Deep Sequence Modeling
MIT 6.S191

Ava Soleimany

January 28, 2019

Given an image of a ball,
can you predict where it will go next?

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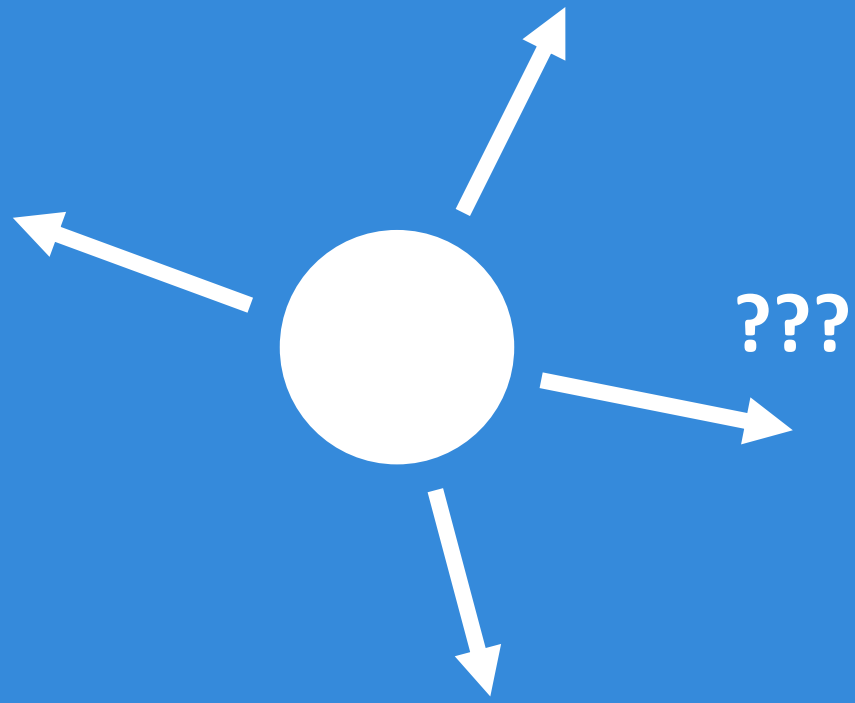

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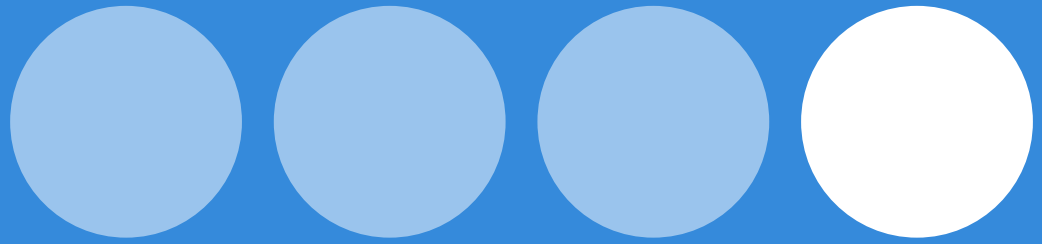

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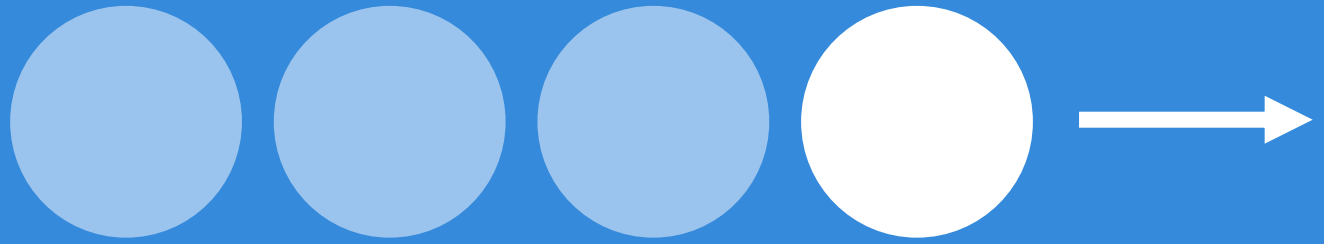
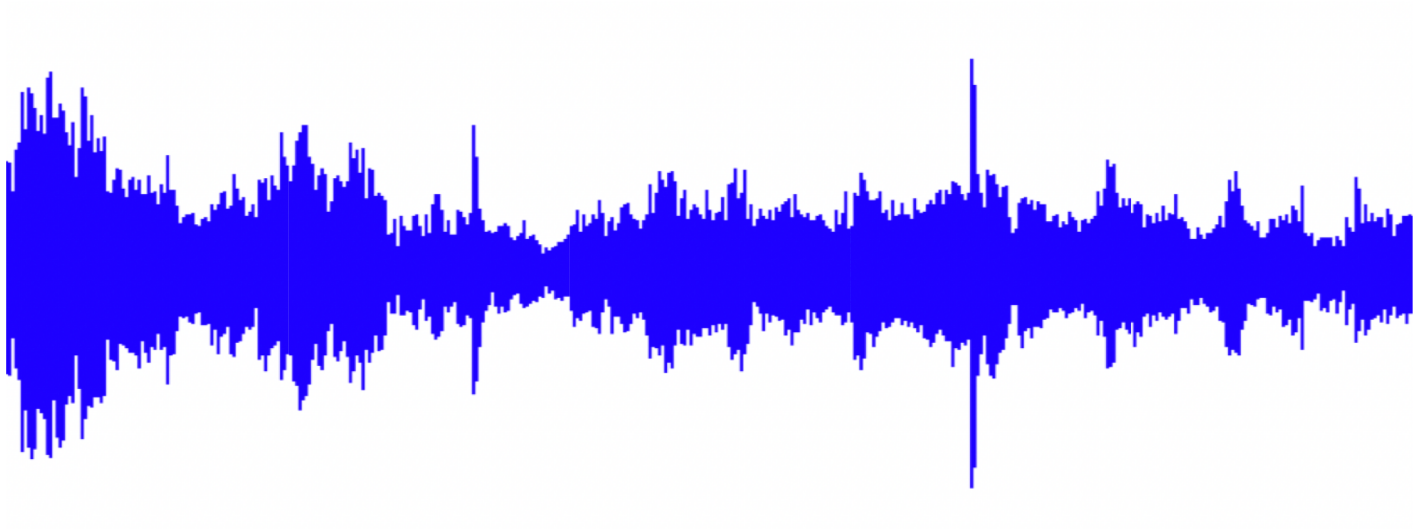


Sequences in the wild



Audio

Sequences in the wild

character:

6.S191 Introduction to Deep Learning

word:

Text

A Sequence Modeling Problem: Predict the Next Word

A sequence modeling problem: predict the next word

"This morning I took my cat for a walk."

Adapted from H. Suresh, 6.S191 2018

A sequence modeling problem: predict the next word

"This morning I took my cat for a walk."

given these words

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Idea #1: use a fixed window

"This morning I took my cat for a walk."

given these two words

predict the next word

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Idea #1: use a fixed window

"This morning I took my cat for a walk."

given these two words

predict the next word

One-hot feature encoding: tells us what each word is

[1 0 0 0 0 0 1 0 0 0]

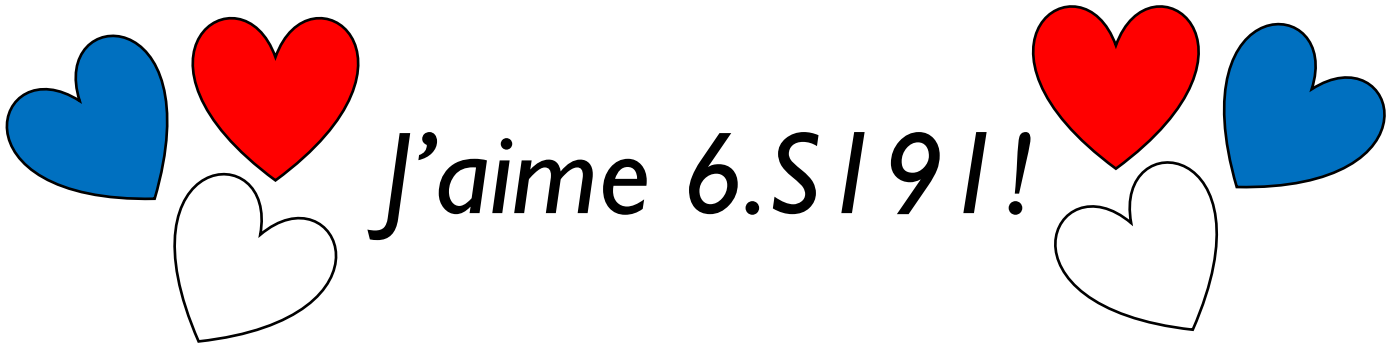
for a

prediction

Adapted from H. Suresh, 6.S191 2018

Problem #1: can't model long-term dependencies

"**France** is where I grew up, but I now live in Boston. I speak fluent ___."



We need information from **the distant past** to accurately predict the correct word.

Adapted from H. Suresh, 6.S191 2018

Idea #2: use entire sequence as set of counts

"This morning I took my cat for a"

"bag of words"

[0 1 0 0 1 0 0 ... 0 0 1 1 0 0 0 1]

prediction

Adapted from H. Suresh, 6.S191 2018

Problem #2: counts don't preserve order

The food was good, not bad at all.

vs.

The food was bad, not good at all.

Adapted from H. Suresh, 6.S191 2018

Idea #3: use a really big fixed window

"This morning I took my cat for a walk."

given these words

predict the next word

[1 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 ...]

morning I took this cat

prediction

Adapted from H. Suresh, 6.S191 2018

Problem #3: no parameter sharing

[1 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 ...]

this morning took the cat

Each of these inputs has a **separate parameter:**

Adapted from H. Suresh, 6.S191 2018

Problem #3: no parameter sharing

[1 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 ...]

this morning took the cat

Each of these inputs has a **separate parameter:**

[0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 1 ...]

this morning

Adapted from H. Suresh, 6.S191 2018

Problem #3: no parameter sharing

[1 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 ...]

this morning took the cat

Each of these inputs has a **separate parameter:**

[0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1 ...]

this morning

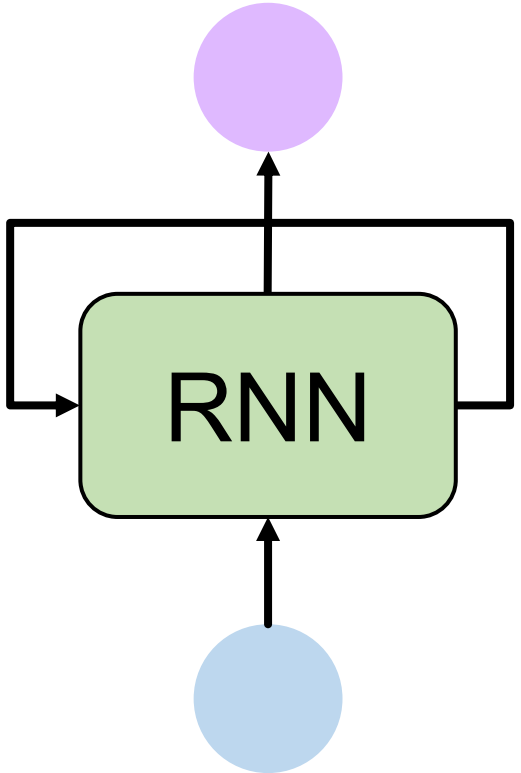
Things we learn about the sequence **won't transfer** if they appear **elsewhere** in the sequence.

Adapted from H. Suresh, 6.S191 2018

Sequence modeling: design criteria

To model sequences, we need to:

1. Handle **variable-length** sequences
2. Track **long-term** dependencies
3. Maintain information about **order**
4. **Share parameters** across the sequence

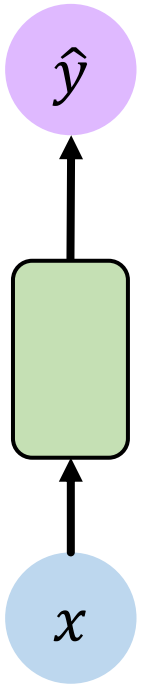


Today: **Recurrent Neural Networks (RNNs)** as an approach to sequence modeling problems

Adapted from H. Suresh, 6.S191 2018

Recurrent Neural Networks (RNNs)

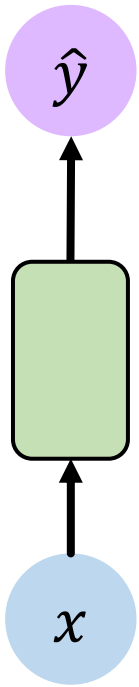
Standard feed-forward neural network



One to One
"Vanilla" neural network

[1]

Recurrent neural networks: sequence modeling



One to One
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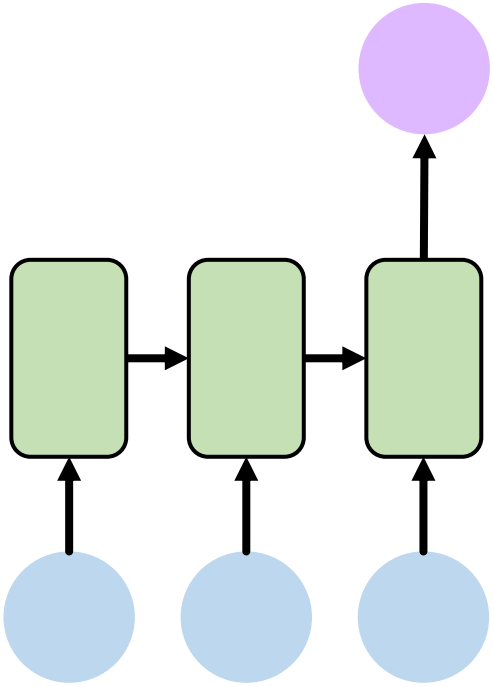
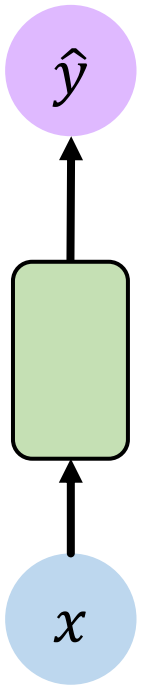


Many to One
Sentiment Classification

Recurrent neural networks: sequence modeling



One to One
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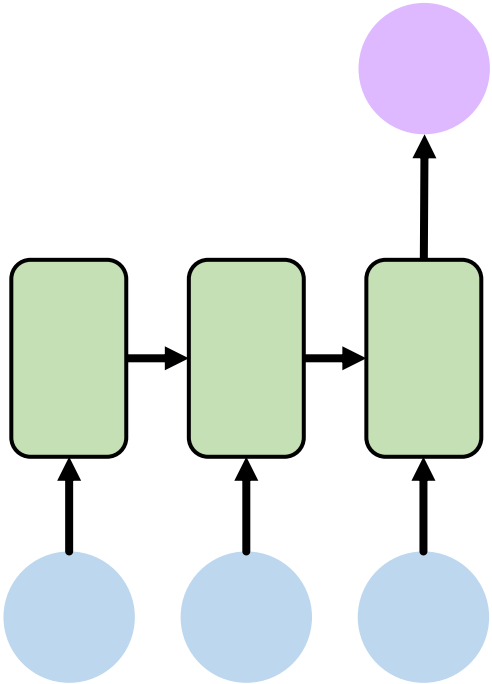

Many to One
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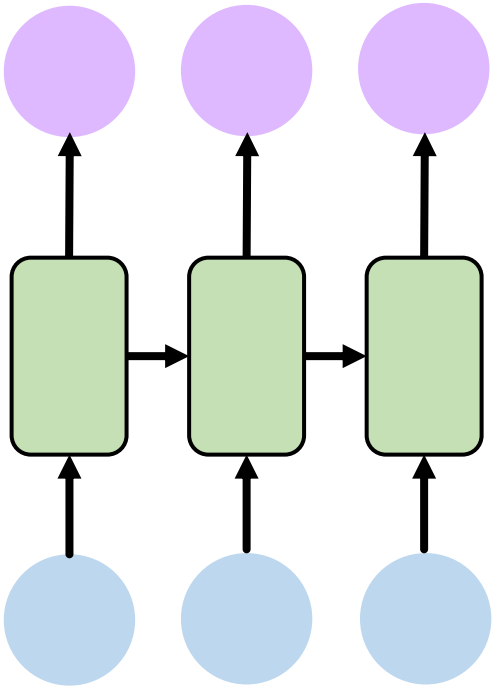
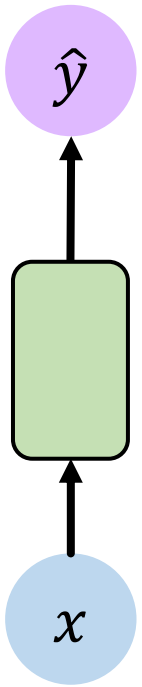


Many to Many
Music Generation

6.S191 Lab!

[1]

Recurrent neural networks: sequence modeling



One to One
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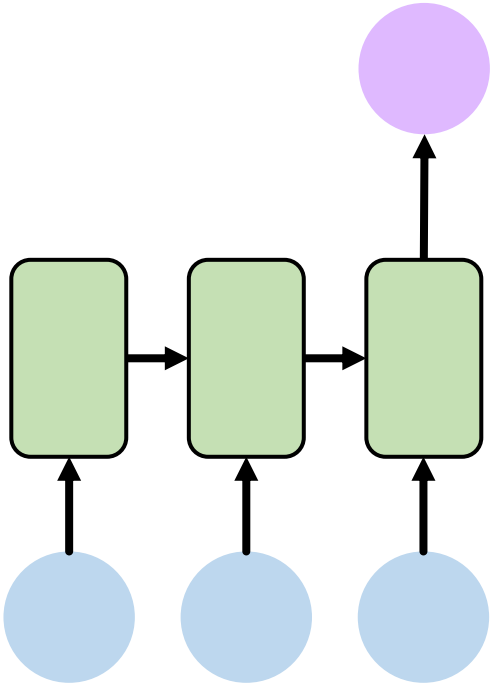

Many to One
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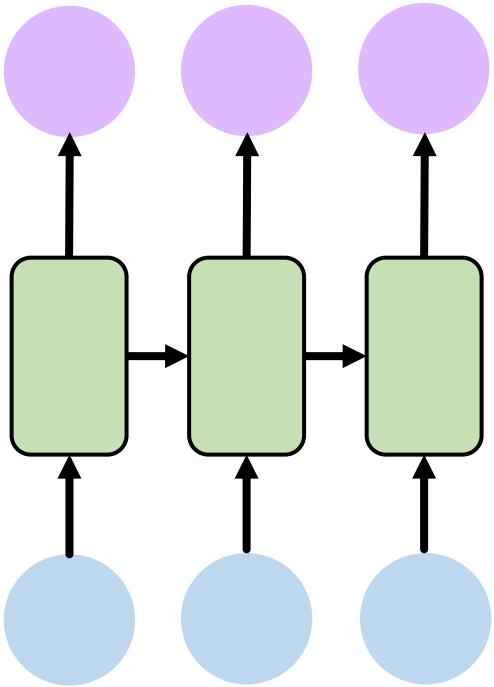


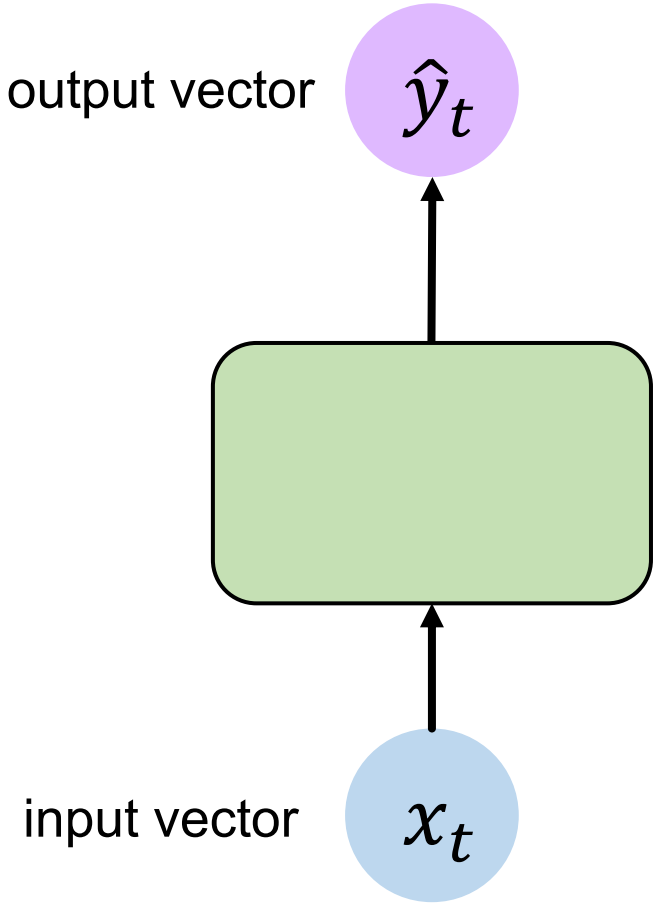
Many to Many
Music Generation

6.S191 Lab!

