# Discounted Cash Flow Valuation III AEM 2241 - Finance

Tibor Jánosi

Cornell University - Fall 2023



Loan Types Fixed, 25%, 5yrs Fixed, 10%, 5yrs Fixed, 10%, 30yrs Var, 25%, 5yrs Var, 10%, 5yrs Var, 10%, 30yrs OOO OOO

### Outline

#### Loan Types

Fixed-Payment Amortized Loans Variable-Payment Amortized Loans Bullet Loans Amortization Tables

Fixed Payments, Short Maturity, High Interest

Fixed Payments, Short Maturity, Lower Interest

Fixed Payments, Long Maturity, Lower Interest

Variable Payments, Short Maturity, High Interest

Variable Payments, Short Maturity, Lower Interest

Variable Payments, Long Maturity, Lower Interest

### Fixed-Payment Amortized Loans

- A fixed-payment annuity loan can be interpreted as being just a regular annuity. The yearly (more generally: per-period) payment will then be  $C = \frac{r}{1 - (\frac{1}{1+r})^t} PV$ .
- While payments are fixed in magnitude their structure is variable, as they consist of variable "mixtures" of interest and (repaid) principal. This is illustrated by analyzing the loan's amortization table. We discussed this matter in class.
- This the structure of a row in the loan's amortization table:

Year	Beg. Balance	Payment	Interest	Principal	End Balance
• • •	• • •	• • •	• • •	• • •	• • •
k	ВВ	С	BB · r	$C - BB \cdot r$	$BB - (C - BB \cdot r) = (1 + r)BB - C$
•••	• • •			• • •	• • •



Loan Types

Fixed, 25%, 5yrs

Fixed, 10%, 5yrs Fixed, 10%, 30yrs

Var, 25%, 5yrs Var, 10%, 5yrs Var, 10%, 30yrs

### Variable-Payment Amortized Loans

- Loan payments do not have to be constant. An important type of loan arises when at the end of each year (period) we pay back a fraction of  $\frac{1}{t}$  of the principal, i.e. an amount of  $\frac{PV}{t}$ .
- In this case interest payments are variable, and each chunk of repaid principal is constant. This makes for a variable total yearly payment.
- This the structure of a row in the loan's amortization table:

Year	Beg. Balance	Payment	Interest	Principal	End Balance
• • •	• • •	• • •			• • •
k	ВВ	$BB \cdot r + \frac{PV}{t}$	BB · r	PV t	$BB - \frac{PV}{t}$
• • •	• • •				



#### **Bullet Loans**

- A bullet loan is set up as if it were amortized over a longer period of time (say, 15 years), but it matures (ends) after a shorter period (say, 5 years).
- Both fixed-payment and variable-payment loans can be structured as bullet loans.
- The amortization table of a bullet loan is the amortization. table of the underlying loan (say, 15 years), but "cut off" at the maturity of the bullet loan (say, 5 years).
- The principal of a bullet loan is not already repaid by the time the loan matures, so the borrower will have a (still-outstanding) principal balance. This is either paid in full in a lump sum, or - more commonly - refinanced using another bullet loan.



Loan Types

Fixed, 25%, 5yrs Fixed, 10%, 5yrs Fixed, 10%, 30yrs Var, 25%, 5yrs Var, 10%, 5yrs Var, 10%, 30yrs

#### Amortization Tables

- For a regular (i.e., non-bullet) loan, the ending balance of the principal should be 0 (if we ignore rounding errors).
- It is possible to write formulas that directly provide the values in a loan's amortization table. For example, consider the beginning balance of a variable-payment loan with principal PV, interest rate r, and maturity t, in year k:  $BB = \frac{t-k+1}{t}PV$  ( $1 \le k \le t$ ). Can you explain why this formula is correct? Formulas for fixed-payment loans are more complicated.
- The terms of loans can be modified to suit particular clients and situations. It is possible to accelerate or delay the repayment of the principal outstanding, either by design, or on a contingent basis (if certain events occur). An amortization table derived from the terms of the loan can then be constructed.



