INFO 2950: Intro to Data Science

Lecture 18
2023-10-30

Agenda

1. Geometric distributions
2. Negative Binomial distributions
3. Poisson distributions

How do you know if you can use a binomial distribution?

1. Each trial has only 2 possible outcomes: “success” (1) or “failure” (0)
2. You conduct each “trial” exactly the same way for a fixed number of times *n*
3. The probability of success *p* is the same for each trial
4. Trials are independent

Can you use a binomial distribution?

Sample students until you've found one who likes cats.
X = number of students sampled.
Probability of a student liking cats is 28%.

https://online.stat.psu.edu/stat414/lesson/10/10.2

Can you use a binomial distribution?

Nope, no *n* defined!

Sample students until you've found one who likes cats.
X = number of students sampled.
Probability of a student liking cats is 28%.

https://online.stat.psu.edu/stat414/lesson/10/10.2

How do you know if you can use a geometric ~~binomial~~ distribution?

1. Each trial has only 2 possible outcomes: "success" (1) or "failure" (0)
2. You conduct each "trial" exactly the same way ~~for a fixed number of times *n*~~
3. The probability of success p is the same for each trial
4. Trials are independent

How do you know if you can use a geometric ~~binomial~~ distribution?

1. Each trial has only 2 possible outcomes: “success” (1) or “failure” (0)

2. You conduct each “trial” exactly the same way ~~for a fixed number of times *n*~~ and count the number of trials until the first success

3. The probability of success *p* is the same for each trial

4. Trials are independent

Geometric distribution

Event space: integers **0** to infinity

Parameters: chance of "success" p

Story: how many trials before the first success?

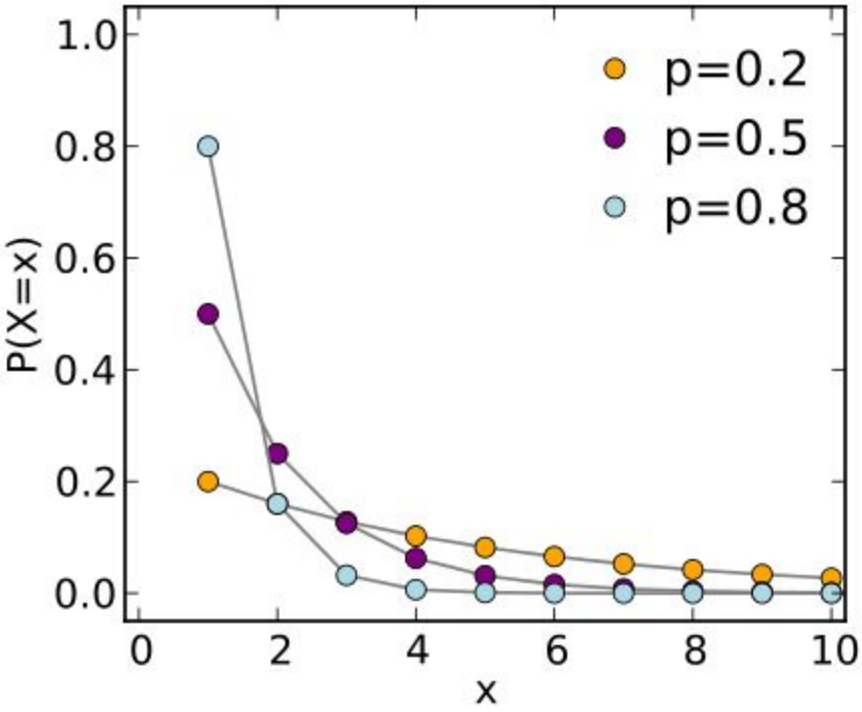
Geometric distribution (shifted)

Event space: integers **1** to infinity

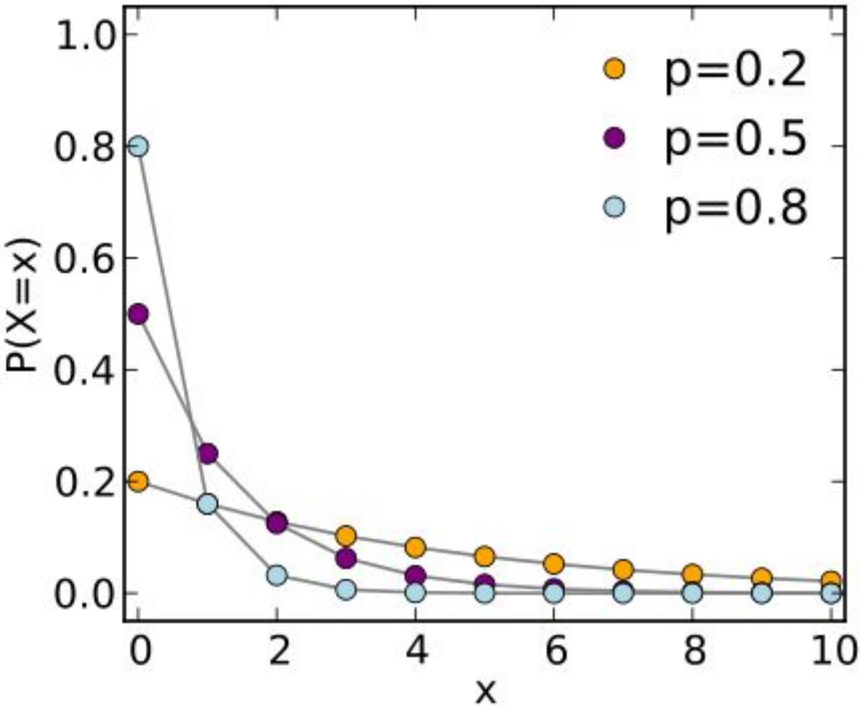
Parameters: chance of "success" p

Story: how many trials **to reach** the first success?

