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A Case Study of Electoral Manipulation: The Mexican Laws of 1989 and 1994

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The Mexican electoral laws of 1989 and 1994 are used as a case study to illustrate a point: the logical and mathematical properties defined in law have significant practical consequences. The 1989 law is an extreme case of logical inconsistency and ignorance of the properties of rules of apportionment, and as such provides a rich menu of the properties rules for apportioning seats should satisfy. The 1994 law, at face value more reasonable, is an illustration of how innocuous looking rules can yield real political advantage. Both laws can engender anomalous and contradictory results; and both reveal an inattention to elementary concepts of equity which cannot go unnoticed in view of Mexico's current turmoils. An electoral law defines the transformation of numbers (populations and votes) into numbers (political representation to political power), and so defines a mathematical function. The political and legal men who devise such functions are perhaps wise to the wiles of men, and sometimes even to the subtleties of arithmetic rules, but the professional advice of those competent in the ways of functions would bring benefits in accuracy, rigor and clarity, if not equity, to the formulation of electoral law. Copyright © 1996 Elsevier Science Ltd.

1. Introduction

National electoral systems are seldom neutral. Typically, they are designed by those who wield the political power in the hope of maintaining their position, restrained only by historical and cultural traditions, and evolving political events. The Mexican electoral reforms of 1989 and 1994, and the changes to the Constitution that these required, are no exception.

It seems that *el Partido Revolutionario Institucional* (PRI), the political party that despite over 60 years of power maintains its deceptive name, awoke to some unpleasant surprises on the morrow of the 1988 election. The government of President Carlos Salinas de Gortari wasted no time in proceeding to a reform of the electoral system, amending the Constitution where necessary, in an attempt to ensure that if new surprises were to arise in the future then, at least, the mathematics of transforming votes into seats would be favorable ... lawyers and politicians write laws. Judging from what they devised as the electoral reform of 1989,

they are illogical, impractical and intellectually arrogant. The style of the writing was so confused and imprecise that one is tempted to say that they were illiterate. That they were 'innumerate' is an inescapable fact: the descriptions of the rules for computing the apportionment of seats to parties on the basis of the vote totals were incomplete in addition to being grotesquely complex. In many real possible cases of the outcome of the voting, it would have been impossible to apply the law: reasonable configurations were unforeseen, contradictions existed. Indeed, under certain outcomes of the voting, it assured *two* parties an absolute majority of the seats, a most congenial paradox: had such been realized imagine the ensuing commotion! The opposition should have roared with derision when the law was proposed. That it did not is only because it paid no attention to the implications of the mechanisms used to apportion seats: their concerns were ballot stuffing and similar tampering in the counting of the votes.

The electoral reform of 1994—adopted just before the election of August 1994 and in the heat of the intense political conflict that followed the insurrection in the state of Chiapas—seems to have been written by somewhat wiser lawyers and politicians, or perhaps by people chastened by the unfolding events. A distinct level of innumeracy remains, the writing again lacks clarity and could be formulated in much simpler terms, but the rules have a much more acceptable and reasonable appearance. In fact, they again contain logically inconsistent requirements, though the likelihood of encountering these impossibilities in practice are minimal. More importantly they reinforce the advantages conferred on the one big political party, though in not so evidently blatant a manner.

The Mexican Constitution, promulgated in 1917, established a representative democracy, in keeping with its Carta Federal of 1857. Its Congress consists of two Houses, the House of Deputies and the House of Senators. The members of the first are to be distributed in proportion to the populations of the states and the Federal District, whereas there is a fixed number of senators in each state independent of the populations. Initially this number was two per state; since 1993 it is four per state. The lower chamber is renewed every three years; the upper chamber every six years.

Initially the House of Deputies had approximately 300 members, and the method of election was based on the majority principle applied to single-member constituencies apportioned among the states. Not surprisingly this had the effect of barring all small parties from having any representation whatever—thus giving to the one large party, the PRI, overwhelming majorities—and there upon the idea of introducing a small dose of 'proportionality' was advanced.

The 1963 reform of the Constitution aimed at giving at least a bit of representation to the small parties. In addition to the direct election of deputies as before (a number which was fixed at precisely 300 beginning in 1977), each party with at least 2.5 per cent of the vote received 5 deputies, plus 1 deputy for each 0.5 per cent of the vote in excess of 2.5 per cent, but not more than 20 seats in all. Those parties electing 20 deputies or more in the 300 districts were not eligible to receive such extra seats. Thus, a party having 10 per cent of the vote and electing fewer than 19 deputies in the districts would add 20 seats to its representation. Experience showed that this adjustment yielded meager results, so in 1972 the 2.5 per cent threshold was replaced by 1.5 per cent.

