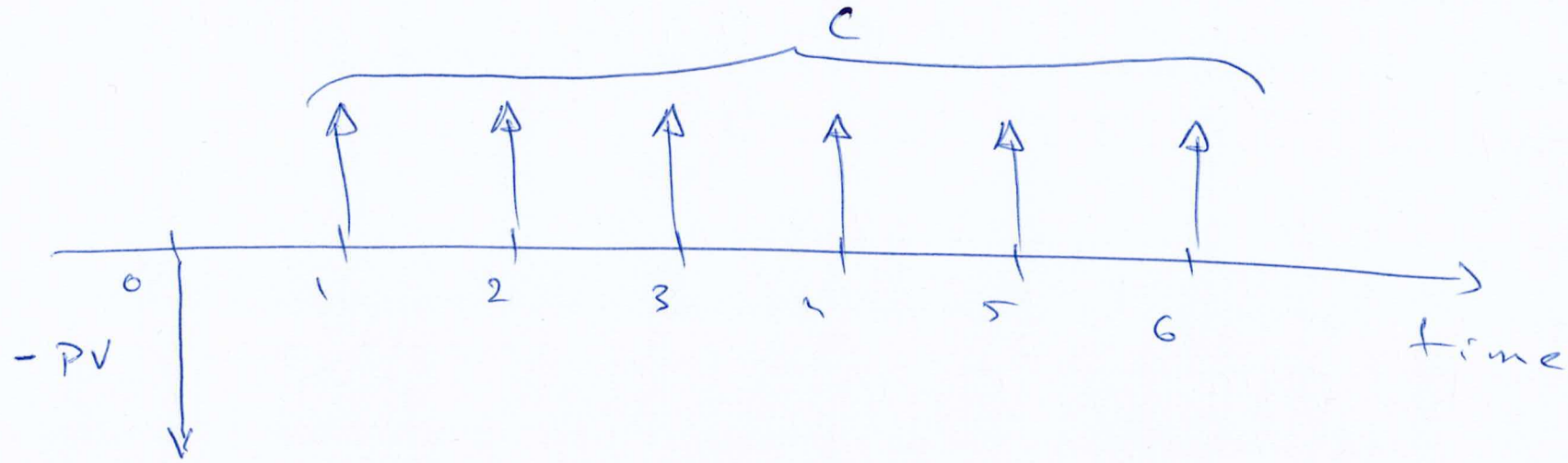
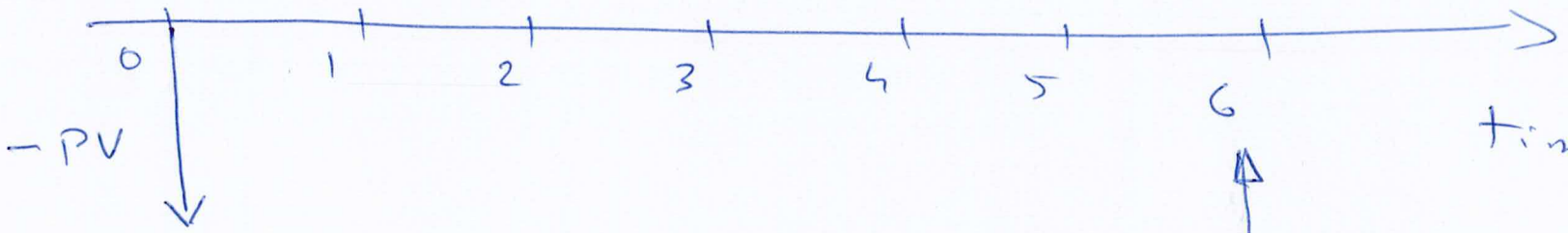
