Sample Problems
AEM 2241 - Finance

1 Finance Matters

1.1. Some of these problems are more complex that the problems you may be expected to solve during the exam. Nevertheless, you should attempt to solve them as **sub-parts** of more complex questions may be posed as exam questions. You will benefit most if you work on these problems before you look at the solutions. The course staff can answer any questions that you may have in relation to these problems.

1.2. The problem set is issued before all the material testable on the exam was taught. It may happen that some topics may not have been covered (yet) in class before the exam. If so, just skip the relevant parts.

1.3. All assumptions, conventions, and notations that we normally use can be relied without further explanations. If you use non-standard notations, explain what they mean.

1.4. Unless stated otherwise, we ask that dollar amounts be rounded to two decimals, and interest rates be rounded to four decimals. For example: \$156,798.38, \$9.75, 0.0315 = 3.15%, 0.1425=14.25%.

1.5. Be careful to distinguish between per-period quantities, such as per-period coupon payments and interest rates, and their annualized versions, which are the ones that must be typically provided as results.

1.6. [When working on paper] Unless we tell you otherwise, you may use either formulas or financial calculators to solve a problem. Whether you are using formulas or calculators, you must show what you did (e.g. what formulas you used, what values you replaced, or what calculator buttons you pushed and what values you entered), and **you must briefly explain the logic of your solutions**.

1.7. [When working on paper] To eliminate any possible ambiguity, whenever using the calculator, you must indicate values for all 5 TVM buttons; except for the value that you compute. Indicate explicitly what buttons you pushed to get a result, and what the result is.

1.8. Whenever possible, interpret the meaning of the results in terms consistent with the problem.

Good Luck!

2 Present Value Calculations

Each row of the table below represents two cash flows, one being the present value of the other when viewed as an investment over a time horizon t, at an interest rate r. One or two numbers are missing in each row.

2.1. For rows (a) and (c) compute the value of the missing number using the formulas given in class. Write down the suitable general formulas first, then replace letter symbols with known numerical values; finally, compute the respective results.

2.2. For rows (b) and (d), use a financial calculator. Similar to what we did in class, show the values that you would set up for the TVM variables, then show what key combinations you would press to get to the solution; also, provide the calculator's answer.

2.3. Is there anything special about the problem in row (e)? If so, what is it? Explain, in no more than two sentences, how a situation like the one shown may arise. For this part you can use formulas or the calculator, but you must state what you did and show the steps that lead to the solution.

2.4. For part (f), where there are two missing numbers, determine, using formulas, **two** combinations of values for PV and r that would make the connection between the four variables in row (e) correct. Note: You need to provide two pairs of numbers, (PV_1, r_1) and (PV_2, r_2), which are both consistent with the data in row (f). You may be able to determine part of the answer by making a choice, as the problem is not fully determined.

Part	**PV [$]**	**t [years]**	**r [%]**	**FV [$]**
(a)	7,513	7	9	
(b)		29	13	48,318
(c)	48,000	15		185,000
(d)	18,400		9	289,715
(e)	200,000	5		175,415
(f)		8		89,980

3 Simple vs. Compound Interest Rates

Consider an investment of \$10,000 that you make at the end of year 0. You are guaranteed an interest rate of 7.5% per annum for 10 years, compounded annually. We say that two interest rates are equivalent over a given time horizon t if the total interest earned by time t is the same in both cases.

3.1. Given a time horizon of $t = 5$ years, what is the simple interest rate that is equivalent to the compound interest rate described above?

3.2. Given a time horizon of $t = 10$ years, what is the simple interest rate that is equivalent to the compound interest rate described above?

3.3. Do the answers to the two questions above depend on your initial investment, assuming that the terms of the investment remain otherwise the same? Why?

3.4. Is it possible to have the same simple interest rate be equivalent to a given positive compound interest rate **simultaneously** over **several** investment periods longer than one year?