Shifted geometric distribution

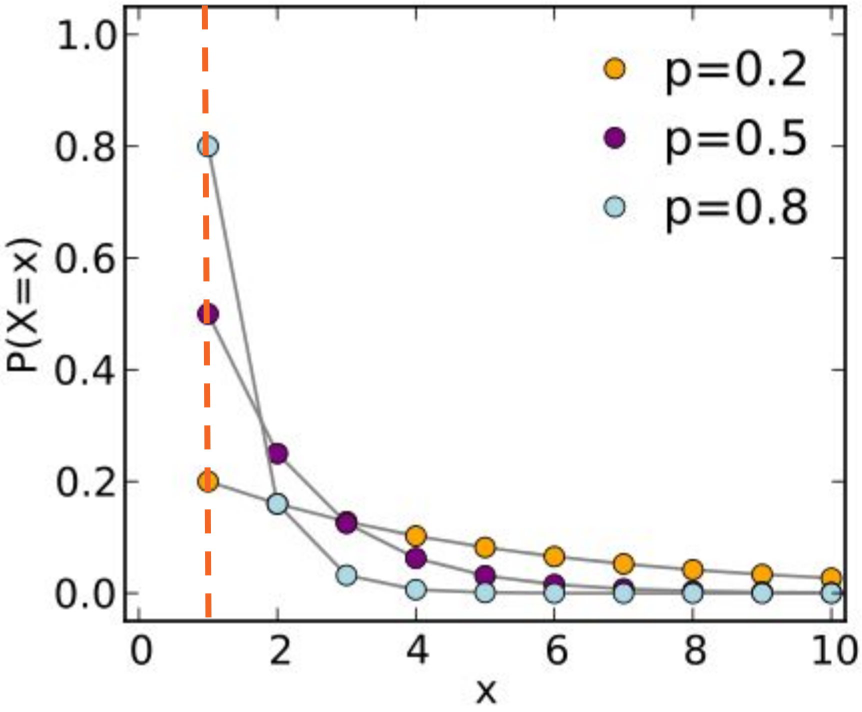


Geometric distribution

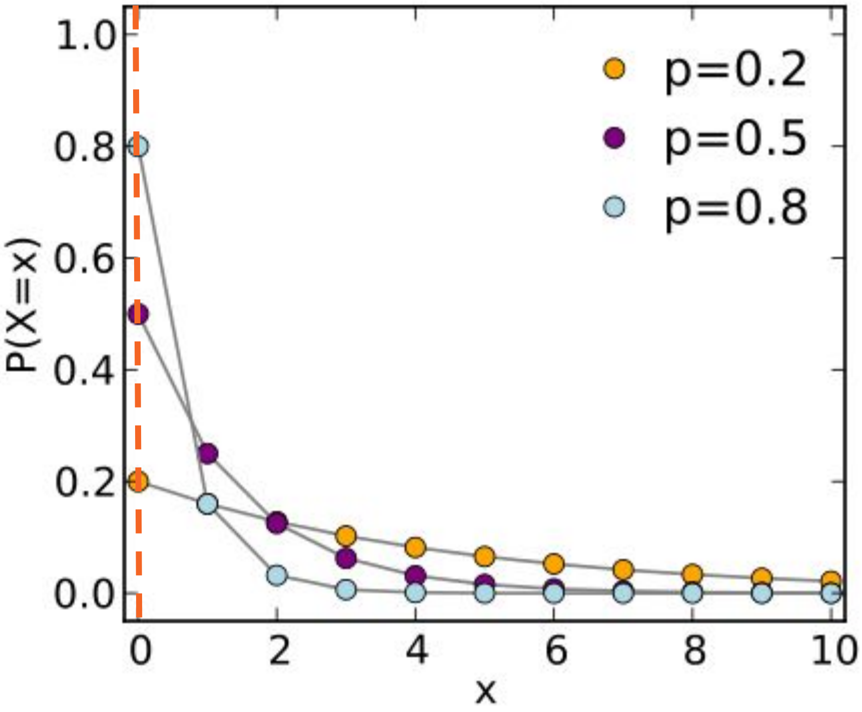


https://en.wikipedia.org/wiki/Geometric_distribution

Shifted geometric distribution



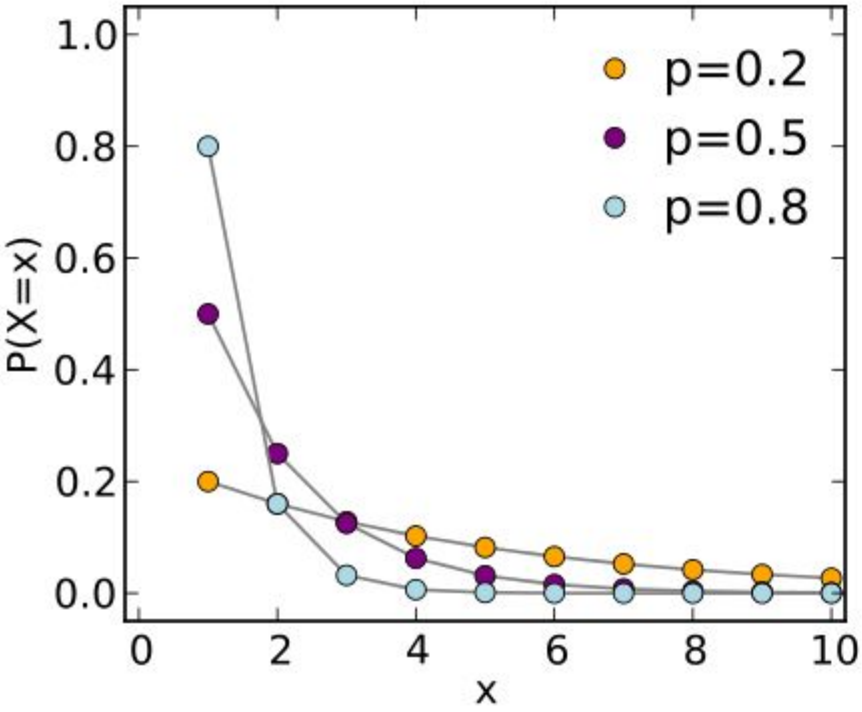
Geometric distribution



