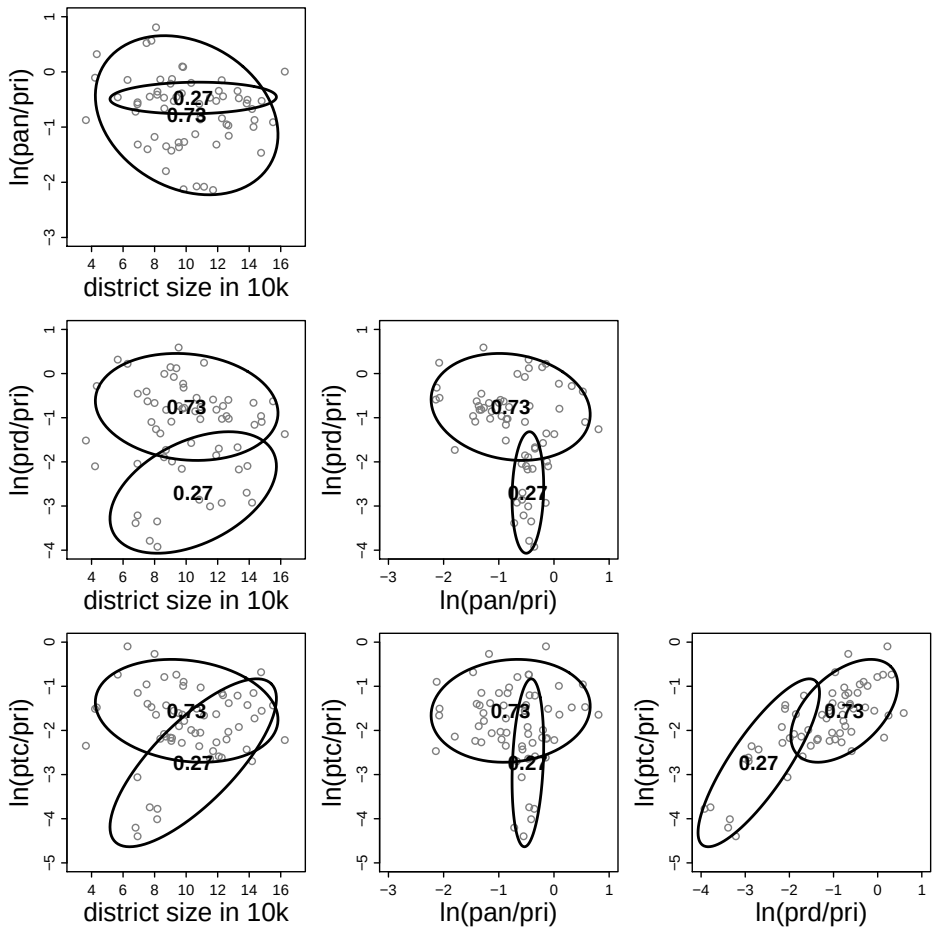


Mixture model

- Combines the properties of two or more prob. density functions: can approximate any arbitrary distribution
- Seek components (multivariate normals) and weights of log-ratio votes shares



