Networks: Fall 2023 **Practice Final Exam**
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The final exam is **Tuesday, December 12 from 2:00 to 4:30 pm**.

Please read the following instructions to see where you should go for the exam. The exam will be held in two rooms:

Room assignment based on the first letter of your netid:

- A in Statler 196
- B thru Z in Statler Auditorium

The final will be a closed-book, closed-notes exam. The final will be cumulative, in that it will draw from all parts of the course, both before and after the midterm. The best guide to the coverage of the exam is the contents of the course lectures; it will also be useful to review the homeworks and the readings.

We will hold two review sessions for the final exam. One will held on Wednesday, Dec. 6, in Uris G01 from 3-4:30pm. The other will be held on Friday, Dec. 8, in Ives 305 from 3-4:30pm. In both review sessions we will go over the practice final exam.

To help in studying, we are providing a practice final exam composed of questions from exams that we've given in previous years. There are more questions on this practice exam than there will be on the real final; we added questions so that the practice exam would cover more (but not all) topics from the course. The composition of questions is structured to approximately resemble the real final, although of course the actual questions on the real final may cover topics from the course that are not explicitly the focus of any question here. The practice final questions are not meant to be handed in; rather, we will discuss them at the upcoming final exam review session and during office hours.

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(1) In a large school the teachers have noticed the outbreak of a bad form of flu. The kids mix a lot in the school during various activities, and each infected child tends to meet 100 other children before his infection is discovered and he is sent home. Infected kids infect any kid they meet independently with probability 0.03. To control the disease two alternatives have been proposed.

Proposal 1. Distribute free vitamins to all kids. The nurses state that this would decrease the probability of infection from 0.03 to 0.015.

Proposal 2. The school could cancel many of the activities where kids mix, and try to keep kids separated in their own classrooms only. This will decrease the number of other kids an infected kid meets to only 25 other kids.

Both proposals have issues. The school administration is debating what may be the best course of action.

(a) They are considering acting only on Proposal 1, as it interferes less with learning. What would be your advice to the school as they are thinking of implementing Proposal 1 only? Do you think this is good enough to control the outbreak alone? Explain your answer.

(b) Do you think Proposal 2 alone may be effective in controlling the outbreak? Explain your answer.

(2) In the payoff matrix below the rows correspond to player A's strategies and the columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

		Player B L	Player B R
Player A	U	$4, 5$	$3, 1$
Player A	D	$1, 3$	$6, 6$

(a) Find all pure strategy Nash equilibria.

(b) Find all mixed strategy Nash equilibria. Let p represent the probability that A plays U and let q represent the probability that B plays L.

(3) Suppose that there are three types of used cars – good ones, medium ones and bad ones – and that sellers know which type of car they have. Buyers do not know which type of car a seller has, and sellers of good and medium cars have no way of proving what kind of car they have.

Buyers know that the fraction of used cars in the total population of used cars that are good quality is 1/3, the fraction of used cars that are medium quality is 1/3, and the fraction of used cars that are bad quality is 1/3 (but as is typical in these situations, it is not necessarily the case that all the owners of these used cars will choose to sell them). The following table summarizes the values of the different cars for sellers and buyers in thousands of dollars

	Value for good car	Value for medium car	Value for bad car
buyer	15	10	5
seller	12	7	2

We assume that buyers are risk-neutral; that is, they are willing to pay their expected value of a car. You may assume that there are many more buyers than sellers.

(a) Is there a self fulfilling expectations equilibria in the market when all kinds of cars are sold (bad, medium or good quality)? If there is one, what price are the cars sold? If there isn't one, explain briefly why not.

(b) Is there a self fulfilling expectations equilibria in the market when only bad and medium quality cars are sold? If there is one, what price are the cars sold? If there isn't one, explain briefly why not.

(c) Is there a self fulfilling expectations equilibria in the market when only bad cars are sold? If there is one, what price are the cars sold? If there isn't one, explain briefly why not.

(d) Are there any other self fulfilling expectations equilibria in the market not considered in (a-c)? If there are, what kind of cars are being sold in each of these other equilibria, and at what price? If there aren't any others, explain briefly why not.

(4) Suppose a search engine has three ad slots that it can sell. Slot a has a clickthrough rate of 6, slot b has a clickthrough rate of 4, and slot c has a clickthrough rate of 1. There are three advertisers who are interested in these slots. Advertiser x values clicks at 5 per click, advertiser y values clicks at 3 per click, and advertiser z values clicks at 1 per click.

(a) What is the socially optimal allocation (which slots should each of the advertisers get)?

