INFO 2950: Intro to Data Science

Lecture 13
2023-10-11

Agenda

1. Overfitting
2. Train / Test Split
3. Evaluation Metrics

“Training a model”: single variable

- Given a df with two columns, x and y
- You run regression y~x in Python
- Python returns α-hat and β-hat
- Are there problems with this?

Overfitting: single variable

- It depends! If you just want to describe the data that you have, this is fine.

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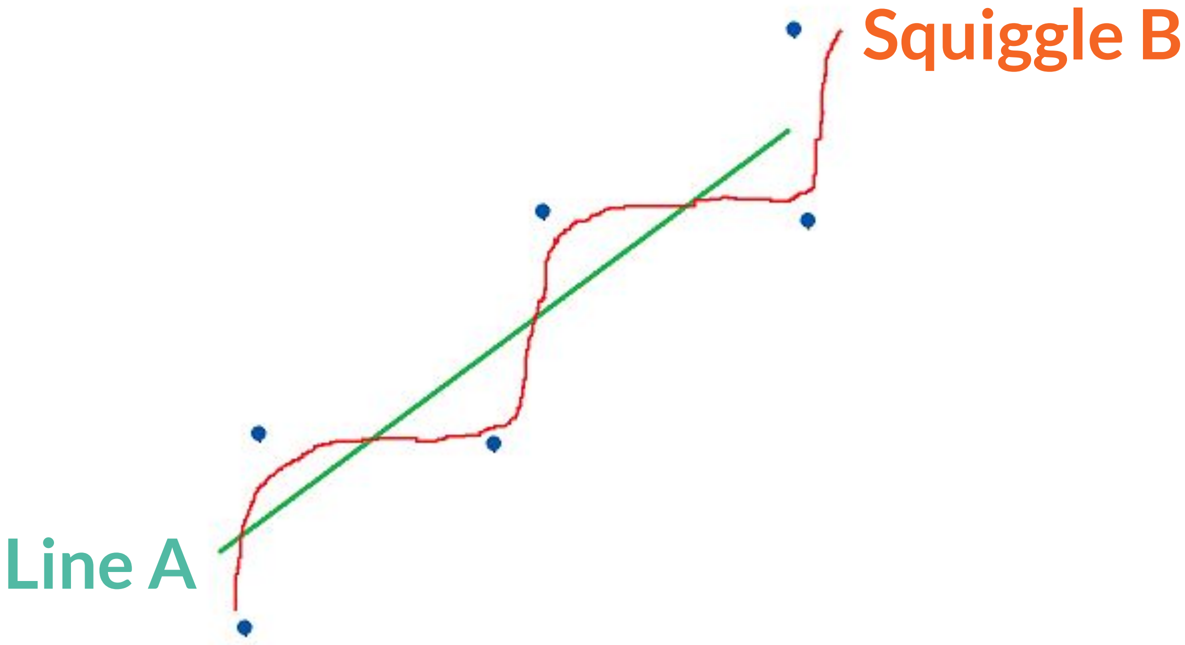
- If you're trying to **generalize your findings** to "new data", what happens if the (x, y) values in your df aren't representative of the "new data"?

Overfitting: single variable

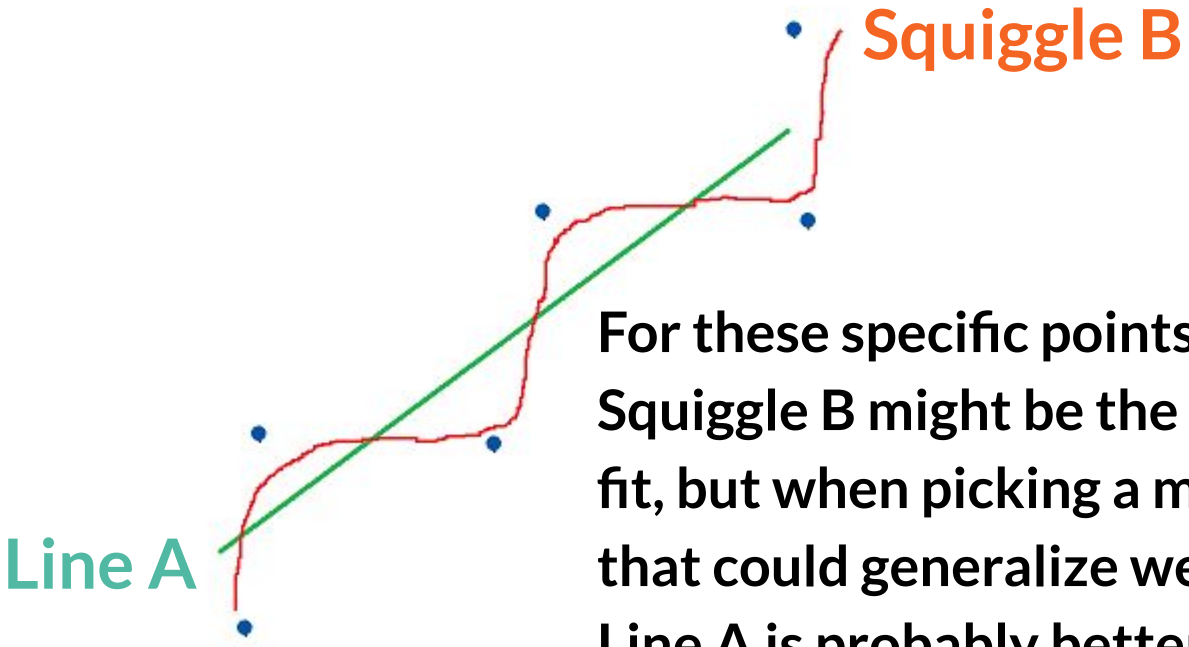
- It depends! If you just want to describe the data that you have, this is fine.
- If you're trying to generalize your findings to "new data", what happens if the (x, y) values in your df aren't representative of the "new data"?
    - You make **bad generalizations**
    - This is called **"overfitting"** to your existing data

Overfitting: which line is "better"?



https://statisticsbyjim.com/regression/overfitting-regression-models/

Overfitting: which line is “better”?



For these specific points, Squiggle B might be the best fit, but when picking a model that could generalize well, Line A is probably better.

https://statisticsbyjim.com/regression/overfitting-regression-models/

Training data: multiple variables

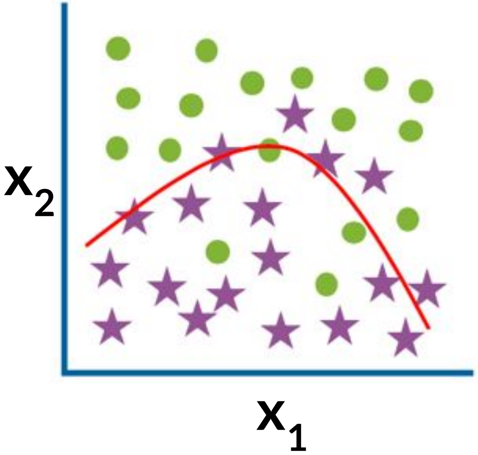
- Given multiple x’s and one y, now we run a multivariable regression
- Is overfitting a problem in this case, too?

Training data: multiple variables

- **Yes**: overfitting in high dimensions is ***much more likely***
- Many different input x's → the model can pick up on more complexity, but then you get **models adhering too much to the *data* instead of the underlying *idea***

Overfitting: multiple variables

Output y (binary) represented by shape



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Overfitting: multiple variables

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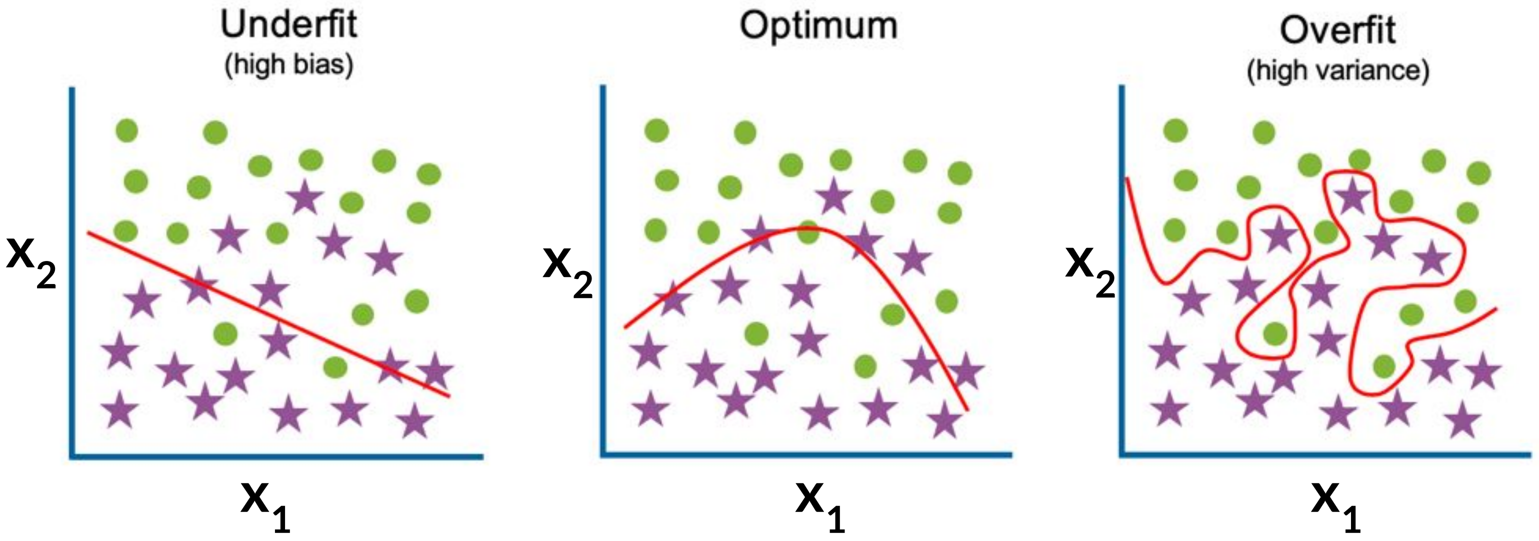

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How to overcome overfitting?

- "**Feature selection**": identify the most important covariates (inputs) for your model and only include those. This allows you to reduce the number of dimensions of your data
- Use a **train / test split**
- **Regularization**
  - Much later in the class

Feature selection review

- Choose covariates that…
  - **make sense given domain expertise**
  - **aren't redundant** (i.e., *aren't collinear* and *don't overfit* the data)
  - allow you (with transformation) to get **random-looking residual plots**

How to overcome overfitting?

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- **Regularization**
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Train / test split

- Train/test split: **before you do anything,** randomly split your df's data rows into two subsets
    - Train set: 70% of your data
    - Test set: 30% of your data
- The 70/30 ratio not set in stone, just a rule of thumb
- *Often you will see a third dataset called the "validation set" that could yield a ~70/15/15 train/val/test split*

Train / test split for overfitting?

- Look at only your training set (a random 70% of your data) when building your model
- Then confirm it generalizes well to the other 30% of your data (the test set)!
    - *This step helps you confirm that you're not overfitting*

Train / test split for overfitting?

- Look at only your training set (a random 70% of your data) when building your model

- Then confirm it generalizes well to the other 30% of your data (the test set)!
    - *This step helps you confirm that you're not overfitting*
    - *Validation sets are nice because you can check for overfitting on just 15% of the data, and if you're overfitting, then you can keep fixing your model until you're ready to check the final 15% of your data (the test set)*

Think, pair, share: explain the meme

https://pbs.twimg.com/media/DdkUUTMV4AAsohU.jpg

Think, pair, share: explain the meme

This "model" was only *trained* on side sleepers! It works great for them, but is a terrible model if *tested* on back sleepers or people who roll around

https://pbs.twimg.com/media/DdkUUTMV4AAsohU.jpg

Think, pair, share: explain the meme

This is why it's important to represent a diversity of sleeping positions in both the *training data* and also the *test set*

https://pbs.twimg.com/media/DdkUUTMV4AAsohU.jpg

https://www.youtube.com/watch?v=DQWI1kvmwRg

How to generate a train / test split?

Step 0. Figure out what data you need to run your model

$$y \sim x_1 + x_2 + x_3 + x_4 + x_5$$

https://builtin.com/data-science/train-test-split

How to generate a train / test split?

Step 0. Figure out what data you need to run your model

$$y \sim x_1 + x_2 + x_3 + x_4 + x_5$$

How many columns *at minimum* should there be in your dataframe, in order to fit this regression?

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How to generate a train / test split?

