Networks: Fall 2023	**Homework 2**
David Easley and Yian Yin	**Due 3:30pm, Thursday, September 14, 2023**

Homework solutions should be submitted by upload to Gradescope, which can be accessed via Canvas. The file you upload **must be typed and submitted in PDF format**. Handwritten assignments will not be graded. However, you can draw graphs and insert them into your pdf. You can create a separate file with the solutions (you don't need to repeat the questions); it is fine to create the homework in any format provided it's typed and handed in as a single PDF file. When you upload your pdf to Gradescope be sure to assign your answers to the correct question.

To be eligible for full credit, your homework must come in by 3:30pm Thursday. We will also accept late homeworks after 3:30pm Thursday until 3:30pm Friday for a deduction of 10% of the total number of points available. Gradescope will stop accepting homework uploads after 3:30pm Friday; after this point we can only accept late homework when it is accompanied by a University-approved reason that is conveyed to and approved by the TA in charge of this homework prior to the due date of the homework. (These include illness, family emergencies, SDS accommodations and travel associated with university activities.)

The TA in charge of this homework is Ruqing Xu rx24@cornell.edu

Reading: The questions below are primarily based on the material in Chapters 6 and 8. Chapter 8 will covered in class next Wednesday.

---

(1) [6 points: 2+2+2] In this question we will consider several two-player games. In each payoff matrix below the rows correspond to player A's strategies and the columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff. Both players prefer higher payoffs to lower payoffs.

(a) Find all pure (non-randomized) strategy Nash equilibria for the game described by the payoff matrix in Figure 1.

		Player B	
		L	R
Player A	U	$5, 1$	$3, 4$
	D	$1, 2$	$2, 3$

Figure 1: The payoff matrix for Question (1a).

(b) Find all pure (non-randomized) strategy Nash equilibria for the game described by the payoff matrix in Figure 2.

(c) Find **all** Nash equilibria for the game described by the payoff matrix in Figure 3. Provide a brief explanation or calculation for your answer.

Player A \ Player B	L	R
U	$3, 1$	$1, 4$
D	$4, 3$	$5, 2$

Figure 2: The payoff matrix for Question (1b).

Player A \ Player B	L	R
U	$0, 6$	$3, 4$
D	$4, 3$	$1, 5$

Figure 3: The payoff matrix for Question (1c).

(2) [3 points] Find **all** Nash equilibria for the game described by the payoff matrix in Figure 4. Provide a brief explanation or calculation for your answer.

Player A \ Player B	L	R
U	$2, 1$	$2, 4$
D	$6, 2$	$1, 0$

Figure 4: The payoff matrix for Question (2).

---

(3) [8 points: 3+2+3] Most of our examples of games had only two strategies for each player, but the concept of Nash equilibrium also applies to games with many strategies for each player. In this question we will consider a two-player game in which player A has four strategies and player B has four strategies. The matrix below describes the payoffs to these strategies. A Nash equilibrium is still a pair of strategies that are best responses to each other.

(a) Does either player have a dominant strategy? If so, which player(s)? Explain briefly (1-3 sentences).

(b) A strategy is said to be **strictly dominated** for a player if the player has another strategy that yields a strictly greater payoff regardless of the strategy chosen by the other player. Clearly, a player should not use a strictly dominated strategy as it cannot be a best response to any of the other player's strategies. Is there a strategy that player A should not use as there is another strategy that always (regardless of what player B does) yields a strictly greater payoff? If so, how many strictly dominated strategies does player A have and which ones are they? Is there a strategy that player B should not use as there is another strategy that always yields a strictly greater payoff, regardless of what player A does? If so, how many of these strictly dominated strategies player B have and which ones are they?

(c) Find **all** pure strategy Nash equilibria for this game. [Your answer to part (b) should

		Player B			
		A	B	C	D
Player A	W	$1,0$	$4,0$	$2,1$	$0,1$
	X	$1,4$	$8,1$	$2,5$	$5,6$
	Y	$0,2$	$6,3$	$1,6$	$3,1$
	Z	$3,3$	$7,4$	$3,7$	$2,3$

Figure 5: Payoff Matrix for Question 3

be helpful in answering this question.]

---

(4) [7 points: 2+1+2+2] In this problem we will consider an attack-defense game. (This particular game is a simple version of the "Colonel Blotto" game which was first proposed by Émile Borel in 1921.) There is a military battle taking place at two nearby mountain passes, which we'll call A and B, and two Colonels from opposing armies are directing the battle. One Colonel — the *attacker* — is trying to break through at least one of these mountain passes to the territory beyond, while the other Colonel — the *defender* — is trying to prevent this from happening.

The defender has to decide which of the two mountain passes to reinforce. His possible actions are to reinforce pass A or to reinforce pass B. He can't defend both passes. The attacker has to decide which pass to attack. He can either attack pass A or attack pass B. He can't attack both passes.

The two Colonels make their decisions simultaneously and independently. The attacker wins the game if the pass he attacks is not reinforced by the defender. The defender wins the game otherwise — that is, the defender wins if he reinforces the pass that the attacker attacks.

