

## HADM 4200 Homework 2: Discounted cash flows and valuation

Goal: the goal of this assignment is to provide additional practice with computing discounted cash flows and to review the three primary approaches for valuing property: (1) sales comparison approach, (2) income approach, and (3) cost approach.

**Deliverables:** please submit the following files:

- **To course Gradescope:** A single PDF file containing a consolidated version of your entire Excel workbook.
  - The PDF file will be named Lastname\_Firstname\_HADM4200\_HW2.pdf. (Insert your last and first names in the relevant spaces, e.g., Kytomaa\_Lauri\_HADM4200\_HW2.pdf).
  - To create a PDF from your Excel file, you should first go to the “View” tab when looking at each worksheet and then enlarge the “Page Break” so that everything is on a single page. After this adjustment, simply enter printer dialogue, select Print “Entire Workbook”, and then save to PDF instead of physically printing the document.
  - You must select the relevant pages for each of the questions to help ensure timely grading.
- **To course Canvas:** A single Excel workbook containing your work and answers.
  - You should create four separate tabs (one for question 1, one for question 2, and one for each part of question 3). The tabs should be labeled “Q1,” “Q2,” “Q3” and “Q4”.
  - The Excel file will be named Lastname\_Firstname\_HADM4200\_HW2.xlsx. (Insert your last and first names in the relevant spaces, e.g., Kytomaa\_Lauri\_HADM4200\_HW2.xlsx).

### Important:

- GROUP SUBMISSIONS ARE NOT PERMITTED FOR HOMEWORK ASSIGNMENTS
- SHOW YOUR WORK AS MUCH AS POSSIBLE
  - In Excel files, all work should be labeled using comments with clear formatting consistent with the course tutorials.
- DO NOT ROUND any numbers (you may format your answers in Excel such that the underlying data appear rounded, but do not actually round the answers). Rounding can make it harder for TAs to follow your work.

**Question 1: Discounted Cash Flow Review – 20 points**

The formula for discounted cash flows is given by:

$$DCF = \sum_{t=1}^N \frac{Cash\ flow_t}{(1 + Discount\ Rate)^t}$$

Where:  $t$  is a particular time period.  $N$  is the total number of periods that we are discounting. *Discount Rate* is the rate that we are using to discount future cash flow to present value. If we make \$1,000 in the first period and \$2,000 in the second period then we have  $Cash\ flow_1 = \$1,000$  and  $Cash\ flow_2 = \$2,000$ .

Remember that the  $\sum$  just represents a sum. To illustrate how this equation works, suppose that we have three periods, the formula yields the following:

$$DCF_3 = \frac{Cash\ flow_1}{(1 + Discount\ rate)^1} + \frac{Cash\ flow_2}{(1 + Discount\ rate)^2} + \frac{Cash\ flow_3}{(1 + Discount\ rate)^3}$$

**Q1 (a) [Manual approach]** Without specific Excel functions, manually calculate the discounted cash flows for three periods if the cash flows in every period are \$1,000 and the discount rate is 3%.

**Q1 (b) [Excel function]** Use the =NPV() function in Excel to complete the same calculation for three periods. Is the answer different or the same as your manual calculation? Explain any differences.

Recall that for a finite horizon geometric series with a discount rate that is less than one  $|r| < 1$ , such as  $r^1 + r^2 + r^3$ , the general formula is given by:<sup>1</sup>

$$\sum_{k=1}^N r^k = \frac{r(1 - r^N)}{1 - r}$$

We can define  $\hat{r} = \frac{1}{1 + \text{Discount Rate}}$  to get this identical geometric series, this new value is often called the **discount factor**. To see this, if our cashflows are the same every period then we can re-write our DCF formula above as:

$$DCF_N = (\text{Cash flows}) \cdot \left[ \frac{1}{(1 + \text{Discount rate})^1} + \frac{1}{(1 + \text{Discount rate})^2} + \dots + \frac{1}{(1 + \text{Discount rate})^N} \right]$$

Using our new definition of  $\hat{r}$  we can just re-write this as:

$$DCF_N = (\text{Cash flows}) \cdot [\hat{r}^1 + \hat{r}^2 + \dots + \hat{r}^N]$$

Finally, applying our geometric series equation:

$$DCN_N = (\text{Cash flows}) \cdot \frac{\hat{r}(1 - \hat{r}^N)}{1 - \hat{r}}$$

**Q1 (c) [Geometric series]** Once again, assume three periods with constant cashflows of \$1,000 and a discount rate of 3%. Use the geometric series formula to compute the discounted cash flows. How does your result compare to the answers in part (a) and part (b)?

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<sup>1</sup> This means that (test it out!):

$$\sum_{k=1}^3 r^k = r^1 + r^2 + r^3 = \frac{r(1 - r^3)}{1 - r} \text{ where } |r| < 1.$$

The interested student can review more about geometric series here:

<https://mathworld.wolfram.com/GeometricSeries.html>

**Question 2: Valuation using cost approach (Based on BF Ch 7 #3) – 20 points**

You have identified a two-year old apartment complex that might be a worthwhile addition to your firm's investment portfolio. Your team has already completed an income-based valuation of the property but would also like to complete a cost-based valuation as an alternative benchmark.

You have collected the relevant square footage information about the complex to complete your calculations. The property has 25 apartments that are each 1,000 square feet. There will also be 4,000 square feet of common space across the whole property. Finally, there is also a 50-spot parking lot next to the property that uses 16,250 square feet.

Your team also has the construction costs per square footage that you need. The apartments themselves would currently cost \$300 per square foot to build. The common areas are a bit less expensive at about \$225 per square foot. The parking lot only costs \$9 per square foot.

