Chapter 5

Regression for Calibration and Interference Filtering

In Section 4.4.2 we evaluated the non-ideal properties of the HMC1053 for their expected impact on measurement repeatability. We summarize the findings of Section 4.4.2 by listing the properties which need to be calibrated, including necessary second order effects:

- Static offset
- Gain
- Linearity
- Cross-axis coupling
- Temperature dependence of gain
- Temperature dependence of offset
- Temperature dependence of cross-axis coupling

5.1 The Measurement Equation

In this section we will use the known non-ideal effects to write an equation that captures the expected mapping of the real incident field to the measured field with non-ideal effects included. The calibration model, which was constructed from the individual calibration steps described in Section 4.4.2, is given in Eq. (5.1), with terms described in Table 5.1.

$$\vec{B_{act}} = \left[\mathbf{S} + \mathbf{K_S} * T\right] * \vec{B}_{meas} + \left[\vec{O} + \vec{K_O} * T\right] \tag{5.1}$$

Table 5.1: Terms in the measurement equation.

Term	Type	Description
$\vec{B_{act}}$	Vector	The real magnetic field at the measurement location
$\mathbf{S}$	Matrix	Sensitivity matrix scales the signal based on the real sensitivity of each axis and includes linear off-axis effects
$\mathbf{K_S}$	Matrix	The linear temperature dependence of each of the terms in the sensitivity matrix
T	Scalar	Temperature of the sensor
$\vec{B}_{meas}$	Vector	The raw three-axis measurement coming from the magnetometer
$\vec{O}$	Vector	Static offset value for each axis
$\vec{K_O}$	Vector	The temperature dependence of the static offset

5.1.1 Verification with Test Data

The predictive power of the calibration equation has been evaluated by applying the entire calibration model to all data collected in the determination of the non-ideal effects as described in Section 4.4.2. The results of this regression are provided in Table 5.2.

Table 5.2: Derived regression coefficients.

Axis	Sx	Sy	Sz	Ksz	Ksy	Ksz	Oz	Koz	RMSE
X	1.026	-0.163	-0.211	0.0032	0.0047	0.0080	-1.210	0.0360	0.0236
Y	-0.160	2.370	0.043	0.0027	-0.0520	-0.0028	-0.071	-0.0009	0.0593
Z	-0.086	0.096	1.214	0.0046	-0.0011	-0.0040	4.323	-0.1607	0.0332

The most important takeaway is that the magnetic field root mean square error for all axes is less than 60 nT, indicating we will meet the 100 nT precision requirement (as long as our collected data is adequately representative of the orbital magnetic environment). The observed error is approximately the environmental noise floor in this location; the calibration results may be better than observed but we do not have the facilities to test to a lower noise floor.[1]

The results are generally as we expected from the discussion of the non-ideal properties of the AMR sensors from Section 4.4.1. The diagonal components of the sensitivity matrix are near unity with the exception of Syy, which can be explained by looking at the sensitivity matrix temperature dependence. The data used in this regression used units of uT for magnetic fields and degrees Celsius for temperature. The Ksyy term is also anomalously large at -0.052 as compared to less than magnitude 0.01 for all other sensitivity terms. Most (but not all) measurements were taken at room temperature of approximately 25 degrees Celsius, and plugging T=25 into the Syy and Ksyy finds a room temperature Syy dependence of 1.07, approximately unity as expected. This shows a that lack of proper calibration data can cause anomalous fitting effects and we will need to be careful to properly capture a

[1]The measurements used to create Table 5.2 were made in a non-shielded room as described in Section 4.4.2.

wide range of calibration data on orbit. The offset values themselves and their own temperature dependence all fall within ranges predicted based on individual analysis as summarized in Table 4.8.

5.2 Extension to Interfering Sources

Here we describe how the measurement equation can be extended to included potential interfering sources, such as material and current-path interference effects.

5.2.1 Material Interference

The most significant magnetic interference will come from ferromagnetic materials (see Section 2.3). Many ferromagnetic materials can be categorized into primarily "hard" materials or "soft" materials based on how the material coercive force compares to the local magnetizing field (see Section 2.3.3). Here we use the magnetically soft and hard limiting approximations to show that the calibration equation which we have already derived can account for arbitrary spatial distributions of soft and hard materials without modification.