(b) Compute the VCG prices for this socially optimal allocation.

(c) Show that if the search engine used a Generalized Second Price auction then truthful bidding is not a Nash equilibrium.

(5) Consider the two groups of close friends, as depicted on the Figure 1 below. Strong ties are indicated by dark edges, and weak ties by lighter dashed edges.

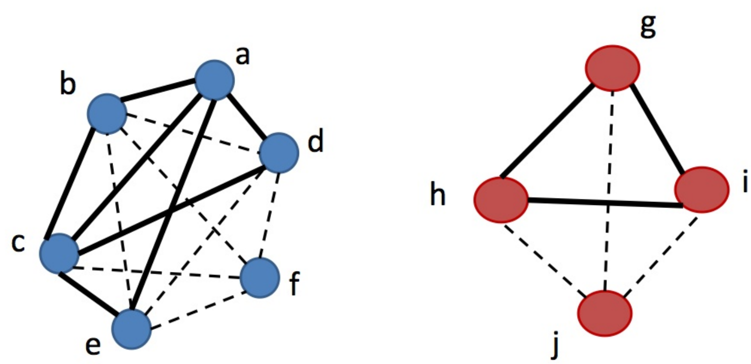

Figure 1: Two groups of close friends

(a) Suppose that in addition to the edges indicated in the figure, there is also a connection between persons d and h. According to the theory of strong triadic closure, is that connection more likely to be strong or weak? Briefly explain your answer.

(b) Which pair(s) of nodes, with one node in the pair from the group on the left and the other from the group on the right, can we connect with a strong tie without the resulting graph violating the strong triadic closure property? List all possible such pairs, and briefly explain your answer.

(6) Consider the network of Web pages shown in Figure 2. We want to determine the equilibrium PageRank values of these pages using the Basic PageRank Update Rule.

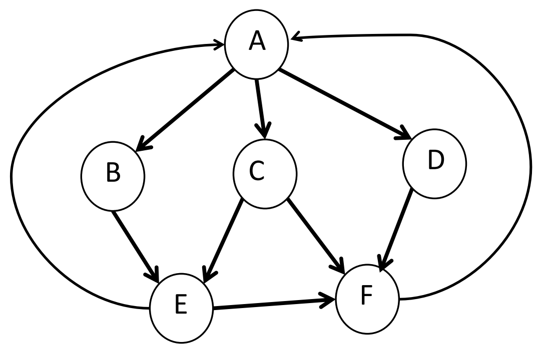

Figure 2: A network of Web pages.

(a) Denote the equilibrium PageRank value of node A by a. What are the equilibrium PageRank values for all the other nodes in terms of a?

(b) Give the equilibrium PageRank values of all nodes (not in terms of a, but as actual numbers).

(c) Suppose the owner of page C decides to divide page C into two parts. We will represent this change by adding a new page C', moving all links from C to C', and linking the old page C only to this continuation page C', as depicted on Figure 3. Will this change increase or decrease the number of pages with PageRank strictly greater than C's PageRank compared to your answer from part (b)? Briefly explain your answer.

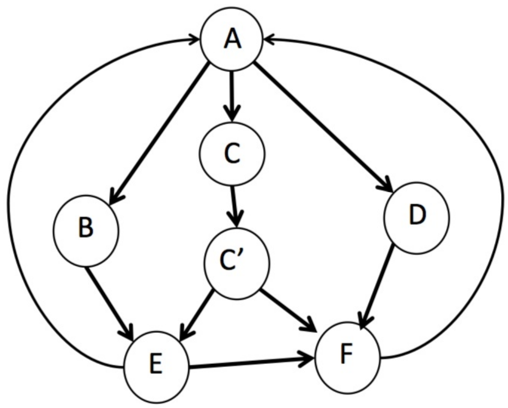

Figure 3: A network of Web pages.

(7) Five hundred travelers begin in city A and must travel to city B. There are two routes between A and B.

Route I starts with a highway from A to C that takes an hour, and then is followed by a local street from city C to city B, which requires a travel time $2 + x/100$ if x travelers are using the road from C to B.

Route II begins with a local street from city A to city D, which requires a travel time per traveler in hours equal to the number of travelers on the street divided by 100, and ends with a highway from city D to city B which requires two hours of travel time per traveler regardless of the number of travelers who use this highway.

All roads are one-way roads.

(a) Draw the network described above and label the edges with the travel time needed to move along the edge. Let x be the number of travelers who use Route I and let y be the number of travelers who use Route II.

(b) Travelers simultaneously choose which route to use. Find Nash equilibrium values of x and y, and explain your answer.

