ORIE 3510/5510 DISCUSSION 3

Question 1: (Elmer Fudd's Rifle) The Rabbit season is about to begin. Elmer Fudd goes to a tavern outside the woods. Including Fudd, there are n hunters all carrying the same type of rifle. All of them put their rifles on a table upon entering the tavern. For some reason, the rifles get mixed together. Upon leaving, each hunter randomly selects one rifle.

- (a) Calculate the expected number of hunters that select their own rifle.
- (b) Suppose that those selecting their own rifles depart, while the others (those without a match) put their selected rifles on the table again, mix them up, and then re-select. If this process continues until each hunter has his or her own rifle, calculate the expected number of rounds that are necessary.

Question 2: (Lunch time for Taz) The Tasmanian "Taz" devil is out hunting small animals (like rodents, gophers, hamsters e.t.c). Imagine there are n distinct small animals that Taz is trying to hunt for food. Every time he catches an animal, he gets one of the n small animals at random, with each small animal being equally likely. (You can think of this as draws with replacement, or you can imagine that there are an infinite number of each of the n small animal types.) Thus it is quite likely that the same animal type will be drawn more than one time. How many trials/draws does it take on average for Taz to get all n distinct small animal types?

CHALLENGE PROBLEM: This is an optional question; Attempt this question after you're done with the previous questions

Question 3: (The Grand Old Duck of York) Bugs Bunny and Daffy Duck are at JFK boarding a plane to Las Vegas. There are n passengers in total (including Bugs and Daffy) and exactly n seats on the plane. Each passenger is assigned a seat number. Daffy is pretty drunk, yet he is the first person to board the plane. Upon boarding, he picks and sits on a random seat. Passengers boarding after him either sit on their assigned seat or sit at any other random empty seat if their assigned seat is taken. Bugs is the last passenger to board the plane. Compute the probability that Bugs sits on his assigned seat. (Assume $n \geq 2$)