Dyson
Cornell
SC Johnson College of Business

Homework 3
Solutions

The solutions below are commented extensively so that you can get insights into how to reason when solving similar problems. Even if you had to present full solutions on paper you would not be expected to provide anything remotely similar in order to get full score. Very concise, correct solutions would be perfectly acceptable.

1 Problem 1

This was not really a "problem," just an acknowledgment that one will abide by Cornell's rules of academic integrity.

2 Problem 2

The annual dividend yield is computed by dividing _____ annual dividend by the current stock price.

Answer:

In our problems we often simplify things so that we can focus on the important issues. In the context of dividend payments, we always assumed that we are at the very beginning of a dividend-payment period, and the that the dividend due for the "current period" has already been paid. These considerations justify the choice of "next year's" for the answer to this question.

3 Problem 3

Book Match just paid an annual dividend of \$1.50 per share. The company will increase its dividend by 7 percent next year and will then reduce its dividend growth rate by 2 percentage points per year until it reaches the industry average of 3 percent dividend growth, after which the company will keep a constant growth rate forever. What is the price of this stock today given a required return of 14 percent?

Answer:

Let $D_0 = \$1.50$ be the "just paid" dividend. At the end of year 1 (next year), the dividend paid will be $D_1 = D_0 \cdot (1 + 0.07) = \$1.605 \approx \% = \$1.61$; at the end of year 2 the dividend paid will be $D_2 = D_1 \cdot (1 + 0.05) = \$1.6905 \approx \$1.69$; at the end of year 3, the dividend paid will be $D_3 = D_2 \cdot (1 + 0.03) = \$1.7407 \approx \$1.74$.

At time t=2, looking toward future dividends, we see a stream of dividends growing at a constant rate, and we get $p_2 = \frac{D_3}{r-g} = \frac{1.74}{0.14-0.03} = \$15.81816 \approx \$15.82$. The dividends at time t=1 and time t=2 do not form a pattern of payments that we studied; hence, we use the formulas for idiosyncratic (irregular, non-constant) dividend payments to get:

$$\begin{aligned}
p_0 &= \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \frac{p_2}{(1+r)^2} \\
&= \frac{1.61}{1.14} + \frac{1.69}{1.14^2} + \frac{15.82}{1.14^2} \\
&= 1.41228 + 1.30040 + 12.17298 \\
&= \$14.88566 \\
&\approx \$14.89.
\end{aligned}$$

Our result is closest to \$14.85 among all the suggested answers. We will choose this one, as the difference is only \$0.04, and such a divergence may easily result when one uses approximate calculations.

4 Problem 4

McKerley Corporation has preferred stock outstanding that will pay an annual dividend of \$4.60 per share with the first dividend exactly 14 years from today. If the required return is 3.78 percent, what is the current price of the stock?

Answer:

The simplest solution is to compute the (estimated) price of the stock at time t=13 years. "Looking out" into the future, we see an infinite sequence of constant (annual) dividends. We can immediately compute the the stock price at time t=13: $p_{13} = \frac{4.60}{0.0378} = \$121.69312$. We now discount back this amount back to time 0 to get $p_0 = \frac{p_{13}}{(1+0.0378)^{13}} = \frac{121.69312}{1.0378^{13}} = \$75.12567 \approx$ \$75.13, which is the answer that we choose.

A question that came up in discussions of this problem is whether we must discount the first dividend that will be paid over 14 time periods or only over 13 time periods. Without ambiguity, as long as we compute a present value at time 0 and the cash flow at time 14 is part of that present value, the respective cash must be discounted over 14 time periods. Now, this discounting can happen in one step or in several steps. In the solution that we chose, we first discount the dividend at time 14 over one time period when we compute p_{13}. This may not be obvious at first sight, because you are using the formula for an ordinary perpetuity to get p_{13};however, if you examine the formulas that give the NPV of ordinary perpetuities, you will find that they aggregate the discounted values of their component cash flows (coupons). After the one-period discounting that is implied by the formula used to compute p_{13}, we discount p_{13}, which includes the payment at t=14, over 13 more periods. In the end, the dividend at time 14 **is discounted over 14 time periods**.

5 Problem 5

The crossover rate for two projects is that discount rate which makes the NPV of the two projects to be equal. Read your textbook to understand how to compute the crossover rate, then solve the following problem:

You are evaluating two projects with the following cash flows:

Year	Project X	Project Y
0	-547,200	-516,500
1	218,600	208,300
2	228,500	218,100
3	235,700	226,000
4	195,400	186,800

What is the crossover rate for these two projects?