(c) Recent construction added a new option. This new Route III offers a direct highway between A and B, avoiding other cities. The direct route takes 6 hours. Let z be the number of travelers who use Route III. Find Nash equilibrium values of x, y and z, and explain your answer.

(d) Improvement to this highway III allowed for faster travel, and cut down the travel time to 4 hours. Find Nash equilibrium values of x, y and z under this new condition, and explain your answer.

(8) Three buyers x, y and z are each considering buying one of three items A, B and C. Their valuations for the items are given in the table below.

Buyer	Value for item A	Value for item B	Value for item C
x	5	5	4
y	5	4	3
z	4	2	3

(a) Suppose the price of each item is 2. Are these prices market clearing? Explain. Your explanation must use the preferred seller graph.

(b) Starting from the prices in part (a), can you make the prices market clearing by raising the price of a single item (without lowering any price)? Explain why you cannot, or show the resulting market clearing prices (and what will be the new preferred seller graph and resulting assignment of items to buyers).

(9) Suppose three voters express their rankings among the alternatives X, Y, and Z, and then a winner is chosen using the Borda count.

The first voter (named 1) expresses the following ranking

$$X \succ_1 Y \succ_1 Z, \tag{1}$$

and the second voter (named 2) expresses the following ranking

$$Y \succ_2 Z \succ_2 X, \tag{2}$$

You aren't told what the third voter's ranking was.

(a) Suppose you know that X emerges as the winner (not in a tie) when the Borda count is applied to the three rankings (the two rankings that you know plus the one you don't know). From this information, can you tell for sure what voter 3's ranking was? If you think the answer is "yes," say what voter 3's ranking must be, with an explanation. If you think the answer is "no," explain why there's not enough information to tell what voter 3's ranking is.

(b) Suppose you know that Y emerges as the winner (not in a tie) when the Borda count is applied to the three rankings (the two rankings that you know plus the one you don't know. From this information, can you tell for sure what voter 3's ranking was? If you think the answer is "yes," say what voter 3's ranking must be, with an explanation. If you think the answer is "no," explain why there's not enough information to tell what voter 3's ranking is.

(10) Consider a scenario of a technology being considered for adoption by a population of users one by one. Assume that the probability that this technology is good (G) is $Pr(G) = 1/2$. Any individual who rejects the technology receives a payoff of 0. The payoff for adopting depends on whether the technology is good or bad. Let's suppose that if it's good, then the payoff obtained from adopting it is $+1$. If the technology is bad, then the payoff is -1.

Each person in turn gets a signal about the quality of the product, and also observes the decisions made by all previous people. When the technology is bad, it is very likely that users get bad signals; when the technology is good, it is very likely that users get good signals. Concretely, the probability of a high signal when the technology is good is $Pr(H|G) = 3/5$ (and hence $Pr(L|G) = 2/5$), while the probability of a low signal when the technology is bad is $Pr(L|B) = 3/5$ (and hence $Pr(H|B) = 2/5$).

For this question, it may be useful to recall Bayes' rule. For two events C and D, Bayes' rule states that

$$Pr(C|D) = \frac{Pr(D|C)Pr(C)}{Pr(D)}.$$

(a) Assume the first person gets a high signal. What is the probability that the technology is good (given the high signal)?

(b) Now assume that the first person gets a low signal. What is the probability that the technology is good now (given the low signal)?

(c) How should the first person make a decision: what should he/she decide conditional on a high signal? and what should he/she decide conditional on a low signal?

(d) Assume the first person adopted the new technology. How should the second person make a decision? State the decision the second person should make conditional on whether they get a high signal or a low signal (and given that the first person adopted the technology). Assume that if the second person is indifferent between adopting and rejecting the new technology, then he follows his own signal. Briefly explain your answer. [You do not need to write a proof, or compute probabilities. A brief argument is sufficient.]

(e) Assume the first two persons adopted the new technology. How should the third person make a decision? State the decision the third person should make conditional on whether they get a high signal or a low signal (and given that the first two persons adopted the technology). Briefly explain your answer. [You do not need to write a proof, or compute probabilities. A brief argument is sufficient.]

(11) Consider a good that has network effects in the sense of our model from Chapter 17. Consumers are named using real numbers between 0 and 1. The reservation price for consumer x when a z fraction of the population uses the product is given by the formula $r(x)f(z)$, where $r(x) = (1-x)$ and $f(z) = 5z$.

(a) Suppose that the good is sold at price of 1.2. What are the possible equilibrium fractions of the population purchasing the good?

(b) Which of the equilibria you found in part (a) are stable? Explain.