4 Discounted Cash Flow Valuation

The present value (at time 0) of the cash flow stream below is \$7,500 when discounted at 9% annually. What is the value of the missing cash flow? What would be the missing cash flow if the present value were \$4,500? Beside magnitude (the amount of dollars), is there anything different between the unknown cash flows in these two situations? Interpret this difference. Use formulas.

Year	**Cash Flow [\$]**
1	1,700
2	x
3	2,450
4	2,980

5 Time Value of Money

Use formulas to solve this problem, except, possibly, for the first and last sub-part.

The 6-month interest rate is 3.5%. Consider two successive six-month periods.

5.1. Fill out the statement below relying on our usual conventions, so that it is consistent with the problem text.

"The interest rate is _____% per year, compounded semi-annually."

5.2. Compute the EAR.

5.3. What is the present value at time 0 of \$500 to be paid at the end of the second 6-month period?

5.4. Now consider two equal payments made at the end of the first and second six-month periods, respectively. How big should these payments be so that their total present value at time 0 is equal to the value obtained in part (5.3) above?

Note: If you need the result of part (5.3), but you cannot calculate it, then use \$400 for the needed total present value.

5.5. More abstractly, consider a payment P made at the end of the second 6-month period and its time-0 present value PV_0. Separately, consider two payments of size P/2 made at the end of the first and second six-month period, respectively, as well as their total present value PV_0'. Can you determine, relying either on a qualitative argument **or** on formulas, whether PV_0 or PV_0' is bigger under ordinary circumstances, when interest rates are positive?

6 Mystery Financial Product

You purchased (invested in) a financial product that will pay you \$500,000 every six months for 4 years, starting exactly six months from today. The relevant interest rate is 12% per year, compounded quarterly.

6.1. Given the frequency of payments and the quarterly compounding interest, which of the following interest rates is closest to the rate you must use to discount the first cash flow you will receive?

(6.1.1) 3%

(6.1.2) 4%

(6.1.3) 6%

(6.1.4) 12%

6.2. The financial product that you purchased will be phased out (eliminated) and replaced with a similar product which has only 4 annual payments of \$1,000,000, with the first payment arriving

exactly one year from now. From your perspective, is the new contract more, or less valuable than the old contract?

(6.2.1) The new contract is more valuable.

(6.2.2) The two contracts are equally valuable.

(6.2.3) The new contract is less valuable.

6.3. In a different scenario, the financial product you invested in initially will be eliminated, and will be replaced with a similar contract. This time, however, you will receive four $1,000,000 payments, with the first one due in exactly 9 months. The second, third, and fourth payments will each be made exactly one, two, and three years after the first payment, respectively (i.e., payments following the first one will be made at the ends of successive one-year intervals following the first payment).

To make the exchange fair, if the value of the new contract to you is greater than that of the old contract, you will have to pay the difference in contract values to your counterparty. Should the value of the new contract be less than the value of the old contract, you will receive the difference in contract values from your counterparty. If the old and new contract have the same value, no money exchanges hands.

What will happen?

(6.3.1) The answer cannot be determined.

(6.3.2) No money will exchange hands.

(6.3.3) You will receive a payment from your counterparty.

(6.3.4) You will make a payment to your counterparty.

7 You Won the Lottery!

You have just won the lottery and will receive 10 yearly payments, as follows: you get $1,500,000 in the first year, after which yearly payments will increase by 2.7% per year. A company specializing in purchasing annuities (yes, they do exist!) offers you instant $14,000,000 in cash to purchase the right to receive your winnings. The relevant interest rate is 3% per year. Will you take the offer?

8 Annuity, But You Have to Wait

Consider an annuity with a yearly payment of $\$75,000$ that makes its first payment at the end of year 5, and consists of 10 payments. Assume that the appropriate interest rate is 11% per year.

8.1. Using formulas for annuities, value this annuity (compute its present value) as of the end of year 4 (i.e. as if time had passed, and "now" were at the end of year 4); name this value PV_4.