Magnetically Hard Interference

As discussed in Section 2.3.3, sources which are very magnetically hard (high coercivity and remanence) can be approximated as constant dipoles based on their constant magnetization state. The interference from any single constant dipole element can be written as in Eq. (5.2), taken from Eq. (2.1).

$$\vec{B}_{dipole} = \frac{\mu_0}{4\pi}\left[\frac{3\hat{r}(\vec{m}\cdot\hat{r}) - \vec{m}}{r^3}\right] \tag{5.2}$$

For constant moment m, and separation from the material in question to the measurement location r, it is apparent that the contribution to the magnetic measurement will be constant, and for the i'th interferer we can consider a B_i, which sums as in Eq. (5.3).

$$\vec{B}_{hard} = \sum_i \vec{B}_i \tag{5.3}$$

If we separately consider the "external" magnetic field with no interference (which is what we want to measure), we can write the following:

$$B_{ext} = B_{act} + B_{hard} \tag{5.4}$$

$$\vec{B_{ext}} = \left[\mathbf{S} + \mathbf{K_S} * T\right] * \vec{B}_{meas} + \left[\vec{O} + \vec{K_O} * T\right] + \vec{B}_{hard} \tag{5.5}$$

So to find $\vec{B_{ext}}$ in the presence of an unknown but constant $\vec{B}_{hard}$ we can use the same measurement equation as before (Eq. (5.1)) but with a different definition of $\vec{O}$ where we use prime to indicate instrument-only offset vector.

$$\vec{O} = \vec{O}' + \vec{B}_{hard} \tag{5.6}$$

Since the original (instrument-only) offset was also unknown prior to fitting, accounting for any number of static interferers requires no new unknown terms in the measurement equations.

Magnetically Soft Interference

For any magnetically soft interferer, the magnetic moment of the interferer can be calculated from the incident environmental field.

$$\vec{m_i} = (\mu_{r_i} - 1) * V_i * \frac{1}{\mu_0} * \vec{B_{env}} \tag{5.7}$$

$$\vec{B}_{soft_i} = \frac{1}{4\pi}\left[\frac{3\hat{r}_i(\chi_i V_i \vec{B_{env}} \cdot \hat{r}_i) - \chi_i V_i \vec{B_{env}}}{r_i^3}\right] \tag{5.8}$$

$$\vec{B}_{soft_i} = \left[\frac{\chi_i V_i 3}{4\pi r_i^3}\right]\left(\vec{B_{env}} \cdot \hat{r}_i\right) - \left[\frac{\chi_i V_i}{4\pi r_i^3}\right]\vec{B_{env}} \tag{5.9}$$

$$\vec{B}_{soft_i} = \left[\frac{\chi_i V_i}{4\pi r_i^3}\right]\left[3\hat{r_i}^T\hat{r}_i - \mathbb{1}\right]\vec{B_{env}} \tag{5.10}$$

Therefore, we can define a sensitivity matrix contribution from each magnetically soft interfering source.

$$\mathbf{S_{soft_i}} = \left[\frac{\chi_i V_i}{4\pi r_i^3}\right]\left[3\hat{r_i}^T\hat{r}_i - \mathbb{1}\right] \tag{5.11}$$

We represent the instrument intrinsic sensitivity matrix from before as $\mathbf{S'}$ and the new total sensitivity matrix can be defined.

$$\mathbf{S} = \mathbf{S'} + \sum_i \mathbf{S_{soft_i}} \tag{5.12}$$

The unknown interference and the unknown instrument sensitivity can be combined so that we do not need any more unknown parameters to fit the interference from the magnetically soft source. This is analogous to how magnetically hard interference was indistinguishable from the instrument-intrinsic offset.

Limitations and Assumptions

Our analysis above holds for materials which are completely magnetically hard (coercivity much larger than the environmental field) or completely magnetically soft (coercivity much lower than the environmental field). For materials with magnetic hardness in between these two limits, hysteresis effects will be present in the data which will make the calibration problem dependent on past events.