### Table: Fixed Payments, Short Maturity, High Interest

r 25.00% C 37,184.67 t 5 Total int: 85,923.37 PV 100,000.00

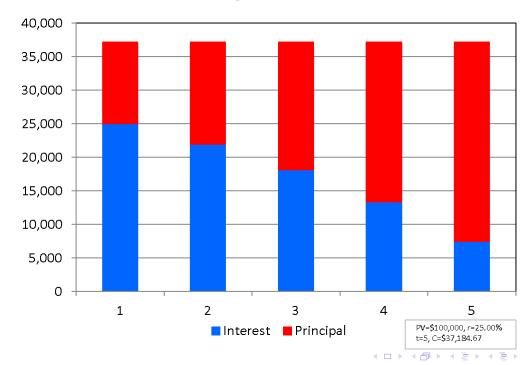
Period	Beg. Balance	Payment	Interest	Principal	End Balance	Cum. Interest	Cum. Principal
0					100,000.00		
1	100,000.00	37,184.67	25,000.00	12,184.67	87,815.33	25,000.00	12,184.67
2	87,815.33	37,184.67	21,953.83	15,230.84	72,584.48	46,953.83	27,415.52
3	72,584.48	37,184.67	18,146.12	19,038.55	53,545.93	65,099.95	46,454.07
4	53,545.93	37,184.67	13,386.48	23,798.19	29,747.74	78,486.44	70,252.26
5	29,747.74	37,184.67	7,436.93	29,747.74	(0.00)	85,923.37	100,000.00

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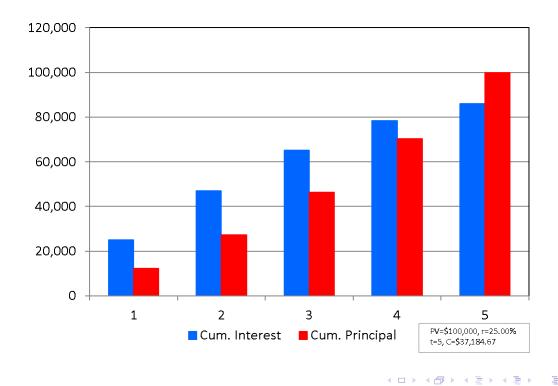
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Loan Types Fixed, 25%, 5yrs OoO Fixed, 10%, 5yrs Fixed, 10%, 30yrs Var, 25%, 5yrs Var, 10%, 5yrs Var, 10%, 30yrs OOO OOO

# Interest vs. Principal: Fixed Payments, Short Maturity, High Interest



## Cumulative: Fixed Payments, Short Maturity, High Interest



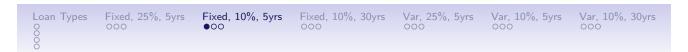


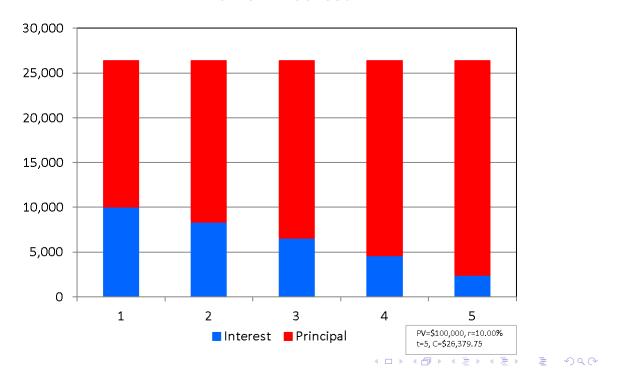
Table: Fixed Payments, Short Maturity, Lower Interest

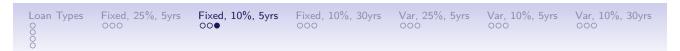
r	10.00%	С	26,379.75
t	5	Total int.:	31,898.74
PV	100,000.00		