(c) Suppose this is a new product, and you discover that the fraction of potential users initially using the product is 0.3. What do you expect the trend in users to be? Is the product likely to become more popular over time, or do you expect that after the initial excitement of trying the new product dies off, the product will become less popular? Explain.

(d) Would your answer to (c) change if you found that the fraction of potential users initially using the product was 0.5? Explain.

(12) Consider the network depicted in Figure 4. Suppose that each node starts with the behavior B, and each node has a threshold of $q = 0.35$ for switching to behavior A.

(a) Let e and f form a two-node set S of initial adopters of behavior A. If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to A? Give a brief (1-2 sentence) explanation for your answer.

(b) Find a cluster of density greater than $1 - q = 0.65$ in $G - S$ that blocks behavior A from spreading to all nodes, starting from S, at threshold q. Give a brief (1-2 sentence) explanation for your answer.

(c) Suppose you were allowed to add a single edge to the given network, connecting one of nodes e or f to any one node that it is not currently connected to. Could you do this in such a way that now behavior A, starting from S and spreading with a threshold of 0.35, would reach all nodes? Give a brief explanation for your answer.

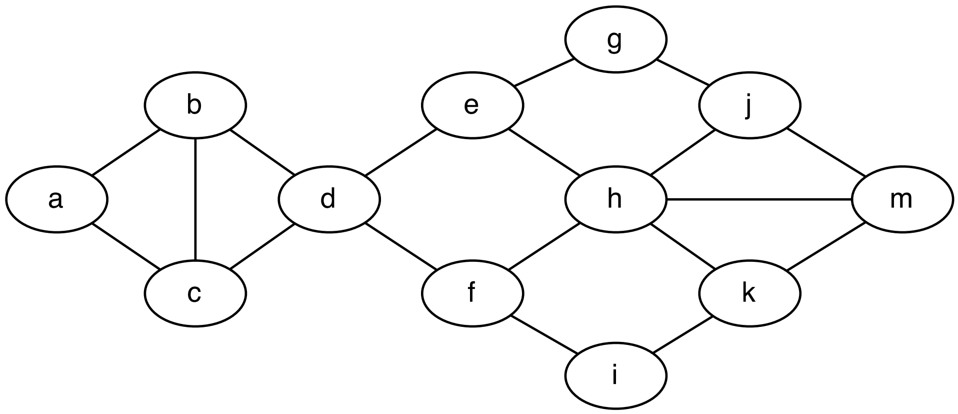

Figure 4: The network for question 12

(13) In this problem we will consider the relationship between Nash equilibria and evolutionarily stable strategies.

(a) First, consider the game in Figure 5:

		Player B X	Player B Y
Player A	X	$2,2$	$4,1$
Player A	Y	$1,4$	$3,3$

Figure 5: The payoff matrix for Question (13a).

Find all of the pure strategy Nash equilibria and all of the evolutionarily stable strategies for this game.

(b) Next, consider the game in Figure 6:

		Player B X	Player B Y
Player A	X	$2,2$	$3,1$
Player A	Y	$1,3$	$3,3$

Figure 6: The payoff matrix for Question (13b).

Find all of the pure strategy Nash equilibria and all of the evolutionarily stable strategies for this game.

(c) There should only be one difference in your answers to parts (a) and (b) of this question. Provide a brief explanation of why the answers differ.

(14)

Consider a betting market with two horses A and B and three bettors 1, 2 and 3, as in Chapter 22. Let's suppose that the true probability that horse A will win is 1/2, and the true probability that horse B will win is 1/2. Let's suppose that each bettor has wealth w, and so the total wealth to be bet on the horses is $3w$. Bettor 1 believes there is a probability of 1/4 that horse A will win, and a probability of 3/4 that horse B will win. Bettor 2 believes there is a probability of 1/2 that horse A will win, and a probability of 1/2 that horse B will win. Bettor 3 believes there is a probability of 3/4 that horse A will win, and a probability of 1/4 that horse B will win. All bettors have logarithmic utility for wealth, and they each choose bets to maximize the expected utility of wealth given their beliefs.

(a) How much money should bettors 1, 2 and 3 each bet on horse A?

(b) Find the equilibrium inverse odds on horse A. How do these inverse odds compare to the true probability that horse A will win the race?

(c) How much money will each bettor have after the race if horse A wins?

(d) Suppose that, as in part (c), horse A wins the race today. Suppose also that there will be another race the next day between horses A and B, that the probabilities are all unchanged (both the true probability and the bettor's beliefs), and that the bettors will bet on this new race using the wealths that you found in part (c). Find the new equilibrium inverse odds on horse A.