Homework 2

Solutions

As explained in class, it is a good idea to set your calculator to use **higher precision** when computing results on a homework or exam question. Generally, setting your computer to use a 4 or 5-decimal precision will be more than enough. It is especially important for you to use high precision if you will be writing down intermediate results. Changing your precision setting was demonstrated in class.

You can also **store and recall intermediate results** by using the STO X and RCL X key combinations, where X is a digit from 0 to 9. This allows you access to the calculator's 10 memory registers. Additionally, X can also be one of the TVM buttons: N, I/Y, PV, PMT, FV when used in combination with RCL. This functionality; also demonstrated in class, can be used to avoid writing down intermediate results altogether.

In the solutions below **we typically show the value stored in the PMT register, eve if it is 0**. As you know, when you clear the TVM registers, PMT is set to 0. To eliminate ambiguities, however, consistent with in-class practice, we set the PMT to 0 explicitly.

We do not present all possible solutions to all problems. Sometimes we present both a formula-based and a calculator-based solution, sometimes we present only one of them. **We encourage you to practice by solving the problems below by also using the method that we did not present explicitly**.

When setting up problems for the calculator, it is critical to keep track of cash flow signs when solving problems with the calculator. If you have present values, payments, and future values, consistency is especially important. However, unless we explicitly request it, we do not ask for signed cash flow amounts **as results** for answers to homework or exam problems **if the context makes clear the direction of the cash flow**. If we asked, however, to show us how would you set up a calculator problem, then signs of quantities input into the TVM registers would have to shown consistent with the question posed.

You may have had to solve problems whose numerical values were different; however, the reasoning and the types of calculations that produce the final result would have been the same.

1 Problem 1

Two annuities have equal present values and an applicable discount rate of 7.25 percent. One annuity pays $2,500 on the first day of each year for 15 years. How much does the second annuity pay each year for 15 years if it pays at the end of each year?

Solution:

We compute the PV of the **annuity due** using the calculator. For this, we must set the cash flows to arrive at the beginning of the time period. We demonstrated this in class. On the BA II Plus you can **flip the cash flow timing** setting between END and BGN (which we need now) by pressing the following buttons: 2ND, BGN, 2ND, SET.

- Set the cash flow timing to BGN.
- N = 15, I/Y = 7.25, PMT = 2,500, FV = 0. Press CPT PV to get -24,039.6144.
- Set the cash flow timing to END.
- N, I/Y, FV are already set correctly. Set PV to -24,039.6144. Press CPT PMT to get 2,681.25.

So the annuity pays $2,681.25 every period.

2 Problem 2

You just acquired a home mortgage for 30 years in the amount of $184,500 at 4.65 percent interest compounded monthly. How much of the first payment will be interest if the loan is repaid in equal monthly payments?

Solution:

The BB for the first month is the actual amount of the loan, so BB = $184,500. The monthly interest rate is $r = \frac{4.65}{12} = 0.3875\%$. The amount of interest is $BB \cdot r = 184,500 \cdot 0.003875 =$ $714.94. We do not need to compute the monthly payment to answer this question!

This is a constant-payment loan - effectively, an annuity. We compute by payment by using the calculator: N=12·30, PV=184,500, I/Y=$\frac{4.65}{12} = 0.3875$ (the per-period - monthly - interest), FV = 0. Pressing CPT PMT we get 2,681.2500, so the monthly mortgage payment is $2,681.25. Again, this calculation was not needed to answer the question.

3 Problem 3

Write riding the Tube, on way to your first day of internship at a prestigious financial firm in the City of London, you found a torn and dirty piece of paper that was almost illegible. The legible parts are shown below, and are sufficient to remind you of the loans you once studied in a Finance course you took.

Assuming that this loan is held to maturity (it does not end early), and assuming that this one of the loan types that we studied, what is the (original) maturity of the loan, in years?

