

# AIRBRAKES CONTROL ALGORITHM

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## 1 Abstract

We wish to choose a target apogee altitude using a variable-area airbrake system. Airbrakes are deployed using a servomotor that can expose a particular fraction of the Airbrake area to the freestream, thereby allowing the drag to depend on time.

This algorithm seeks to enable a rocket having some default without-Airbrakes apogee altitude to lower this altitude to a close but particular height, to be chosen by the designer. Overall flow is as follows:

1. A linear approximation (coefficient  $k$ ) of the vertical velocity of the rocket as a function of time is characterized.
2. A non-Airbrakes apogee altitude is determined according to this approximation.
3. The difference between default apogee altitude and desired apogee altitude is calculated according to the pre-determined altitude choice. Let us denote it  $\delta x$ .
4. A difference in the velocity decay slope,  $\delta k$ , is calculated based on  $\delta x$ .
5. A difference in the drag  $\dot{x}^2$  coefficient,  $\delta B$ , is calculated according to  $\delta k$ .
6. The Airbrakes' deployed area  $A(t)$  is calculated from  $\delta B$ .
7. Once Airbrakes are activated, they are to be extended according to  $A(t)$ , with a PIDF controller on position accuracy and acceleration.

## 2 Derivation: Energy Conservation

Consider the rocket to undergo purely vertical motion. In the case in which airbrakes are never deployed, the rocket will reach an apogee height of  $x_{\text{apog}}$ , corresponding to a gravitational potential energy of  $mgx_{\text{apog}}$ . Airbrakes deployment, with area some  $A(t)$ , will do work on the rocket:

$$W = \int F \, dx = \int \frac{C_D \rho A(t) \dot{x}^2}{2} \, dx \quad (1)$$

This work will contribute to some loss of altitude in the apogee, given by

$$\int \frac{C_D \rho A(t) \dot{x}^2}{2} \, dx = mg(\Delta x) \quad (2)$$

We recall that  $\dot{x} = dx/dt$ , and so  $dx = \dot{x}dt$ :

$$\int \frac{C_D \rho A(t) \dot{x}^3}{2} \, dt = mg(\Delta x) \quad (3)$$

The governing differential equation is

$$\ddot{x} = -g - B\dot{x}^2 \quad (4)$$

where

$$B = \frac{C_D \rho A_{\text{ref}}}{2m} \quad (5)$$

We approximate that acceleration varies linearly in time, of the form

$$\ddot{x} = -g + \alpha(t - t_{\text{apog}}) \quad (6)$$

We solve for  $\dot{x}$ :

$$\dot{x} = \sqrt{-\frac{g + \ddot{x}}{B}} \quad (7)$$

Thus,

$$\dot{x}^3 = \left(-\frac{g + \ddot{x}}{B}\right)^{3/2} \quad (8)$$

Substituting in (6) gives

$$\dot{x}^3 = \left(-\alpha \frac{t_{\text{apog}} - t}{B}\right)^{3/2} \quad (9)$$

Thus, the integral becomes

$$\frac{C_D \rho \alpha^{3/2}}{2B^{3/2}} \int A(t)(t_{\text{apog}} - t)^{3/2} dt = mg(\Delta x) \quad (10)$$

We move all terms to the other side:

$$\int A(t)(t_{\text{apog}} - t)^{3/2} dt = \frac{2mg(\Delta x)B^{3/2}}{C_D \rho \alpha^{3/2}} \quad (11)$$

Let  $\eta$  be the right side expression, such that

$$\int A(t)(t_{\text{apog}} - t)^{3/2} dt = \eta \quad (12)$$

Values of  $\eta$  for different altitudes are given below according to current Zephyrus statistics:

$\Delta x$	$\eta$
300 m	1.875
200 m	1.249
100 m	0.625

We are free to choose the bounds of  $A(t)$ 's nonzero interval, as well as the kind of function  $A(t)$  is. We will choose a ramp-like function, of the form

$$A(t) = \begin{cases} 2A_0(t - t_0) & t \in \left[t_0, t_0 + \frac{1}{2}\right] \\ A_0 & t \geq t_0 + \frac{1}{2} \end{cases} \quad (13)$$

