

Measuring malapportionment, gerrymander, and turnout effects in multi-party plurality systems*

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Abstract

Mexico’s federal electoral authority has carried out automated redistricting since the mid 1990s. We inspect two recent machine-generated maps for evidence of partisan bias in the translation of votes to seats for the lower chamber of Congress. Based on Monte Carlo simulations of recent elections, we develop a new procedure to measure three potential sources of partisan bias in a multiparty system separately: malapportionment, gerrymandering, and turnout. Analysis reveals how, relative to the right-of-center PAN, the former hegemonic PRI, but especially the left-of-center PRD have enjoyed advantageous seats:votes ratios. We also show that systematic and often large turnout-based bias in favor of PRI has been countered by district boundaries substantively helping one or both other major parties. Despite nominally neutral district drawing, gerrymandering remains a key ingredient of partisan bias in representation.

This paper describes an empirical method for the analysis of the difference between the vote share that a party receives in the electorate and the seat share it subsequently wins in a plurality election. This difference is at the heart of debates about electoral reform and has received much scholarly attention in two-party competition from political scientists, economists, sociologists, geographers, mathematicians, and statisticians.¹ The concern has

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¹See Altman and McDonald (2011), Balinski and Young (2001), Brady and Grofman (1991), Cain (1985), Cox and Katz (2002), Dahl (1956), Engstrom (2006), Erikson (1972), Gelman and King (1994), Grofman (1983), Grofman, Koetzle and Brunell (1997), Gudgin and Taylor (1979), Johnston (2002), Kendall and Stuart (1950), King and Browning (1987), Niemi and Fett (1986), Rae (1967), Rossiter, Johnston and Pattie (1997), Taagepera (1973), Trelles and Martínez (2012), Tufte (1973).

been the derivation of votes–seats curves assessing the electoral system’s most fundamental function: the conversion of electoral support into assembly representation (Lijphart 1994).

The standard approach measures two distinct votes–seats curve characteristics, responsiveness and partisan bias (King and Browning 1987, Tufte 1973). The system’s *responsiveness* is a measure of the seat bonus granted to large parties in detriment of the small. That plurality systems grant a substantial, if variable majority seat bonus, and that bonus size drops markedly when the formula is proportional representation, is well established (Linzer 2012, Taagepera and Shugart 1989). Responsiveness is a symmetric distortion, in the sense that substituting the top vote-getting party shifts the seat bonus earner accordingly. In contrast, *partisan bias* introduces an asymmetry in the seats–votes relationship. Partisan bias grants the beneficiary undue advantage in the ability to win assembly seats. Parties favored by systematic bias buy seats with fewer votes than others, countering a central tenet of democracy.

Theory highlights three sources of partisan bias. One is *malapportionment*—differences in district populations. A party with stronger voting bases in smaller districts receives a seat bonus nationwide (Jackman 1994, Johnston 2002). Another is *gerrymandering*—the practice of drawing district boundaries to achieve partisan advantage. Packing adversaries in few districts is a classic way to force them into vote wasting (Cox and Katz 2002, Engstrom 2006). The other is *turnout differences* across districts. A party enjoying stronger support in high- than low-turnout districts pays a seat penalty (Campbell 1996, Rosenstone and Hansen 1993). Our method measures the independent contribution to partisan bias of the three sources.

The procedure builds upon work by Grofman, Koetzle and Brunell (1997). Our contribution is three-fold. Firstly, unlike Grofman, Koetzle and Brunell (and unlike every work cited in footnote 1) our approach drops the restrictive assumption of two-party competition. National two-party systems remain exceptional even among plurality systems (Cox 1997), so extending measurement to multi-party competition clears the way to empirical verification of theoretical propositions in numerous systems that were beyond reach. Secondly, we take “creeping malapportionment” (Johnston 2002), often ignored, into account. District size differentials arise by commission when cartography is adopted that deliberately underrepresents some citizens, but also by omission when failure to redistrict perpetuates secular demographic imbalance. Thirdly, we inspect the case of Mexico for application of the measuring method proposed to a multi-party case. *Brief summary of findings*.

The paper proceeds thus. Replicas of three models on which we build our argument are described in sections 1–3. Each model removes obstacles: King (1990) the estimation of partisan bias in multi-party systems; Grofman, Koetzle and Brunell (1997) the challenge

of measuring the size and polarity of three additive sources of partisan bias independently; and Linzer (2012) how to estimate quantities of interest with few observation points given that general elections are not really comparable. Our method stands at the intersection of this model trio.

Section 4 then introduces the case of Mexican Chamber of Deputies' elections to test our method. Elaborates on redistricting and turnout. Presents the data.

Section 5 is on creeping malapportionment. Empirical expectations

Section 6 reports results.

The next section discusses partisan bias, the three sources behind it (the distributive or gerrymandering-, the turnout-, and the malapportionment-based), and how operationalizing parties' national vote shares in slightly different ways clear the way to measure them empirically. The third section investigates malapportionment in Mexican elections, showing that it is substantial due to belated redistricting—when carried five or more years after the census, as has been done, population information is outdated. The fourth section derives expectations of bias from the partisan model of electoral regulation. District boundaries in Mexico are not drawn by politicians but by experts under the aegis of a collegiate electoral regulator where all three major parties share power (Estévez, Magar and Rosas 2008). This removes major suspicions of flagrant manipulation, but the question of partisan gerrymandering subsists. The fifth section describes estimation methods and the sixth section elaborates on our data. The seventh section reports our party bias results, both in terms of raw party bias and in terms of its three, additive components. An eighth section concludes.

1 Partisan bias in the multi-party context

Formalizing partisan bias and responsiveness is the first step towards measurement. The two-party case is simpler (King and Browning 1987, Taagepera 1973, Tufte 1973) and extends to multiparty competition. It is a generalization of the cube law stipulating that

$$\frac{s}{1-s} = e^{\lambda} \left(\frac{v}{1-v} \right)^{\rho} \iff \text{logit}(s) = \lambda + \rho \text{logit}(v) \quad (1)$$

where s is the seat share won by the left party with vote share v ; λ is the left party's bias relative to the right party (positive values favor the left, negative values favor the right); and ρ is responsiveness. Set $\lambda = 0$ and a system without partisan bias ensues. Figure 1 shows how parameters affect the votes-seats translation function.

Black lines illustrate variable responsiveness without partisan bias. A system with $\rho =$

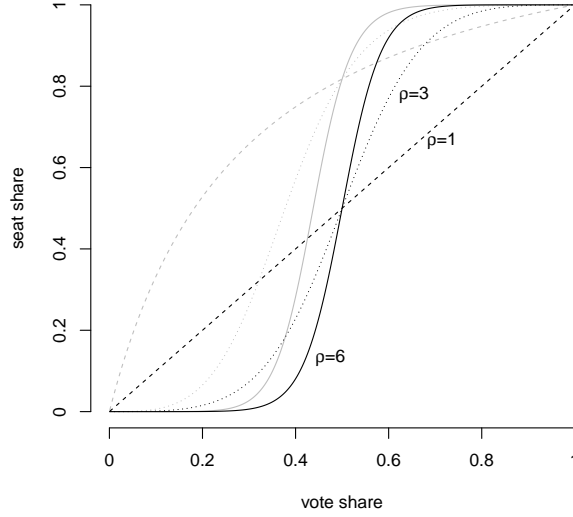


Figure 1: Illustration of model parameters. Partisan bias is set to $\lambda = 0$ in black lines. Gray lines replicate the black one-by-one with $\lambda = +1.5$.

1 is perfect PR, the ideal type against which real districts are often contrasted. It appears as the dotted diagonal line: every party winning $x\%$ of the vote gets, precisely, $x\%$ of seats. $\rho = 3$ characterizes the classic cube law, the dotted curve over-representing the winner (points above the diagonal). Here a party with 55% of the vote wins two-thirds of the seats, but with 33% it wins only one-tenth of the seats. As responsiveness heightens, the curve grows steeper, until barely crossing the majority threshold suffices to win all the seats available.

Partisan bias is asymmetric party treatment in the votes–seats conversion. Gray lines replicate the values of ρ just discussed but with $\lambda = +1.5$ added. Bias in favor of the left party achieves a leftward pull of lines. In other words, a bias-favored party will require less effort to reach the threshold for large-party over-representation, cooking artificial parliamentary majorities with substantially less than a vote majority—as routinely occurred in the United Kingdom since the 1970s. (The gray dotted convex line shows how, due to logit links in Equation 1, partisan bias also reshapes the function’s trace.)