Period	Beg. Balance	Payment	Interest	Principal	End Balance	Cum. Interest	Cum. Principal
0					100,000.00		
1	100,000.00	26,379.75	10,000.00	16,379.75	83,620.25	10,000.00	16,379.75
2	83,620.25	26,379.75	8,362.03	18,017.72	65,602.53	18,362.03	34,397.47
3	65,602.53	26,379.75	6,560.25	19,819.50	45,783.03	24,922.28	54,216.97
4	45,783.03	26,379.75	4,578.30	21,801.44	23,981.59	29,500.58	76,018.41
5	23,981.59	26,379.75	2,398.16	23,981.59	-	31,898.74	100,000.00

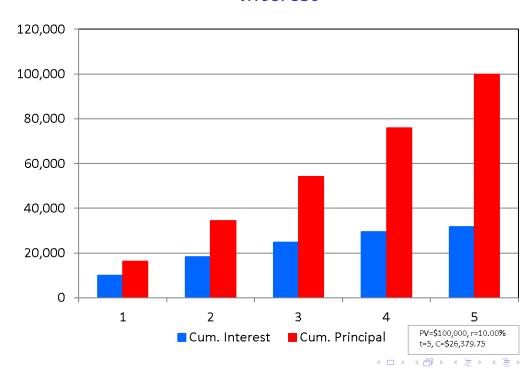
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### Interest vs. Principal: Fixed Payments, Short Maturity, Lower Interest





# Cumulative: Fixed Payments, Short Maturity, Lower Interest



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# Table: Fixed Payments, Long Maturity, Lower Interest

r t PV	10.00% 30 100,000.00		C Total interest:	10,607.92 218,237.74			
Period	Beg. Balance	Payment	Interest	Principal	End Balance	Cum. Interest	Cum. Principal
0					100,000.00		
1	100,000.00	10,607.92	10,000.00	607.92	99,392.08	10,000.00	607.92
2	99,392.08	10,607.92	9,939.21	668.72	98,723.36	19,939.21	1,276.64
3	98,723.36	10,607.92	9,872.34	735.59	97,987.77	29,811.54	2,012.23
4	97,987.77	10,607.92	9,798.78	809.15	97,178.62	39,610.32	2,821.38
5	97,178.62	10,607.92	9,717.86	890.06	96,288.56	49,328.18	3,711.44
- 6	96,288.56	10,607.92	9,628.86	979.07	95,309.49	58,957.04	4,690.51
7	95,309.49	10,607.92	9,530.95	1,076.98	94,232.51	68,487.99	5,767.49
8	94,232.51	10,607.92	9,423.25	1,184.67	93,047.84	77,911.24	6,952.16
9	93,047.84	10,607.92	9,304.78	1,303.14	91,744.70	87,216.02	8,255.30
10	91,744.70	10,607.92	9,174.47	1,433.45	90,311.24	96,390.49	9,688.76
11	90,311.24	10,607.92	9,031.12	1,576.80	88,734.44	105,421.62	11,265.56
12	88,734.44	10,607.92	8,873.44	1,734.48	86,999.96	114,295.06	13,000.04
13	86,999.96	10,607.92	8,700.00	1,907.93	85,092.03	122,995.06	14,907.97
14	85,092.03	10,607.92	8,509.20	2,098.72	82,993.31	131,504.26	17,006.69
15	82,993.31	10,607.92	8,299.33	2,308.59	80,684.72	139,803.59	19,315.28
16	80,684.72	10,607.92	8,068.47	2,539.45	78,145.27	147,872.06	21,854.73
17	78,145.27	10,607.92	7,814.53	2,793.40	75,351.87	155,686.59	24,648.13
18	75,351.87	10,607.92	7,535.19	3,072.74	72,279.13	163,221.78	27,720.87
19	72,279.13	10,607.92	7,227.91	3,380.01	68,899.12	170,449.69	31,100.88
20	68,899.12	10,607.92	6,889.91	3,718.01	65,181.11	177,339.60	34,818.89
21	65,181.11	10,607.92	6,518.11	4,089.81	61,091.29	183,857.71	38,908.71
22	61,091.29	10,607.92	6,109.13	4,498.80	56,592.50	189,966.84	43,407.50
23	56,592.50	10,607.92	5,659.25	4,948.68	51,643.82	195,626.09	48,356.18
24	51,643.82	10,607.92	5,164.38	5,443.54	46,200.28	200,790.47	53,799.72
25	46,200.28	10,607.92	4,620.03	5,987.90	40,212.38	205,410.50	59,787.62
26	40,212.38	10,607.92	4,021.24	6,586.69	33,625.69	209,431.74	66,374.31
27	33,625.69	10,607.92	3,362.57	7,245.36	26,380.34	212,794.31	73,619.66
28	26,380.34	10,607.92	2,638.03	7,969.89	18,410.45	215,432.34	81,589.55
29	18,410.45	10,607.92	1,841.04	8,766.88	9,643.57	217,273.39	90,356.43
30	9,643.57	10,607.92	964.36	9,643.57	0.00	218,237.74	100,000.00