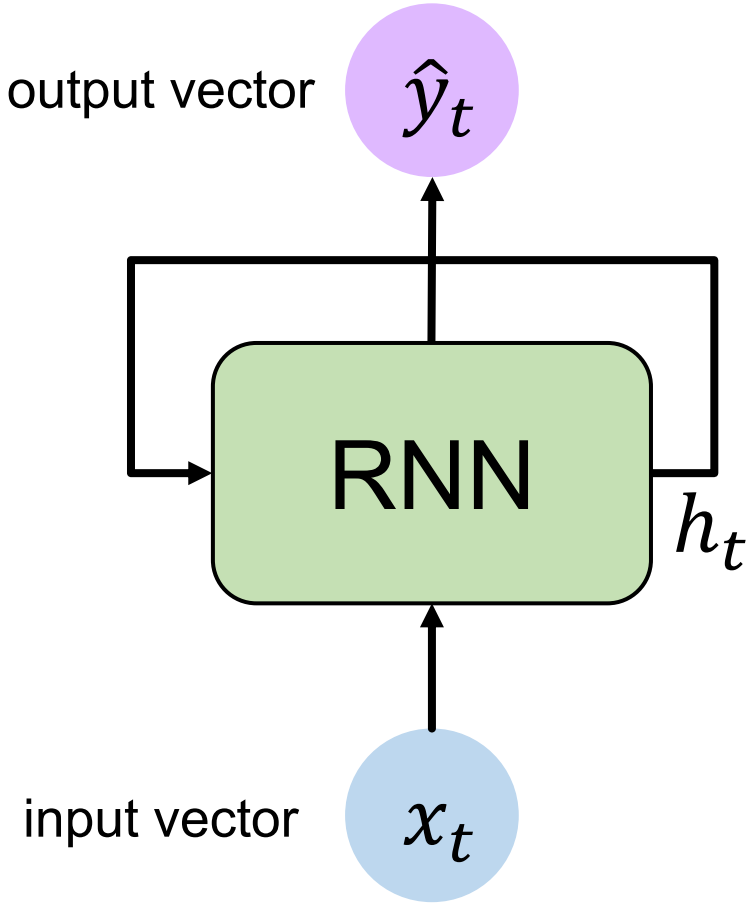
… and many other architectures and applications

[1]

A standard "vanilla" neural network



A recurrent neural network (RNN)



[2]

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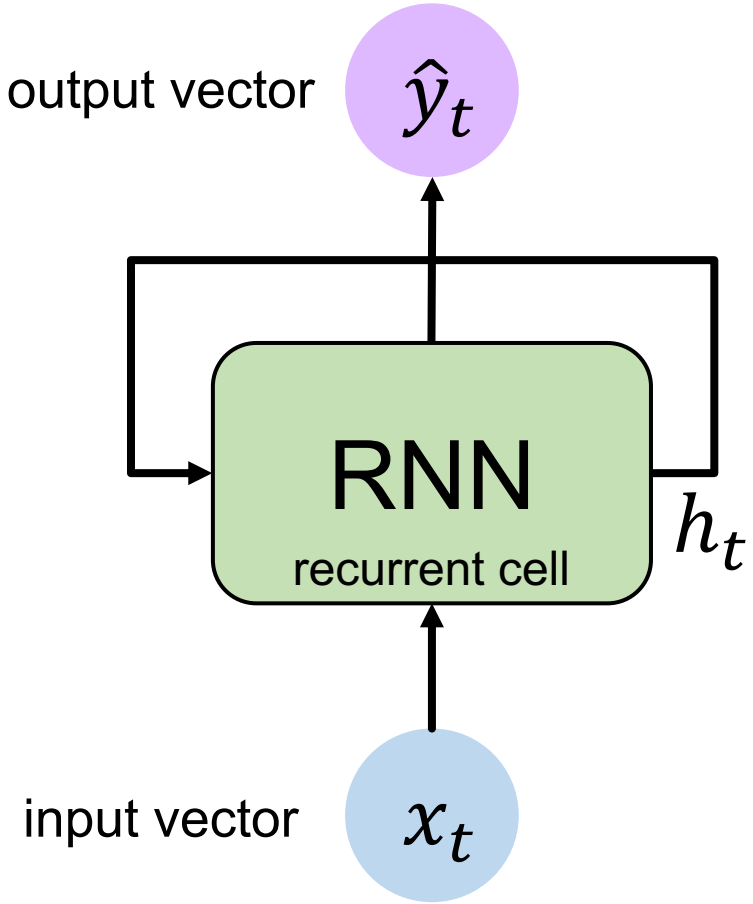

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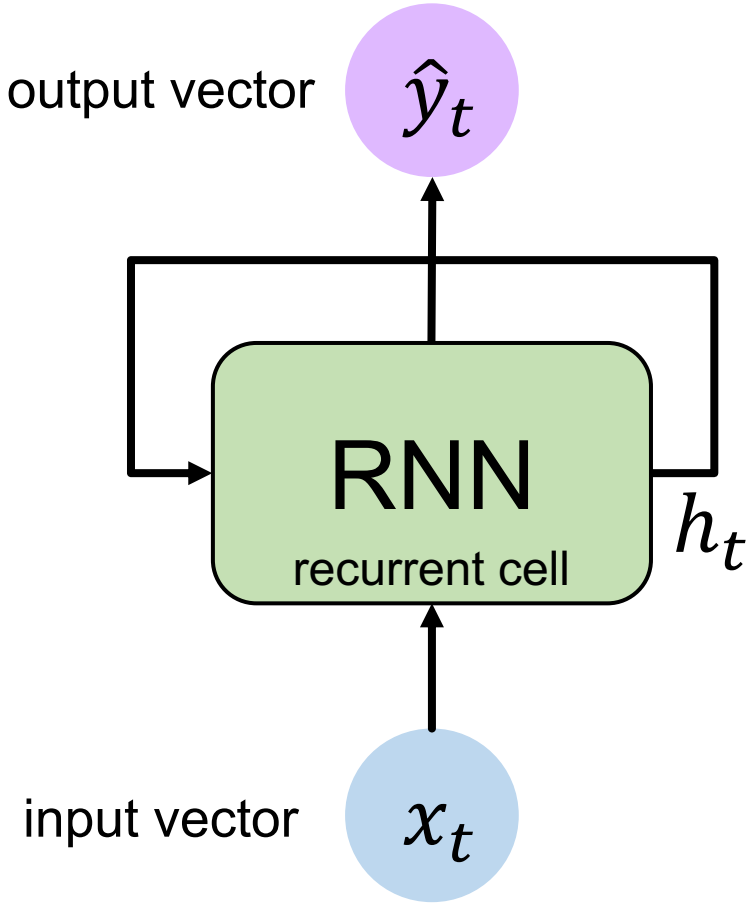

Apply a **recurrence relation** at every time step to process a sequence:

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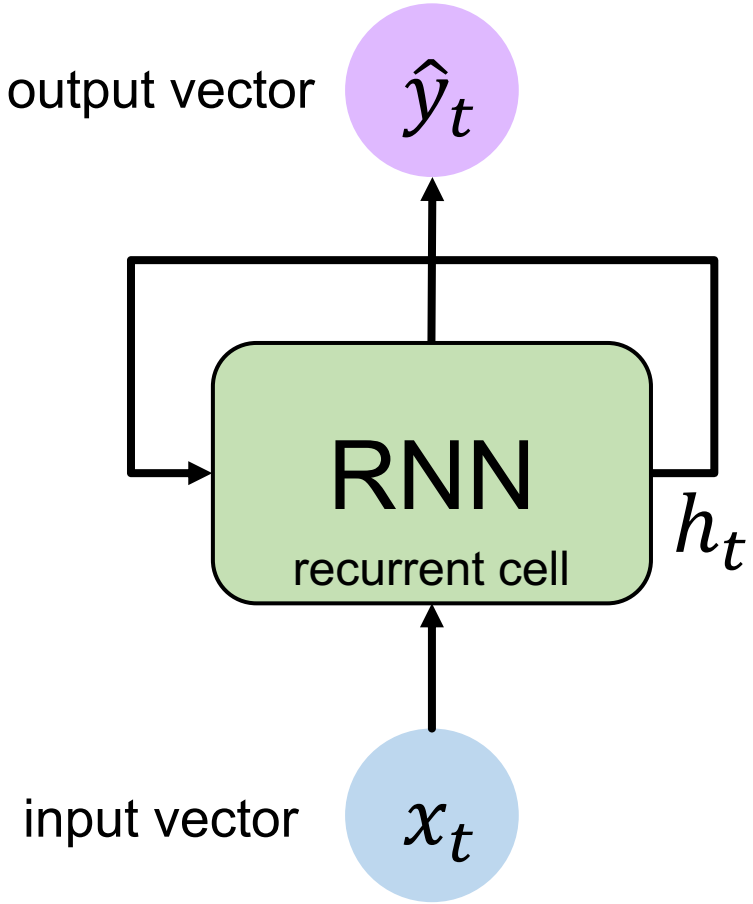


Apply a **recurrence relation** at every time step to process a sequence:

$$h_t = f_W(h_{t-1}, x_t)$$

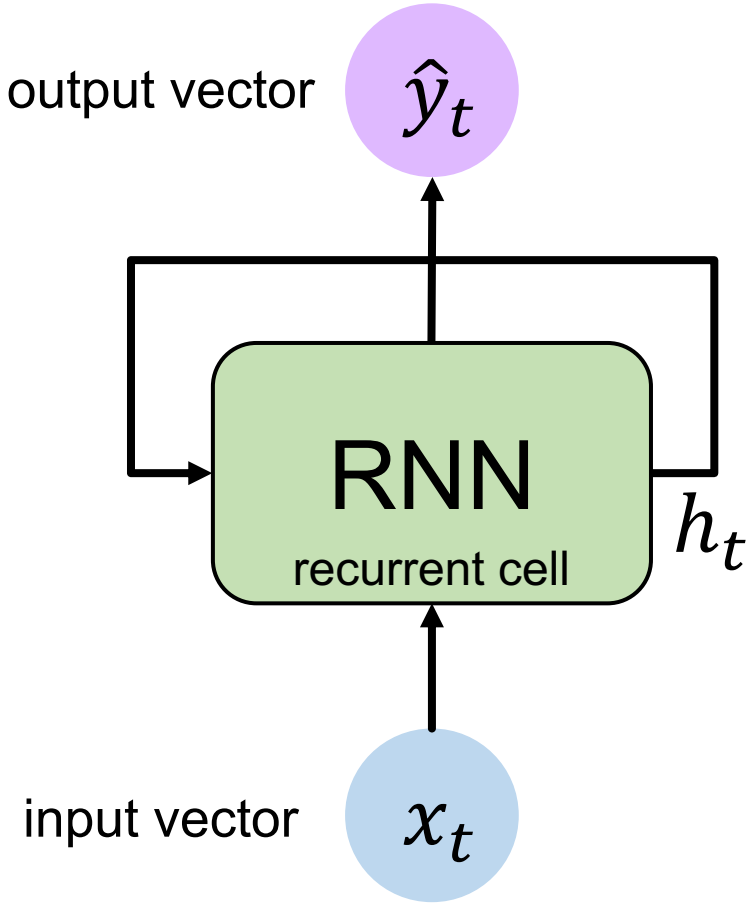
cell state

function parameterized by W

old state

input vector at time step *t*

A recurrent neural network (RNN)



Apply a **recurrence relation** at every time step to process a sequence:

$$\boxed{h_t} = \boxed{f_W}\left(\boxed{h_{t-1}}, \boxed{x_t}\right)$$

new state

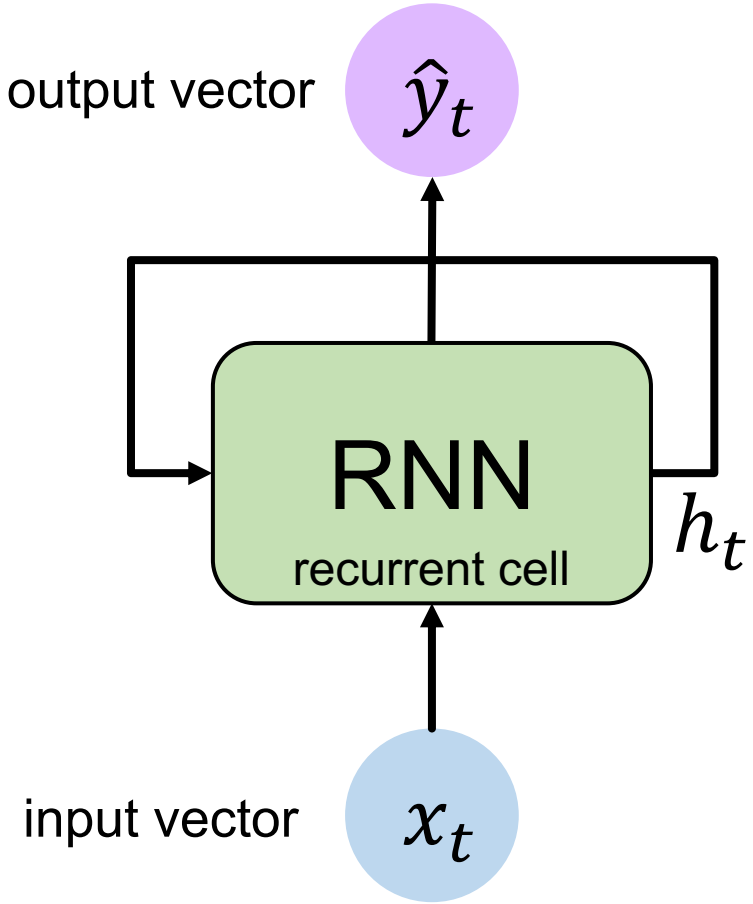
function parameterized by W

old state

input vector at time step *t*

Note: the same function and set of parameters are used at every time step

RNN state update and output



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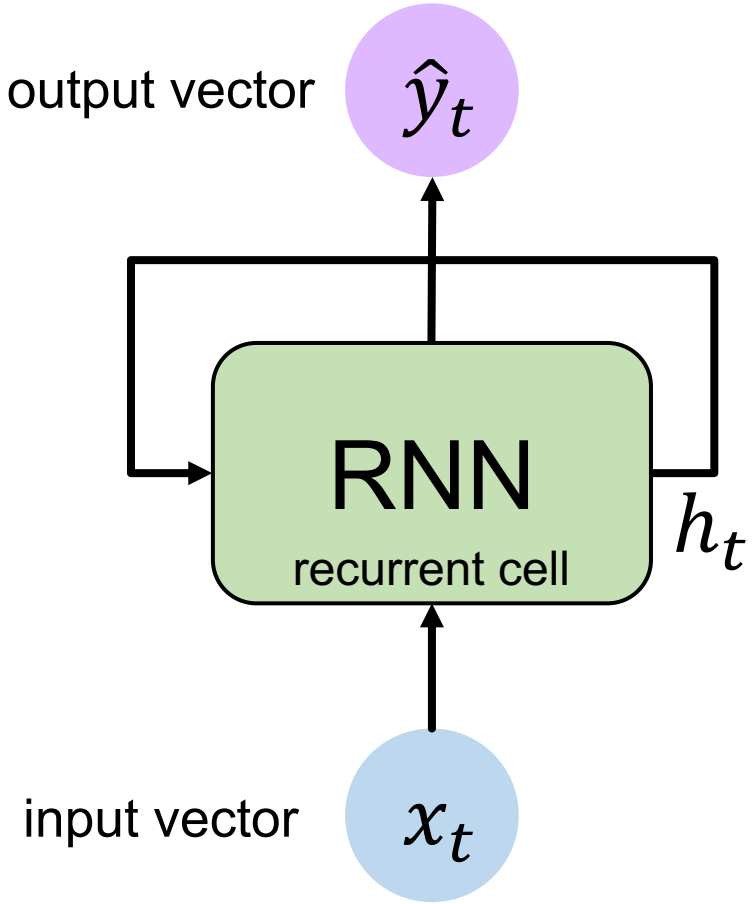
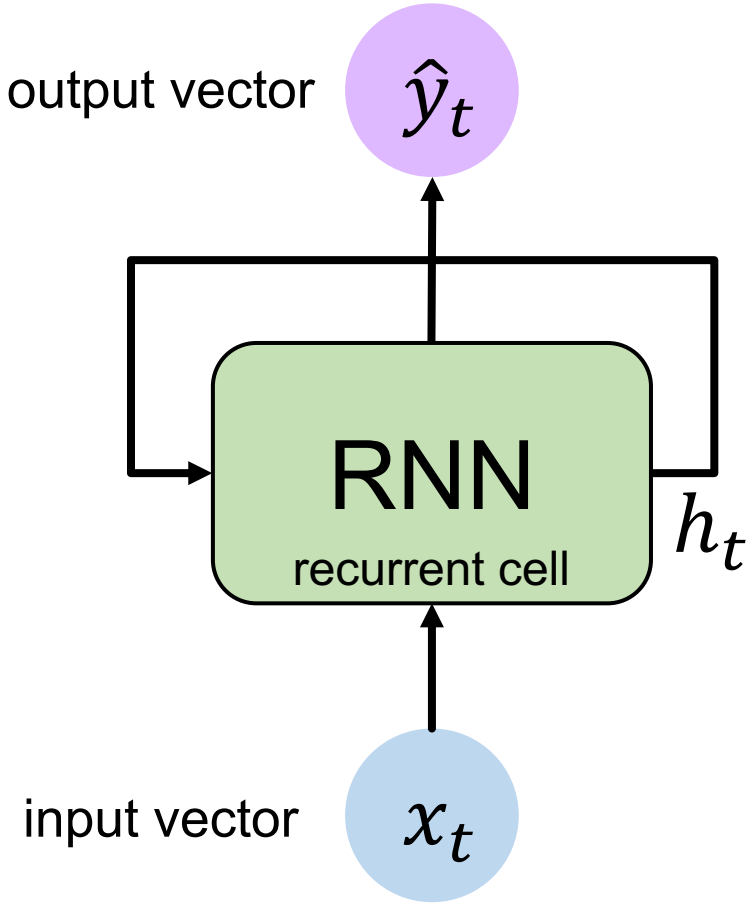


Input Vector

RNN state update and output

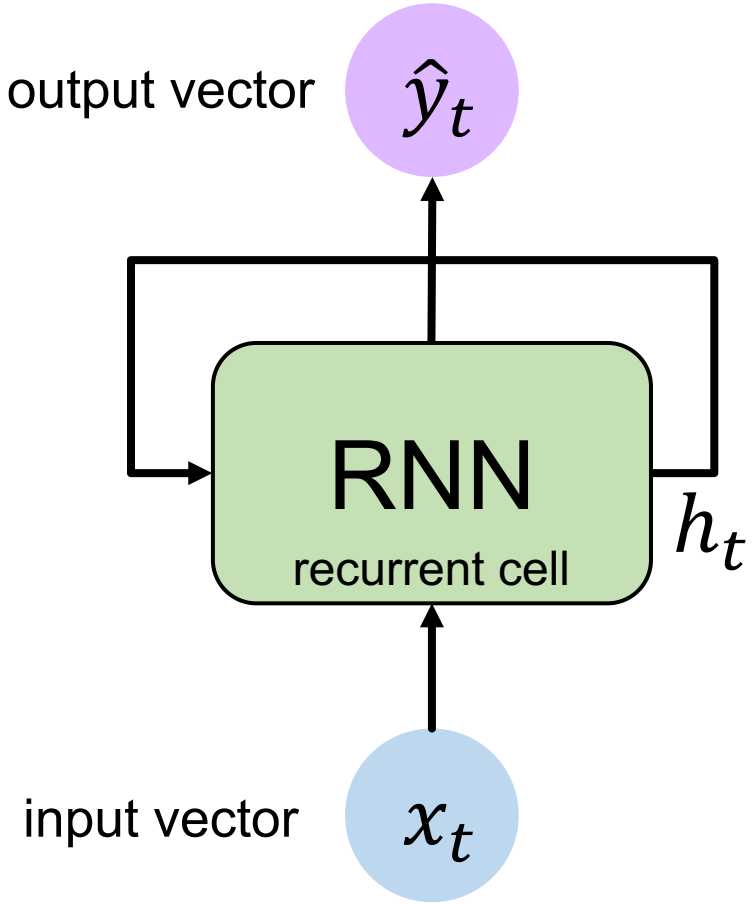


Update Hidden State

$$h_t = \tanh(\boldsymbol{W_{hh}} h_{t-1} + \boldsymbol{W_{xh}} x_t)$$

Input Vector

RNN state update and output



Output Vector

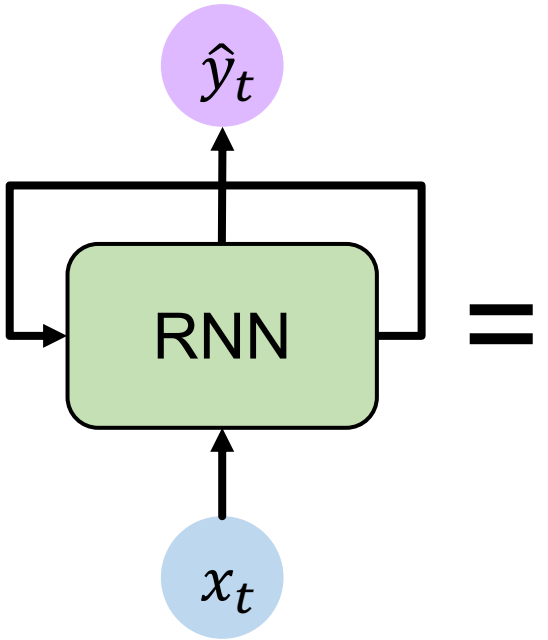
$$\hat{y}_t = \boldsymbol{W}_{\boldsymbol{hy}} h_t$$

Update Hidden State

$$h_t = \tanh(\boldsymbol{W}_{\boldsymbol{hh}} h_{t-1} + \boldsymbol{W}_{\boldsymbol{xh}} x_t)$$

