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Measuring bias: Moving from two-party to three-party elections

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Abstract

One method for assessing the extent of electoral bias is that first developed by Brookes. This method decomposes bias into different elements, including efficiency of vote distribution as well as effects separately produced by electorate size and turnout. Brookes' method is used to measure electoral bias largely in two-party systems but the rise of third parties, particularly in recent UK elections, has prompted the search for a reliable alternative. This paper reports upon findings from an on-going research programme. The nature and theoretical underpinnings of different procedures that might be used for decomposing bias in the three-party case are outlined. Two main procedures are constructed and then tested against the results from actual elections. The evidence shows that these procedures produce similar findings in respect of the 2005 general election but differences emerge when earlier elections are considered. Research continues to assess whether these differences follow from the nature of party competition at each election or the particular procedure employed.

Keywords: Electoral bias; Electoral geography; Third parties

1. Introduction

A well-known feature of simple plurality electoral systems is that, irrespective of any explicit political involvement in drawing district boundaries (the malapportionment and gerrymandering strategies characteristic of much redistricting in the United States), legislative contest outcomes are almost invariably disproportional—more so than with many other types of electoral system. Such disproportionality usually favours the largest of the two main parties which—as identified from Duverger's classic work (Duverger, 1954) onwards—tend to dominate such systems. What is not as well attested is whether that disproportionality is unbiased by not treating these two largest parties differentially. A system that gives the largest party a 'winner's bonus', with, say, a ten percentage points greater share of the seats than of the votes, is disproportional. However, if main party *A* obtains that bonus but main party *B*, with the *same* vote share, gets a bonus of only five points, then the system is not only disproportional but also biased towards *A*. Such a winner's bonus is sometimes termed exaggeration (Johnston et al., 2002) or responsiveness (King, 1990), or majoritarian bias (as in Calvo and Micozzi, 2005), and it differs from 'electoral bias' (Johnston

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et al., 2001) or 'partisan bias' (Grofman and King, 2007) which refers to an asymmetry in the way party vote share is translated into seats (the same share of the votes cast can result in substantially different shares of the seats). We will consider electoral bias only and refer to it as simply bias.

An unbiased system, according to Grofman and King (2007), is characterised by what they term *partisan symmetry* (see also King et al., 2005), a requirement that

> '… the electoral system treat similarly-situated parties equally, so that each receives the same fraction of legislative seats for a particular vote percentage as the other party would have received if it had the same percentage'.

Grofman and King (2007, p. 6) claim that this definition of partisan symmetry has been virtually the consensus position of social scientists as a means of assessing the partisan fairness of a districting scheme. Measuring it has been a cause of considerable experimentation and debate, however: as King et al. (2005, p. 9) note:

> 'A consensus exists about using the symmetry standard to evaluate partisan bias in electoral systems. But such a consensus does not answer the subsidiary question: how to measure symmetry itself in order to determine whether partisan bias exists'.

(Hence the experimentation—as in King, 1990; Gelman and King, 1994; Grofman et al., 1997; Gelman et al., 2004—some of which has sought only to identify the extent of bias, without also decomposing it to uncover its sources.[1])

One procedure used to measure that bias was developed in New Zealand by Ralph Brookes (1953, 1959, 1960), which has the major benefits of using a readily-appreciated metric and being decomposable into the various bias sources that he identified (variations in constituency size, abstention rates, voting for third parties, and the distributions of party support).[2] It has been widely applied to the analysis of British election results in the last two decades (e.g. Johnston et al., 2001, 2002, 2006).[3]

Brookes' method was ideally suited to the analysis of a system where two parties predominated—as was the case in New Zealand until the 1990s: it assumes that other parties gain a proportion of votes cast that *cannot* be translated into seats. Its application to British elections since 1974 is thus constrained by the growth of 'third party' votes in England and 'fourth party' votes in Scotland and Wales that were subsequently translated into seats. Although a third party victory component was added later by Mortimore (1992): (see Johnston et al., 1999) to get a more realistic appreciation of the extent, direction and sources of any observed bias, nevertheless the method essentially remains focused on the two-party situation. Brookes' method—along with most others seeking both to identify and decompose the level of bias—treats third parties as, in effect, the source of relatively small amounts of 'noise' in a predominantly two-party system. Our goal here is to undertake a further modification of the method of decomposing bias that will make it better suited to the realities of three-party competition, where each of the parties is competing with the other two (perhaps in different places) for substantial numbers of votes and all three are potential seat-winners. This paper represents the initial stages in this process.

2. Reformulating Brookes' measure

In this paper, we re-formulate the two-party Brookes' method in such a way that will subsequently allow extension to the three-party case. More formally, let x be the number of seats the leading party wins with given share, k, of the two-party vote, and y the number of seats the second party could win if it got the same share of the votes. Then, bias towards the first party is defined as the difference between the number of seats gained by this party, x, and the mean of seats gained by both parties, i.e. the mean of x and y. Thus, for the two-party competition, bias is a function of one variable, vote share k.