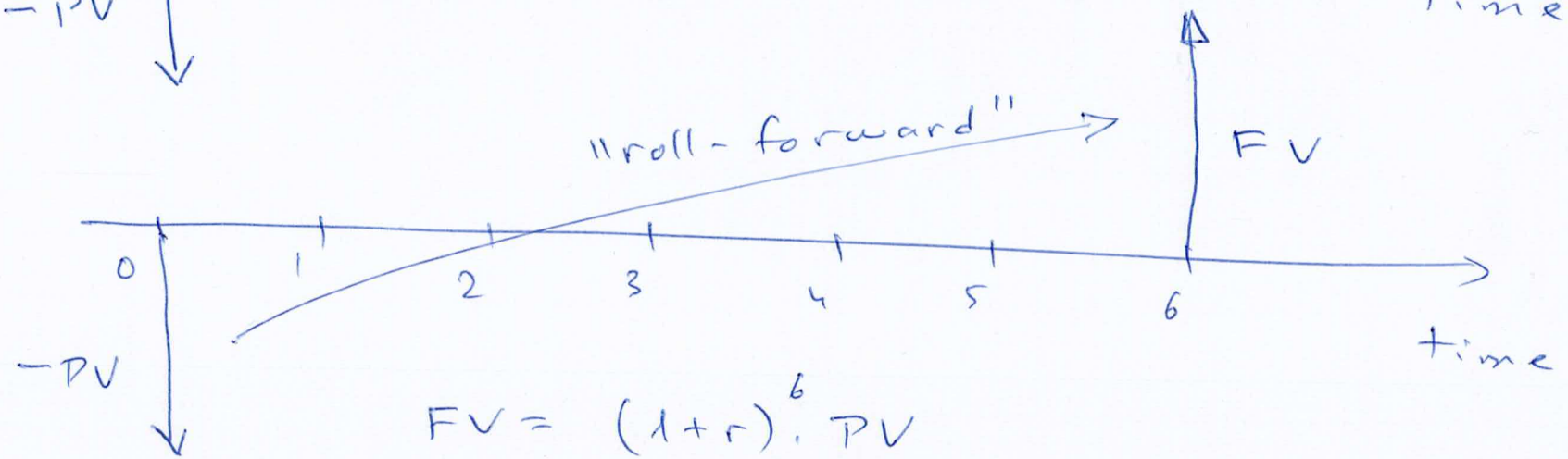
Ordinary Annuities