Input Vector

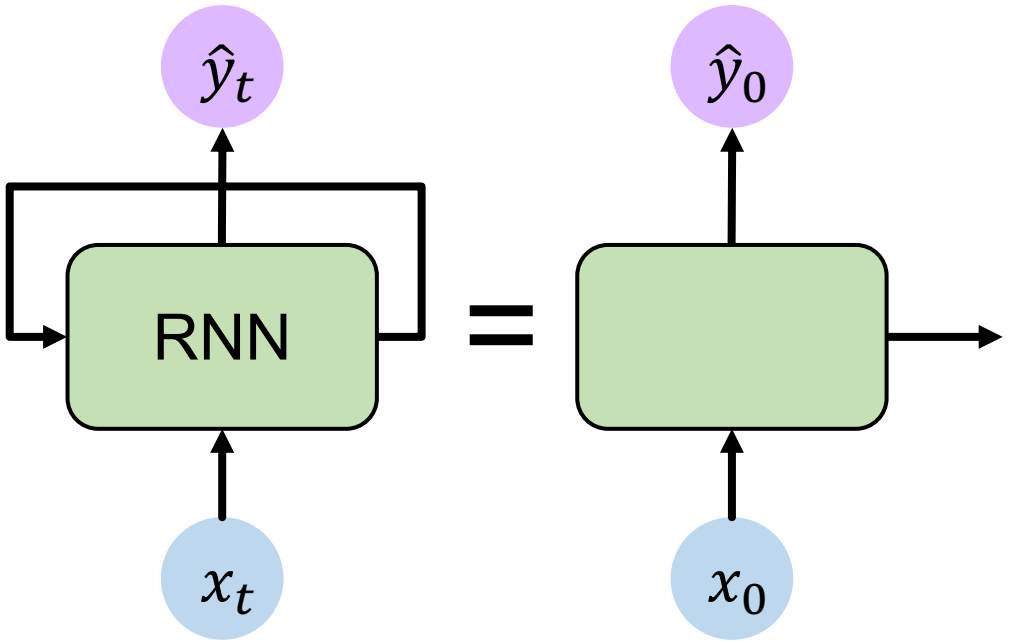
RNNs: computational graph across time



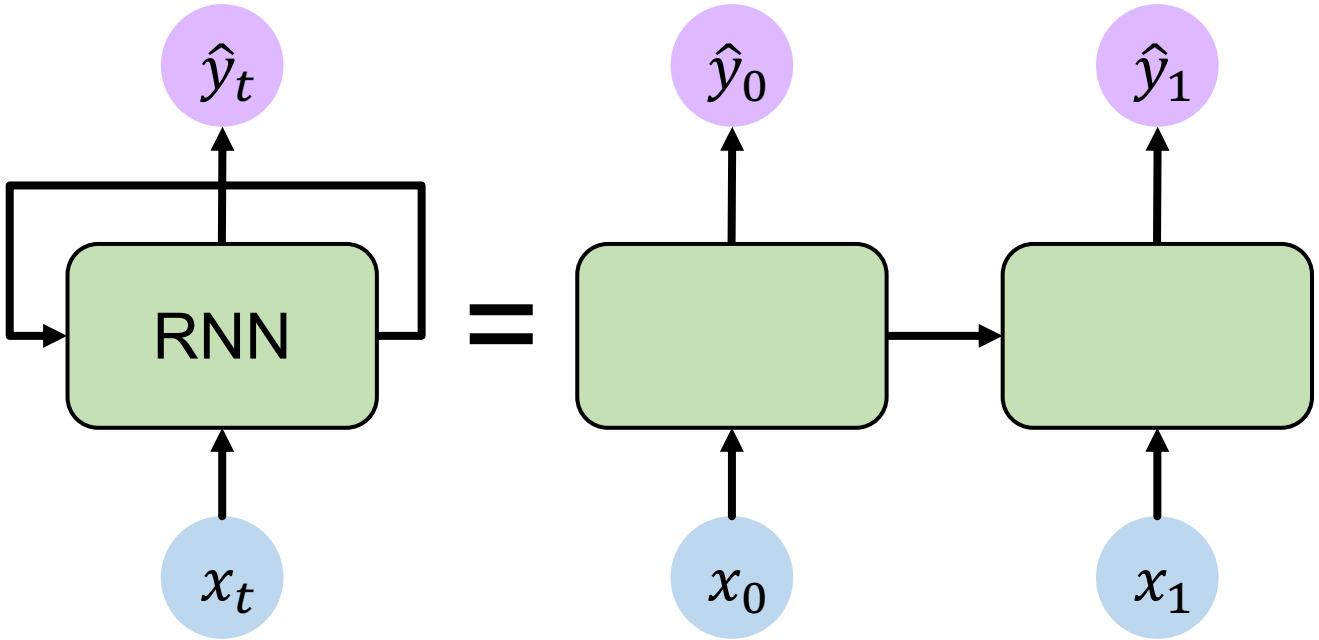
=

Represent as computational graph unrolled across time

RNNs: computational graph across time



RNNs: computational graph across time



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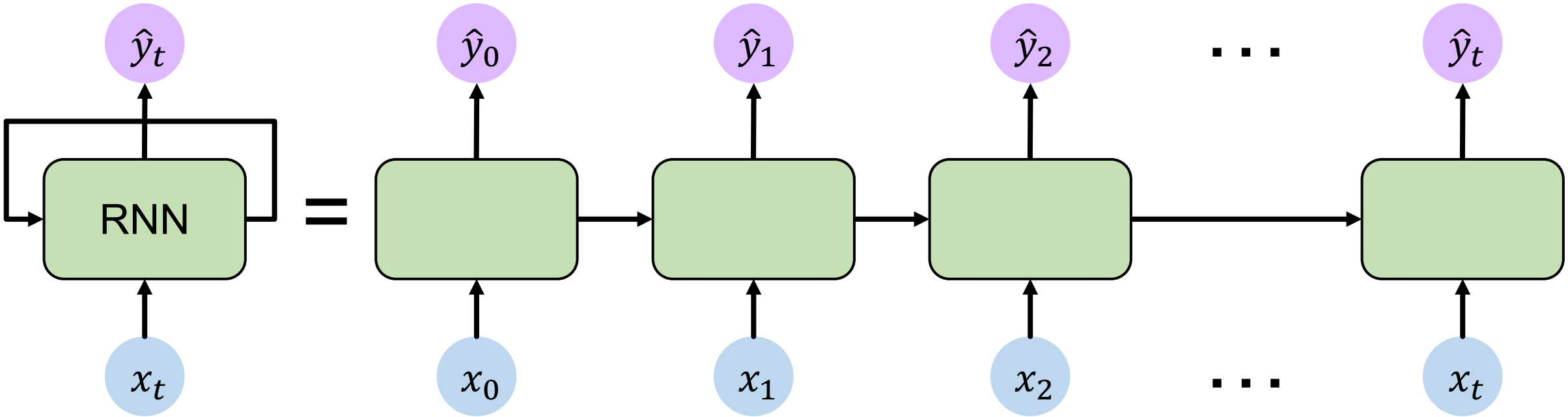

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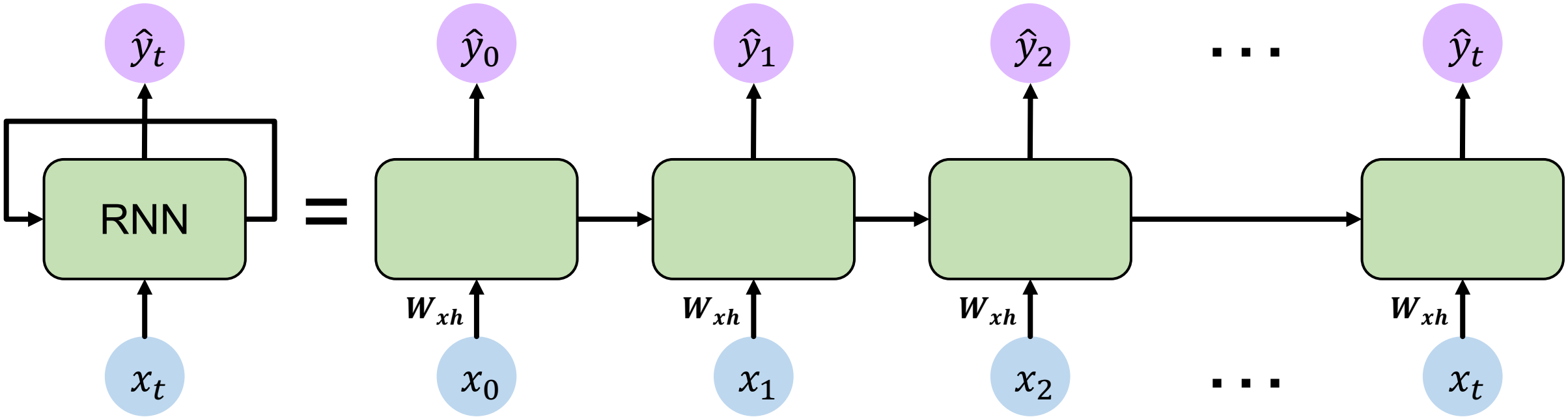
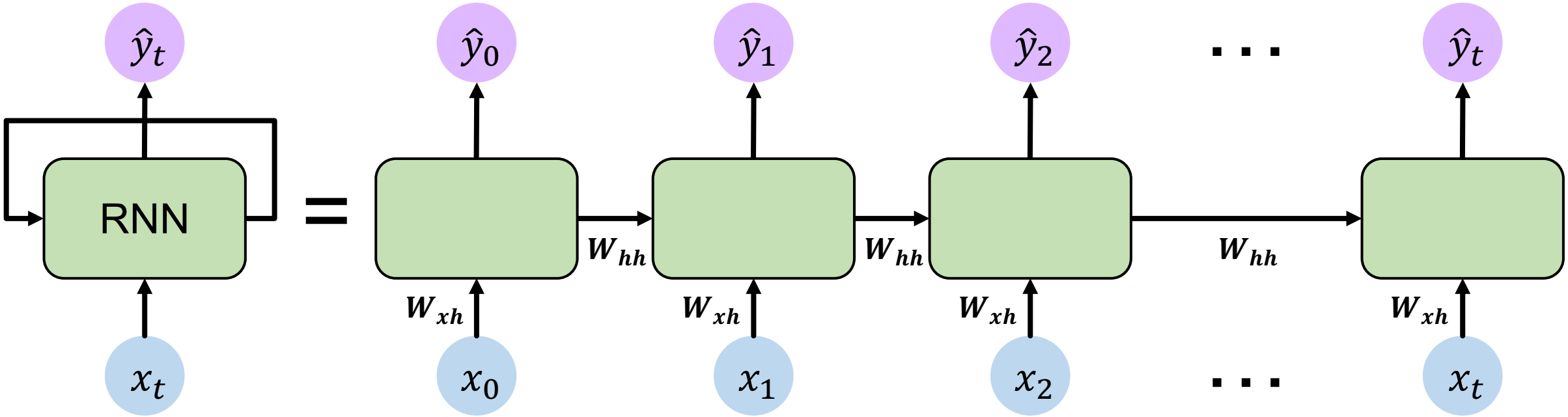
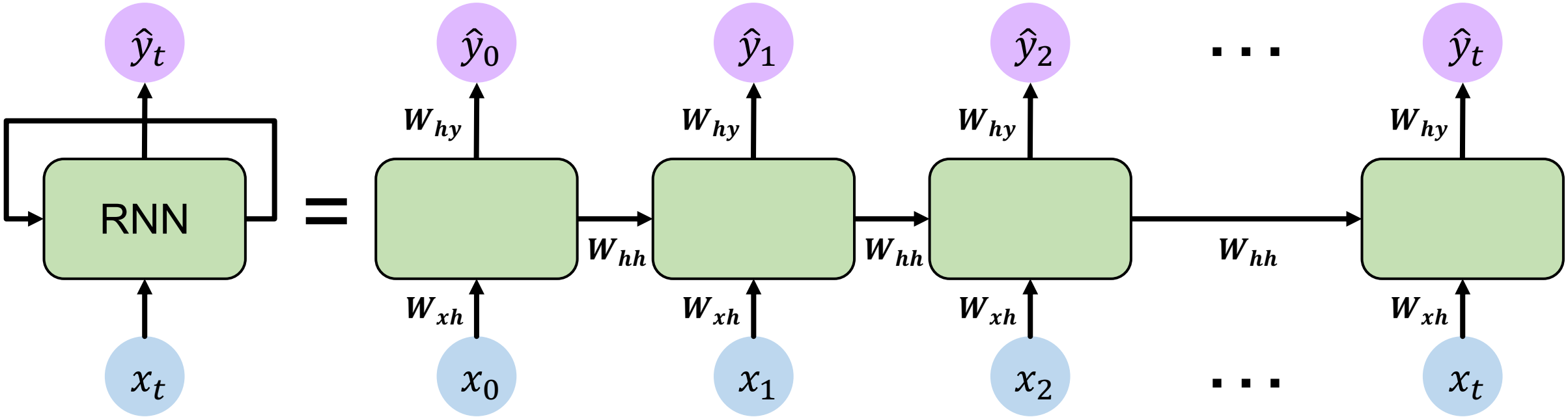


RNNs: computational graph across time

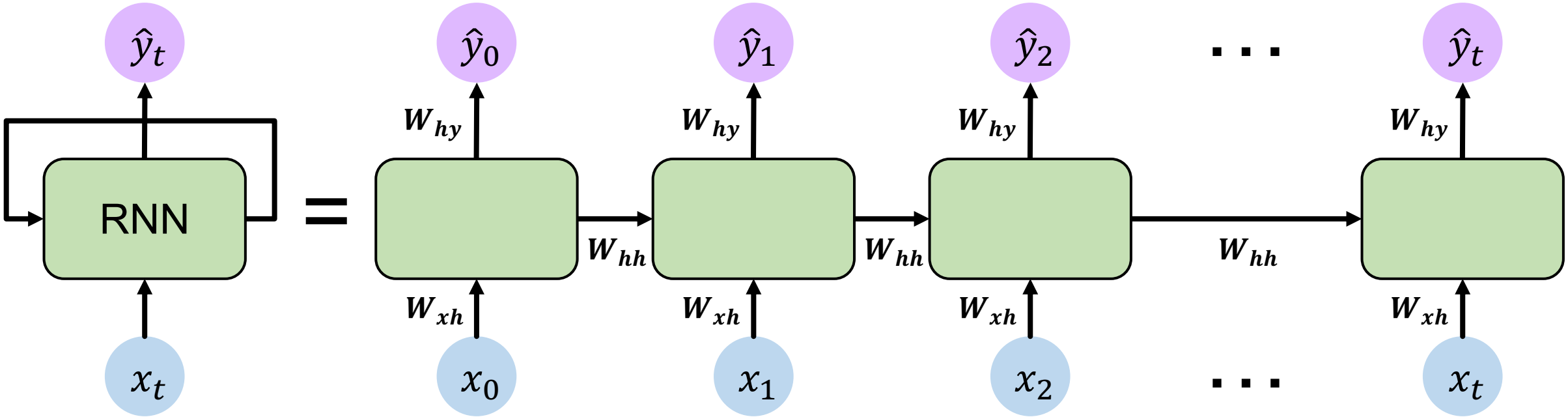


RNNs: computational graph across time



RNNs: computational graph across time

Re-use the **same weight matrices** at every time step



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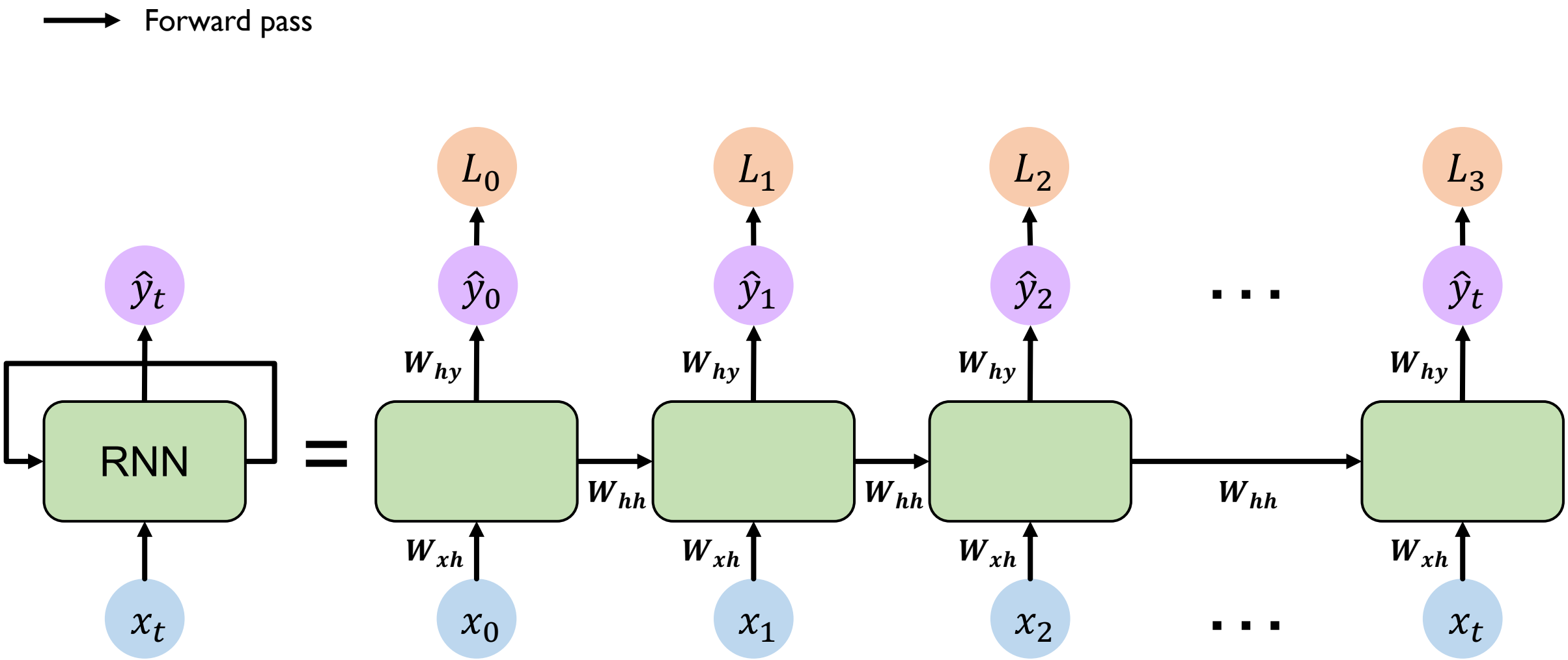
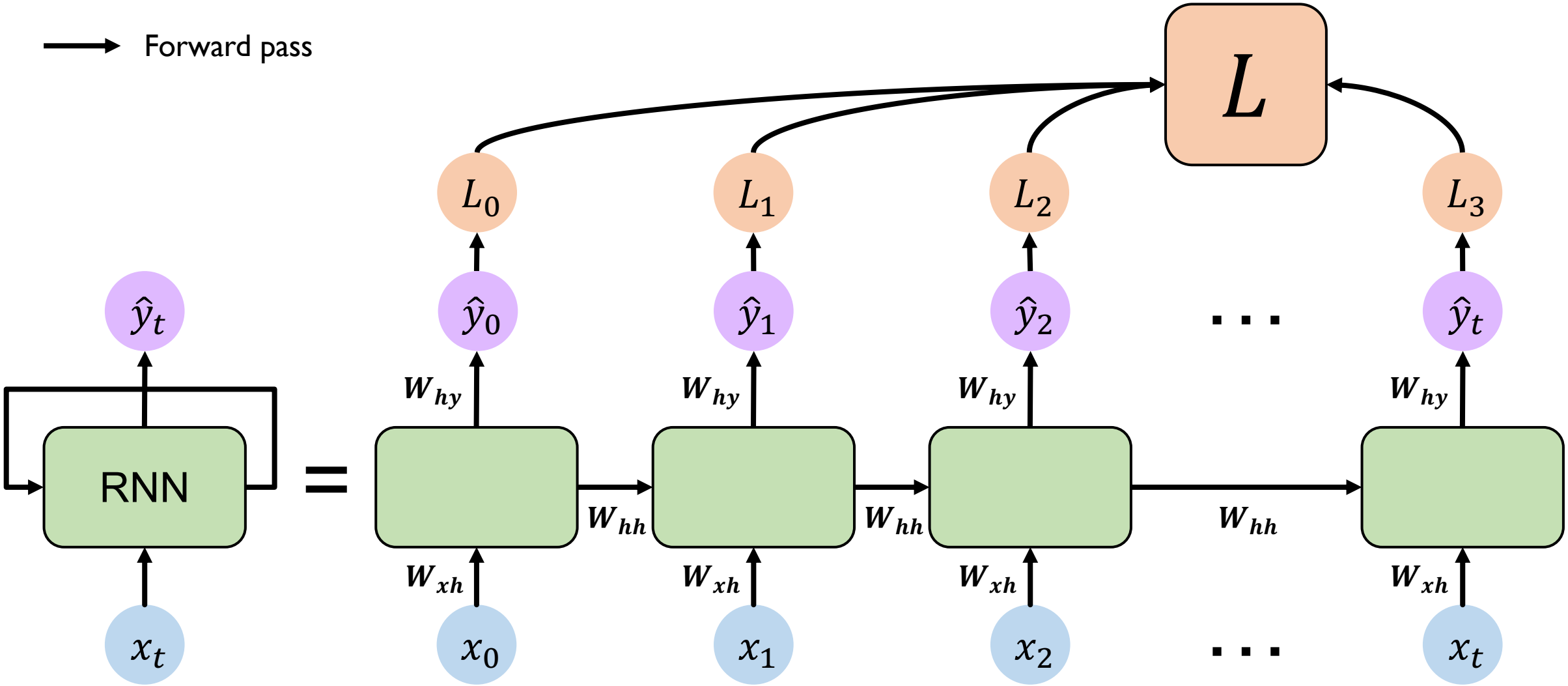
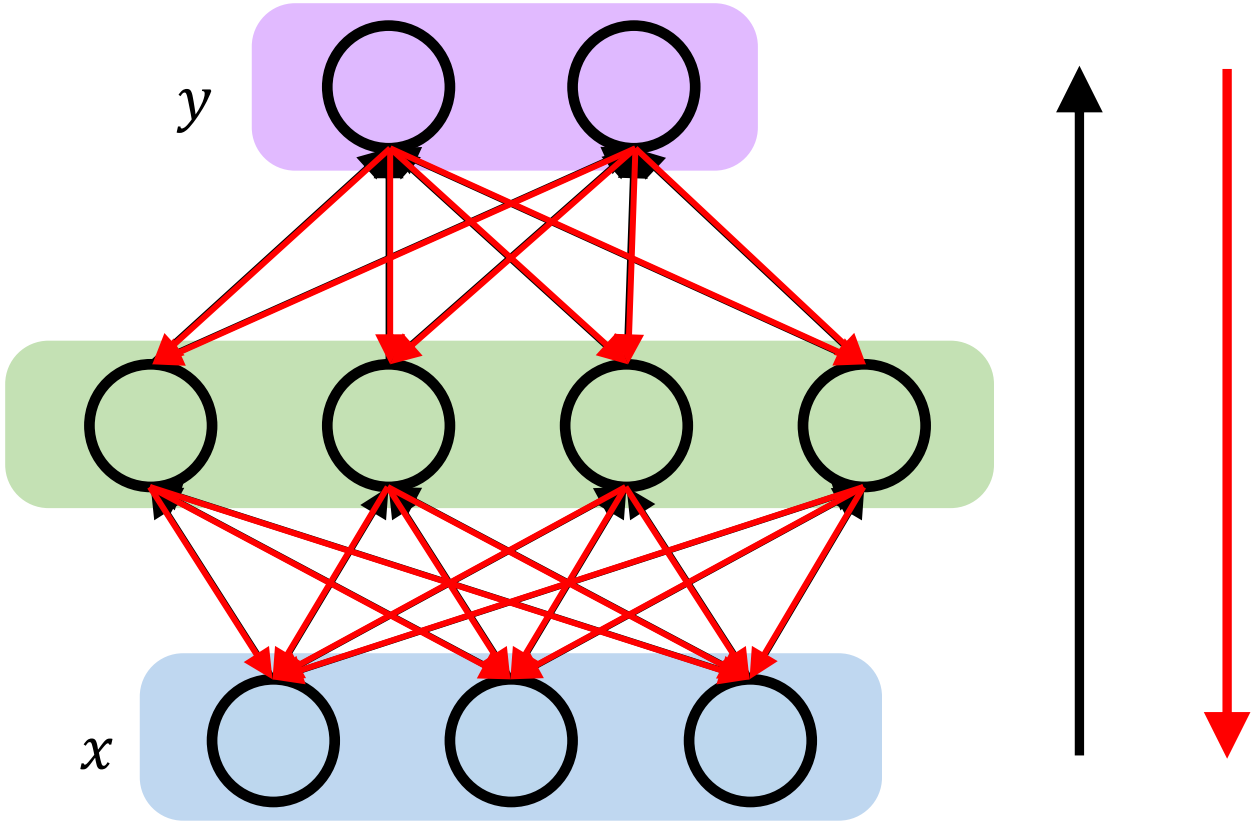