Bias to party A is defined as

$$bias_{\mathrm{A}}(k) = x - \mathrm{MEAN}(x,y) = x - (x+y)/2 = (x-y)/2 \tag{1a}$$

which is simply the negative of bias towards its rival, B:

$$bias_{\mathrm{B}}(k) = y - \mathrm{MEAN}(x,y) = y - (x+y)/2 = (y-x)/2. \tag{1b}$$

Although Brookes' method of measuring bias is often considered as based on electoral outcomes of uniform

[1] Grofman and King (2007, p. 32) do claim, however, that 'The degree of deviation from symmetry of treatment is known as *partisan bias*, and is easily quantified, and made specific as to direction'.

[2] An alternative approach, developed almost contemporaneously with Brookes', identifying the same basic bias components, is Soper and Rydon (1958), who developed early ideas of Brookes (1953).

[3] The only other attempts to measure and account for bias in the UK have been those by Curtice (2001) (see also Curtice and Steed, 1986), which although it identified the various sources of bias did not quantify their relative importance in terms of seats, and Blau's important critique of Brookes' approach (Blau, 2001) and his suggestion to use an 'integrated method' (Blau, 2001, 2004).

swing of support from one party to another across all constituencies (Rossiter et al., 1999) or cited almost exclusively in the course of criticism of the assumption of uniform swing (Blau, 2001), here we distinguish between:

1. the quantitative measure of bias towards a party (difference between x and the mean of x and y);
2. the derived formulae for decomposition of bias into vote distribution effect, constituency size effect, etc.;
3. what magnitude of k should be used if we are interested in measuring bias for a particular election; and
4. the process whereby we might obtain the figures x and y.

As part of our extension of Brookes' method, therefore, we explicitly address these four issues.

The first two issues are independent of the 'uniform swing' procedure. For example, if the two main parties get equal vote shares at an election then we can calculate bias without even addressing the fourth point, the process for obtaining x and y. The reason for this is that the two parties have equal vote shares naturally and the actual seats won can be used as x and y. Therefore, we know that we can deal with this part of the problem (defining and decomposing bias) without discussing issues 3 and 4 above.

The last two issues, the magnitude of k and the process for obtaining x and y, are however concerned with 'notional' elections—it is doubtful that two main parties would get exactly equal vote shares at any election! So, we have to 'construct' an imaginary election with 'equal conditions' for two parties.

Issue 3 concerns the magnitude of k that should be used when measuring bias for a particular election. In practise two methods have been used. The first of these, the 'equal shares' approach, compares the number of seats won by each of the two parties respectively if the votes actually cast were redistributed equally between them. The 'reverse shares' method, on the other hand, considers what would have happened to the distribution of seats had the second-placed party (in terms of votes) obtained the vote share won by the first-placed party and the first-placed party obtained the vote of the second-placed party. In principle, it is possible to calculate bias for any k in a range between 0 and 100 (as in Johnston et al., 2002). Nevertheless, bias is usually calculated at either $k = 50\%$, i.e. equal vote shares, or at $k =$ actual vote share of the leading party, i.e. reverse vote shares. Both approaches have merit. The former allows easy interpretation of bias—if two main parties get an equal share of votes but non-equal numbers of seats then the bias towards one of those parties is its 'excess'/'deficiency' in seats. The latter, it might be argued, retains more features from the actual electoral outcome—size of constituencies (electoral units), turnout and minor party support variations across constituencies, as well as magnitude of national vote share, k, of the leading party. The only difference is that two main parties swap their positions—actual second party becomes new leading party with k vote share.

This leaves only the process for arriving at the values of x and y at our notional election. Brookes used uniform swing, applying change in vote share for both parties in each available electoral district (constituency).[4] In principle other approaches might be used, for example, Monroe's (1998) variable swing or the method of simulation ('*approximate uniform partisan swing*') favoured by Gelman, King and others (Gelman and King, 1994; Grofman and King, 2007).

3. Measuring three-party bias

Whereas Brookes' method performs well in evaluating bias in electoral situations where two parties predominate, it is less satisfactory in systems where there is a strong third-party attracting support from either or both of the main contestants. This has been the situation in Great Britain since the mid-1970s, and although it is possible to use Brookes' method to compare the situation of any pair of parties in a three-party system (as in Johnston et al., 2006) this is not particularly satisfactory. Hence our search for a better procedure, based on Brookes, in which we allow for shifts in support across all three parties rather than one being treated as a 'constant' thereby restricting the focus to a contest between the other two.

The original definition of bias permits an extension to the three-party case. Bias towards the leading party

[4] Swing is a measure of the net change in support for two parties, A and B, compared across two elections and can be defined as: $[(ShareA_2 - ShareA_1) + (ShareB_1 - ShareB_2)]/2$. Two variants of swing are known: Butler swing and Steed swing. Both rely on the same formula but apply different methods for calculation of parties' share of vote. For Butler swing, vote share is calculated on the basis of the total vote cast for a party whilst Steed swing uses total two-party vote (or total three-party vote). In this paper we apply 'Steed' rather than 'Butler' swing for precisely the same reasons as discussed by Blau, 2001, p. 62 (see also Mortimore, 1992). The former is preferred because it preserves the shape of the distribution of votes of the relevant parties which feature in our estimations. Under Butler's approach parties' shares are changed uniformly across constituencies; Steed's method imposes uniform change on *relative* two/three-party shares.