A multiparty, estimable version of equation 1 is King (1990) (another is Calvo and Micozzi 2005). A transformation akin to a multinomial logit’s departure from the dichotomous kind is used, formulating that party p ’s ($p = 1, 2, \dots, P$) expected seat share is

$$E(s_p) = \frac{e^{\lambda_p} \times v_p^\rho}{\sum_{q=1}^P e^{\lambda_q} \times v_q^\rho} \quad (2)$$

with data and parameters now indexed for party identification. Setting $\lambda_1 = 0$ restricts the remainder $\lambda_{p \neq 1}$ to express partisan bias in relation to party $p = 1$ without loss of generality. This is convenient. Partisan bias in two-party competition is asymmetry in a votes–seats curve centered on $v = .5$, as in Figure 1. While the partisan bias shift operates likewise, there is no reason to expect a .5-centered curve in multi-party competition. Nor is it a priori evident what vote share serves as center point, a difficulty towards expressing a partisan bias estimate $\hat{\lambda}_p$ as a percentage points advantage or handicap for party p in the votes-to-seats conversion—as routinely done in analysis of two-party systems (e.g., Cox and Katz 2002). Finding $\lambda_{p \neq 1} < 0$ is evidence of bias against party $p \neq 1$ relative to party $p = 1$.

2 Three sources of partisan bias

At the root of partisan bias in systems with multiple districts are differences in the geographic concentration of parties’ vote strength. One party with 20 percent of the vote nationwide evenly spread across districts may fail to win a single seat, but another with much less support that is concentrated in few districts will, in fact, get seats. In general, vote concentration helps smaller parties while hurting larger ones through vote (and therefore seat) wasting (Calvo and Rodden 2015). In the end, several forces add up and interact to yield partisan bias (Gudgin and Taylor 1980).

Grofman, Koetzle and Brunell (1997, henceforth GKB) demonstrate that what we shall now call *raw partisan bias* (λ) has three clear and distinct sources and offer a procedure to separate empirically the independent, additive contribution of each.²

- *Gerrymanders* (GKB call this source of partisan bias ‘distributional’) correspond to different party distributions of vote wasting across districts. Vote wasting may be deliberate (e.g., the strategy of packing opponent in few districts mentioned above), but it may also arise by the accidents of geography (e.g., when districts cannot cross state boundaries and the state is a party stronghold).
- *Turnout* differentials across districts. Those who abstain from voting are passively lowering the bar to win a district’s seat. Parties stronger in the lower turnout districts will achieve victories with fewer votes than others, improving their votes:seats ratio. Turnout differentials arise when correlates of participation, such as socio-economic

²Other elements highlighted by Gudgin and Taylor (1980) that our analysis of raw partisan bias ignores are the cube-law’s bonus, large third-party votes, and possible interactions between all the elements. The bonus is, in fact, captured by the system’s responsiveness parameter and therefore distinct from partisan bias in our framework (more on this in section 3). Calvo (2009) models departures from bipartism explicitly. Interactions remain interesting venues for future research.

Districts	Pop.	Turnout	Raw votes			Vote shares		Seat shares	
			left	right	total	left	right	left	right
Gerrymander-based partisan bias only									
1 and 2	420	.5	147	63	210	.7	.3	1	0
3, 4 and 5	420	.5	84	126	210	.4	.6	0	1
nationwide	2100	.5	546	504	1050	.52	.48	.4	.6
Turnout-based partisan bias only									
1 and 2	420	.70	200	100	300	.67	.33	1	0
3, 4 and 5	420	.35	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6
Malapportionment-based partisan bias only									
1 and 2	600	.5	200	100	300	.67	.33	1	0
3, 4 and 5	300	.5	50	100	150	.33	.67	0	1
nationwide	2100	.5	550	500	1050	.52	.48	.4	.6

Table 1: Illustrative five-district system scenarios

status, vary systematically across districts, or when parties mobilize more effectively in some areas than in others.

- *Malapportionment* arises when sparsely populated regions get the same representation as more densely populated ones. It can be found whenever multiple districts are drawn for the purpose of seat allocation. District size differentials may be designed, adopting cartography that deliberately underrepresents some citizens. Senates often grant states equal representation, regardless of population. But it inevitably germinates passively by failure to redraw district boundaries that compensate for secular demographic imbalance. Creeping malapportionment ensues.

The scenarios in Table 1, which draw heavily from examples in GKB, illustrate the sources operating in isolation from one another. The division of vote and seat shares nationwide and the degree of partisan bias remain constant in all scenarios: the left party suffers a 12 percentage point deficit in representation, with 52% of votes but just 40% of seats (it won two of five districts); and the right party enjoys 12 percent overrepresentation, winning 60% of seats with just 48% of votes. Other traits change, one at a time. The first scenario has equal-sized and constant-turnout districts³ that nonetheless manifest partisan differences in votes wasted, the left party winning seats by wider margins (+.4) than the right (+.2). The sole source of partisan bias is gerrymanders. Shifting district boundaries might re-allocate wasted votes in such way that another district tips towards the left.

³A less restrictive scenario allows size and turnout differences across districts with distributions that are independent of the distribution of partisan support.

The next scenario has equal-sized districts and winning margins uncorrelated with the vote distribution, but varying turnout that is not orthogonal to vote shares. Right and left are winning seats with the exact same margins, but the right wins in lower-turnout districts—half, in fact, the turnout of districts won by the left. As a consequence, right seats come quite cheaper than the left’s. In this case, partisan bias is the product of turnout differentials alone, against the left. Extend the reach of the left’s mobilization effort beyond districts 1 and 2, and other seats might be won.

The other scenario has equal-turnout districts and winning margins uncorrelated with party vote strength, but different district sizes that do correlate with the latter. Again, both parties are winning with equal margins, but the right is doing it in districts half as populous as those won by the left. The consequence is a more efficient conversion of quite similar total votes into seats for the right than the left. This is partisan bias attributable to malapportionment by itself.

The formalization of the votes-seats curve in section 1 assumed that votes in Equations 1 and 2 are the party’s share of the national vote v_p —the party’s vote aggregated across districts divided by the total raw vote nationwide. This standard mode of national aggregation of district-level vote returns measures raw partisan bias. Noting that party p ’s raw vote in district d is the product of its district vote share v_{dp} and the district’s raw vote, the party’s vote share nationwide can be expressed as

$$v_p = \sum_d v_{dp} \times \frac{\text{raw vote}_d}{\text{total raw vote}}. \quad (3)$$

This algebraic transformation eases consideration of two alternative national aggregations of district returns in GKB’s separation argument. One is party p ’s mean district vote share, defined as

$$\bar{v}_p = \sum_d v_{dp} \times \frac{1}{\text{total districts}}. \quad (4)$$

The other is party p ’s population-weighted mean district vote share, defined as

$$\bar{w}_p = \sum_d v_{dp} \times \frac{\text{population}_d}{\text{total population}}. \quad (5)$$

Following the insight of Tufte’s (1973) seminal work (further elaborated in Gelman and King 1994), fitting the votes–seats curve using \bar{v}_p instead of v_p yields gerrymander-based partisan bias. This is so because \bar{v}_p aggregates district vote shares with disregard to district size and voter turnout. In the same spirit, GKB show that relying on \bar{w}_p (an aggregate compounding district vote shares and relative district populations) yields estimates conflating

gerrymander- and malapportionment-based partisan bias. So subtracting partisan bias estimated with \bar{v}_p from partisan bias estimated with \bar{w}_p yields pure malapportionment-based partisan bias. And, because raw partisan bias conflates all three sources, subtracting partisan bias estimated with \bar{w}_p from partisan bias estimated with v_p yields pure turnout-based partisan bias.⁴

In sum, the GKB procedure consists of repeatedly fitting equation 2, alternatively using v_p , then \bar{v}_p , and \bar{w}_p . Denoting λ_p^v , $\lambda_p^{\bar{v}}$, and $\lambda_p^{\bar{w}}$ party p 's partisan bias parameter from each fitting, the following subtractions bring forth the quantities of interest:

- a. raw partisan bias = λ_p^v ,
- b. gerrymander-based partisan bias = $\lambda_p^{\bar{v}}$,
- c. malapportionment-based partisan bias = $\lambda_p^{\bar{w}} - \lambda_p^{\bar{v}}$, and
- d. turnout-based partisan bias = $\lambda_p^v - \lambda_p^{\bar{w}}$.

It is easy to verify that raw partisan bias is the sum of the three components in the GKB framework ($a = b + c + d$).

3 Measurement via Monte Carlo simulation

The final obstacle is fitting the votes–seats curve to data of interest. The general problem is one of scarcity of data points to achieve this. Each party fielding candidates in a general election offers one point in a votes–seats coordinate system, and relatively few parties do in each election. One common approach to overcome this limitation is pooling several elections in the historical record for estimation (e.g., Márquez 2014a). Multi-election studies inevitably sacrifice comparability, especially over the longer haul (Jackman 1994). Single-election studies are therefore preferable (Niemi and Fett 1986), with alternative procedures to multiply data points desirable. Dis-aggregation in a federal case is one strategy: reliance on state-level congressional election votes and seats, instead of standard national-level aggregates. But by treating state party systems as the national, this has pitfalls similar to longitudinal multiplication.

