

Lecture 4

Random scenario *lost chess!*

- Outcome space Ω
- Probability measure P on Ω
- Events: subsets $A \subseteq \Omega$
- Random variables $X: \Omega \rightarrow \mathbb{R}$

Ex: Flip two p-coins
 Could use $\Omega = \{00, 01, 10, 11\}$
 Could use $\Omega = \{0, 1\}^2$

$X: \Omega \rightarrow \mathbb{R}$
 = counting dice

Obvious model (Specific to alternating)

Ω = possible sequences of wins that actually happen

$= \{AAAA, AAEEA, BEAAAB, \dots\}$

$P(\{AAAA\}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
 $P(\{BEAAAB\}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
 schedule loses here!

Not Scully's obvious model

Idea 1: think about games that would have happened if series kept going

$P(\{w\}) = P(\{w, w, w, w\})$

Idea 2: think about schedule-independent

$A: \{w, l\} \rightarrow \{0, 1\}$
 $B: \{w, l\} \rightarrow \{0, 1\}$

$\{w, l\} \rightarrow \{0, 1\}$
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Best-of-seven series (first 4 win)

- Home + away gamewin A and B
 → wins 3/5 prob
- A gets 4 home games

Q: Schedule doesn't matter for P(A wins)!

Counter: alternating schedule
 Here team A BABABA

$W_A = \{w \in \Omega \mid \text{last letter of } w \text{ is } A\}$
 (i.e. "A wins in w")

$N(w) = \text{length of } w$
 $P(W_A) = ???$

$W_A \subseteq \Omega \rightarrow W = \{w \in \Omega \mid \dots\}$
 $N: \Omega \rightarrow \mathbb{R} \rightarrow N(w) = \dots?$
 "realization"

$P(W_A, W_B) = \left(\frac{1}{2}\right)^{2w_A + 2w_B} \cdot \left(\frac{1}{2}\right)^{4 - 2w_A - 2w_B}$
 → 4th is home game
 → 5th 1st away game

$W_A = \text{sequence of results of A home games}$
 (e.g. 1011)
 $W_B = \dots$ → B's home games

Let $f(w) = \sum w_i = w_1 + \dots + w_n$
 "4th B's win" →

[Faint handwritten notes]

$W_A = \{w \in \Omega \mid \sum w_i \geq 4\}$
 $N_A(w) = ???$
 Schedule here!