$$FV = (1+r)^{6} \cdot PV$$

"signs are context-dependent"
(problem/transaction dependent)

"Infinity is not too far."

Annuity

Perpetuity

$C = 100$

$r = .08$

$n = 10$ $PV \approx 671$

$n = 20$ $PV \approx 981$

$n = 40$ $PV \approx 1,192$

$n = 50$ $PV \approx 1,223$

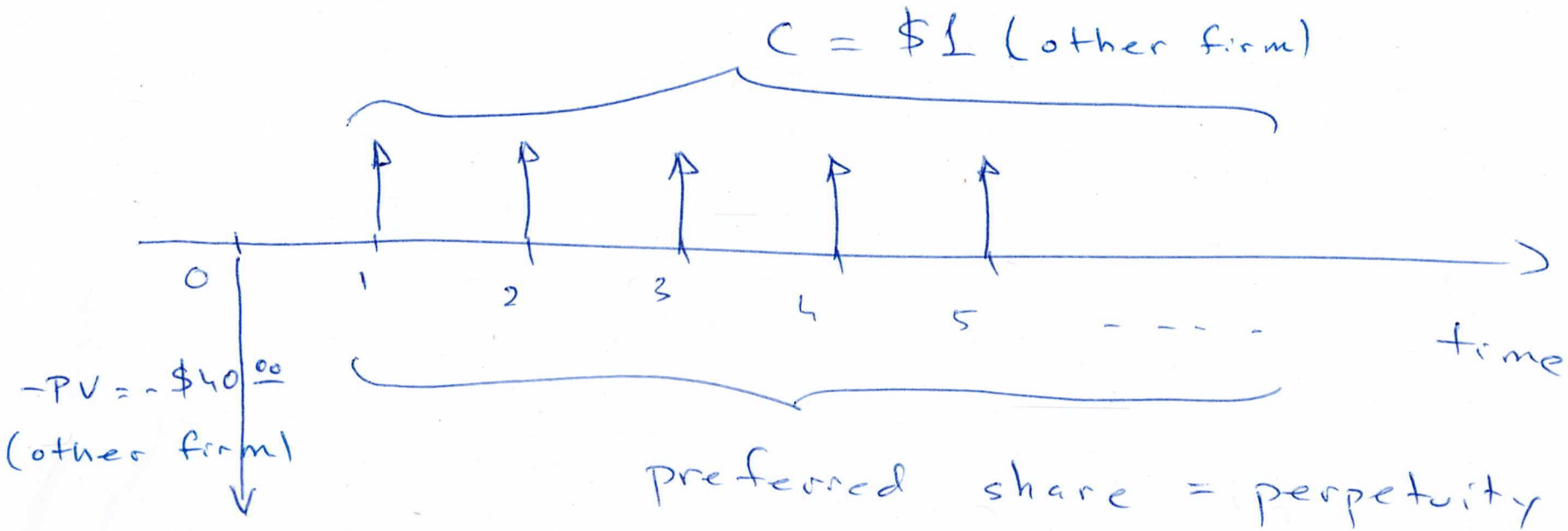
$n \to \infty$

$$PV = \frac{C}{r} = \frac{100}{.08} = 1,250.00$$

Challenge:

$r = 0 \Rightarrow$ then what?

$r < 0 \Rightarrow$



preferred share = perpetuity

$$\$40 = PV_{other} = \frac{C_{other}}{r} = \frac{\$1}{r_{other}}$$

r_{other} ← what is this?

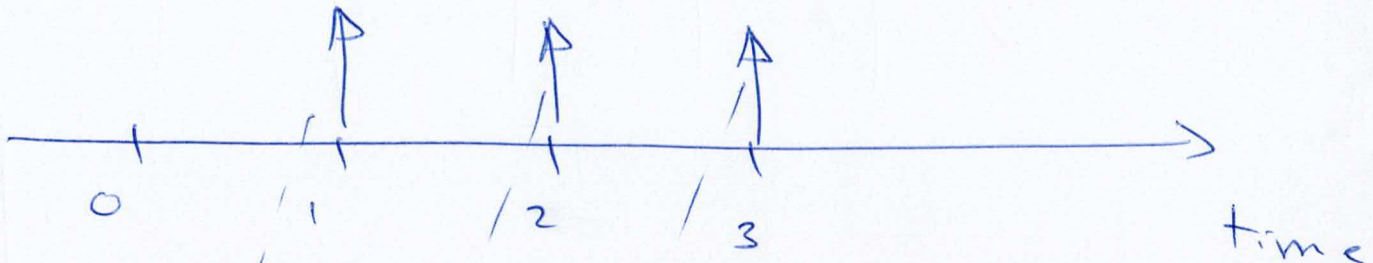
$$r_{other} = \frac{\$1}{\$40} = 2.5\% \quad \text{"per quarter"}$$

Use r_{other} for "our r".

