

HW 2.

1.

$$P(A) = \frac{r}{r+b} \cdot \frac{r+a}{r+b+a} \cdot \frac{b}{r+b+a+a}$$

$$= \frac{r}{r+b} \cdot \frac{r+a}{r+b+a} \cdot \frac{b}{r+b+2a}$$

$$= \frac{r}{r+b} \cdot \frac{r+a}{r+b+a} \cdot \frac{b}{r+b+2a} \cdot \frac{b+a}{r+b+3a}$$

2.

设  $A =$  “抽到正品”

$$(a) P(A) = 1\% \times \frac{1}{2} + 1\% \times \frac{1}{3} + 2\% \times \frac{1}{6}$$

$$\approx 0.5\% + 0.33\% \times 2 + 0.33\% = 0.99\%$$

(b). 设  $B_i =$  “是  $i$  号车间生产”

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)}$$



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3.

解: 设  $A = \{\text{男性}\}$

$\bar{A} = \{\text{女性}\}$

$B = \{\text{此人为色盲}\}$

$P(A) = P(\bar{A}) = 0.5$  (当人群足够大时).

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = \frac{0.025}{0.025 + 0.00125} \approx 0.9523 \quad \#$$

4.

解:

(1) 设事件  $A_i = \{\text{抽到第 } i \text{ 个笔筒}\}$

事件  $B = \{\text{抽到红色笔芯}\}$

$$\Rightarrow P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$$

$$= \frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{2}$$

$$(2) \quad P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B)} = \frac{\frac{1}{9}}{\frac{1}{2}} = \frac{2}{9}$$

$$P(A_2|B) = \frac{P(B|A_2)P(A_2)}{P(B)} = \frac{\frac{2}{9}}{\frac{1}{2}} = \frac{4}{9}$$

$$P(A_3|B) = \frac{P(B|A_3)P(A_3)}{P(B)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

所以在第一个  
笔筒中取出  
概率最大



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5. 证明: 若  $P(B|A) = P(B|\bar{A})$ , 则  $A, B$  独立.

已知:  $P(B|A) = P(B|\bar{A})$

$$\Rightarrow P(B|A) = \frac{P(AB)}{P(A)} = P(B|\bar{A}) = \frac{P(\bar{A}B)}{P(\bar{A})}$$

$$\Rightarrow P(AB) \cdot P(\bar{A}) = P(A) \cdot P(\bar{A}B)$$

$$\Rightarrow P(AB) \cdot (1 - P(A)) = P(A) \cdot (P(B) - P(AB))$$

$$\Rightarrow P(AB) = P(A)P(B)$$

由事件独立性定义可知.

•  $A, B$  独立

6.

$$P(A) = P(A|C)P(C) + P(A|\bar{C})P(\bar{C})$$

$$= 0.9 \times 0.5 + 0.2 \times 0.5 = 0.55$$

$$P(B) = P(B|C)P(C) + P(B|\bar{C})P(\bar{C})$$

$$= 0.9 \times 0.5 + 0.1 \times 0.5 = 0.5$$

$$P(AB) = P(AB|C)P(C) + P(AB|\bar{C})P(\bar{C})$$

$$= P(A|C)P(B|C)P(C) + P(A|\bar{C})P(B|\bar{C})P(\bar{C})$$



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$$= 0.81 \times 0.5 + 0.02 \times 0.5 = 0.415$$

由于  $P(A) \cdot P(B) \neq P(AB)$

因此,  $A, B$  不独立。