is measured as the difference between the number of seats gained by the party and a norm which is the mean of seats gained by three parties under equal conditions:

$$bias_{\mathrm{A}}(k) = x - \mathrm{MEAN}(x, y, z), \tag{2a}$$

where x, y, and z are the number of seats won by parties A, B, and C respectively under 'equal conditions', i.e. with the same percentage of votes cast (in actual or notional elections). In turn, bias towards party B is measured as

$$bias_{\mathrm{B}}(k) = y - \mathrm{MEAN}(x, y, z), \tag{2b}$$

and it follows that bias towards party C is measured as

$$bias_{\mathrm{C}}(k) = z - \mathrm{MEAN}(x, y, z). \tag{2c}$$

4. How to identify different components of bias if *x*, *y*, *z* are known

Let us suppose that we know/can calculate x, y, and z. Then the formulae (2a)–(2c) given in the definition of bias can be rearranged in the following form

$$x - \mathrm{MEAN}(x,y,z) = x - \frac{x+y+z}{3} = \frac{x-y}{3} + \frac{x-z}{3} \tag{3a}$$

$$y - \mathrm{MEAN}(x,y,z) = \frac{y-x}{3} + \frac{y-z}{3} \tag{3b}$$

$$z - \mathrm{MEAN}(x,y,z) = \frac{z-x}{3} + \frac{z-y}{3} \tag{3c}$$

We can see from these equations that bias towards party A, for example, is partitioned into two terms—bias resulting from a non-symmetry between party A and B, $(x-y)/3$, plus bias derived from a non-symmetry between A and C, $(x-z)/3$. These may both move in the same direction, or one may partly cancel out the other if they move in opposite directions.

We follow Brookes' method when decomposing terms $(x-y)/3$, $(x-z)/3$ etc. in equations (3a)–(3c).

Four main components of bias were scrutinized in Brookes' and subsequently in Johnston et al. papers:

- 'gerrymandering' (also termed the 'vote distribution' or 'efficiency' effect), i.e. asymmetry in the distribution of partisan voting strength across constituencies as indicated by skewed frequency distributions (Gudgin and Taylor, 1979);
- malapportionment, i.e. differences in electorate size across constituencies;
- abstainers/turnout effect; and
- minor party vote effect.

A fifth component, minor party victory, was introduced by Mortimore (1992) (see formulae in Johnston et al., 1999) to take into account the fact that in recent British elections the 'third' party became much more prominent since 1970 there than it was in New Zealand when Brookes was writing. Naturally, we do not need to include this fifth term in the decomposition of bias because our analysis specifically considers three-party competition from the outset.

As a result of decomposition, we get the following formulae for different bias components towards party A:

Vote distribution effect[5]

$$G_{\mathrm{toward\_A}} = \frac{y}{3}(P_x/Q_y - 1) + \frac{z}{3}(P_x/Q_z - 1) \tag{4}$$

Malapportionment or electorate size effect

$$E_{\mathrm{toward\_A}} = \frac{y}{3}(S_y/R_x - 1) + \frac{z}{3}(S_z/R_x - 1) \tag{5}$$

Abstention or turnout related effect

$$A_{\mathrm{toward\_A}} = \frac{y}{3}\left[\frac{R_x}{R_x - C_x}(C_x/R_x - D_y/S_y)\right] + \frac{z}{3}\left[\frac{R_x}{R_x - C_x}(C_x/R_x - D_z/S_z)\right] \tag{6}$$

Minor party vote effect

$$M_{\mathrm{toward\_A}} = \frac{y}{3}\left[\frac{R_x}{R_x - U_x}(U_x/R_x - V_y/S_y)\right] + \frac{z}{3}\left[\frac{R_x}{R_x - U_x}(U_x/R_x - V_z/S_z)\right] \tag{7}$$

where: x = number of seats won by party A, y = number of seats won by party B, z = number of seats won by party C; P_x = average number of combined votes for three major parties where party A won seats, Q_y = average number of combined votes for three major parties where party B won seats, Q_z = average number of combined votes for three major parties where party C won seats; R_x = average electorate where party A won seats, S_y = average electorate where

[5] Notation for the equations follows the traditional form where G represents gerrymander (Brookes' original term), E is electorate size, A is abstention and M for minor party votes. We do not believe that gerrymander is the most appropriate term ('distributional effect' is more accurate perhaps and throughout the rest of the paper we prefer this term) but we have resisted the temptation to alter the notation from Brookes' original. By so doing we believe we make more transparent our alterations to the original algebra.

party B won seats, S_z = average electorate where party C won seats; C_x = average number of abstentions where party A won seats, D_y = average number of abstentions where party B won seats, D_z = average number of abstentions where party C won seats; U_x = average number of minor party votes where party A won seats, V_y = average number of minor party votes where party B won seats, V_z = average number of minor party votes where party C won seats.

It is important to note that we compare places where parties ***win*** seats. This is the method used by Brookes but was subsequently modified by Johnston et al. (1999). Their chosen method was for, say, party A, to examine all seats where A's vote share was greater than B's vote share and not simply those seats won by party A. This was done to accommodate the principle of the 'third' party winning seats. Because we are dealing with the three-party case we can now ignore this procedure and revert to the original.

Decomposition of biases towards party B and C yields formulae analogous to those stated in equations (4)–(7) above.

5. Does it work?

To illustrate how this extension of the Brookes method takes into consideration the third party, we consider two examples where a 'third' party participates and wins seats. We compare the results of two-party bias decomposition (including a 'third party victory' component) with our three-party bias analysis.

