

Steps

- 1). Assumptions & Definitions  $\diamond$  need to make sure we are careful about ft vs m units when creating controller
  - No thrust, mass = const.
  - $F_d = \frac{1}{2} \rho(h) C_d(u) A(u) v |v|$ 
    - $u \in [0, 1]$  is airbrake deployment fraction
    - use lookup table for  $\rho(h) \rightarrow \rho(h) = \text{interp1}(\text{htable}, \rho_{\text{table}}, h) \rightarrow$  can use 1976 Standard Atmosphere
  - $g(h) = g_0 \left( \frac{R_e}{R_e + h} \right)^2$
  - Avionics provide current  $h, v, \alpha$  at predetermined control update times

## 2). Physics Model

Variables:

- $h(t) = \text{altitude}$
- $v(t) = \text{vertical velocity}$
- $m = \text{rocket dry mass}$
- $g(h) = \text{gravity at altitude } h$
- $\rho(h) = \text{air density at altitude } h$
- $C_d(\alpha, u) = \text{drag coeff. as a function of a.o.a. } \alpha \text{ and airbrake deployment } u \rightarrow$  can choose either  $C_d$  or  $A$  to vary as a function of  $\alpha, u$  depending on what data we have
- $A = \text{reference area}$
- $u = \text{airbrake deployment (0-1)}$

## Equations of Motion:

$$\begin{aligned} \dot{h} &= v \\ \dot{v} &= -g(h) + \frac{1}{2m} \rho(h) v |v| C_d(\alpha, u) A \end{aligned}$$

## Drag Coefficient Model

$$C_d(\alpha, u) = C_{D_0} + k_\alpha \alpha^2 + (C_{D, \text{brake}}(u) - C_{D_0})$$

- $C_{D_0} = \text{base drag coefficient}$
- $k_\alpha$  models increase w/ a.o.a.
- $C_{D, \text{brake}}(u)$  from lookup table

## 3). Apogee Prediction

- Simulate until  $v(t) = 0$
- Use numerical integration (ode45 in MATLAB, scipy.integrate.solve\_ivp in Python) of above dynamics

## 4). MPC Objective

At each timestep (in Hz?):

1. Measure  $h, v, \alpha$
2. Predict apogee  $h_{\text{apogee}}(u)$  using current model & possible future  $u$  values
3. Compute target apogee logic:

$$h_{\text{target}} = \begin{cases} h_{\text{desired}}, & h_{\text{apogee}}(0) > h_{\text{desired}} \\ \max(\text{floor}(h_{\text{apogee}}(0)/2500) \times 2500, \text{min altitude}), & \text{otherwise} \end{cases}$$

4. Optimize  $u$  to minimize error cost function:

$$J(u) = |h_{\text{pred}}(u) - h_{\text{target}}| + R_u (u - u_{\text{prev}})^2 \rightarrow R_u \text{ penalizes large deployment changes}$$

5. Apply optimal  $u^*$  for next control cycle