8.2. Using formulas for annuities, value this annuity as of the end of year 5; name this value PV_5. What kind of annuity is this when viewed from the end of year 5?

8.3. Compute the present value of this annuity at time 0 in two different ways, starting separately from PV_4 and PV_5, respectively. Compare and comment very briefly on the two answers that you get.

9 One or Several?

Consider an annuity payment that will pay 75,000 every six months for 5 full years. Afterwards, the payment will increase to 100,000 every six months, for 5 more years. The relevant interest rate is 7.5% per annum, compounded semi-annually.

9.1. Explain how this annuity is equivalent to the difference of **two** regular annuities, each starting at time 0, one with a maturity of 10 years, one with a maturity of 5 years. Specify in full the details of the two component annuities.

9.2. Using the observation made in the previous part, value the component annuities, and combine these values to get the value of the composite annuity. If you could not identify the two annuities into which the initial annuity can be decomposed, feel free to use any other method to value the initial annuity.

Note: Many complex financial instruments can be seen as collections of simpler instruments that can be understood and/or valued in isolation. In such cases the value of the complex financial instrument emerges by the suitable aggregation of the values of its respective components.

10 Amortizing loans with variable payments

Consider an amortizing loan with **variable** payments. The initial loan amount is $10,000,000, while the initial loan maturity is 30 years. The borrower must make monthly payments, with payments due

at the end of each month. The borrower's total payment on this loan that is due at the end of the 73^{rd} month equals \$87,777.78.

10.1. What is the monthly principal repayment on this loan?

10.2. What is the balance outstanding on this loan at the beginning of the 73^{rd} month?

10.3. What is the interest rate that would have been quoted on this loan when it was issued?

Note: If you need to use results from either parts (10.1) or (10.2) above, or from both, but you were not able to determine the respective values, you may use \$50,000 for the answer to part (10.1), and \$5,000,000 for the answer to part (10.2), respectively.

11 Balloons

You are an important local real estate investor; you just got a \$10,000,000 balloon loan to buy a new office building in your home town. The nominal maturity of the loan is 30 years, but the loan has a 10-year balloon payment. In other words, the loan will end at the end of the 10th year, and the outstanding balance will be paid off in a lump sum at that time. The interest on the loan is 6.7% per annum, compounded monthly.

11.1. Assume that the loan has fixed payments.

(11.1.1) What is the monthly fixed payment that you have to make?

(11.1.2) Let PV be the present value at time 0 of the fixed payments made over the 10-year life of the loan. What is PV?

(11.1.3) Can you find a connection between the \$10,000,000 principal, PV, and the balloon payment at the end of year 10? Compute, using this connection, the size of the balloon payment at the end of year 10. If you cannot find the connection, a more work-intensive approach is to use an amortization table to compute the answer (use Excel!).

11.2. Assume now that this loan has variable payments. At the end of each month fixed, equal portions of the loan's principal are paid down, such that the principal would be fully amortized at the end of the hypothetical 30-year loan period. What is the size of the balloon payment at the end of year 10 in this case?

12 Better Late Than Never

You work for a bank. Exactly five years ago, you helped Al Kapon, a well-known local businessperson, to get a $15,000,000, 20-year variable-payment amortizing loan in order to build a "soft drink bottling facility." The loan carries an interest of 7% per annum, has monthly payments, and is structured like similar loans discussed in class; in particular, Kapon is expected to pay down the same amount of principal every month for the duration of the loan, in addition to the interest due monthly.

12.1. What is the payment due at the end of the very first month of this loan?

12.2. On the fifth anniversary of the loan Kapon comes to your office unexpectedly, and states that his business is in trouble. However, he hopes that difficulties are temporary, and that his business and finances will recover within one year. After some back and forth, you agree on behalf of the bank to forsake principal payments due for the next 12 months. However, Kapon still must pay in full the interest due at the end of each month.