The treatment of the magnetically soft material also assumes that the change in magnetization, and therefore magnetic moment, is linear with the amplitude of the applied field. Given that the incident magnetic field will dominantly be the Earth's magnetic field with a relatively constant amplitude this is a reasonable assumption in our limited application.

5.2.2 Current Paths

The current paths on the spacecraft also produce interfering magnetic fields. The total magnetic field can be found for a known path and current with a path integral of the Biot-Savart law (originally used in Eq. (2.5)).

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_C \frac{I\mathbf{dl} \times \mathbf{r}'}{|r'|^2} \tag{5.13}$$

The current magnitude I can be removed from the integral as it is constant over the path. The solution to this integral is not known without knowing the path, but we do know that the integral is constant.

$$\vec{B}_{current} = \vec{D}I \tag{5.14}$$

Where we have introduced the "dependency vector" $\vec{D}$ which is the solution to the path integral in the Biot-Savart law. For multiple interfering current paths (indexed by i) we can write the sum in Eq. (5.15).

$$\vec{B}_{current} = \sum_i \vec{D}_i I_i \tag{5.15}$$

The actual environmental field can be calculated from the measured field given this interfering field (or vice versa).

$$\vec{B_{act}} = \vec{B_{env}} + \vec{B}_{current} \tag{5.16}$$

$$\vec{B_{env}} = \vec{B_{act}} - \vec{B}_{current} \tag{5.17}$$

These results can be combined with the main measurement equation, Eq. (5.1), to create Eq. (5.18). This is the measurement equation extended to included current path interference sources. The sum over interfering current sources can be performed over as many current sources are available to measure, such as those in the spacecraft housekeeping telemetry data products.

$$\vec{B_{act}} = \left[\mathbf{S} + \mathbf{K_S} * T\right] * \vec{B_{meas}} + \left[\vec{O} + \vec{K_O} * T\right] - \sum_i \vec{D}_i I_i \tag{5.18}$$

5.3 Implementation

The measurement equation represents all the complexity of the measurement device and interference sources (at least to a level that meets performance requirements as shown in Table 5.2). To actually implement the calibration we also need to load data from spacecraft instruments, to be able to differentiate from measurement and calibration data sets, and to be able to perform the minimization with the calibration datasets to determine the unknowns in the measurement equation. We have implemented these functions in Python3 in three main modules: a calibration module, a magnetic measurement class, and a file loader.[2]

- **Calibration**: This module contains a Python class which implements the measurement equation and stores the unknown measurement equation parameters as class variables. This also provides functions which will apply the derived measurement equation to new data, and will also perform least squares minimization to find its best fit parameters.
- **Mag Measurement**: This module defines the Measurement class. The class values are all the measured terms in the measurement equation. The class provides functions which help to populate the data in a consistent format.
- **File Loader**: This module contains a list of functions which will load data from files and properly format the data in the measurement module class structure. These functions are generally called by the user or by a short user-written program. Each function in this module maps one-to-one with a method of generating and storing data, such as each of the test hardware devices described in Appendix A.
- **User Script**: This file is written by the user and is only a few lines long. This

[2]The software is hosted on STARLab's Enterprise GitHub (`AEROVISTA_ASP/GroundTools/cal_software/`).

contains information like the file path for the data and which data products are to be plotted and saved.

5.3.1 Calibration Class

The calibration class contains a numpy array for every unknown parameter in the matrix measurement equation. The calibration class also contains a variable called `useFlags` which indicates which unknown variables are actually used in the calibration. By defining different elements in this python dictionary to be Boolean True or False, the user can mask the effects of different unknown terms in the measurement equation. This is useful when testing with hardware which may not generate all of the data required for the full measurement equation. For example, the RM3100 magnetometers in the magEval board do not have any associated temperature data, and without an external interfering source, there will be no current sources to calibrate against. The calibration object stores this information as a flag to properly implement the desired calibration during regression and application, and also to document the data used to generate the calibration object. With this flag stored as a class parameter, the user knows which data should be provided if this calibration object is going to be used again to apply to new data.