In 1977 a more significant reform was introduced. The Constitution established a lower house of 400 deputies, with 300 to be elected by the majority principle in single member districts and 100 in multi-candidate constituencies on the basis of

the principle of proportional representation. In 1987 the size was increased to 500 deputies, 300 to be elected in single member districts and 200 in five multi-candidate constituencies, and these numbers of deputies have not been changed since.

The 200 proportional representation deputies were elected in each of five regional constituencies, each consisting of several not necessarily contiguous states. Within each, parties presented lists, with those candidates elected who were highest on the list: we will call them *party deputies*. However, the apportionment of the 40 party deputies in the five regional constituencies was not made in proportion to the votes within each, but rather on the basis of the total votes of the parties across the five regional constituencies, and the idea was to apportion party seats so that the total of the directly elected deputies—the *majority deputies*—plus the party deputies in the house of 500 members would be proportional to the vote totals of the respective parties. But the unexpected results of the 1988 election provoked grave fears and the PRI promulgated the reform of 1989. Its object seems to have been to guarantee that party which elects the most majority deputies and which has at least 35 per cent of the national vote for party deputies, an absolute majority in the House of Deputies. The effect was to introduce potentially aggravated disproportionalities, augmenting the big party's share and simultaneously reducing the important opposition parties' shares.

The reform of 1994 is not so obviously flawed. Slight ambiguities remain, but a method of apportionment is consistently used (though the method itself is a poor choice). Its main impact is to ensure that so long as there is *one* relatively strong party opposed by several much smaller parties and a strong dose of deputies elected in single member districts by the 'majority' principle, as is the case today, that one party will have absolute majorities in the House of Deputies. If the opposition is unable to mount an alliance whose electoral vote can match the PRI's, this state of affairs will continue. This stranglehold on seats was extended to the House of Senators in 1993: within each state that party receiving the most votes elects three senators, whereas the party next in votes receives one.

Our analysis, and conclusions, are based on nothing more than the laws themselves, the constitutional amendments they inspired, and numbers. The evolving political and historical events, the traditions and compromises, the accommodations and ballot stuffing and the like, are ignored. Why? To show the importance of logic and arithmetic. To suggest that lawyers and politicians are not professionally competent to define an electoral system all by themselves, because the transformation of numbers (votes in this case) into other numbers (seats in a legislature in this case) is mathematics, and it would be well advised to obtain the advice of professional mathematicians when such transformations or 'functions' are to be chosen. We do not deny the importance of national legal practices and historical traditions, but too often decisions are made in total ignorance of the properties of the functions that are consecrated as law.

Why Mexico? On the one hand because the 1989 law is so amazingly bad, which opens the door wide to a discussion of the properties of laws; on the other hand because together, the two laws, as 'bad' as they may be, illustrate how even clumsy manipulation may prove profitable in holding on to the reins of power. Judging on the basis of the last four elections (1985, 1988, 1991, 1994) both the 1989 and 1994 laws were kinder to the PRI than their predecessors. Estimates of the percentage of the total vote necessary for the one big party to assure itself of an absolute majority by the 1994 law have been made. They suggest that 39.5 per cent will do.

Perhaps this analysis can serve as an example to others faced with the task of establishing or modifying an electoral system.

2. The Common Background to the Laws

Articles 52-54 of the Mexican Constitution concern the size and the election of the members of the House of Deputies, Articles 12-18 of both laws concern the apportionment of the seats. What is common to both is described in this section. Where and how they differ in their 1989 and 1994 versions, is explained anon.

The House of Deputies contains 300 deputies elected in single member districts according to the principle of 'relative majority', and 200 deputies elected by a system of lists in multi-member constituencies according to the principle of proportional representation. For clarity we will consistently refer to *majority deputies* from single member *districts* and *party deputies* from multi-member *constituencies.*

The 300 single member districts are supposed to be apportioned among the states proportionally to their respective populations according to the last available national census, except that each state is guaranteed at least two seats (in fact, the apportionment has not changed since 1977). The 200 party deputies are elected in five national multi-member constituencies, each with 40 seats, whose territorial boundaries are determined by law. In fact, neither law specifies how or by what criteria the 300 districts are to be apportioned among the states, nor how or by what criteria the territorial boundaries of districts and constituencies are to be determined.

A voter casts two votes: one for a majority deputy candidate in his district, one for a party list in his constituency.