(12.2.1) What is the loan balance at the end of 5 years, when Kapon asks for the modification of the loan?

(12.2.2) What payments will be made at the end of each month for the duration of the year when principal payments are suspended?

12.3. Principal repayments resume after the 12 months elapse. For the remainder of the loan's original term, the same amount of principal will be repaid every month, so that by the loan's original maturity date the principal is fully paid off.

(12.3.1) What will be the monthly principal payments due after the end of the principal repayment suspension?

(12.3.2) Provide the row of the loan's updated amortization table corresponding to the first month in which after principal payments have resumed.

13 Beginning Balance at the End...

Assume that you have a fixed-payment amortized loan with a principal of $8,000,000, a yearly interest rate of 9% compounded monthly, and a maturity of 10 years.

To the closest thousand, what is the beginning balance of the loan at the start of the last month (i.e., at the beginning of the month at the end of which the very last payment due on the loan is made)?

13.1. $100,000

13.2. $101,000

13.3. $102,000

13.4. None of the numbers above can be the answer.

14 Collecting Coupons

Assume that a bond sells for $948; it has semi-annual coupons, a maturity of 8 years, yields 5.1%, and has a face value of $1,000. What is the coupon rate of this bond?

15 But How Much Do I Make?

Assume that a bond with a maturity of 10 years, face value of $1,000, coupon rate of 5%, with semi-annual coupons, has a market price of $903.25. You have already determined that the yield of the bond is between 6% and 7%. Show yields as percentages with two decimals precision.

15.1. Assuming that the yield of the bond were 6.5%, show how you would set up your calculator to compute the implied bond value and provide this respective value.

15.2. Set up a table similar to that given in class to determine an approximate value for the yield of the bond. Use your calculator to compute bond values, but do not show the details of your calculator operations. Stop when the mid-yield produces a price within $0.50 of the bond's true price.

Hints: You only need to compute the total value of the bond for various yields, not also the part attributable to coupons or principal. Also, since only the yield changes, you do not have to re-enter all the values into the TVM worksheet - just change the yield and recompute the value.

16 Treasuries

On February 28, 2023, the financial press announced that yields for US Treasury bonds with 10-year maturities were at record low levels, never before seen. Indeed, early in the day, 10-year Treasury yields were as low as 1.18%, corresponding to a price of $1,029.50 per $1,000 face value. Typical Treasury bonds pay semi-annual coupons.

16.1. Without performing any computations, can you provide a **lower bound** (lower limit) for the yearly coupon rate of this 10-year US Treasury bond? If yes, state what this lower bound is, and how you know it is correct. A trivial lower bound of 0 is not an acceptable answer.

16.2. What is the implied annual coupon rate for this bond?

17 Bond Decomposition

You are studying a bond that has a leftover maturity of 7 years, has a face value of $1,000, and a stated coupon rate of 5% per year, payable semiannually. The yield of the bond is 4.7% per annum; further, you may assume that per-period rates are constant (they do not depend on the time horizon).

The bond contract includes covenants (agreements) meant to protect bond investors' interest. In the recent past the company breached one of these covenants. Even though some improvements did occur since then, the breach is expected to persist for the next two years, after which it is expected to be cured (eliminated). While the breach persists, the coupon rate will be double the normal (stated) one. If default were to occur, such an accelerated payment schedule assures that investors get more of their money back; if default does not occur, then the increased coupons act as a penalty for the breach of the covenant.

17.1. Decompose this bond into simpler financial instruments that were studied in class. Specify what these simpler instruments are, what their parameters are, and how do you know your decomposition is correct.

17.2. Determine the current price of the bond.

18 NPV Mystery I

The NPV of a regular coupon bond's cash flows, when the discount rate is equal to the bond's yield, is equal to the bond's price (value). This statement is...

18.1. True

18.2. False

19 NPV Mystery II

For the types of bonds discussed in lectures, the NPV of the bond's principal (face value) always exceeds half of the bond's current price (value). This statement is...