The application of the calibration object to new data is achieved by implementing the measurement equation using numpy array math. The minimization of the error using calibration data uses the numpy `least_squares` function. This function minimizes the output of an error function, and this error function is defined as the difference between the calibrated data and the reference data at every measurement point. To provide the most robust minimization, every axis is minimized independently. The least_squares library expects and returns a simple list of coefficients so the Calibration class also provides functions help format the calibration parameters into this simple list. One function packs the class parameters into the list so that the minimization can be started with the default coefficients. Another function updates the class values based on a list which is returned from the minimization routine.

5.3.2 Measurement Class

The measurement class stores measurement data in a standard format. This allows the calibration class to be written without regard to the original format of the raw data from the instrument. The calibration class assumes that all data is sampled on the same time base and that all necessary data is populated into the measurement objects. The measurement class itself implements functions which make it easier to properly format the data from a variety of hardware sources. The main class variables of the measurement class are time, magnetometer, temperature, and interfering current source data. This class implements interpolation so that each variable, as stored by the class objects, will be sampled on the same time base. There is also a class list that keeps track of the names of all the current path data which is stored in the current paths data array. Finally, there is a simple string text description in the class object to document the source of the data, which is filled in manually by the user. The general procedure for populating the measurement class is as follows:

1. Load time stamps from file, and add these to the class object.
2. Load magnetometer data and associated time stamps, add to measurement class calling function for time interpolation.
3. Load temperature data and associated time stamps, add to measurement class calling function for time interpolation.
4. Load interferer data and associated time stamps, add to measurement class calling function for time interpolation.

5.3.3 File Loader

This module stores functions that load data from file locations. This software maps one-to-one with instrument programs which store data to files. These files will return a measurement class object which is properly formatted with the available data. The user can then use the returned measurement class objects (data magnetometer, and optionally reference data), and call the calibration class objects to create the calibration object or apply the calibration data. Any future use for the calibration method described in this work should be able to use this software with whatever data format is available by only adding a new file loading function to the file loader module.

5.4 Considerations for System Integration

The calibration method we have described is performed exclusively on the ground. This is advantageous because the relative complexity of the calibration software does not need to be verified to flight standards, and the user can easily modify parameters and source data to achieve the best results. However, collecting all the system data needed to run the proposed calibration does impose some requirements on the mission and the rest of the spacecraft system.

First, to filter out the interference caused by spacecraft current paths, the magnetometer ground processing software must be provided with accurate measures of the current flowing through the spacecraft. This data is usually collected anyway as part of the spacecraft's housekeeping or health and status telemetry sets. This data is collected on AERO-VISTA at a rate of 10 Hz. As long as the current does not change significantly during the measurement period, this rate should be sufficiently high. Very high frequency variations (on the order of 100s of Hz or above) will be filtered by the analog low pass filters in the magnetic sensor. Additionally, the housekeeping data must be synchronized with the magnetometer data to at least better than a measurement period (approximately 20 ms). This is already achieved on AERO-VISTA as all data must be time stamped with 1 ms accuracy.

Finally, we need a source for calibration data. We expect to perform calibration on orbit for AERO-VISTA by collecting magnetometer data at low latitudes. At these low latitudes the magnetic field predicted by models such as the World Magnetic Map (WMM) is accurate to approximately 150 nT [8]. This accuracy is above our measurement requirement, but since we require *precision* of <100 nT, not absolute accuracy, these models should be sufficient for determination of calibration parameters. To extract the WMM prediction of the magnetic field as seen in the spacecraft's body frame we will use global position information from a combination of GPS and orbital ephemeris data. The WMM provides magnetic field data with respect to an Earth centered Earth fixed (ECEF) coordinate system. This can be rotated into the spacecraft's coordinate system with data from the spacecraft ADCS telemetry. This is reported with respect to an Earth centered inertial (ECI) coordinate system, so we will additionally need to rotate the ECEF magnetic vector into an ECI magnetic vector using the absolute time stamp of the measurement.

AERO-VISTA generally is not running science observations with the main vector sensor at low altitudes, so the magnetometer sensor will be separately turned on at low latitudes to capture calibration data. We expect to capture 500 seconds of calibration data so that the data-take parameters are the same for calibration as they are for science data measurement.