Suppose n is an integer, then $n=3q$, $n=3q+1$ or $n=3q+2$ where $q \in \mathbb{Z}$ by the quotient-reminder-theorem

Case 1) Let $n=3q$, then $n+1=3q+1$.

thus $n(n+1) = 3q(3q+1)$

$= 9q^2+3q$

$= 3(3q^2+q)$

Let $k=3q^2+q$, then $n(n+1) = 3k$

Case 2) Let $n=3q+1$, then $n+1=3q+2$

thus $n(n+1) = (3q+1)(3q+2)$

$= 9q^2+6q+3q+2$

$= 3(3q^2+2q+q)+2$

Let $k=3q^2+2q+q$, therefore $n(n+1)=3k+2$

Case 3 Let $n=3q+2$, then $n+1=3q+3$

thus, $n(n+1)=(3q+2)(3q+3)$

$$=9q^2+9q+6q+6$$

$$=3(3q^2+3q+2q+2)$$

Let $k=3q^2+3q+2q+2$, which is an integer, therefore, $n(n+1)=3k$

Hence, in any case, the product of any two consecutive intergets has the form $3k$ or $3k+2$

Suppose m is an integer, then $m=3q$, or $m=3q+1$ or $m=3q+2$

where $q \in \mathbb{Z}$ by the Quotient-reminder-theorem

Case 1) Let $m=3q$

then $m^2=(3q)^2$

$= 9q^2$

$= 3(3q^2)$

Let $k=3q^2$, which is an integer, therefore $m^2=3k$

Case 2) Let $m=3q+1$

then $m^2=(3q+1)^2$

$= 9q^2+6q+1$

$= 3(3q^2+2q)+1$

Let $k=3q^2+2q$, which is an integer, therefore $m^2=3k+1$

Case 3 Let $m=3q+2$,

then $m^2=(3q+2)^2$

$$=9q^2+12q+3+1$$

$$=3(3q^2+4q+1)+1$$

Let $k=3q^2+4q+1$, which is an integer

Therefore $m^2=3k+1$

Suppose there is an integer a, $7a+4$ is divisible by 7

then there exist $n \in \mathbb{Z}$ s.

then $7a+4 = 7n$

$4 = 7(n-m)$

$\frac{4}{7} = n-m$

But $n-m$ is an integer while $\frac{4}{7}$ is not an integer

This is a contradiction, so $7a+4$ is not divisible by 7 for every integer a

Neg: The product of any irrational num and any non zero rational number is rational

b. Suppose n is a irrational number and m is non-zero rational number and mn is rational.

There exist $a, b \in \mathbb{Z}$ and $b \neq 0$, that $m = \frac{a}{b}$

Also exist $c, d \in \mathbb{Z}$ and $d \neq 0$, that $mn = \frac{c}{d}$

Then

$$mn = \frac{c}{d} = \frac{a}{b} \cdot n$$

$$\frac{cb}{da} = \left(\frac{a}{b} n\right) \cdot \frac{b}{a}$$

$$mn = \frac{cb}{da}$$

Let $k = cb$, which is an integer and $l = da$, which is an integer and $da \neq 0$, then

$n = \frac{k}{l}$ which is rational.

Since n can't be both rational and irrational at the same time

Therefore it's be an contradiction, so the orignal statement is true

03:19 Wed Feb 28 37%

Calculus 2 | Untitled Notebook | Untitled Noteboo... | discreet math

For all integer a, b and c, if $a \mid bc$, then $a \mid b$

Suppose a, b and c are integers, Let $a \nmid bc$ and $a \mid b$, there there exists $k, \in \mathbb{Z}$ s.t. $b = ak$

Then

$$b = ak$$
$$bc = a(kc)$$

Let $q = kc$, which is an integer

then $bc = aq$

So. $a \mid bc$

Therefore, we have prove the contrapositione of the original statement which is true

16 of 16