The multiplication approach adopted here, inspired by Linzer (2012) and explored by Márquez (2014b), relies on Monte Carlo simulation instead.⁵ Towards this goal, a prob-

⁴The notation (subscripts dropped) that GKB use for v , \bar{v} , and \bar{w} is R , P , and M , respectively.

⁵We did not pursue Linzer's swing ratios. The relation of that quantity with the notion of partisan bias adopted here is straightforward in balanced two-party competition (see Linzer 2012:410), but not when multiple parties compete. We therefore followed his method part of the way, borrowing his code to generate hypothetical national party vote and seat pairs, then fitting a standard votes–seats curve on those.

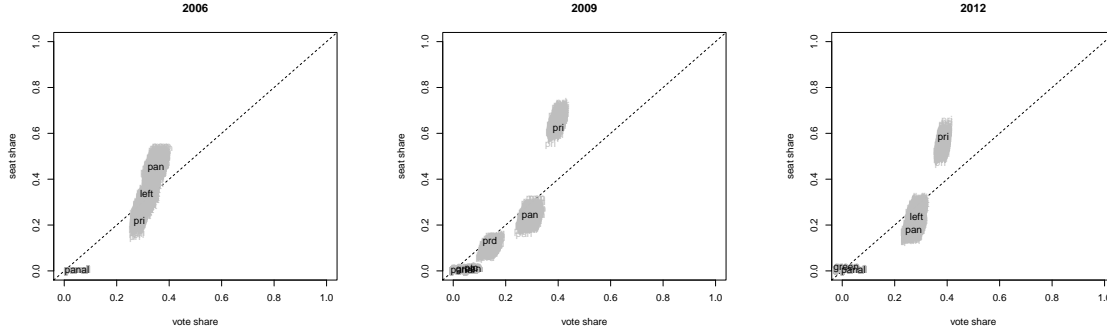


Figure 2: Votes and plurality seats in three elections. Actual data in black, simulated elections in gray. Source: prepared with data from www.ine.org.mx.

ability density of national party vote returns is approximated from observed district outcomes with a finite mixture model. The FMM works with district-level data, assuming sub-populations with known distributions are present (e.g., some districts where party 1 vote grows at the expense of party 2's, others where they grow jointly) but information to match districts to sub-populations is unavailable. A mix of the known distributions describes the unknown distribution. Repeated draws of hypothetical district outcomes from the mix reflect variation in the sources of partisan bias: in district size, in turnout, and in vote choice (information fed to the FMM); and aggregating them nationwide yields a large sample of vote–seat simulations that are supported by the data.⁶

Figure 2 describes the output of the simulation process for the case study that section 4 presents in detail. Observed national votes received and seats won appear as black labels for three consecutive elections. Simulated elections are surrounding clouds of gray labels. The mixture offers reliable counterfactual predictions about party seats shares for vote shares near observed points only (about ± 10 percent Linzer 2012:fn. 8). The single-election approach is not suited for extreme counterfactual prediction (Gelman and King 1994). We show below that the approach is nonetheless very attractive.

Another problem is parties fielding candidates in less than all legislative districts. Strategic parties often withdraw in anticipation of a hopeless race. The mix can handle this challenge by considering patterns of district contestation separately. More problematic are partial coalitions between parties—when parties A and B field joint candidates in some districts but run against each other in the rest, complicating national votes and seats aggregation. We now turn attention to a description the case study. We return to elaborate on the solution adopted for partial coalitions after this. A better grip of the minutiae of Mexican

⁶An online appendix describing the multiplication procedure in detail, and with commented code extending Linzer's procedure will be posted upon publication.

congressional elections eases exposition of the problem at hand.

4 Mexican Cámara de Diputados elections

We demonstrate the procedure’s usefulness by applying it to recent elections of Mexico’s lower chamber of Congress. The Cámara de Diputados has been elected with a mixed-member electoral system for decades. Systems of this nature give voters a direct role in the election of representatives from single-member districts (SMDs), while also using some form proportional representation (PR) to improve the odds that parties will receive about as many seats as they are worth in votes (Shugart and Wattenberg 2001a). Underneath the PR-tier, however, lies a standard plurality system. We study it in isolation of the PR-tier. This is justified because campaigns and voting takes place at the plurality tier.⁷

As in other plurality systems, we suspect strong distortions in the form of partisan bias to be arising. Substantial malapportionment is one cause for concern. The procedure offers a way to contrast malapportionment to gerrymanders and turnout effects, answering theoretically interesting questions.

Márquez (2014a) used a multi-election approach of votes and seats won over two decades, uncovering a degree of responsivity characteristic of plurality systems and substantive partisan bias against the PAN.

Only minor parties failed to contest all districts in the case study, and many were in fact removed from the dataset to avoid complicating the field with irrelevant players.⁸ More problematic are partial coalitions between parties. Twice in the period portrayed, two parties (PRI and Green) fielded joint candidates in some districts, while running against each other in the rest, complicating national votes and seats aggregation. The option of computing separate aggregates for PRI, for Greens, and for PRI+Greens, while attractive for faithfully describing the situation, was abandoned due to result sensitivity. The PRI wrongly appears as having fielded no candidate in in Party A is substantially larger than the partner, and ; and (taking advantage that party B is substantially smaller than party A), adding A&B totals to party A, producing aggregates , national aggregation was achieved by considering

In the case study, only minor parties failed to contest all districts, and some were in fact removed from the dataset to avoid complicating the field with irrelevant players.⁹

⁷Each voter has a unique vote to choose among candidates in 300 SMDs with seats allocated by plurality. Votes then transfer to candidates’ parties in order to allocate seats in five second-tier districts of magnitude 40, by closed-list Hare proportional representation, using a 2 percent threshold.

⁸Linzer (2012:fn. 4) recommends likewise. Vote shares of the major parties were rescaled to sum 100 percent in selected districts.

⁹Minor parties receiving only a marginal vote share in some districts were actually removed from the

Consider the relation between congressional party votes (x-axis) and seats (y-axis) portrayed in Figure ?? . Panel (a) relates shares at the state level (i.e., votes/seats won by the party in each of the state’s federal districts, summed and divided by the total votes/seats in the state) and panel (b) at the national level (i.e., 300-district aggregates). Each point reports one party’s vote-seat relation at the unit of aggregation in one election between 2006 and 2012. For instance, the dot floating to the left of the cloud and above the dashed diagonal line in panel (a) is the Green party in the state of Chiapas in 2012, where it won 3 out of 12 seats. The chart shows that 9 percent of the vote statewide awarded the party 25 percent of the seats, an outstanding achievement for any party. Both clouds manifest a steep upwards slope characteristic of plurality systems (Taagepera 1973).¹⁰ Points below the diagonal indicate under-representation, those above over-representation. Differences are notable among major parties: the PRI achieved over-representation in three-fifths of election-states in the period, the PAN in two-fifths, and the PRD in one-fourth only. Might the system confer undue advantage to the PRI?

5 One Mexican, one vote?

The second condition [of majority rule] is that each individual be treated the same as far as his influence on the outcome is concerned
—May’s Theorem (1952)

5.1 Malapportionment in a mixed-member system

The lower chamber elects 300 members in SMDs and 200 members by PR (Weldon 2001). It could be argued that malapportionment in mixed-member systems is of little, if any consequence. The PR tier is, after all, specifically designed to compensate for imbalances ensuing from SMDs. We debunk this claim.

Compensating *parties* bears relation to, but is not the same as compensating *citizens* of more densely populated districts. Keeping these compensations distinct is important. Much of the evidence presented here, as in the scholarly literature, deals with party vote:seat ratios. From the normative standpoint, however, it is the ‘one person, one vote’ principle—one of Dahl’s (1972) preconditions of democratic government—that malapportionment an-

dataset to avoid complicating the field with irrelevant parties—as [[:fn. 4]Linzer (2012) recommends. Vote shares of the major parties were rescaled to sum 100 percent in districts where this was done.

¹⁰Adding the excluded PR seats would level the slope considerably. Doing this would be easy with national aggregates. It is not evident how to carry it with state aggregates, since PR seats are awarded in five second-tier districts grouping several states each.

tagonizes, and party compensation is not designed to redress this imbalance.¹¹

Malapportionment also has implications for reelection and incumbency. Mexico recently removed single-term limits for legislators. Ambitious deputies elected in 2018 will, again, be allowed to seek reelection, restoring the electoral connection that was severed in the 1930s (Dworak 2003). The reform should reinvigorate the relation between citizens and their representatives in the SMD tier. The persistence of substantial variations in district size, however, acts against realizing the full potential of the electoral connection (Mayhew 1974)—widespread dissatisfaction and even protests in past years underscore how bad this is needed for the consolidation of Mexico’s young democracy.

