

AIRBRAKES CONTROL ALGORITHM

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1 Abstract

We wish to choose a target apogee altitude using a variable-area airbrake system. Airbrakes are deployed using a servomotor that can expose a particular fraction of the Airbrake area to the freestream, thereby allowing the drag to depend on time.

This algorithm seeks to enable a rocket having some default without-Airbrakes apogee altitude to lower this altitude to a close but particular height, to be chosen by the designer. Overall flow is as follows:

1. A linear approximation (coefficient k) of the vertical velocity of the rocket as a function of time is characterized.
2. A non-Airbrakes apogee altitude is determined according to this approximation.
3. The difference between default apogee altitude and desired apogee altitude is calculated according to the pre-determined altitude choice. Let us denote it δx .
4. A difference in the velocity decay slope, δk , is calculated based on δx .
5. A difference in the drag \dot{x}^2 coefficient, δB , is calculated according to δk .
6. The Airbrakes' deployed area $A(t)$ is calculated from δB .
7. Once Airbrakes are activated, they are to be extended according to $A(t)$, with a PIDF controller on position accuracy and acceleration.

2 Derivation: Perturbation Theory

We observe from simulations and real launches that after engine burnout, velocity is roughly linear, following

$$\dot{x}(t) = -kt + v_0 \quad (1)$$

where k and v_0 are obtained from the rocket onboard sensors: a linear fit to several $\dot{x}(t)$ datapoints quickly obtains these.

This must satisfy the differential equation obtained from Newton's Second Law:

$$m\ddot{x} = \sum F_i = -mg - \frac{C_D A \rho \dot{x}^2}{2} \quad (2)$$

We write the coefficient of \dot{x}^2 as some $-mB$:

$$\ddot{x} = -g - B\dot{x}^2 \quad (3)$$

We now substitute $\ddot{x} = -k$ and $\dot{x} = -kt + v_0$:

$$k = g + B(v_0 - kt)^2 \quad (4)$$

We wish to see how a perturbation δk constrains a perturbation δB , removing all second/third order terms:

$$k + \delta k = g + (B + \delta B)(v_0 - (k + \delta k)t)^2 \quad (5)$$

$$\Rightarrow \frac{k + \delta k - g}{B + \delta B} = v_0^2 - 2(k + \delta k)tv_0 + (k + \delta k)^2 t^2 \quad (6)$$

$$= v_0^2 - 2ktv_0 - 2\delta ktv_0 + (k^2 + 2k\delta k + \delta k^2)t^2 \quad (7)$$

$$= v_0^2 - 2ktv_0 - 2\delta ktv_0 + k^2 t^2 + 2k\delta k t^2 \quad (8)$$

$$\Rightarrow k + \delta k - g = (B + \delta B)(v_0^2 - 2ktv_0 - 2\delta ktv_0 + k^2 t^2 + 2k\delta k t^2) \quad (9)$$

$$= Bv_0^2 - 2Bktv_0 - 2B\delta ktv_0 + Bk^2 t^2 + 2Bk\delta k t^2 \quad (10)$$

$$+ \delta Bv_0^2 - 2\delta Bktv_0 - \underline{2\delta B\delta ktv_0} + \delta Bk^2 t^2 + \underline{2\delta Bk\delta k t^2} \quad (11)$$

We factor out δB :

$$\delta B(v_0^2 - 2ktv_0 + k^2t^2) + Bv_0^2 - 2Bktv_0 - 2B\delta ktv_0 + Bk^2t^2 + 2Bk\delta kt^2 = k + \delta k - g \quad (12)$$

Solving for δB function of δk gives

$$\delta B = \frac{k + \delta k - g}{v_0^2 - 2ktv_0 + k^2t^2} - B \frac{v_0^2 - 2ktv_0 + k^2t^2 + 2k\delta kt^2 - 2\delta ktv_0}{v_0^2 - 2ktv_0 + k^2t^2} \quad (13)$$

$$= \frac{k + \delta k - g}{v_0^2 - 2ktv_0 + k^2t^2} - B \left(1 + 2t\delta k \frac{kt - v_0}{(kt - v_0)^2} \right) \quad (14)$$

$$= \frac{k + \delta k - g}{(kt - v_0)^2} - B \left(1 + \frac{2t\delta k}{kt - v_0} \right) \quad (15)$$

$$(16)$$

We had defined B as

$$B = \frac{C_D A_{\text{ref}} \rho}{2m} \quad (17)$$

Thus, δB is

$$\delta B = \frac{C_D A_{\text{airbrakes}} A(t) \rho}{2m} \quad (18)$$

Thus, the airbrakes area is

$$A(t) = \frac{2m}{\rho C_D A_{\text{airbrakes}}} \delta B \quad (19)$$

The max apogee altitude is found from the linear velocity approximation:

$$x_{\text{apog}} = -\frac{k}{2} t_{\text{apog}}^2 + v_0 t_{\text{apog}} + x_0 = \frac{v_0^2}{2k} + x_0 \quad (20)$$

Thus, the perturbation is

$$\delta x = \frac{\partial}{\partial k} \left(\frac{v_0^2}{2k} + x_0 \right) \delta k = -\delta k \frac{v_0^2}{2k^2} \quad (21)$$

which gives

$$\delta k = -\frac{2k^2}{v_0^2} \delta x \quad (22)$$

Thus, for a desired apogee altitude x_d , we have

$$\delta x \equiv x_d - x_{\text{apog}} = x_d - v_0^2/2k + x_0 \quad (23)$$

and so

$$A(t) = \frac{2m}{\rho C_D A_{\text{airbrakes}}} \delta B \quad (24)$$

$$= \frac{2m}{\rho C_D A_{\text{airbrakes}}} \left(\frac{k + \delta k - g}{(kt - v_0)^2} - B \left(1 + \frac{2t\delta k}{kt - v_0} \right) \right) \quad (25)$$

$$= \frac{2m}{\rho C_D A_{\text{airbrakes}}} \left(\frac{k - \frac{2k^2}{v_0^2} \delta x - g}{(kt - v_0)^2} - B \left(1 + \frac{2t \left(-\frac{2k^2}{v_0^2} \delta x \right)}{kt - v_0} \right) \right) \quad (26)$$

$$= \frac{2m}{\rho C_D A_{\text{airbrakes}}} \left(\frac{-2k^2 \delta x + v_0^2(k - g)}{v_0^2(kt - v_0)^2} - B \left(1 + \frac{-4tk^2 \delta x}{v_0^2(kt - v_0)} \right) \right) \quad (27)$$

$$\Rightarrow A(t) = \frac{2m}{\rho C_D A_{\text{airbrakes}}} \left(\frac{-2k^2(x_d - v_0^2/2k + x_0) + v_0^2(k - g)}{v_0^2(kt - v_0)^2} - B \left(1 + \frac{-4tk^2(x_d - v_0^2/2k + x_0)}{v_0^2(kt - v_0)} \right) \right) \quad (28)$$

Thus, given a k , v_0 obtained from a linear fit to $\dot{x}(t)$, an x_0 from matching to a locally-averaged altitude point, a B from the drag of the rocket without Airbrakes, and ρ and $C_D A_{\text{airbrakes}}$ hardcoded, the $A(t)$ function can be fully obtained. The formula is not very computationally intensive.

3 Implementation, Characterizations