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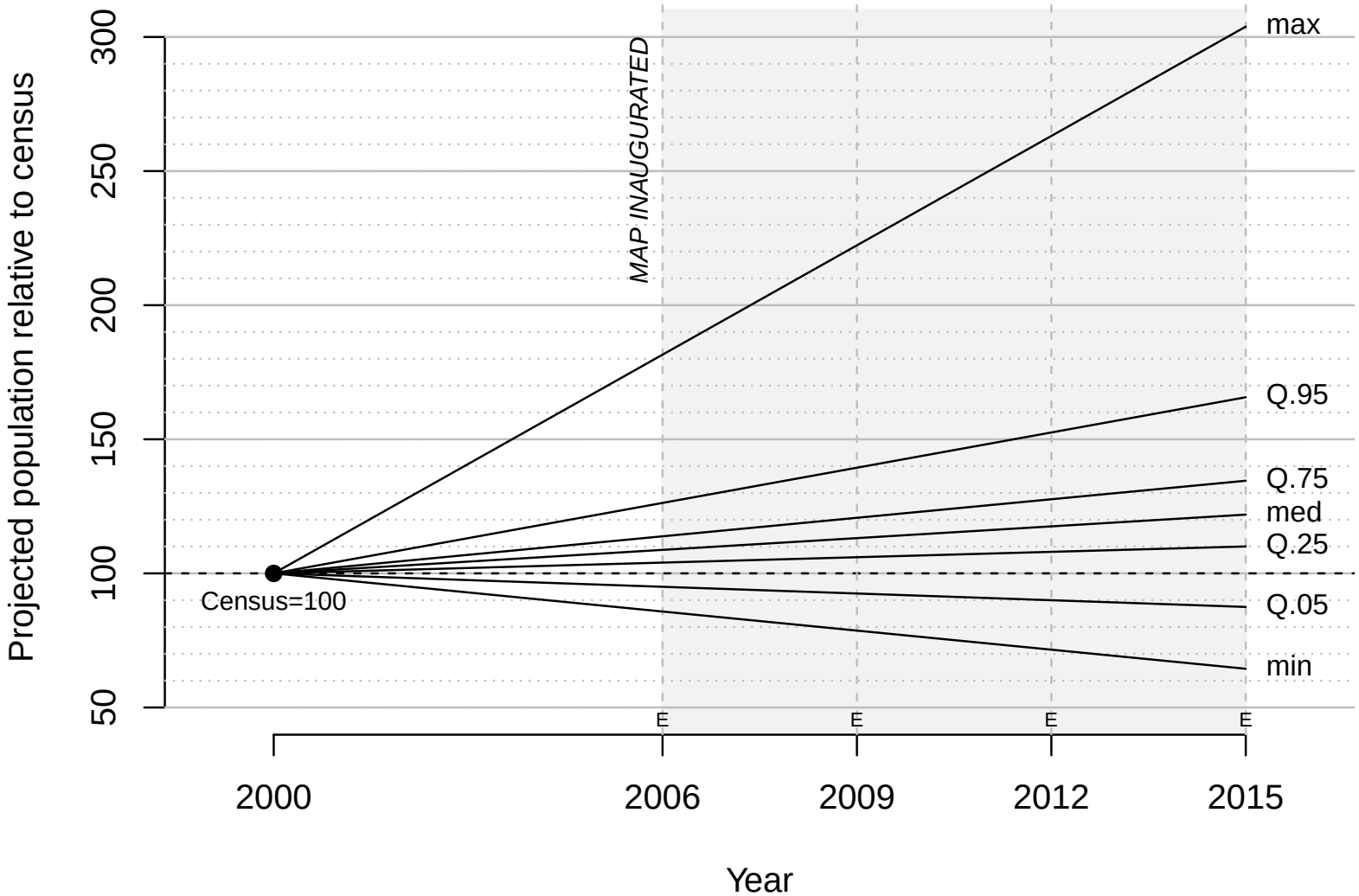
One person–one vote?

Malapportionment –definition When more densely-populated regions get the same representation as the less densely populated
Can arise by comission or by omission
Census lag