To avoid any unnecessary discussion at this point about the approach for defining 'equal conditions' required for calculation of three-party bias (see equations 2) and whether 'equal' or 'reverse' vote shares are preferable for two-party bias, we consider examples where all parties get a virtually equal number of 'actual' votes overall, thereby avoiding the need to construct any notional elections. Furthermore, for the purpose of these examples we assume that turnout in each constituency is 100% (thereby reducing the number of components in the decomposition).

The chosen examples are admittedly rather extreme but they are used to make important general points and to demonstrate more clearly how the two approaches treat the 'third' party. Both examples show an unbiased electoral outcome when three participating parties win the same number of seats and receive virtually the same share of votes. Although bias toward any single party equals 0 in each case (the parties get an equal share of votes *and* an equal share of seats) we nevertheless can partition bias using the above formulae and find that the bias components differ from 0, operate in opposite directions, and cancel out each other. When calculating two-party bias, the party that receives a slightly smaller number of votes than the others is considered as the 'third' party.

In the first example (Table 1) parties A and B (with 225 votes each) are the two main parties with equal two-party vote shares and party C is the third party (223 votes). Parties A and B win two seats each, both have 15 surplus votes (i.e. votes cast where a party wins the seat but receives more votes than are necessary to defeat the second-placed party), and 115 wasted votes (votes that bring no return because they are cast in constituencies where the party loses). Of these 115 wasted votes, 95 are wasted in a constituency where the rival main party wins, and 20 are wasted where the third party C is the winner. The positions of parties A and B look identical and unbiased. To investigate whether there are any sources of bias that operate in opposite directions, cancelling each other out and so not immediately apparent, decomposition formulae were applied.

Unsurprisingly, Brookes' two-party method performs badly in this extreme situation when the 'third' party (smallest in terms of vote share) becomes an equal player in terms of seat distribution. As the calculations below show, the algorithm gives a false impression that party A has a more efficient vote distribution than party B (the G component is positive for A and negative for B); that the electorate size component also gives party A the advantage over B; and that A loses from third party votes whilst party B gains from that effect.

Vote distribution effect

$$G_{\text{toward\_A}} = 0.08,\ G_{\text{toward\_B}} = -0.08,$$

Electorate size effect

$$E_{\text{toward\_A}} = 0.15,\ E_{\text{toward\_B}} = -0.14,$$

Table 1
Example 1 of a hypothetical election

Constituencies	Electorate	Votes			Winner
		Party A	Party B	Party C	
I	150	60	55	35	A
II	100	50	40	10	A
III	150	55	60	35	B
IV	100	40	50	10	B
V	103	5	9	89	C
VI	70	15	11	44	C
		Overall share of vote (%)			
		33.4	33.4	33.1	

Table 2
Decomposition of three-party bias (Example 1 election)

	Toward party A			Toward party B			Toward party C		
	overall	from B	from C	overall	from A	from C	overall	from A	from B
Vote Distribution	0.30	0.00	0.30	0.30	0.00	0.30	−0.41	−0.21	−0.21
Electorate Size	−0.21	0.00	−0.21	−0.21	0.00	−0.21	0.59	0.30	0.30

Third party votes effect
$M_{toward\_A} = -0.21$, $M_{toward\_B} = 0.25$, and
Third party victories effect
$W_{toward\_A} = 0.00$, $W_{toward\_B} = 0.00$.

But we have already demonstrated that the positions of parties A and B at this election are virtually identical. Therefore, the Brookes method of decomposition in this case appears counter-intuitive.

Next, we consider for the same election how a measure of three-party bias operates. We can calculate biases toward each party using the formulae from equations (2a)–(2c) and then partition them applying the algorithm for decomposition of three-party bias, (equations (4)–(7)). The outcome is presented in Table 2.

Of course, Table 2 does not consider any turnout effect (turnout is 100%) nor does it consider a minor/fourth party effect (because the example uses only three parties). From the results of decomposition using this new method we can now see that parties A and B have equal advantage over party C in terms of the vote distribution and are equally disadvantaged in terms of size component. The total number of ineffective votes for both A and B equals 130 (surplus plus wasted votes) and for C it equals 199, (109 surplus and 90 wasted votes). Parties A and B won seats in constituencies with an average size of 125 electors while party C won in constituencies with a smaller average, just 86.5 electors. It was shown earlier that each component of bias toward a party, let's say party A, can be further partitioned regarding the source of asymmetry—an asymmetry between A and B, i.e. 'bias toward A from B', and an asymmetry between A and C, i.e. 'bias toward A from C'. Thus, we can see that the electoral disadvantages for both A and B came from C rather than from each other. Party C gets its gains and losses equally from A and B. In short, the pattern of bias decomposition using the three-party method lies in the expected direction.

The next example is used to demonstrate how the two-party Brookes method is sensitive to the precise distribution of votes between one of the major parties and the third party in constituencies won by the third party. It differs from the example in Table 1 only in the distribution of votes (bold figures in Table 3) between party A and party C in seats won by party C—in all other respects (overall vote and seat share) the two examples are identical.

The components of two-party competition bias are very different from those for example 1. For party A we can see an even larger 'distributional advantage' relative to B (0.14 vs. 0.08 in example 1) and 'minor party vote disadvantage' (−0.26 vs. −0.21).