A majority deputy who obtains a plurality of the valid votes is definitely elected. The seats attributed to a party list in a constituency are assigned to physical persons according to the order of the list.

To be eligible to receive party seats a party must present majority deputy candidates in at least 200 of the 300 districts, and it must receive at least 1.5 per cent of the total vote (including invalid or blank votes, which are significant: 4.18 per cent in 1989 and 3.18 per cent in 1994) over all constituencies. The 'votacion nacional emitida' or vne is the total vote of all *eligible* parties over all constituencies. It is the basis for all subsequent calculations.

The two laws differ in how they use this data—the numbers of majority deputies elected by each party and the vne of each party in each constituency and in total—to determine how to apportion the 200 party deputy seats. But both laws share a serious difficulty often encountered in practice: in the event of contested results in the election of majority deputies it is impossible to compute definitive allocations of the party deputies.

The detailed clauses of both laws are badly written, often confusing, and sometimes incomplete. Our explanations are given in terms of logical possibilities and are not an article by article rendition of the laws themselves.

3. The Meaning of the 1989 Law

The 1989 law seems to have been written in haste, in a state of nerves, after the near defeat of Salinas (which some claim was in fact a defeat at the polls). Its main

preoccupation was to assure the PRI a majority of the seats even if it had a small percentage of the vne. It pretends to continue to pay attention to the idea that all the seats of a party—majority and party deputies—should be proportional to the vne of the party. But it is a curious law that takes as its base point the one big party, determines how many seats it deserves, and then handles all the others as a kind of afterthought—except that the formulas for distributing the seats to the others gives advantages to the smaller parties so as to penalize the potential challengers.

The presumption is that there will be the one 'big' party, though the opposite is considered. The law gives different rules for calculating the seats depending upon different possible outcomes of the vote. The point to keep in mind is that first the number of seats to be apportioned to the one big party (if it exists) is computed; then, how the one big party's seats are to be distributed among the constituencies; and once these numbers are known the remaining seats in each constituency are apportioned among the other eligible parties.

Logically, four mutually exclusive possibilities may be distinguished: either (i) every party receives less than 35 per cent of the vne; or (ii) exactly one party receives at least 35 per cent but not more than 60 per cent of the vne; or (iii) exactly one party receives at least 60 per cent of the vne; or (iv) exactly two parties receive at least 35 per cent of the vne.

Possibility (i). In this case—the exception when there is no 'big party'—the law (Article 13(a)) states that each eligible party must be apportioned a number of party deputies so that its total number of deputies represents the same percentage of 500 seats as its percentage of the vne. No specific rule by which to do this is given. Nevertheless, this may be impossible. For suppose there are four eligible parties and that they win the percentages of the vne and the numbers of majority deputies as indicated in Table 1.

TABLE 1. Article 13(a) of 1989 Electoral Law inconsistent

Party	A	B	C	D	Total
Per cent vne	32	28	22	18	100
Majority deputies	170	121	8	1	300

In this imaginary situation party *A* has 170 majority deputies. In the view of the strong advantage given to the largest party in a system with single member districts it is not at all surprising for it to win 56.67 per cent of the seats with around 32 per cent of the votes (of course the per cent of the vne is not necessarily the same as the per cent of the votes it receives in the single member districts, but one might expect these percentages not to be dissimilar—and in fact the 1994 election results show that the actual numbers are almost exactly the same within each district). But party *A* with 32 per cent of the vote should have exactly 160 of the 500 deputies and it already has more: so it is impossible to meet the principle.

This inconsistency is proven with an admittedly 'toy' example. It is an easy matter to concoct a more complex and 'realistic looking' example, but then one would have more difficulty in seeing the reason for the inconsistency. For this reason our analysis will consistently use simple toy examples designed to make transparently clear the reasons for the difficulties. In every case it is possible to formulate a 'realistic' example that displays precisely the same phenomenon.

Possibility (ii) may be divided into three mutually exclusive cases: exactly one party (say *A*) receives at lease 35 per cent but not more than 60 per cent of the vne *and* either (iia) *A* has at least 251 majority deputies, or (iib) *A* has the largest number of majority deputies but less than 251, or (iic) *A* does not have the largest number of majority deputies.

Possibility (iia). The law (Article 13(c)) stipulates that party *A* will be apportioned two additional party deputies for each 1 per cent of the vne over 35 per cent, but the total number of deputies for party *A* cannot exceed 251 plus twice the number of percentage points of the vne over 35 per cent. In the example of Table 2, this means at most 251 plus 4 or 255 seats. But this is impossible since *A* already has 280 deputies.

