Solution Sketch for PS4

1. **Solution**

(a) The unique Nash is for the traders to buy and sell for 0 and 1 respectively, and hence make 1 profit each.

(b) Bids are 0 and asks are 1, both goods go from sellers, and to one of the original buyers and the new one, one original buyer (B1 or B2) is no longer getting the good. Bids and asks are all 0 and 1 as in (a) and traders make the same profits. The reason is that the two traders compete for the new buyer (of higher value) and this pushes the price down to what they had before.

2. **Solution**

(a) There are 4 SCCs: AEF (ii, IN); BCGH (i, giant SCC), D (iii, OUT), I (iii, OUT)

(b) DA (or DE, DF), giant SCC: ABCDEFGHI

(c) CB (or GC, HG, BH), giant SCC: AEF

3. **Solution**

(a) I. Starting from I one can eventually reach all pages in the system. Starting from any other page one can never reach I.

(b) First round: $p = (1/18, 1/18, 2/9, 1/18, 5/18, 1/9, 1/9, 1/9, 0)$

Second round: $p = (1/36, 1/9, 2/9, 1/36, 2/9, 5/18, 1/9, 0, 0)$

(c) 0. Because there will be no long paths (longer than 2 steps) pointing to G.

(d) 1/9. Because the probability to visit G, H, I will always be 1/3. G, H, I form a symmetric relationship, which means each of them will get a share of 1/9.

4. **Solution**

(a) Authority scores for A-G: (0,0,0,0,17/49,19/49,13/49)

Hub scores for A-G: (36/134,17/134,49/134,32/134,0,0,0)

(b) In option 1

Authority scores for A-G, X-Y: (0,0,0,0,17/50,19/50,13/50,1/50,0)

Hub scores for A-G, X-Y: (36/135,17/135,49/135,32/135,0,0,0,0,1/135)

In option 2

Authority scores for A-G, X-Y: (3/58,3/58,0,0,17/58,19/58,13/58,3/58,0)

Hub scores for A-G, X-Y: (36/143,17/143,49/143,32/143,0,0,0,0,9/143)

X gets a higher authority score in option 2 since Y gets a higher hub score by pointing to more webpages.