AEM 2240: Finance for Dyson Majors
Time Value of Money

Dyson
Cornell
SC Johnson College of Business

Overview

- **Basic Concept and Terminology**
- Present Value and Future Value
- Cash Flow Streams
- Case Study

Time Value of Money

Basic premise:

It is easy to compare the **values** of two **cash flows** when those cash flows occur at the **same point in time**.

E.g., would you prefer $100 today versus $102 today?

It is harder to compare the **values** of two **cash flows** when those cash flows occur at **different points in time**.

E.g., would you prefer $100 today versus $102 in one year?

Why? Isn't $102 better than $100? (Let's assume you don't need the money right now)

Time Value of Money

Basic premise:

In this lecture block, we will develop a **quantitative framework** for evaluating **cash inflows and outflows** that occur at **different points in time**.

Some Terminology

FV = **Future Value**: Value of cash flow **in the future** given the level of interest/rate of return in the marketplace.

PV = **Present Value**: Value of future cash flow **in the present** given the level of interest/rate of return in the marketplace.

r = **Interest Rate**: “exchange rate” between “earlier money” and “later money”

r is also known as the (opportunity) cost of capital, (required) rate of return, discount rate.

Where does r come from? We will discuss this later in the course.

The Time Line of Cash Flows

Because the timing of cash flows is so crucial, it is always a good idea to draw **time lines of cash flows**.

Common practice is to put:

- time on top of each tick,
- cash flows below the ticks, and
- interest rates on top of the time line.

Having said that, you can do whatever works best for you (e.g., time below the ticks and cash flows on top of each tick).

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Example 1: Future Value

- Suppose you invest $1,000 for one year at a rate of 5% per year. What is the future value in one year?

- Suppose you leave all your money in for another year. How much will you have two years from now?

Future Value: General Formula

$FV = PV\ (1 + r)^t$

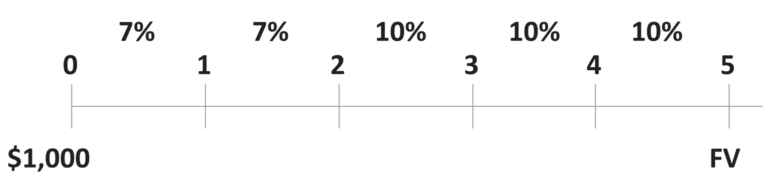
- FV = future value
- PV = present value (<u>Note</u>: PV is sometimes also referred to as C_0)
- r = interest rate, expressed as a decimal
- t = number of periods (<u>Note</u>: Some textbooks use “n” to denote the number of periods. This is somewhat unusual. In this lecture block, we will use “t” to denote the number of periods.)

Example 2: Different Interest Rates

What if the interest rate is not the same each period?

Suppose you invest $1,000 for 5 years.
The interest rate is 7% per year for the first two years and 10% per year afterwards.

How much would you have?



Example 3: Solving for *t*

Suppose you have $1,000 today.
Investing at 10% per year, when do you double your money?

- What inputs do we have?
  - PV = 1,000; r = 0.10; FV = 2,000
- What are we missing?
  - *t*

How do we solve this problem?

Present Value: General Formula

We've talked a lot about future values.

Now let's switch things around and ask ourselves the following:
How much should I be willing to pay today in order to get X in the future
[Or what is the present value of the future cash flow X?]

The formula we use to address these kinds of questions is:

$PV = FV / (1 + r)^t$.

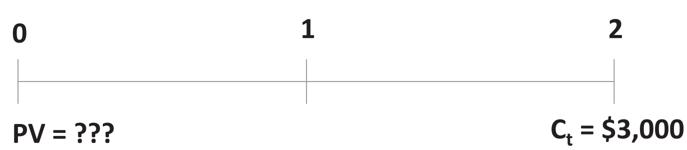
$[1 / (1 + r)^t]$ is called the **discount factor**.