RNNs: computational graph across time



Backpropagation Through Time (BPTT)

Recall: backpropagation in feed forward models

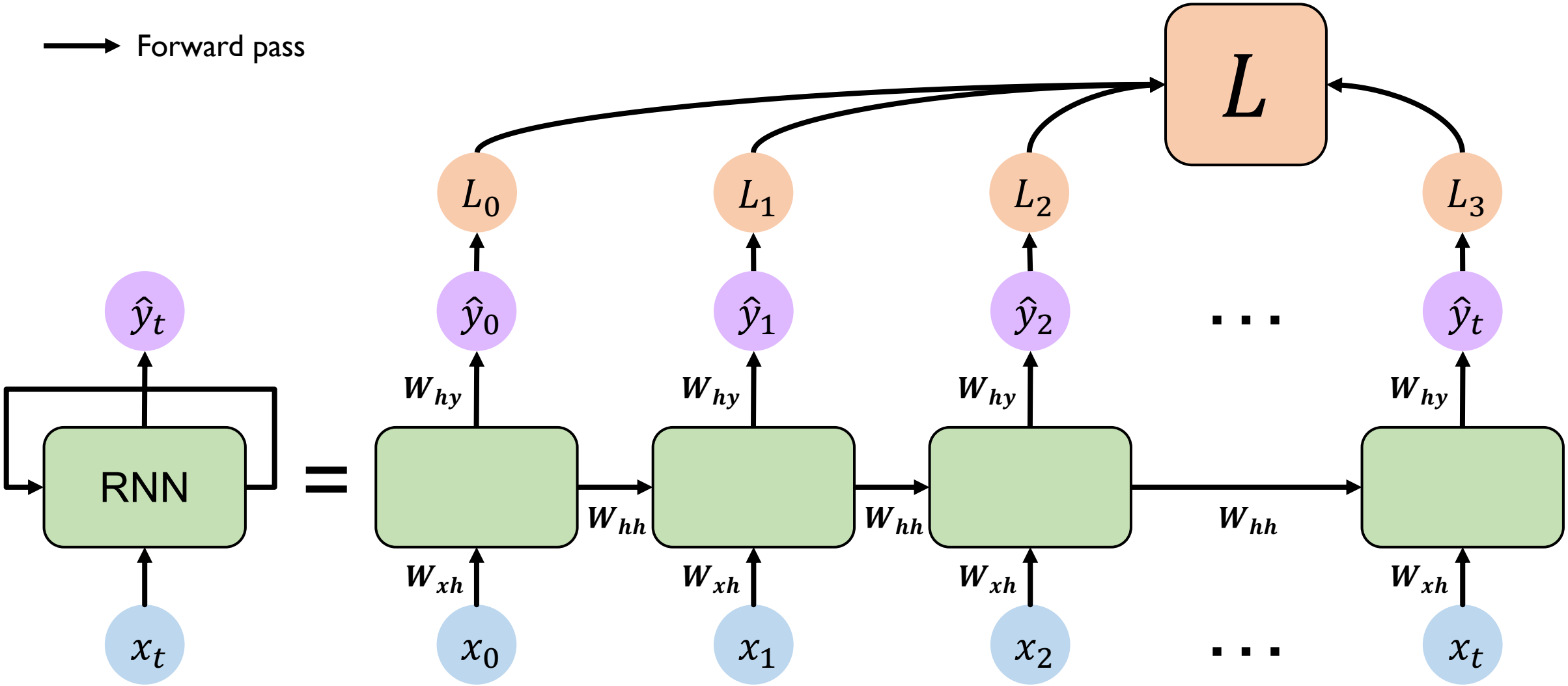


Backpropagation algorithm:

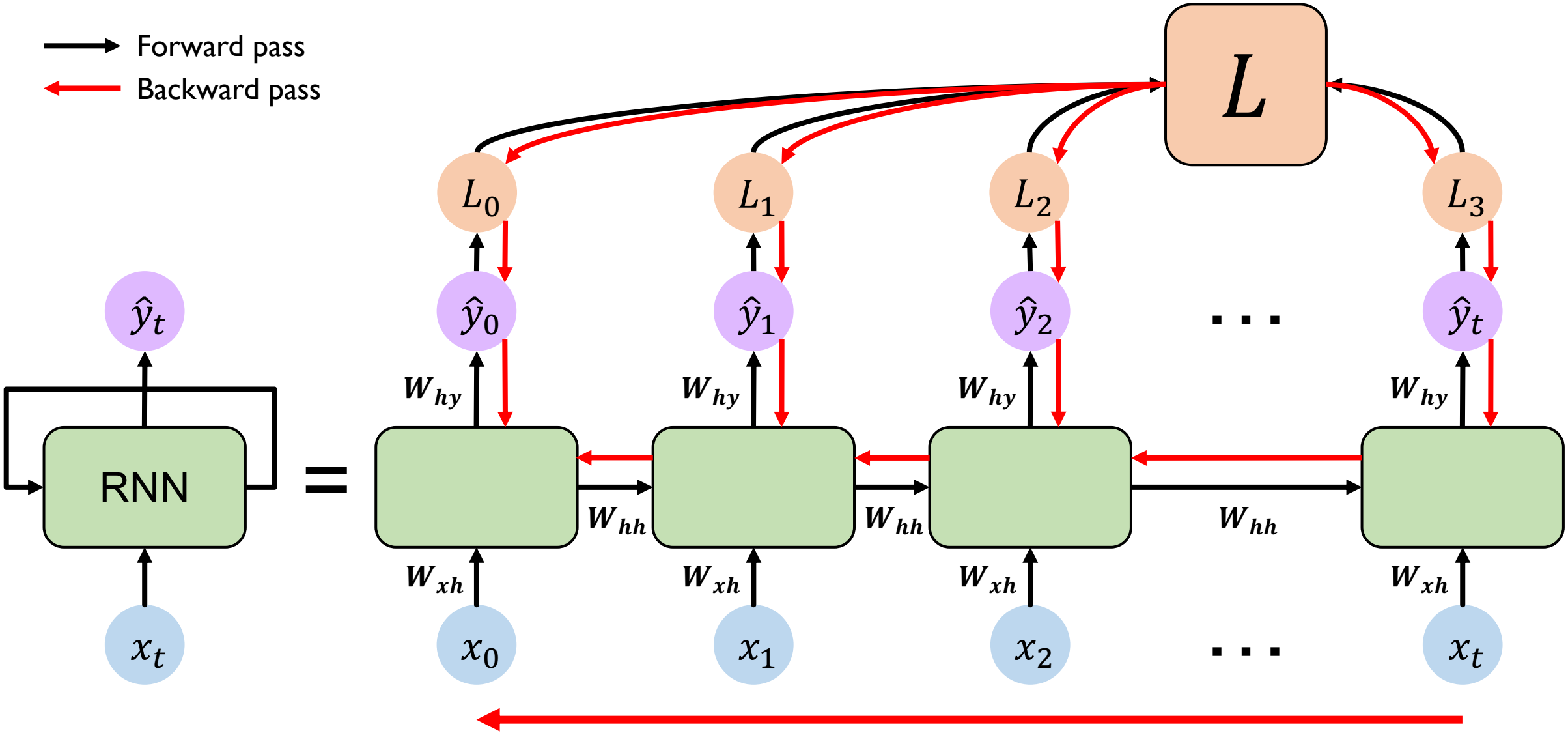
1. Take the derivative (gradient) of the loss with respect to each parameter
2. Shift parameters in order to minimize loss

[3]

RNNs: backpropagation through time

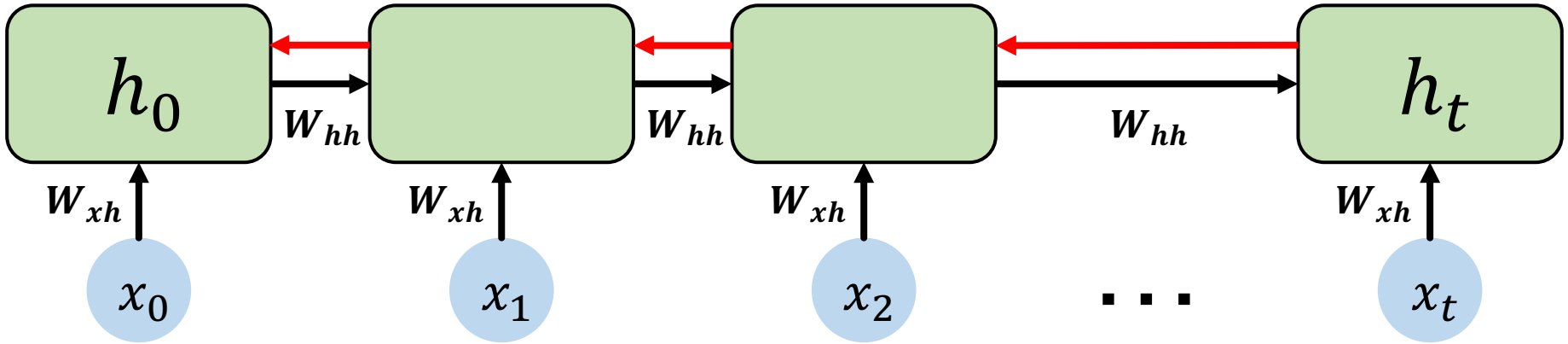


RNNs: backpropagation through time



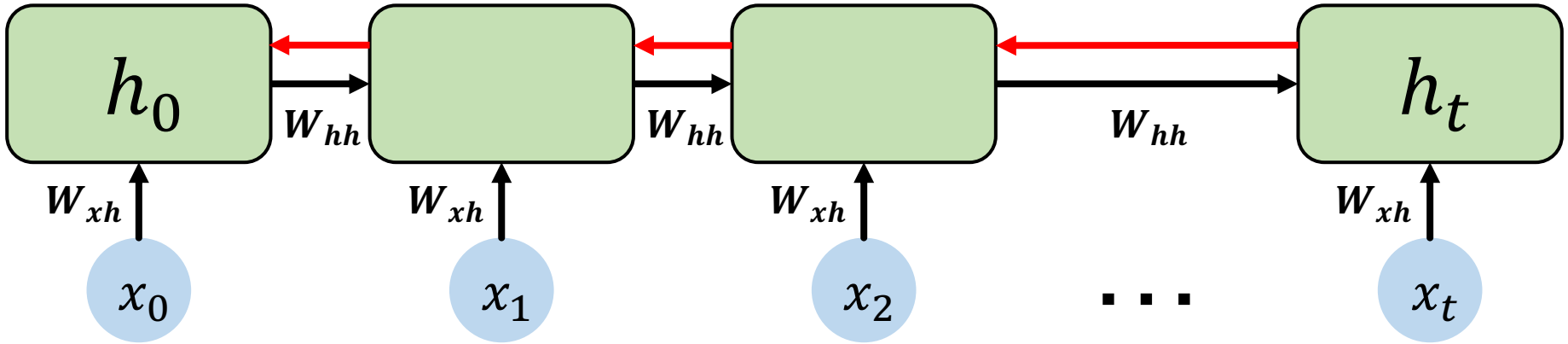
[4]

Standard RNN gradient flow



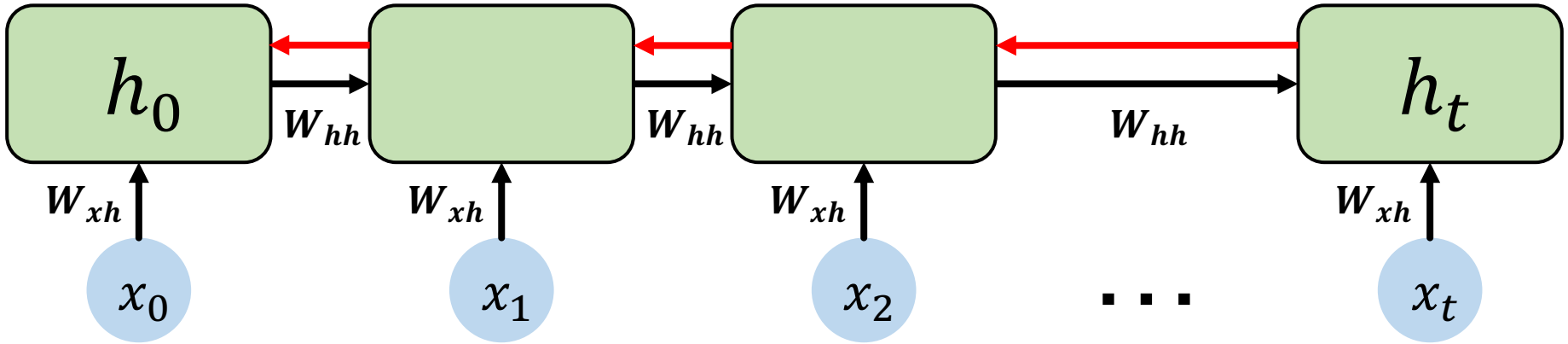
[1]

Standard RNN gradient flow



Computing the gradient wrt h_0 involves **many factors of W_{hh}** (and repeated f'!)

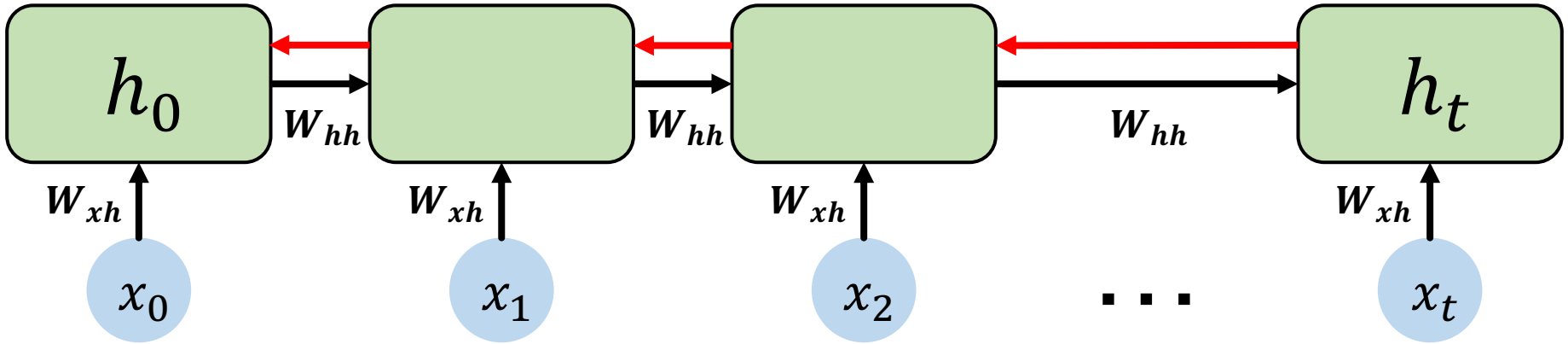
Standard RNN gradient flow: exploding gradients



Computing the gradient wrt h_0 involves **many factors of W_{hh}** (and repeated f'!)

Many values > 1:
exploding gradients

Standard RNN gradient flow: exploding gradients



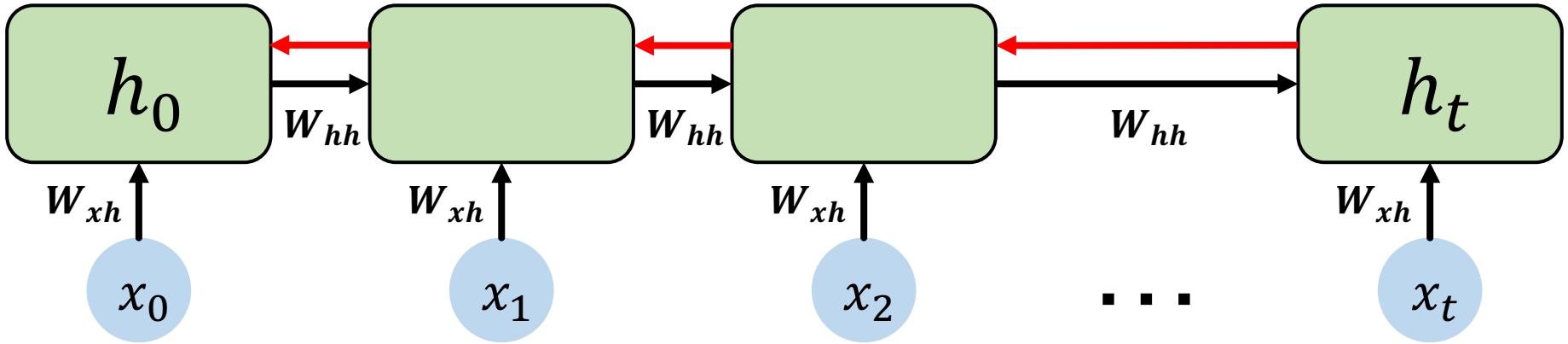
Computing the gradient wrt h_0 involves **many factors of W_{hh}** (and repeated f'!)

Many values > 1:
exploding gradients

Gradient clipping to scale big gradients

[1]

Standard RNN gradient flow: vanishing gradients



Computing the gradient wrt h_0 involves **many factors of W_{hh}** (and repeated f'!)

Many values > 1:
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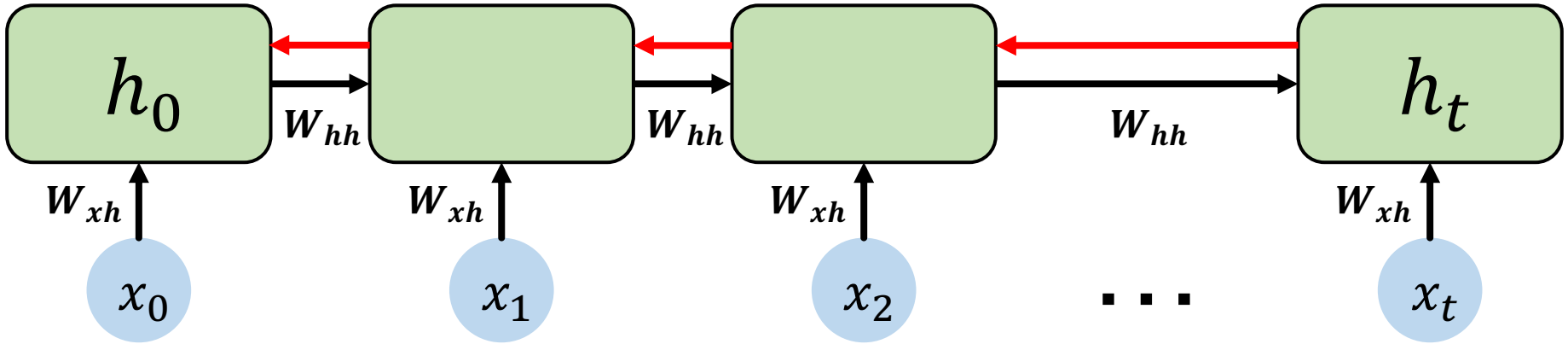
Gradient clipping to scale big gradients

Many values < 1:
vanishing gradients

[1]

Standard RNN gradient flow: vanishing gradients



Computing the gradient wrt h_0 involves **many factors of W_{hh}** (and repeated f'!)