Outside of the construction costs, you need to consider a few other factors. The apartment building has special insulation that costs \$850,000 to install and has had landscaping improvements worth \$15,000. The land is appraised at \$2,000,000.

As a part of your calculation, you also assume that the building has experienced \$200,000 of depreciation.

**Q2 Complete the cost approach to valuing the apartment complex given the information above. What is the value implied by this approach?**

**Question 3: Valuation using sales comparison approach (Based on LA Ch 7 #13) – 30 points**

You are working as an analyst with a major single-family housing investor and you have been assigned the task of appraising a particular property in the Mountain Home neighborhood of Asheville, North Carolina. Specifically, your firm would like to appraise the property using a comparable sales approach. You have identified three properties that are very similar to the property that you are planning to purchase based on their financial, physical, and geographic features. The relevant factors that you want to employ to adjust the sales price of the comparables are listed in Table 1 below.

**Table 1: Comparable property adjustment factors**

Elements of comparison	Subject property	Comp 1 Sale 1	Comp 2 Sale 2	Comp 3 Sale 3
Sale price of comparison		\$395,000	\$415,000	\$375,000
<i>Transaction characteristics</i>				
Property Rights conveyed	Fee simple	Same	Same	Same
Financing terms	Conventional	Same	Same	Same
Condition of sale	Arm's Length	Same	Same	Same
Immediate expenditures		\$0	\$4,000	\$0
Added price growth		3%	6%	6%
<i>Location</i>				
Neighborhood	Mountain Home	Mountain Home	Bent Creek	Balfour
<i>Physical characteristics</i>				
Construction quality	Typical	Typical	Typical	Typical
Condition	Average	Average	Average	Average
Site/lot size (sq. ft.)	4,662	4,700	4,450	4,500
Effective age (years)	4.0	8.0	1.0	5.0
Living area (sq. ft.)	2,800	3,200	2,900	3,150
Number of baths	4.0	3.0	4.5	3.0
Garage Spaces (cars)	2	2	2	1
Wood deck	Yes	No	No	No
Pool	No	No	Yes	Yes
<i>Economics characteristics</i>				
Use	Single-family	Single-family	Single-family	Single-family
Nonrealty components	None	None	None	None

Note that:

- The property rights being conveyed in the acquisition of the subject property are fee-simple absolute;
- Conventional mortgage financing will be used by the purchaser; and
- the acquisition appears to be an arm's length transition.

Thus, no adjustments need to be made to the sale prices of the comparable properties for the type of property rights conveyed, financing terms, or conditions of sale. Nonetheless, there are property specific adjustments that must be made for each property:

- For comparable one:
  - Sold three months ago, you expect that it will sell for 3% more now.
  - Located in the same neighborhood as the subject property (Mountain Home)
- For comparable two:
  - Sold six months ago, you expect that it will sell for 6% more now.
  - Located in Bent Creek, which is considered a more desirable location. Homes sell at a 8% price premium relative to Mountain Home.
  - The buyer of comparable two was aware that she would have to remove rotting carpeting immediately after acquiring the property (which she did); thus, she was able to negotiate a \$4,000 price reduction from the seller.
- For comparable three:
  - Sold six months ago, you expect that it will sell for 6% more now.
  - Located in Balfour, which is considered a less desirable location. Homes sell at a 4% price discount relative to Mountain home.

An incremental square foot of lot size or living area is worth about \$25 per square foot or \$85 per square foot respectively. Each year of effective age reduces the value of the properties in all of the markets by \$3,000 per year.

Each additional half-bath is worth \$500, and an additional full bath is worth \$1,000. Additional garage spaces, wood decks, and pools in these three neighborhoods are worth \$8,000, \$1,000, and \$12,000 respectively. No significant non-realty items were included in the comparable transactions.

**Q3 Based on the above information, complete an adjustment grid for the three comparable properties. What is the final adjusted price (indication of the subject's value) for comparable 1, 2, and 3? What is the average implied price from these comparables?**

**Hint – Use Exhibit 7-7 in the Ling & Archer textbook to get started.**

**Question 4: Valuation using the income approach (Based on LA Ch 8 #11) – 30 points**

You need to estimate the market value of a small office building in downtown Houston, Texas. The property has 12,750 square feet of leasable space that was fully leased (no vacancy) for five years to an exciting software company named BBqueue at the start of this year. Terms of the lease call for rent payments of \$9,725 per month for each of the five years. The tenant must also pay all operating expenses as a part of this agreement.

Since the property has been leased for five years to a reliable tenant, there will be no vacancy or collection losses during that period. Once the contract expires, you expect BBqueue to either shrink its operations or expand to a larger office space elsewhere – leading to a vacancy period which is typically five months in the office district. When you find new shorter-term tenants, you also expect a vacancy loss allowance of 6% per year. You’ve had trouble completely filling office space in the past.

The current market rental for this type of property is \$10.50 per square foot when tenants are paying their own operating expenses, and this rent is expected to grow 4 percent per year in the future (*remember that for the first five years, you are committed to the rental agreement*). For the property that you are considering, use a market discount rate of 12 percent. For cap rates, use 1) a “going in” cap rate of 8.5 percent, and 2) a terminal cap rate of 9.75 percent.

**Q4 Prepare a multi-year pro forma using Excel showing the annual rental income, expense reimbursements, NOIs, and the net proceeds from the sale of the property at the end of a 9-year holding period. Then use the information provided to estimate the market value of the property. Make sure your solution includes a discounted cash flow analysis.**