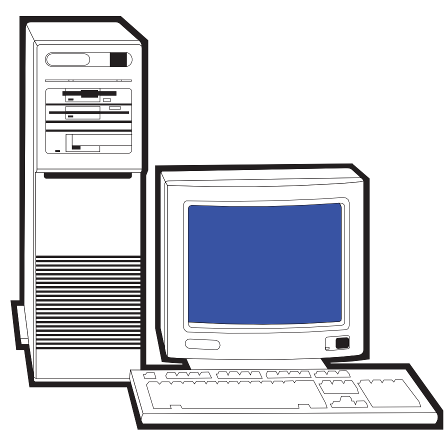
Example 4: Present Value

Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?

Example 5: Present Value

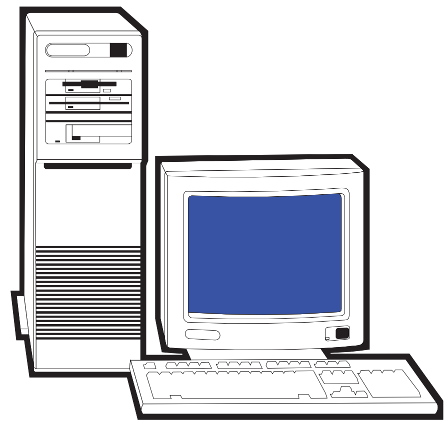
You just bought a retro PC for $3,000. The payment terms are “pay cash in 2 years.” If you can earn 8% per year on your money, how much money should you save today to make the payment when due in two years?





Example 5: Present Value

What happens if you are given the opportunity to pay for the PC in three years instead?



Example 6: Solving for the Interest Rate

Van Gogh's "Sunflowers" sold for $125 in 1889.
In 1987, it sold for $36,000,000.
Suppose you bought the painting in 1889 and sold it in 1987, was it a good investment (based on the annual rate of return)?

You held the painting for 98 years (1987 – 1889).

Example 7: Solving for the Rate of Return

"I just had the best idea ever… Postage rates keep going up… So why don't you buy a bunch of forever stamps now, wait for postage rates to go up, and sell your forever stamps then!!!"

Example 7: Solving for the Rate of Return

History of postage rates:

On March 26, 2007, the US Postal Service unveiled the first forever stamp, for 41 cents ($0.41).

Dates	Rates
May 12, 2008	$0.42
May 11, 2009	$0.44
January 22, 2012	$0.45
January 27, 2013	$0.46
January 26, 2014	$0.49 (dropped to $.47 in 2016; came back to $.49 in 2017)
January 21, 2018	$0.50
January 27, 2019	$0.55
August 29, 2021	$0.58

Example 7: Solving for the Rate of Return

In April 2022, you could sell a forever stamp for \$0.58. If you bought it at \$0.41 in April 2007 (about fifteen years ago), was this a good investment…?

Recap

$PV = FV / (1 + r)^t$

- There are four parts to this equation:
    - PV, FV, r and t.
    - If we know any three, we can solve for the fourth:
        - FV $= PV (1 + r)^t$,
        - r $= (FV / PV)^{1/t} – 1$,
        - t $= \ln(FV / PV) / \ln(1 + r)$.

- If you are using a financial calculator, remember the sign convention or you will receive an error when solving for r or t.

Overview

- Basic Concept and Terminology
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- **Cash Flow Streams**
- Case Study

Example: Introduction to Cash Flow Streams

You deposit $4,000 at the end of each year over the next three years in a bank account that pays 8% interest per year.
You currently have $7,000 in the account.

How much money will you have in three years? In four years?

Valuation of Multiple Cash Flows

It helps to categorize the different types of “multiple cash flows:”

- multiple cash flows
  - “finite”
    - “even”
      - “constant”
        - Hard way
        - Shortcut*
        - Fin. calc.*
      - “growing”
        - Hard way
        - Shortcut*
    - “uneven”
      - Hard way
      - Fin. calc.*
  - “infinite”
    - “even”
      - “constant”
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* Denotes the recommended method

Valuation of Multiple Cash Flows

Let’s go through them one by one…

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Finite, Even, Constant

“Hard way” to compute future values:

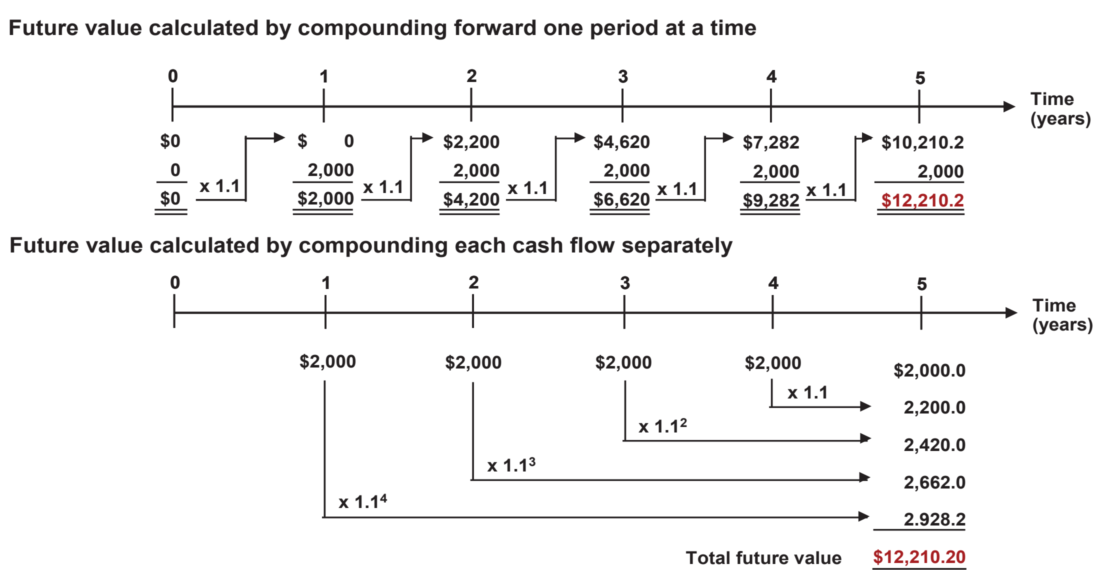
Option 1 (recommended):	Calculate the future value of each cash flow and add them up.
Option 2:	Compound the accumulated balance forward one year at a time.

“Hard way” to compute present values:

Option 1 (recommended):	Calculate the present values of each cash flow and add them up.
Option 2:	Discount back the balance one period at a time.

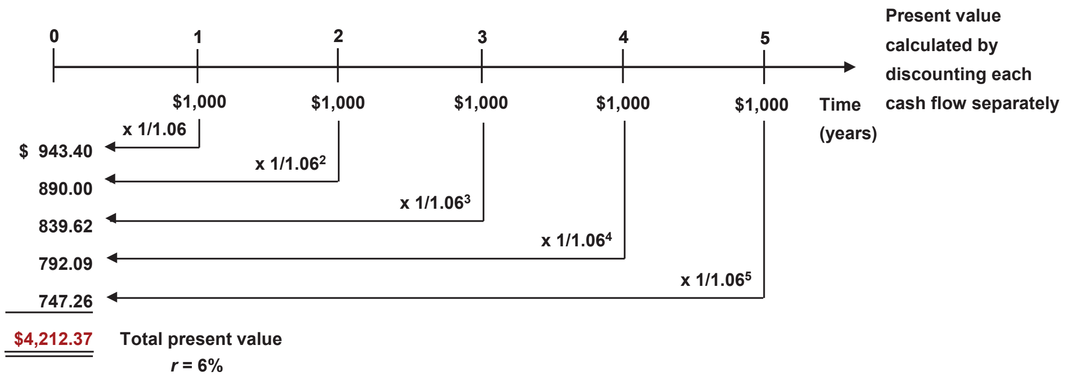
Finite, Even, Constant

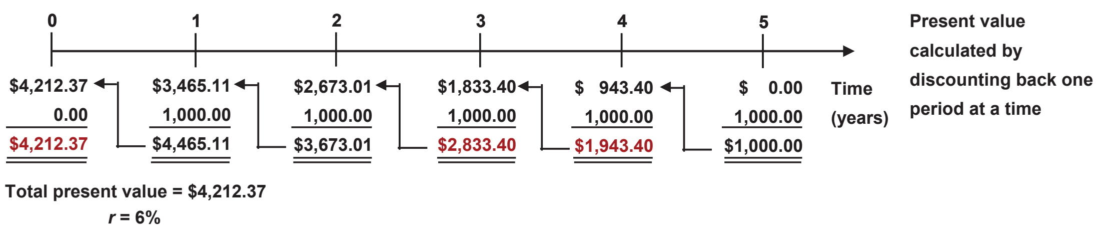
We invest $2,000 at the end of each year over the next 5 years at 10% per year. What is the future value in five years?



