Bonds and Bond Valuation

AEM 2241 - Finance

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Outline

Bonds: Introduction

- Bonds are generic debt instruments that are used by governments, corporations, and other organizations to borrow money ("to raise money").
- There are many kinds of bonds, and their many features are described by specialized terminology.
- In aggregate, the face value of bonds well exceeds the capitalization of the stock market. Understanding bond markets is critical for ...
  - borrowers (bond issuers);
  - lenders (investors in bonds);
  - policymakers at federal, state and local government level, as well as at the level of international organizations;
  - bond traders;
  - consumers who may wish to invest in certain bonds (e.g. municipal bonds, see later);
  - insurers, since bonds can be insured;
  - derivatives issuers and traders.

Bond Basics | Yields Intro | Yields vs. Prices | Computing Yields | Yield Curves | Treasuries

Plain-Vanilla Bonds

- Consider an annuity paying a fixed amount C at the end of successive equal periods, as usual, with a maturity of t. At time t another, separate, payment, denoted by P is also made. C is most often expressed as a percentage p of P : $C = p \cdot P$.
- The annuity payments and the additional payment, taken together, define a (plain-vanilla) bond.
- Typical notation:
  - C is the bond's annual coupon payment (could also be the per-period coupon, depending on context);
  - P is the bond's principal (also known as face value, or par value);
  - t, expressed in years, is the bond's maturity (often denoted using T).

Coupons

- Coupons can be paid semi-annually (most frequently in the US), or, more rarely, annually or quarterly.
- The **annual** coupon divided by P is called the **coupon rate** of the bond. Informally, we can say that a bond "has a coupon of 10% [per year]."
- A bond with a face value of $\$1,000$ and a coupon (rate) of 8% will have...
  - a single coupon payment of \$80 if the coupon is "paid annually;"
  - two coupon payments of \$40 each, paid at six-month intervals, if coupons are "paid semi-annually."
- Which of the two bonds above will be more valuable?
- For a real bond, coupon payment dates will be adjusted to account for weekends and (bank) holidays.

Yield to Maturity

- A bond is a package of cash flows, which has an aggregated present value.
- For (real) traded bonds this present value is observable in the market, it is the very price of the bond.
- One can always determine an interest rate which, when used to discount the bond's cash flows, makes their total present value equal to the bond's price.
- This interest rate is called the **yield to maturity** (YTM) or, simply, the **yield** of the bond.
- The yield is a compound interest rate; it is typically compounded as many times as many coupons are paid in a year.

Yield: Numerical Example 1

- Consider a bond with a face value of \$1,000, having a coupon (rate) of 8%, with coupons **paid annually**, and a maturity of 10 years.
- What is the value of the bond if the yield is 10% per year?
- The bond's value is the sum of the present value of the annuity (the coupon payments) and the present value of the face value:

$$
\begin{aligned}
B &= C \cdot \frac{1-\left(\frac{1}{1+y}\right)^t}{y} + P \cdot \left(\frac{1}{1+y}\right)^t \\
&= 80 \cdot \frac{1-\left(\frac{1}{1+0.10}\right)^{10}}{0.10} + 1,000 \cdot \left(\frac{1}{1+0.10}\right)^{10} \\
&= 491.57 + 385.54 = \$877.11.
\end{aligned}
$$

Yield: Numerical Example 2

- Now assume that the same bond's yield is 8%. What is its value?

$$
\begin{aligned}
B &= C \cdot \frac{1-\left(\frac{1}{1+y}\right)^t}{y} + P \cdot \left(\frac{1}{1+y}\right)^t \\
&= 80 \cdot \frac{1-\left(\frac{1}{1+0.08}\right)^{10}}{0.08} + 1,000 \cdot \left(\frac{1}{1+0.08}\right)^{10} \\
&= 536.81 + 463.19 = \$1,000.
\end{aligned}
$$

- The bond's price (value) is the same as its face value. Such a bond **trades at par**. If the price is below the face value, the bond trades **under par**; otherwise it trades **above par**.
- For a bond to trade at par, the annual coupon rate must be equal to the yield to maturity (compounded appropriately).