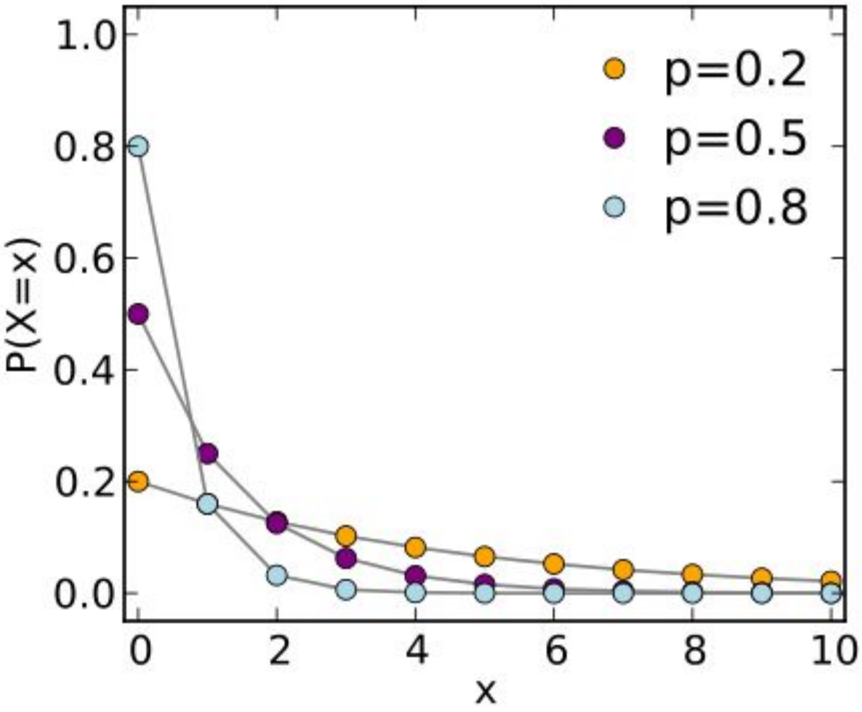
https://en.wikipedia.org/wiki/Geometric_distribution

Does *n* matter?

Shifted geometric distribution



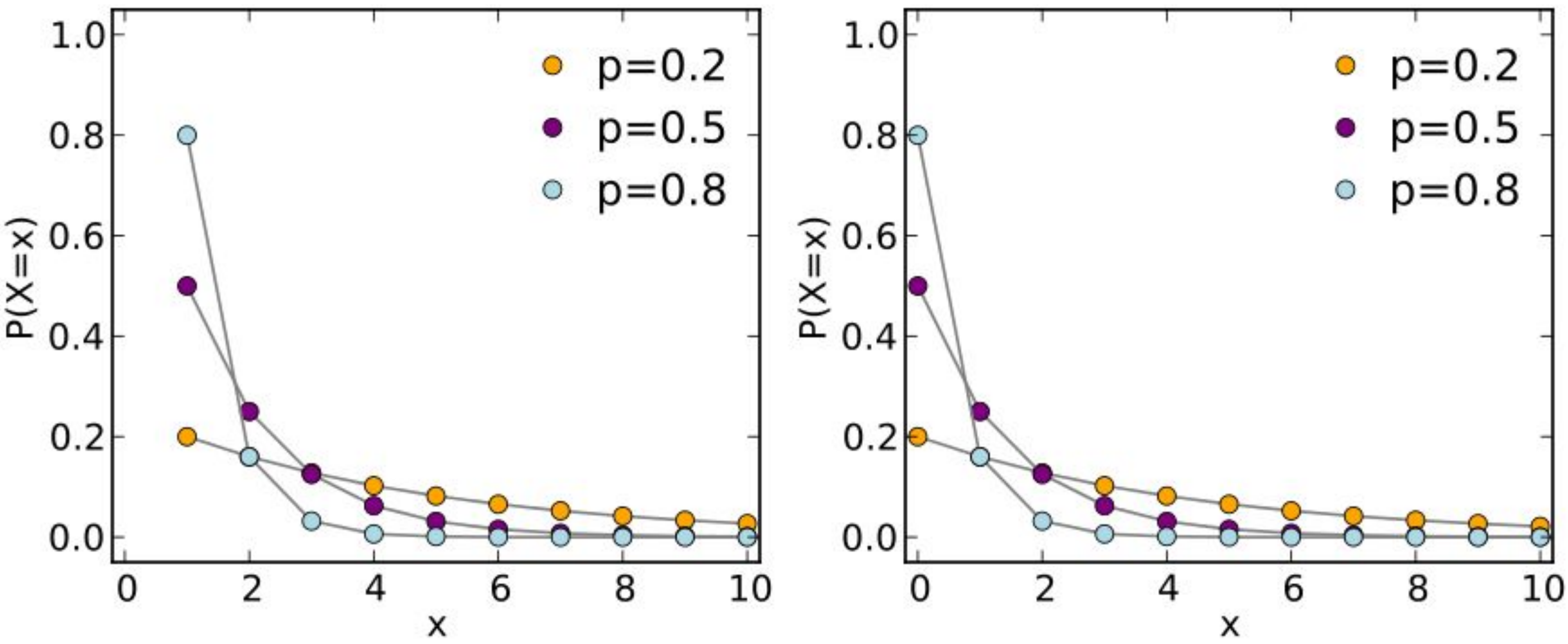
Geometric distribution



https://en.wikipedia.org/wiki/Geometric_distribution

Does *n* matter? No!

