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partisanship has very little importance for British voters. Perhaps answers to the party identification questions are seriously influenced by short-term forces so that, at least in Britain, asking about partisanship is little different from asking about vote. Or perhaps, as Dreyer suggests for the United States, most people have extremely stable party preferences, with apparent short-run changes being random answers to a question some respondents found meaningless.[15] In this case, unless we can sort the random responders from the true changers, our ability to explain party conversions, either from recall or from panel data, will be questionable.

If these data raise questions about the validity even of current reports of partisanship, they also fit into a growing pattern of results which suggest that all sorts of political recall questions suffer from a systematic bias toward the respondent's current preference.[16] We have shown here that respondents tend to recall that both they and their parents always preferred the party that they (the respondents) now prefer. In a recent note, Himmelweit *et al.* have shown that recall of vote tends to be distorted to conform to current vote.[17] Similarly, it is well known that a noticeable proportion of survey respondents misreport votes cast only a few days before so as to claim to have voted with the majority. The effect has been to exaggerate the stability of the vote, the stability of partisanship, and the cross-generational transmission of party preference, as well as the strength of relationships involving these variables. Taken altogether, these results suggest that the burden placed on the memories of survey respondents may be excessive. Although the temptation to ask respondents to remember things we would like to know is great, if politics is only of marginal concern to most people most of the time – and most of our theories suggest that it is – then asking for prior vote, and even more for prior or parental partisanship, may simply be inviting many respondents to make up answers about matters they do not recall.

[15] Edward C. Dreyer, 'Change and Stability in Party Identifications', *Journal of Politics*, XXXV (1973), 713–22.

[16] Reports of the contemporary political attitudes of parents and others are also biased towards respondents' own preferences. See Richard G. Niemi, *How Family Members Perceive Each Other* (New Haven, Conn.: Yale University Press, 1974), Chaps. 3, 4, 7.

[17] Hilde T. Himmelweit, Marianne Jaeger Biberian and Janet Stockdale, 'Memory for Past Vote: Implications of a Study of Bias in Recall', *British Journal of Political Science*, VIII (1978), 365–75.

The Decomposition of Electoral Bias in a Plurality Election

G. GUDGIN AND P. J. TAYLOR*

This paper describes an empirical procedure for analysing the difference between the proportion of votes a major party attracts and the proportion of seats it subsequently wins in a plurality election. This difference will be referred to as the *electoral bias* with respect to that particular party at the election being analysed:

$$B = S - V, \tag{1}$$

where S is the seat proportion and V the vote proportion gained by the party. This bias is the basic concept underlying much of the debate about electoral reform but curiously it has been an under-researched topic in political science literature. Apart from a small

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continuing debate over the cubic law of seat proportions[1] and some analyses of Australian and New Zealand elections[2] most attention in election studies has been concerned with determining the proportion of votes rather than the subsequent proportion of seats.

It is generally agreed that the basic reason why vote proportions and seat proportions tend to differ significantly in plurality elections is because of the territorial basis of this form of electoral law. Hence it is not just the quantity of votes a party attracts that counts but also *where* those votes are located.[3] This spatial basis to translating votes into seats has attracted some research from geographers in recent years[4] and this current offering is intended to provide improved procedures to those we have previously developed.[5] The core of this Note is an algebraic argument which starts with party vote proportion and concludes with party seat proportion. The steps between these two ends of the argument are presented in two parts – deriving the effective vote proportion from the actual vote proportion and then deriving the proportion of seats from the effective vote proportion. The final product is a single argument decomposing electoral bias into seven components. By definition the sum of these components and the vote proportion equals the seat proportion. This equation is the main contribution of this paper. The procedures are illustrated by application to the 1979 British general election (excluding Northern Ireland).

FROM ACTUAL VOTE PROPORTION TO EFFECTIVE VOTE PROPORTION

In a country composed of n constituencies let there be two major parties and a collection of minor parties who win few or zero seats. We concentrate on the electoral bias experienced by one of the major parties. Now we define the following:

e_i is the *total* electorate in each constituency i; t_i is the *proportion* of e_i that voted – the turnout; p_i is the *proportion* of t_i that voted for the two major parties – the two-party vote; and v_i is the *proportion* of p_i that voted for the party being studied. In any

[1] M. G. Kendall and A. Stuart, 'The Law of Cubic Proportions in Elections Results', *British Journal of Sociology*, I (1950), 183–97; D. E. Butler, *The British General Election of 1951* (London: Macmillan, 1952); E. R. Tufte, 'The Relationship Between Seats and Votes in Two Party Systems', *American Political Science Review*, LXVII (1973), 540–54; and R. Taagepera, 'Seats and Votes: A Generalisation of the Cube Law of Elections', *Social Science Research*, II (1973), 257–75.