Finite, Even, Constant

What is the present value of a cash flow stream of $1,000 at the end of each year over the next 5 years when the appropriate discount rate is 6% per year?





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Finite, Even, Constant

Finite, even, constant cash flow streams are also referred to as "ordinary annuities" or as "annuities due:"

- Ordinary annuity: series of identical cash flows occurring at the end of each period for some fixed number of periods.
- Annuity due: an annuity for which the cash flows occur at the beginning of each period.
- Unless further specific, ordinary annuity will be the default.

Finite, Even, Constant

There are shortcut formulas that make our life easier (Note: Most financial calculators use "PMT" and "CF" to denote cash flows. In this lecture block, we will mostly use "C".). We can also use a financial calculator.

Ordinary annuity:

$$PV = C_1 \times \left[\frac{\left(1 - \frac{1}{(1+r)^t}\right)}{r}\right]$$

$$FV = C_1 \times \left[\frac{\left((1+r)^t - 1\right)}{r}\right]$$

Finding the FV: Retirement

Your current retirement account balance is \$0. Suppose you start saving for retirement by depositing \$2,000 at the end of each year.
Suppose you can earn 7% per year, how much will you have in 40 years?

Finding the PV: Buying a House

You are ready to buy a house and you have $20,000 for a down payment. You have an annual salary of $36,000 and the bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income. The interest rate on the loan is 0.5% per month; it's a 30-year fixed rate loan.

How much money will the bank loan you?

How much can you offer for the house?

Finding the PV: Buying a House

How much money will the bank loan you?

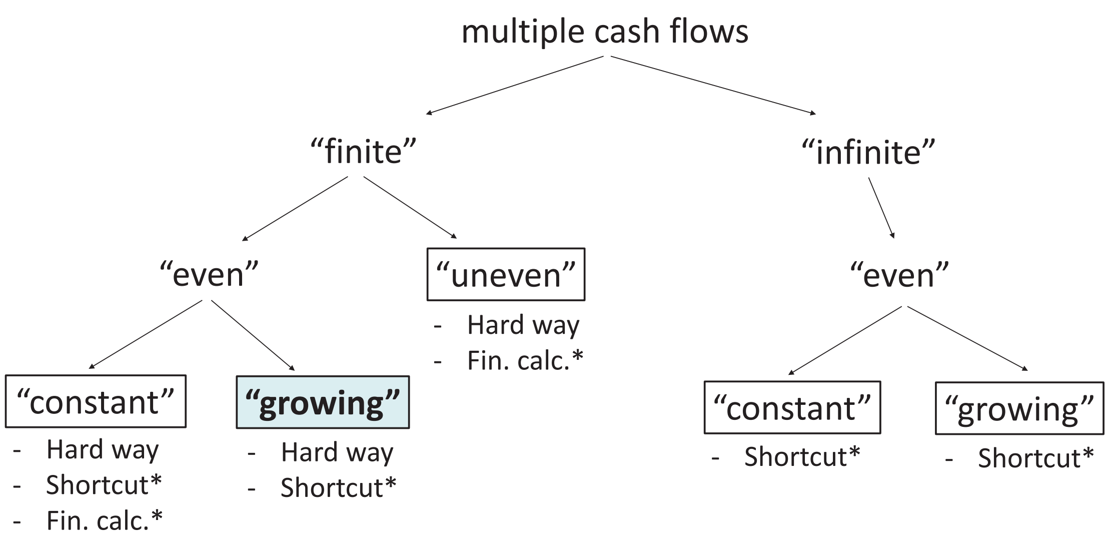
How much can you afford?

Finding the Payment C

Suppose you want to borrow $20,000 for a new car.
You can borrow at 0.75% per month.
If you take a 4 year loan, what is your monthly payment?