Yield: Numerical Example 3

- Consider a bond with a face value of $1,000, having a coupon (rate) of 8%, with coupons **paid semi-annually**, and a maturity of 10 years.
- What is the value of the bond if its yield is 10% per year?
- The yield must now be compounded semi-annually, consistent with the coupon payments:

$$
\begin{aligned}
B &= \frac{C}{2} \cdot \frac{1 - \left(\frac{1}{1+\frac{y}{2}}\right)^{2t}}{\frac{y}{2}} + P \cdot \left(\frac{1}{1+\frac{y}{2}}\right)^{2t} \\
&= 40 \cdot \frac{1 - \left(\frac{1}{1+0.05}\right)^{20}}{0.05} + 1,000 \cdot \left(\frac{1}{1+0.05}\right)^{20} \\
&= 498.49 + 376.89 = 875.38.
\end{aligned}
$$

- If the yield were 8% compounded semi-annually, the bond would trade at par. Check this!

Technical Note: Par Bonds

- Consider a bond with face value of P, maturity T, a yearly coupon rate of p,and k coupon payments per year. Let y be the yield of this bond. Per the usual convention, the yield is compounded k times a year.
- The yearly coupon is $C = p \cdot P$, the per-period coupon payment is $\frac{C}{k} = \frac{p}{k} \cdot P$, the per-period yield (interest rate) is $\frac{y}{k}$, and the number of compounding (or coupon) periods is $k \cdot t$.
- The value of the bond is given by the usual formula, particularized for the case at hand:

$$
B = \frac{p \cdot P}{k} \cdot \frac{1 - \left(\frac{1}{1+\frac{y}{k}}\right)^{k \cdot t}}{\frac{y}{k}} + P \cdot \left(\frac{1}{1+\frac{y}{k}}\right)^{k \cdot t}.
$$

Technical Note: Par Bonds (2)

- Now assume that $p = y$, i.e. that the yield is numerically equal to the annual coupon rate. From the general formula we immediately get:

$$
\begin{aligned}
B &= \frac{p \cdot P}{k} \cdot \frac{1-\left(\frac{1}{1+\frac{y}{k}}\right)^{k\cdot t}}{\frac{y}{k}} + P \cdot \left(\frac{1}{1+\frac{y}{k}}\right)^{k\cdot t} \\
&= \frac{y \cdot P}{k} \cdot \frac{1-\left(\frac{1}{1+\frac{y}{k}}\right)^{k\cdot t}}{\frac{y}{k}} + P \cdot \left(\frac{1}{1+\frac{y}{k}}\right)^{k\cdot t} \\
&= .P \cdot \left[1-\left(\frac{1}{1+\frac{y}{k}}\right)^{k\cdot t} + \left(\frac{1}{1+\frac{y}{k}}\right)^{k\cdot t}\right] \\
&= P.
\end{aligned}
$$

- **A bond whose annual coupon rate is equal to its yield trades at par!**

Bond Prices vs. Yields

- The yield of the bond is the interest rate, compounded appropriately, that makes the price of the bond equal to the present value of its future cash flows.
- As with any interest rate, the higher the yield, the lower the present value (price) of the bond. The opposite is also true.
- **Yields and bond prices always move in opposition.**
- Given a yield, computing the bond price is easy (see above). Given a bond price, computing the yield implies solving the following equation for y: $B = \frac{C}{2} \cdot \frac{1-\left(\frac{1}{1+\frac{y}{2}}\right)^{2t}}{\frac{y}{2}} + P \cdot \left(\frac{1}{1+\frac{y}{2}}\right)^{2t}$. There are no general formulas; a solution can be found by trial and error or by using numerical methods.

Prices vs. Yields: Which is Primary?

