Bonds and Bond Valuation II
AEM 2241- Finance

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Outline

Inflation
    Generalities
    Real vs. Nominal Interest Rates
    The Fisher Effect
    Inflation and Present Values

Term Structure of Interest Rates

Bond Risks
    Yields vs. Risk
    Interest Rate Risk

Important Assignment

Inflation

- Inflation is an increase in the general price level of goods and services.
- Historically, inflation has occurred in many countries throughout history.
- Prices do not increase uniformly; the price of some goods and services will increase faster than that of others.
- Because firms and individuals do not consume the same mix of products and services, they do not experience inflation in exactly the same way.
- The opposite of inflation is deflation - this is a more rare economic phenomenon. Hyperinflation is inflation that has reached extreme levels. Both deflation and hyperinflation can gravely impact the affected economies.
- Low levels of inflation are generally considered positive (or, at least, neutral) by economists.

Real vs. Nominal Interest Rates

- Statisticians define the level of inflation by estimating the consumer price index (CPI) - which is the weighted average price of a basket of consumer goods and services. Many other specialized inflation measures exist, none of them providing the full picture on its own.
- Assume that a basket of goods and services costs D_0 dollars at time 0, and costs $D_1 > D_0$ at time 1. Then set $\frac{D_1}{D_0} = 1 + h$, where h is the inflation rate over the one-year period considered.
- Now assume that you invest \$1 and earn an interest of R over the same year. At time 1 you will have $1 + R$ dollars, but these dollars will **buy** fewer products and services as they would otherwise, because of inflation. In terms of **time-0 purchasing power**, you only have $\frac{1+R}{1+h}$ dollars at time 1.

The Fisher Effect

- If you only have $\frac{1+R}{1+h}$ dollars in terms of purchasing power, you did not really earn a return of R, but a return $r < R$. We determine r from the equation $1 + r = \frac{1+R}{1+h}$, and get $r = \frac{1+R}{1+h} - 1 = \frac{R-h}{1+h}$. If inflation levels are low, we can approximate $1 + h \approx 1$, thus obtaining $r \approx R - h$.

- We can also rewrite $1 + r = \frac{1+R}{1+h}$, to get $1 + R = (1 + r) \cdot (1 + h)$. Then $R = r + h + r \cdot h$; if both r and h are small, then $R \approx r + h$, or $r \approx R - h$, which is the same result as before.

- We say that R is the **nominal rate**, while r is the **real rate**. Understanding the difference between these rates is critical for investors.

- If $R = 10\%$, $h = 3\%$, then $r \approx 7\%$. The precise calculation gives $r = \frac{0.10 - 0.03}{1 + 0.03} = 6.80\%$.

Inflation and Present Values

- When we consider inflation, we deal with cash flows that differ both in their timing and purchasing power.

- If we use nominal dollars, i.e. dollars that are not equal in purchasing power, we must use nominal rates to compute present values.

- If we use constant-purchasing-power dollars (real dollars), we must use real rates to compute present values.

- As long as you use these rules consistently, you will get the same (correct) result every time.

Technical Note: Inflation and Present Values

Consider a one-period example, where the nominal rate is R, the real rate is r, the inflation rate is h, and you earn D dollars at time 1. The present value of this amount can be computed as follows:

- Nominal dollars: $PV = \frac{D}{1+R}$.

- Real dollars: D "inflated dollars" correspond to $\frac{D}{1+h}$ real dollars. We have

$$PV = \frac{\frac{D}{1+h}}{1+r} = \frac{D}{(1+r)\cdot(1+h)} = \frac{D}{1+R},$$

because $1 + R = (1+r)\cdot(1+h)$.

As claimed above, discounting nominal dollars using nominal rates, or real (i.e., constant-purchasing-power) dollars using real rates, yields the same result.

Inflation and Present Values: Example

- Consider a regular annuity with a constant annual payment of $C = \$25,000$ per year, for 10 years. The (nominal) interest rate is $R = 10\%$, the inflation rate is $h = 4\%$; both are constant.

- $r = \frac{R-h}{1+h} = \frac{0.10-0.04}{1+0.04} = 5.77\%$.