Malapportionment has the potential to substantially distort representation even in mixed-member systems. How much does this matters in practice? We examine this through an analysis of new data from Mexican congressional elections.

5.2 Apportionment to the states

Redistricting in Mexico occurs in two steps. The three-hundred national legislative seats are distributed between the states, then district boundary maps are drawn for each of the thirty-two states. Distortions can be introduced in each step (see Snyder and Samuels 2004). We highlight structural insinuation of malapportionment when the census counts that are used for redistricting are allowed to become out of date.

In contrast with the U.S., there has been negligible public debate about methods of apportionment (Balinski and Ramírez González 1996, Szpiro 2010). The Hamilton method of largest remainders is used (Balinski and Young 2001:10). The quota Q (or price of a seat) is the nation’s population divided by 300, the number of seats. A first allocation is made by dividing each state’s population in the last general census by Q , rounded down. States with populations $< 2Q$ nonetheless get two seats (with Colima and Baja California Sur in this group in earlier apportionment rounds, but no more). Unallocated seats, if any, are awarded to states with largest fractional remainders.

Figure 3 summarizes states’ congressional representation. In order to take creeping malapportionment into account, we projected each state’s population at the time of a federal election between 1997 and 2015.¹² With moving population estimates for election years,

¹¹The exception would be a system with perfectly district-based parties, where measures to achieve party proportionality are, in fact, compensations for the district citizenry only. Shift away from fully local parties, towards party nationalization, and party compensation stops accruing to citizens of the underrepresented districts only.

¹²Projections up to 2010, inclusive, was by linear interpolation of census populations, and by linear extrapolation for 2012 and 2015. The years 1995 and 2005 had population counts, 2000 and 2010 had general censuses. The former collect much fewer respondent attributes than the latter, but report a population figure

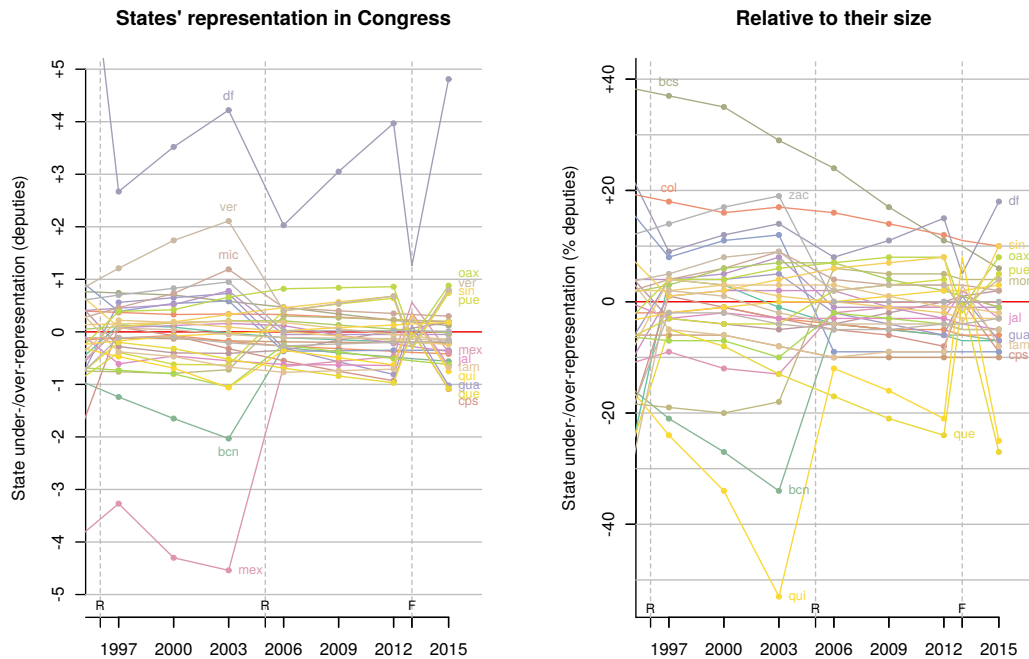


Figure 3: Demography and state apportionment in Congress (lower chamber). Lines connect the seats that each state had relative to those it should have had on a population basis. Population projections from censuses used for this computation. Letters R in the horizontal axis indicate redistricting, letter F a redistricting failure (the effect that the proposed map would have had is reported on the dotted line). Source: prepared with data from www.inegi.org.mx and www.ine.mx.

a time-series of states' fair share of congressional seats was then computed (i.e., time-varying Q_s). Subtracting the actual number of seats apportioned to the state yields the state's over- or under-representation in election years. A state with perfect apportionment appears at zero in Figure 3. The slope in the line indicates deviations, downward for states losing relative weight in the lower chamber, positive for winners. The left panel reports this measure in absolute terms (so that +3 indicates a state with three deputies more than its fair share), the right panel relative to seats apportioned to the state (+20 indicates twenty percent more deputies than its fair share). Vertical dashed lines mark redistricting events (the "F" towards 2015 indicates a failed proposal to redistrict).

Creeping malapportionment is manifest, points drifting between redistricting events. Since the first term of the subtraction is a real number and the second an integer, a fraction is bound to remain. Yet, upon redistricting, fractions in a well-apportioned systems should all be less than 1 quota in absolute value. The left panel shows that this is mostly the case, but exceptions are, and have always been present. The 2006 redistricting corrected the most egregious deviations from the ideal. The states of Baja North and of Mexico, with deficits of 2 and 4.5 deputies relative to population in 2003, respectively (deficits of 30 and less than 10 percent of the deputies they actually held), achieved near-parity in 2006. Over-represented Veracruz behaved near symmetrically to Baja in absolute terms (not relative). And a distortion strongly favoring the Federal District (+4 deputies) was somewhat attenuated in 2006, but never removed.

The notorious case of the DF reveals that relying on obsolete population data is problematic. The constitution mandates the use of the census for redistricting, but has no obligation to redistrict as soon as it becomes available. As a consequence, drawing a new map for 2006 with data from the year 2000 injected a fair amount of creeping malapportionment into the new plan right at its inception. Compared to the 2000 census, our projected state population are off by 9.7 percent on average, with a standard deviation of 6.9 percent. Most states remained within the $[-1, 1]$ range, but not the DF. With one extra year away from the reference census, the 1997 map had even less success achieving proper apportionment. One year closer to census makes a huge difference, as the abandoned map for 2015 reveals (although this is probably compounded with a less dynamic demography): the dotted vertical line at year 2013 reports how the new plan might have looked, have it been adopted. With this in mind, it is puzzling that the redistributive nature of apportionment has not pushed for the adoption of alternative methods. The seventeen states that were under-represented in 2006 jointly controlled a majority (162) of single-member districts; fourteen of them have always been below the red line indicating fair representation.

that we used.

5.3 Intra-state malapportionment

Unequally-sized districts are common practice in Mexico in spite of the automated application of clear quantitative redistricting criteria since 1997. In fact, Mexican parties in general, and IFE in particular, have been remarkably tolerant of this practice. This form of malapportionment is related, in some degree, to states' imbalanced apportionment in Congress, which perforce creates size differences across states' mean district sizes. But size inequality within states is also prevalent and substantial.

Small deviations around a state's mean district population are unavoidable, especially as a map ages. But what constitutes significant deviation depends on the political context. Courts in the U.S. have struck down new district maps bearing less than 1% differences without proper justification (Tucker 1985). Redistricting authorities generally view a *de minimus* population deviations of as little as one or zero persons between congressional districts as desirable to inoculate against litigation. In stark contrast, IFE has considered deviations between 10% (in 2006) and 15% (in 1997 and 2015) above or below mean state district size perfectly normal (Lujambio and Vives Segl 2008, Trelles and Martínez 2012). If a pair of districts had populations exactly at the bounds of the ± 15 spread, the citizens at the bottom end would be worth one-third more in Congress than those at the top end. Surprisingly, no party has ever challenged this practice in Court.

We inspect district population imbalance. As before, time-changing population estimates are behind this exercise. Following Ansolabehere, Gerber and Snyder (2002), we measure a district's relative representation index as $RRI = \frac{1/\text{district size}}{300/\text{national population}} = \frac{Q}{\text{district size}}$, where the numerator is the number of congressional seats per person in the district and the denominator is the average number of congressional seats per person nationwide (300 is the number of congressional seats).¹³ Since the denominator represents the inverse of the apportionment quota Q , the second equality is a convenient algebraic transformation. A district with unity index value has representation matching the 'one person, one vote' ideal perfectly. Values above one indicate over-representation, values below one under-representation, and the measure is continuous. An example shows how the index is interpreted. The size of the 3rd district of Aguascalientes in 2012 was about 306,000 inhabitants, and the (projected) quota about 387,000, so the district in question had 26 percent more representation than the national average, for an index value of 1.26. As before, we used population projections to compute *RRIs*.¹⁴

¹³Ansolabehere et al.'s *RRIs* were devised to measure how well are represented counties (some of which have several districts) in U.S. state assemblies. In our case, all units are a whole districts.