Possibility (iib). Under this hypothesis the law (Article 13(b)) says that party *A* will be apportioned a number of party deputies sufficient to assure them of an absolute majority, and in addition, two seats for each 1 per cent of the vote over 35 per cent. Logically this cannot in all cases be realized, but the examples that show the inconsistency are not realistic, so are omitted.

Possibility (iic). The electoral law (Article 13) does not foresee this possibility, yet it decidedly exists as a realistic possibility, as can be seen in Table 3.

In possibility (iii) the law is ambiguous. (iiia) If exactly one party has at least 60 per cent of the vne and it is the party with the largest number of majority deputies, then the same prescription as under possibility (iib) applies (Article 13(b)). On the other hand, (iiib) if it has at least 60 per cent but less than 70 per cent of the vne then the law (Article 13(d)) assures it a total number of deputies equal to that percentage of 500, so whatever number of party seats that is necessary to give this result. In practice it is certain that in case (iiib) the same party will have the largest number of majority deputies, so there is a conflict of clauses. Whatever the case an absolute total limit of at most 350 seats is imposed on every party.

This ambiguity causes trouble. Suppose that one party (say *A*) had 71 per cent of the vne and the largest number of majority deputies. This is not case (iiib), so the prescription of possibility (iiia) applies and party *A* should receive in total 251 plus 2 x (60 - 35) or 301 seats. On the other hand, if party *A* had 69 per cent of the vne and the same number of majority deputies then case (iiib) could apply and it should receive in total 345 seats. Thus it is better for party *A* to win only 69 per cent of the vote than 71 per cent, *fewer votes give more representation.*

TABLE 2. Article 13(c) of 1989 Electoral Law inconsistent

Party	*A*	*B*	*C*	*D*	Total
Per cent vne	37	26	20	17	100
Majority deputies	280	20	0	0	300

TABLE 3. Article 13 of 1989 Electoral Law incomplete

Party	*A*	*B*	*C*	*D*	Total
Per cent vne	35.1	34.9	20	10	100
Majority deputies	140	142	16	2	300

Possibility (iv) may be divided into three mutually exclusive cases: (iva) two parties (say *A* and *B*) receive at least 35 per cent of the vne and one (say *A*) obtains more majority seats than any other party; or (ivb) *A* and *B* each receive at least 35 per cent of the vne, the same number of majority seats, and this number of majority seats is more then any other party; or (ivc) *A* and *B* each receive at least 35 per cent of the vne but another party (say *C*) obtains the largest number of majority seats.

Possibility (iva) is what the authors had in mind. The law (Article 13(b)) says that party *A* will be apportioned a number of party deputies sufficient to assure *A* an absolute majority; and in addition, two seats for each 1 per cent of the vote over 35 per cent.

Possibility (ivb) is what the authors were unable to image, for the law gives the same prescription as in (iva). But if the example of Table 4 were to occur then parties *A* and *B* would each be guaranteed absolute majorities—a formula for an irreconcilable power struggle!

TABLE 4. Article 13(b) of 1989 Electoral Law inconsistent

Party	*A*	*B*	*C*	*D*	Total
Per cent vne	36	35	16	13	100
Majority deputies	140	140	20	0	300

Possibility (ivc). This possibility, logically possible but not in practice realizable, is not envisioned by the law.

The next stage of the calculation now begins. Either there is no big party (the exception (i)), or there is one big party and its number of party deputies has already been determined (the remaining possibilities).

In possibility (i) the intent of the law (Article 14) is clearly to arrive at total allocations to parties that are proportional to the party's vne's, though the rules are not sufficiently precise to define definite outcomes. Then, each party's attribution is apportioned among its five constituency lists according to the same set of rules (Article 15(2)) in proportion to the vne's in each of the constituencies. Note that this procedure nowhere uses the fact that a constituency should consist of 40 party deputies, so it is entirely possible (and is confirmed by realistic examples) for some to receive more than 50 and others less than 30 seats.

In the remaining possibilities, assuming that no logical impediment intervenes, party *A*'s total number of party deputies is known. The law (Article 15(1)) says that they are to be apportioned among its five constituency lists again according to the same set of rules.

This implies that in each constituency there will remain 40 or fewer party deputy seats to be apportioned among the remaining lists. The law says (Articles 16, 17 and 18) that 'la formula de primera proporcionalidad' is to be applied on a constituency by constituency basis to apportion the remaining seats among the other eligible parties. It is ironic indeed that in this one case very precise rules are given for the calculation, although the description is involved and confusing (and it is logically possible for them to be inapplicable). It gives a definite and not proportional advantage to the smaller parties. 'La formula' is given in the Appendix. In this case each constituency is guaranteed to be apportioned exactly 40 party deputy seats.