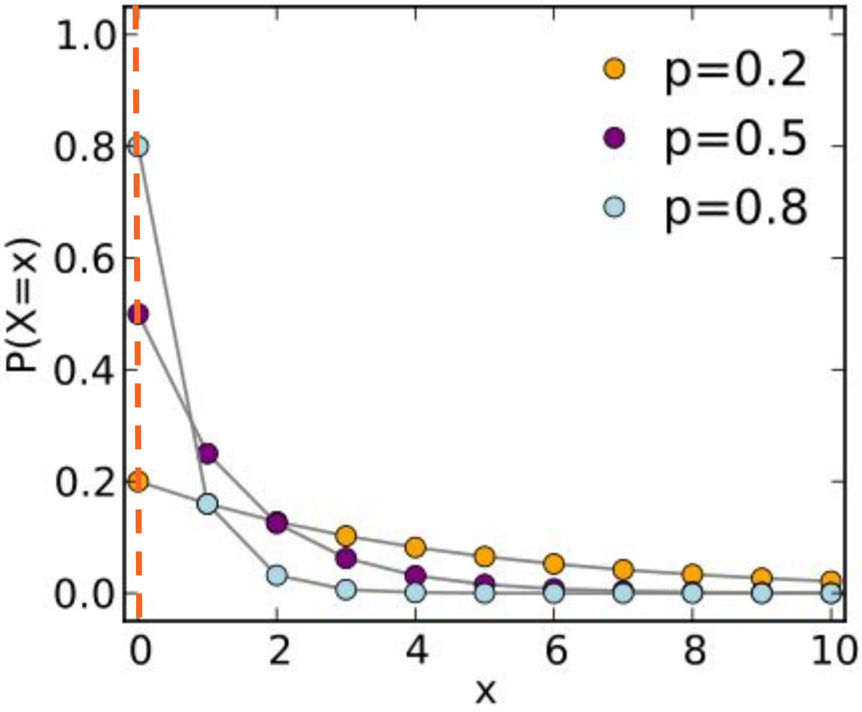
Shifted geometric distribution | **Geometric distribution**



https://en.wikipedia.org/wiki/Geometric_distribution

Geometric distribution

- x represents the number of failures before you get your first success
- P(X=10) = the probability that you survey 11 people, the first 10 people prefer dogs, and only the 11th person prefers cats



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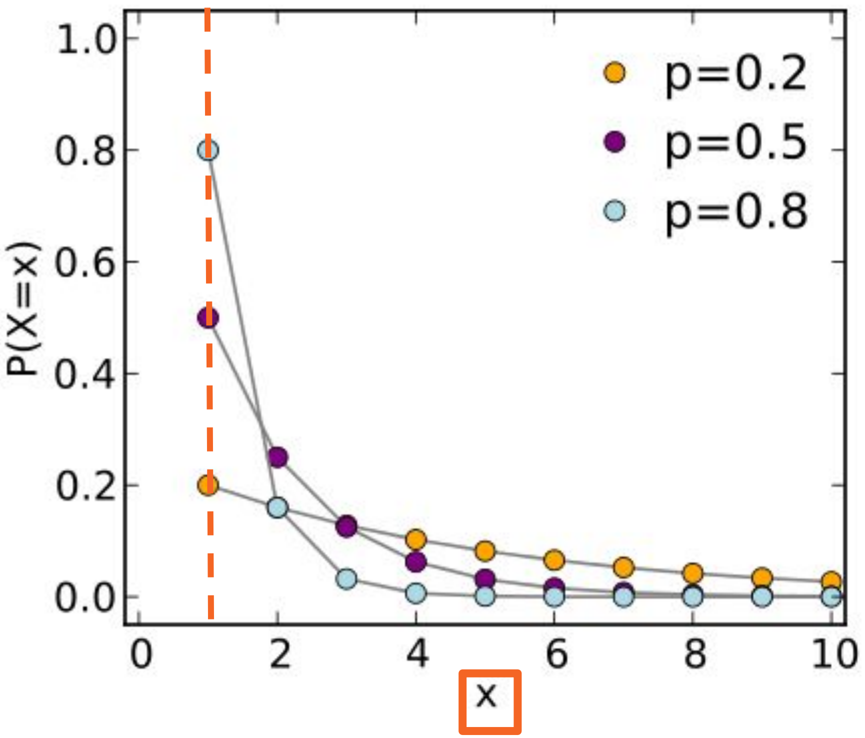
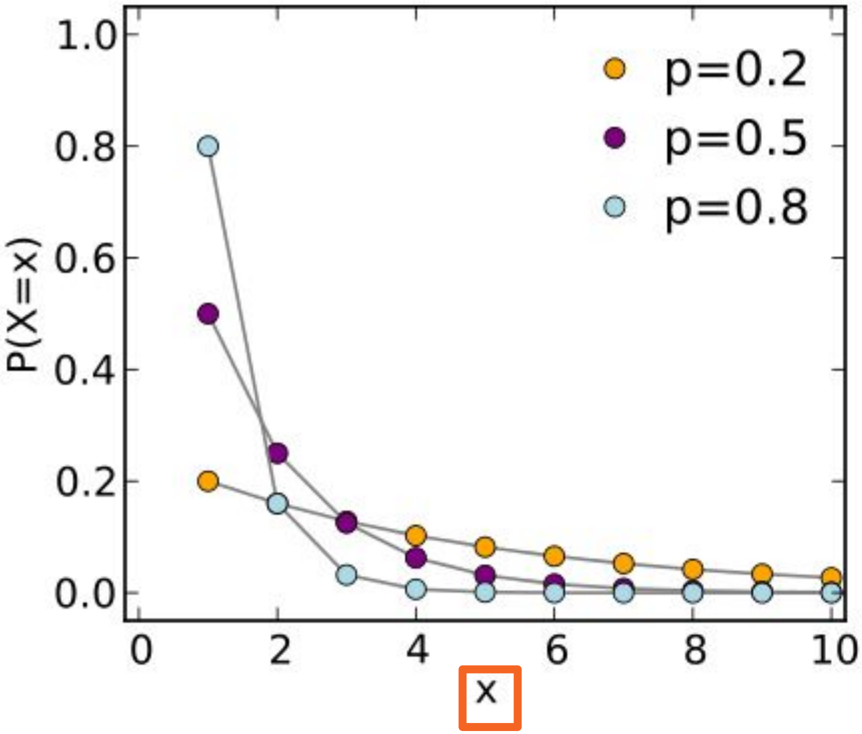
Shifted geometric distribution



