Networks: Fall 2023	Homework 8
David Easley and Yian Yin	Due 3:30pm, Thursday, November 16, 2023

Homework solutions should be submitted by upload to Gradescope, which can be accessed via Canvas. The file you upload **must be typed and submitted in PDF format**. Handwritten assignments will not be graded. However, you can draw graphs and insert them into your pdf. You can create a separate file with the solutions (you don't need to repeat the questions); it is fine to create the homework in any format provided it's typed and handed in as a single PDF file. When you upload your pdf to Gradescope be sure to assign your answers to the correct question.

To be eligible for full credit, your homework must come in by 3:30pm Thursday. We will also accept late homeworks after 3:30pm Thursday until 3:30pm Friday for a deduction of 10% of the total number of points available. Gradescope will stop accepting homework uploads after 3:30pm Friday; after this point we can only accept late homework when it is accompanied by a University-approved reason that is conveyed to and **approved by the TA in charge of this homework prior to the due date of the homework**. (These include illness, family emergencies, SDS accommodations and travel associated with university activities.)

The TA in charge of this homework is Ruqing Xu (rx24@cornell.edu).

Reading: The questions below are primarily based on the material in Chapter 19.

---

(1) [8 points] Consider the model we've discussed in class and in Chapter 19 for the diffusion of a new behavior through a social network. Suppose that initially everyone is using behavior B in the social network in Figure 1, and then a new behavior A is introduced. This behavior has a threshold of $q = 0.4$: any node will switch to A if at least 40% of its neighbors are using it. We will consider this process in two networks (X and Y), each of which has 9 nodes and 12 edges.

(a) [3 points] Consider Network X on the left hand side. Suppose the new behavior A is introduced at node 1. Which nodes will eventually adopt the behavior A beginning from this starting point? Give an explanation for your answer.

Solution: Starting from node 1, nodes 2 and 9 will adopt, then 3 and 8, and then 4 and 7, and 5 and 6.

(b) [3 points] Consider Network Y on the right hand side. Suppose the new behavior A is introduced at node 2. Which nodes will eventually adopt the behavior A beginning from this starting point? Give an explanation for your answer.

Solution: Starting from node 2, node 1 will adopt, then 9.

(c) [2 points] Consider Network Y on the right hand side. Can you find two nodes with the property that if these two nodes in the network act as the only two initial adopters of A, then A will spread to all nodes? If you think the answer is yes, name two nodes with this

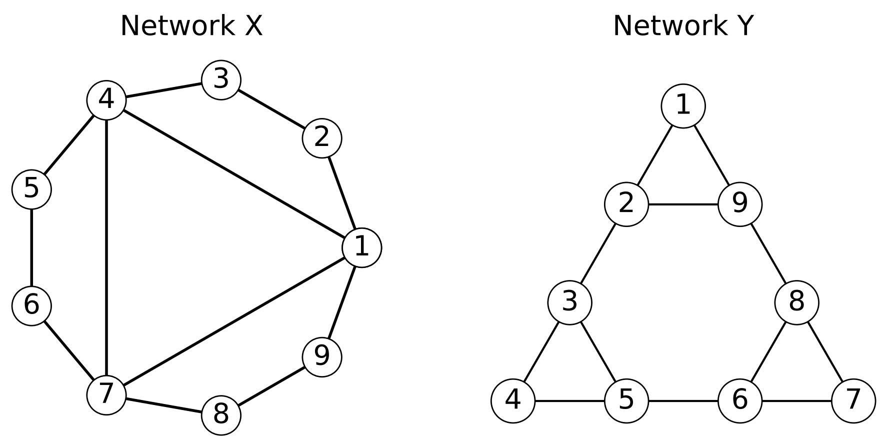

Figure 1:

property. If you think the answer is no, give an explanation for why there is no such set of two nodes.

Solution: It is not possible: We have three clusters with density $2/3 > 1 - q = 0.6$: $\{1, 2, 9\}$, $\{3, 4, 5\}$ and $\{6, 7, 8\}$. Hence in order for A to spread to all nodes, we will need at least one seed node in each cluster.

---

(2) [6 points] Consider the social network in Figure 2. Suppose that everyone is initially using some behavior B. Then a new behavior A is introduced, initially at node 3.