¹⁴Census data reported at the electoral *sección* level (used to aggregate district populations in two maps) are available since 2005 only. We projected the 2005–2010 changes to 2006 and 2015 for this exercise. Since many districts are made of full municipalities, we could use that unit to interpolate all the way to 1997.

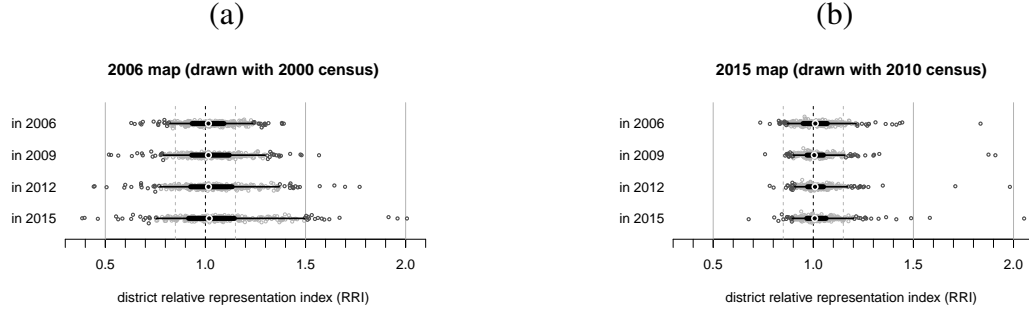


Figure 4: Representation in four elections and two maps. Panel (a) portrays the 2006 map (status quo), panel (b) the hypothetical 2015 map. Finer horizontal lines connect the 5th and 95th percentiles, thicker lines the quartiles, and white circles indicate the median. Points represent districts, hollow for outliers.

Redistricting automation should make districts within states tend towards zero malapportionment. The electoral bureaucracy claims that population balance is an important criterion, and assigns it a nominal weight higher than all other criteria (Altman, Magar, McDonald and Trelles 2014). However, in practice the system allows population balance to easily be trumped by other criteria.¹⁵ *RRIs* can also be expressed relative to state populations by replacing the denominator with the average number of congressional seats per person in the district's parent state (i.e., the number of seats in the state over the state's population). With three seats apportioned to the state of Aguascalientes and a total population of about 1,200,000, the 3rd district had 33 percent more representation than the state average, for an index value of 1.33. This is instructive to inspect within-state representation. The percentiles corresponding to state *RRIs* equal to .85 and 1.15 (the bounds of IFE's ± 15 percent tolerance range) in 2006 were 6 and 89, respectively, implying that $6 + 100 - 89 = 17$ percent of districts were outside the range relative to the average state population. That was the map's inaugural year. By 2012, as many as 27 percent districts off the population tolerance range, and by 2015 one third will be. Drawing equal-sized district boundaries with old population data is no easy task, and distorts representation within states over and above between-state imbalances.

Figure 4 returns to *RRIs* computed with the national average. Nationwide, the ± 15 band has been substantively surpassed. Plots summarize the distribution of citizen representation as population has fluctuated. Consider the top plot first. Each point represents one district. The fine horizontal line connects the *RRI* values corresponding to the 5th and 95th percentiles—both well outside the tolerance band, represented by dashed gray lines, since

¹⁵The other criteria are the preservation of municipal boundaries, travel times within the district, and district compactness. All are achieved by exploiting tolerated population leeway.

the map's inauguration. The thick horizontal line is the inter-quartile range, nearly covering the full tolerance band by 2015—implying that soon half the districts will be off-range! For the 2015 midterm congressional election, citizens in the plot's right-most districts (in central Monterrey and two in battered Juárez) will be worth *four times more* in Congress than citizens in the left-most districts (one each in suburban Monterrey and Mexico City, the other in Cancún). Politically, citizens at one quartile will be nearly worth twice as much as those at the other quartile.

The bottom plot offers a counterfactual, adopting the hypothetical map that was drawn towards the 2015 election but abandoned. It is quite clear that, even if outliers remain in its inauguration, the hypothetical map would have represented Mexicans much better than the status quo (note the narrower horizontal lines). Taking the mental experiment further, it is possible to aggregate section-level population projections for 2009 into the hypothetical map to assess its performance in the year closest to the census on which the map was prepared. Like fish, demographic information must be fresh: all but a handful of hypothetical districts in that year are within the ± 15 band.

The evidence is unambiguous: malapportionment in Mexico is systematic and substantial.¹⁶

5.4 Redistricting

5.5 Describe data

The data section should briefly discuss the data sources, their caveats, and how they were recoded/matched/integrated by us—and provide a citation to a replication file (or at least a placeholder).

5.6 Expectations

On paper, IFE is an autonomous, non-partisan election regulator. Members of its Council-General are thoroughly vetted and recruited from among professionals without party affiliation and admitted after winning consensual endorsement in the Chamber of Deputies. Once in office, councilors enjoy secure tenure. Their budget, which includes generous public financing for political parties and their electoral campaigns, is subject to few political whims. Yet closer inspection of the structure and process reveals how the Council-General

¹⁶The comparative survey by Snyder and Samuels (2004) ranked Mexico among well-apportioned cases. The measure reported is for the 1997 map, but no guidance is offered about the population figures used in denominators. We suspect reliance—as IFE did then and still does now—on raw 1990 census data, severely underestimating malapportionment.

is, in practice, a power-sharing agent of the major congressional parties (Estévez, Magar and Rosas 2008). The systematic partisan segmentation of the Council-General is quite conclusive evidence that major-party influence in electoral regulation is unremitting.

If the Council-General acts as an agent of the major parties, then one should observe the following:

1. When a map is adopted, districts have no bias favoring one major party relative to another. If a new map has bias, it must be against minor parties relative to major ones.
2. As maps age and district populations drift away from census relative levels, partisan bias (malapportionment-based) can creep in. partisan bias should therefore grow with each failure to redistrict.
3. There is no partisan gerrymandering. Gerrymander-based advantage for a party one year will be compensated by gerrymander-based handicap in other years, changes from one year to next according to parties' electoral fortunes

6 The procedure at work

6.1 Raw partisan bias

We now report results of putting all this to work with MCMC estimation (Jackman 2000).¹⁷

The responsiveness parameter is of secondary interest here, but useful to assess general model fit. Judging the 90-percent Bayesian confidence intervals (i.e., the 5th to 95th percentile range of $\hat{\rho}$'s posterior sample) reveals sizable shifts in the estimate between congressional elections: from a low of [2.2, 2.4] in 2012 to a high of [2.6, 3] in 2006. (*Is inter-election drift in ρ mentioned in the literature?*) The large-party premium of recent Mexican plurality congressional races is about one-sixth smaller than the power of the putative cube law of plurality elections (Taagepera 1973).

Raw partisan bias results (i.e., the λ_p parameters estimated using v_p) bear direct interest. Figure 5 reports summaries of the posterior sample for different parties. The absence of the PRI is due to its choice are reference to express partisan bias measures. Several patterns

¹⁷Three chains were iterated 10 thousand times, taking every fiftieth observation of the last 5 thousand to sample the posterior distribution. The Gelman–Hill $\hat{R} \approx 1$ (Gelman and Hill 2007), evidence that the chains had reached a steady state. Convergence was also inspected visually in chain traceplots of each of the model's parameters. Estimation performed with open-source software: Jags (Plummer 2003), implemented in R (R Dev. Core Team 2011) with library R2jags (Su and Yajima 2012). Data and commented code to replicate the analysis will be posted on-line upon publication.