Step 0. Figure out what data you need to run your model

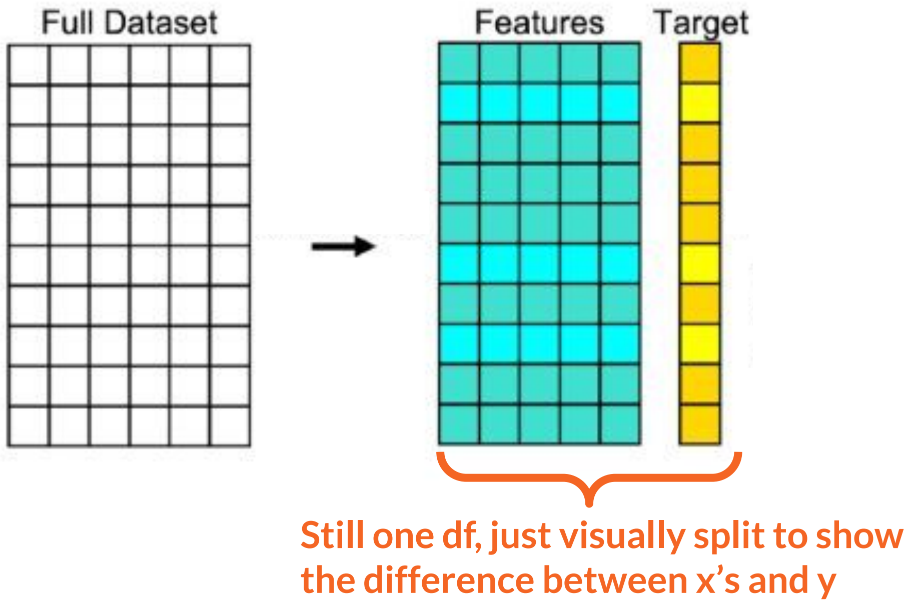
$$y \sim x_1 + x_2 + x_3 + x_4 + x_5$$

Answer: at least 5 columns for each of the x's plus 1 column for the y, so there should be at least 6 total columns present in your df to run this linear regression (without needing to reshape first)

https://builtin.com/data-science/train-test-split

How to generate a train / test split?

Step 1. Given your df, use a randomizer to assign 70% of your rows to the “train set” and 30% of your rows to the “test set”



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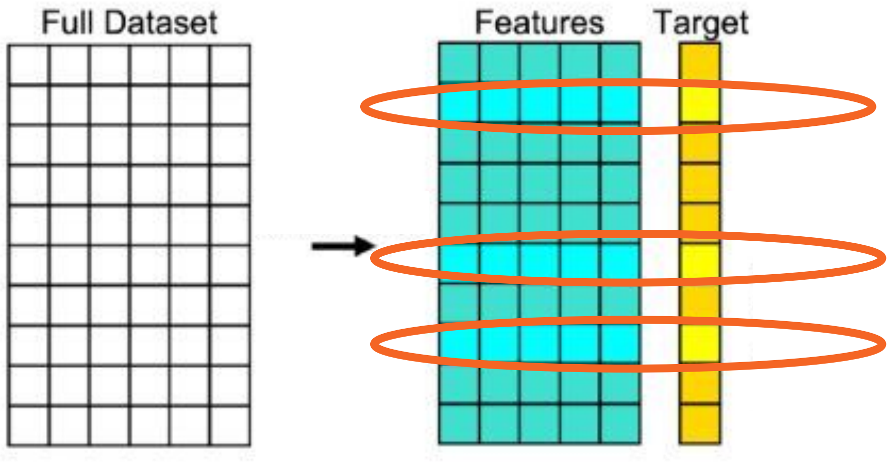

We’ve identified that these three rows will go into our test set

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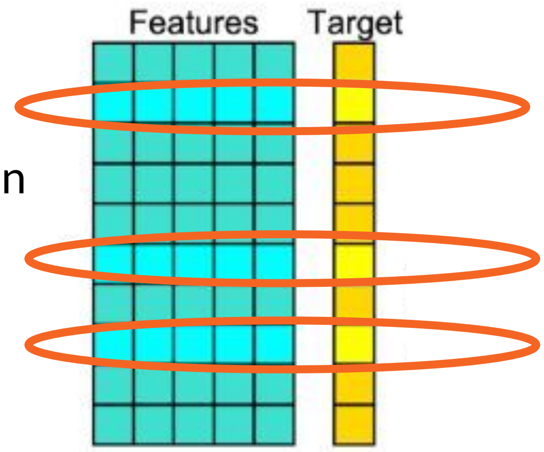
How to generate a train / test split?

Step 2. Split your df into two separate df's based on randomized values



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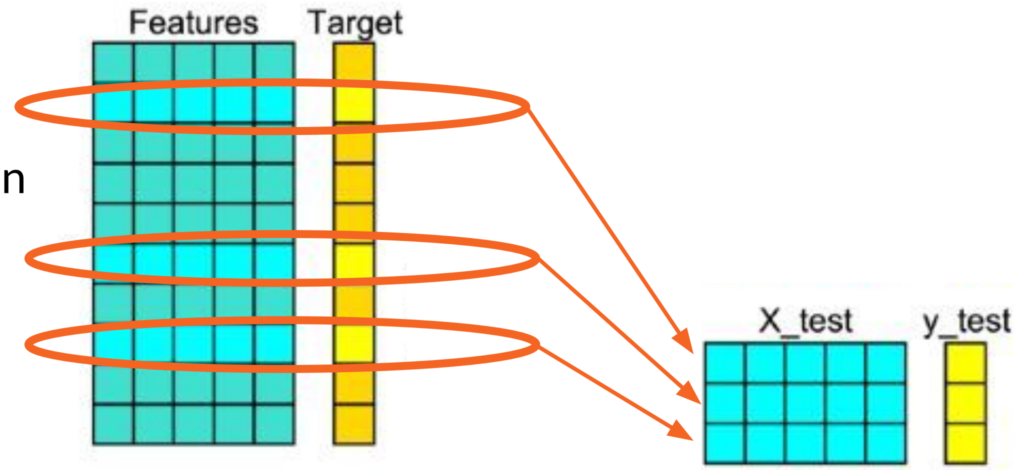

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Test set (1 dataframe with 3 rows, 6 columns)

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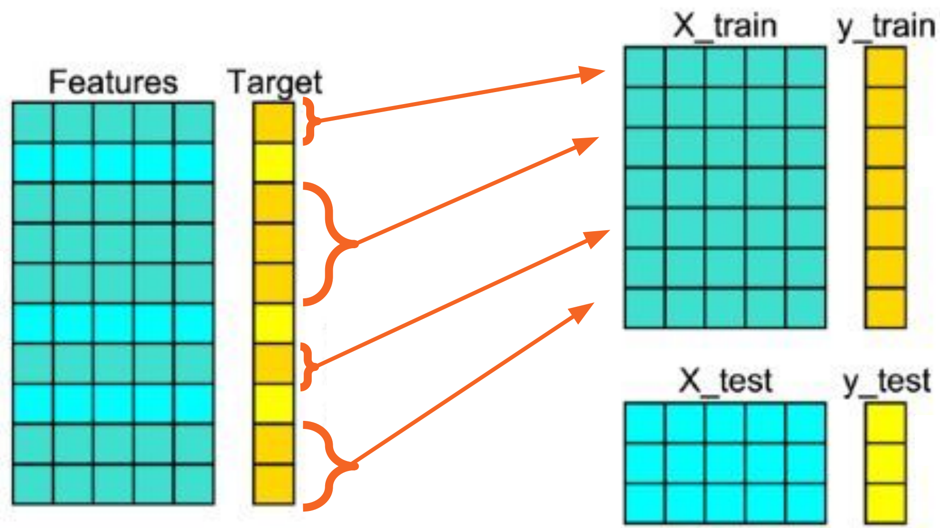

Train set (1 df, 7 rows, 6 cols)

Test set

The other seven rows go to the "train set"

https://builtin.com/data-science/train-test-split

How to generate a train / test split?

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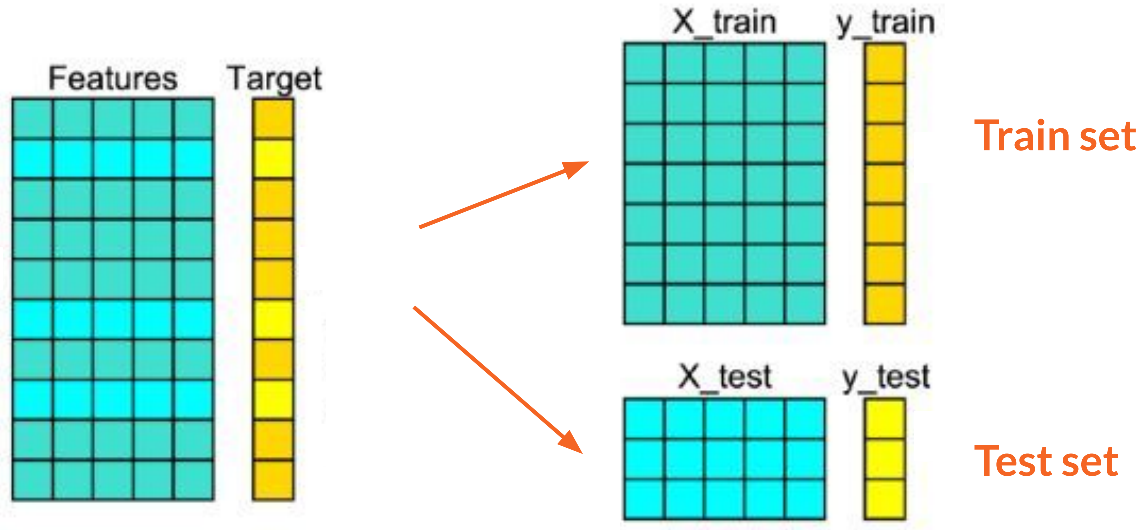

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Four components generated

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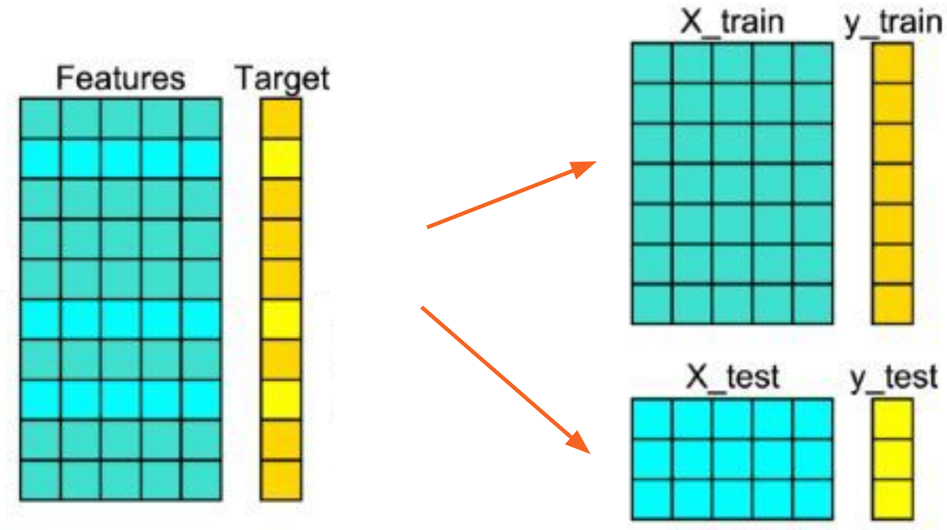

Train set

Test set

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Four components generated

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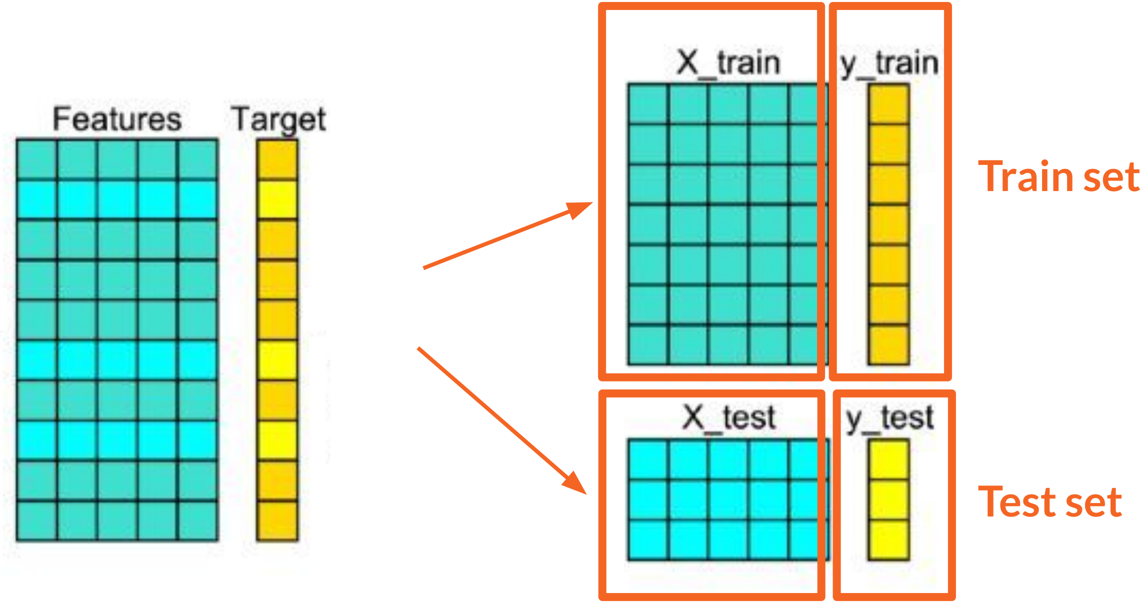

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Generate train/test in Python

- `from sklearn.model_selection import train_test_split`
- `X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.3, random_state = 42)`

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Inputs to train_test_split are X (can be multiple columns of x's) and y (one column)

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70% train set, 30% test set split

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set seed to a specific integer so you can reproduce your “random” results

Generate train/test in Python

- `from sklearn.model_selection import train_test_split`
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output: 4 different datasets

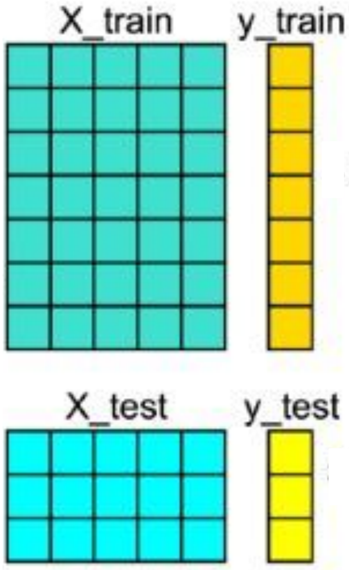
Pros/cons of train/test split?