Largest singular value > 1:
exploding gradients

Gradient clipping to scale big gradients

Largest singular value < 1:
vanishing gradients

1. Activation function
2. Weight initialization
3. Network architecture

[1]

The problem of long-term dependencies

Why are vanishing gradients a problem?

The problem of long-term dependencies

Why are vanishing gradients a problem?

Multiply many **small numbers** together

The problem of long-term dependencies

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Multiply many **small numbers** together

Errors due to further back time steps have smaller and smaller gradients

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Bias network to capture short-term dependencies

The problem of long-term dependencies

"The clouds are in the ___"

Why are vanishing gradients a problem?

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Bias network to capture short-term dependencies

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↓

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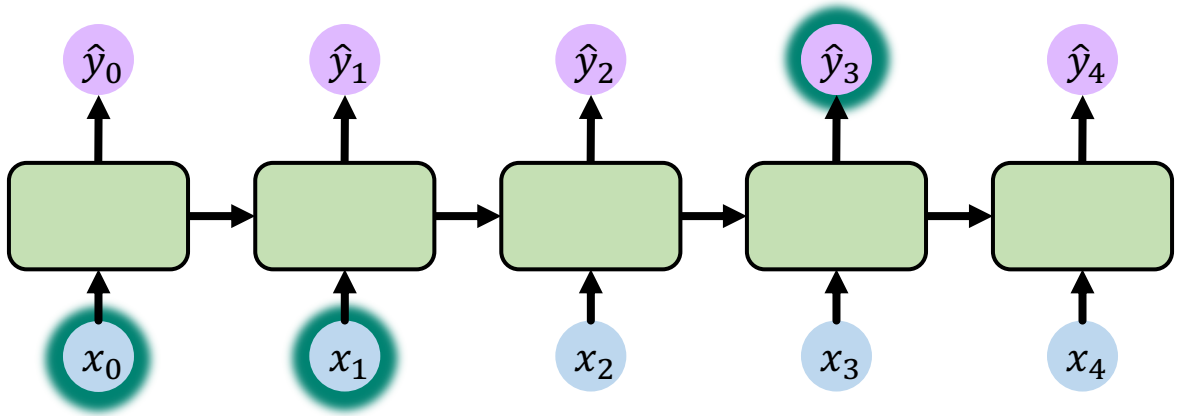
↓

Bias parameters to capture short-term dependencies

"The clouds are in the ___"



The problem of long-term dependencies

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Errors due to further back time steps have smaller and smaller gradients

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Bias parameters to capture short-term dependencies

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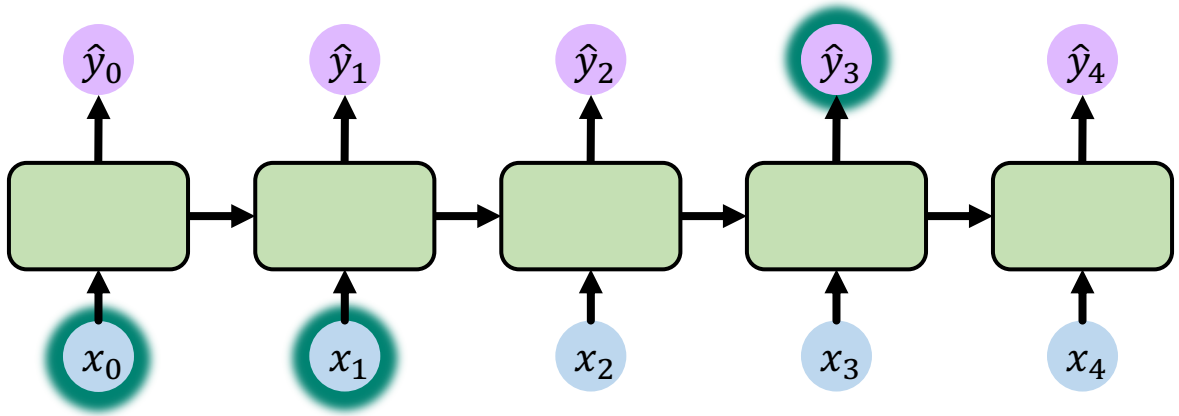

"I grew up in France, … and I I speak fluent___ "

The problem of long-term dependencies

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↓

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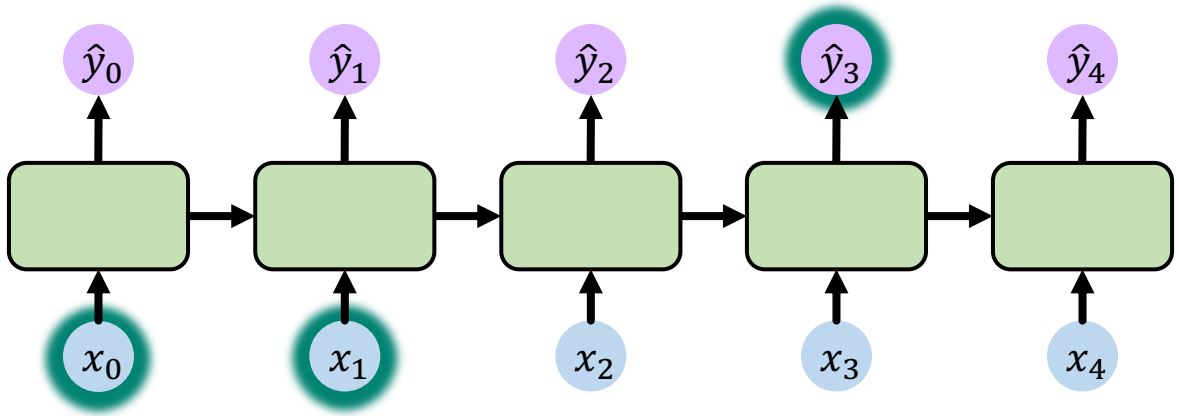
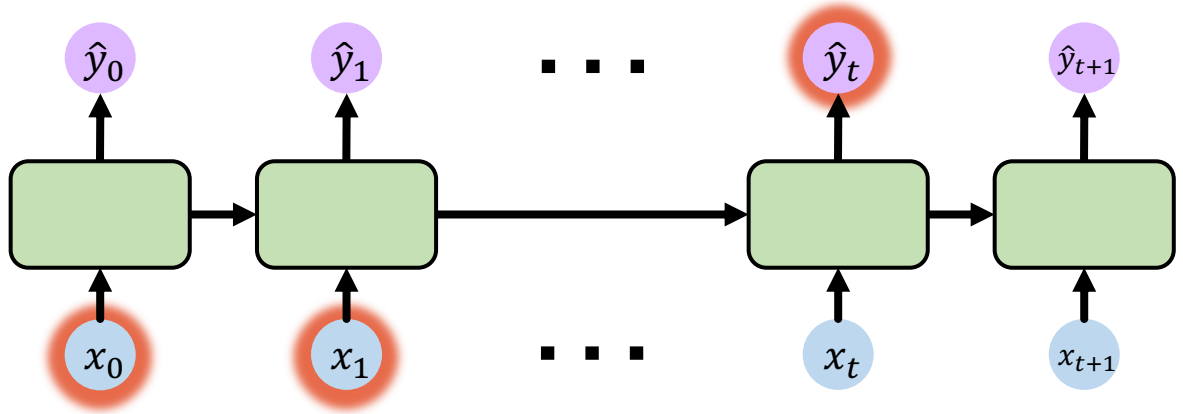
↓

Bias parameters to capture short-term dependencies

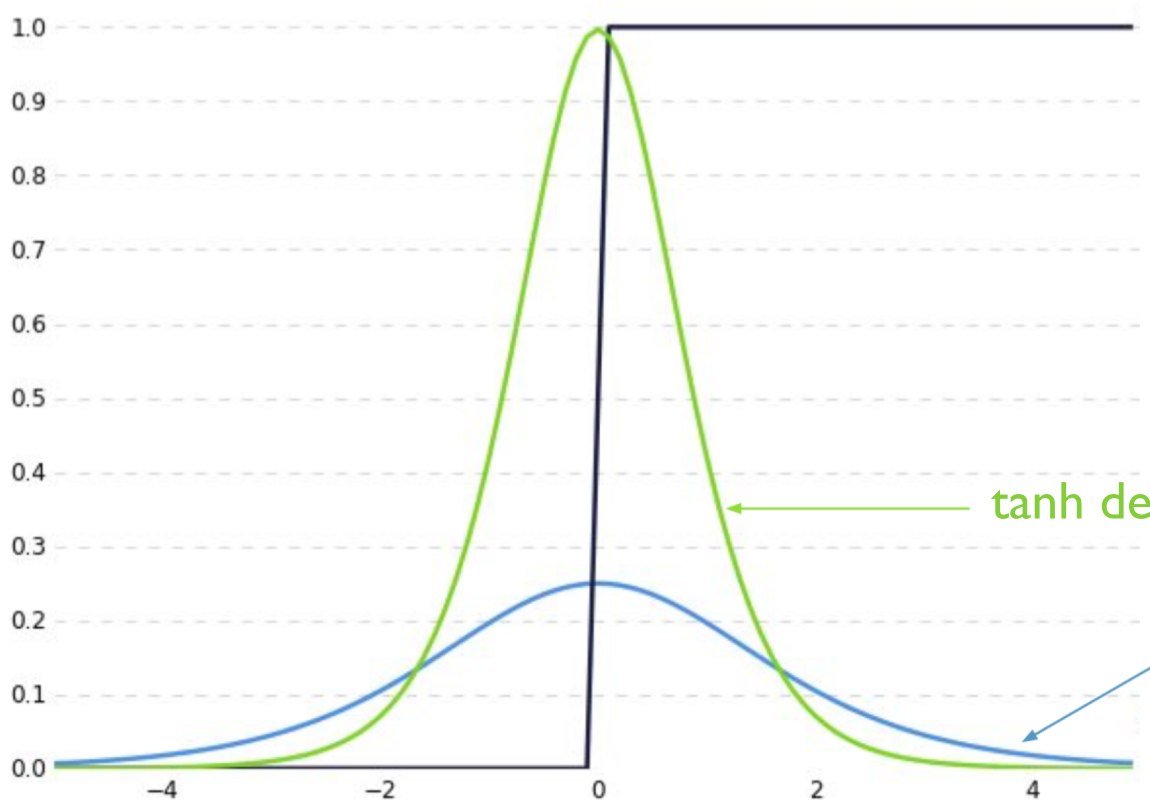
"The clouds are in the ___"



"I grew up in France, … and I I speak fluent___ "



Trick #1: activation functions



Using ReLU prevents f' from shrinking the gradients when $x > 0$

Adapted from H. Suresh, 6.S191 2018

Trick #2: parameter initialization

Initialize **weights** to identity matrix

Initialize **biases** to zero

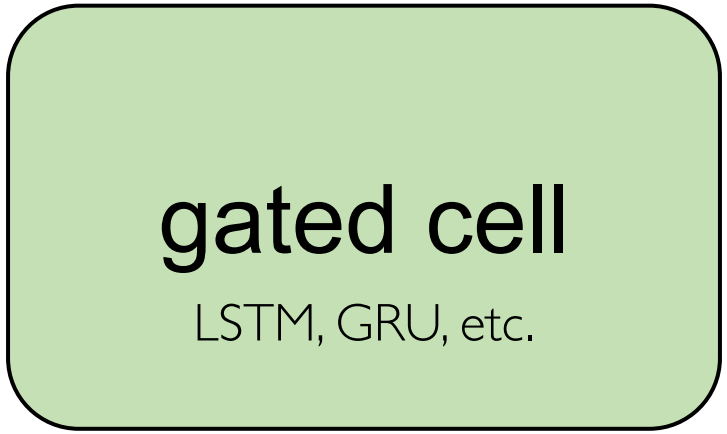
$$I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

This helps prevent the weights from shrinking to zero.

Adapted from H. Suresh, 6.S191 2018

Solution #3: gated cells

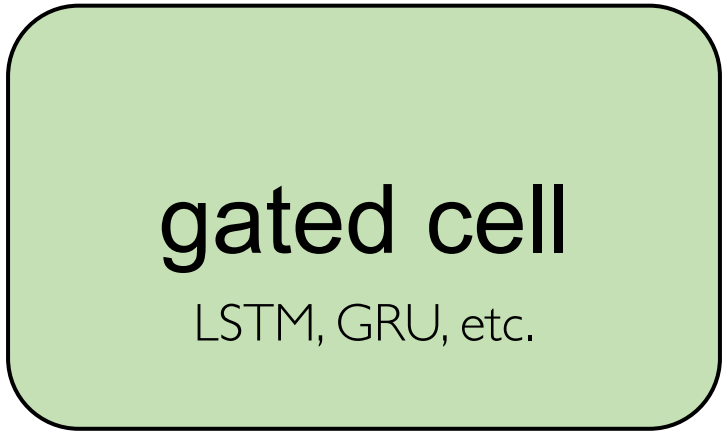
Idea: use a more **complex recurrent unit with gates** to control what information is passed through



Adapted from H. Suresh, 6.S191 2018

Solution #3: gated cells

Idea: use a more **complex recurrent unit with gates** to control what information is passed through



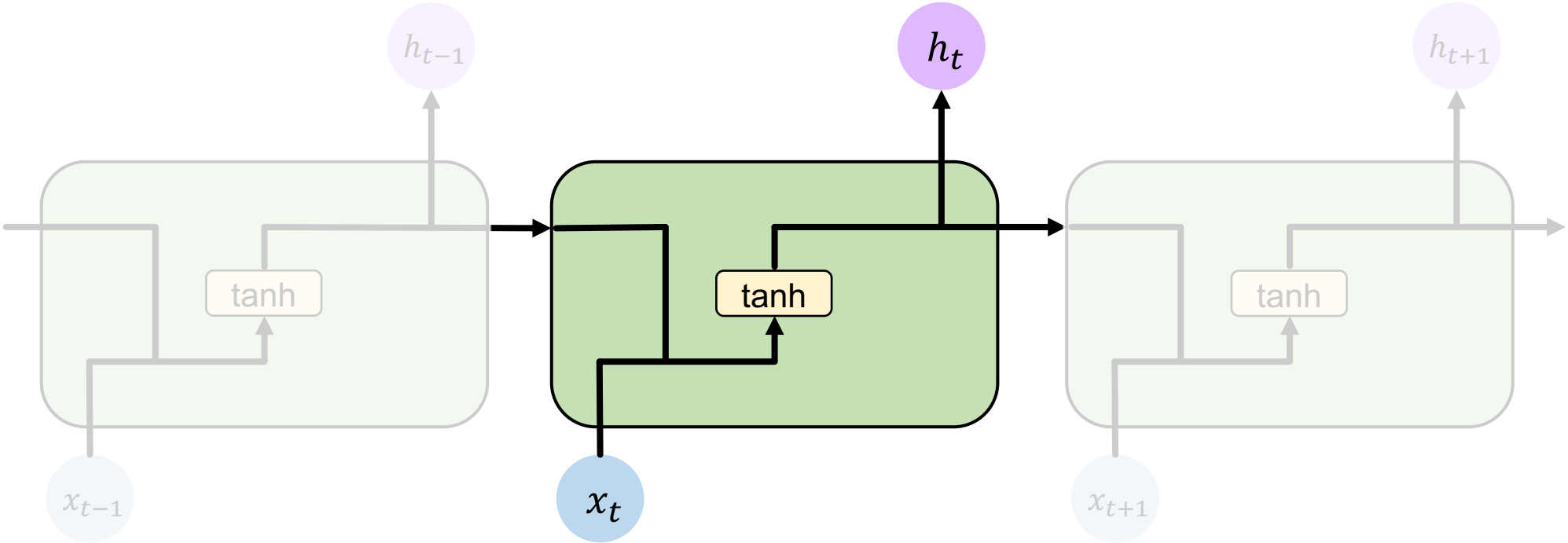
Long Short Term Memory (LSTMs) networks rely on a gated cell to track information throughout many time steps.

Adapted from H. Suresh, 6.S191 2018

Long Short Term Memory (LSTM) Networks

Standard RNN

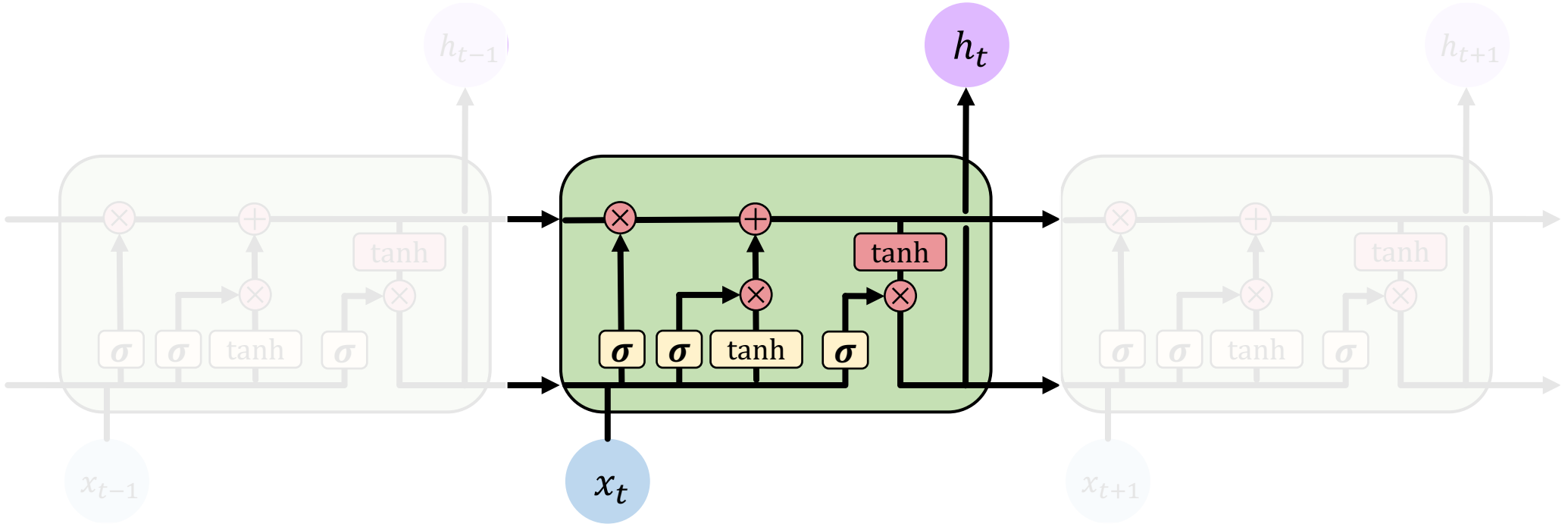
In a standard RNN, repeating modules contain a **simple computation node**



[2]

Long Short Term Memory (LSTMs)

LSTM repeating modules contain **interacting layers** that **control information flow**

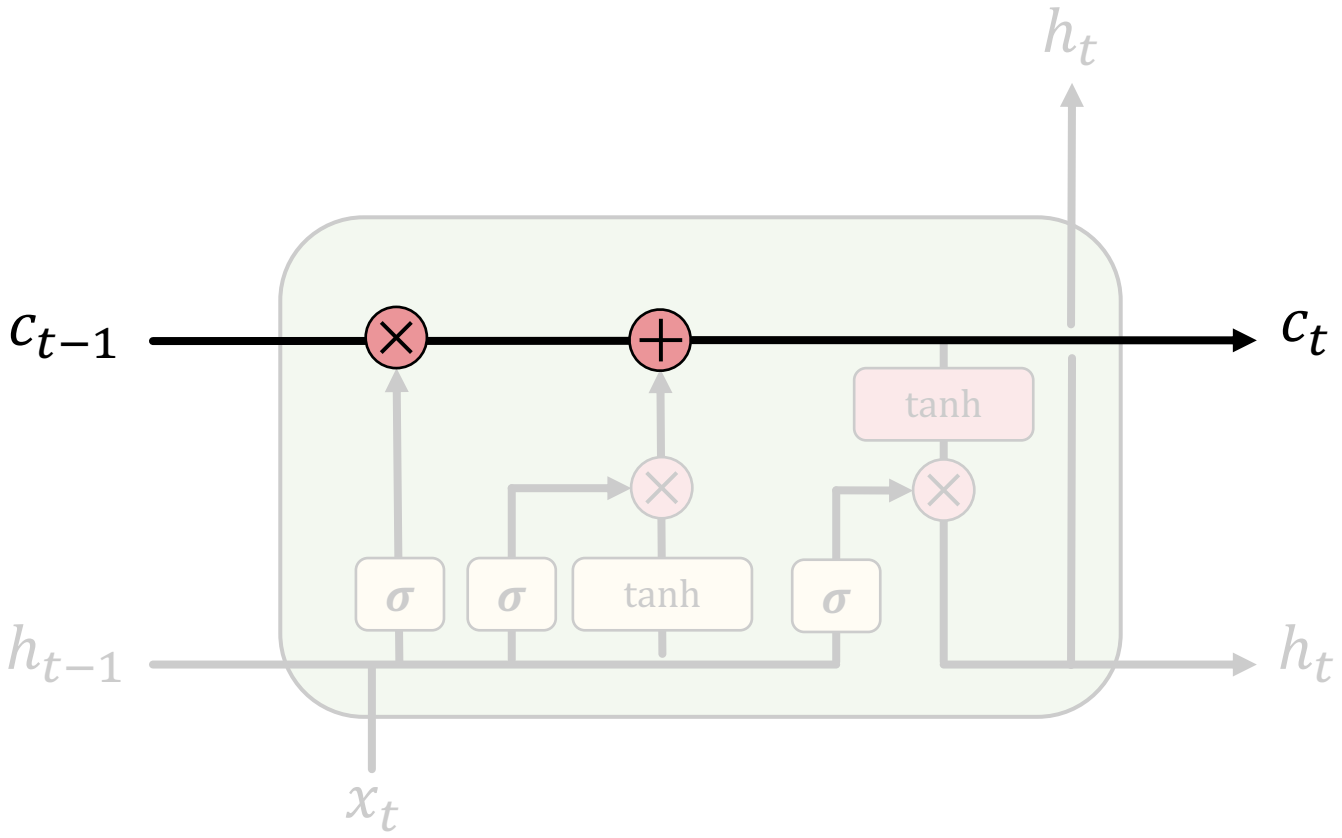


LSTM cells are able to track information throughout many timesteps

Hochreiter & Schmidhuber, 1997 [2, 5]