Period	BB		Interest		EB
1	\$ 75,000.00	\$ 11,007.25	\$ 7,500.00	\$ 3,507.25	\$ 71,492.75
2	\$ 71,492.75	\$ 11,007.25	\$ 7,149.28		\$ 67,634.78
3	\$ 67,634.78	\$ 11,007.25	\$ 6,763.48	\$ 4,243.77	\$ 63,391.01

Assuming that this loan is held to maturity (it does not end early), and assuming that this one of the loan types that we studied, what is the (original) maturity of the loan, in years?

Solution:

The third column must be the monthly payment, the fifth column must be the amount of principal paid. But the values in the monthly column are constant, so this is a fixed-payment loan (an annuity, really). The balance of the loan is \$75,000, the annuity's coupon is \$11,007.25. We can determine the interest rate from the first row of the table, since $Interest = BB \cdot r$. So $r = \frac{7{,}500}{75{,}000} = 10\%$. We can now set up the calculator to compute the number of periods to the full maturity of this loan.

I/Y = 10, PV = 75,000, PMT = -11,007.25, FV = 0. We press CPT N and we get 12 (years), which is the final answer.

We can also use formulas: $t = -\frac{\ln\left(1-\frac{PV}{C}\cdot r\right)}{\ln(1+r)} = -\frac{\ln\left(1-\frac{75{,}000}{11{,}007.25}\cdot 0.10\right)}{\ln(1+0.10)} = 12.0000$.

4 Problem 4

Gugenheim, Incorporated, has a bond outstanding with a coupon rate of 6.4 percent and annual payments. The yield to maturity is 7.6 percent and the bond matures in 20 years. What is the market price if the bond has a par value of \$2,000?

Solution:

This bond is atypical, as its face value is \$2,000. But the general formulas and methodology still holds.

C $= \$2,000 \cdot 0.064 = \$128.00$. This number represents the amount of dollars paid out as coupons per year. But the payments (coupons) are annual, so there is only one payment per year.

We use the calculator: N = 20, I/Y = 7.6, PMT = 128.00, FV = 2,000. CPT PV yields -1,757.18. So the price of the bond is \$1,757.18.

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5 Problem 5

A 22-year, semiannual coupon bond sells for \$965.18. The bond has a par value of \$1,000 and a yield to maturity of 6.96 percent. What is the bond's coupon rate?

Solution:

The bond has a par value of \$1,000. The bond trades at par when its price is equal to its face value (or principal). So P = \$1,000. As seen here, sometimes the face value is also called "par value."

The per-period yield is $\frac{6.96\%}{2} = 3.48\%$. How do we know? The bond pays semi-annual coupons.

We use the calculator: N = 44 ($22 \cdot 2$), I/Y = 3.48%, PV = -965.18, FV = 1,000. We press CPT PMT and we get 33.2425. This is the semi-annual coupon. To get the annual amount of coupon payments, we multiply by 2: $\$33.2425 \cdot 2 = \$66.4851$. The coupon rate is thus $\frac{66.4851}{1{,}000} = 6.64851\% \approx 6.65\%$.

6 Problem 6

There are zero coupon bonds outstanding that have a yield to maturity of 5.19 percent and mature in 16 years. The bonds have a par value of \$10,000. If we assume semiannual compounding, what is the price of the bonds?

Solution:

The semi-annual yield is $\frac{5.19\%}{2} = 2.5950\%$. The number of periods is $16 \cdot 2 = 32$. We can use the formula for direct discounting, as we are dealing with a single cash flow:

$PV = \frac{10{,}000}{(1+0.0259)^{32}} = \$4,405.1605 \approx \$4,405.16$.

We can do this using the calculator: N = 32, I/Y = 2.5950, PMT = 0, FV = 10,000. We press CPT PV and we get -4,405.1605, which is the same result as above.

7 Problem 7

An investment had a nominal return of 9.5 percent last year. If the real return on the investment was only 6.6 percent, what was the inflation rate for the year?

Solution:

R = 9.5%, r = 6.6%. We know that $\frac{1+R}{1+h} = 1 + r$ or, equivalently, $(1 + R) = (1 + r)(1 + h)$. We get that $h = \frac{1+R}{1+r} - 1 = \frac{R-r}{1+r}$. Replacing values we get $h = \frac{0.095-0.066}{1+0.066} = 2.72\%$.