Valuation of Multiple Cash Flows

Let's go through them one by one…



* Denotes the recommended method

Finite, Even, Growing

These cash flow streams are also referred to as “growing annuities”

$$PV = \frac{C_1}{(1+r)} + \frac{C_1(1+g)}{(1+r)^2} + \frac{C_1(1+g)^2}{(1+r)^3}$$

Finite, Even, Growing

You can solve them the “hard way” or through another shortcut formula (Technically, you can also use the financial calculator, but I don’t recommend that.)

$$PV = C_1 \times \left[\frac{\left(1 - \left(\frac{1+g}{1+r}\right)^t\right)}{r-g} \right]$$

g is the growth rate of C_1;
Note: C_1 is the cash flow that occurs exactly one year from today.

Valuation of Multiple Cash Flows

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      - **“constant”**
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      - **“growing”**
        - Shortcut*

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Infinite, Even, Constant or Growing

These cash flow streams are also referred to as “perpetuities,” or, more specifically, as “constant perpetuities” and “growing perpetuities.”

You can only solve for them using a shortcut formula.

Constant Perpetuity:

$$PV = \frac{C_1}{r}$$

Growing Perpetuity:
(also known as **Gordon growth model**)

$$PV = C_1 \times \left[\frac{1}{r-g}\right]$$

g is the growth rate of C_1;
Note: C_1 is the cash flow that occurs exactly one year from today.

Valuation of Multiple Cash Flows

Let's go through them one by one…

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Finite, Uneven

You can solve for them either the "hard way" or with the help of your financial calculator.

Let's do an example….
Suppose your school currently charges you $20,000 in tuition (due at the end of year 1). Unfortunately, you expect tuition to rise, in particular, to $22,000 in year 2 and to $25,000 in year 3.
Suppose you can invest your money and earn 8% per year.
How much money do you need to have in the account today?

C_1 = \$20,000
C_2 = \$22,000
C_3 = \$25,000

Finite, Uneven

Now, start discounting:

Alternatively, use a financial calculator:

- **CF_0** = 0; **CF_1** = 20,000; **CF_2** = 22,000; **CF_3** =25,000;
  I = 8 (on my calculator, you have to press **NPV** to get to **I**).
- Then solve for **NPV**.
- The answer will be the same.

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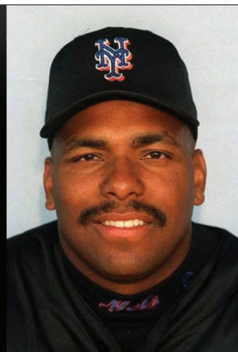
- **Case Study**

Case Study

How Bobby Bonilla Landed The Luckiest Baseball Contract Ever

15 Worst Contracts in American Sports History



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7. Bobby Bonilla, New York Mets

Contract: $5.9 million buyout

The Worst Contract In Sports History Begins Today

Dashiell Bennett | Jul. 1, 2011, 2:47 PM | 14,601 | 17

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AP

The New York Mets gave 48-year-old third baseman Bobby Bonilla a $1,193,248 check today – and will eventually give him another one every July 1 *for the next 25 years.*

How did a retired player end up on the Mets payroll for the next quarter-century ... and 11 years after he last played a game for them?

Well, after the 1999 season, Bonilla had one-year left on his contract. The Mets wanted to get rid of him and he wanted to leave, but it would have cost them $5.9 million to buy him out. That was $5.9M more than they wanted to spend.

Case Study

The facts:

“He hit just .160 with four home runs in 60 games in 1999. He feuded with manager Bobby Valentine. In perhaps his most memorable transgression, he played cards with teammate Rickey Henderson as the Mets lost the deciding game of the National League Championship Series in Atlanta. The Mets didn't want him around any longer, and Mr. Bonilla wanted the freedom to pursue another contract with another team, but the Mets were still on the hook for the **$5.9 million** due Mr. Bonilla in 2000…

… starting on **July 1, 2011**, Bobby Bonilla will remain on the franchise's payroll for **25 years**, collecting an annual salary of **$1,193,248.20**. Those are the terms the Mets agreed on in 2000 [let’s say it was **July 1, 2000**] when they bought out the final year of Mr. Bonilla's contract.”

Case Study

The question:

Was this indeed such a foolish business decision?
Let’s say the Mets’ discount rate is 10%/year.