- Pros of splitting your data:
  - Better out-of-sample generalizability
  - Confirmation that you're not overfitting across variables
  - Usually results in a more meaningful interpretation
- Cons of splitting your data:
  - Less training data means your initial model might not be as accurate; this isn't great if you're *definitely really truly* not trying to make broader generalizations (**rare**)

What do you do with train / test sets?

Train set

Test set



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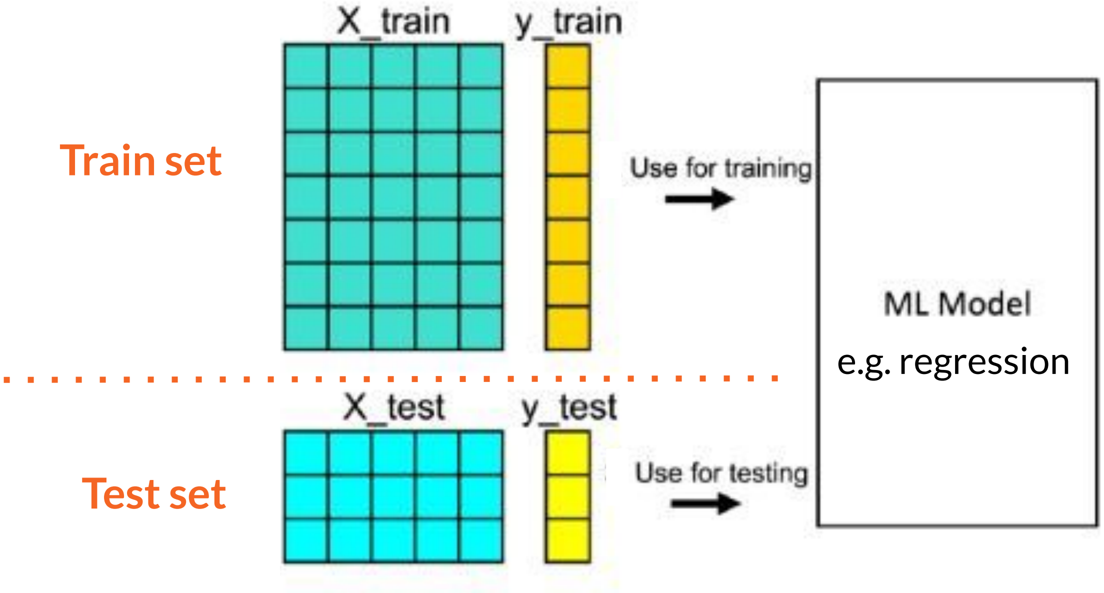

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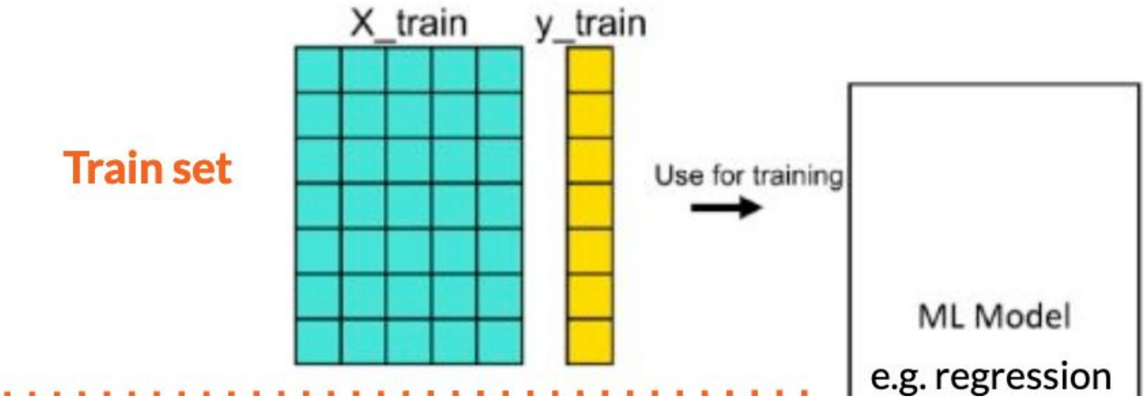
What do you do with train / test sets?

- **Step 1**: experiment with your regression on your training set. **Make any adjustments you need to here** (e.g. try different models, transformations, etc.)



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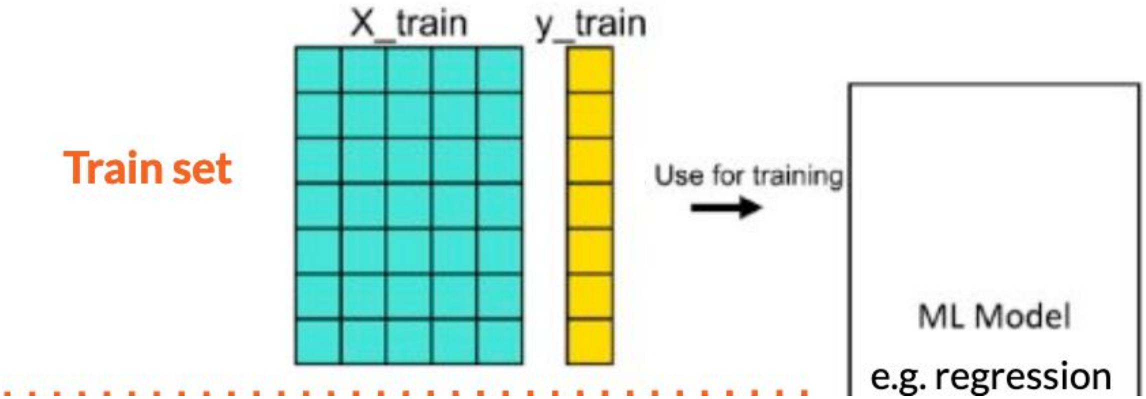



```
model = LinearRegression().fit(X_train,y_train)
```

https://builtin.com/data-science/train-test-split

What do you do with train / test sets?

- **Step 2**: make predictions using the train set only

```
model = LinearRegression().fit(X_train,y_train)

y_hat_train = model.predict(X_train)
```

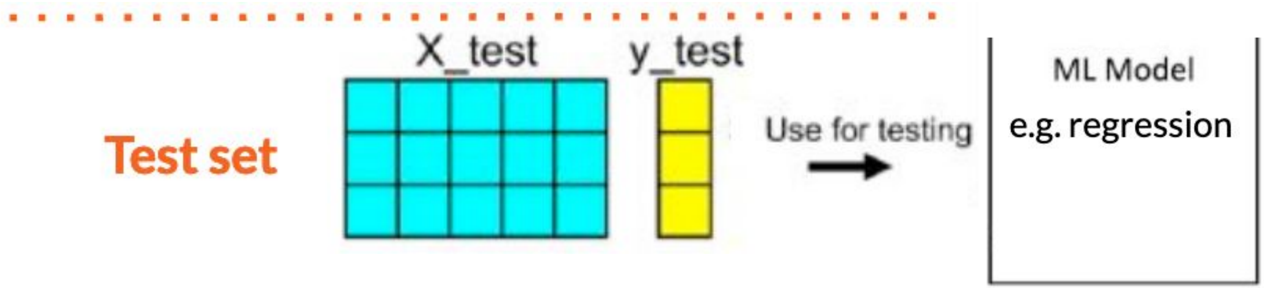
Why do we do this? **Because we want to compare our true y values (y_train) to the values predicted by our model (y_hat_train).**

Details: stay tuned for Step 4!

https://builtin.com/data-science/train-test-split

What do you do with train / test sets?

- **Step 1**: fit model on train set
- **Step 2**: predict y-hats from using train set model on X_train
- **Step 3**: predict y-hats from using train set model on **test set**



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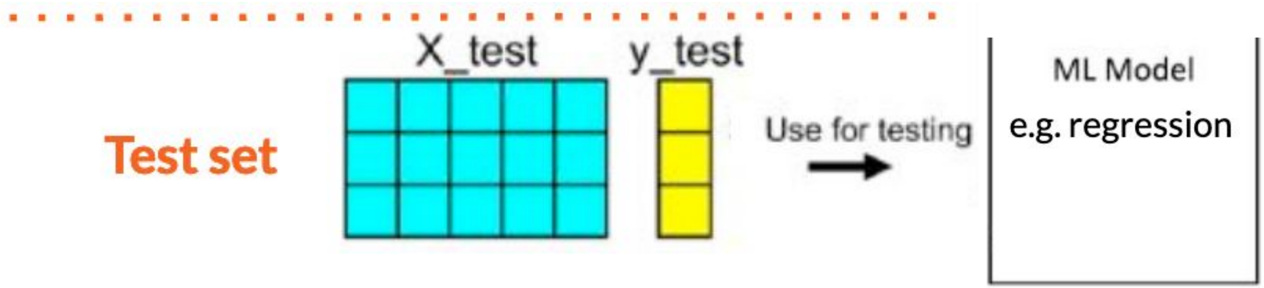


```
y_hat_test = model.predict(______)
```

https://builtin.com/data-science/train-test-split

What do you do with train / test sets?

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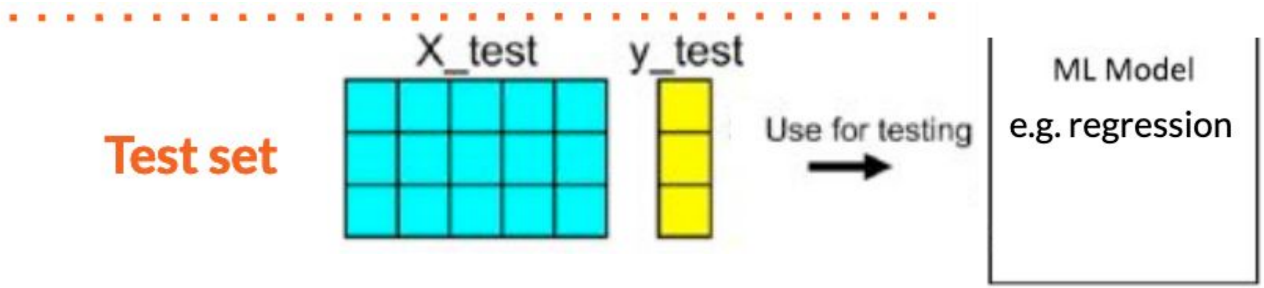


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y_hat_test = model.predict(X_test)
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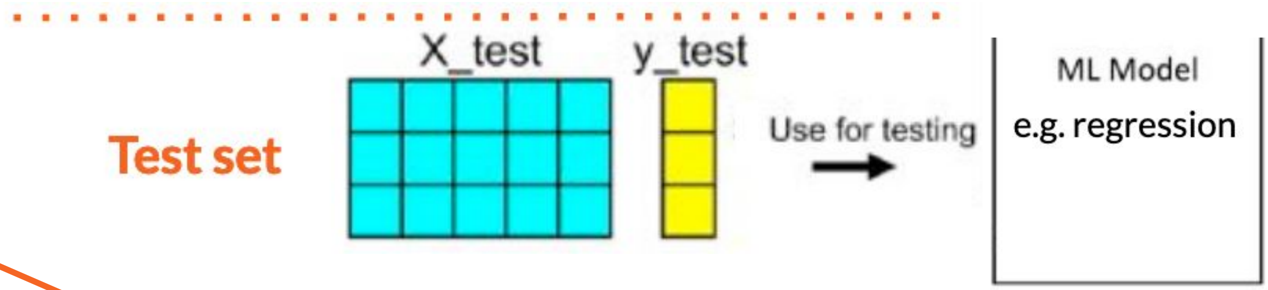

What is the "true" value of y that we want to compare y_hat_test to?

```
y_hat_test = model.predict(X_test)
```

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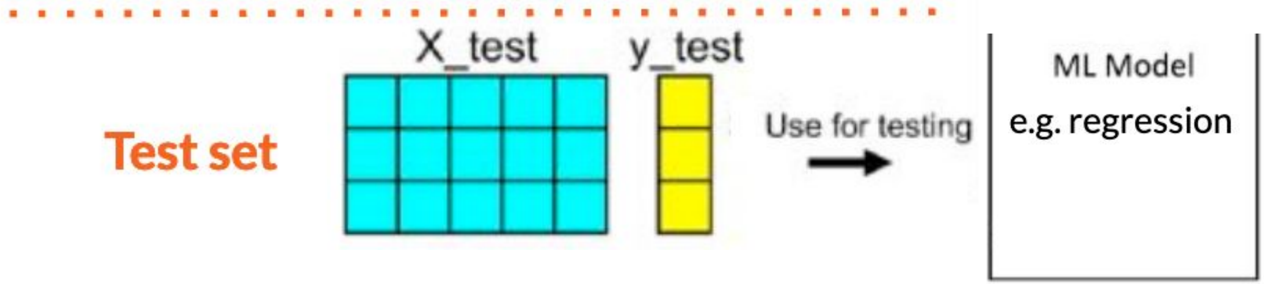



```
y_hat_test = model.predict(X_test)
```

y_test is the true output when the input is X_test (they are both in the test set)

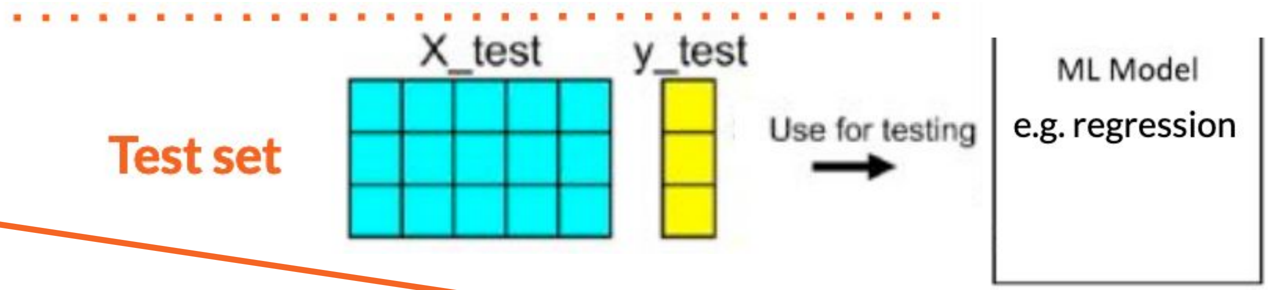
https://builtin.com/data-science/train-test-split

What do you do with train / test sets?

