

## Steps

need to make sure we are careful about ft vs m units when creating controller

### 1). Assumptions & Definitions

- No thrust, mass = const
- $F_D = \frac{1}{2} \rho(h) C_D(u) A(u) v |v|$ 
  - $u \in [0, 1]$  is airbrake deployment fraction
  - use lookup table for  $\rho(h) \rightarrow \rho(h) = \text{interp1}(h_{\text{table}}, \rho_{\text{table}}, h) \rightarrow$  can use 1976 Standard Atmosphere
- $g(h) = g_0 \left( \frac{R_e}{R_e + h} \right)^2$
- Avionics provide current  $h, v, \alpha$  at predetermined control update times

### 2). Physics Model

#### Variables:

- $h(t)$  = altitude
- $v(t)$  = vertical velocity
- $m$  = rocket dry mass
- $g(h)$  = gravity at altitude  $h$
- $\rho(h)$  = air density at altitude  $h$
- $C_D(\alpha, u)$  = drag coeff. as a function of a.o.a.  $\alpha$  and airbrake deployment  $u$  } can choose either  $C_D$  or  $A$  to vary as a fn of  $\alpha, u$  depending on what data we have
- $A$  = reference area
- $u$  = airbrake deployment (0-1)

#### Equations of Motion:

$$\dot{h} = v$$
$$\dot{v} = -g(h) + \frac{1}{2m} \rho(h) v |v| C_D(\alpha, u) A$$

← ?

#### Drag Coefficient Model

$$C_D(\alpha, u) = C_{D_0} + k_\alpha \alpha^2 + (C_{D, \text{brake}}(u) - C_{D_0})$$

- $C_{D_0}$  = base drag coefficient
- $k_\alpha$  models increase w/ a.o.a.
- $C_{D, \text{brake}}(u)$  from lookup table

### 3). Apogee Prediction

- Simulate until  $v(t) = 0$
- Use numerical integration (ode45 in MATLAB, scipy.integrate.solve\_ivp in Python) of above dynamics

### 4). MPC Objective

At each timestep (in Hz?):

1. Measure  $h, v, \alpha$
2. Predict apogee  $h_{\text{apogee}}(u)$  using current model & possible future  $u$  values
3. Compute target apogee logic:

$$h_{\text{target}} = \begin{cases} h_{\text{desired}}, & h_{\text{apogee}}(0) > h_{\text{desired}} \\ \max(\text{floor}(h_{\text{apogee}}(0)/2500) \times 2500, \text{min altitude}), & \text{otherwise} \end{cases}$$

4. Optimize  $u$  to minimize error cost function:

$$J(u) = |h_{\text{pred}}(u) - h_{\text{target}}| + R_u (u - u_{\text{prev}})^2 \rightarrow R_u \text{ penalizes large deployment changes}$$

5. Apply optimal  $u^*$  for next control cycle