District populations: linear projection



Plus: bureaucratic leeway in new district sizes → substantial malapportionment

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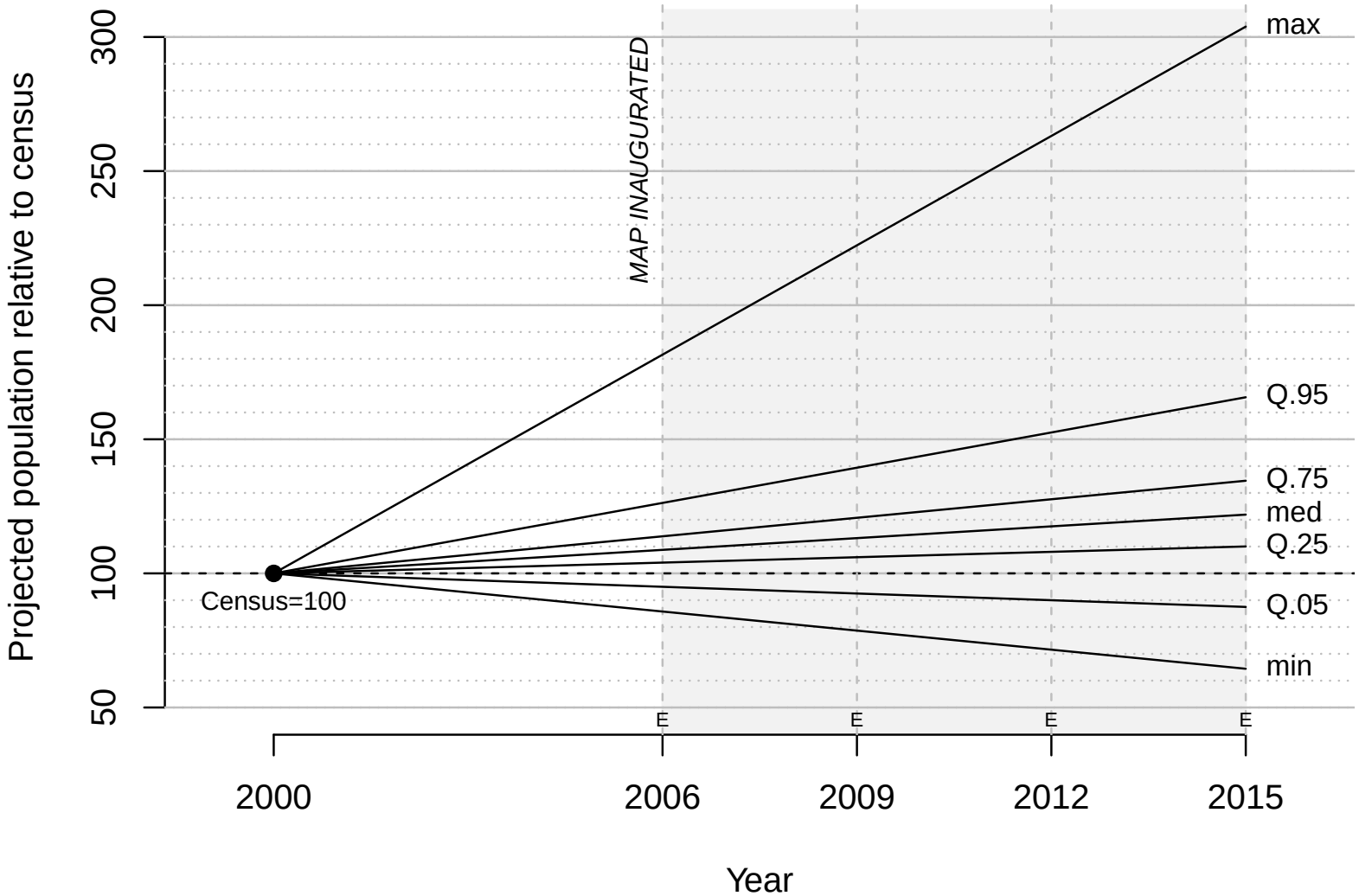
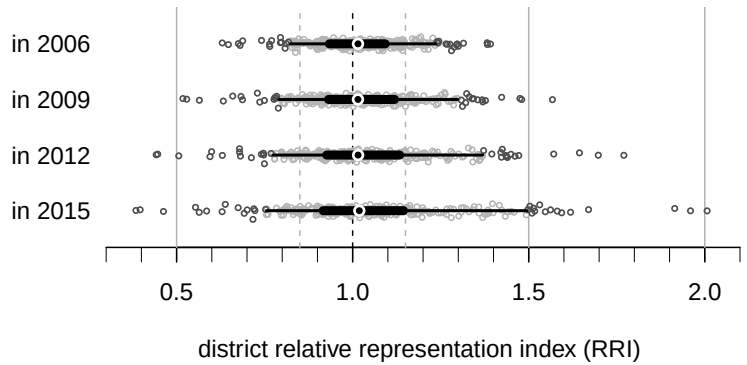


Plus: bureaucratic leeway in new district sizes $\rightarrow$ substantial malapportionment

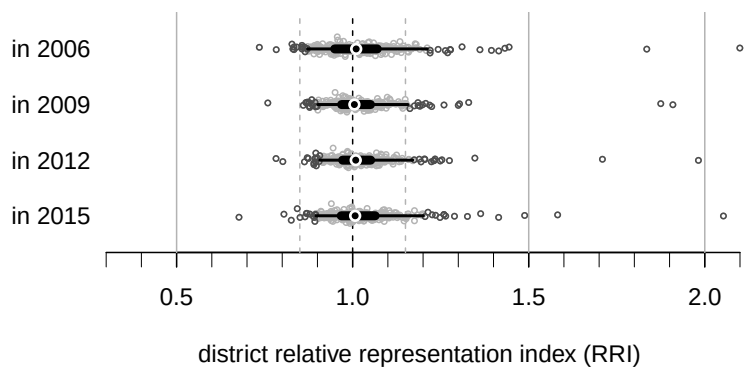
Malapportionment is substantial

$$RRI = \frac{nat.pop./300}{\text{district size}}$$

2006 map (drawn with 2000 census)