- **Step 1**: fit model on train set
- **Step 2**: predict y-hats from using train set model on X_train

- **Step 3**: predict y-hats from using train set model on **test set**

Notice we use the train set model when predicting the test set ŷ

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```

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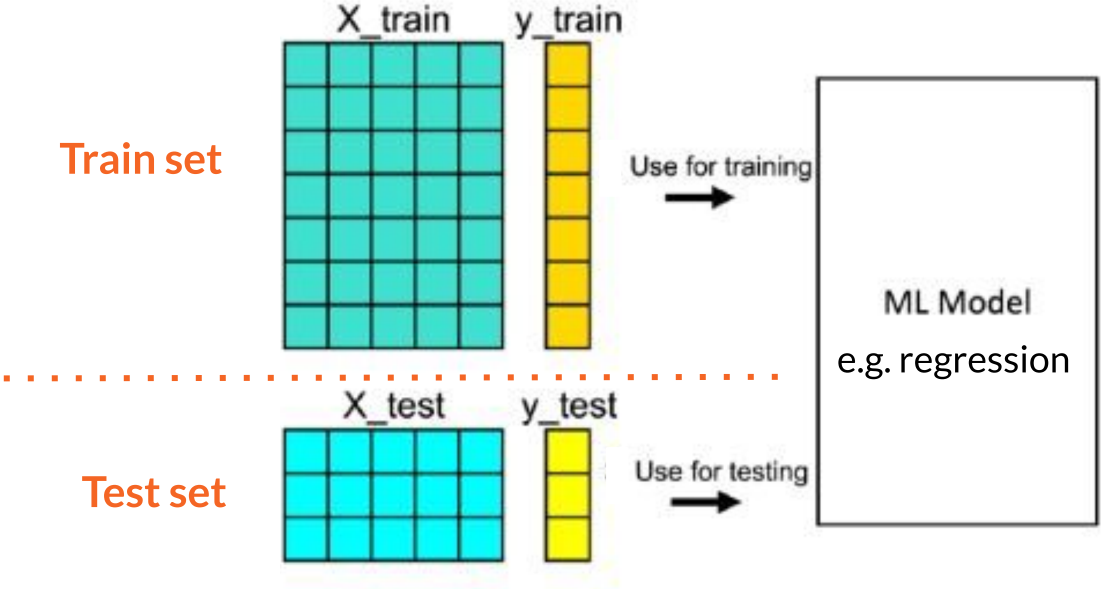
Why do you predict both train and test set y-hats with the same ML model?



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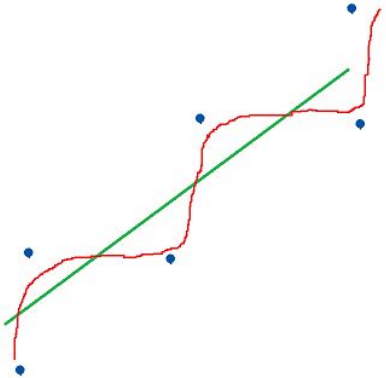
Think, pair, share: why don't we want to predict y_hat_test by fitting a new model on the test set?

https://twitter.com/deeplearningai_/status/1242160243204689920

Why don't we want to predict y_hat_test by fitting a new model on the test set?

We'd be 1. training a model on an even smaller dataset (only 30% of the data – so probably less generalizable) and 2. artificially reporting "good" results because y_hat_test should be pretty accurate if you trained on the test set!



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Comparing evaluation metrics

- **Step 1**: fit model on train set
- **Step 2**: predict y-hats from using train set model on X_train
- **Step 3**: predict y-hats from using train set model on test set

- **Step 4:** calculate "**evaluation metrics**"
  - For now, think of this as "accuracy" of the model
  - Evaluate accuracy metrics on the **train** set
  - Evaluate accuracy metrics on the **test** set

Comparing evaluation metrics

- **Step 4:** calculate "**evaluation metrics**" on...
    - **Train set**:
        - Compare **y_hat_train** to **y_train**
        - Gives you a sense of whether your model is good on your train set only
    - **Test set:**
        - Compare **y_hat_test** to **y_test**
        - Gives you a good sense of how your model would generalize to "other" data *(are you overfitting?)*

Danger zone: your model might look “good”, but actually be overfitting

https://9gag.com/gag/aOYo6Ly

Match answers to the grid

Keep experimenting with your regression model – the good test set metrics are a fluke!	A
LGTM	B
Keep experimenting with your regression model – it doesn't seem to do well on any data!	C
Your model isn't generalizing well to out-of-sample data. Figure out how to fix this in your model!	D

- **What does it mean if…**

	Train set metrics "bad"	Train set metrics "good"
Test metrics "good"	?	?
Test metrics "bad"	?	?

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Straightforward cases where either both results are good, or both results are bad

Match answers to the grid

- **What does it mean if…**

	Train set metrics "bad"	**Train set metrics "good"**
Test metrics "good"	Keep experimenting with your regression model – the good test set metrics are a fluke! *(or, you accidentally trained on the test set)*	LGTM
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Match answers to the grid

- **What does it mean if…**

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The "overfitting" case!

You should never ignore the test set in an attempt to get good accuracy!

https://twitter.com/deeplearningai_/status/1375100156186611712

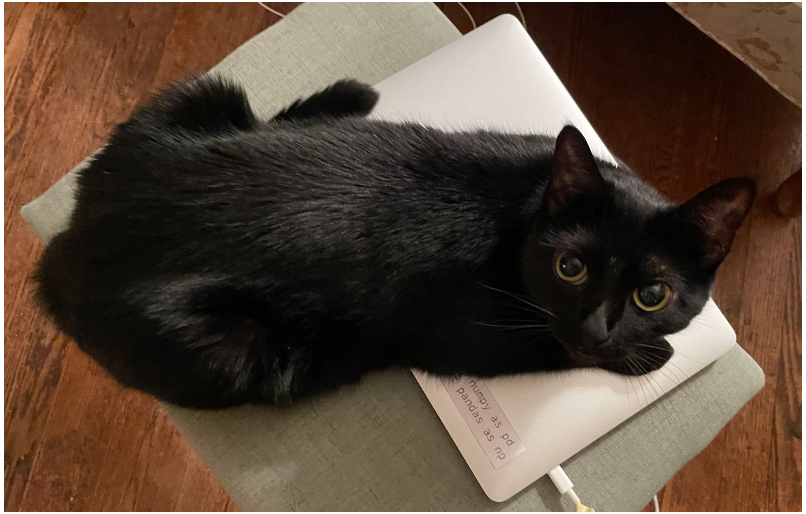
Takeaways: reg evaluation

- Use a train/test split if you want your model to be generalizable (~70/30%)
- Don't peek at the test set until you're ready to do a final confirmation that your model is generalizable
  - Train set evaluation: compare y_hat_train to y_train
  - Test set evaluation: compare y_hat_test to y_test

Caution: train/test

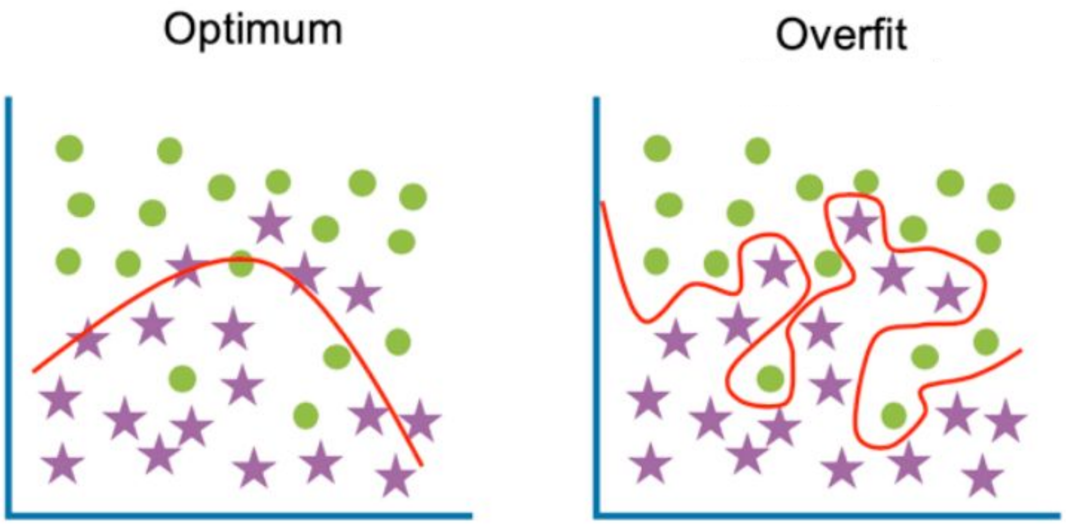
- Need to make sure your train/test sets have similar distributions
- Need to be careful if you have time series data (can't just randomly pick different times!)
- What if you just get lucky with your train set choice?
- We'll address these + discuss cross validation in a future lecture

1 minute break & attendance



tinyurl.com/z5wm2vcw

Motivation: I want to be able to numerically find that the left/right model is good/bad.



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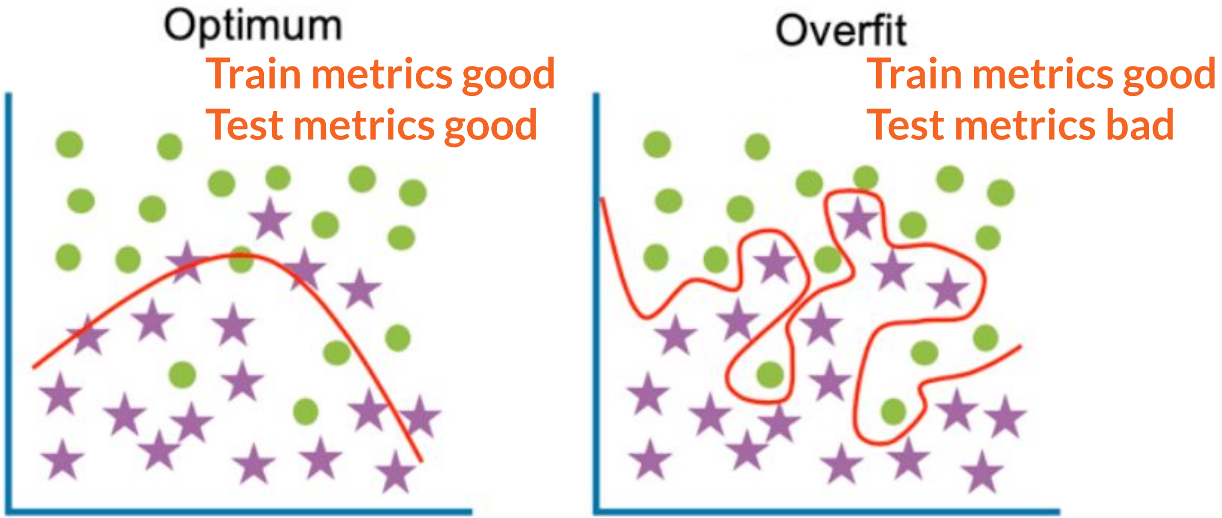

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Train metrics good
Test metrics bad

Evaluating Regressions

- How do we know if our regressions are any good?
    - Checking residual plots, correlation matrices for inputs, interaction plots, etc.

https://www.facebook.com/RegressiveRegressionMemesForHeteroskedasticTeens/

Evaluating Regressions

- How do we know if our regressions are any good?
  - Checking residual plots, correlation matrices for inputs, interaction plots, etc.
  - **Evaluation metrics**

$$R^2 = \frac{\sum(\hat{y} - \bar{\hat{y}})^2}{\sum(y - \bar{y})^2}$$

What is R^2?