Loan Types

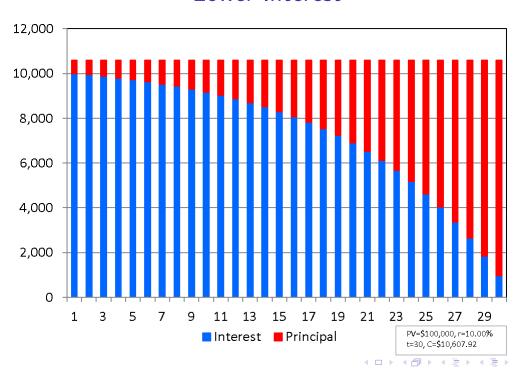
Fixed, 25%, 5yrs Fixed, 10%, 5yrs Fixed, 10%, 30yrs

Var, 25%, 5yrs Var, 10%, 5yrs

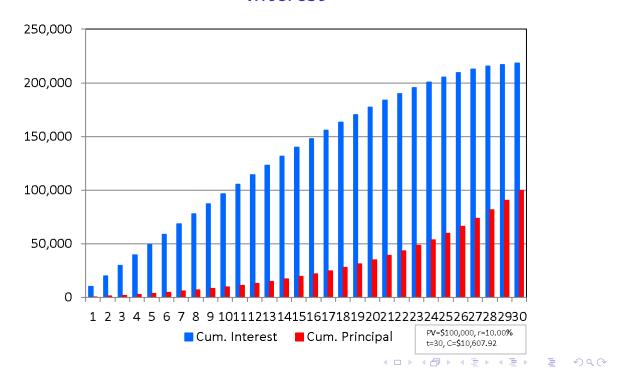
Var, 10%, 30yrs

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## Interest vs. Principal: Fixed Payments, Long Maturity, Lower Interest



# Cumulative: Fixed Payments, Long Maturity, Lower Interest



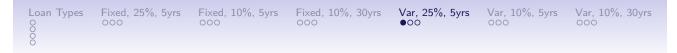
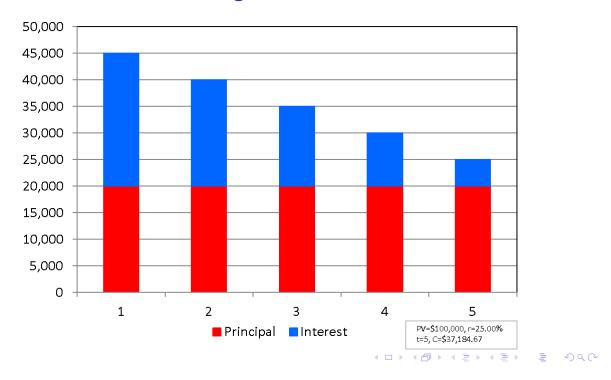