Thus,

$$\frac{\eta}{A_0} = \int_{t_0}^{t_0+1/2} 2(t - t_0)(t_{\text{apog}} - t)^{3/2} dt + \int_{t_0+1/2}^{t_{\text{apog}}} (t_{\text{apog}} - t)^{3/2} dt \quad (14)$$

We perform a change of variables from  $\tau = t_{\text{apog}} - t$  which gives  $d\tau = -dt$  and  $t = t_{\text{apog}} - \tau$ . The integration limits become

$$t_0 \rightarrow t_{\text{apog}} - t_0 ; \quad t_0 + \frac{1}{2} \rightarrow t_{\text{apog}} - t_0 - \frac{1}{2} ; \quad t_{\text{apog}} \rightarrow 0 \quad (15)$$

Thus,

$$\frac{\eta}{A_0} = -2 \int_{t_{\text{apog}} - t_0}^{t_{\text{apog}} - t_0 - 1/2} (t_{\text{apog}} - \tau - t_0)\tau^{3/2} d\tau - \int_{t_{\text{apog}} - t_0 - 1/2}^0 \tau^{3/2} d\tau \quad (16)$$

$$= 2 \int_{t_{\text{apog}} - t_0 - 1/2}^{t_{\text{apog}} - t_0} (t_{\text{apog}} - \tau - t_0)\tau^{3/2} d\tau + \int_0^{t_{\text{apog}} - t_0 - 1/2} \tau^{3/2} d\tau \quad (17)$$

We expand the expression into three integrals, and solve them one by one. The first is

$$2(t_{\text{apog}} - t_0) \int_{t_{\text{apog}} - t_0 - 1/2}^{t_{\text{apog}} - t_0} \tau^{3/2} d\tau = 2(t_{\text{apog}} - t_0) \left( \frac{\tau^{5/2}}{5/2} \right) \Big|_{t_{\text{apog}} - t_0 - 1/2}^{t_{\text{apog}} - t_0} \quad (18)$$

$$= \frac{4}{5}(t_{\text{apog}} - t_0) \left( (t_{\text{apog}} - t_0)^{5/2} - (t_{\text{apog}} - t_0 - 1/2)^{5/2} \right) \quad (19)$$

The second integral evaluates to

$$-2 \int_{t_{\text{apog}} - t_0 - 1/2}^{t_{\text{apog}} - t_0} \tau^{5/2} d\tau = -2 \left( \frac{\tau^{7/2}}{7/2} \right) \Big|_{t_{\text{apog}} - t_0 - 1/2}^{t_{\text{apog}} - t_0} \quad (20)$$

$$= -\frac{4}{7} \left( (t_{\text{apog}} - t_0)^{7/2} - (t_{\text{apog}} - t_0 - 1/2)^{7/2} \right) \quad (21)$$

The third integral evaluates to

$$\int_0^{t_{\text{apog}} - t_0 - 1/2} \tau^{3/2} d\tau = \left( \frac{\tau^{5/2}}{5/2} \right) \Big|_0^{t_{\text{apog}} - t_0 - 1/2} = \frac{2}{5}(t_{\text{apog}} - t_0 - 1/2)^{5/2} \quad (22)$$

Thus,  $A_0$  evaluates to

$$\boxed{\frac{4}{5}(t_{\text{apog}} - t_0) \left( (t_{\text{apog}} - t_0)^{5/2} - (t_{\text{apog}} - t_0 - \frac{1}{2})^{5/2} \right) - \frac{4}{7} \left( (t_{\text{apog}} - t_0)^{7/2} - (t_{\text{apog}} - t_0 - \frac{1}{2})^{7/2} \right) + \frac{2}{5}(t_{\text{apog}} - t_0 - \frac{1}{2})^{5/2}} \quad (23)$$

The maximum area of  $A(t)$  is 0.0066 m<sup>2</sup>. A table of results is below. Note that apogee time is 31.89 seconds.

$t_0$	$\Delta x = 300$ m	$\Delta x = 200$ m	$\Delta x = 100$ m
15 s	63%	42%	21%
17.5 s	94%	63%	32%
19 s	124%	83%	42%
20 s	153%	102%	51%

**Table 1.** Values of  $\frac{A_0}{A_{\max}}$  requisite for the labeled altitude difference, according to Zephyrus Test Launch Simulations.

### 3 Implementation, Characterizations