In addition to the inconsistencies and anomalies of the Mexican Electoral Law of 1989 there are other 'sins of omission': instructions that are incomplete and computations that must be made but are not specified.

In several instances when calculations are made, the resulting numbers are not integers or whole numbers and yet the answers must be whole numbers of seats: for example in the 1991 election the PRI was to receive 320.88 seats, but nothing was said about what should be done to obtain a whole number. The electoral commission chose to round down: but the prescription might just as well have been round to the closest integer, or to round up, or some other rule.

The law requires that the 300 single member districts are to be apportioned among the states proportionally to their populations except that a minimum of two seats is guaranteed each state: but no rule is specified as to how this is to be computed.

Exactly the same mathematical problem is to be solved in two other instances: (1) in apportioning the party deputies among the five constituencies (possibility (i)), and (2) in apportioning the leftover seats of each multi-member constituency among the smaller parties. In the first instance the description of the computation is incomplete, and in the second ('la formula de primera proporcionalidad') it is flawed.

How the five constituencies are to be defined is left unspecified. And how and by what standards are the geographical boundaries of the single member districts to be defined is also left unspecified. These are problems that are at once politically significant and mathematically difficult to solve.

4. The Meaning of the 1994 Law

The 1994 law is a calmer document: no state of nerves here. The most important change is that the pretence of allocating party deputy seats so that the total of all 500 seats is proportional to the vne's of the parties is abandoned—except when it can help the one party that has more than 60 per cent of the vne. Instead, a party's percentage of the vne determines its claim on the 200 party deputy seats—subject to a more modest overall maximum—also an advantage to the big party.

The law is relatively simple to explain. First, use the method of largest remainders (also known as Hamilton's method) to determine the claim on the 200 party deputy seats of each eligible party on the basis of their vne's. The *method of largest remainders* may be described as follows: compute the exact proportional share of each party (called its *quota*), give to each the whole number contained in its quota, and assign any seats that remain—one per party—to those having the largest remainders.

Two clauses override the claims determined above. (i) If one party has more than 60 per cent of the vne it is assigned party deputy seats so that its total number of seats is proportional to its percentage of the vne, except that the total cannot exceed 315. (ii) A party with 60 per cent or less of the vne can have at most 300 seats. If one applies, the party concerned is given its due and the remaining party deputy seats are apportioned among the remaining eligible parties by the method of largest remainders; if neither applies, the claims stand as is.

This law harbors several logical inconsistencies, albeit unrealistic ones, so it represents an improvement over its predecessor. However, it permits a minuscule increase of the vne of one party to translate into a whopping increase in its number

TABLE 5. An implication of Article 14 of 1994 Electoral Law

Parties	Majority seats	*Before correction* % vne	Total seats	*After correction* % vne	Total seats
A	165	59.99	285	60.01	300
B	135	40.01	215	39.99	200

of seats, as the two party example of Table 5 shows. Imagine party *B*'s furor in seeing an overcount of .02 per cent of the vote (representing .04 seats) lead to a loss of 15 seats.

As before, no rule is given to determine the number of majority seats apportioned to states; and no criteria are given for how the 300 majority districts or the five constituencies are to be drawn. The rules for determining the apportionment of seats to constituency lists nowhere take account of the fact that they should add to 40 in each: indeed, by the results of the 1994 election, the five constituencies received 36, 39, 39, 43 and 43 seats, respectively. Finally, although it is an improvement that a method for allocating party seats to parties and constituencies is specified, the method used harbors inherent anomalies that could provoke new inconsistencies (which would have been realized in 1994 had the vne's been very slightly different).

Is the 1994 law an 'improvement' over the 1989 law for the one big party? The facts speak for themselves. The results are given for each of the last four elections, those of 1985, 1988, 1991 and 1994: the actual election outcomes and those that would have obtained if the two laws under review had been used. Note that in 1985 the electoral law foresaw only 100 party deputies, so a total of 400 and not 500 seats.

If one concedes that 315 seats of 500 confers the same essential power as 320 seats, the 1994 law is uniformly the same or better than the 1989 law for the PRI.

