Solution Sketch for PS2

1. **Solution**

1a (U,R). These are both dominant strategies

1b (D, L). Here D is a dominant strategy and L is the best response to D

1c No pure strategy equilibrium. There is a mixed strategy equilibrium with Pr(U)=Pr(D)=1/2 and Pr(L)=1/3 and Pr(R)=2/3.

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2. **Solution**

Two pure strategy equilibria: (U,R), (D,L). Also a mixed strategy equilibrium with Pr(U)=2/5, Pr(D)=3/5, Pr(L)=1/5 and Pr(R)=4/5.

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3 **Solution:**

3a No. For each player there is no strategy that is a best reply to every strategy by the other player. [It's OK to go into more detail, but just restating the definition of dominance and noting why it is not satisfied here is OK.]

3b For player A: strategy W is (strictly) dominated by by strategy Z and strategy Y is strictly dominated by strategy X. For player B: strategies A and B are (strictly) dominated by strategy C. These are the only dominated strategies in the full game. [No explanation is needed.]

3c The pure Nash equilibria are: (X,D) and (Z,C). This is easy to see once we eliminate the strictly dominated strategies. [No explanation is needed.]

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4 **Solution:**

		Defender A	Defender B
Attacker	A	l, w	w, l
	B	w, l	l, w

Figure 1: The payoff matrix for Question (4a).

4a

4b No. [No explanation needed.]

4c Each player puts probability 1/2 on each strategy. To see this let p be the probability that the Attacker attackers A and let q be the probability that the defender defends A. In a mixed strategy equilibrium must have:

$$lq + w(1-q) = wq + l(1-q)$$

$$wp + l(1-p) = lp + w(1-p)$$

Solving gives $p = q = 1/2$. [No explanation or calculations are necessary.]

4d There are several ways to "explain" this. One is to note that the payoff to A is a q and (1-q) mixture of l and w while the payoff to B is the same q and (1-q) mixture of now w and l. For them to be equal q must equal (1-q). So $q = 1/2$. Alternatively, it's OK to just note that in solving for the mixed strategy equilibrium, w and l drop out of the equations in part (c).

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5 **Solution**

5a x=50, y=20. This is calculated by equalizing the time used on route ACB and ADB.

5b y=0, x=20, z=50. Observe that the new route AB is strictly faster than route ADB for any number of travelers on these two routes. Therefore, route ADB can't be used in equilibrium, so $y = 0$. Equalizing time used on route ACB and AB gives the values of x and y.

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6 **Solution**

6a x=320, y=520, z=260. This is calculated by equalizing the time used on the three routes and solve the system of equations.

6b x=525, y=575, z=525, and no one using AD or CB. This problem illustrates Braess Paradox as the initial travel time was 76 per traveler and after CD is added it increases to 78.75 per traveler.

Consider the equilibrium in (a) and add in the new route CD (and DC). Observe that people on route CB have an incentive to switch to route CDB, and people on route AD have an incentive to switch to route ACD. Suppose that all of them switch, and let the traffic on route ACDB be $t = x = z$. Equalizing the time used on ACDB and route AEB gives $t = 525$ and $y = 575$. Check that under this traveler distribution no one indeed wants to use route AD or CB.