Both Colonels obviously want to win. Let's suppose that for each Colonel the payoff to winning is w and the payoff to losing is l (the Colonel who does not win is the loser), with $w > l$.

(a) Set up the payoff matrix for this game.

(b) Is there a pure strategy equilibrium? Find all such equilibria, if there are any.

(c) Find a mixed strategy equilibrium.

(d) Your answer to part (c) should not depend on the values of w and l (as long as $w > l$). Can you explain why the actual values of w and l do not affect the probabilities for the mixed strategy in (c)?

---

(5) [5 points: 2+3] Seventy (70) travelers begin in city A and must travel to city B. There are two routes between A and B. Route I begins with a highway from city A to city

C which takes four hours of travel time per traveler regardless of how many travelers use it. It ends with a local street from city C to city B; this local street requires a travel time per traveler in hours equal to the number of travelers on the street — let's call this number x — divided by 10. Route II begins with a local street from city A to city D, which requires a travel time per traveler in hours equal to the number of travelers on the street — let's call this number y — divided by 20. It ends with a highway from city D to city B which requires eight hours of travel time per traveler regardless of the number of travelers who use this highway. All roads are one-way roads. The road network is depicted in Figure 6.

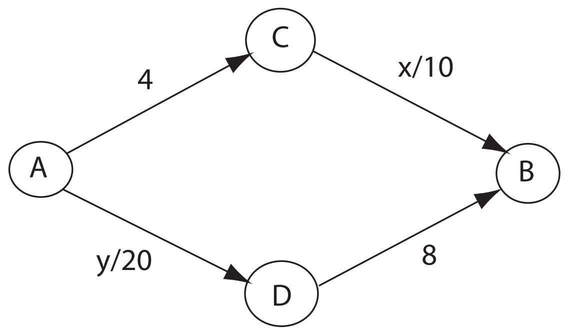

Figure 6: Graph for Question 5

(a) Travelers simultaneously chose which route to use. Find Nash equilibrium values of x and y.

(b) Now the government builds a new highway directly connecting cities A and B. The travel time per traveler on this road is six hours regardless of the number of travelers who use it. Let's call the number of travelers who use this new AB route, z. Find a new Nash equilibrium.

---

(6)[7 points: 3+4] One thousand and one-hundred (1,100) travelers begin in city A and must travel to city B. There are three routes between A and B. Route I begins with a road from city A to city C which takes $x/20$ minutes of travel time per traveler, where x is the number of travelers use it. It ends with a highway from city C to city B which takes 60 minutes of travel time regardless of the number of travelers on it. Route II begins with a highway from city A to city D, which takes 50 minutes of travel time regardless of the number travelers on it. It ends with a street from city D to city B which requires $z/10$ minutes of travel time per traveler, where z is the number of travelers who use it. Route III begins with a highway from city A to city E, which takes 50 minutes of travel time regardless of the number of travelers on it. It ends with a street from city E to city B which requires $y/20$ minutes of travel time per traveler, where y is the number of travelers who use it. All roads are one-way roads. The road network is depicted in Figure 7.

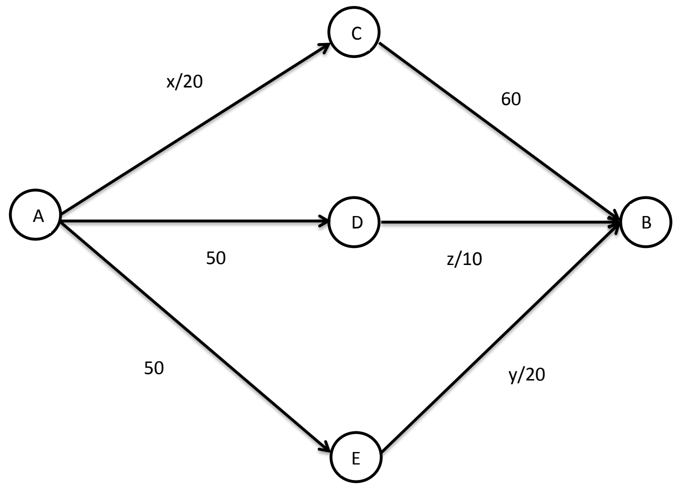

Figure 7: Graph for Question 6

(a) Travelers simultaneously chose which route to use. Find Nash equilibrium values of x, y and z.

(b) Now the government builds a new highway directly connecting cities C and D; this is a two-way highway, travelers can go from C to D or from D to C and both directions of travel take no time. The travel time per traveler on this highway is 0 minutes regardless of the number of travelers who use it. Find the new Nash equilibrium. Be sure to describe not just x, y and z, but also how many travelers use the AD and CB roads. [Hint: It may help to start with the Nash equilibrium you found in part (a) and ask if any traveler(s) would want to deviate to the new routes. Also remember that Nash equilibrium does not require that all roads are used; it only requires that no individual traveler can benefit by switching to a different route.]