TABLE 6. Distribution of seats by different laws: 1985 electoral data

Parties	PRI	PAN	PSUM	PST	PDM	PPS	PARM	PMT	PRT	Total
Actual	289	41	12	12	12	11	11	6	6	400
1989 law	323	71	18	18	17	16	17	9	11	500
1994 law	315	91	18	17	15	13	14	9	8	500

Table 7. Distribution of seats by different laws: 1988 electoral data

Parties	PRI	PAN	PFCRN	PPS	PARM	PMST	Other	Total
Actual	260	101	35	32	31	19	22	500
1989 law	283	92	30	28	27	18	22	500
1994 law	300	89	27	26	23	13	22	500

TABLE 8. Distribution of seats by different laws: 1991 electoral data

Parties	PRI	PAN	PRD	PFRCN	PARM	PPS	Total
1989 law	320	89	41	23	15	12	500
1994 law	315	101	42	22	11	9	500

Table 9. Distribution of seats by different laws: 1994 electoral data

Parties	PRI	PAN	PRD	PT	Other	Total
1989 law	285	123	75	15	2	500
1994 law	300	118	70	10	2	500

As the PRI's fortunes wax and wane one might expect that the same would happen to its representation. Table 10 shows that the fluctuations are minimal: a large difference in the percentage of the vne is reflected in a drastically dampened difference in the total number of seats. It is clear that if there is one big party in the presence of several smaller ones, the big party will be fairly sure to obtain upwards of 300 seats. Indeed, if in the 1994 election the PRI elected the same number of majority deputies, it could have garnered as little as 13.5 per cent of the vne (instead of its actual 52.70 per cent) and been allocated the same 300 seats.

One might ask how small a percentage of the vne would still assure the PRI of an absolute majority under the present 1994 law. Two approaches to this question yield nearly identical answers. Assume, as was essentially the case in 1994, that within a district the number of votes cast for a party's majority deputy candidate is the same as the number of votes cast for that party's list. The PRI's vne represented 52.70 per cent of the vne, whereas the actual percentages in the districts varied above and below; and similarly for the other parties. Take the actual election results in each district, decrease the PRI's percentage (by 3λ per cent) and increase every other party's percentage equally (by λ per cent) in *each* district. For each λ the majority deputy winner in each district can be determined directly, and the percentage of the total vne of each party changes by the same amount. The results are given in Table 11 (more details are given in the Appendix): with as little as 44 per cent of the vne the PRI keeps its 300 seats, that is, 60 per cent of the seats; and with as little as 39.5 per cent the PRI maintains its absolute majority.

The second approach (described in the Appendix) takes a naive statistical point of view and comes up with almost the same conclusions: with about 46 per cent of the vne the PRI keeps its 300 seats; and with as little as 39.2 per cent the PRI maintains its absolute majority.

TABLE 10. PRI seats by 1994 law

	1985	1988	1991	1994
PRI % vne	64.77	51.92	64.18	52.70
PRI seats 1994 law	315	300	315	300

TABLE 11. Estimates of numbers of seats, 1994 law

PRI % vne	52.7%	50%	47%	44%	41%	40%	39.47%
PRI maj. seats	277	266	247	222	194	179	172
PRI party seats	23	34	53	78	82	80	79
PRI total seats	300	300	300	300	276	259	251

One infers from these estimates that the 1994 electoral law gives to the one big party a near assurance of maintaining an absolute majority in the House of Deputies with about 39.5 per cent of the vne, and some 45 per cent gives it 300 seats. If there were more parties this advantage would be more pronounced.

5. What Can Be Done?

The Mexican Electoral Law of 1989 may be a particularly glaring example of the logical inadequacies of the legal and/or political mind, but there are others in other countries. The reform of 1994, though seemingly a vast improvement, is occasionally vague, unnecessarily complex in wording, and where precise gives rules that are flawed and whose solutions may be at variance with the stated constitutional intent.

A law should give a complete and precise set of instructions to compute who gets how many seats and in which specific districts and constituencies. This is rarely realized. Instructions should be analyzed logically in order to foresee its consequences in every possible situation. Thus, if one state's population is bigger than another's then the first should not have fewer single member districts than the second. If one party's vote total increases and nothing else changes then it is unacceptable that that party should lose seats; it is also unacceptable that a very slight increase in the vote results in a huge increase in the seats. If more seats are to be shared among a set of parties and their proportional shares of the whole are the same, then no party should lose seats ("it ain't necessarily so" with the method of largest remainders!). Finally, the instructions should be relatively simple and straightforward, permitting anyone to verify the results.