- R^2 = Explained Variation / Total Variation
- Summarizes the % variation in the output that is explainable by the regression model
- R^2 is between 0 to 1, we generally want our models to have higher R^2

$$R^2 = \frac{\sum(\hat{y} - \bar{\hat{y}})^2}{\sum(y - \bar{y})^2}$$

Reasons to not use R^2

- R^2 can be low even when the model is correct
  - E.g., when variance increases, the R^2 value goes to 0
- R^2 can be high even when the model is wrong
  - E.g., non-linear data

https://library.virginia.edu/data/articles/is-r-squared-useless

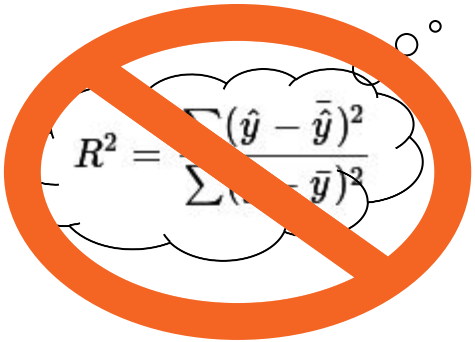
Reasons to not use R^2

$$R^2 = \frac{\sum(\hat{y} - \bar{\hat{y}})^2}{\sum(y - \bar{y})^2}$$

- R^2 can be low even when the model is correct
  - E.g., when variance increases, the R^2 value goes to 0
- R^2 can be high even when the model is wrong
  - E.g., non-linear data
- R^2 can get worse if you keep your model the same, but change the range of x, or use transformations of y
- R^2 is symmetric between x and y

https://library.virginia.edu/data/articles/is-r-squared-useless

Use R^2 with *extreme caution*



- It's relatively uncommon for data science practitioners to use R^2 in many real-world applications

$$R^2_{adjusted} = 1 - \frac{(1 - R^2)(N - 1)}{N - p - 1}$$

where

R^2 = sample R-square

p = Number of predictors

N = Total sample size.

- If you ***really must*** report R^2, use the ***adjusted*** R^2, which at least accounts for having multiple inputs (regular R^2 increases in # inputs x)

https://www.listendata.com/2014/08/adjusted-r-squared.html https://library.virginia.edu/data/articles/is-r-squared-useless

What to use if not R^2?

- There are lots of other **evaluation metrics** to use
    - More common in practice
    - Better for doing model selection of linear (and nonlinear) regressions, e.g. telling you if you're overfitting

Recall that...

- **What does it mean if...**

	Train set metrics "bad"	Train set metrics "good"
Test metrics "good"	Keep experimenting with your regression model – the good test set metrics are a fluke!	LGTM
Test metrics "bad"	Keep experimenting with your regression model – it doesn't seem to do well on any data!	Your model isn't generalizing well to out-of-sample data. Figure out how to fix this in your model!

What does “good” and “bad” mean??

- **How do you quantify your evaluation metrics?**
    - Depends on whether your y is:
        - **Numeric variable (non-binary)**
        - **Binary variable**
    - Intuition: metric should be related to residuals, so big error → “bad” metric

Nomenclature for these slides

General case	**"y_true", "y_i"**	**"y_hat", "$\hat{y}_i$"**
Train set	y_train	y_hat_train
Test set	y_test	y_hat_test

Nomenclature for these slides

We want to make this comparison

General case	“y_true”, “y_i”	“y_hat”, “$\hat{y}_i$”
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Nomenclature for these slides

General case	**"y_true", "y_i"**	**"y_hat", "$\hat{y}_i$"**
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We also want to make this comparison

Nomenclature for these slides

When providing formulas in these slides, we'll generalize to these

General case	**"y_true", "y_i"**	**"y_hat", "$\hat{y}_i$"**
Train set	y_train	y_hat_train
Test set	y_test	y_hat_test

If our y's are numerical non-binary

- **Mean Squared Error (MSE)** $\frac{1}{n}\sum_{i}^{n}(y_i - \hat{y}_i)^2$
- **Root Mean Squared Error (RMSE)** $\sqrt{\text{MSE}}$
- **Mean Absolute Error (MAE)** $\frac{1}{n}\sum_{1}^{n}|y_i - \hat{y}_i|$
- **Mean Absolute Percent Error (MAPE)** $\frac{100}{n}\sum_{i}^{n}\frac{y_i - \hat{y}_i}{y_i}$

If our y's are numerical non-binary

All 4 of these metrics are often used in the real world!

- **Mean Squared Error (MSE)** $\frac{1}{n}\sum_{i}^{n}(y_i - \hat{y}_i)^2$
- **Root Mean Squared Error (RMSE)** $\sqrt{\text{MSE}}$
- **Mean Absolute Error (MAE)** $\frac{1}{n}\sum_{1}^{n}|y_i - \hat{y}_i|$
- **Mean Absolute Percent Error (MAPE)** $\frac{100}{n}\sum_{i}^{n}\frac{y_i - \hat{y}_i}{y_i}$

If our y's are numerical non-binary

- **Mean Squared Error (MSE)** $\frac{1}{n}\sum_{i}^{n}\boxed{(y_i - \hat{y}_i)}^2$
- **Root Mean Squared Error (RMSE)** $\sqrt{\text{MSE}}$
- **Mean Absolute Error (MAE)** $\frac{1}{n}\sum_{1}^{n}\boxed{\left|y_i - \hat{y}_i\right|}$
- **Mean Absolute Percent Error (MAPE)** $\frac{100}{n}\sum_{i}^{n}\left|\frac{\boxed{y_i - \hat{y}_i}}{y_i}\right|$

All based on residuals!

Numerical prediction metrics in Python

- `from sklearn.metrics import mean_squared_error, mean_absolute_error, mean_absolute_percentage_error`

- `mse = mean_squared_error(y_true,y_hat)`
- `rmse = np.sqrt(mse)`
- `mae = mean_absolute_error(y_true,y_hat)`
- `mape = mean_absolute_percentage_error(y_true,y_hat)`

Comparing evaluation metrics

- **Step 1**: fit model on train set
- **Step 2**: predict y-hats from using train set model on X_train
- **Step 3**: predict y-hats from using train set model on **test set**

- **Step 4:** calculate "**evaluation metrics**"
  - For now, think of this as "accuracy" of the model
  - **Evaluate "accuracy" metrics on the train set**
  - **Evaluate "accuracy" metrics on the test set**

Make sure your inputs are what you really want (overall, train set only, test set only)

- `from sklearn.metrics import mean_squared_error, mean_absolute_error, mean_absolute_percentage_error`

- `mse = mean_squared_error(y_true,y_hat)`
- `rmse = np.sqrt(mse)`
- `mae = mean_absolute_error(y_true,y_hat)`
- `mape = mean_absolute_percentage_error(y_true,y_hat)`

If our y's are binary

- There are just a few very common metrics (stay tuned in three slides!)
- But to understand them, we have to think more carefully about 0's and 1's

Binary classification outcomes

“Given this item’s customer review and price, do I predict that it’s a nose pack?”

	Model predicts 1	Model predicts 0
True value is 1		
True value is 0		

Binary classification outcomes

"Given this item's customer review and price, do I predict that it's a nose pack?"

	Model predicts 1	Model predicts 0
True value is 1	**True positive** Correct prediction	**False negative**
True value is 0	**False positive**	**True negative** Correct prediction

(Lots of options for classification)

		Predicted condition		Sources: [24][25][26][27][28][29][30][31][32] view · talk · edit	
	Total population $= P + N$	Positive (PP)	Negative (PN)	Informedness, bookmaker informedness (BM) $= TPR + TNR - 1$	Prevalence threshold (PT) $= \frac{\sqrt{TPR \times FPR} - FPR}{TPR - FPR}$
Actual condition	Positive (P)	**True positive (TP),** hit	**False negative (FN),** type II error, miss, underestimation	True positive rate (TPR), recall, sensitivity (SEN), probability of detection, hit rate, power $= \frac{TP}{P} = 1 - FNR$	False negative rate (FNR), miss rate $= \frac{FN}{P} = 1 - TPR$
	Negative (N)	**False positive (FP),** type I error, false alarm, overestimation	**True negative (TN),** correct rejection	False positive rate (FPR), probability of false alarm, fall-out $= \frac{FP}{N} = 1 - TNR$	True negative rate (TNR), specificity (SPC), selectivity $= \frac{TN}{N} = 1 - FPR$
	Prevalence $= \frac{P}{P + N}$	Positive predictive value (PPV), precision $= \frac{TP}{PP} = 1 - FDR$	False omission rate (FOR) $= \frac{FN}{PN} = 1 - NPV$	Positive likelihood ratio (LR+) $= \frac{TPR}{FPR}$	Negative likelihood ratio (LR−) $= \frac{FNR}{TNR}$
	Accuracy (ACC) $= \frac{TP + TN}{P + N}$	False discovery rate (FDR) $= \frac{FP}{PP} = 1 - PPV$	Negative predictive value (NPV) $= \frac{TN}{PN} = 1 - FOR$	Markedness (MK), deltaP (Δp) $= PPV + NPV - 1$	Diagnostic odds ratio (DOR) $= \frac{LR+}{LR-}$
	Balanced accuracy (BA) $= \frac{TPR + TNR}{2}$	F_1 score $= \frac{2PPV \times TPR}{PPV + TPR} = \frac{2TP}{2TP + FP + FN}$	Fowlkes–Mallows index (FM) $= \sqrt{PPV \times TPR}$	Matthews correlation coefficient (MCC) $= \sqrt{TPR \times TNR \times PPV \times NPV} - \sqrt{FNR \times FPR \times FOR \times FDR}$	Threat score (TS), critical success index (CSI), Jaccard index $= \frac{TP}{TP + FN + FP}$

https://en.wikipedia.org/wiki/Sensitivity_and_specificity

False positives vs false negatives

- Which metric do we care more about?
  - Depends on the application!
- Sometimes you want to prioritize minimizing fp over fn, and sometimes vice versa
  - It can be very difficult to find a method that minimizes both – sometimes you must make a trade-off!

	$\hat{y}=1$	$\hat{y}=0$
$y=1$	tp	fn
$y=0$	fp	tn

False positives or false negatives?

- It’s Halloween and some annoying kids want to egg your house without causing damage to your house. They use an “egg classifier” for whether an egg-shaped item is an egg (egg=1, low damage to house) or a rock (egg=0, high damage to house).
    - *Do the kids care more about fp or fn?*

https://www.reddit.com/r/mildlyinteresting/comments/5uvn8r/i_collect_rocks_that_look_like_eggs/

https://towardsdatascience.com/false-positives-vs-false-negatives-4184c2ff941a

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  - **If the kids think something is an egg but it’s actually a rock, that’s bad! If they think something is a rock and don’t throw it (but it’s really an egg), it doesn’t matter. Consequences of a Type I error are costlier, so they’ll want to minimize fp.**

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False positives or false negatives?

True Value	Model predicts... Egg	Model predicts... Rock
Egg	**True positive** Correct prediction	**False negative** (kids' model predicts rock so it doesn't get thrown, but it's actually just an egg)
Rock	**False positive** (kids' model predicts egg, so it'll get thrown, but it's actually a rock!)	**True negative** Correct prediction

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This is the most dangerous case! → False positive

https://www.reddit.com/r/mildlyinteresting/comments/5uvn8r/i_collect_rocks_that_look_like_eggs/

	ŷ=1	ŷ=0
y=1	tp	fn
y=0	fp	tn

False positives or false negatives?

- The kids discover that the neighborhood egg stash contains some golden eggs. They use the same “egg classifier” for whether an egg-shaped item is an egg (egg=1, potentially high $ value) or a rock (egg=0, definitely $0).
    - *Do the kids care more about fp or fn?*

https://www.increasemyefficiency.com/how-to-build-a-golden-egg/

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False positives or false negatives?

- The kids discover that the neighborhood egg stash contains some golden eggs. They use the same “egg classifier” for whether an egg-shaped item is an egg (egg=1, potentially high $ value) or a rock (egg=0, definitely $0).
    - **If the kids think something is an egg but it’s actually a rock, that’s okay – they just get $0 out of it. If they think something is a rock but it’s really a $$$ golden egg, they’re missing out on a ton of money. Consequences of a Type II error are costlier, so they’ll want to minimize fn.**

https://www.increasemyefficiency.com/how-to-build-a-golden-egg/

https://towardsdatascience.com/false-positives-vs-false-negatives-4184c2ff941a

False positives or false negatives?

https://www.increasemyefficiency.com/how-to-build-a-golden-egg/

Model predicts...

True Value	Egg	Rock
Egg	**True positive** Correct prediction	**False negative** (kids' model predicts rock so they don't try to cash it in, but it's actually worth $$$!)
Rock	**False positive** (kids' model predicts egg, so they try to cash it in, but it's only worth $0)	**True negative** Correct prediction

False positives or false negatives?

This is the worst case!

		Model predicts...	
		Egg	Rock
True Value	Egg	**True positive** Correct prediction	**False negative** (kids' model predicts rock so they don't try to cash it in, but it's actually worth $$$!)
	Rock	**False positive** (kids' model predicts egg, so they try to cash it in, but it's only worth $0)	**True negative** Correct prediction

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(Lots of options for classification)

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https://en.wikipedia.org/wiki/Sensitivity_and_specificity

Evaluating with accuracy

- **Accuracy = (tp + tn) / (p + n)**
  = % things you predicted correctly
- Seems intuitive…
- But rarely used in ML. Why not?