Table: Variable Payments, Short Maturity, High Interest

r	25.00%	С	37,184.67
t	5	Total interest:	75,000.00
PV	100,000.00	Principal pymt	20,000.00

Period	Beg. Balance	Payment	Interest	Principal	End Balance	Cum. Interest	Cum. Principal
0					100,000.00		
1	100,000.00	45,000.00	25,000.00	20,000.00	80,000.00	25,000.00	20,000.00
2	80,000.00	40,000.00	20,000.00	20,000.00	60,000.00	45,000.00	40,000.00
3	60,000.00	35,000.00	15,000.00	20,000.00	40,000.00	60,000.00	60,000.00
4	40,000.00	30,000.00	10,000.00	20,000.00	20,000.00	70,000.00	80,000.00
5	20,000.00	25,000.00	5,000.00	20,000.00	-	75,000.00	100,000.00

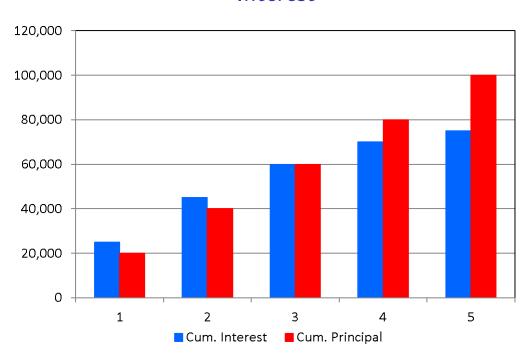
# Interest vs. Principal: Variable Payments, Short Maturity, High Interest



 Loan Types
 Fixed, 25%, 5yrs
 Fixed, 10%, 5yrs
 Fixed, 10%, 30yrs
 Var, 25%, 5yrs
 Var, 10%, 5yrs
 Var, 10%, 30yrs

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# Cumulative: Variable Payments, Short Maturity, High Interest





### Table: Variable Payments, Short Maturity, Lower Interest

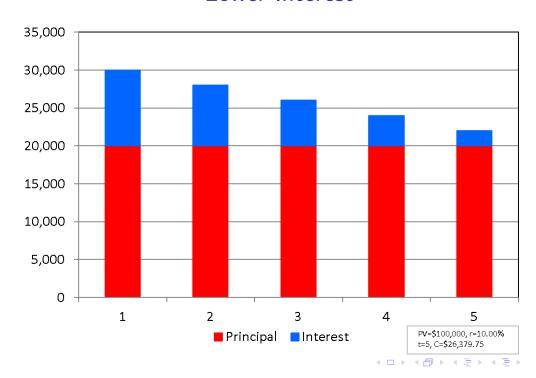
r	10.00%	С	26,379.75
t	5	Total interest:	30,000.00
PV	100,000.00	Principal pymt	20,000.00

Beg. Balance	Payment	Interest	Principal	End Balance	Cum. Interest	Cum. Principal
				100,000.00		
100,000.00	30,000.00	10,000.00	20,000.00	80,000.00	10,000.00	20,000.00
80,000.00	28,000.00	8,000.00	20,000.00	60,000.00	18,000.00	40,000.00
60,000.00	26,000.00	6,000.00	20,000.00	40,000.00	24,000.00	60,000.00
40,000.00	24,000.00	4,000.00	20,000.00	20,000.00	28,000.00	80,000.00
20,000.00	22,000.00	2,000.00	20,000.00	-	30,000.00	100,000.00
	100,000.00 80,000.00 60,000.00 40,000.00	100,000.00 30,000.00 80,000.00 28,000.00 60,000.00 26,000.00 40,000.00 24,000.00	100,000.00 30,000.00 10,000.00 80,000.00 28,000.00 8,000.00 60,000.00 26,000.00 6,000.00 40,000.00 24,000.00 4,000.00	100,000.00 30,000.00 10,000.00 20,000.00 80,000.00 28,000.00 8,000.00 20,000.00 60,000.00 26,000.00 6,000.00 20,000.00 40,000.00 24,000.00 4,000.00 20,000.00	100,000.00           100,000.00         30,000.00         10,000.00         20,000.00         80,000.00           80,000.00         28,000.00         8,000.00         20,000.00         60,000.00           60,000.00         26,000.00         6,000.00         20,000.00         40,000.00           40,000.00         24,000.00         4,000.00         20,000.00         20,000.00	100,000.00       100,000.00     30,000.00     10,000.00     20,000.00     80,000.00     10,000.00       80,000.00     28,000.00     8,000.00     20,000.00     60,000.00     18,000.00       60,000.00     26,000.00     6,000.00     20,000.00     40,000.00     24,000.00       40,000.00     24,000.00     4,000.00     20,000.00     20,000.00     28,000.00