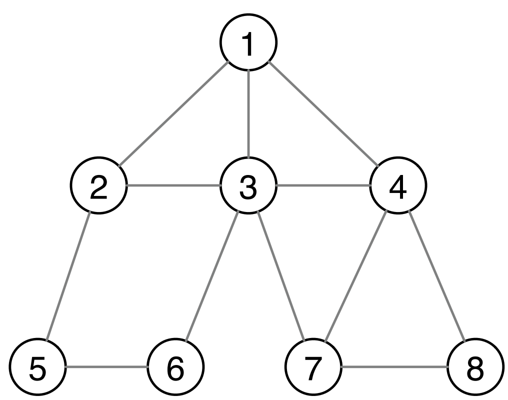

Figure 2:

You observe that the behavior spreads to other nodes over time. You do not know whether this spread can be explained by the model from Chapter 19, but you would like to try to determine this from what you observe.

(a) [2 points] In particular, suppose that you observe behavior A spread to all other nodes as follows. In the first step, nodes 1, 2, 6 and 7 switch to A; in the second step, nodes 4, 5 and 8 switch to A.

Could there be a value q so that in each step, each node decides whether to switch to A using a threshold of q? If yes, give one such value of q that would produce the observed behavior, and explain why your choice of q works. If no, explain why there is no single choice of q that all nodes could use for which you would see the observed sequence of switches to A.

Solution: Yes, if we set q to be greater than $1/4$ and at most $1/3$; then 1, 2, 6 and 7 will switch in the first step but 4, 5, and 8 won't; after that, the nodes will switch as described.

(b) [2 points] Suppose you observe a different scenario playing out on the same network, in which the behavior spreads more slowly. In this scenario, the behavior again starts just at node 3. In the first step, node 6 switches to A; in the second step, node 5 switches to A. The behavior continues to spread after the second step, but the exact information is not available to you.

Given the information of the first two steps, give a value q so that in each step, each node decides whether to switch to A using a threshold of q? Explain why your choice of q works. (If you think there's a value of q that works in multiple parts of the question, it's not necessary that it be the same value of q in each part.)

Solution: Yes, q should be greater than $1/3$ and at most $1/2$.

(c) [2 points] Now let us use the q value you give in part **(b)** and infer what happens after the second step. Your answer should include which nodes will eventually switch to A. Give an explanation for your answer.

Solution: All nodes will adopt. Step 3: 2, Step 4: 1, Step 5: 4, Step 6: 7 and 8.

---

(3) [5 points] Suppose that a group of co-workers are each members of one of two possible messaging systems, A or B. A is generally an easier system to use than B; and also, two workers have an easier time communicating when they are on the same system than when they are on different systems. But the two systems are compatible, so any two workers are able to communicate regardless of which systems they're on; it's just that the experience may be better or worse.

(a) [2 points] We'll represent the quality of the experience for two co-workers v and w using the following payoff matrix, which shows the payoffs received by v and w respectively when they each use A or B.

Note that unlike the main model that we use in class and Chapter 19, v and w still receive positive payoffs even if they communicate using opposite platforms (since we want to represent the idea that communication is still possible in this case, just a worse experience).

It turns out that even with this richer payoff structure, the type of threshold rule we saw in Chapter 19 still applies, but adapted to handle these new payoffs. Specifically, suppose that a person u in the network is trying to decide whether to use system A or system B.

		w	
		A	B
v	A	$6, 6$	$2, 2$
	B	$2, 2$	$4, 4$

Figure 3: Payoffs for communicating using A or B for part a

Let x be the fraction of u's neighbors in the network who are using A. (So a $1 - x$ fraction of u's neighbors are using B.)

Specify a number q so that u should choose A if $x > q$, and should choose B if $x < q$. (You should find that your choice of q has the property that if $x = q$, then u is indifferent; our convention in these cases has been that u chooses A, but that's not crucial for the current question.) Give an explanation for your answer, including how you derived the value of q.

Solution: If u has d neighbors, then its payoff from using A is $6dx + 2d(1-x)$; its payoff from using B is $2dx + 4d(1 - x)$. Therefore u should use A if and only if $6dx + 2d(1 - x) > 2dx + 4d(1 - x)$, which solves to $d > 1/3$. So we can choose $q = 1/3$.

(b) [3 points] Now let us consider the case where System A no longer supports compatibility functions for System B, resulting in the revised payoff matrix presented below. Calculate the value of q under this condition. Comparing your answer with **(a)**, do you think that it would be a good idea for the developers of System A to discontinue the compatibility function? Briefly explain why or why not?

		w	
		A	B
v	A	$6, 6$	$0, 0$
	B	$0, 0$	$4, 4$

Figure 4: Payoffs for communicating using A or B for part b

Solution: u should use A if and only if $6dx > 4d(1 - x)$, which solves to $d > 2/5$. So we can choose $q = 2/5$. Since q is getting larger, stopping is a bad idea.