Vote distribution effect
$G_{toward\_A} = 0.14$, $G_{toward\_B} = -0.13$,
Size effect
$E_{toward\_A} = 0.15$, $E_{toward\_B} = -0.14$,
Minor party vote effect
$M_{toward\_A} = -0.26$, $M_{toward\_B} = 0.31$, and
Minor party victories' effect
$W_{toward\_A} = 0.00$, $W_{toward\_B} = 0.00$.

By contrast, using the three-party bias method the components of the decomposition remain unaltered, as we should expect.

The purpose of these examples was to construct elections for which the Brookes' method was never intended—where there is real three-party competition and where, as in the example above, the third party actually wins most seats. That method, of course, is primarily concerned with the relationship between

Table 3
Example 2 of a hypothetical election

Constituencies	Electorate	Votes			Winner
		Party A	Party B	Party C	
I	150	60	55	35	A
II	100	50	40	10	A
III	150	55	60	35	B
IV	100	40	50	10	B
V	103	**1**	9	**93**	C
VI	70	**19**	11	**40**	C
		Overall share of vote (%)			
		33.4	33.4	33.1	

party A and party B. In an election including a significant third party the Brookes method tries to assign any special features to one or other of the two main parties. The three-party bias method, however, is precisely designed to measure relationships across three and not two parties.

6. What is 'equal conditions'? How to derive *x*, *y*, and *z*?

To calculate bias we have to know *x*, *y*, and *z*—the number of seats each of three parties could get, given share of vote, *k*. In the real world, of course, we have just one actual election. Therefore, we need to consider at least one notional election with 'similar conditions'. Analogous to the two-party case, we have a choice between constructing some form of 'equal' or 'reverse' share situations. If we construct a notional election with equal three-party votes, then we lose information about the leading party's actual vote share. Our current thinking, therefore, suggests a preference for redistribution of shares using some variant of reverse shares. Under this notional election procedure a new leading party will be awarded the same vote share as the leading party at the actual election while at the same time the relative weights of the other two parties will be retained. The attraction of this approach is that it retains more memory of the actual election.

In the three-party case, therefore, we need to consider *two* notional elections. One of these would see the actual second-placed party awarded the same vote share as the actual first-placed party. The second notional election would see the actual third-placed party awarded the same vote share as the actual first-placed party. For the calculation of bias we will use actual number of seats won by leading party, *x*, and *y* and *z* will be the respective seats won by the new leading parties in our two notional elections.

Assuming that we follow this procedure we still have to consider *how* votes are re-distributed within each constituency. The standard procedure is to assume uniform swing within each constituency. A number of objections can and have been raised with this procedure, not least the fact that when applied to some constituencies a uniform swing might take a party's vote share above 100 or below 0. These problems are magnified when it comes to the three-party case.

We need to consider adopting a different procedure, or two as it happens for this paper. The first of these methods begins by applying a uniform/homogeneous increase in share of vote for the new leading party across all constituencies. Next, it reduces the votes of the notional two non-leading parties though retaining the ratio of votes won at the constituency level between those parties at the actual election.[6] This means, in effect, that different constituencies may see a non-uniform reduction of votes for the two non-leading parties, as the example discussed below shows. The main attraction of this procedure is that it is perhaps closest to the uniform swing procedure that is currently used for the two-party bias method.

For the second approach the most important concern is to retain the ratio between national vote shares for the two non-leading parties. It does this by calculating a coefficient that becomes the proportion of decrease for the vote share of the two notional non-leading parties. This coefficient is arrived at by summing the vote share of the actual two non-leading parties and dividing the resulting total by the sum of the vote shares for the actual leading party and that party which is neither the leading party at the actual election nor the leading party at the notional election. These new votes are then added to the vote of the new leading party at the notional election.[7] As a result of

[6] The formal procedure is as follows. Assume that party A is the leading party and party B is second leading party with a difference between their national vote shares of delta_{AB}, where $\mathrm{delta}_{AB} = \mathrm{sh}_A - \mathrm{sh}_B$, and where sh_A, sh_B, sh_C are national vote shares for parties A, B, and C respectively. Constructing notional election, for each constituency, ***i***, we add delta_{AB} percentage points to party B: $\mathrm{sh}^{NEW}_{B\,i} = \mathrm{sh}_{B\,i} + \mathrm{delta}_{AB}$ and then redistribute remaining three−party votes between parties A and C in such a way that preserves the initial ratio between their shares (in each constituency, ***i***), $\mathrm{sh}^{NEW}_{A\,i}/\mathrm{sh}^{NEW}_{C\,i} = \mathrm{sh}_{A\,i}/\mathrm{sh}_{C\,i}$. As a result of the procedure, the new national vote share for party B (new leading party), will be the same as that for leading party A in the actual election, $\mathrm{sh}^{NEW}_B = \mathrm{sh}_A$. Party A becomes second largest party and party C stays the third. Notional election with party C as the new leading party is constructed in a similar way, increasing vote share of party C by delta_{AC} percentage points, $\mathrm{delta}_{AC} = \mathrm{sh}_A - \mathrm{sh}_C$ and so on as above.