[2] J. Rydon, 'The Relationship of Votes to Seats in Elections for the Australian House of Representatives, 1949–54', *Political Science*, IX (1957), 49–61; C. S. Soper and J. Rydon, 'Under-representation and Election Prediction', *Australian Journal of Politics and History*, IV (1958), 94–106; R. H. Brookes, 'Electoral Distortion in New Zealand', *Australian Journal of Politics and History*, V (1959), 218–33; R. H. Brookes, 'The Analysis of Distorted Representation in Two Party, Single Member Elections', *Political Science*, XII (1960), 158–67; and R. J. Johnston, 'Spatial Structure, Plurality Systems and Electoral Bias', *Canadian Geographer*, XX (1976), 310–28. These methods are applied to Britain in R. J. Johnston, 'Parliamentary Seat Redistribution: More Opinions on the Theme', *Area*, VIII (1976), 30–4.

[3] P. J. Taylor, 'Some Implications of the Spatial Organisation of Elections', *Transactions, Institute of British Geographers*, LX (1973), 121–36.

[4] P. J. Taylor and R. J. Johnston, *Geography of Elections* (Harmondsworth, Midx.: Penguin, 1979).

[5] P. J. Taylor and G. Gudgin, 'The Myth of Non-Partisan Cartography: A Study of Electoral Biases in the English Boundary Commission's Redistribution for 1955–1970', *Urban Studies*, XIII (1976), 13–25; G. Gudgin and P. J. Taylor, *Seats, Votes and the Spatial Organisation of Elections* (London: Pion, 1979).

constituency the votes polled for the party is given by $e_i t_i p_i v_i$ so that their national *total* is

$$\sum_i^n e_i t_i p_i v_i$$

and their national *proportion* of the two-party vote is

$$V = \sum_i^n e_i t_i p_i v_i \Big/ \sum_i^n e_i t_i p_i. \tag{2}$$

This *actual* proportion of the two-party vote can be compared to a hypothetical proportion where all constituencies have the same size electorates, the same turnout proportion and the same two-party vote proportion (i.e. e, t and p are constants)

$$V^* = \sum_i^n etpv_i \Big/ \sum_i^n etp, \tag{3}$$

$$V^* = \sum_i^n v_i/n. \tag{4}$$

This hypothetical proportion is simply the constituency average two-party vote proportion. In plurality elections this is the *effective* vote proportion since votes are counted and seats are allocated only at this scale. The difference between the actual vote proportion and the effective vote proportion indicates the effect of constituency size, turnout and minor party intervention on that party's vote. Clearly a party has an advantage if its vote is located in small constituencies with low turnout and low two-party voting since it is in such places that seats can be won with relatively few votes. Because fewer votes are required to win seats we refer to this generally as a *size effect* which is given by

$$L = V^* - V. \tag{5}$$

The size effect can be divided into its component parts by assuming two of the three elements constant and allowing the third to vary. If we assume equality of turnout and equality of two-party vote among all constituencies then only variations in the electorate remain to produce the *malapportionment effect* which is given by;

$$M = V^* - \left(\sum_i^n e_i v_i \Big/ \sum_i^n e_i \right). \tag{6}$$

Similarly the *turnout effect* is given by

$$T = V^* - \left(\sum_i^n t_i v_i \Big/ \sum_i^n t_i \right) \tag{7}$$

and the *minor party effect* by

$$P = V^* - \left(\sum_i^n p_i v_i \Big/ \sum_i^n p_i \right). \tag{8}$$

This argument could be continued to find specific interactions between pairs of these effects by holding just one element constant at a time but we prefer here to define a general interaction effect as the residual:

$$I = L - M + T + P. \tag{9}$$

Hence we have decomposed the size effect into four component parts

$$L = M + T + P + I \tag{10}$$

and the actual vote converts into the effective vote in an election as follows,

$$V + M + T + P + I = V^*. \tag{11}$$

In the 1979 British general election the Conservative Party attracted 0·5423 proportion of the two-party vote. However, its constituency mean proportion was only 0·5350. Hence this party suffered a size-effect penalty of −0·0073, i.e. its effective vote proportion at constituency level was below its actual vote proportion. This value can itself be divided up into its components. The major reason for the Conservative disadvantage was the malapportionment effect of −0·0113 although this was nearly compensated for by a minor party effect of +0·0080, with the turnout effect and interaction term lower at +0·0021 and −0·0061 respectively. These can be summarized as measures of the degree to which the Conservatives were disadvantaged by attracting votes disproportionally in larger constituencies but were partially compensated by tending to obtain votes where minor parties were relatively strong.