19.1. True

19.2. False

20 Congratulations, You're an Analyst

You have just been hired as a junior analyst working for a bond trader. Your first assignment is to value a corporate bond paying semi-annual coupons at an annual rate of 9.5%, with a maturity of exactly 2 years. The bond has a face value of $1,000. A senior analyst has already processed the current Treasury price data and provided you with an up-to-date term structure chart, shown in Figure 1. You are told to treat this corporate bond similarly to government bonds; i.e. you will ignore all default, liquidity, and similar risks, which will be analyzed by more experienced colleagues.

20.1. The term structure of interest rates chart has a horizontal axis labeled "Maturity," and a vertical axis labeled "[Annual[1]] Yield." What kind of government bonds have their yields and maturities plotted on this chart?

20.2. From earlier problems you solved while in college, you learned to decompose more complex financial instruments into sums - or differences - of simpler instruments. Explain how you can decompose this corporate bond into a collection of zero-coupon bonds of different maturities, perhaps having atypical face values. Provide a brief statement explaining the decomposition and show, in a table, what would be the bond's maturities and face values, respectively.

20.3. You learned that in realistic settings cash flows that arrive later must be discounted at (usually) higher per-period interest rates. For each zero-coupon bond listed in part 20.2 above, use the term-structure chart and look up its corresponding yield. Next, compute the present value of each zero-coupon bond. Using these zero-coupon bond prices, and also relying your earlier insights, provide a computed (theoretical) price for your corporate bond.

20.4. You now have a price for your corporate bond - what is its yield?

20.5. You provide the result computed in item 20.4 to one of your colleagues, who explains that in the practice of your firm, in order to adjust for the risks that a bond like yours bears in addition to government bonds, its yield must be changed by 0.50% per year. The colleague did not say

[1] As you will note, the chart only uses "Yield" as a label for the vertical axis. The usual bond terminology expresses yields in annualized terms, and you should do the same in this class, as well as in other finance-related work that you do. We provide a reminder here, but you should **not** assume that similar reminders will also be present when taking an exam.