Evaluating with accuracy

- $y = 1$ if someone has an extremely rare disease (with 1% incidence in your dataset)
- You make a naive model that simply predicts $y = 0$ for every single input x
- **What accuracy would you get with this naive model?** *Accuracy = (tp + tn) / (p + n)*

Evaluating with accuracy

- y = 1 if someone has an extremely rare disease (with 1% incidence in your dataset)
- You make a naive model that simply predicts y = 0 for every single input x
- **Your naive model would get 99% accuracy. This isn't useful for what matters: making good predictions even if there are only a few true y = 1 values.**

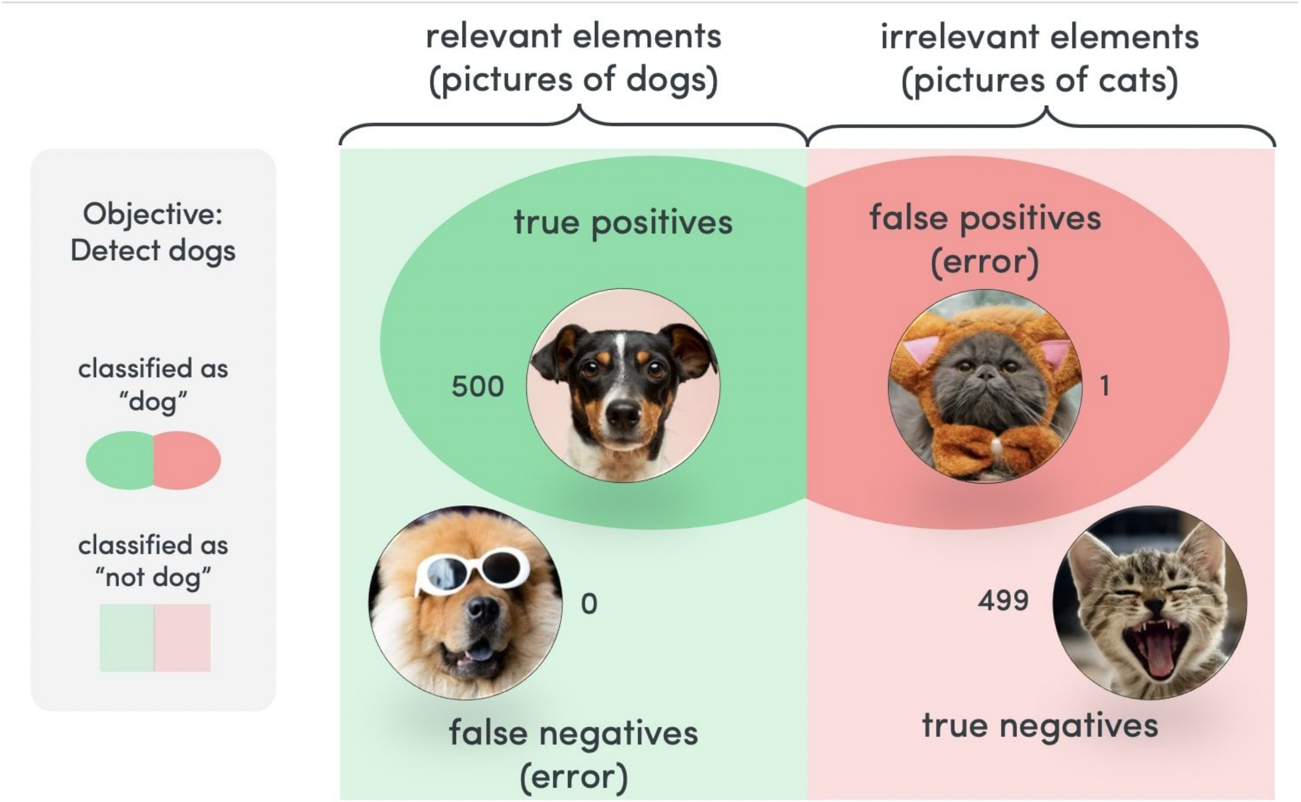
https://www.facebook.com/AnalyticsVidhya/photos/a.457177331040819/2826021784156350/?type=3

Binary classification outcomes

	Model predicts 1	Model predicts 0
True value is 1	**True positive** Correct prediction	**False negative**
True value is 0	**False positive**	**True negative** Correct prediction

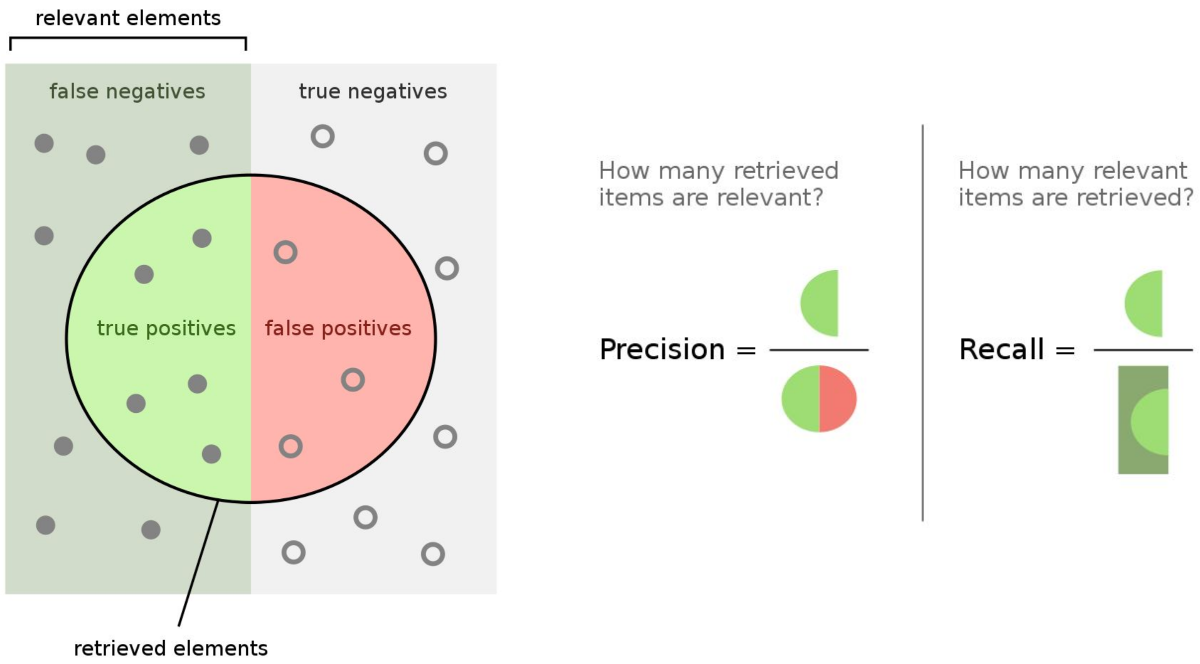
Binary evaluation metrics, part 1

- **Precision** = tp / (tp + fp)
- **Recall** (a.k.a. sensitivity) = tp / (tp + fn)
- Higher is better!



https://levity.ai/blog/precision-vs-recall

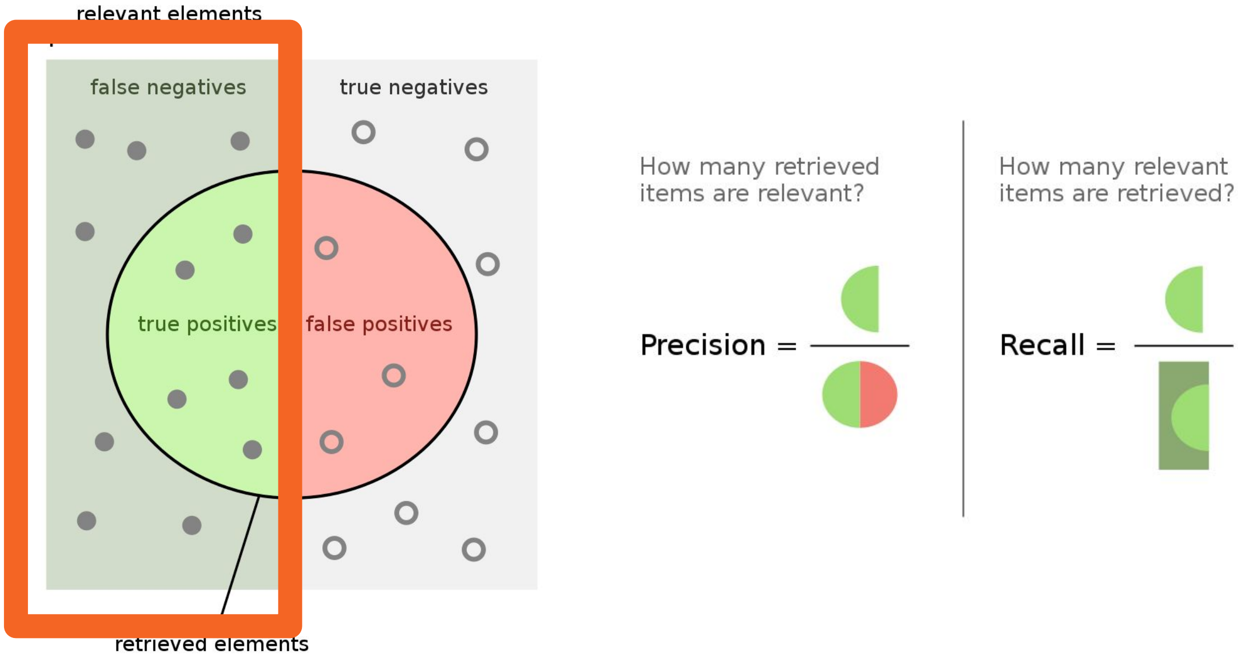
Precision, Recall



https://en.wikipedia.org/wiki/Precision_and_recall

Precision, Recall

True 1's



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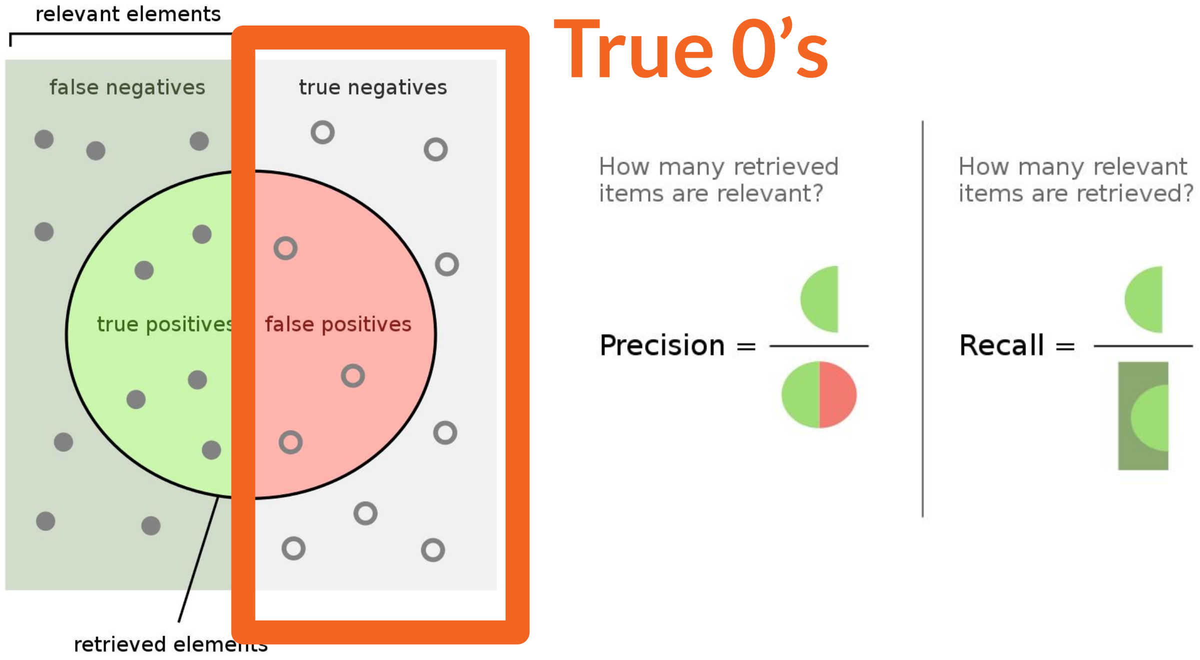
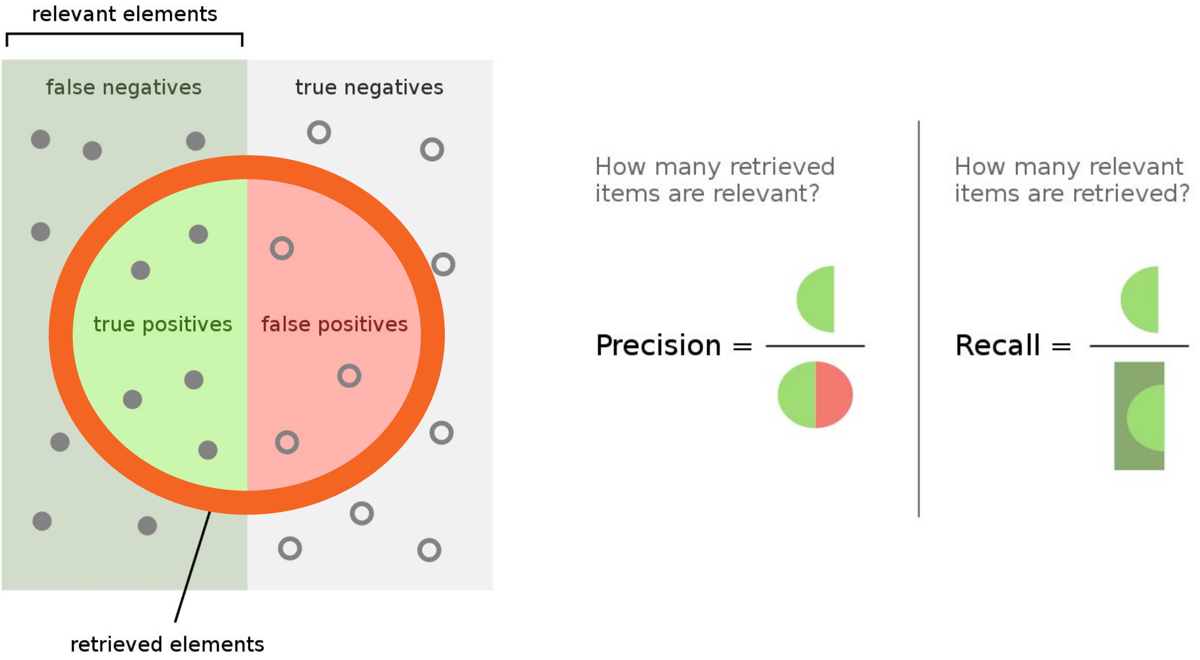
Precision, Recall