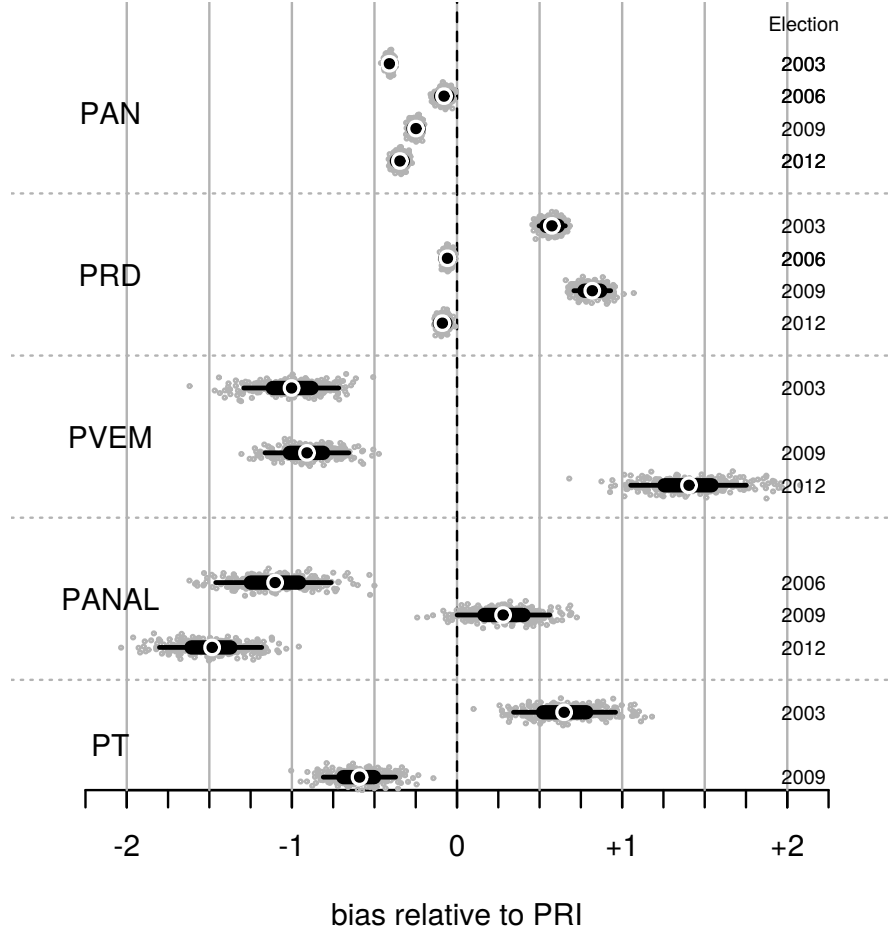


Figure 5: Raw partisan bias in four elections. The plot describes the posterior samples (small gray points) of estimated parameters $\hat{\lambda}_p$ for five parties. Some parties did not run in some years. Finer horizontal black lines connect the 5th and 95th percentile values of the posterior sample, thicker lines the quartiles, and white circles indicate the median value.

are noteworthy. Estimate precision (i.e., how concentrated the posterior sample’s cloud appears) is consistently lower for minor parties. Among the major ones, the PAN’s estimates excel, variation around the median posterior value (taken as the point estimate) nearly indistinguishable every year. The PRD’s generally precise estimates are slightly less so in midterm elections (2003 and 2009) than in presidential election years. And the size and polarity of estimates reveal important party differences. The PAN experienced negative, albeit small bias vis-à-vis the PRI in every year observed. Leaving aside the question of how meaningful the estimated quantities—we analyze swing ratios below with this object—the near exception was 2006, when the cloud nearly overlaps with the zero’s vertical line and the estimate is tiny.

In stark contrast, the left experienced favorable and substantive bias relative to the PRI in some years but not in others. In this respect, bias for the left is a mirror image of its electoral fortunes: it vanished when its candidates for Congress rode López Obrador’s presidential campaign coattails (the party’s national congressional vote was 30 percent on average in 2006 and 2012), but materialized otherwise (when vote averaged 16 percent).¹⁸ In spite of losing about half of its electoral support, the PRD converted votes into seats much more efficiently than either other major party in midterm election years. Whether this was due to winning smaller districts or lower-turnout districts we answer next.

6.2 Components of partisan bias

Table 2 breaks down raw partisan bias into GKB’s additive components: the distributive, the turnout, and the malapportionment bases. Ignore the right column for now. Relative bias estimates for the PAN, the PRD, and a selected minor party vis-à-vis the PRI are included. The minor party in 2006, 2009, and 2012 is the Green, and in 2006 the PANAL (close to the teachers’ union). The Green ran in pre-election coalition with the PRI in a changing number of districts every year—one-fifth of all in 2009, one-third in 2003, two-thirds in 2012, and in all districts in 2006—hence the choice of another minor party to report that year. Numbers in parentheses report the share of the posterior sample that has a sign opposite to the one reported, and therefore serves as a test of the estimate’s statistical significance (the probability that the estimate’s sign is wrong).

Turnout played favorably for PRI relative to other major parties in every election in the period, as indicated by systematic negative figures. An edge in lower-turnout districts

	Year	PAN	PRI	PRD	Green	PANAL
¹⁸ Parties’ national vote shares in the period were:	2003	.32	.38	.18	.04	—
	2006	.34	.28	.31	—	.05
	2009	.29	.40	.14	.06	.04
	2012	.27	.38	.28	.02	.04

partisan bias	Actual map			Hypothetical map		
	PAN-PRI	PRD-PRI	min-PRI	PAN-PRI	PRD-PRI	min-PRI
2003 election				(with 2006 map)		
raw	−.37 (0)	+.72 (0)	−1.01 (0)	−.41 (0)	+.57 (0)	−1.00 (0)
gerrym.	−.09 (0)	+.69 (0)	−.88 (0)	−.13 (0)	+.62 (0)	−.90 (0)
turnout	−.26 (0)	−.11 (0)	−.08 (0)	−.26 (0)	−.09 (0)	−.09 (0)
malapp.	−.01 (.11)	+.14 (0)	−.05 (0)	−.02 (.12)	+.05 (0)	−.02 (0)
2006 election						
raw	−.08 (0)	−.06 (0)	−1.10 (0)			
gerrym.	+.28 (0)	+.30 (0)	−.62 (0)			
turnout	−.36 (0)	−.41 (0)	−.43 (0)			
malapp.	−.00 (.42)	+.05 (0)	−.05 (0)			
2009 election						
raw	−.25 (0)	+.82 (0)	−.91 (0)			
gerrym.	−.11 (0)	+1.01 (0)	−.79 (0)			
turnout	−.14 (0)	−.24 (0)	−.12 (0)			
malapp.	−.00 (.36)	+.05 (0)	−.00 (0)			
2012 election				(with 2015 map)		
raw	−.35 (0)	−.09 (0)	+1.40 (0)	−.32 (0)	−.13 (0)	+1.03 (0)
gerrym.	−.28 (0)	−.07 (0)	+1.41 (0)	−.24 (0)	−.05 (.06)	+1.02 (0)
turnout	−.07 (.02)	−.08 (0)	+0.02 (0)	−.08 (.26)	−.09 (0)	+0.01 (0)
malapp.	+0.01 (.42)	+0.06 (0)	−0.02 (0)	−0.00 (.38)	+0.01 (0)	+0.00 (0)

Table 2: Relative major-party raw bias and its additive components. Entries report the median of the posterior sample of parameters estimated with the single-election models. Numbers in parentheses are the share of the posterior sample with sign opposite to the reported point estimate's. The right column reports bias estimates using election data re-arranged according to district boundaries adopted after an election (i.e., a hypothetical 2003 election with the 2006 map and a hypothetical 2012 election with the 2015 map).

gave the PRI a springboard to more efficient votes-to-seats transformation. The apex was the 2006 election, which concurred with a presidential race and the PRI's arrival in distant third place in it. By retracting to its core support districts, when negative presidential coattails depressed the party's congressional vote in swing districts nationwide, the low-turnout effect grew in size quite remarkably. The opposite happened in 2012, when favorable presidential coattails to some extent helped the party's congressional candidates.

That the gerrymander element often predominates in the mix is notable, as was the PRD's case in midterm elections, the PAN's in 2012, and systematically for the selected minor parties. Since, owing to formidable entry barriers in the election law, no minor party is regional-based, this pattern is quite expected for them: spreading meager support across the board returns few, if any seats—hence the negative sign. 2012 is the exception, when Greens nominated the coalition's candidate in a concurrent gubernatorial race whose coattails returned three congressional seats (Magar 2012). And the distributive component's volatility for major parties, in size and in sign, is consistent with the absence of partisan gerrymanders—as expected. Also notable is how the raw sum hides large components that are pushing in opposite directions and therefore cancel out. The PAN and PRD's extraordinary performances in 2006 led them to the lowest measures of raw partisan bias (in absolute value) in the period. Separation makes it possible to note that a gerrymander advantage was key to compensate for even larger turnout disadvantages. District boundaries played to their advantage. The PRD experienced something similar, but less balanced, in 2009, when a superb gerrymander advantage was only partially offset by the turnout disadvantage.