- x represents this number being the first time success happens
- P(X=10) = the probability that you survey ___ people, the first ___ people prefer dogs, and only the ___th person prefers cats

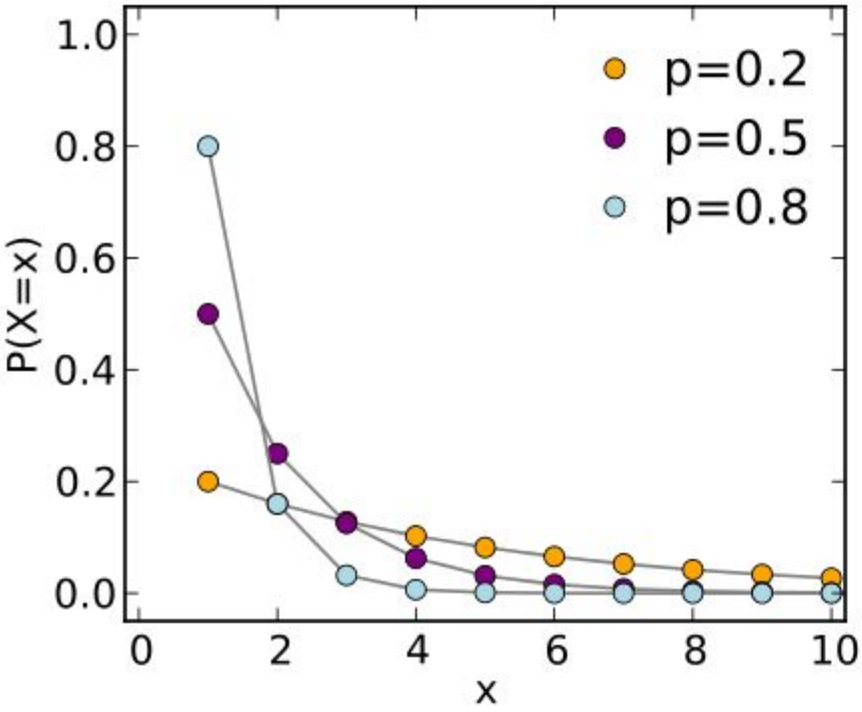
Shifted geometric distribution



- x represents this number being the first time something happens
- P(X=10) = the probability that you survey 10 people, the first 9 people prefer dogs, and only the 10th person prefers cats

https://en.wikipedia.org/wiki/Geometric_distribution

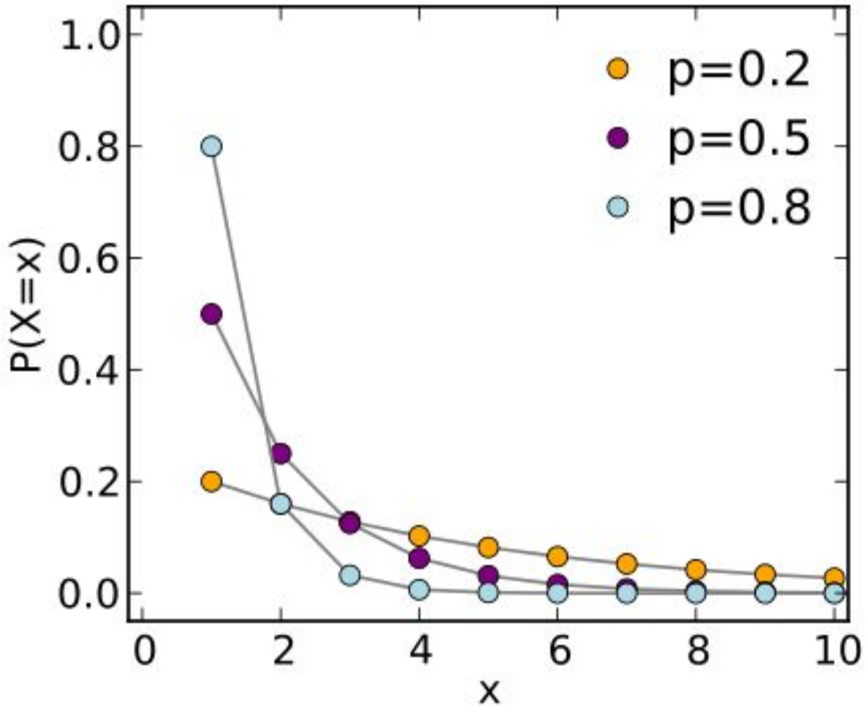
Shifted geometric distribution



Properties:

- Probability always decreases as x increases!