[7] Constructing Version 2 notional election with party B as a new leading party, we compute a ratio, gamma_{AB}, where $\mathrm{gamma}_{AB} = (\mathrm{sh}_B + \mathrm{sh}_C)/(\mathrm{sh}_A + \mathrm{sh}_C)$, which effectively is the ratio of vote share of party A and party C together in the new (notional) election to their vote share in the actual one. For each constituency, *i*, vote shares for party A and party C are reduced proportionally: $\mathrm{sh}^{NEW}_{A\,i} = \mathrm{sh}_{A\,i} * \mathrm{gamma}_{AB}$, and $\mathrm{sh}^{NEW}_{C\,i} = \mathrm{sh}_{C\,i} * \mathrm{gamma}_{AB}$ (that preserves the ratio between A and C shares: i.e. $\mathrm{sh}^{NEW}_{A\,i}/\mathrm{sh}^{NEW}_{C\,i} = \mathrm{sh}_{A\,i}/\mathrm{sh}_{C\,i}$). Then vote share of party B is increased by a certain amount of percentage points: $\mathrm{sh}^{NEW}_{B\,i} = \mathrm{sh}_{B\,i} + (1 - \mathrm{gamma}_{AB}) * (\mathrm{sh}_{A\,i} + \mathrm{sh}_{C\,i}) = \mathrm{sh}_{B\,i} + * (\mathrm{sh}_A - \mathrm{sh}_B) * (\mathrm{sh}_{A\,i} + \mathrm{sh}_{C\,i}) /* (\mathrm{sh}_A + \mathrm{sh}_C)$. This value means that different percentage points will be added to the new leading party in each constituency. New leading party B will get the same national share of votes as the leading party A in the actual election: $\mathrm{sh}^{NEW}_B = \mathrm{sh}_A$. New ratio of vote shares of non-leading (new) parties, $\mathrm{sh}^{NEW}_A/\mathrm{sh}^{NEW}_C$, will be exactly the same as the initial one, $\mathrm{sh}^{NEW}_A/\mathrm{sh}^{NEW}_C = \mathrm{sh}_A/\mathrm{sh}_C$.

Table 4
Example 3 of a hypothetical election, actual election outcome

	Actual election (Party A is the leader)				
	Votes			Ratio: share A to share B	Ratio: share A to share C
	Party A	Party B	Party C		
Constituency I	500	250	250	2.00	2.00
Constituency II	500	400	100	1.25	5.00
	National share of votes (%)				
	50.0	32.5	17.5		
	Ratio: share A to share B 1.54				
	Ratio: share A to share C 2.86				

this procedure the new leading party is guaranteed to receive the same national vote share as that received by the lead party at the actual election. Another attraction of this second approach is that it avoids completely the problem of constructing party vote shares that rise above 100 or fall below zero.

Example 3 hypothetical election (Table 4) has just two constituencies, with a thousand votes cast in each. Overall, the national vote sees party A win with 50% of the vote share, party B is placed second with 32.5% and party C wins 17.5%.

Table 5 sets out two notional elections based on Table 4, using the first procedure described above. When using this procedure, party B becomes the new leading party. To take it to 50% of the overall vote we apply a uniform 17.5% percentage point increase across constituencies. The non-leading parties see a reduction in their vote share that maintains the original constituency level ratios (see the final two columns in the first part of the table).

As a result of this procedure, the new leading party gets exactly the same 'national' (overall) share of vote as that of the leading party at the actual election. The new non-leading parties keep the same finishing order that they have in the actual election. However, as we noted earlier, with this procedure we cannot guarantee the ratio of national shares of non-leading parties. Nevertheless, in this example the new national ratio of 2.81 (or 1.52 in the case when party C becomes the leading party) is close to the original value of 2.86 (1.54).

Next, in Table 6 we construct two further notional elections based on Table 4 but this time instead use the second procedure described above. Once again, we describe the notional election where the new leading party is B. In this example the use of procedure two means that parties A and C will retain 74% of their votes in each constituency (obtained by summing 32.5 and 17.5 and dividing by 50.0 plus [in this case] 17.5). These votes are then assigned to party B. In the notional election this means that in constituency I party B will receive a 19.4 percentage point increase in vote share but a 15.6 percentage point increase in constituency II.

In effect, using these two procedures means that certain (though different) information about the original election has been retained for the two constructed notional elections. The first procedure retains two

Table 5
Notional elections using procedure 1

	Party B as the leader				Party C as the leader			
	Votes			Ratio: share A to share C	Votes			Ratio: share A to share B
	Party A	Party B	Party C		Party A	Party B	Party C	
I	383	425	192	2.00	283	142	575	2.00
II	354	575	71	5.00	319	256	425	1.25
	National share of votes (%)				National share of votes (%)			
	36.9	50.0	13.1		30.1	19.9	50.0	
	Ratio: share A to share C 2.81				Ratio: share A to share B 1.52			

Table 6
Notional elections using procedure 2

	Party B as the leader				Party C as the leader			
	Votes			Ratio: share A to share C	Votes			Ratio: share A to share B
	Party A	Party B	Party C		Party A	Party B	Party C	
I	370	444	185	2.00	303	152	545	2.00
II	370	556	74	5.00	303	242	455	1.25
	National share of votes (%)				National share of votes (%)			
	37.0	50.0	13.0		30.3	19.7	50.0	
	Ratio: share A to share C 2.86				Ratio: share A to share B 1.54			

characteristics: the shape of the vote distribution for the new leading party; and the relative weight of the two non-leading parties at the constituency level. The second procedure also retains two characteristics: the relative weight of the two non-leading parties at the constituency *and* national levels.