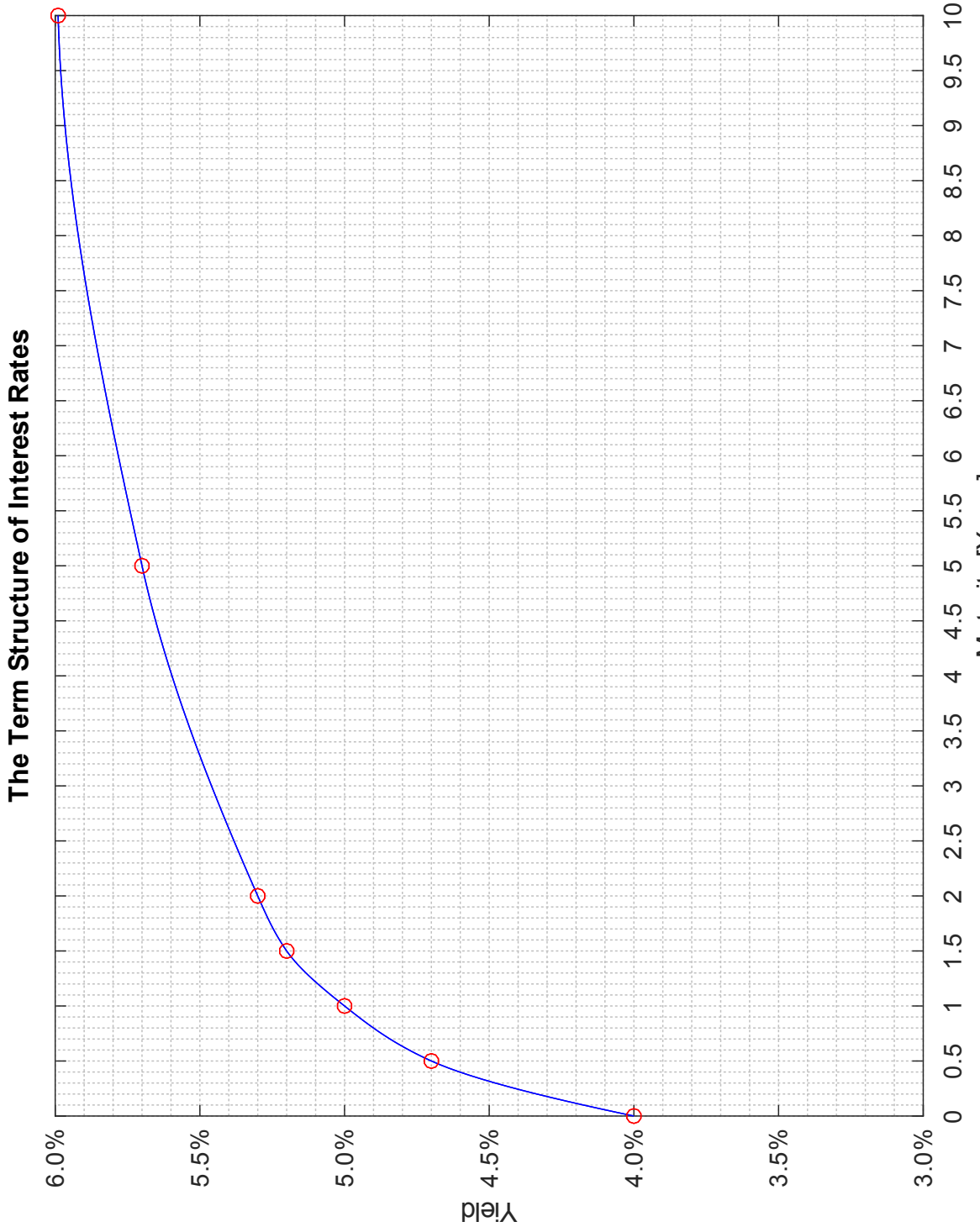

Figure 1: Term structure of interest rates on your first day as an analyst, as determined by a senior analyst using current government bond data.

explicitly whether the yield should be increased or decreased. State whether the yield must be increased or decreased, explain why, and then compute the new bond price. Determine what is the percentage change in the bond price when comparing prices before and after the risk adjustment, respectively.

20.6. Now adjust the yield in the **opposite direction** to that you decided was necessary in part 20.5 above. Compute the new price and the percentage price change when comparing the original (riskless) price and the price after the newest change in yield. Compare the percentage changes in the price of the corporate bond when the yield has been increased and decreased by the same amount, respectively. Which relative change is bigger? Could you have predicted which change is bigger by examining any of the slides discussed in class, without resorting to formulas?

21 We Have No Money, We're a Startup

Next Wonder, Inc. is a startup that cannot afford to pay dividends, since it has to finance its rapid growth, as it aims to take over the technology world. The company will pay no dividends for 5 years, but then it plans to pay \$2.5 per share, per year for the next 5 years. After these 10 years, the stock dividend will jump to \$5 in year 11, and will keep increasing by 3% per year indefinitely. The required return is 8% per year.

21.1. Assume that you are computing the price of the stock, P_{10}, at the end of year 10, just after the dividend due at the end of year 10 has been paid. What is P_{10}?

21.2. Assume that you are computing the price of the stock, P_5, at the end of year 5. What is P_5?

21.3. What is the price of the stock at time 0, P_0?

22 Stock price

You are a new analyst following the stock of company Big Break, Inc., valued at \$15 per share on the morning of March 31, 2023. The company has been paying dividends of 35 cents on the very last day of each calendar quarter, and this was expected to continue indefinitely.