Also distinctive is how generally small the malapportionment component of partisan bias is relative to other sources. The PAN experienced no bias attributable to district size differentials over the period. The party's success was therefore not likelier in districts confined at one end of the RRI distribution, as the left was to some extent. The PRD was slightly advantaged relative to the PRI in every year observed. This is very likely due to overrepresentation of the Federal District—a PRD stronghold—but the effect is easily eclipsed by the other components of partisan bias. (The drop from +.14 to +.05 between 2003 and 2006 actually coincides with reapportionment and the accessory reduction—not removal—of the capital's overrepresentation in Congress.) Malapportionment-driven bias is not much larger for minor parties, whose perennial small vote shares locate at the wrong end of the system's responsiveness to size. For further perspective on malapportionment, we repeated the 2003 and 2012 estimations with counter-factual outcomes using the district boundaries of the map that was re-drawn immediately after those elections (reported in the right column of Table 2).¹⁹ As expected, redistricting mitigated malapportionment-based

¹⁹Counter-factual outcomes were generated with *sección*-level vote returns. *Secciones electorales* are anal-

Year	Variable	PAN		PRI		PRD	
		β	(SE)	β	(SE)	β	(SE)
2003	v	1.84	(.06)	2.44	(.07)	1.75	(.05)
	$v \times \text{reMap}$	+.06	(.08)	+.08	(.10)	−.12	(.06)
2006	v	2.18	(.07)	2.17	(.10)	1.73	(.05)
	$v \times \text{reMap}$	+.13	(.10)	−.36	(.14)	−.07	(.08)
2009	v	1.67	(.07)	2.18	(.07)	1.42	(.04)
	$v \times \text{reMap}$	+.44	(.10)	+.26	(.11)	+.18	(.07)
2012	v	2.31	(.07)	3.86	(.13)	2.35	(.06)
	$v \times \text{reMap}$	−.24	(.09)	+.03	(.17)	−.11	(.09)

Table 3: Vote–seat swing ratios. Also in the right side, but not reported, were a dummy indicating data simulated with the hypothetical map, and a constant. Method of estimation: OLS.

partisan bias systematically: under hypothetical, more balanced districts, the statistically nil quantity against the PAN actually observed (the probability that the estimate reported has wrong sign is .11) remained so; and the pro-PRD’s discernible bias relative to the PRI shrank to about one third its original size. The same is true inspecting the 2012 election in light of districts re-drawn with updated population figures.

6.3 The size of partisan bias

We close by elaborating how meaningful partisan bias has been in recent congressional elections. As said, translating the estimates of λ_p discussed into a percentage point advantage or handicap for party p in the votes-to-seat conversion is not straightforward. We therefore gauge this with an alternative quantity of substantive interest: vote–seat swing ratios (Niemi and Fett 1986, Tufte 1973). Swing ratios (or the vote elasticity of seats) the percentage change in seats associated with a one-percent change in the party’s national congressional vote. If the previous section aimed to measure this type of effect system-wide (the responsiveness parameter), swing ratios measure the sensitivity of individual parties’ seat shares to changes in voter preferences. A party with unity swing ratio can expect to receive its fair share of seats. Anything else indicates that parties can expect to win more (> 1) or less (< 1) than one percent of seats for a unit percentage change in vote share. (We rule out negative swing ratios corresponding to a party losing seats as it wins votes; for

ogous to U.S. census tracts, but bigger (median population in the 2010 census was 1,280, with a maximum at 79,232). The 1997, 2006, and 2015 maps relate more than 66 thousand *secciones*, the basic units for district cartography, to congressional districts. This makes reconstitution of hypothetical election outcomes in the period possible. Altman et al. (2014) describe the redistricting process in detail.

violations of the monotonicity principle of representation, see Balinski and Young 2001).

We derive swing ratios by regressing a party's seat shares in simulated elections on the party's simulated vote shares (Linzer 2012). To also gauge the effects of redistricting, we pool the latter with simulated elections using not the actual district boundaries but those in the map that supplanted it (i.e., the 2006 map for the 2003 election and the 2015 map for the 2006–12 elections). Interacting this with dummy reMap (equal 1 for hypothetical simulated elections, 0 otherwise) yields the fitted equation: $s_p = \beta_0 + \beta_1 v + \beta_2 \text{reMap} + \beta_3 v \times \text{reMap} + \text{error}$. Coefficient β_1 is the swing ratio, coefficient β_3 the swing ratio change after redistricting.

Table 3 reports major party results. In general, major parties enjoyed quite favorable swing ratios in the period—2.14 on average, indicating a hike in seats surpassing 2 percentage point attributable to an extra percentage point in votes. But a good deal of change, both between parties and between elections, is palpable. The left enjoyed the smallest four-election average swing ratios (1.8), the PRI the largest (2.7), the PAN somewhere in between (2.0). Given 300 Congressional seats, the PRI at its most elastic (in 2012) would have earned nearly 12 more seats by winning just one percent more support in the electorate nationwide. A dozen seats would have amply sufficed to give the coalition majority status that it failed to achieve in the chamber of deputies! Contrast this with the nearly 2.5 and 3 additional percentage points in votes, respectively, that it would have taken the PAN and the PRD at their least elastic (in 2009) in order to earn the same dozen extra seats.

7 Conclusion

What bias does the plurality component of Mexican elections produce? Do three sources push the same way, each one adding bias in favor of one same party? Who gains, who suffers from partisan bias? Do parties specialize in some component, in such way that, by canceling, the resulting partisan bias is more modest than its components?

Inspection of Mexican congressional districts uncovered non-trivial malapportionment—nearly half of district for the midterm congressional in June 2015 will have populations that deviate from the mean district size by more than $\pm 30\%$. The problem is built into rules commanding the use of census data to redraw district boundaries, while remaining silent about when drawing must be done. The last three redistricting rounds dragged by more than a half a decade. By then, population data was outdated.

Fitting a votes–seats curve to measure partisan bias revealed that the plurality component of Mexico's mixed electoral system gave persistent advantage to some parties in recent congressional elections. Relative to the PAN, there is evidence of small, but systematic bias

in favor of the PRI in the votes-to-seats conversion, and of larger, but more volatile bias favorable to the PRD throughout the period. These findings, derived from simulated data to bridge methodological complications, are in contrast with other studies finding no evidence of partisan bias in state-level data (Magar, Altman, McDonald and Trelles 2014) or evidence of substantive anti-PRI bias in multi-election data (Márquez 2014a). Our findings here are more solidly grounded.

A breakdown of the components of partisan bias with Grofman, Koetzle and Brunell's method adds further depth to our findings. Malapportionment added surprisingly little to the mix. It helped the left relative to other major parties, and with growing strength as maps aged, but the contribution is much smaller than, and easily canceled by those of turnout differences and the geographic distribution of party strength. Despite nominally neutral redistricting, the left in most years, and the PAN in 2006 were able to overcome a large turnout disadvantage through more favorable line drawing. Gerrymandering merits closer inspection in the future.

Appendix 1: BUGS code

```
bugsModel <- function() {
  for (i in 1:I){      # loop over state-years
    for (p in 1:P){ # loop over parties (dummy selects those who ran that year)
      S[i,p] ~ dbin(pi[i,p], D[i]) # D is number SMD seats in obs. i's state
    }
    numerator[i,1] <- dummy[i,1] * exp( lambda[1] + rho * log(v[i,1]) )
    numerator[i,2] <- dummy[i,2] * exp(          rho * log(v[i,2]) )
    for (p in 3:P){
      numerator[i,p] <- dummy[i,p] * exp( lambda[p-1] ) * v[i,p]^rho
    }
    for (p in 1:P){ # loop over parties (dummy=1 selects those who ran that year)
      d1[i,p] <- dummy[i,1] * exp( lambda[1] ) * v[i,1]^rho
      d2[i,p] <- dummy[i,2] * v[i,2]^rho
      d3[i,p] <- dummy[i,3] * exp( lambda[2] ) * v[i,3]^rho
      d4[i,p] <- dummy[i,4] * exp( lambda[3] ) * v[i,4]^rho
      d5[i,p] <- dummy[i,5] * exp( lambda[4] ) * v[i,5]^rho
      d6[i,p] <- dummy[i,6] * exp( lambda[5] ) * v[i,6]^rho
      d7[i,p] <- dummy[i,7] * exp( lambda[6] ) * v[i,7]^rho
      denominator[i,p] <- d1[i,p]+d2[i,p]+d3[i,p]+d4[i,p]+d5[i,p]+d6[i,p]+d7[i,p]
      pi[i,p] <- numerator[i,p] / denominator[i,p]
    }
  }
  ### priors
  for (q in 1:6){ # there are 7 party labels in the 3-election data, PRI is reference
    lambda[q] ~ dnorm( 0, tau.lambda )
  }
  tau.lambda <- pow(.25, -2)
  rho ~ dexp(.75) # this has positive range, median close to 1, mean 1.25, max 4.5
}
```

References

- Altman, M., E. Magar, M. P. McDonald and A. Trelles. 2014. "The Effects of Automated Redistricting and Partisan Strategic Interaction on Representation: The Case of Mexico." Presented at the 2014 Annual Meeting of the American Political Science Association, Washington DC.
- Altman, M. and M. P. McDonald. 2011. "BARD: Better Automated Redistricting." *Journal of Statistical Software* 42(4):1–28.
URL: <http://www.jstatsoft.org>
- Ansolabehere, S., A. Gerber and J. Snyder. 2002. "Equal votes, equal money: Court-ordered redistricting and public expenditures in the American states." *American Political Science Review* 96(04):767–77.
- Balinski, M. L. and H. P. Young. 2001. *Fair representation: Meeting the ideal of one man, one vote*. 2nd ed. Washington D.C.: Brookings.
- Balinski, M. L. and V. Ramírez González. 1996. "A case study of electoral manipulation: The Mexican laws of 1989 and 1994." *Electoral Studies* 15(2):203–17.
- Brady, D. W. and B. Grofman. 1991. "Sectional differences in partisan bias and electoral responsiveness in U.S. House elections." *British Journal of Political Science* 21:247–56.
- Cain, B. E. 1985. "Assessing the partisan effects of redistricting." *American Political Science Review* 79(2):320–33.
- Calvo, E. 2009. "The competitive road to proportional representation: Partisan biases and electoral regime change under increasing party competition." *World Politics* 61(2):254–95.
- Calvo, E. and J. P. Micozzi. 2005. "The Governor's Backyard: A Seat-Vote Model of Electoral Reform for Subnational Multiparty Races." *The Journal of Politics* 67(4):1050–74.
- Calvo, E. and J. Rodden. 2015. "The Achilles Heel of plurality systems: geography and representation in multiparty democracies." *American Journal of Political Science* forthcoming.
- Campbell, J. E. 1996. *Cheap seats: Democratic party advantage in U.S. House elections*. Columbus, OH: Ohio State University Press.