FROM EFFECTIVE VOTE PROPORTION TO PROPORTION OF SEATS

In plurality elections the effective vote proportion does not usually equal the seat proportion won by a party. Usually the major party which wins most votes obtains a disproportionate quantity of seats. We will refer to such 'winner's bias' as the *distribution effect* defined by

$$D = S - V^*, \tag{12}$$

where S is the proportion of seats won by the party so that if $V^* > 0{\cdot}5$ D will usually be positive. It is the purpose of this second section to decompose D in a manner similar to that achieved above for L.

The approach we adopt follows Kendall and Stuart's discussion of the cube law and the normal distribution underlying it.[6] They show that the cube law results from a normal distribution of constituency vote proportions with a standard deviation of 0·137. We have previously developed these ideas by showing that a normal distribution with approximately the required standard deviation will result from the arbitrary placing of constituency boundaries upon a class-based voting map.[7] Hence the cube law has been derived as a consequence of class-based voting and class-based residential segregation. Of course no election will exactly conform to the cube law and so we use the deviation from such prediction to derive further components of electoral bias in this section. We start by assuming the operation of the cube law and then relax its assumptions to derive separate components of the distribution effect.

In a 'pure' cube-law election the proportion of seats is given by

$$S_c = \Phi(V^*, 0{\cdot}137), \tag{13}$$

where Φ is a cumulative normal distribution with mean V^* and standard deviation 0·137.[8] We now define a *cube-law winner's bias* as

$$V = S_c - V^*. \tag{14}$$

Some deviations from the cube-law prediction will occur where the actual standard deviation is not 0·137. This is an additional effect due to a different level of segregation than that specified precisely by the cube law. Using the empirically-derived standard deviation, σ, and maintaining our normality assumption, we can predict the proportion of seats won from

$$S_N = \Phi(V^*, \sigma). \tag{15}$$

[6] Kendall and Stuart, 'The Law of Cubic Proportions in Election Results'.
[7] Gudgin and Taylor, *Seats, Votes and the Spatial Organization of Elections*.
[8] Gudgin and Taylor, *Seats, Votes and the Spatial Organization of Elections*, Table 2.1, p. 23.

The *additional normal segregation effect* is then given by

$$N = S_N - S_C. \tag{16}$$

The *residual distribution effect*

$$R = S_N - S \tag{17}$$

is due to the non-normality of the actual distribution of constituency vote proportions. This may be either due to skewness (lack of symmetry) or kurtosis (lack of normal bell-shape, i.e. too 'peaked' or 'flat' to be normal) in the distribution.[9]

We can now decompose the distribution effect as

$$D = C + N + R, \tag{18}$$

i.e.
$$(S - V^*) = (S_C - V^*) + (S_N - S_C) + (S_N - S) \tag{18a}$$

and the effective vote proportion converts into the seat proportion as follows,

$$V^* + C + N + R = S. \tag{19}$$

In the 1979 British general election the Conservative Party's effective vote was 0·5350, as we have seen. With this vote they won 0·5585 proportion of the seats giving a distribution effect of +0·0235. Under cube-law conditions, however, they would have

TABLE I *Statistical Parameters for Conservative Vote Proportions for Selected Elections*

Election*	$\bar{x}$	σ	β_1	β_2
1929	·512	·180	·01	−·69
1955	·516	·149	−·29	−·12
1966	·458	·146	−·05	−·14
1970	·502	·146	−·30	−·12
1974 (Feb.)	·505	·165	·03	−·68
1974 (Oct.)	·472	·173	−·06	−·86
1979	·535	·171	−·01	−·81
Cube law	—	·137	·00	·00

* Source for elections 1929–1974 (October) is Table 4.3 (p. 77) in Gudgin and Taylor, *Seats, Votes and the Spatial Organization of Elections*.