Long Short Term Memory (LSTMs)

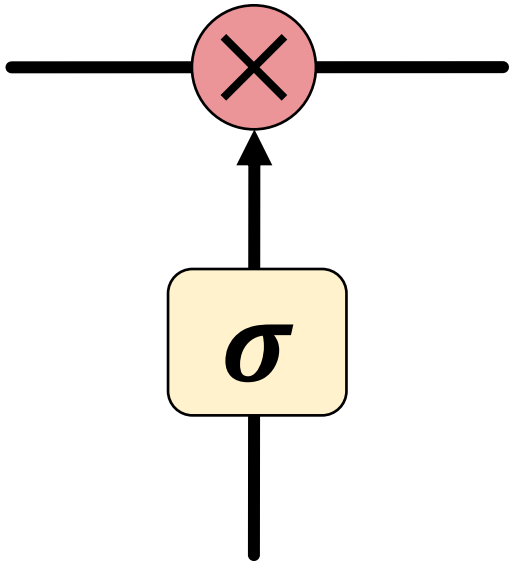
LSTMs maintain a **cell state** c_t where it's easy for information to flow



[2, 5]

Long Short Term Memory (LSTMs)

Information is **added** or **removed** to cell state through structures called **gates**

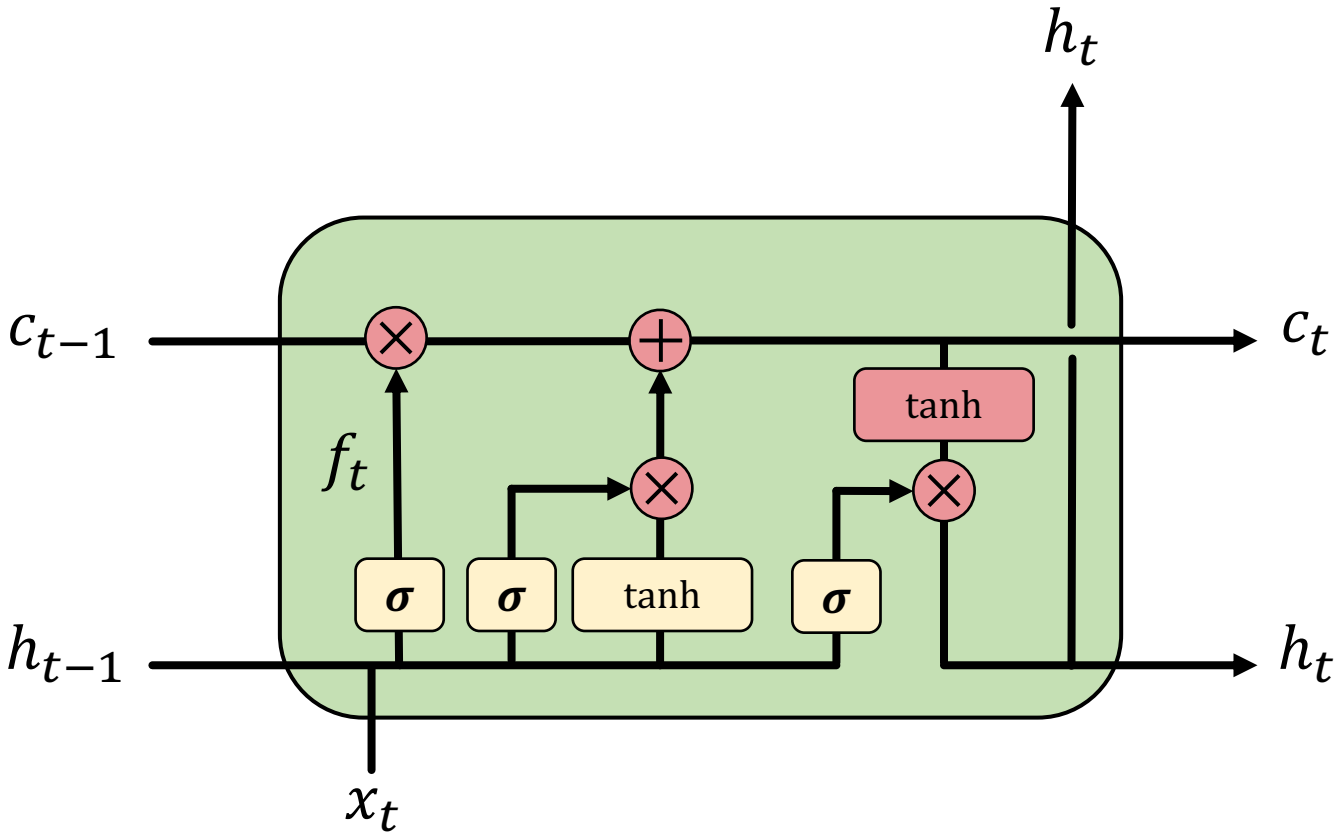


Gates optionally let information through, via a sigmoid neural net layer and pointwise multiplication

[2, 5]

Long Short Term Memory (LSTMs)

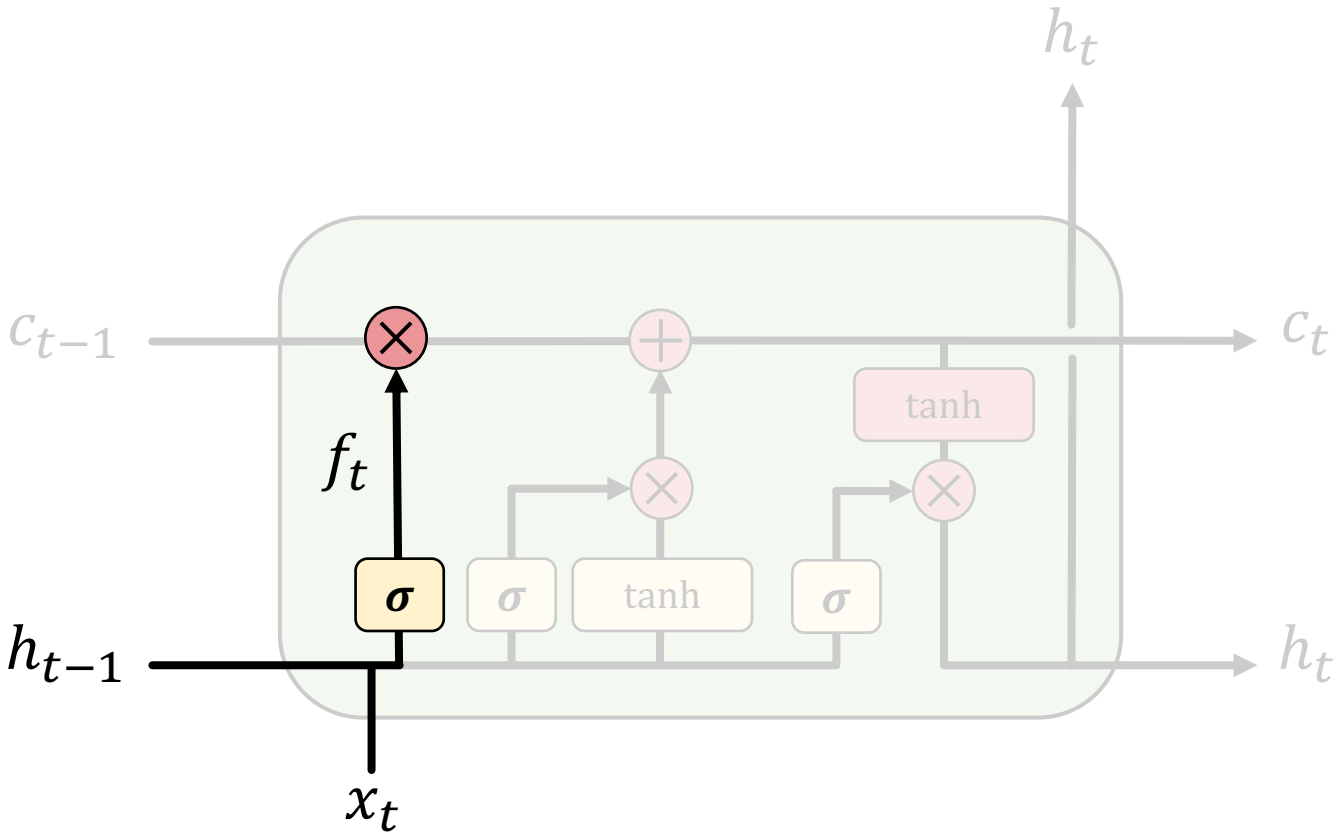
How do LSTMs work?



[2, 5]

Long Short Term Memory (LSTMs)

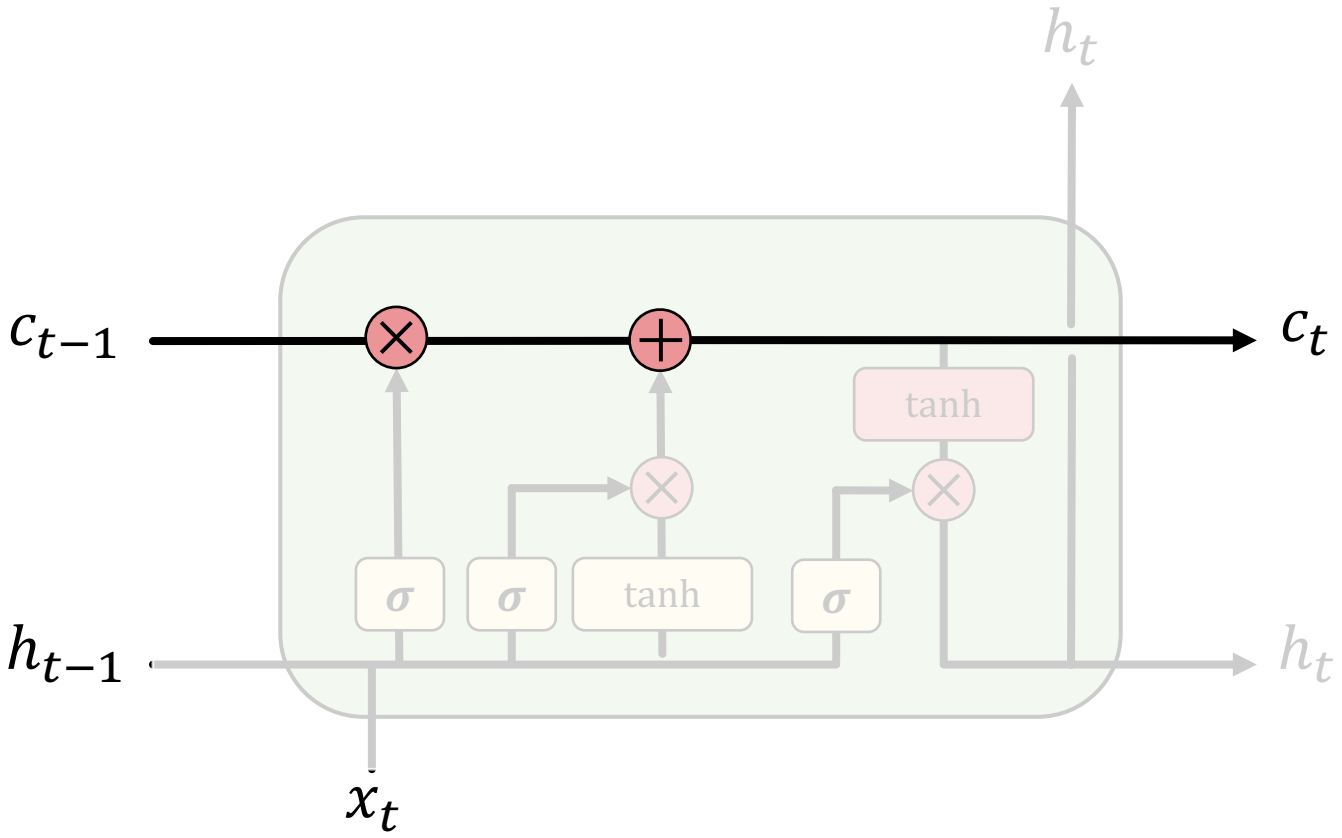
LSTMs **forget irrelevant** parts of the previous state



[2, 5]

Long Short Term Memory (LSTMs)

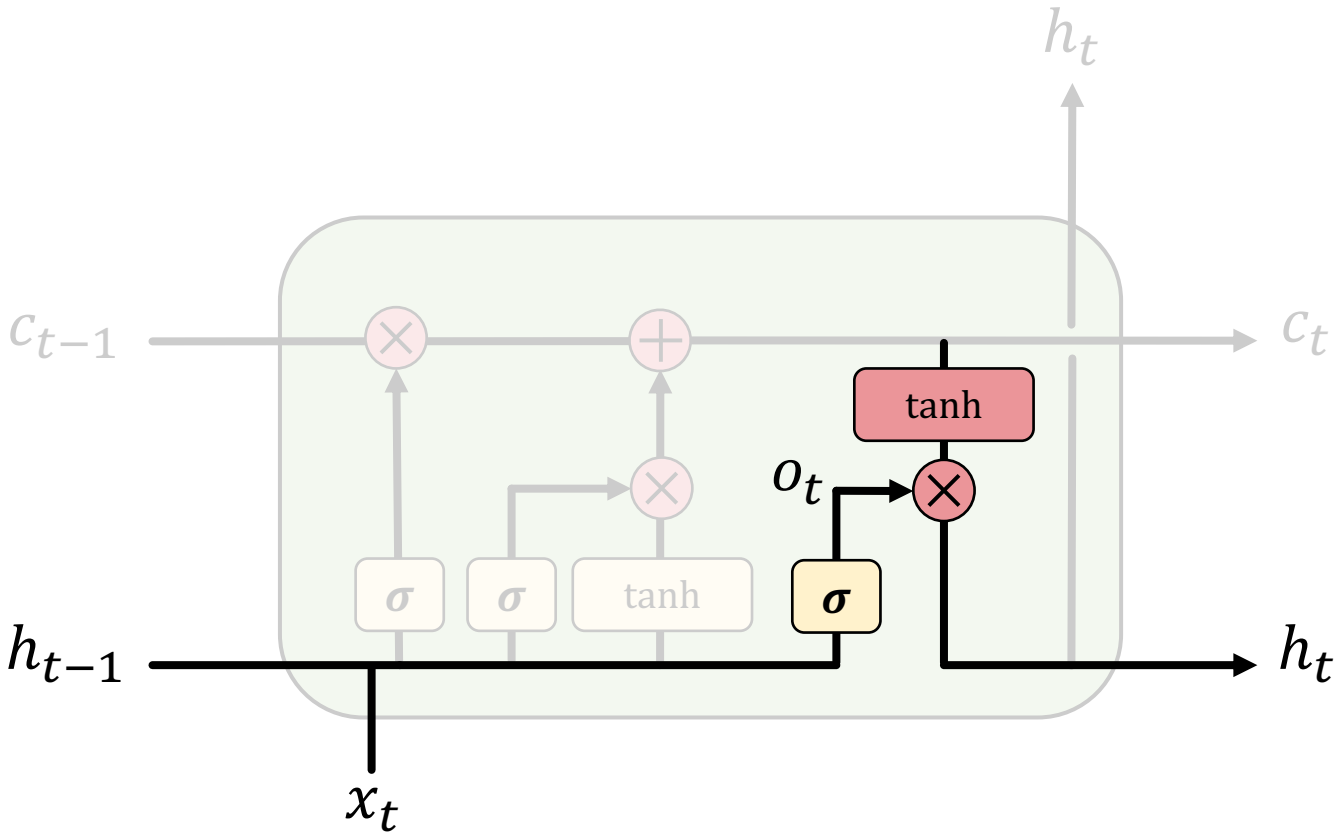
LSTMs **selectively update** cell state values



[2, 5]

Long Short Term Memory (LSTMs)

LSTMs use an **output gate** to output certain parts of the cell state

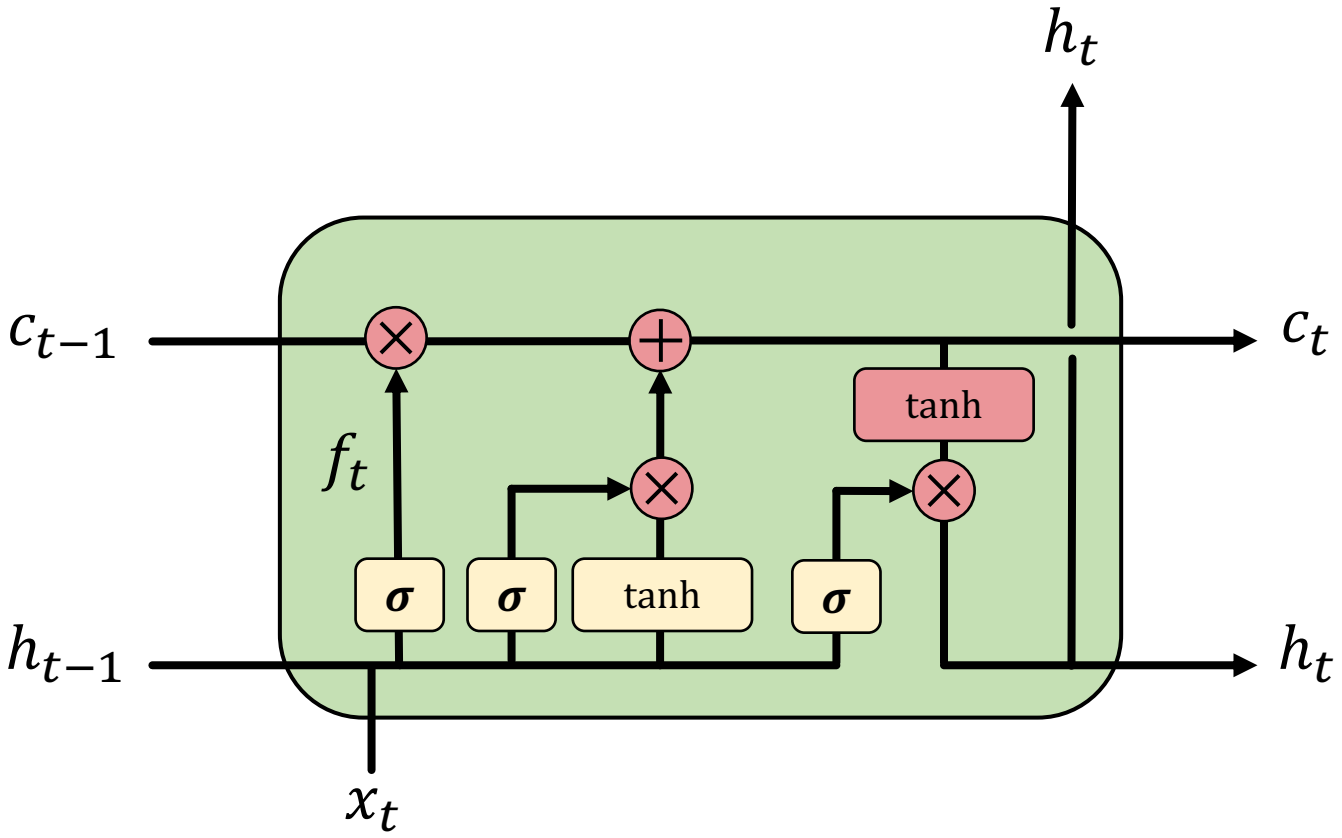


[2, 5]

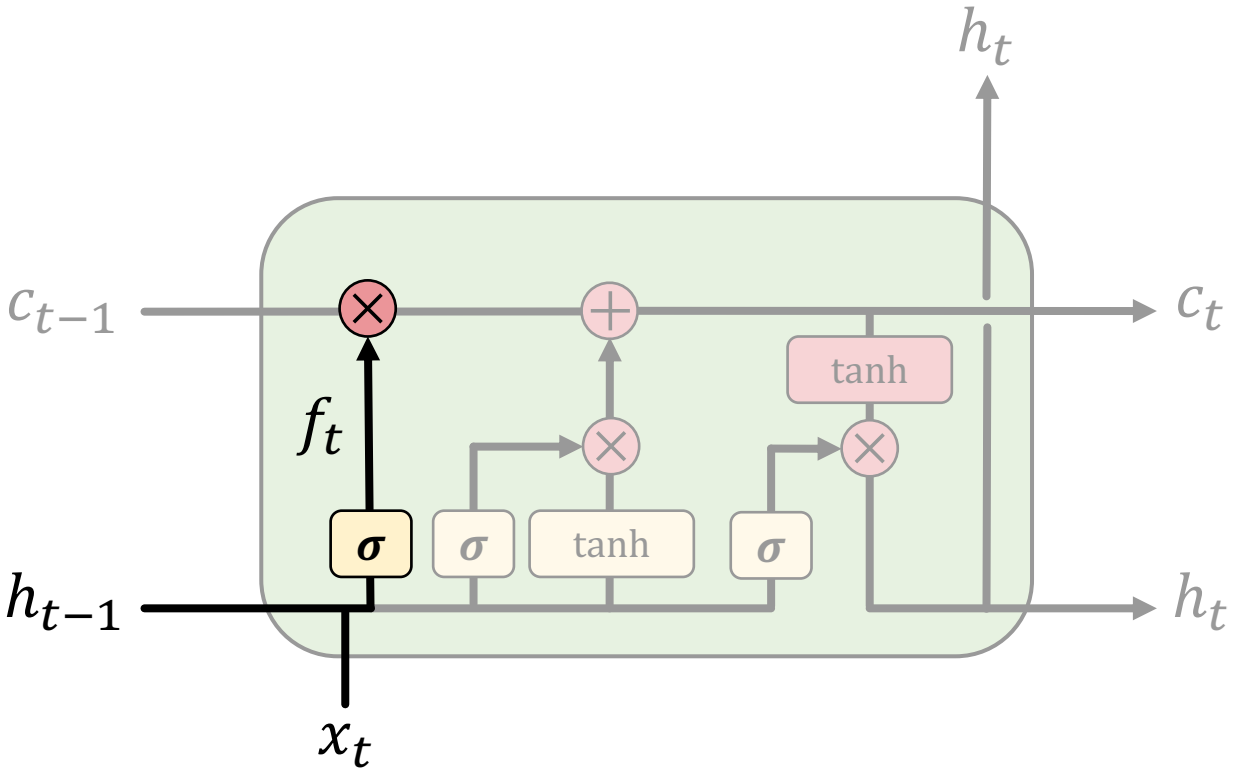
Long Short Term Memory (LSTMs)

How do LSTMs work?

