**Networks: Fall 2023** **Homework 1 Solutions**
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**(1)**
**(a)** Degree 1 for nodes A, B, C, E, F, G, H, and I. Degree 4 for D. Degree 6 for J.
**(b)** 1 for nodes A, B, C, E, F, G, H, and I. 4 for D. 6 for J.
**(c)** D and J.
**(d)** A highest degree node need not be centrally located. A simple example is to add a node K between nodes D and J with D and J linking to K but not each other. Node J still has the highest degree. But node K is centrally located, not node J. All nodes can be reached in at most 2 hops from K but it takes 3 hops to reach nodes A, B or C from J.

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**(2)**
**(a)** Yes. One way to see this is that there is cycle that contains all nodes. The cycle is ABEFEDCA.
**(b)** It is connected and we can remove at most 2 edges to leave the remaining graph connected. Those edges are AE and EC.

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**(3)**
Be aware: Bridges are also local bridges.
**(a)** Bridges: AC, BC, CD. Local bridges: AC, BC, CD, DE, DF, EG, FG.
**(b)** Hidden friendships: BD, CF, DG.
**(c)** Bridges: AC. Local bridges: AC.

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**(4)**
We can first calculate the shortest distance between cities: AB 120, AC 90, AD 160, AE 270, BC 130, BD 105, BE 240, CD 235, CE 180, DE 135.
**(a)** Strong ties: AB, AC, BC, BD, DE. Weak ties: AD, CE.
**(b)** A, C, and E satisties. B violates because CD doesn't have a tie. D violates because BE doesn't have a tie.
**(c)** 240 miles (or any number higher than 240 miles). To satisfy STC, we need to make sure CD and BE are in the network. So the threshold should be larger than or equal to the shortest distance between these cities.

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**(5)**

**(a,b)** See Fig. 1 for answers.

**(c)** The possible number of negative edges should take the form of $x(n-x)$, where $x$ can any integer values in the interval $[0, n/2]$. (Note: If an answer gives a duplicated version of the same set, e.g. $x(n-x)$ for $x = 0, \cdots, n$, it should also receive full points.)

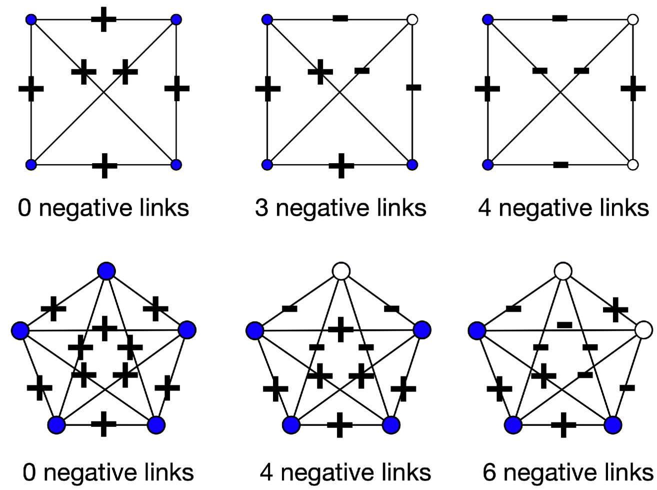


Figure 1: Q5 Balanced Networks