ORIE 3510/5510
DISCUSSION 2

Question 1. A red die is rolled a single time. A green die is rolled repeatedly. The game stops the first time that the sum of the red and green die is either 4 or 7. What is the probability that the game stops with a sum of 4?

Question 2. You are enrolled in ORIE 3510/5510. This course has k homeworks which counts towards your letter grade. Every time you attempt a homework for an hour, you solve it completely with probability $p \in [0,1]$. Compute the expression for probability you solve all k homeworks in n hours? (*Hint: Think about the probability space of an infinite sequence of* p*-coin flips*)

Question 3. Let $Y \sim Binom(n,p)$ and $Z \sim Binom(n,q)$, where $n \geq 1$ is an integer and $0 < p < q < 1$. Because $p < q$, there is some sense in which Y should be smaller than Z. Your aim in this problem is to show that for all $k \in \{0, \ldots, n\}$,

$$P\{Y > k\} \leq P\{Z > k\}.$$

(a) Come up with an outcome space Ω, a probability measure on it, and define random variables $Y, Z : \Omega \to \mathbb{N}$ such that all of the following hold:
  - $Y \sim Binom(n,p)$
  - $Z \sim Binom(n,q)$
  - $P\{Y \leq Z\} = 1$. It suffices to have $Y(\omega) \leq Z(\omega)$ for all $\omega \in \Omega$.

(b) Using the fact that $P\{Y \leq Z\} = 1$, prove $P\{Y > k\} \leq P\{Z > k\}$.