Shifted geometric distribution

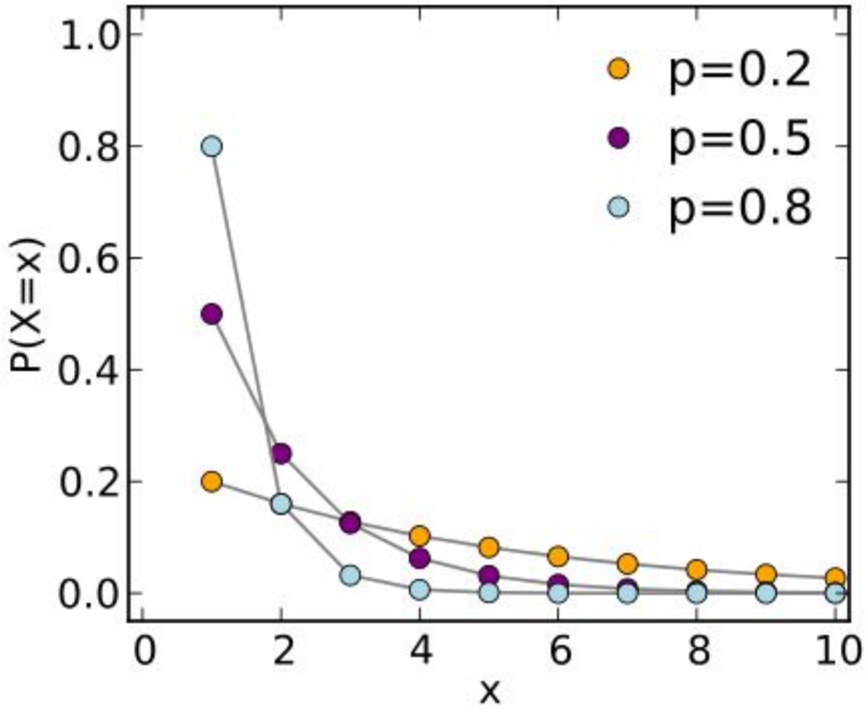


Properties:

- Probability always decreases as x increases!
- Memorylessness: the distribution of “waiting time” X doesn’t depend on how much time x has already elapsed.

https://en.wikipedia.org/wiki/Geometric_distribution

Shifted geometric distribution

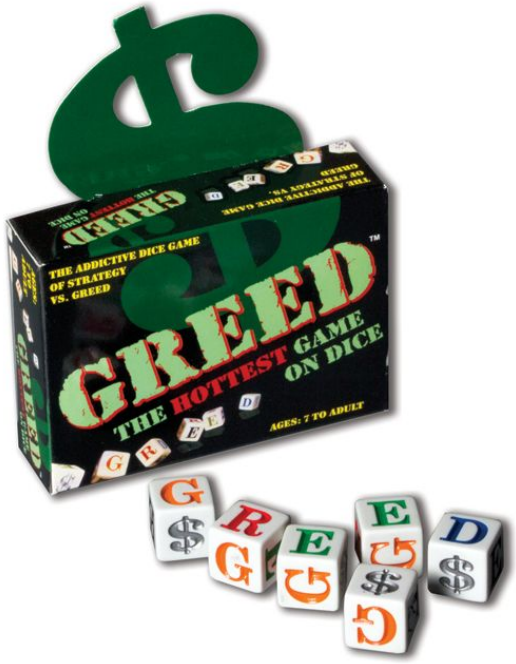


Properties:

- Probability always decreases as x increases!
- Memorylessness: the coin doesn't remember its past flips!

https://en.wikipedia.org/wiki/Geometric_distribution

The Greed Game: each round

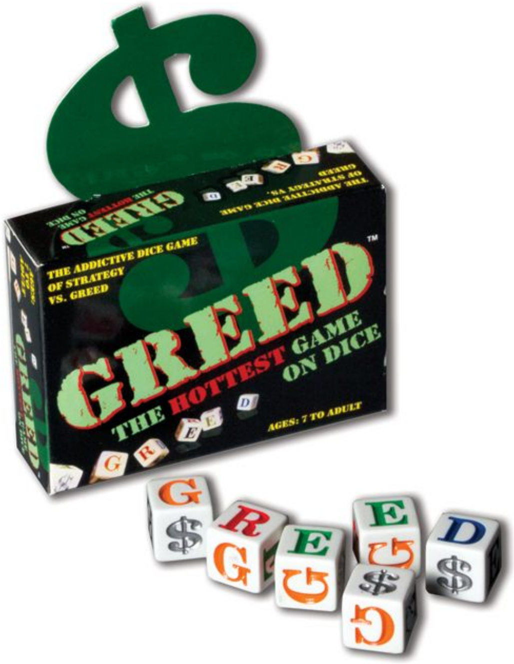


- If I roll a 1-5, you get +1 point
- If I roll a 6, you reset to 0 points and you're out of the game
- You can sit down at any time (if a 6 has not yet been rolled) and keep the points you've accumulated

https://woodexpressions.com/wp-content/uploads/2017/03/Greed.jpg

https://www.statsmedic.com/apstats-chapter6-day9

The Greed Game: rules



- Everyone starts by standing
- I roll a dice simulator once each round
- **On your whiteboard:** track your points (update each round): +1 for any 1-5 roll
- Sit down **when you choose to stop** playing, or **when I roll a 6** (reset to 0 points)
- Game ends when a 6 is rolled; winner has most points