- Cox, G. W. 1997. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. New York: Cambridge University Press.
- Cox, G. W. and J. N. Katz. 2002. *Elbridge Gerry's Salamander: The Electoral Consequences of the Reapportionment Revolution*. New York: Cambridge University Press.
- Dahl, R. A. 1956. *A Preface to Democratic Theory*. Chicago: Chicago University Press.
- Dahl, R. A. 1972. *Polyarchy: Participation and Opposition*. New Haven: Yale University Press.
- Dworak, F. F., ed. 2003. *El legislador a examen: El debate sobre la reelección legislativa en México*. Mexico City: Fondo de Cultura Económica.
- Engstrom, E. J. 2006. "Stacking the states, stacking the House: The partisan consequences of congressional redistricting in the 19th century." *American Political Science Review* 100(3):419–427.
- Erikson, R. S. 1972. "Malapportionment, gerrymandering, and party fortunes in congressional elections." *The American Political Science Review* 66(4):1234–1245.
- Estévez, F., E. Magar and G. Rosas. 2008. "Partisanship in Non-Partisan Electoral Agencies and Democratic Compliance: Evidence from Mexico's Federal Electoral Institute." *Electoral Studies* 27(2):257–71.
- Gelman, A. and G. King. 1994. "A unified method of evaluating electoral systems and redistricting plans." *American Journal of Political Science* 38(2):514–54.
- Gelman, A. and J. Hill. 2007. *Data Analysis Using Regression and Multilevel/Hierarchical Models*. Cambridge University Press.
- Grofman, B. 1983. "Measures of bias and proportionality in seats-votes relationships." *Political Methodology* 9:295–327.
- Grofman, B., W. Koetzle and T. Brunell. 1997. "An integrated Perspective on the three potential sources of partisan bias: malapportionment, turnout Differences, and the geographic distribution of party vote shares." *Electoral Studies* 16(4):457–70.
- Gudgin, G. and P. J. Taylor. 1979. *Seats, Votes and the Spatial Organisation of Elections*. London: Pion.
- Gudgin, G. and P. J. Taylor. 1980. "The Decomposition of Electoral Bias in a Plurality Election." *British Journal of Political Science* 10(4):515–21.

Jackman, S. 1994. "Measuring electoral bias: Australia, 1949–93." *British Journal of Political Science* 24(3):319–57.

Jackman, S. 2000. "Estimation and Inference via Bayesian Simulation: An Introduction to Markov Chain Monte Carlo." *American Journal of Political Science* 44(2):375–404.

Johnston, R. J. 2002. "Manipulating Maps and Winning Elections: Measuring the Impact of Malapportionment and Gerrymandering." *Political Geography* 21(1):1–31.

Kendall, M. and A. Stuart. 1950. "The law of cubic proportion in election results." *Journal of Sociology* 1(1):183–96.

King, G. 1990. "Electoral responsiveness and partisan bias in multiparty democracies." *Legislative Studies Quarterly* 15(2):159–81.

King, G. and R. X. Browning. 1987. "Democratic representation and partisan bias in congressional elections." *American Political Science Review* 81(4):1251–73.

Lijphart, A. 1994. *Electoral Systems and Party Systems: A Study of Twenty-Seven Democracies, 1945–1990*. Oxford: Oxford University Press.

Linzer, D. A. 2012. "The relationship between seats and votes in multiparty systems." *Political Analysis* 20(3):400–16.

Lujambio, A. and H. Vives Segl. 2008. From politics to technicalities: Mexican redistricting in historical perspective. In *Redistricting in Comparative Perspective*, ed. L. Handley and B. Grofman. Oxford: Oxford University Press.

Magar, E. 2012. "Gubernatorial coattails in Mexican congressional elections." *The Journal of Politics* 74(2):383–99.

Magar, E., M. Altman, M. P. McDonald and A. Trelles. 2014. "Malapportionment and representation: party bias and responsiveness in Mexico." Presented at the Analyzing Latin American Politics conference, University of Houston, 14 Nov. 2014.

Márquez, J. 2014a. "Curva histórica de curules y votos (1991–2012) con simulación bayesiana." <http://javier-marquez.com/2014/02/11/curva-historica-de-curules-y-votos-1991-2012-con-simulacion-bayesiana/>.

Márquez, J. 2014b. "Medición del sesgo partidista con modelos de mezclas finitas." <http://javier-marquez.com/2014/02/14/medicion-del-sesgo-partidista-con-modelos-de-mezclas-finitas/>.

- May, K. O. 1952. "A set of independent necessary and sufficient conditions for simple majority decision." *Econometrica* 20(4):680–4.
- Mayhew, D. R. 1974. *Congress: The Electoral Connection*. New Haven: Yale University Press.
- Niemi, R. G. and P. Fett. 1986. "The swing ratio: An explanation and an assessment." *Legislative Studies Quarterly* 11(1):75–90.
- Plummer, M. 2003. JAGS: A Program for Analysis of Bayesian Graphical Models Using Gibbs Sampling. In *Proceedings of the 3rd International Workshop on Distributed Statistical Computing*. Vienna, Austria: .
URL: <http://www.ci.tuwien.ac.at/Conferences/DSC-2003>
- R Dev. Core Team. 2011. "R: A Language and Environment for Statistical Computing." R Foundation for Statistical Computing <http://www.R-project.org>.
- Rae, D. 1967. *The Political Consequences of Electoral Laws*. New Haven: Yale University Press.
- Rosenstone, S. J. and J. M. Hansen. 1993. *Mobilization, Participation, and Democracy in America*. New York: Macmillan.
- Rossiter, D., R. Johnston and C. Pattie. 1997. "Estimating the Partisan Impact of Redistricting in Great Britain." *British Journal of Political Science* 27(2):319–31.
- Shugart, M. S. and M. P. Wattenberg. 2001a. Introduction: The electoral reform of the Twenty-First Century? in *Mixed-Member Electoral Systems: the Best of Both Worlds?* Shugart and Wattenberg (2001b) pp. 1–6.
- Shugart, M. S. and M. P. Wattenberg, eds. 2001b. *Mixed-Member Electoral Systems: the Best of Both Worlds?* Oxford: Oxford University Press.
- Snyder, R. and D. J. Samuels. 2004. Legislative malapportionment in Latin America. In *Federalism and Democracy in Latin America*, ed. E. L. Gibson. Washington, DC: Johns Hopkins University Press.
- Su, Y.-S. and M. Yajima. 2012. *R2jags: A Package for Running jags from R*. R package version 0.03-08.
URL: <http://CRAN.R-project.org/package=R2jags>

- Szpiro, G. G. 2010. *Numbers Rule: The Vexing Mathematics of Democracy from Plato to the Present*. Princeton NJ: Princeton University Press.
- Taagepera, R. 1973. "Seats and Votes: A generalization of the cube law of elections." *Social Science Research* 2(3):257–75.
- Taagepera, R. and M. S. Shugart. 1989. *Seats and Votes: The Effects and Determinants of Electoral Systems*. New Haven: Yale University Press.
- Trelles, A. and D. Martínez. 2012. "Electoral boundaries: Lessons for California from Mexico's redistricting experience." *Política y Gobierno* 19(2):199–241.
- Tucker, H. J. 1985. "State legislative apportionment: legal principles in empirical perspective." *Political Geography Quarterly* 4(1):19–28.
- Tufte, E. R. 1973. "The relationship between seats and votes in two-party systems." *American Political Science Review* 67:540–554.
- Weldon, J. A. 2001. The consequences of Mexico's mixed-member electoral system, 1988–1997. in Shugart and Wattenberg (2001b) pp. 447–76.