Homework 1
Solutions

As explained in class, it is a good idea to set your calculator to use **higher precision** when computing results on a homework or exam question. Generally, setting your computer to use a 4 or 5-decimal precision will be more than enough. It is especially important for you to use high precision if you will be writing down intermediate results. Changing your precision setting was demonstrated in class.

You can also **store and recall intermediate results** by using the STO X and RCL X key combinations, where X is a digit from 0 to 9. This allows you access to the calculator's 10 memory registers. Additionally, X can also be one of the TVM buttons: N, I/Y, PV, PMT, FV when used in combination with RCL. This functionality; also demonstrated in class, can be used to avoid writing down intermediate results altogether.

In the solutions below **we typically show the value stored in the PMT register, eve if it is 0**. As you know, when you clear the TVM registers, PMT is set to 0. To eliminate ambiguities, however, consistent with in-class practice, we set the PMT to 0 explicitly.

We do not present all possible solutions to all problems. Sometimes we present both a formula-based and a calculator-based solution, sometimes we present only one of them. **We encourage you to practice by solving the problems below by also using the method that we did not present explicitly**.

When setting up problems for the calculator, it is critical to keep track of cash flow signs when solving problems with the calculator. If you have present values, payments, and future values, consistency is especially important. However, unless we explicitly request it, we do not ask for signed cash flow amounts **as results** for answers to homework or exam problems **if the context makes clear the direction of the cash flow**. If we asked, however, to show us how would you set up a calculator problem, then signs of quantities input into the TVM registers would have to shown consistent with the question posed.

1 Problem 1

Myca Corporation has a project with the following cash flows. What is the value of the cash flows today assuming an annual interest rate of 10.6 percent?

Year	Cash Flow
1	$1,940
2	$2,480
3	$2,850
4	$2,860

Solution:

Use the present value equation to discount back the cash flows from each year then add them up. Use 10.6% as the interest rate, so 1+r=1+0.106=1.106. $PV = 1,940/1.106 + 2,480/1.106^2 + 2,850/1.106^3 + 2,860/1.106^4 = 7,799.44$.

2 Problem 2

You can invest in an account that pays simple interest or an account that pays compound interest. In either case, you plan to invest $3,000 today and both accounts have an annual interest rate of 9 percent. How much more interest will you receive in the 11th year in the account that pays compound interest?

Solution:

Use the equation for simple interest to calculate the simple interest per year since that value doesn't change from year to year. This gives you the interest per year = $3,000 × .09 = $270. Since you are looking for the interest on the 11th year, calculate the value after 10 years using compound interest:

Value after 10 years = $3,000 \times 1.09^{10} = 7,102.09$.

Then use this value to calculate the interest on the 11th year:

Compound interest in Year 11 = $7,102.09 × .09 = $639.19.

Then find the difference between the simple and compound interest to get your final answer:

Difference = (compound interest) - (simple interest) = $639.19 − $270.00 = $369.19.

3 Problem 3

You are going to deposit $26,000 today. You will earn an annual rate of 6.1 percent for 11 years, and then earn an annual rate of 5.5 percent for 14 years. How much will you have in your account in 25 years?

Solution:

First, find the future value at the end of the first 11-year period:
$\text{FV} = 26,000 \times 1.061^{11} = \$49,870.37$.
To solve this using you calculator, input N=11, I/Y=6.1, PV=26,000, PMT=0, CPT FV.
Then, find the future value of the next 14-year period. You use the future value from the first 11 years as your new present value (at time t=11) because you have already received returns. To solve this using your calculator, input N=14, I/Y=5.5, PV= \$49,870.37, PMT=0, CPT FV. Also, $FV = 49,870.37 \times 1.055^{14} = 105,530.26$.

4 Problem 4

You want to have \$14,000 in 7 years for a dream vacation. If you can earn an interest rate of .6 percent per month, how much will you have to deposit today?

Solution:

Since you want \$14,000 in 7 years, you use the present value equation:
$PV = 14,000/\left(1+0.006\right)^{7\cdot 12} = \$8,470.28$.
To solve this equation in your calculator input $N = 7 \cdot 12$, I/Y=0.6, PMT=0, FV=14,000, CPT PV.
This is not the only way to solve the problem. For example, you could compute the EAR:
$EAR = (1+0.006)^{12} - 1 = 7.4424\%$.
You can now set up the problem to discount over one-year periods: N=7, I/Y=7.4424, PMT=0, FV=14,000, CPT PV; we get the same result.