Intuitively, it appears as though we should prefer the second procedure over the first. From a formal mathematical point of view however no such preference can be made.

7. Applying methods for calculating bias to the 2005 British general election

Johnston et al. (1999), (2002) prefer the 'equal' vote shares procedure for calculating two-party bias but because we propose here a three-party method that concentrates on the investigation of bias for/against the leading party we consider the 'reverse' shares two-party procedure as more appropriate for the comparison of the two-party/three-party analyses. We calculate three-party bias for the 2005 British general election (Northern Ireland is, as usual, excluded, as is the Speaker's seat) using the two variants of constructing notional elections described earlier and compare the results with the two-party bias method.

7.1. Two-party bias (reverse shares)

In this section, we revisit the 2005 general election using the standard Brookes two-party method (as in Johnston et al., 2006), except that we use the reverse share rather than equal share approach (see Table 7). Labour won 0.52 of the two-party vote at that election, and the Conservatives 0.48, with Labour getting 355 seats ($x = 355$ in our notation): if those shares were reversed (using the Steed swing) the Conservatives would get 255 seats (i.e. $y = 255$). Using formulae (1a) and (1b) above, this indicates a bias towards the Labour party of 50 seats.[8] Decomposing that bias we find that all the components favour Labour apart from minor party votes. For the Conservatives this feature works in the party's favour but rewards them with less than nine seats.

7.2. Three-party bias. Decomposition of bias (using procedure 1 notional elections)

We now use the first version of our three-party method to evaluate the 2005 general election result with three notional results. These produce a substantial pro-Labour bias of 89.7 seats, and biases against the Conservatives and Liberal Democrats of 21.3 and 68.3 seats respectively (Table 8). According to these estimates, Labour was clearly the main beneficiary of how the system operated (or was being operated through the geography of party vote-winning strategies: Johnston et al. (2006) distinguish between system operation and operation of the system) and the Liberal Democrats the main losers.

The bias decompositions (using formulae (4)–(7)) show not only which component favoured which party but also which of its opponents it gained that advantage from. Labour's greatest advantage came from the vote distribution effect, with three-quarters of that advantage (34 of 45 seats) coming at the expense of the Liberal Democrats, which clearly have the least efficient distribution of votes across the country's constituencies. The Conservatives, too, have a net advantage from this component (7 seats); however, this net figure comprises a loss to Labour of 15

[8] In our notation $\mathrm{bias}_{\mathrm{Lab}}(0.52) = +50$ and $\mathrm{bias}_{\mathrm{Con}}(0.52) = -50$. We thank the anonymous referee for his/her suggestion that we should characterise overall bias as $\mathrm{bias}_{\mathrm{Lab}}(0.52) - \mathrm{bias}_{\mathrm{Con}}(0.48)$. But because we are mostly interested in bias for/against *the leading party* we do not concern ourselves with overall bias.

Table 7
Electoral bias at the 2005 general election: Brookes' two-party method

	Lab	Con
Number of seats won with 52% of two-party vote share	355	255
Bias at 52%	+50	−50
	Decomposition of bias	
Vote distribution	11.6	−13.6
Electorate size	12.5	−14.5
Abstention	17.8	−20.0
Third party votes	−6.5	8.6
Third party victories	11.0	−11.0
Net interaction	3.6	0.6

seats but a gain from the Liberal Democrats of 21 seats. It should be noted that there are important differences between the three-party and two-party bias methods in respect of minor party votes and victories. Under the two-party Brookes method Labour was disadvantaged by the minor party votes component but this disappears when the Liberal Democrats are excluded from the minor party category with the three-party method.

7.3. Three-party bias. Decomposition of bias (using procedure 2 notional elections)

Turning to our second method of estimating bias in a three-party situation, the net figures in Table 9 are very similar to those in Table 8. Voter distribution was the main source of bias giving us 49 seats to Labour compared with 45 seats under the first procedure. The further partition of this particular term shows that Labour gained 12 seats from the Conservatives under both procedures and 37 and 34 seats from the Liberal Democrats using the second and first procedures respectively. Variation across constituencies in abstentions was the second largest component in both procedures and variation in constituency size the third. The impact of minor party votes (i.e. mainly the two nationalist parties) is small. Overall, Labour is the main beneficiary, gaining a bias advantage from each of the four components, and the Liberal Democrats are the most disadvantaged, again especially because of the vote distribution effect. The Conservatives are also largely disadvantaged, although they have a more efficient vote distribution than the Liberal Democrats; the abstention and constituency size components both work to the Conservatives disadvantage against each of the other parties.

8. Discussion and conclusion

In constructing a method for measuring three-party bias we gave close attention to four issues that relate to the original Brookes method: the definition of bias; formulae for the decomposition of that bias; and (the third and fourth issues), construction of some 'norm' for comparison, by choosing equal or reverse shares, for example, and how exactly we derive this norm. We have suggested a definition of bias for the three-party case with reference to mean values. Formulae for the decomposition of bias were derived.