2015 map (drawn with 2010 census)



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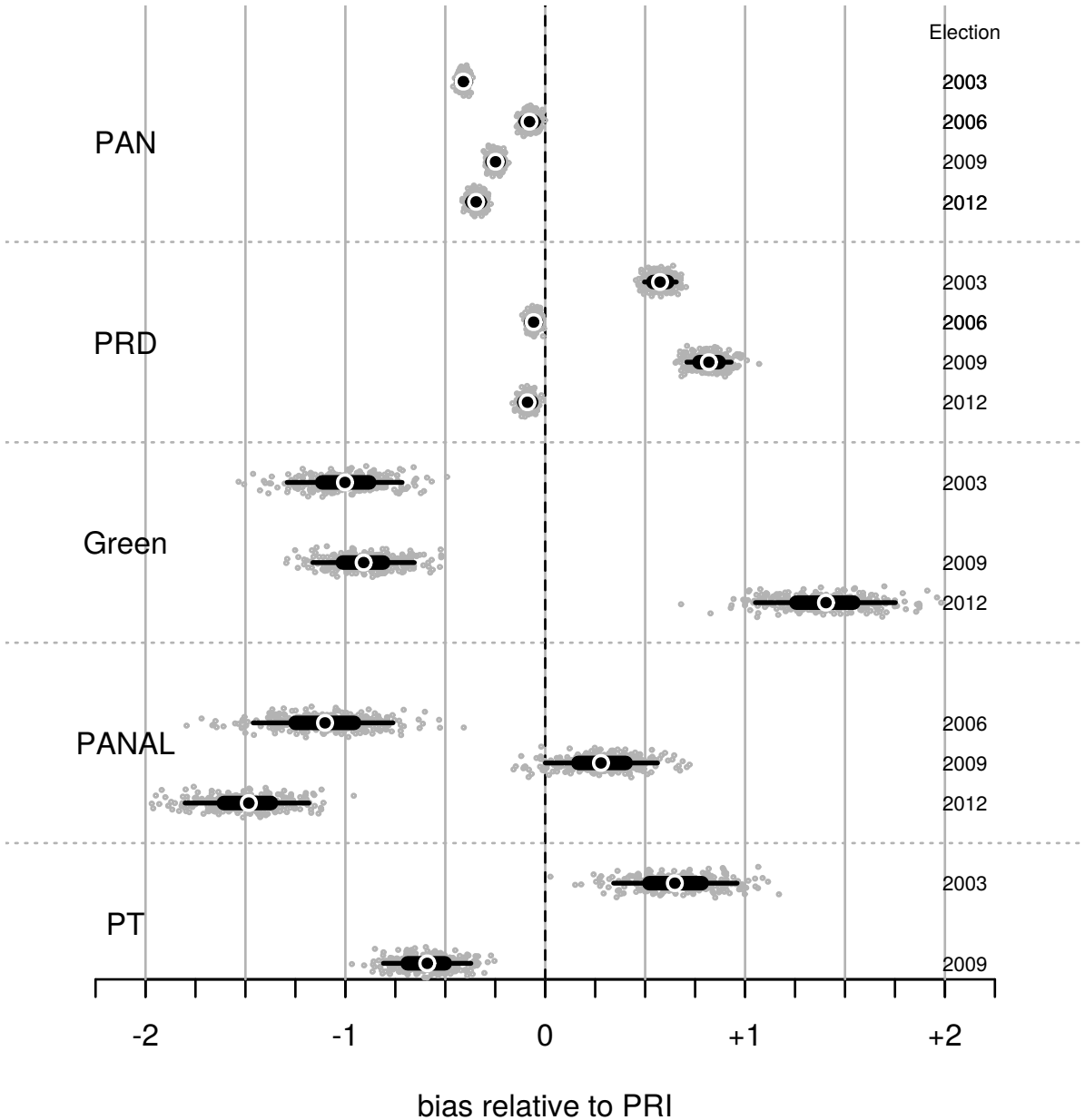
Estimation

Votes–seats curve fitted with MCMC (`Jags` via `R`)
100k iterations of 3 chains, every 500th obs. of last 50k used to sample posterior density
Visual inspection of model parameters to verify chain convergence