- In the real economy, it is cash flows (dollars) that can be observed. As such bond prices are the primary, objective data that one should rely on.
- Interest rates, including yields, are generally not directly observable - they are just a way to establish connections, or to convert between cash flows.
- Yet bond traders will often think of bond "prices" in terms of yield. "I will not pay more [than a yield of] y% for this bond." Here, "more" means "lower yield."
- Yields are also a convenient way to compare bonds that otherwise may differ in several features, such as maturity, coupon, or riskiness, that make them hard to compare directly.
- Yield is a simple, but somewhat crude way to value bond cash flows. In reality, different interest rates should be used for individual cash flows over different horizons.

What Changes Bond Prices?

- Interest rates change as time passes, and the same is true for bond yields. A bond with exactly the same parameters (maturity, coupon, face value) may have a different price tomorrow than today, because yields may have changed.
- Even if yields do not change, bond prices may change for many reasons, including ...
  - ... the passage of time. If there are fewer coupons to receive and/or they are closer in time, the bond price will change.
  - ... the change in the riskiness of the bond issuer (borrower). Riskier cash flows require a higher yield.
- Bonds are typically traded in-between coupon payment dates. The coupon is split between the seller and the buyer. Special rules for **accumulated (accrued) interest** specify how much of a coupon has built up up to the moment of sale. These partial coupons are very close in time, so they have a measurable impact on bond prices.

Computing Bond Prices: Calculators

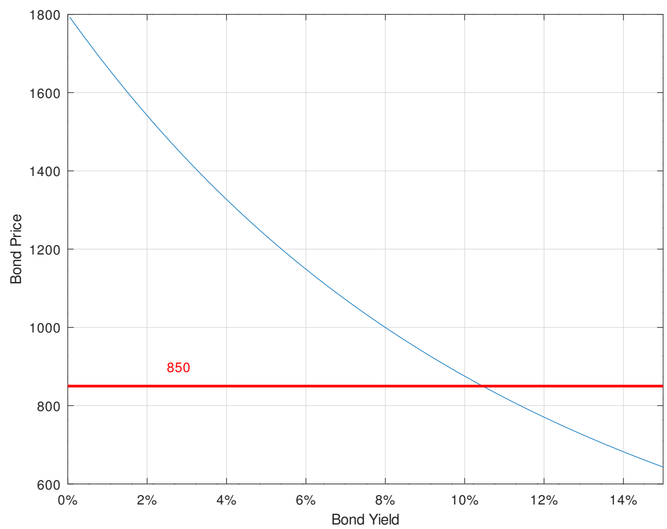
- You can use the TVM registers of your calculator to compute bond prices, given their respective yields.
- Consider the same bond as before: face value of \$1,000, maturity of 15 years, a semi-annual coupon of 9%, and a yield of 7.5%.
- We will price an annuity with $30 = 2 \cdot 15$ payments of \$45 per period (six months), a future value equal to the face value, and a per-period interest rate (yield) of $3.75\% = \frac{7.5\%}{2}$.
- Watch the cash flow signs!
- $N = 30$, $I/Y = 3.75\%$, $PMT = 45$, $FV = 1,000$. Now press $CPT\ PV$ to get $-1,133.72$ for the price of the bond. Notice the sign of the cash flow! We would have to pay \$1,133.72 today to buy the right to receive the "package" of cash flows represented by the bond.

Computing Bond Yields Using Calculators

- You can use the TVM registers of your calculator to compute bond yields.
- Consider the same bond: face value of \$1,000, maturity of 10 years, a semi-annual coupon of 8%, and a price of \$850.
- We will price an annuity with $20 = 2 \cdot 10$ payments of \$40 per period (six months), a future value equal to the face value, and a present value equal to the bond price.
- Watch the cash flow signs!
- $N = 20$, $PV = -850$, $PMT = 40$, $FV = 1,000$. Now press $CPT\ I/Y$ to get the **per-period yield** of 5.23%. Since this is the per-period yield, we must annualize it. Thus the yield of the bond is approximately $2 \cdot 5.23\% = 10.46\%$.