22.1. Assuming that the dividend model is reasonably accurate for this stock, what is the implied return rate demanded by investors in this stock, expressed in "per annum, compounded quarterly" terms?

22.2. In a news conference late in the day on March 31, 2023, the company's CEO announces "temporary difficulties related to external funding needs" that will force the company to cancel the

dividend for four successive quarters, "in order to conserve cash." The dividend due on June 30 will be paid, however, as it had already been declared, so the first missed payment will occur on September 30, 2023. The CEO, a highly credible, seasoned industry veteran, states that the dividend policy will be reinstated in its current form after the temporary suspension ends. Note:

If you need to use the value that was requested in part (22.1) above, but you were not able to compute it, you may use 5.5% for the "per annum, compounded quarterly" rate.

(22.2.1) Estimate the price of stock as of March 31, 2024.

(22.2.2) Estimate the price of the stock as of March 31, 2023, just after the announcement.

23 Dividends

Fly Over Airlines just paid an annual dividend of 80 cents, and a new analysis revealed that dividends will grow long term at a rate of 4% per year. The company's stock price is $70. Estimate, to the closest percent, the rate of return demanded by investors who buy Fly Over's shares.

23.1. 5%

23.2. 6%

23.3. 7%

23.4. None of the above is the correct answer.

24 What's an Option Worth?

Over the last few weeks we got into the habit of describing more complex financial instruments as collections of simpler instruments. For example, we can decompose a typical bond into an annuity (the coupon payments), and a separate single payment (the principal). Similarly, we have seen atypical annuities described as sums or differences of regular annuities. In this problem you will build a synthetic bond (see below), by suitably combining fractional amounts of two other bonds.

Consider the following information about three Treasury bonds:

Maturity Date	Coupon Rate	Price	Bond Type
05/15/2024	6.500	106.31250	regular
05/15/2024	8.250	103.43750	callable
05/15/2024	12.000	134.78125	regular

All the bonds above have a face value of \$1,000. Prices are expressed as percentages of face value. We do not know the precise date when these prices were quoted; as such, it is possible that we are not exactly at the beginning of a coupon period. We do know, however, that all coupon payment dates for all these bonds are the same; the coupons, of course, are not. The bond in the middle is a so-called callable bond, while the other two are regular Treasuries.

24.1. Read the textbook and/or research the web to understand what callable Treasuries are. Summarize your findings in a **brief** paragraph.

24.2. Given what you learned about callable bonds, under what conditions would the option to call the callable Treasury be exercised and by whom (the bond holder, or the bond's issuer)? In other words, when would the callable Treasury be called and by whom?
Hint: Assume that the call decision is rational; consider changes in interest rate levels as time passes.

24.3. From the **holder**'s perspective, would a callable Treasury bond be more, or less valuable than an otherwise identical non-callable, i.e., regular Treasury bond? Briefly state your opinion and justify it qualitatively.

24.4. Now consider the pricing information provided in the table above. Use the information on the regular bonds to construct a bond that is identical to the the callable bond in all respects, but for the callable feature. Use this artificial bond to compute the cost of the call option. From the perspective of the bond holder (the lender), is this cost positive or negative?

Hint: Let B_1, B_2, and B_3 be the three bonds shown in the table above, starting with the bond at the top. If we ignore the call feature, the listed Treasuries are characterized by a small number of parameters: their coupon sizes, coupon payment times, and face values. You should build a portfolio (combination) of the two regular bonds B_1 and B_3, such that their combination produces B_2 (again, except for the call feature). Let B_{13} be an artificial (often called "synthetic") bond that you can create from a combination of B_1 and B_3, and assume that B_{13} consists of a

fraction f of B_1 and a fraction $1-f$ of bond B_2, where $0 \leq f \leq 1$. Write down equations so that the resulting coupon and face value of B_{13} (seen as a mix/combination/portfolio of B_1 and B_3) matches the respective parameters of B_2. Use these equations to determine f, and then use f to determine an appropriate price for B_{13}.