$\bar{x}$ is the mean constituency vote proportion;

σ is the standard deviation of those proportions;

β_1 is the measure of skewness of the distribution;

β_2 is a measure of its kurtosis – negative values indicate a 'flatter' distribution than normal.

won 0·6000 proportion of the vote giving a cube-law winner's bias of +0·0650. Unfortunately for the Conservative Party the 1979 election did not closely conform to cube-law assumptions. In Table I the parameters of the distribution of 1979 Conservative constituency vote proportions are given alongside a selection of distributions from previous elections. Notice that it is the 'middle' set of elections (1955, 1966 and 1970)

[9] For further discussion see G. Gudgin and P. J. Taylor, 'Electoral Bias and the Distribution of Party Voters', *Transactions, Institute of British Geographers*, LXIII (1974), 53–73 and Gudgin and Taylor, *Seats, Votes and the Spatial Organization of Elections*, Chap. 4.

with their two-party dominance that most closely conform to cube-law assumptions. The 1974 elections seem to mark a return to a distribution similar to 1929 with a relatively high standard deviation and negative kurtosis. Such a distribution indicates fewer marginal constituencies in the centre of the distribution and hence produces a winner's bias smaller than that predicted by the cube law. This situation was maintained in 1979 despite the renewed dominance of the two main parties. Using the actual standard deviation of 0·171 we find $N = 0{\cdot}0191$ leaving a residual non-normality effect of $R = -0{\cdot}0224$. Hence the cube-law winner's bias of 0·0650 has been reduced to the actual distribution effect of 0·0235 by the additional normal segregation of −0·0191 plus a non-normality effect of −0·0224.

FROM ACTUAL VOTE PROPORTION TO SEAT PROPORTION

The argument from the two previous sections can now be combined. Electoral bias has been decomposed into seven component parts as follows:

$$B = L + D \tag{20}$$

so that
$$B = M + T + P + I + C + N + R. \tag{20a}$$

For the Conservative Party in 1979

$$0{\cdot}0162 = -0{\cdot}0113 + 0{\cdot}0021 + 0{\cdot}0080 - 0{\cdot}0061 + 0{\cdot}0650 - 0{\cdot}0191 - 0{\cdot}0224.$$

Alternatively we can present the analysis as a derivation of seat proportion from actual vote proportion as follows

$$S = V + B, \tag{21}$$

$$S = V + M + T + P + I + C + N + R \tag{21a}$$

which with respect to the Conservative Party in 1979 can be written

Actual vote proportion − Malapportionment effect + Turnout effect +
0·5423 0·0113 0·0021
Minor party effect − Interaction term = Effective vote proportion +
0·0080 0·0061 0·5350
Cube-law winner's bias − Additional normal segregation − Non-normality effect
0·0650 0·0191 0·0224
= Seat proportion.
0·5585

The results are summarized in Table 2.

CONCLUSION

The purpose of this paper has been to introduce simple measurement procedures for decomposing electoral bias. The next step is obviously to attempt to account for the individual measures generated by the analysis. In the above treatment of the 1979 British general election many widely observed features can be seen to underlie the measures we have presented. The size effect clearly reflects disproportionate Conservative successes in county constituencies in the South which have both large electorates and sizeable Liberal vote proportions. Similarly the relatively small distribution effect reflects increased polarization of the party voters among constituencies. The non-normality is much more difficult to account for: in 1974 it seemed to be due to the rise in the Liberal

TABLE 2 *Decomposition of Electoral Bias for the Conservative Party in the 1979 Election*

Proportion of the votes	·5423
Proportion of the seats	·5585
Electoral bias	+·0162
Constituency mean vote (effective vote proportion)	·5350
Over-all size effect	−·0073
which decomposes into	
Malapportionment effect	−·0113
Turnout effect	+·0021
Minor party effect	+·0080
Interaction term	−·0061
Over-all distribution effect	+·0235
which decomposes into	
Cube-law winner's bias	+·0650
Additional normal segregation	−·0191
Non-normality effect	−·0224

vote as the distribution reverted to the 1929 pattern but the 1979 fall in the Liberal vote has not led to any return in the distribution to the situation of 1970 (Table 1). Clearly a change in voting patterns is indicated which is more stable than Liberal support.[10] Hence the procedures we have outlined calibrate known effects and point to the need for more research on little understood effects.

[10] For some alternative suggestions see P. J. Taylor, 'The Changing Geography of Representation in Britain', *Area*, XI (1980), 289–94 and R. J. Johnston, 'Regional Variations in the 1979 General Election Results for England', *Area*, XI (1980), 294–7.

A Crucial Test of Alphabetic Voting: The Elections at the University of Leiden, 1973–1978

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Politicians and political scientists have long known that there is a slight tendency among voters to prefer candidates whose names appear at the top of the ballot compared with lower-placed candidates, and hence that *ceteris paribus* the former have a somewhat better chance of being elected than the latter. When the candidates' names appear on the ballot in alphabetical order, this positional voting bias is usually called 'alphabetic voting'. It is of special importance and interest in preferential voting systems where the voters may indicate their first, second, third, etc., preferences among a list of candidates, as in the

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