- In terms of **purchasing power**, the payments are worth less and less; they have a negative, constant "growth" rate:

$$\frac{C}{1+h} = C\cdot(1+g),$$

thus

$$g = \frac{1}{1+h} - 1 = -\frac{h}{1+h} = -\frac{0.04}{1+0.04} = -3.85\%.$$

Inflation and Present Values: Example (2)

- **Nominal dollars**:
$PV = C \cdot \frac{1-\left(\frac{1}{1+R}\right)^{10}}{R} = 25,000 \cdot \frac{1-\left(\frac{1}{1+0.1}\right)^{10}}{0.1} = \$153,614.18.$

- **Real dollars**: The first payment is worth $\frac{C}{1+h}$ in real dollars.
$PV = \frac{C}{1+h} \cdot \frac{1-\left(\frac{1+g}{1+r}\right)^{10}}{r-g} = \frac{25,000}{1+0.04} \cdot \frac{1-\left(\frac{1-0.0385}{1+0.0577}\right)^{10}}{0.0577+0.0385} = \$153,586.01.$

- The values are close, but not equal, due to rounding percentages to two decimals. Using a computer to get many more decimals we get: $PV_{nominal} = \$153,614.1776426171$ and $PV_{real} = \$153,614.1776426171$, for a difference of $0.000000000029104.

- The corresponding example in the textbook refers to an annuity that has a positive growth rate in nominal dollars. This example is slightly different, in order to illustrate that the principle discussed here is applicable universally.

Term Structure of Interest Rates

- In real life, interest rates depend on time: longer time horizons (typically) require higher per-period interest rates. This is because investing money for longer periods involves assuming more risk; investors need to be paid more to assume this risk.

- The yield of a coupon bond is used to discount cash flows that are paid at different times. These yields obscure the dependence of interest rates on maturity.

- The yields of zero-coupon bonds, however, only depend on one cash flow each. As such, there is no amalgamation of cash flows, and the dependence of interest rates on maturity is clearly observable.

- The yield curve built using the yields of zero-coupon bonds is called the term-structure of interest rates.

Bond Risks and Yields

- The yield of bonds is a measure of the promised nominal return on investment for bond investors (buyers).

- Many bonds have long maturities, and over these long periods investors will be exposed to many risks, for which they demand compensation.

- The higher the perceived risks, the higher the yield demanded by investors.

- The main risks faced by bond investors are:
  - inflation;
  - interest rate risk;
  - default risk;
  - liquidity risk;
  - tax & legal changes (taxes are a factor that impacts yields even if they do not change; not all investors face the same taxes on all investments).

Interest Rate Risk

- One of the major risks associated with bonds is interest rate risk, i.e. sensitivity to changes in yields. Another major risk is default risk, which we ignore (in this class).

- All else being equal interest rate risk is ...
  - ... greater, the longer the maturity of the bond.
  - ... lower, the higher the coupon rate.

- These statements are illustrated by the figures shown on the next two slides - examine them carefully.

- The same increase in maturity has a lower and lower impact on the increase in interest rate risk as the maturity of the bond increases. For a given coupon rate, there is must less difference between the risk of bonds with a maturity of 20 and 30 years, respectively, as between bonds of 1 and 10 years, respectively.