5 Problem 5

You need to have \$30,750 in 5 years. You can earn an annual interest rate of 6 percent for the first 3 years, and 6.6 percent for the next 2 years. How much do you have to deposit today?

Solution:

There are two time periods with different interest rates, so it would be more efficient to divide the calculation into two parts. First, calculate the value of the deposit at the end of the third year, $PV_3 = \frac{FV_5}{(1+r_2)^{t_2}} = \frac{30.750}{(1+0.066)^2} = \$27,060.18$. Then, calculate the value of the deposit today, $PV_0 = \frac{PV_3}{(1+r_1)^{t_1}} = \frac{20,060.18}{(1+0.06)^3} = \$22,720.25$.

Financial calculator: set N=2, I/Y = 6.6, PMT=0, FV = +30,750. Press CPT PV to get -27,060.18; set N = 3, I/Y = 6%, PMT=0, FV = 27,060.18. Press CPT PV to get -22,720.25.

6 Problem 6

An investor who was not as astute as he believed, invested $272,000 into an account 12 years ago. Today, that account is worth $210,000. What was the annual rate of return on this account?

Solution:

Knowing the present value ($210,000), future value ($272,000) and the number of periods (12 years), we calculate $r = \left(\frac{FV}{PV}\right)^{\frac{1}{t}} - 1 = \left(\frac{210{,}000}{272{,}000}\right)^{\frac{1}{12}} - 1 = -0.0213$, or -2.13%.

Financial calculator: set N = 12, PV = -272,000, PMT = 0, FV = 210,000. Press CPT I/Y to get -2.13%.

7 Problem 7

Maxxie purchased a tract of land for $31,000. Today, the same land is worth $49,300. How many years have passed if the price of the land has increased at an annual rate of 5.4 percent?

Solution:

Knowing the present value ($31,000), future value ($49,300) and the interest rate (5.4%), we calculate number of periods: $t = \frac{\ln \frac{FV}{PV}}{\ln(1+r)} = \frac{\ln \frac{49{,}300}{31{,}000}}{\ln(1+0.054)} = 8.82$ years.

Financial calculator: set I/Y = 5.4, PV = -31,000, PMT=0, FV=49,300. Press CPT N to get 8.82.

Note: As discussed in class, we generally do not use a fractional number of years (periods) as **inputs** to our problems. However, when we get a fractional number of periods as a result, we report them as such, unless the context makes it clear that we have to round the result.

8 Problem 8

You have just deposited $9,000 into an account that promises to pay you an annual interest rate of 6.1 percent each year for the next 7 years. You will leave the money invested in the account and 15 years from today, you need to have $25,650 in the account. What annual interest rate must you earn over the last 8 years to accomplish this goal?

Solution:

To calculate the Future Value of the $9,000 invested today and the number of periods (7 years), $FV = PV\,(1+r)^t = 9{,}000 \cdot (1+0.061)^7 = \$13{,}622.29$. To make this value reach

$25,650 at 15 years from today, 15-7=8 years pass after the end of the 7th year. We get $r = \left(\frac{FV}{PV}\right)^{\frac{1}{t}} - 1 = \left(\frac{25{,}650}{13{,}622.29}\right)^{\frac{1}{8}} - 1 = -0.0823\%$, or 8.23%.

Financial calculator: set N = 7; I/Y = 6.1; PV = -9,000; PMT = 0. Press CPT FV to get $13,622.29. Now set N = 7; PV = -$13,622.29; PMT = 0; FV= $25,650. Press CPT I/Y to get 8.23%.

9 Problem 9

You have just deposited $9,000 into an account that promises to pay you an annual interest rate of 6.1 percent each year for the next 7 years. You will leave the money invested in the account and 15 years from today, you need to have $25,650 in the account. What annual interest rate must you earn over the last 8 years to accomplish this goal?

Solution:

First, compute the FV of $9,000 (the money you deposited today) at time = 7 years. Using the financial calculator, set N = 7, I/Y = 6.1, PV = – 9,000, PMT = 0. Press CPT FV to get $13,622.29236. You then have to set $13,622.29236 as the new PV. Using the financial calculator, set N = 8; PV = – 13,622.29236; PMT = 0; FV = 25,650. Note that the signs of PV and FV have to be opposite in the financial calculator. Press CPT I/Y to get 8.23%.