https://en.wikipedia.org/wiki/Precision_and_recall

Precision, Recall

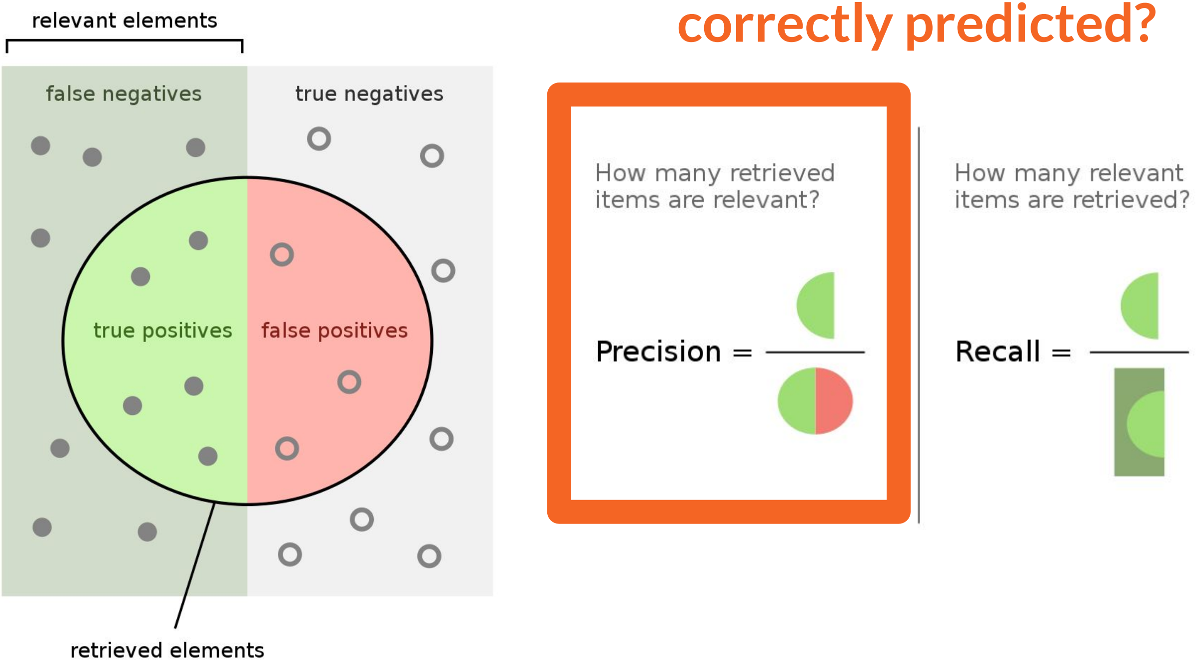
Predicted 1's



https://en.wikipedia.org/wiki/Precision_and_recall

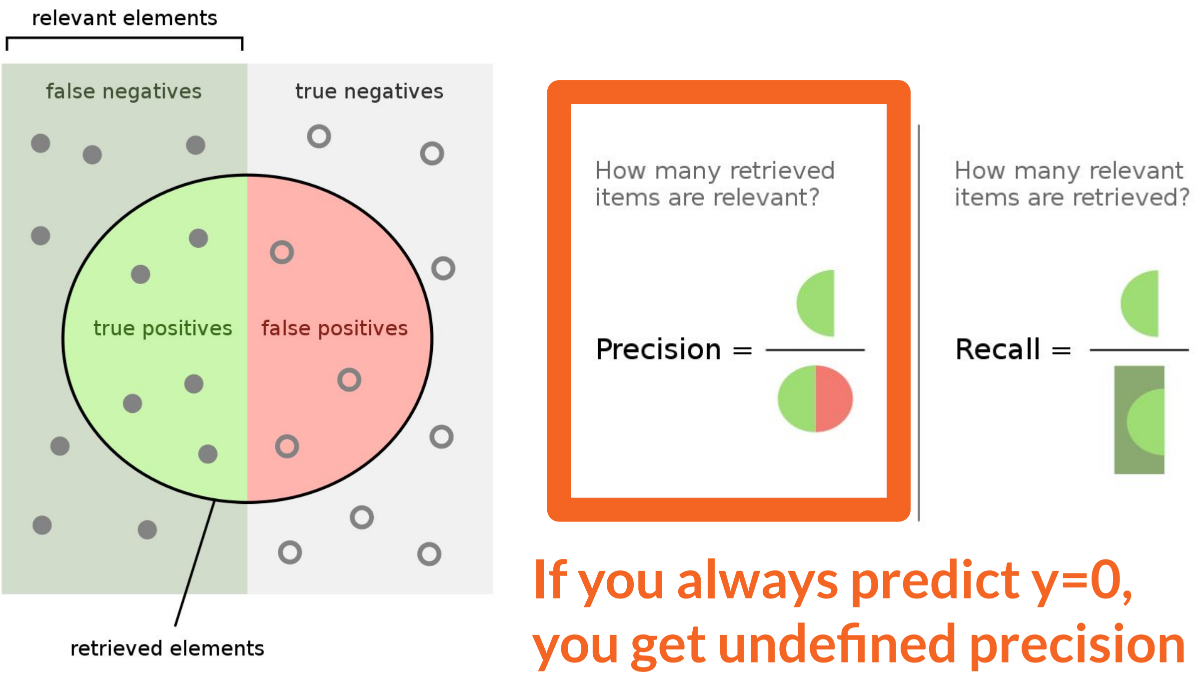
Precision, Recall

How many of your predicted y=1's were correctly predicted?



https://en.wikipedia.org/wiki/Precision_and_recall

Precision, Recall

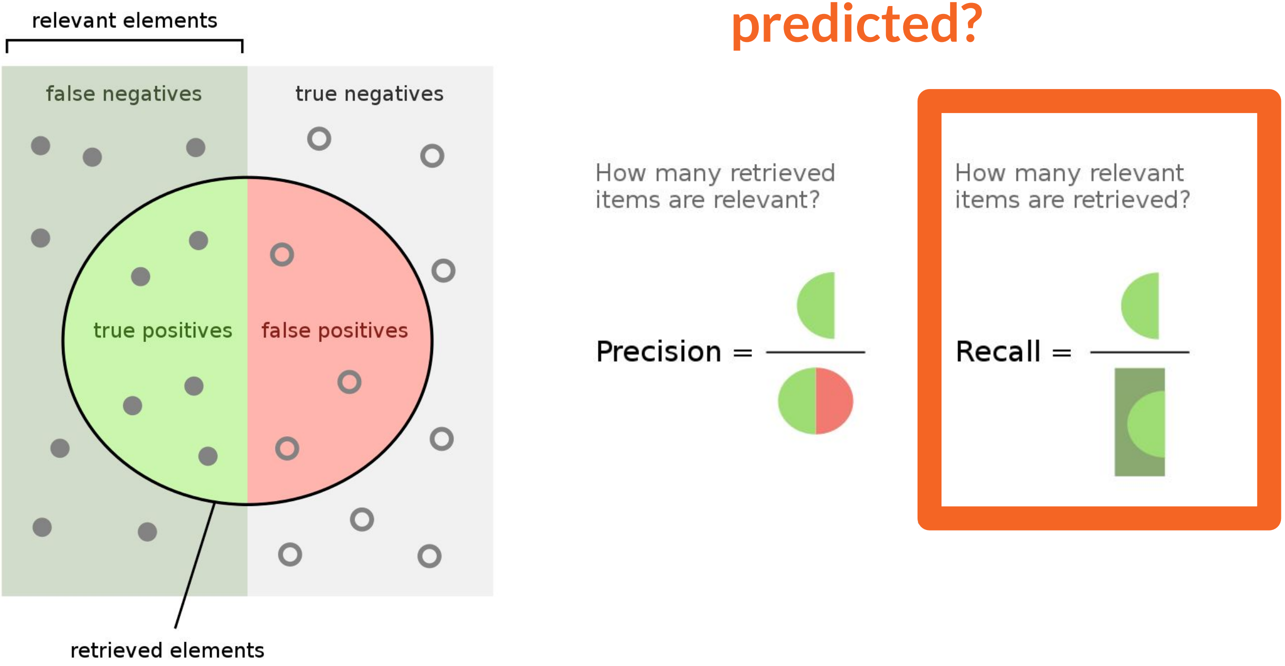


If you always predict y=0, you get undefined precision

https://en.wikipedia.org/wiki/Precision_and_recall

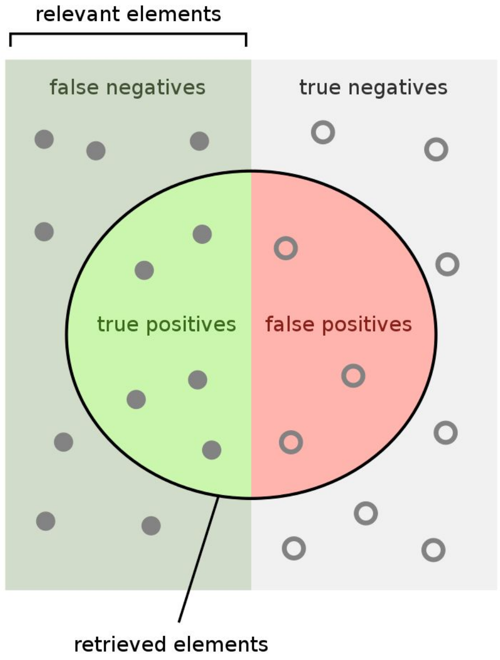
Precision, Recall

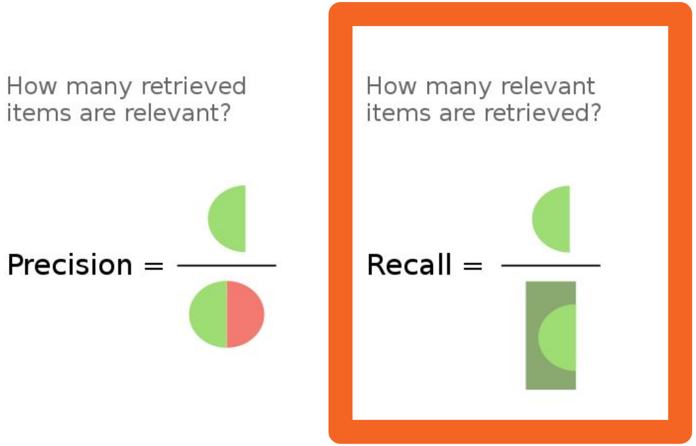
How many of the true y=1's were correctly predicted?