Table 8
Electoral bias at the 2005 general election: three-party method, procedure 1

	Lab		Con		LD	
Number of seats won with 39% of three-party vote share	$x = 355$		$y = 244$		$z = 197$	
Expected unbiased number of seats won with 39% vote share			MEAN(x,y,z) = 265.3			
Bias at 39%	x − MEAN(x,y,z) = 89.7		y − MEAN(x,y,z) = −21.3		z − MEAN(x,y,z) = −68.3	
Decomposition of bias						
Vote distribution	45.3		6.8		−60.2	
From	Con	11.5	Lab	−14.7	Lab	−40.2
	LD	33.8	LD	21.4	Con	−20.0
Electorate size	10.8		−11.4		−3.8	
From	Con	7.1	Lab	−9.5	Lab	−6.3
	LD	3.7	LD	−1.9	Con	2.4
Abstention	16.5		−14.4		−14.4	
From	Con	10.0	Lab	−13.0	Lab	−13.0
	LD	6.5	LD	−1.4	Con	−1.4
Third party votes	2.4		−3.0		−0.4	
From	Con	1.7	Lab	−2.4	Lab	−1.2
	LD	0.7	LD	−0.6	Con	0.8

Table 9
Electoral bias at the 2005 general election: three-party method, procedure 2

	Lab		Con		LD	
Number of seats won with 39% of three-party vote share	$x = 355$		$y = 243$		$z = 187$	
Expected unbiased number of seats won with 39% vote share			MEAN(x,y,z) = 261.7			
Bias at 39%	x − MEAN(x,y,z) = 93.3		y − MEAN(x,y,z) = −18.7		z − MEAN(x,y,z) = −74.7	
Decomposition of bias						
Vote distribution	48.6		9.0		−66.4	
From	Con	11.8	Lab	−15.1	Lab	−43.9
	LD	36.7	LD	24.1	Con	−22.6
Electorate size	10.7		−11.2		−4.2	
From	Con	7.1	Lab	−9.5	Lab	−6.5
	LD	3.8	LD	−1.7	Con	2.43
Abstention	16.2		−14.3		−14.3	
From	Con	10.0	Lab	−13.0	Lab	−13.0
	LD	6.2	LD	−1.3	Con	−1.3
Third party votes	2.3		−2.9		−0.6	
From	Con	1.6	Lab	−2.4	Lab	−1.3
	LD	0.7	LD	−0.6	Con	0.7

In terms of the construction of some norm for comparison we suggested two procedures. Each was then used to re-examine the components of bias at the 2005 general election. The results showed only small differences between them with each clearly indicating a pro-Labour bias largely derived from a vote distribution effect followed by abstention and size effects. Compared with the two-party bias method the three-party approaches find a strong bias towards Labour as expected. Although the absolute size of that bias differs (50 seats in the case of two-party and nearer 90 for three-party bias) that is of no real concern because we are taking into account the disadvantage suffered by the Liberal Democrats. Moreover, there do not appear to be any substantial differences between the two procedures for constructing notional elections. When we consider the different components of the bias again the findings appear intuitively correct and potentially offer a big advantage over the two-party method since we can now identify the partitions (the direction of bias in respect of two other parties) within each bias component. As expected, the Liberal Democrats suffer, as third parties frequently do, from an inefficient distribution of votes. The calculations show the Conservatives are advantaged by an effective vote distribution but this advantage is still smaller than that enjoyed by Labour in its vote distribution.. Regarding the effects from abstention it is apparent that Labour benefits while both Conservative and Liberal Democrats are net losers.

However, and it is a rather important however, when we applied the three-party procedures to the previous five general elections (1983, 1987, 1992, 1997 and 2001) the congruence between them was reduced. The initial findings that give most cause for concern come from an examination of the 1983 general election. Both three-party methods give a different picture of the bias and moreover give quite different findings to those obtained from Brookes' two-party method. Whilst the two-party bias method shows a pro-Conservative bias (+5) the three-party methods both show a negative bias for the Conservatives (−12 for the first procedure; −53 for the second procedure). We are still assessing which of the particular features of the 1983 compared with the 2005 election are responsible for contributing to such differences in the operation of our procedures. It is worth noting that the 1983 election is the one that reveals the largest discrepancy between Brookes' two-party (see, for example, Rossiter et al., 1999) and integrated methods (Blau, 2004). While the former estimates a pro-Conservative bias the latter finds a pro-Labour bias of some 46 seats. It is possible that the 1983 election is simply unusual, but it may be that the problem is more fundamental and we have to return to what is a rather complex question: what are the most important features of the actual election that should inform the process of constructing any notional elections that are necessary for the decomposition of electoral bias?

This paper reports on research in progress and the findings demonstrate that more work is needed to develop a robust method for measuring three-party bias. However, our initial research has led us to believe that Brookes' method does permit extension to the three-party case, something that is important given the current context of UK elections. It is also clear to us that we have to give serious thought to what features of the actual election we retain when constructing alternative equal condition notional elections. The initial findings from the 2005 general election persuaded us that the impact of making different kinds of choices about those features was not significant. The findings from applying those procedures to earlier elections perhaps suggest otherwise. Currently, we are still trying to determine which of the many features of the actual election should be retained when constructing the norm for comparison when estimating bias for the three-party case.

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