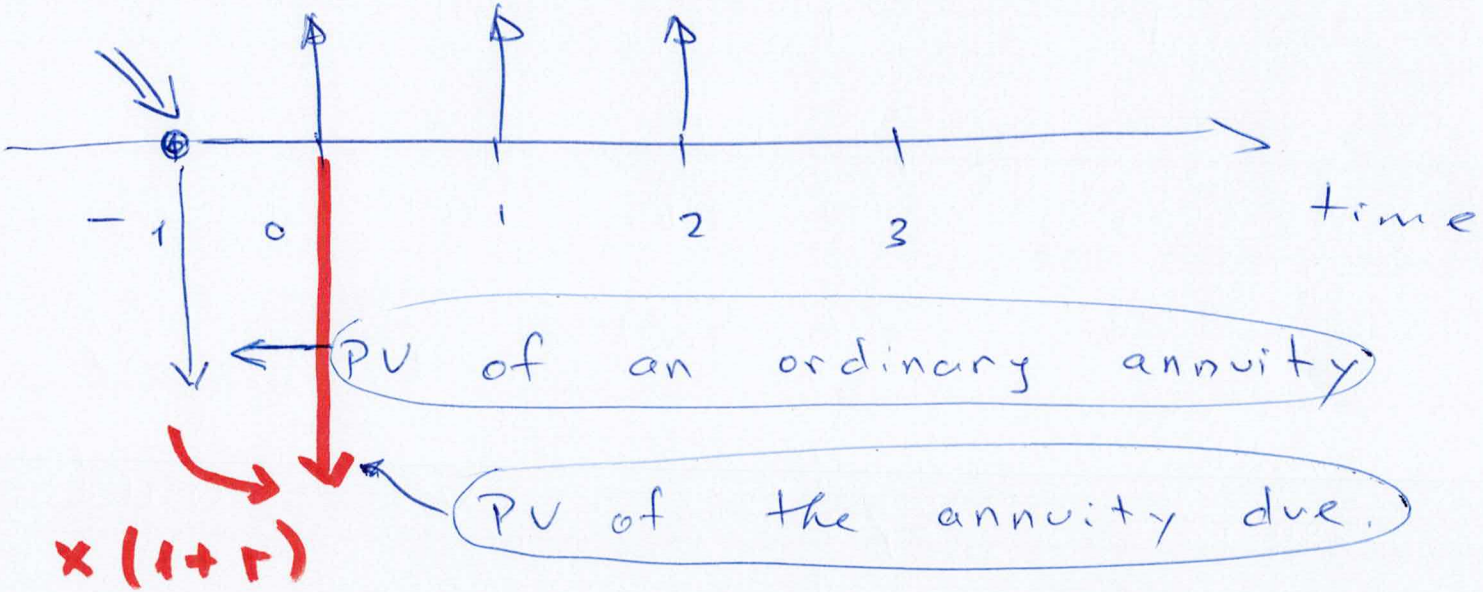
$$\$100 = PV_{our} = \frac{C_{our}}{r} = \frac{C_{our}}{0.025} \Rightarrow$$

$$\Rightarrow C_{our} = \frac{\$100 \cdot 0.025}{0.025} = \$100 \cdot 0.025 = \$2.5$$

Ordinary Annuity



Annuity Due



Annuity Due

$C = \$250$

$r = 7\%$

$n = 10$

* ordinary annuity ("at time -1")

$$PV_{ord} = \$1,755.90$$

* roll forward by 1 period:

$$PV_{due} = PV_{ord}(1+r)$$

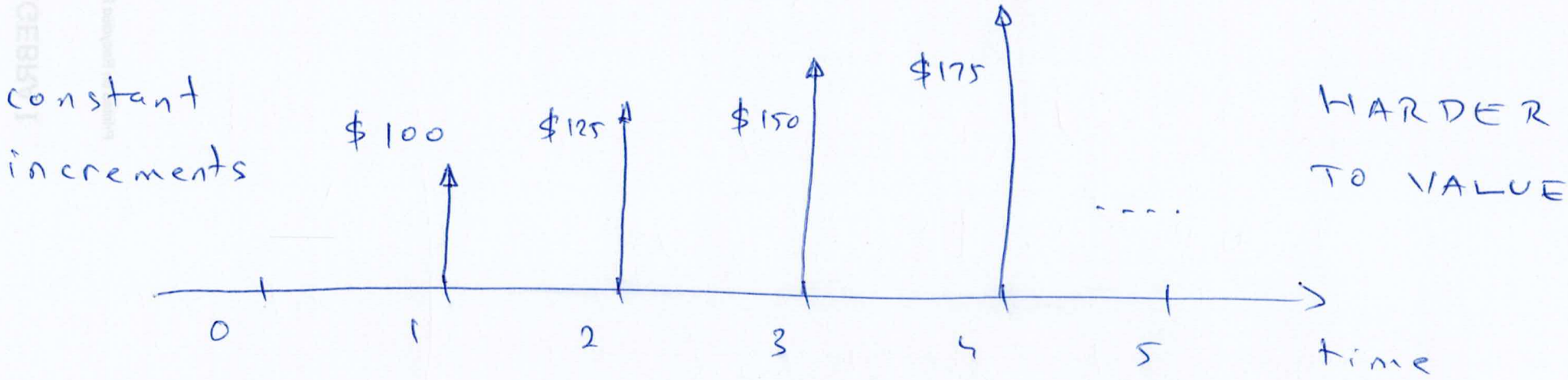
$$= 1,755.90 \times (1 + 0.07)$$

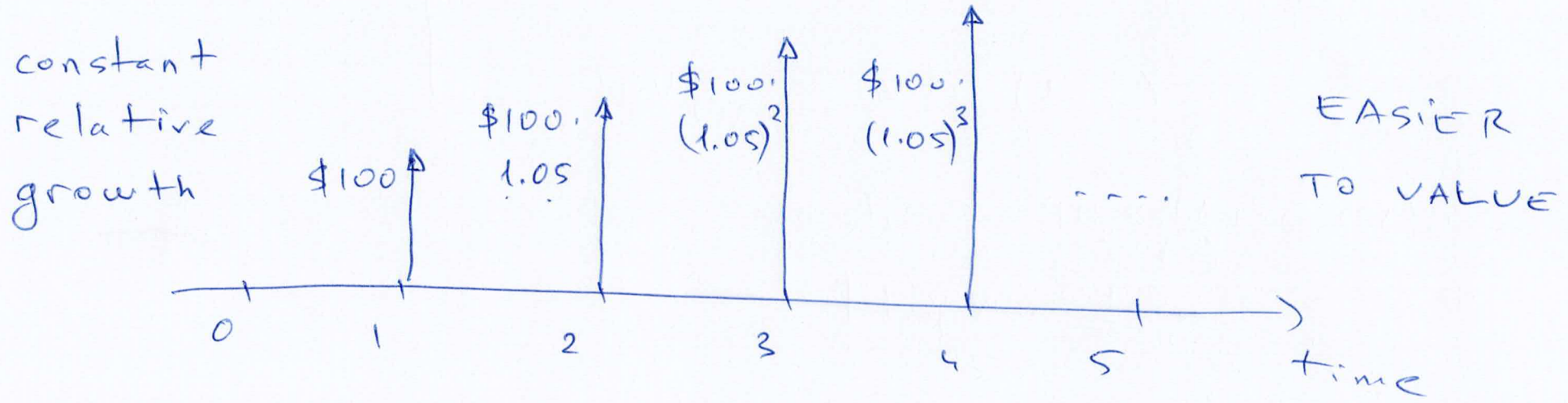
$$= \$1,878.81$$

ALTERNATIVE: Set calculator to use cash flows that occur at BGN!

$PV_{due} = \$1,878.81$ same as before.

Non-Constant Payments





$$1.05 = 1 + .05 = 1 + g.$$

$$g = 5\%$$

Growing Annuity

$C = \$500$ $\quad C_1 = \$500$

$g = 8\%$ $\quad C_2 = \$500 \cdot (1+.08) = \$540.00$

$r = 15\%$ $\quad C_3 = \$500 \cdot (1+.08)^2 = \$583.20$

$\ldots$

$n = 20$

$$PV_{growth} = 500 \cdot \underbrace{\frac{1 - \left(\frac{1+.08}{1+.15}\right)^{20}}{.15 - .08}}_{PV_{factor} = 10.22} = \$5,108.57$$

$$PV_{\substack{growth \\ perpetuity}} = \frac{500}{.15 - .08} = \frac{500}{.07} = \$7,142.86$$

PROBLEMS

- FV of a growth ~~perpetuity~~ annuity?
- PV of a growth annuity due?
- FV of a perpetuity? Assume $r > 0$.