Note: Make sure you remember how to compute the crossover rate, as this question may come up on later exams.

Answer:

In class we discussed several times charts that had discount rates on the horizontal axis and NPVs that corresponded to these discount rates on the vertical axes. We explained that one can build a (continuous) NPV curve for a project by, for example, evaluating the projects NPV for various discount rates within a chosen range (e.g., for discount rates between 0$ and, say, 30% or 40%), an then uniting these points to get a curve.

Of course, we can build such NPV curves for 2 (or more) projects, and we can plot these curves on the same chart. A crossover will occur when the NPV curves corresponding to the two projects intersect. The discount rate that corresponds to the intersection point is the crossover rate.

Let r be the cross-over rate and NPV_X and NPV_Y the NPVs of Project X and Project Y, respectively. We have:

$$
\begin{aligned}
NPV_X &= -547,200 + \frac{218,600}{1+r} + \frac{228,500}{(1+r)^2} + \frac{235,700}{(1+r)^3} + \frac{195,400}{(1+r)^4} \\
NPV_Y &= -516,500 + \frac{208,300}{1+r} + \frac{218,100}{(1+r)^2} + \frac{226,000}{(1+r)^3} + \frac{186,800}{(1+r)^4}
\end{aligned}
$$

At the cross-over rate, the two curves intersect because $NPV_X = NPV_Y$. We move the terms from the RHS to the LHS and we group the corresponding terms:

$$
\begin{aligned}
&-547,200 - (-516,500) \\
&+\frac{218,600 - 208,300}{1+r} \\
&+\frac{228,500 - 218,100}{(1+r)^2} \\
&+\frac{235,700 - 226,000}{(1+r)^3} \\
&+\frac{195,400 - 186,800}{(1+r)^4} \quad = \quad 0
\end{aligned}
$$

In effect, we take the **difference** of the cash flows of the two projects and we compute the present value of these differences. This present value must equal 0. We get:

$$
-30,700 + \frac{10,300}{1+r} + \frac{10,400}{(1+r)^2} + \frac{9,700}{(1+r)^3} + \frac{8,600}{(1+r)^4} = 0
$$

What rate makes the NPV of the cash flow differences equal to 0? The IRR!

So all we need to do is to take the cash flow differences, enter them into our financial calculator's Cash Flow Worksheet, and then compute the IRR of these cash flow differences. We do that and we get $r = IRR = 10.65026\% \approx 10.65\%$, which is the answer that we choose.

6 Problem 6

Blinding Light Company has a project available with the following cash flows:

Year	Cash Flow
0	-34,110
1	8,150
2	9,810
3	13,980
4	15,850
5	10,700

What is the project's IRR?

Answer:

This problem is very straightforward - we set up the cash flows using the Cash Flow Worksheet of our financial calculators. We then hit IRR, followed by CPT. The financial calculator reports an IRR of 19.42%. This is a conventional cash flow series; there is only one sign change, so we can be confident that the calculator has found the true, unique IRR. Obviously, we choose the answer "19.42%."

7 Problem 7

Consider the series of cash flows in the prior problem. Can you find the discount rate that makes the NPV of the cash flows equal of $7,500?

Hint 1: Consider the formula that you would write to compute the NPV of the cash flows as given, let this formula be FNPV; write this formula using r for the unknown rate. Write the equality FNPV = $7,500, then "move" the $7,500 to the LHS of the equation to get FNPV - 7,500 = 0. Combine the $7,500 term with the term from FNPV to which is is closest to in timing. Use the resulting equation to determine r.

Hint 2: You should follow hint 1 to understand the logic of the solution, but once you understand what you need to do, you can use the calculator to solve the problem. Use at least 5 decimals precision in all your calculations and enter the answer below in percentages (i.e. 5 for five percent, not as 5%).