Computing Bond Yields

- Consider a bond with a face value of \$1,000, maturity of 10 years, a semi-annual coupon of 8%, and a price of \$850. What is the yield of the bond?
- The higher the yield, the lower the bond price. The yield must be around 10%. Take a look:



Technical Note: Computing Bond Yields (2)

- We first "bracket" the true yield. We can use a graph like the one above to find a yield **lower** and a yield **higher** than the true yield. In terms of prices computed from these yields, we must find a price higher, and a price lower than the market price.
- We can also just guess and try a few too-low values for the yield (including, say, 0%) and a few too-high values for the yield (including, say, 25%, 50%, or 100%).
- Given a pair of yields that bracket the true yield, we compute their average yield, as well as the bond prices associated with these three yields. We use a table like the one on the next slide to come up with a narrower bracketing yield interval.

Technical Note: Computing Bond Yields (3)

- Given a low yield and a high yield, we compute the average yield (mid-yield).
- We then choose the mid-yield and **either** the low yield and the high yield so that the corresponding prices bracket the bond's market price. This pair of yields then becomes the new pair of bracketing yields. This is called the **bisection method**.
- We continue until bracketing yields are very close; their mid-yield is (a good approximation of) the true bond yield.

Low Yield	Annuity	Principal	Total	High Yield	Annuity	Principal	Total	Mid-Yield	Annuity	Principal	Total
8.00%	543.61	456.39	1,000.00	12.00%	458.80	311.80	770.60	10.00%	498.49	376.89	875.38
10.00%	498.49	376.89	875.38	12.00%	458.80	311.80	770.60	11.00%	478.02	342.73	820.74
10.00%	498.49	376.89	875.38	11.00%	478.02	342.73	820.74	10.50%	488.09	359.38	847.47
10.00%	498.49	376.89	875.38	10.50%	488.09	359.38	847.47	10.25%	493.25	368.03	861.27
10.25%	493.25	368.03	861.27	10.50%	488.09	359.38	847.47	10.38%	490.66	363.68	854.34
10.38%	490.66	363.68	854.34	10.50%	488.09	359.38	847.47	10.44%	489.37	361.52	850.89
10.44%	489.37	361.52	850.89	10.50%	488.09	359.38	847.47	10.47%	488.73	360.45	849.18
10.44%	489.37	361.52	850.89	10.47%	488.73	360.45	849.18	10.45%	489.05	360.99	850.04
10.45%	489.05	360.99	850.04	10.47%	488.73	360.45	849.18	10.46%	488.89	360.72	849.61

Technical Note: Computing Bond Yields (4)

- The bisection method may seem hard to implement, but it is both simple and quick. In each row you must have three yields, and three bond values. However, only one value is actually new. Either the high or the low yield is taken over from the prior row, while the other element of the bracketing yields is the the prior rows mid-yield. So only the new mid-yield and its associated bond value must be computed!
- You can set up your calculator to compute bond values by changing only the yield at each step.

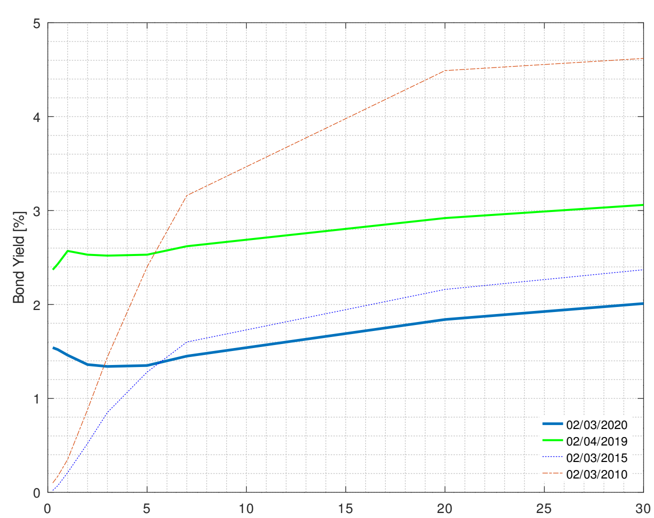
Low Yield	Bond Value	High Yield	Bond Value	Mid-Yield	Bond Value
8.00%	1,000.00	12.00%	770.60	10.00%	875.38
10.00%	875.38			11.00%	820.74
		11.00%	820.74	10.50%	847.47
		10.50%	847.47	10.25%	861.27
10.25%	861.27			10.38%	854.34
10.38%	854.34			10.44%	850.89
10.44%	850.89			10.47%	849.18
		10.47%	849.18	10.45%	850.04
10.45%	850.04			10.46%	849.61