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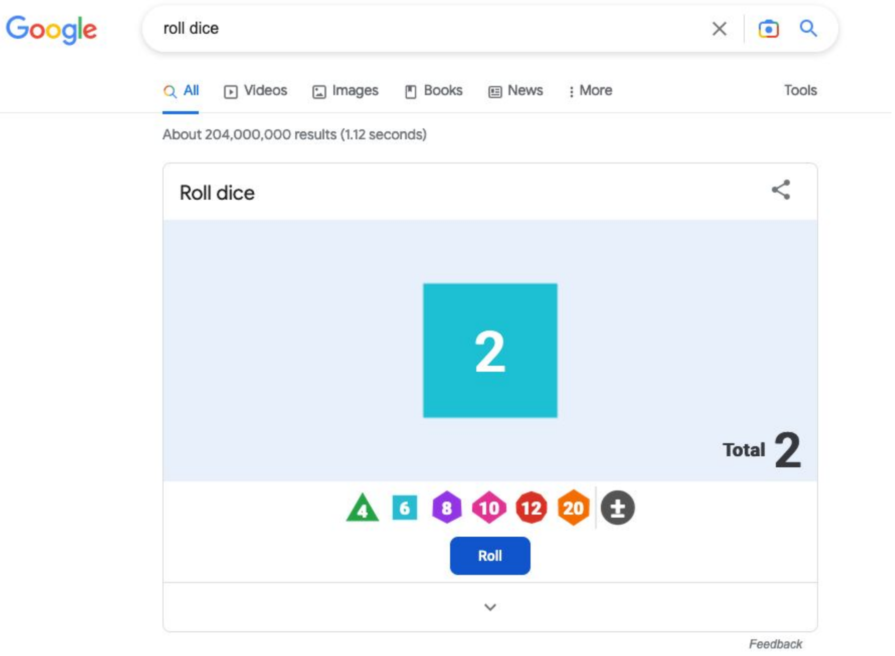
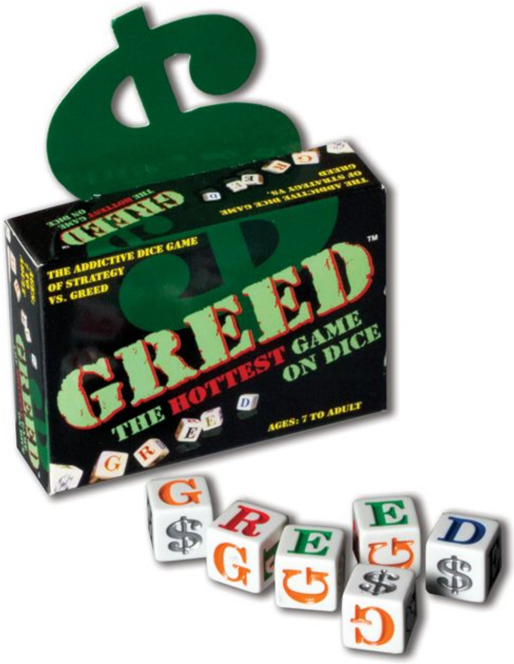
I will be using Google's unbiased 6-sided dice



Was there cheating?!



- How many points did the winner(s) have?
- How expected/unexpected is this?
- How can we tell?

https://woodexpressions.com/wp-content/uploads/2017/03/Greed.jpg

(Shifted) Geometric Distribution

Probability mass function	$P(X=k)=p(1-p)^{k-1}$	
Cumulative Distribution Function	$P(X\le k)=1-(1-p)^k$ $P(X\ge k)=(1-p)^{k-1}$ $P(X>k)=1-P(X\le k)=(1-p)^k$	$p = probability\ of\ success$ $k = \#\ of\ trials$
Mean:	$\mu=E(X)=\frac{1}{p}$	
Variance:	$\sigma^2=V(X)=\frac{(1-p)}{p^2}$	

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/

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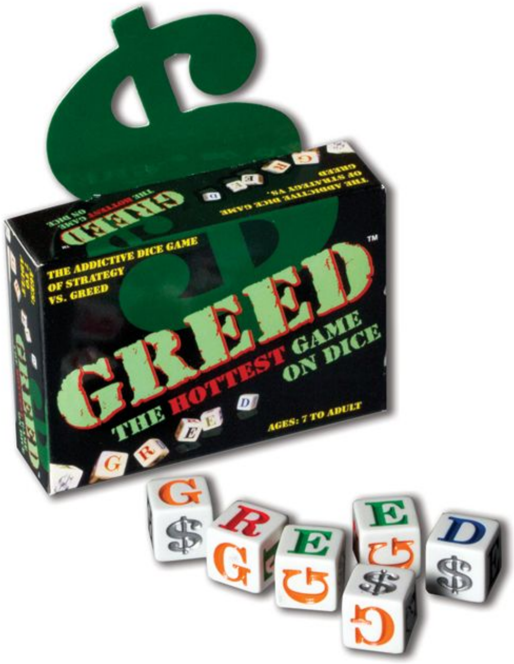
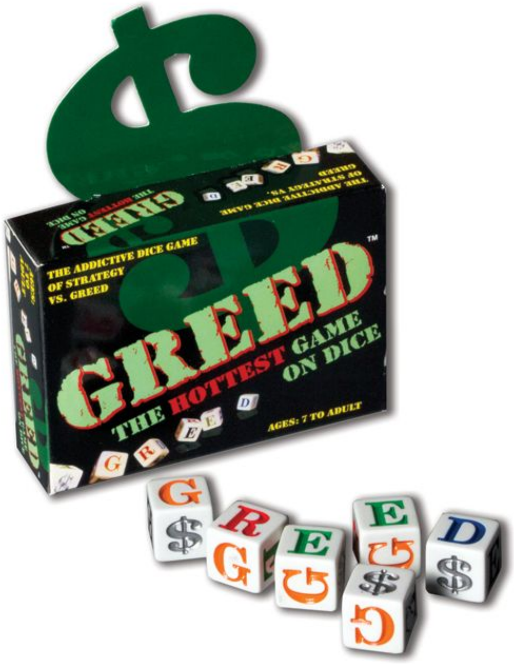
PMF vs. CDF



- How do we express the probability that we're looking to calculate?
- Let's assume our story X = the time to reach "success"
    - "Success" = # rolls until game end (which could be way after the winner of the game sits down)

https://woodexpressions.com/wp-content/uploads/2017/03/Greed.jpg

PMF vs. CDF



- X = the time to reach the end of the game
- Let *k* = some integer
- What is the difference between P(X=k) and P(X≥k)?