Answer:

Let us follow Hint 1 from above: given a discount rate r, the NPV of the cash flows from Problem 6 is given by the expression

$$FNPV = -34,110 + \frac{8,150}{1+r} + \frac{9,810}{(1+r)^2} + \frac{13,980}{(1+r)^3} + \frac{15,850}{(1+r)^4} + \frac{10,700}{(1+r)^5}$$

As advised, we set $FNPV$ equal to the value that we want the NPV to take:

$$-34,110 + \frac{8,150}{1+r} + \frac{9,810}{(1+r)^2} + \frac{13,980}{(1+r)^3} + \frac{15,850}{(1+r)^4} + \frac{10,700}{(1+r)^5} = 7,500$$

and then we move the quantity on the right-hand side to the LHS to get

$$-34,110 - 7,500 + \frac{8,150}{1+r} + \frac{9,810}{(1+r)^2} + \frac{13,980}{(1+r)^3} + \frac{15,850}{(1+r)^4} + \frac{10,700}{(1+r)^5} = 0$$

Now, as we ponder this formula, we note that we can combine the cash flow at time 0 (-34,110) with the NPV value that we are targeting (-7,500; we changed the sign when we moved this amount from the RHS to the LHS). The NPV target can be (re)interpreted as an additional cash outflow that occurs at time 0. We rewrite the formula accordingly:

$$-41,610 + \frac{8,150}{1+r} + \frac{9,810}{(1+r)^2} + \frac{13,980}{(1+r)^3} + \frac{15,850}{(1+r)^4} + \frac{10,700}{(1+r)^5} = 0$$

The formula above represents a **modified** series of cash flows whose NPV forms the LHS of the equation. But this NPV, which is a function of the unknown discount rate r, is set to be equal to 0. This means that r is really the IRR of this modified series of cash flows!

So now we set up the modified series of cash flows using our calculator's Cash Flow Worksheet functionality. All we need to change is CF0, really, the rest of the cash flow amounts are the same as stated in the problem. After we set up the **modified** series of cash flows, we compute its IRR - we get $IRR = 11.62392\% \approx 11.62\%$.

How can we check the correctness of the solution? Well, we change CF0 back to the original -\$34,110, and then we compute the NPV of the cash flows using the rate $r = 11.62\%$. When we do this, we get that $NPV = 7,504.39$, which is very close to our target value. Indeed, if we used a higher-precision value for r, e.g., $r = 11.62392\%$, we would get $NPV = \$7,500.00109$, which is within about 0.1 cents from the target value.

We conclude by pointing out that this problem teaches you how to determine a discount rate that makes a given series of cash flows have a given NPV. All you need to do is to change the time-0 cash flow of your cash flow series by analogy to the discussion above, and the compute the IRR of the modified series of cash flows. This IRR is the discount rate you are looking for. While we wrote down and rewrote some formulas several times in the above, these formulas we only used to develop the reasoning, they are not part of the actual solution, as long as you set up your problem on your financial calculator.

8 Problem 8

Rossdale Flowers has a new greenhouse project with an initial cost of $275,000 that is expected to generate cash flows of $41,900 for 7 years and a cash flow of $57,300 in Year 8. If the required return is 7.5 percent, what is the project's NPV?

Answer:

We set up the series of cash flows on our calculator using the procedure demonstrated in class, remembering to explicitly set the cash flow multiplicities when they are greater than 1. So we set CF0=-275,000 (this is a "cost," which produces a negative cash flow); CF1=41,900; F01=7; CF2=57,300. F02 will be implicitly set to 1, so we do not need to change it. We then press the NPV button, set I=7.5 (remember, the financial calculator always expresses rates in percents, not as decimals), then hit "arrow down" to get the NPV field. We finish by hitting the CPT button to get that NPV=-$20,944.17, which is the answer that we must choose.

9 Problem 9

A project has a cash flow of -$50,000 at time 0, a cash flow of $10,000 at time 1 year, and a cash flow of $15,000 at time 2 years. Assuming ordinary economic conditions, what is the IRR of these cash flows?

Answer:

It is quite straightforward to set up the cash flows on the calculator, and to compute their IRR; just follow the procedure indicated in class. When we do so, the financial calculator reports an IRR of -34.32%. This is a conventional series of cash flows (one negative cash flow followed by positive cash flows); there is one sign change, so we expect one IRR, which our calculator found. The problem, of course, is that the IRR is both large and negative. Under ordinary economic conditions (large) negative discount rates are not acceptable (realistic).

Given the above, we choose the answer "the cash flows do not have an IRR under normal economic conditions."

10 Problem 10

Your company has a project available with the following cash flows:

Year	Cash Flow
0	−81,200
1	21,450
2	24,900
3	30,700
4	25,950
5	19,700

If the required return is 14 percent, should the project be accepted based on the IRR?

Answer:

This problem has a straightforward solution. Set up the cash flows on your calculator following the procedure indicated in class, then compute the IRR of the cash flows. The financial calculator reports that the IRR is 15.57%. We also note that we deal with a conventional series of cash flows - there is only one cash flow sign change, so we expect one IRR, whose value we already know.