1) Forget 2) Update 3) Output



LSTMs: forget irrelevant information



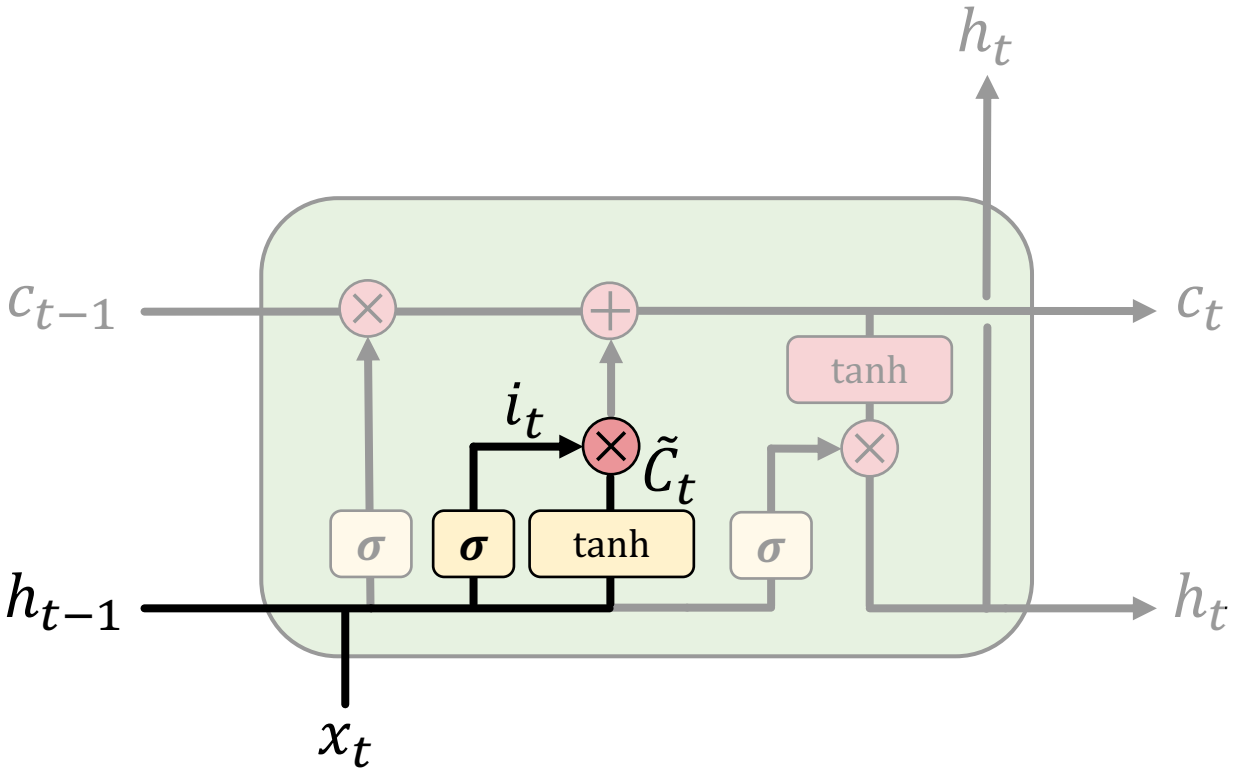
$$f_t = \sigma(\boldsymbol{W_i}[\, h_{t-1}, x_t \,] + b_f)$$

- Use previous cell output and input
- Sigmoid: value 0 and 1 – "completely forget" vs. "completely keep"

ex: Forget the gender pronoun of previous subject in sentence.

[2, 5]

LSTMs: identify new information to be stored



$$i_t = \sigma(\boldsymbol{W_i}[\, h_{t-1}, x_t \,] + b_i)$$

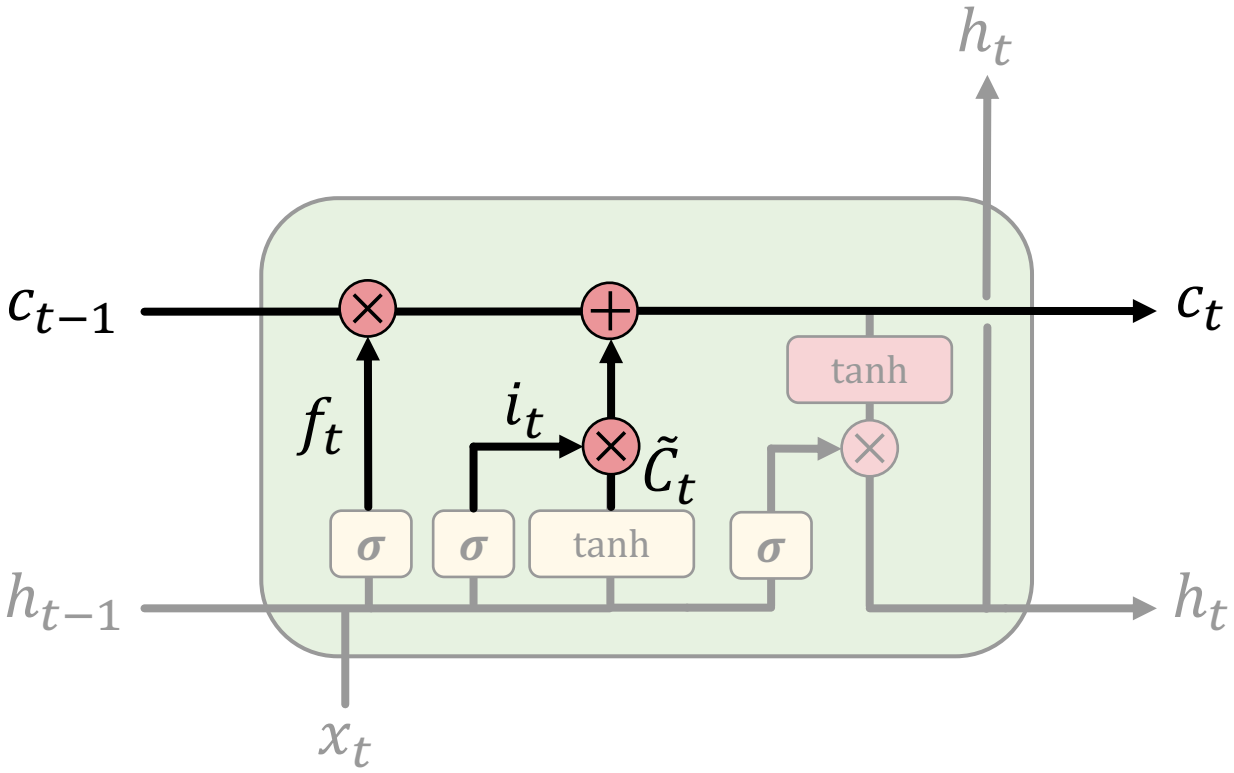
$$\tilde{C}_t = \tanh(\boldsymbol{W_C}[\, h_{t-1}, x_t \,] + b_C)$$

- Sigmoid layer: decide what values to update
- Tanh layer: generate new vector of "candidate values" that could be added to the state

ex: Add gender of new subject to replace that of old subject.

[2, 5]

LSTMs: update cell state



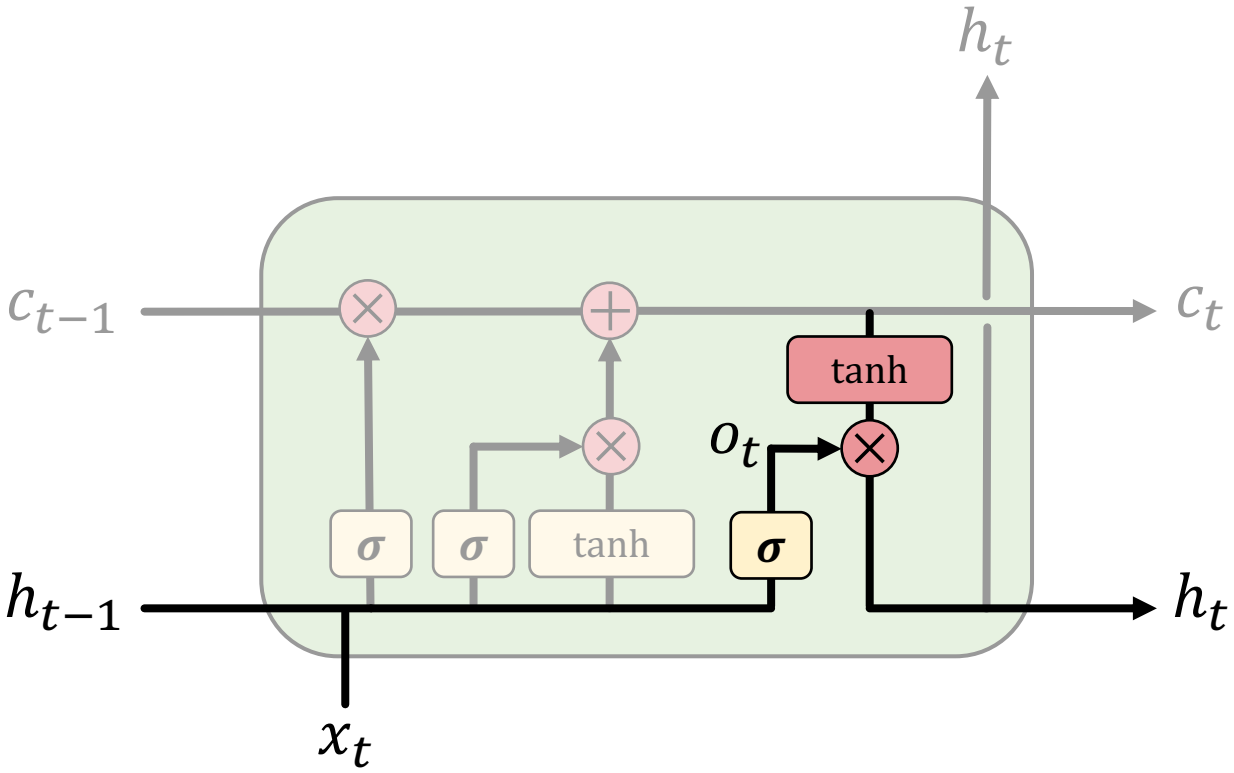
$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

- Apply forget operation to previous internal cell state: $f_t * C_{t-1}$
- Add new candidate values, scaled by how much we decided to update: $i_t * \tilde{C}_t$

ex: Actually drop old information and add new information about subject's gender.

[2, 5]

LSTMs: output filtered version of cell state



$$o_t = \sigma(\boldsymbol{W_o}[\, h_{t-1}, x_t \,] + b_o)$$

$$h_t = o_t * \tanh(C_t)$$

- Sigmoid layer: decide what parts of state to output
- Tanh layer: squash values between -1 and 1
- $o_t * \tanh(C_t)$: output filtered version of cell state

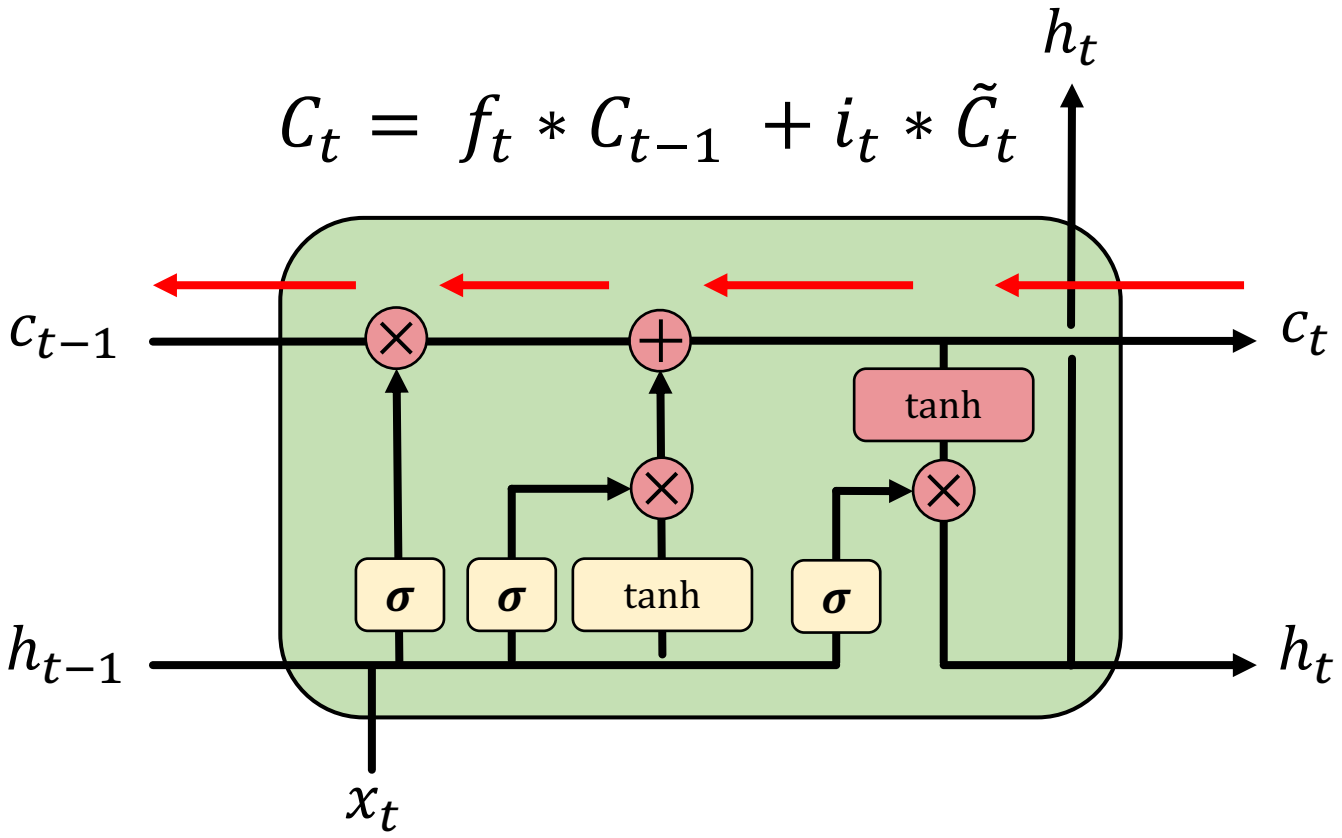
ex: Having seen a subject, may output information relating to a verb.

[2, 5]

LSTM gradient flow

Backpropagation from C_t to C_{t-1} requires only elementwise multiplication!
No matrix multiplication → avoid vanishing gradient problem.

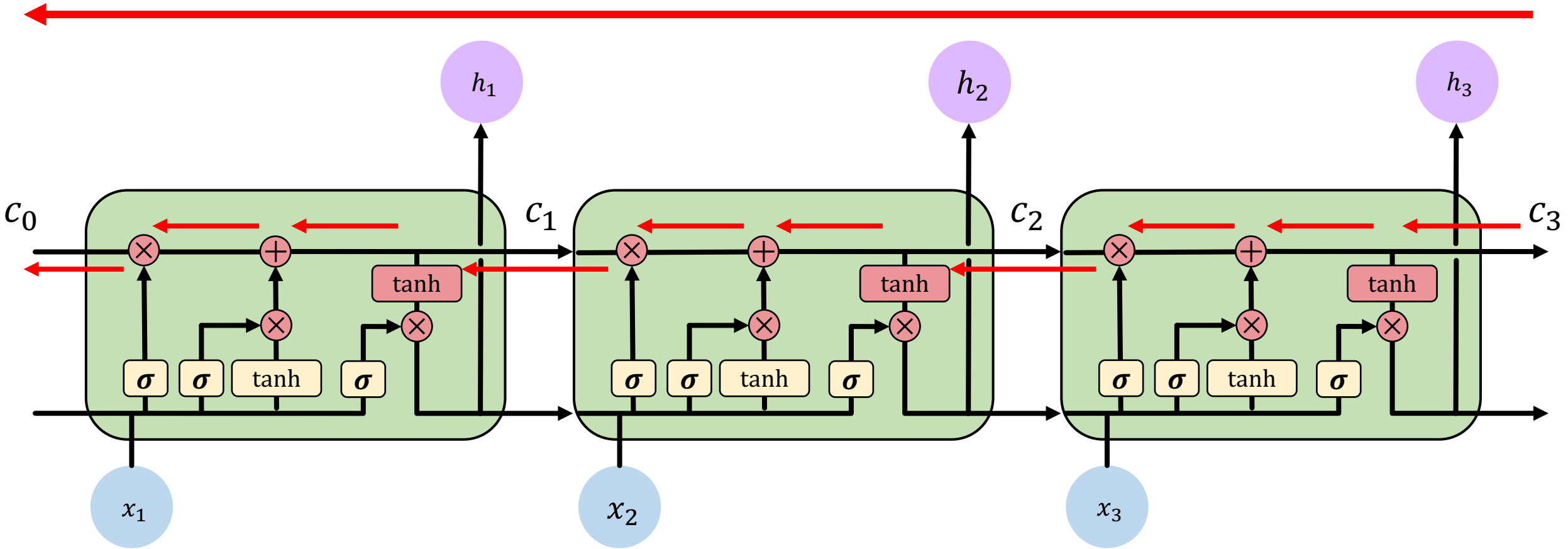
$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$



[2, 5]

LSTM gradient flow

Uninterrupted gradient flow!

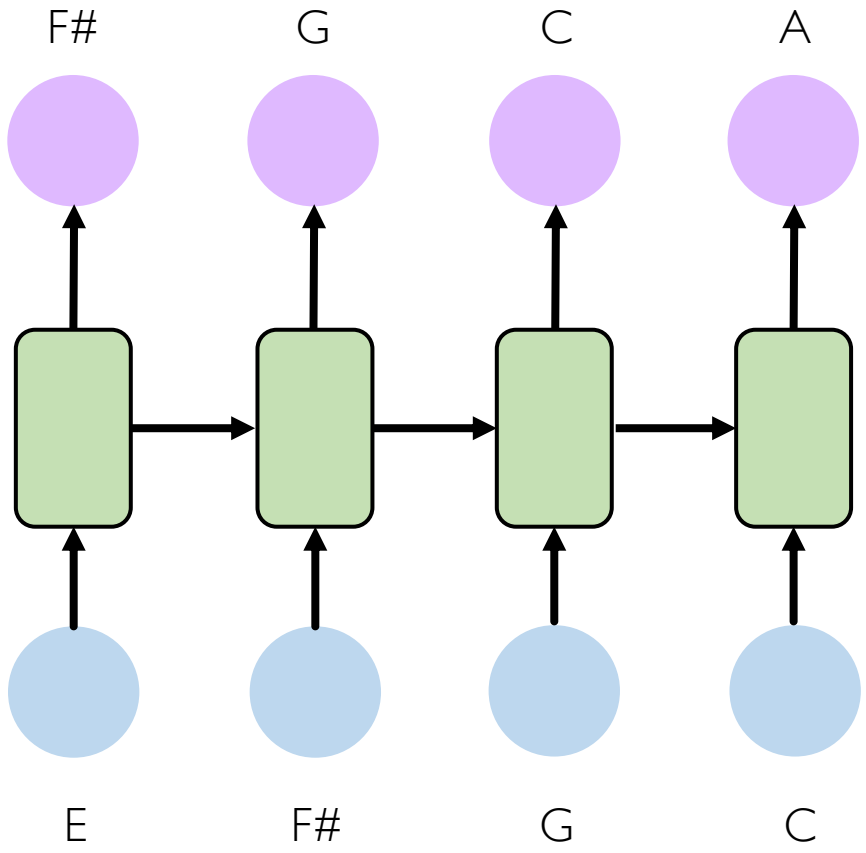


LSTMs: key concepts

1. Maintain a **separate cell state** from what is outputted
2. Use **gates** to control the **flow of information**
   - Forget gate gets rid of irrelevant information
   - Selectively update cell state
   - Output gate returns a filtered version of the cell state
3. Backpropagation from c_t to c_{t-1} doesn't require matrix multiplication: **uninterrupted gradient flow**

RNN Applications

Example task: music generation



Input: sheet music

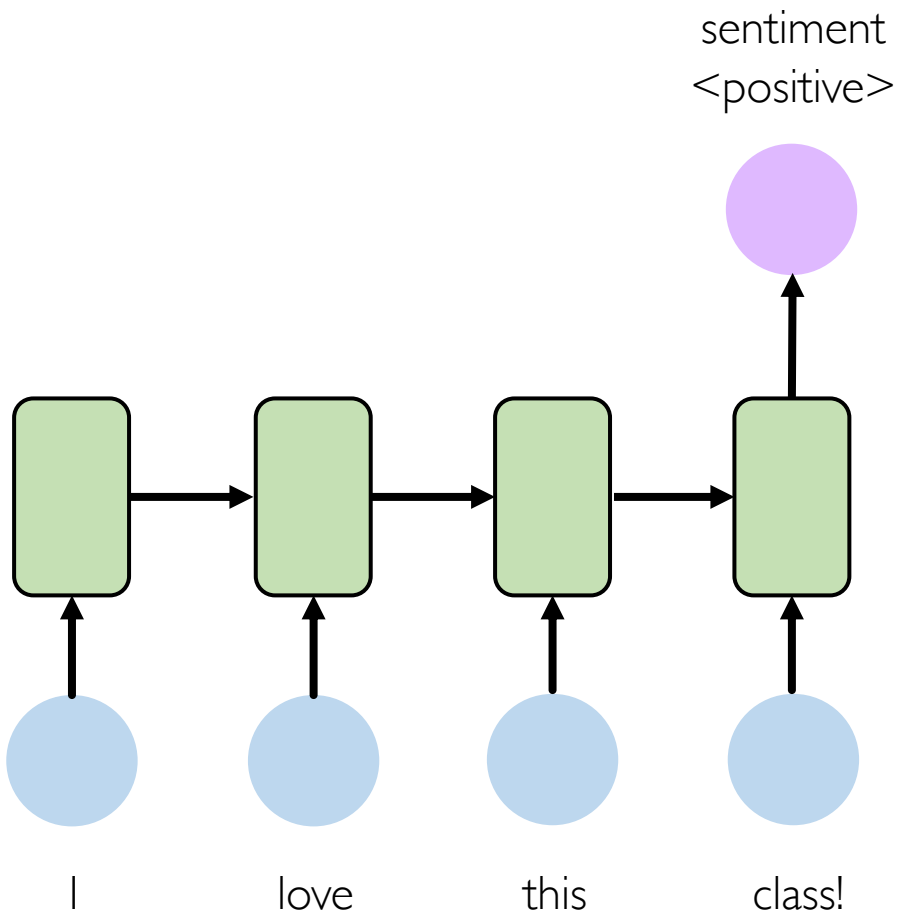
Output: next character in sheet music

6.S191 Lab!