By Article 53 of the Constitution (1989 and 1994) the 300 majority deputies are to be distributed among the states proportionally to their populations, except that each is to have at least 2 seats. But *exactly* how this is to be done is not specified. The actual apportionment is not proportional. In fact, the current attribution has not changed since 1977 when it was done on the basis of *projected* 1980 populations. Table 12, which gives the 1990 census populations, the proportional share of states, their actual numbers of seats and the recommended apportionment, shows that the actual distribution does not meet the constitutional standard. The *fair share* of a state is the state's proportional share of the 300 seats given that every state must have at least two seats. States whose proportional share of 300 is less than two have a right to at least two and the others' shares must be adjusted

Table 12. Actual versus recommended apportionment of seats to states (selection of states)

State	Population	Fair share	Actual seats	Recommended apportionment
Mexico	9,815,795	36.064	34	36
Distrito Federal	8,235,744	30.259	40	30
Guanajuato	3,928,593	14.633	13	15
Chiapas	3,210,496	11.796	9	12
Chihuahua	2,441,873	8.972	10	9
Yucatan	1,362,940	5.008	4	5
Durango	1,349,378	4.958	6	5

proportionally downward so that the resulting sum of all the fair shares is exactly 300 (see Appendix).

The recommended apportionment is found by applying what is known as *Webster's method, or Saint Lagüe's method*, or *the method of rounding to the closest integer* and may be described as follows. (i) Divide the population of each state by a common divisor x and round it to the closest whole number or to 2, whichever is largest; (ii) if the sum of these numbers is less than that wished (namely, 300) then adjust x downward, if it is greater than wished then adjust x upward, until an x is found that yields exactly what is wished: these numbers give the apportionment. Why this is the recommended method has been developed elsewhere in a 'theory of apportionment' based on practical axiomatics (Balinski, 1993; Balinski and Young, 1982, 1978).

The city of Mexico's population is larger than that of the Distrito Federal (DF), yet receives many fewer seats; the same phenomenon occurs between the Yucatan and Durango, and also between Chiapas and Chihuahua: this surely does not satisfy the intent of the Constitution. Chiapas has 9 seats, but deserves 12: no wonder that there are discontented citizens in the state of Chiapas! In fact, the actual apportionment deviates from the constitutional standard in 18 out of 32 states.

Neither the 1989 nor the 1994 law says how the five constituencies are to be defined, and neither respects the number of party deputies that are to be elected in each constituency, facts that seem to violate the spirit of the Constitution. Each party is supposedly entitled to have a number of seats proportional to its total party vote. Thus, given the total vote of each eligible party in each constituency the number of party deputies to be assigned to each constituency list should be determined so that each constituency has in total the number of party deputies it should have (40, it seems, is the correct number) *and* so that each party has in total the number of party deputies to which it is entitled (as previously computed). In fact, this poses a mathematical problem of some difficulty (which is explicitly confronted in Belgium and Holland), for which methods—albeit complicated ones—exist, see Balinski and Demange (1989).

The law of 1994 cannot be fixed by changing a clause here or there. A reformulation of the basic principles in the Constitution is necessary. Well defined methods are needed for assigning numbers of districts to states according to their populations and/or seats to parties according to their votes, and specific criteria for determining the boundaries of districts and constituencies. But the choice of a system for Mexico must also depend on its history, culture, legal and administrative practices, and traditions, and so must rely on politicians, lawyers and historians. It is possible to define an electoral system that combines a majority system based on single-member districts and a dosage of proportional representation to give to smaller parties at least some voice in the House of Deputies. The German electoral system is an example. But the problem of designing an electoral system is—as we hope our analysis has shown—a subtle exercise, with many hidden traps and potential contradictions, and a logical mathematical mind is needed to understand the implications of the choices that are made, to help design a system that answers to whatever stated objectives and requirements there may be. This job cannot be abandoned to mere lawyers, magistrates or politicians in France, Spain or the United States, in the United Kingdom, Mexico or elsewhere: an electoral system is in the end a mathematical transformation of numbers into numbers.

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Appendix

Apportionment of Seats to States

Call p_i the population of state i, h the number of deputies to be distributed, and $p = \Sigma_i p_i$ the total population of the country. The *quota* q_i of state i is $q_i = hp_i/p$: it is the exact proportional share of state i in absence of any minimum required representation. The *fair share* f_i of state i is the exact proportional share of state i when a minimum representation is required in each state. It may be calculated as follows (when 2 is the minimum required). Chose $x>0$ so that $\Sigma_i max\{2, p_i/x\} = 300$, and take $f_i = max\{2, p_i/x\}$.