The IRR rule discussed in class states that for conventional cash flows, we should accept the project if the required return is **below** the IRR. This is the case here, so the project should be accepted.

11 Problem 11

You own a house that you rent for $1,600 per month. The maintenance expenses on the house average $300 per month. The house cost $239,000 when you purchased it 4 years ago. A recent appraisal on the house valued it at $261,000. If you sell the house you will incur $20,880 in real estate fees. The annual property taxes are $3,500. You are deciding whether to sell the house or convert it for your own use as a professional office. What value should you place on this house when analyzing the option of using it as a professional office?

Answer:

This is an example of an opportunity cost. IF we use the house as our office, we give up the opportunity to sell the house.

We must determine how much money we could get if we sell the house. We assume that we'll sell the house for (about) its appraised value of $\$261,000$; however this is not the amount we get in hand - we must subtract the real estate fees that we will pay. So the net comes out at $\$261,000 - \$20,880 = \$240,120.00$. This is the answer that we choose.

As almost always, this problem is a simplification. In real life, we may have to wait for a while until we can sell the house, and we would have to cover the house's maintenance expenses during this time. There may be income tax implications associated with the same. We may have to (try to) break the lease, and the timeline of this may be different, and may involve different costs depending on whether we'll sell the house or convert it to our office. All these complexities are beyond our scope.

12 Problem 12

Consider a project that generates cash flows of $1,000,000.00, -$3,443,000.00, $3,949,800.00, and -$1,509,788.70 that occur at times 0, 1, 2, and 3 (years), respectively.

How many IRRs does this series of cash flows have?

Answer:

Please refer to the workbook "20231024 HW3.xlsx," worksheet "12, 13, 14."

The rule of thumb given in class is that a sequence of cash flows MAY have as many IRRs as many sign change there are within the respective series. We also pointed out that "under ordinary economic conditions" some purported IRRs may not be acceptable, despite these being solutions of the equation NPV=0. For example, negative interest rates (discount rates) are not ordinarily acceptable. Also, very large interest rates do not arise under ordinary economic conditions.

Depending on your background you could attempt to find the IRRs of these cash flows using several methods. The method that we touched upon in class is to take a "reasonable" range of discount rates, compute the NPVs that correspond to these discount rates, plot the points that you get on a chart that has the discount rates on the horizontal axis and the NPVs on the vertical axis, create a curve by uniting these points, and then find the (approximate) places where the curve crosses the horizontal axis (where NPV is 0).

Once you set up your cash flows, you can easily compute NPVs associated with these cash flows using your financial calculator. Still, if you were to examine a range from, say, 0% to 50% in increments of 1% this could become tedious. It is probably best to use a tool like Excel, that allows you to easily generate tables of values. We did just that in the workbook we provided, with the observation that we purposefully did not use any of Excel's more specialized functions - we implemented the NPV calculation exactly as we would have written it on the whiteboard. We challenge you to solve the problem using Excel's specialized NPV function.

We did not create a plot of NPVs as a function of discount rates either, as we can, in fact, solve the problem by just examining the raw data. Still, creating a plot allows you to more intuitively identify the IRRs - we again challenge you do it yourselves.

You will note that we show NPV values using 6 decimals of precision. As we examine the table, we note that discount rates of 11% and 19% seem to generate NPV values equal to 0 (with six decimals precision, at least!). These two values are (very good approximations) of IRRs for this series of cash flows. Could there be only two cash flows? No! As you examine the table, you will note that NPV for 14% is about $3 and the NPV for 15% is about -$7. Arguably, both these NPV values could be considered as being approximately 0, especially when compared the to size of the cash flows themselves, which are in the millions. The fact that the NPV changes signs between a discount rate of 14% and 15%, respectively, indicates that the third IRR will be found in this interval.

We create an additional table, where we show values for the discount rate only within the range [14%, 15%]. WE use steps of 10 basis points (0.1%), as the problem suggested that we provide the answer with this level of precision. This additional table indicates that a discount rate of 14.3% leads to an NPV equal to 0; this must be our third IRR!

So the given series of cash flows has 3 IRRs: 11.0%, 14.3%, and 19.0%.

13 Problem 13

Based on the discussion above, the highest IRR is 19.0%

14 Problem 14

Based on the discussion above, the second-highest IRR is 14.3%.