Interest Rate Risk (2)
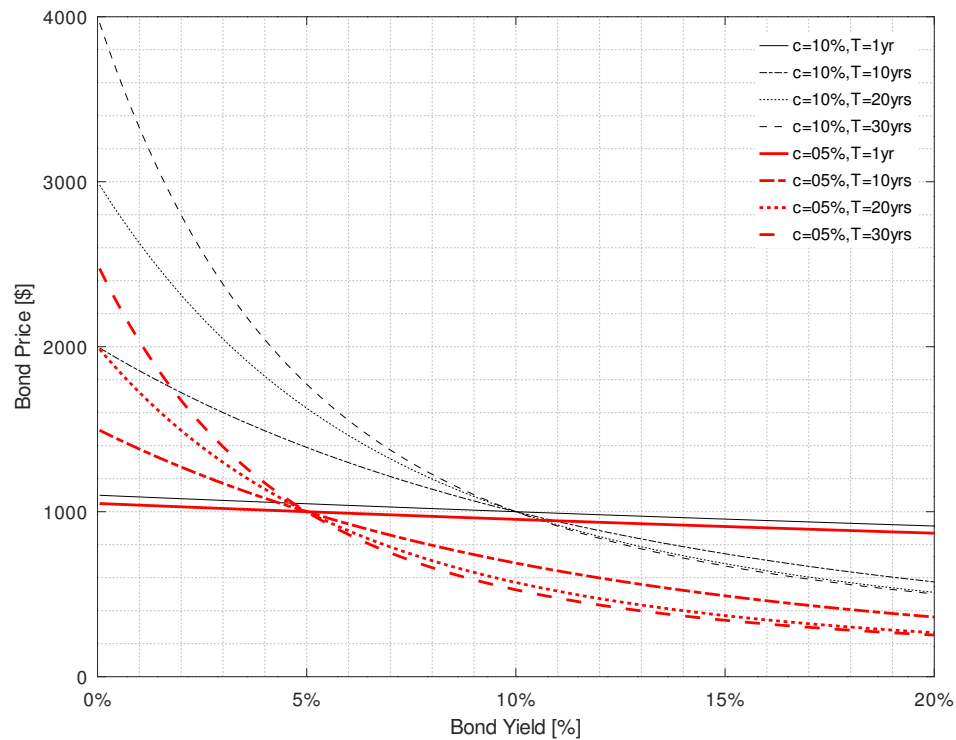
Interest Rate Risk (3)
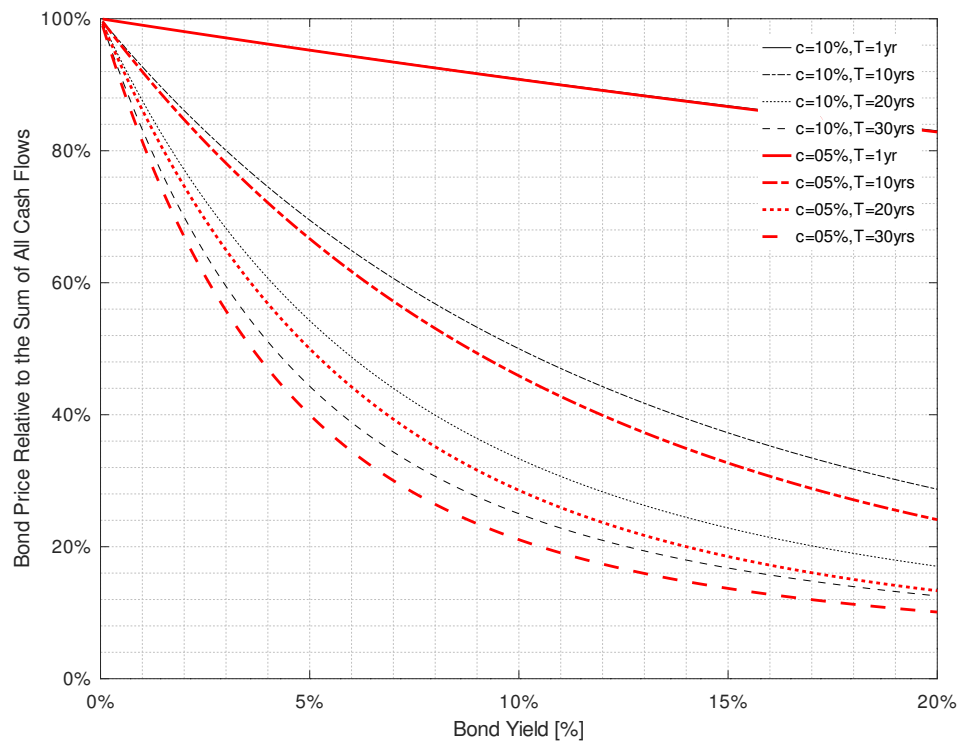
Important: To Do!

As noted earlier, bonds are crucial for the financial market. We focused on fundamentals, with an emphasis on matters closely related to the time-value of money. This knowledge will make it possible for you to approach and understand more sophisticated bond-related topics that you may study later.

To become familiar with the great variety of bonds that exist, as well as issues related to bond ratings and markets, read the following sections from your textbook:

- **7.2: More about Bond Features;**
- **7.3: Bond Ratings;**
- **7.4: Some Different Types of Bonds;**
- **7.5: Bond Markets.**

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