Results: raw partisan bias



Results: components

	Actual map			Hypothetical map		
partisan bias	pan–pri	prd–pri	min–pri	pan–pri	prd–pri	min–pri
2003 election				(with 2006 map)		
raw	−.37 (0)	+.72 (0)	−1.01 (0)	−.41 (0)	+.57 (0)	−1.00 (0)
gerrym.	−.09 (0)	+.69 (0)	−.88 (0)	−.13 (0)	+.62 (0)	−.90 (0)
turnout	−.26 (0)	−.11 (0)	−.08 (0)	−.26 (0)	−.09 (0)	−.09 (0)
malapp.	−.01 (.11)	+.14 (0)	−.05 (0)	−.02 (.12)	+.05 (0)	−.02 (0)
2006 election						
raw	−.08 (0)	−.06 (0)	-1.10 (0)			
gerrym.	+.28 (0)	+.30 (0)	−.62 (0)			
turnout	−.36 (0)	−.41 (0)	−.43 (0)			
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gerrym.	−.28 (0)	−.07 (0)	+1.41 (0)	−.24 (0)	−.05 (.06)	+1.02 (0)
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malapp.	+.01 (.42)	+.06 (0)	−.02 (0)	−.00 (.38)	+.01 (0)	+.00 (0)

- Turnout always pro-PRI
- Malapp. always pro-left
- Redistricting abates malapp.
- Possibly cancelling effects

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Use swing ratios to compare two maps

1. Simulate 5k 2009 elections with status quo map for each party
2. Repeat using abandoned map
3. Pool simulations together and regress
4. Coefficient β_3 measures and tests change in party's swing ratio *due to redistricting*

$$s = \beta_0 + \beta_1 v + \beta_2 dis2015 + \beta_3 v \times dis2015 + \texttt{error}$$

Results

	PAN	PRI	Left
2006 election			
v	1.94^{***}	2.27^{***}	1.91^{***}
$v \times$ dis2015	$+.45^{***}$	$-.01$	$-.04$
2009 election			
v	1.95^{***}	2.27^{***}	1.67^{***}
$v \times$ dis2015	$-.13^{***}$	$+.04$	$-.02$
2012 election			
v	2.24^{***}	3.99^{***}	2.39^{***}
$v \times$ dis2015	$+.02$	$+.03$	$-.06^{*}$

$^{***} < .01$, $^{*} < .1$

Road map

1. Partisan bias: sources, measurement
2. Mexican Cámara de Diputados elections
3. Malapportionment
4. Results

Findings, next steps

1. Rel. to the right, persistent pro-PRI, and esp. pro-left raw bias
2. Malapportionent effects are (surprisingly) small
3. Pro-PRI turnout-based bias
4. Gerrymander effects large and volatile
5. District lines can compensate for turnout disadvantage
6. To-do: add PR-tier to analysis
7. To-do: inspect inter-election volatility

Thank you!