https://en.wikipedia.org/wiki/Precision_and_recall

Precision, Recall

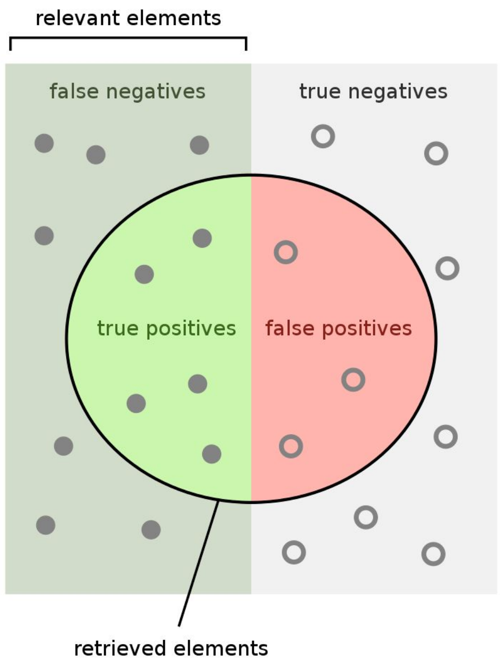


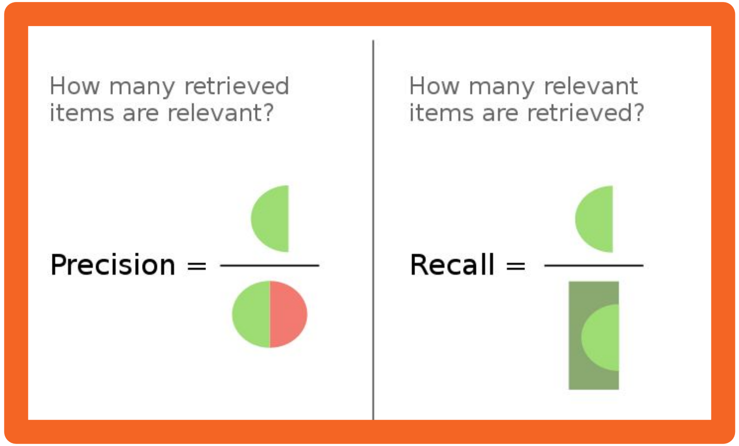


If you always predict y=0, you get 0 recall

https://en.wikipedia.org/wiki/Precision_and_recall

Precision, Recall



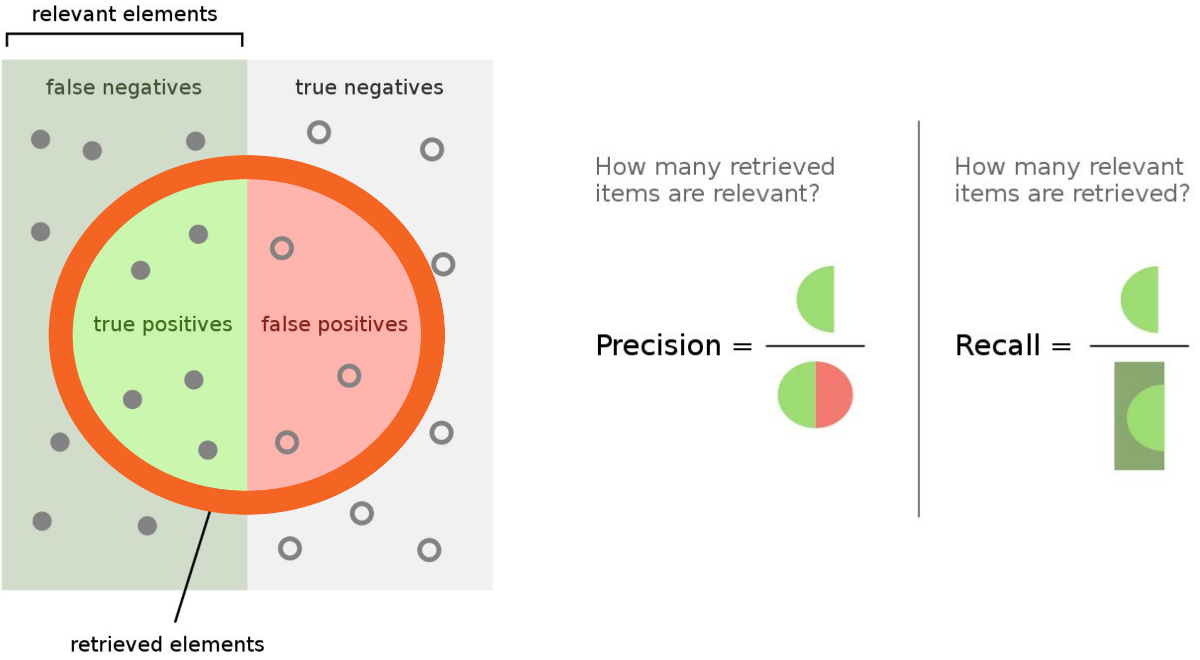


Higher is better for both precision and recall

https://en.wikipedia.org/wiki/Precision_and_recall

Can you get high precision AND recall?

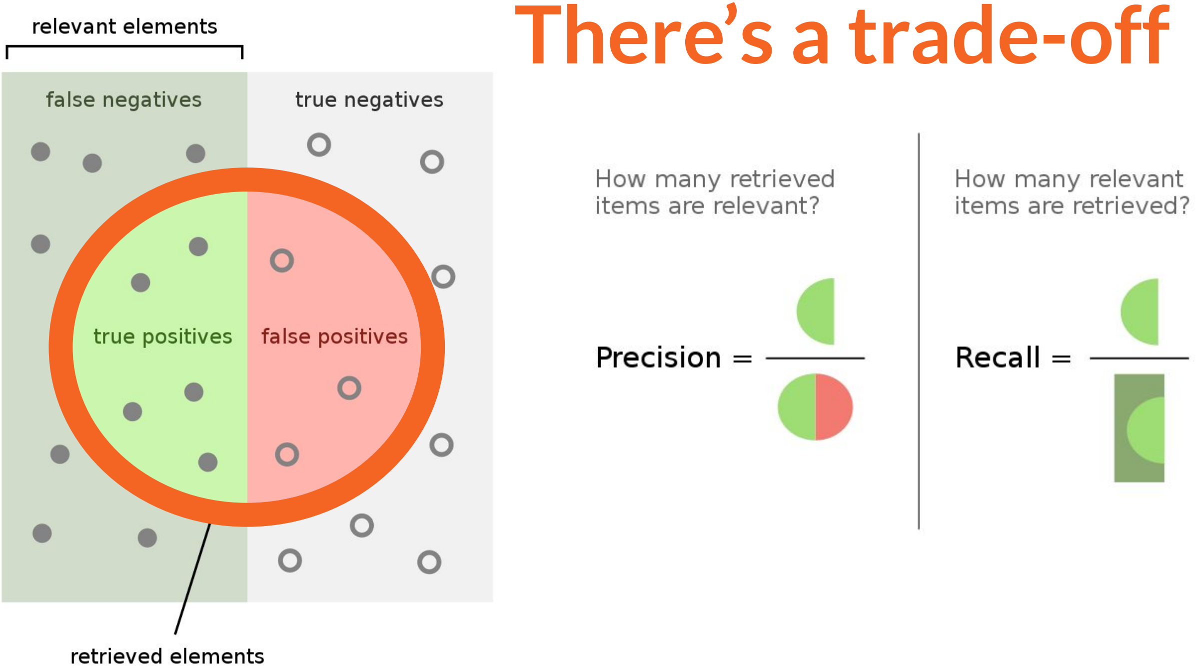
Predicted 1's



https://en.wikipedia.org/wiki/Precision_and_recall

Can you get high precision AND recall?

There's a trade-off



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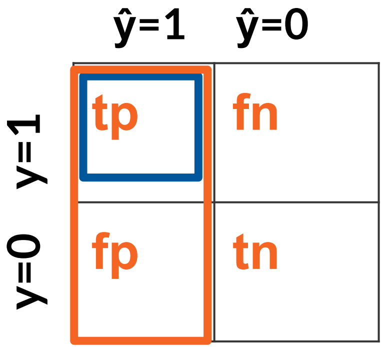
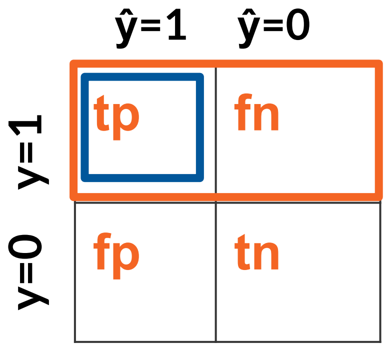
Precision and Recall



- Precision = tp / (tp + fp)



- Recall = tp / (tp + fn)

Calculate precision, recall, accuracy

	ŷ=1	ŷ=0
y=1	tp 40	fn 10
y=0	fp 10	tn 40

- **Precision** = tp / (tp + fp)
- **Recall** = tp / (tp + fn)
- **Accuracy** = (tp + tn) / (tp + fp + fn + tn)

Calculate precision, recall, accuracy

	$\hat{y}=1$	$\hat{y}=0$
$y=1$	tp 40	fn 10
$y=0$	fp 10	tn 40

- Precision = tp / (tp + fp)
- Recall = tp / (tp + fn)
- Accuracy
  = (tp + tn) / (tp + fp + fn + tn)

Precision = 4/5
Recall = 4/5
Accuracy = 4/5

Calculate precision, recall, accuracy

	ŷ=1	ŷ=0
y=1	tp 4	fn 1
y=0	fp 5	tn 90

- **Precision** = tp / (tp + fp)
- **Recall** = tp / (tp + fn)
- **Accuracy** = (tp + tn) / (tp + fp + fn + tn)

Calculate precision, recall, accuracy

	$\hat{y}=1$	$\hat{y}=0$
$y=1$	tp 4	fn 1
$y=0$	fp 5	tn 90

- Precision = tp / (tp + fp)
- Recall = tp / (tp + fn)
- Accuracy
  = (tp + tn) / (tp + fp + fn + tn)

Precision = 4/9
Recall = 4/5
Accuracy = 94/100

Calculate precision, recall, accuracy

	$\hat{y}=1$	$\hat{y}=0$
$y=1$	tp 0	fn 5
$y=0$	fp 0	tn 95

- **Precision** = tp / (tp + fp)
- **Recall** = tp / (tp + fn)
- **Accuracy**
  = (tp + tn) / (tp + fp + fn + tn)

Calculate precision, recall, accuracy

	$\hat{y}=1$	$\hat{y}=0$
$y=1$	tp 0	fn 5
$y=0$	fp 0	tn 95

- Precision = tp / (tp + fp)
- Recall = tp / (tp + fn)
- Accuracy
  = (tp + tn) / (tp + fp + fn + tn)

Precision = NaN
Recall = 0
Accuracy = 95/100

Calculate precision, recall, accuracy

	$\hat{y}=1$	$\hat{y}=0$
$y=1$	tp 4	fn 1
$y=0$	fp 5	tn 9000

- **Precision** = tp / (tp + fp)
- **Recall** = tp / (tp + fn)
- **Accuracy**
  = (tp + tn) / (tp + fp + fn + tn)

Calculate precision, recall, accuracy

	ŷ=1	ŷ=0
y=1	tp 4	fn 1
y=0	fp 5	tn 9000

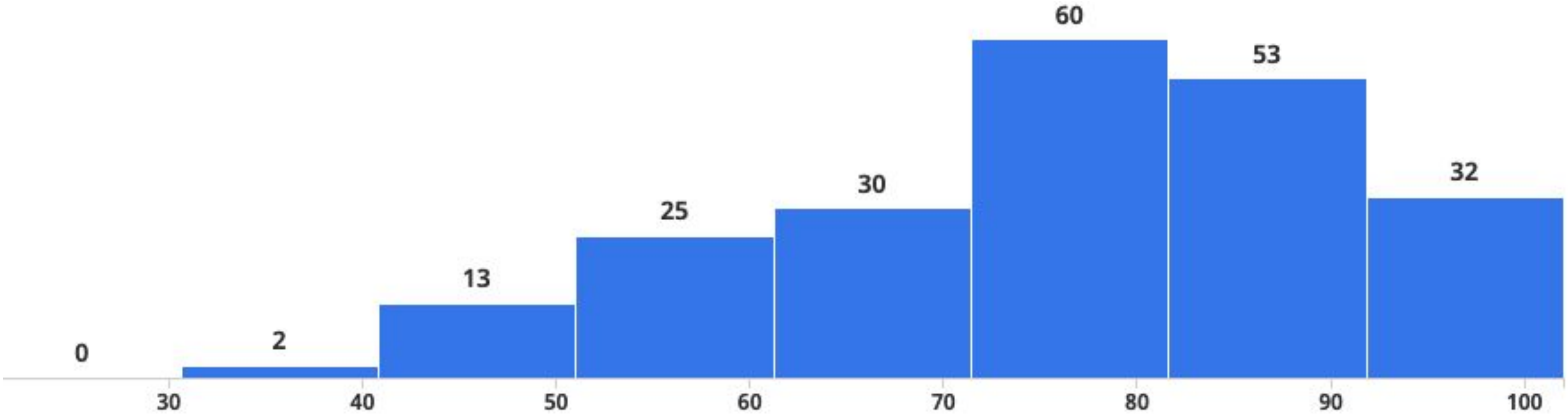
- Precision = tp / (tp + fp)
- Recall = tp / (tp + fn)
- Accuracy = (tp + tn) / (tp + fp + fn + tn)

Precision = 4/9
Recall = 4/5
Accuracy = 9004/9010

Reminder: Extra Credit! 🌟

- Two surveys, +10 points towards HW2 grade for filling out each:
  - Mid-Semester Course Feedback
  - Midterm TA Evaluations
- Surveys due on Oct 13

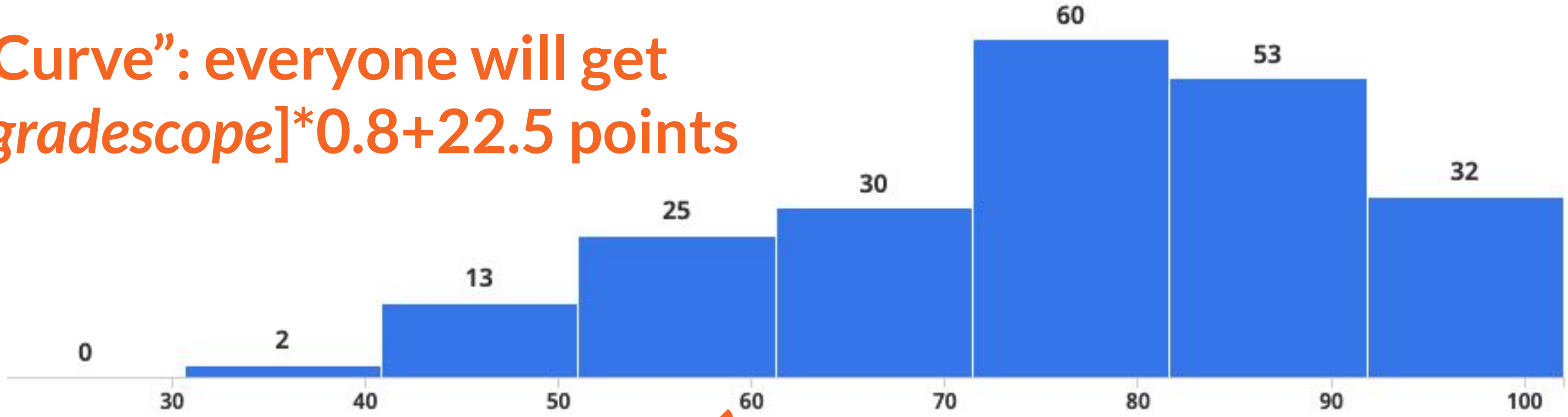
Prelim Grades will be released on Gradescope



Median	Maximum	Mean	Std Dev
78.0	101.5	76.03	14.95

Prelim Grades will be released on Gradescope

"Curve": everyone will get [*gradescope*]*0.8+22.5 points



Median	Maximum	Mean	Std Dev
78.0	101.5	76.03	14.95

84.9

103.7