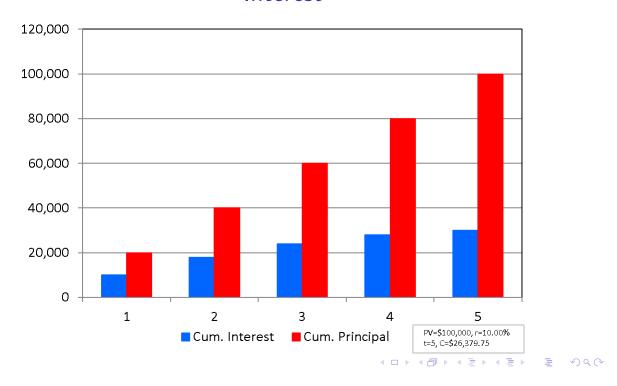
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Loan Types Fixed, 25%, 5yrs Fixed, 10%, 5yrs Fixed, 10%, 30yrs Var, 25%, 5yrs Var, 10%, 5yrs OOO OOO OOO

## Interest vs. Principal: Variable Payments, Short Maturity, Lower Interest



# Cumulative: Variable Payments, Short Maturity, Lower Interest

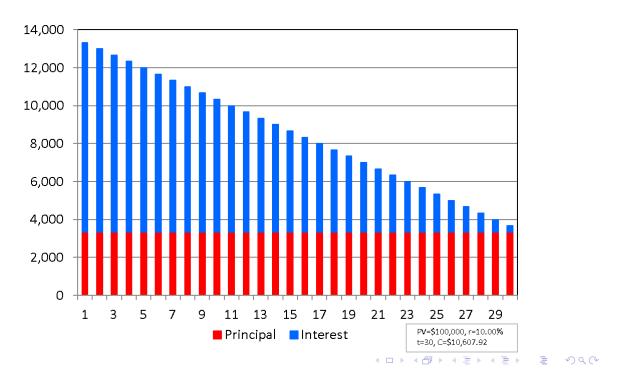


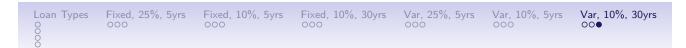
Loan Types Fixed, 25%, 5yrs OOO Fixed, 10%, 5yrs Fixed, 10%, 30yrs Var, 25%, 5yrs OOO Var, 10%, 30yrs OOO Var, 10%, 30yrs