10 Problem 10

Your parents will be giving you $240 a month for 4 years, for all the months when you are in college. At an interest rate of .55 percent per month, what are these payments worth to you today, knowing that you first start college in one month?

Solution:

This is a regular annuity with a monthly payment of $240 that makes its first payment in one month from today. You will receive these payments for 4 years, so there will be a total of 48 annuity payments. The per-month interest rate is 0.55%. To determine what these payments are worth to you today, you need to compute the PV of this regular annuity. Using the financial calculator, set N = 48; I/Y = 0.55; PMT = 240; FV = 0. Press CPT PV to get $10,100.54.

11 Problem 11

To fund your dream around-the-world vacation, you plan to save $1,150 per year for the next 14 years starting one year from now. If you can earn an interest rate of 5.47 percent, how much will you have saved for your vacation at the end of the 14th year?

Solution:

This is a regular annuity. Each annuity payment is \$1,150 paid at the end of each year, for 14 years. Therefore, there will be a total of 14 annuity payments. To figure out how much you will have saved at the end of the 14th year, you must compute the FV of this annuity. Using the calculator, set N = 14; I/Y = 5.47; PV = 0; PMT = – 1,150. Press CPT FV to get \$23,287.66. PV = 0 because you do not have any money saved at time = 0.

12 Problem 12

You have just purchased a car and, to fund the purchase, you borrowed \$20,000. If your monthly payments are \$473.82 for the next 4 years, what is the stated (annual) interest rate on the loan?

Solution:

Using the financial calculator, set N = 48; PV = 20,000; PMT = – 473.82; FV = 0. Press CPT I/Y to get 0.53733748. However, this is the monthly rate. To convert this into the stated (annual) interest rate, multiply the monthly rate by 12. $0.53733748 \cdot 12 = 6.45\%$. Therefore, the stated interest rate on this loan is 6.45% per year, compounded monthly.

13 Problem 13

Assume that interest rates are at 7% per annum, compounded semiannually. You will retire exactly 20 years from today. Your financial adviser suggested that you buy an annuity that will start on the day of your retirement, and which will pay as follows:

- At the end of every whole year after your retirement, starting with the first year and ending on the 10th year after your retirement, you will get \$20,000.
- In the middle of each year after your retirement, for all 10 years that this annuity will run, you also get \$10,000. In other words, \$10,000 payments would be made at 6, 18, 30, 48, ... months after your retirement, the last payment being made exactly 6 months before the 10th anniversary of your retirement.

What is a fair price today for this annuity?

Solution:

There are multiple ways of solving this problem; the approach described below is just one of them.

You can decompose the annuity given in this problem into two regular annuities:

- A1: A regular annuity with 20 payments of \$10,000, spaced at equal 6-month intervals, starting in 6 months from the retirement date.
- A2: A regular annuity with 10 payments of \$10,000, spaced at equal yearly intervals, starting in one year from the retirement date.

Then, you can compute the PVs of A1 and A2 at the retirement date:

When computing the PV of A1, you must use the per-6-month-period rate. This is because the annuity payments for A1 occur on a semiannual basis. To calculate the per-6-month-period rate, divide the stated interest rate by 2: 7% / 2 = 3.5% (per 6-month period). Using the financial calculator, set N = 20; I/Y = 3.5; FV = 0; PMT = -10,000. Press CPT PV to get \$142,124.0330.

When computing the PV of A2, you must use the EAR. This is because the annuity payments for A2 occur on a yearly basis. Use the EAR formula ($EAR = \left(1 + \frac{r}{m}\right)^m - 1$, where r = 0.07 and m = 2) to get 7.1225. Using the financial calculator, set N = 10; I/Y = 7.1225; PMT = 10,000; FV = 0. Press CPT PV to get \$69,839.81967.

The original annuity is the sum of the two annuities. Add PV of A1 and PV of A2 to find the PV of the entire annuity at the retirement date: \$142,124.0330 + \$69,839.81967 = \$211,963.8527.

Finally, discount the PV of the entire original annuity back to time = 0. Use the EAR and discount over 20 years. Using the financial calculator, set N = 20; I/Y = 7.1225%; PMT = 0; FV = 211,963.8527. Press CPT PV to get \$53,536.23344. If you round to 2 decimal places, you get \$53,536.23 as the final answer.