Adapted from H. Suresh, 6.S191 2018

[6]

Example task: sentiment classification



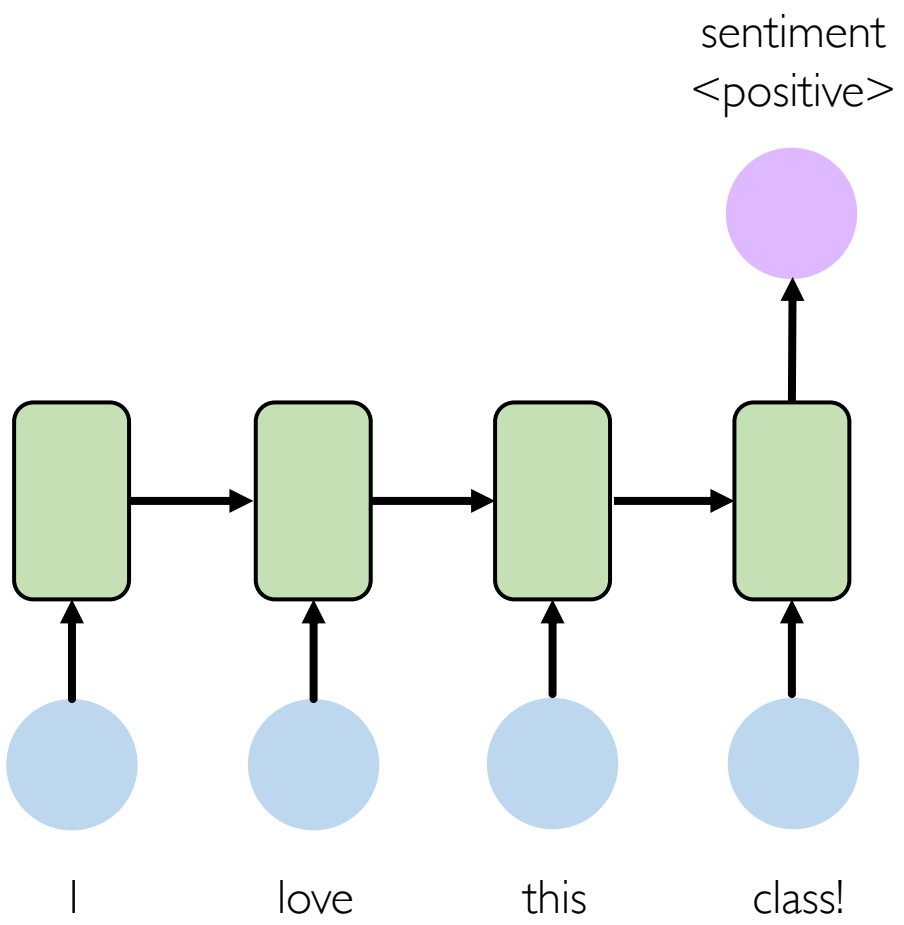
Input: sequence of words

Output: probability of having positive sentiment

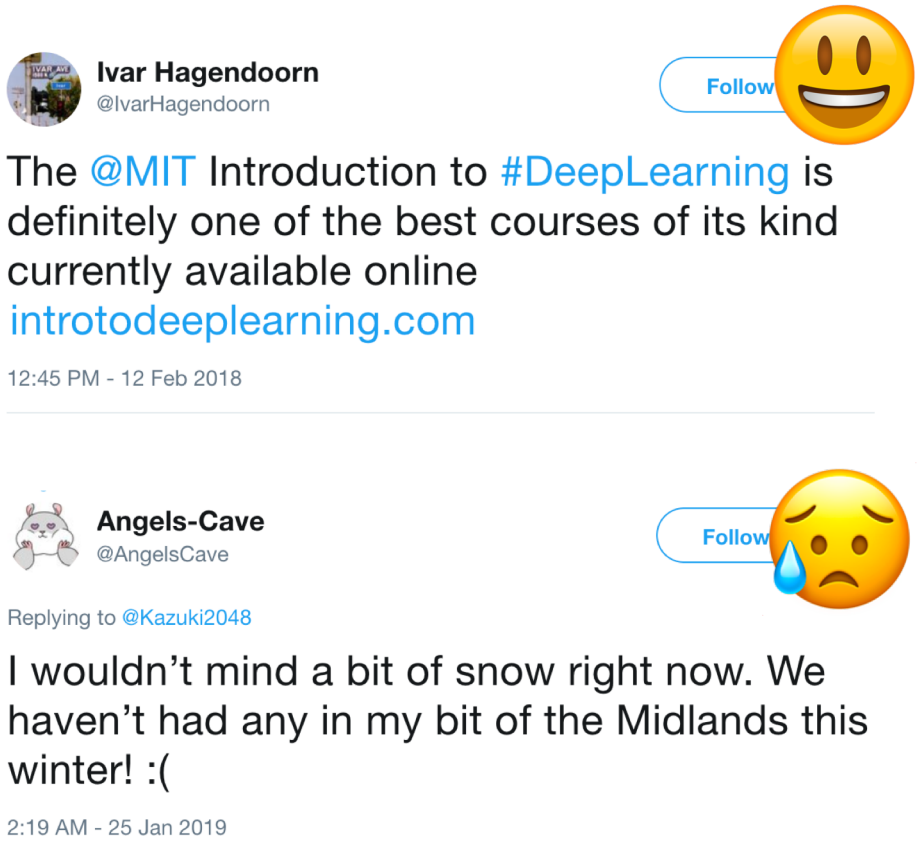
```
loss = tf.nn.softmax_cross_entropy_with_logits(
    labels=model.y, logits=model.pred
)
```

Adapted from H. Suresh, 6.S191 2018

Example task: sentiment classification

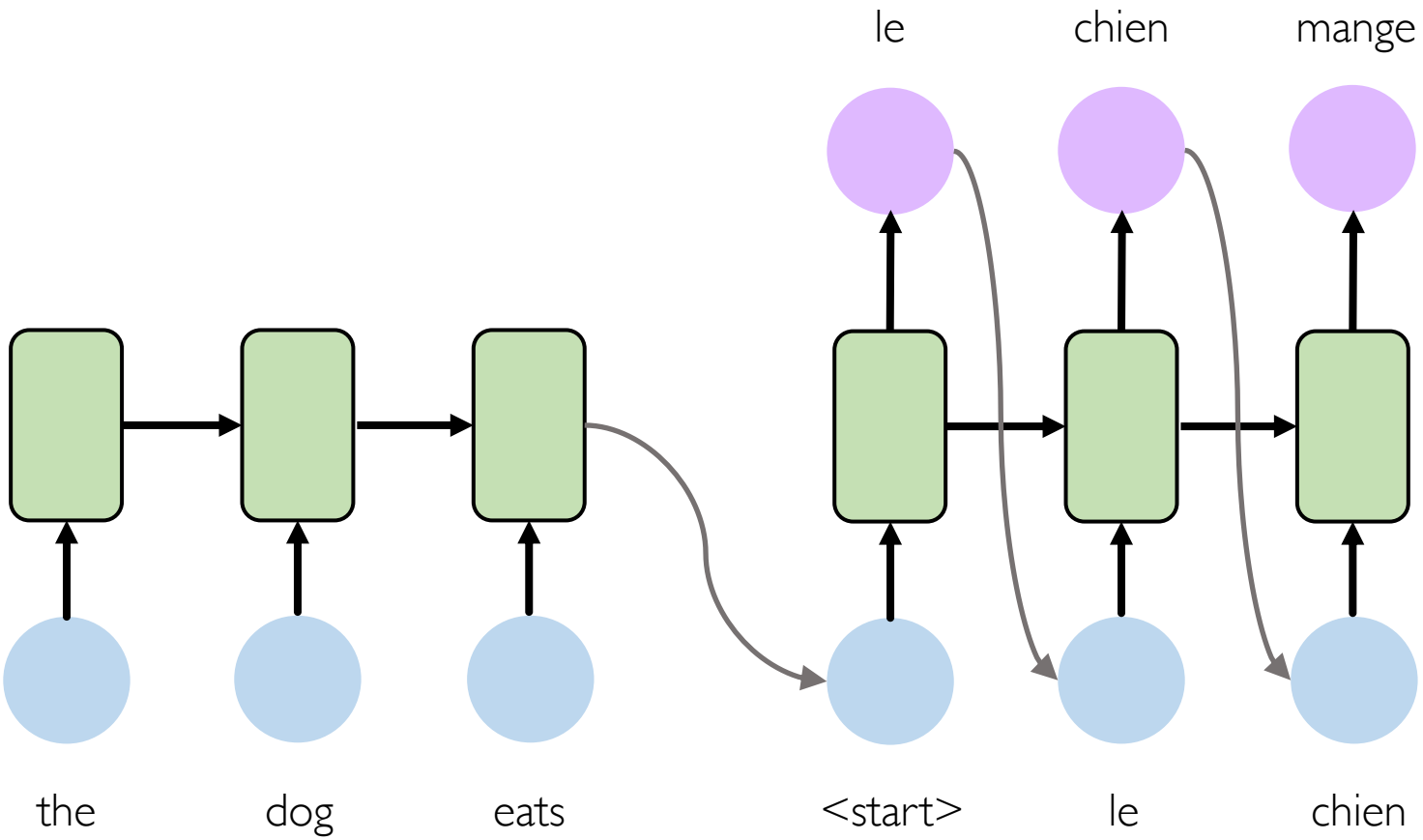


Tweet sentiment classification



Adapted from H. Suresh, 6.S191 2018

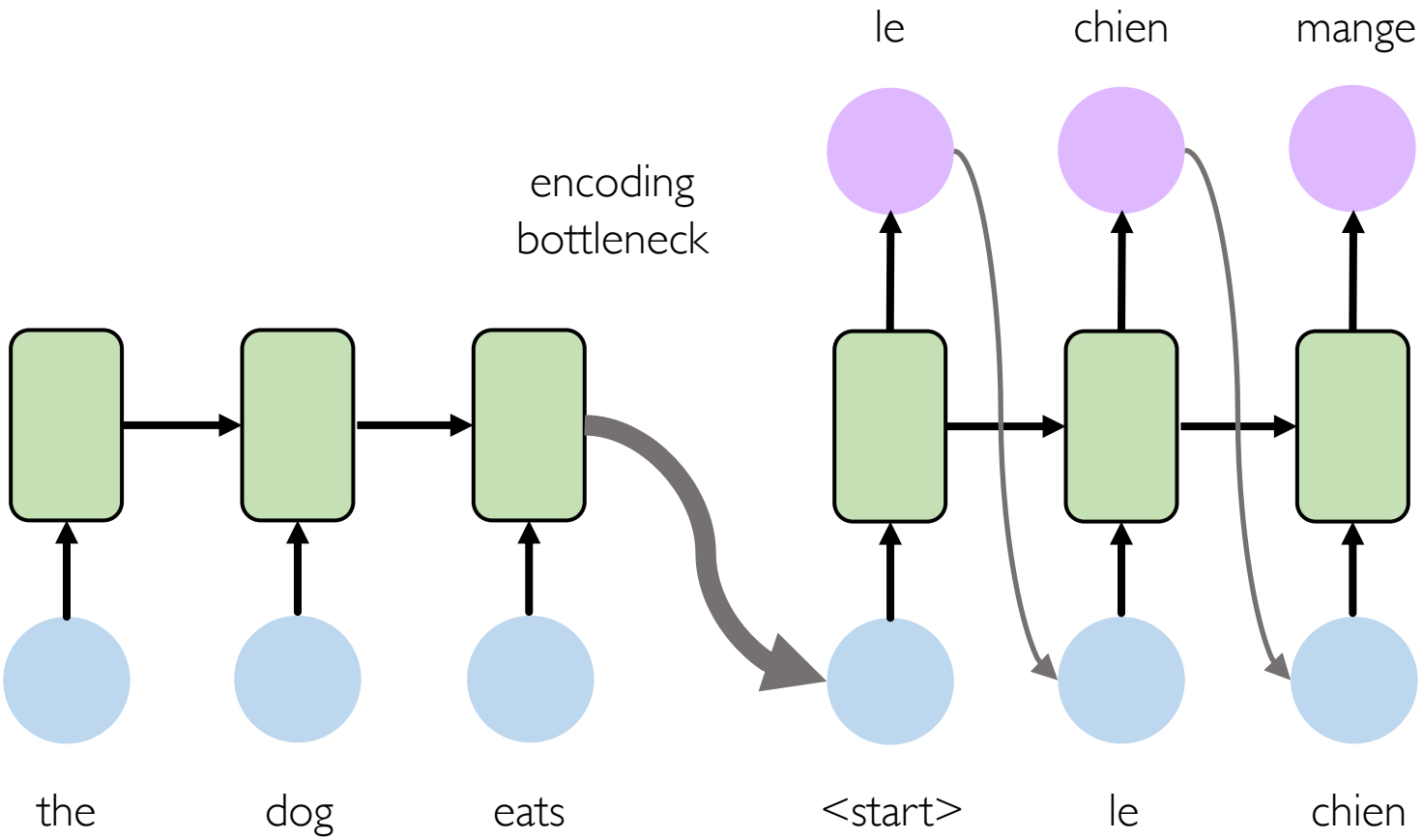
Example task: machine translation



Adapted from H. Suresh, 6.S191 2018

[8,9]

Example task: machine translation

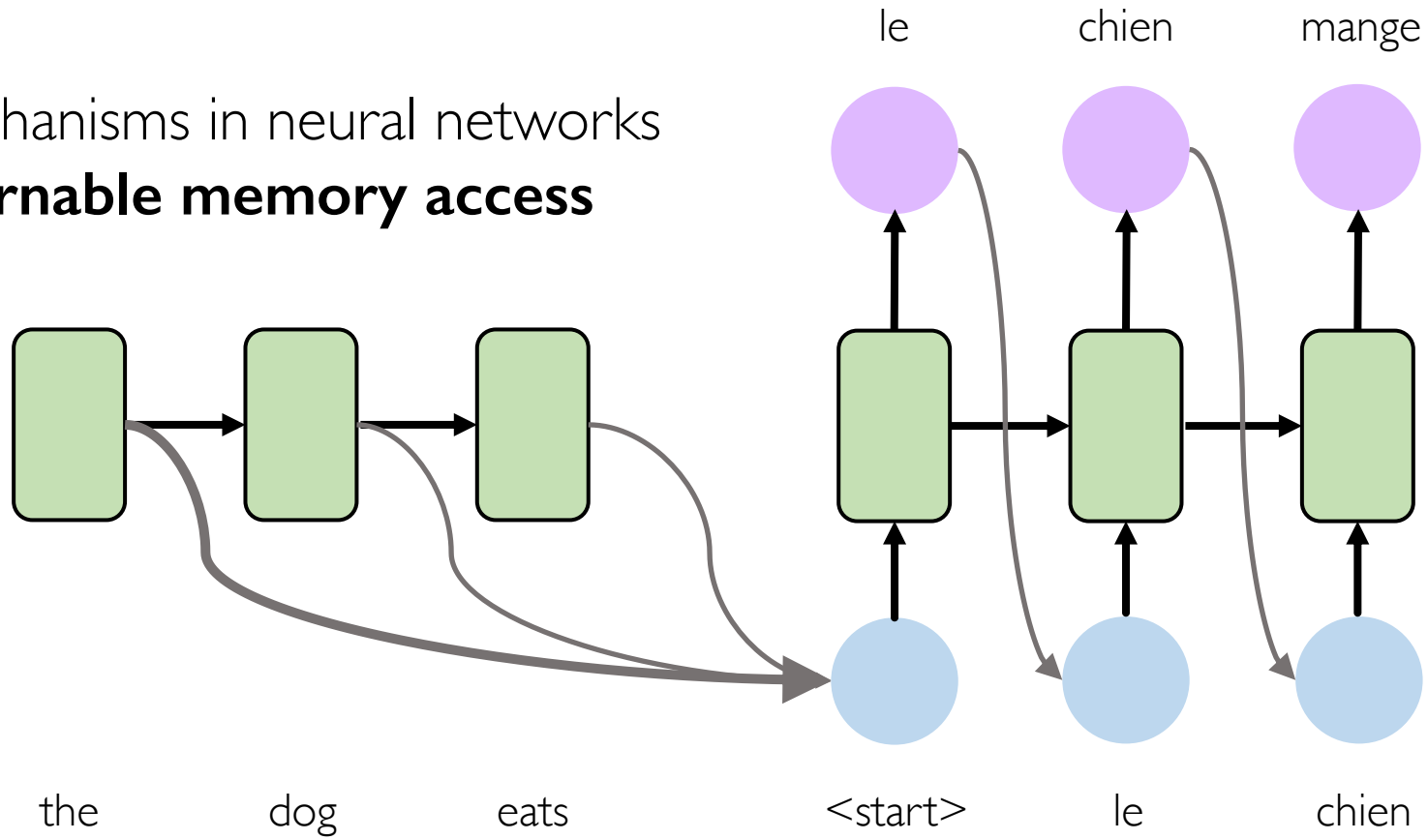


Adapted from H. Suresh, 6.S191 2018

[8,9]

Attention mechanisms

Attention mechanisms in neural networks provide **learnable memory access**

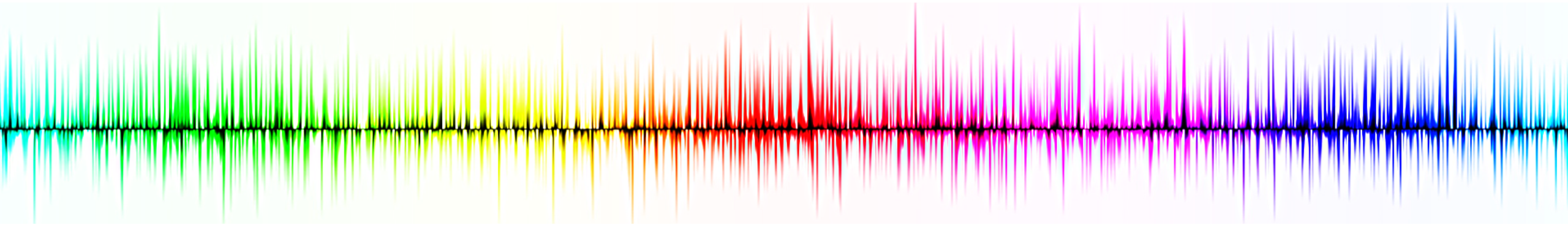


Adapted from H. Suresh, 6.S191 2018

[8,9]

Recurrent neural networks (RNNs)

1. RNNs are well suited for **sequence modeling** tasks
2. Model sequences via a **recurrence relation**
3. Training RNNs with **backpropagation through time**
4. Gated cells like **LSTMs** let us model **long-term dependencies**
5. Models for **music generation**, classification, machine translation



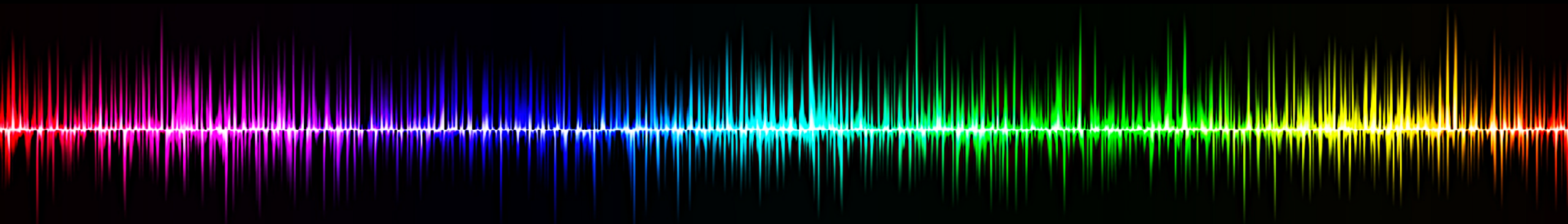
References:
goo.gl/hbLkF6

6.S191: Introduction to Deep Learning

Lab 1: Introduction to Tensorflow and Music Generation with RNNs

Link to download labs:
http://introtodeeplearning.com#schedule

1. Open the lab in Google Colab
2. Start executing code blocks and filling in the #TODOs
3. Need help? Find a TA or come to the front!!



End of Slides