# Table: Variable Payments, Long Maturity, Lower Interest

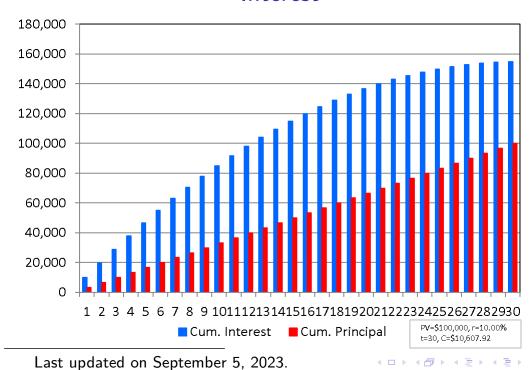
r	10.00%		С	10,607.92			
t	30		Total interest:	155,000.00			
PV	100,000.00		Principal pymt	3,333.33			
Period	Beg. Balance	Payment	Interest	Principal	End Balance	Cum. Interest	Cum. Principal
0					100,000.00		
1	100,000.00	13,333.33	10,000.00	3,333.33	96,666.67	10,000.00	3,333.33
2	96,666.67	13,000.00	9,666.67	3,333.33	93,333.33	19,666.67	6,666.67
3	93,333.33	12,666.67	9,333.33	3,333.33	90,000.00	29,000.00	10,000.00
4	90,000.00	12,333.33	9,000.00	3,333.33	86,666.67	38,000.00	13,333.33
5	86,666.67	12,000.00	8,666.67	3,333.33	83,333.33	46,666.67	16,666.67
- 6	83,333.33	11,666.67	8,333.33	3,333.33	80,000.00	55,000.00	20,000.00
7	80,000.00	11,333.33	8,000.00	3,333.33	76,666.67	63,000.00	23,333.33
- 8	76,666.67	11,000.00	7,666.67	3,333.33	73,333.33	70,666.67	26,666.67
9	73,333.33	10,666.67	7,333.33	3,333.33	70,000.00	78,000.00	30,000.00
10	70,000.00	10,333.33	7,000.00	3,333.33	66,666.67	85,000.00	33,333.33
11	66,666.67	10,000.00	6,666.67	3,333.33	63,333.33	91,666.67	36,666.67
12	63,333.33	9,666.67	6,333.33	3,333.33	60,000.00	98,000.00	40,000.00
13	60,000.00	9,333.33	6,000.00	3,333.33	56,666.67	104,000.00	43,333.33
14	56,666.67	9,000.00	5,666.67	3,333.33	53,333.33	109,666.67	46,666.67
15	53,333.33	8,666.67	5,333.33	3,333.33	50,000.00	115,000.00	50,000.00
16	50,000.00	8,333.33	5,000.00	3,333.33	46,666.67	120,000.00	53,333.33
17	46,666.67	8,000.00	4,666.67	3,333.33	43,333.33	124,666.67	56,666.67
18	43,333.33	7,666.67	4,333.33	3,333.33	40,000.00	129,000.00	60,000.00
19	40,000.00	7,333.33	4,000.00	3,333.33	36,666.67	133,000.00	63,333.33
20	36,666.67	7,000.00	3,666.67	3,333.33	33,333.33	136,666.67	66,666.67
21	33,333.33	6,666.67	3,333.33	3,333.33	30,000.00	140,000.00	70,000.00
22	30,000.00	6,333.33	3,000.00	3,333.33	26,666.67	143,000.00	73,333.33
23	26,666.67	6,000.00	2,666.67	3,333.33	23,333.33	145,666.67	76,666.67
24	23,333.33	5,666.67	2,333.33	3,333.33	20,000.00	148,000.00	80,000.00
25	20,000.00	5,333.33	2,000.00	3,333.33	16,666.67	150,000.00	83,333.33
26	16,666.67	5,000.00	1,666.67	3,333.33	13,333.33	151,666.67	86,666.67
27	13,333.33	4,666.67	1,333.33	3,333.33	10,000.00	153,000.00	90,000.00
28	10,000.00	4,333.33	1,000.00	3,333.33	6,666.67	154,000.00	93,333.33
29	6,666.67	4,000.00	666.67	3,333.33	3,333.33	154,666.67	96,666.67
30	3,333.33	3,666.67	333.33	3,333.33	0.00	155,000.00	100,000.00

### Interest vs. Principal: Variable Payments, Long Maturity, Lower Interest





# Cumulative: Variable Payments, Long Maturity, Lower Interest



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