Yield Curves

- In practice, different interest rates are charged for investments over different time horizons. Generally, the longer the investment period, the higher the interest rate the market requires.
- A bond is a package of cash flows that are paid at regular intervals. In some sense, the yield is (some kind of) an average interest rate associated with these cash flows.
- When looking at bonds that are substantially identical but for their maturity, we note that yields have the same behavior as regular interest rates: bonds with longer maturities typically require higher yields.
- Plotting the yield of bonds as a function of their maturities given the bonds' yield curve.
- This is most often done with government bonds, but it can be done, for example, with a series of comparable corporate bonds.

Historical Yield Curves

These curves were build using Federal Reserve official data for 3-month, 6-month, and 1, 2, 3, 5, 7, 20, and 30-year constant-maturity Treasuries. Yield curves evolve in time. More sophisticated methods exist to compute yield curves that embed information from hundreds of outstanding Treasury bonds.



Government Bonds

- Many governments around the world operate at a deficit, so they need to borrow on financial markets. They do this by issuing bonds.
- Bonds are also issued to deal with the mismatch between the timing of expenses and inflows (e.g. taxes).
- The outstanding debt (in terms of face value) of the US federal government exceeds $23 trillion dollars.
- The US government issues debt denominated in dollars and backed by “the full faith and credit of the United States.” These bonds are considered very safe, virtually risk-free. As such, investors require very low yields to invest in these instruments.
- US government bonds are called “Treasury bonds” or “Treasuries” (some insist on writing “Treasurys”).

Government Bonds (2)

- Foreign governments sometimes issue bonds in currencies other than their own. For example, Argentina may issue bonds denominated in dollars. These can be risky, as the Argentine government may run out of dollars before they run out of their own currency.
- When analyzed through the prism of yields, US Treasuries may not be the safest investment in the world.
- For example, the German 5-year Bund had a yield of -0.60% as of early February 2020. In effect, investors were paying the German government to hold their money. Japanese 5-year government bonds yielded -0.15%. On the same day 5-year Treasuries had a yield of 1.45%.
  See `https://www.bloomberg.com/markets/rates-bonds/government-bonds` for up-to-date details.

Government Bonds (3)

- Treasuries have initial maturities that range from 3 months to 30 years, and bear names like (Treasury) bills, notes, and bonds, depending on their maturities. Other governments issue bonds that may have maturities of 100 years.
- State and local governments also issue bonds, to finance deficits, to deal with income/expense timing mismatches, or to finance specific projects.
- In the US, such instruments are called “municipal” bonds, also known as “munis.”
- Municipal bonds are not riskless; historically, both state and local governments have defaulted.

Government Bonds (4)

- Treasuries are exempt from state taxes, but munis are exempt from federal taxes. This makes them very attractive for high-income individuals; it also makes their (nominal) yield much lower than that of comparable corporate bonds. Yields on an after-tax basis are much closer, however.
- Some governments issue **zero-coupon bonds**. These are bonds with no coupon payments; at maturity, the face value is paid in full. Treasuries with initial maturity under one year are zero coupon bonds. Longer-maturity “zeros” also exist. Because of their simplicity, zeros are highly sought after, as they make certain kinds of risk management simpler. Zeros can be created by “stripping” coupons off regular coupon-bearing Treasuries and selling the right to these coupons (and the principal) separately.

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