The *round up method of apportionment* (the method of *Adams*) is defined as follows, where the a_i are seats apportioned to state i: chose $x>0$ so that $\Sigma_i max\{2,[p_i/x]^+\} = 300$, where $[y]^+$ is the real number y rounded up, and take $a_i = max\{2,[p_i/x]^+\}$.

The *round to the closest integer method of apportionment* (the method of *Webster*, of *Saint Lagüe*, of *odd numbers* and others) is: chose $x>0$ so that $\Sigma_i max\{2,[p_i/x]\} = 300$, where $[y]$ is the real number y rounded to the closest integer, and take $a_i = max\{2,[p_i/x]\}$.

The *round down method of apportionment* (the method of *Jefferson*, of *d'Hondt* and others) is: chose $x>0$ so that $\Sigma_i max\{2,[p_i/x]^-\} = 300$, where $[y]^-$ is the real number y rounded down, and take $a_i = \max\{2,[p_i/x]^-\}$.

These three methods, all reasonable, are used in different countries, but only the round to the closest integer method yields solutions that are neutral as concerns favoring large versus small states (or parties). The round up method strongly favors small states (or parties), the round down method strongly favors large states (or parties). For a detailed analysis of the properties of these and other methods see Balinski (1993); Balinski and Young (1982, 1978).

Estimating Seats: Model Based on 1994 Elections

Let $\mu = (\mu_{PRI}, \mu_{PAN}, \mu_{PRD}, \mu_{PT})$ be percentages of the vne's of the eligible parties. By the 1994 election results, μ = (52.70 per cent, 27.01 per cent, 17.51 per cent, 2.78 per cent). Subsequent figures in Table A1 are based on $\mu = (52.70 - 3\lambda, 27.01 + \lambda, 17.51 + \lambda, 2.78 + \lambda)$.

Estimating Seats: Statistical Model

Suppose as before that $\mu = (\mu_{PRI}, \mu_{PAN}, \mu_{PRD}, \mu_{PT})$ are the percentages of the vne's of the parties. Assume that the 'percentages' of the votes in the districts of each party are independently and uniformly distributed over the interval $[\mu_i-\alpha, \mu_i+\alpha]$, where $\alpha_i = min\{.25, \mu_i\}$. On the basis of this very simple and admittedly unrealistic model it is easy to calculate explicitly the expected number of majority seats of

TABLE A1. Estimates of numbers of seats, 1994 law

Party	μ	Exp. Maj. Seats	Total Seats	μ	Exp. Maj. Seats	Total Seats
PRI	52.70%	277	300	50.00%	266	300
PAN	27.01%	18	119	27.91%	26	119
PRD	17.51%	5	71	18.41%	8	69
PT	2.78%	0	10	3.68%	0	12
PRI	48.00%	253	300	46.00%	243	300
PAN	28.58%	37	121	29.24%	47	125
PRD	19.08%	10	66	19.74%	10	62
PT	4.34%	0	13	5.02%	0	13
PRI	44.00%	222	300	42.00%	206	290
PAN	29.91%	60	125	30.58%	72	133
PRD	20.41%	18	63	21.08%	22	64
PT	5.68%	0	12	6.34%	0	13
PRI	40.00%	179	259	39.47%	172	251
PAN	31.24%	90	152	31.42%	96	159
PRD	21.75%	31	75	21.92%	32	76
PT	7.01%	0	14	7.19%	0	14

each party as a function of μ. The number of party seats according to the 1994 law is then computed on the basis of μ, so the expected total number of seats of each party is the sum of the two. Calculations were based on $\mu = (53 - 3\lambda, 27 + \lambda, 17 + \lambda, 3 + \lambda)$. For example, $\mu = (44, 30, 20, 6)$ yields the distribution of seats (296, 133, 59, 12), and $\mu = (39.2, 31.6, 21.6, 7.6)$ yields (251, 160, 74, 15).

Neither of the two models pretend to 'accuracy': both give qualitative expectations, but are in substantial agreement.

'La formula de primera proporcionalidad'

Call p_i the vote total of party i, h the number of seats to allocate, $p = \Sigma_i p_i$ the total vote, and $d = p/h$ the average number of votes per seat. First (to favor the small parties), a special allocation is made to each party, and its votes are adjusted to p'_i, as follows:

if $p_i < d/2$ party i gets no special seats and $p'_i = p_i$;
if $d/2 \leq p_i < d$ party i gets 1 special seat and $p'_i = p_i - d/2$;
if $d \leq p_i$ party i gets 2 special seats and $p'_i = p_i - d$.

Second, use the method of largest remainders based on the adjusted votes p'_i to allocate the remaining seats among the parties. Each party receives the sum of its two allocations.

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