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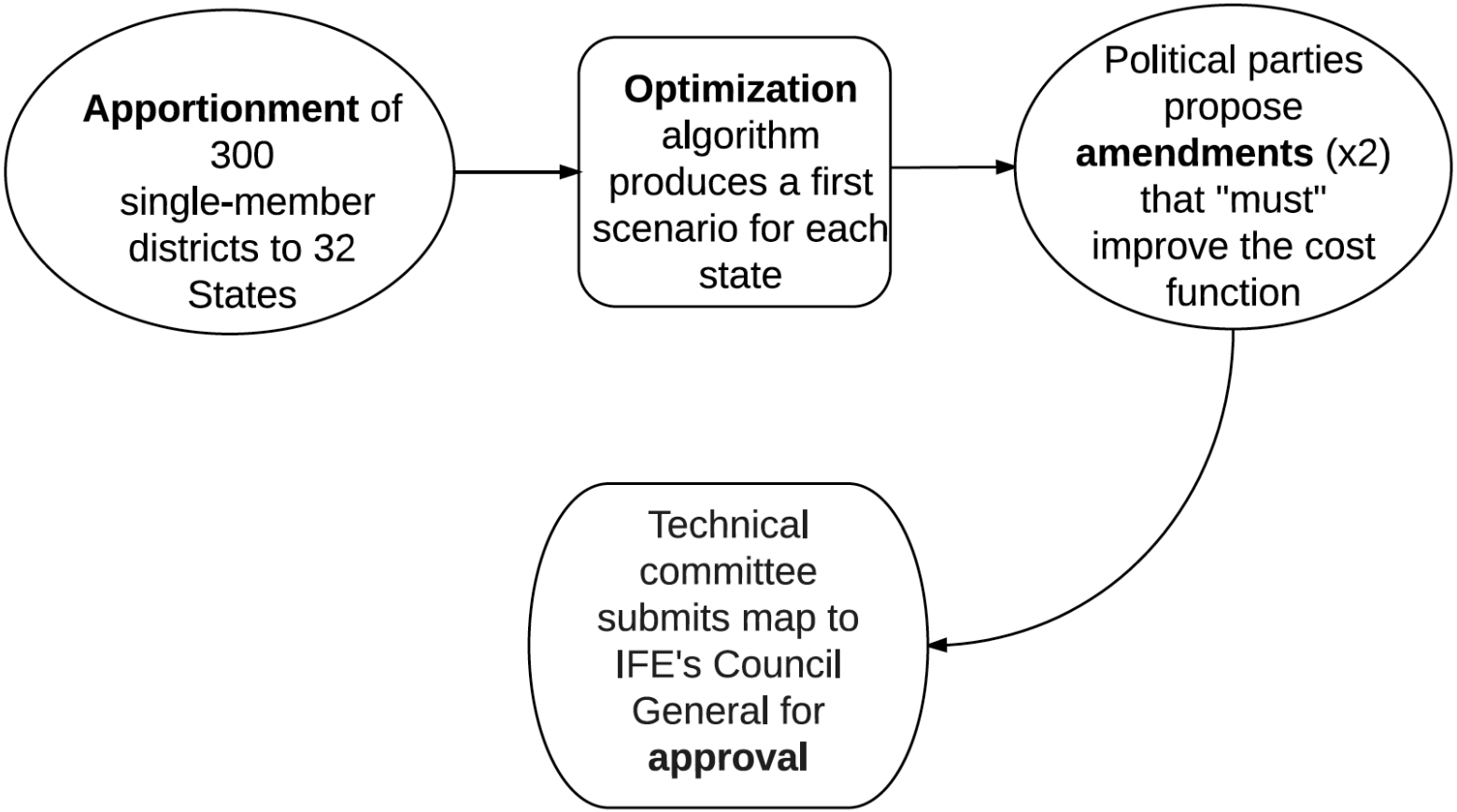
The redistricting process



The redistricting process

Redistricting by experts in 1997, 2006, 2015 (abandoned), and now 2018

1. apportionment of 300 seats to 32 states
2. optimization algorithm $\rightarrow$ proposal
3. parties propose amendments ("must" improve score)
4. new map

$$\texttt{Score} = .4 \times \texttt{PopBalance} + .3 \times \texttt{MunicBoundaries} + .2 \times \texttt{TravelTime} + .1 \times \texttt{Compactness}$$

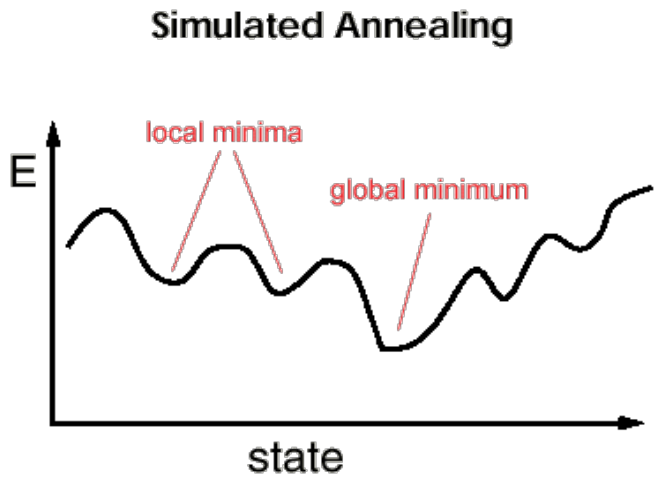
IFE considers $\pm 15\,\%$ imbalance normal (!)

Optimization algorithm

Simulated annealing = probabilistic meta-heuristic for optimization locates a good approximation to the global optimum of the cost function in a large search space

Thousands of iterations using electoral *secciones*

Combinatorial optimization algorithm used to generate the first scenario in each state



IFE claims that this is a public process, but the operation and procedures are done behind closed doors

Party amendments