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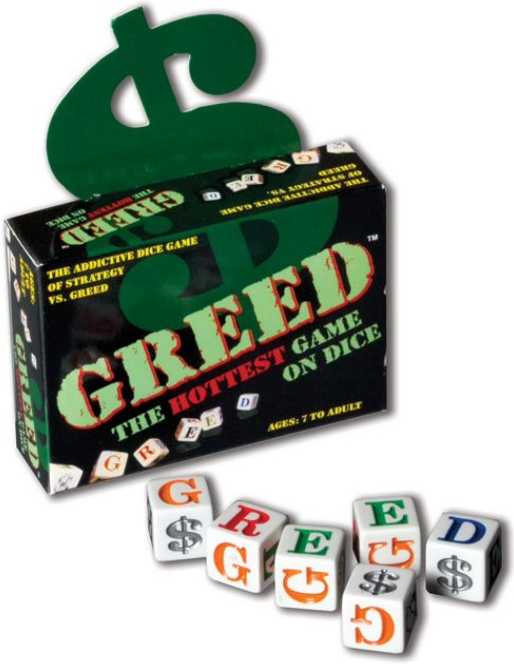
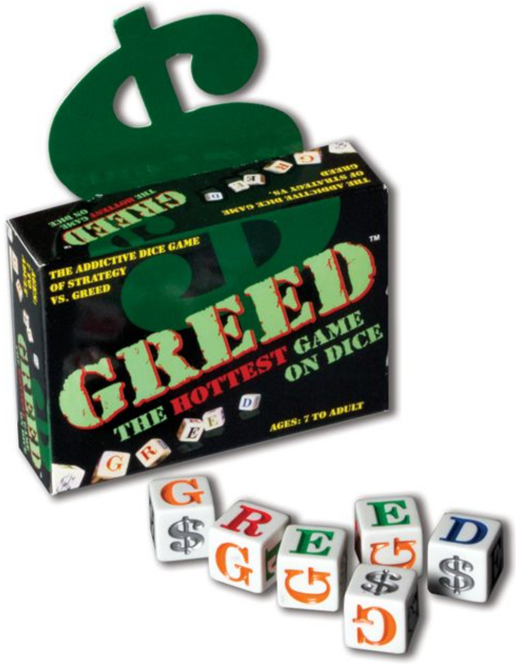
PMF vs. CDF



- X = the time to reach the end of the game
- Let *k* = some integer
- What is the difference between P(X=k) and P(X≥k)?
  - P(X=k) is the probability that it takes k time exactly for the game to end (i.e. a 6 is rolled on the k^{th} round). P(X≥k) is the probability that it takes **at least** k time for the game to end (i.e., a 6 was definitely **not** rolled in the first k-1 rounds).

https://woodexpressions.com/wp-content/uploads/2017/03/Greed.jpg

PMF vs. CDF



- X = the time to reach the end of the game
- To model the probability that someone wins the Greed Game with *k-1* points, should we use P(X=k) or P(X≥k)?

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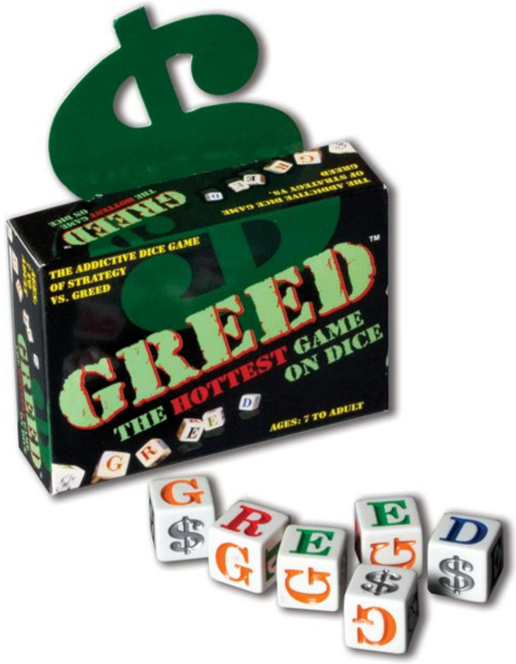
PMF vs. CDF



- X = the time to reach the end of the game
- To model the probability that someone wins the Greed Game with *k-1* points, should we use P(X=k) or **P(X≥k)**?

Someone wins by necessarily sitting down *before* a 6 is rolled. They must also be the last person to sit down, meaning no one else will be standing after the winner sits. So, <u>if the winner sits at time *k*, the 6 could be rolled anytime after k</u>, hence ending the game. This means we care more about P(X≥k).

https://woodexpressions.com/wp-content/uploads/2017/03/Greed.jpg

How likely is a winning result?

Shifted Geometric Distribution Cumulative Distribution Function

$$P(X \le k) = 1 - (1 - p)^{k}$$

$$\boxed{P(X \ge k) = (1 - p)^{k-1}}$$

$$P(X > k) = 1 - P(X \le k) = (1 - p)^{k}$$

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/

How likely is a winning result?

Shifted Geometric Distribution Cumulative Distribution Function	$P(X \le k) = 1 - (1-p)^k$ $\boxed{P(X \ge k) = (1-p)^{k-1}}$ $P(X > k) = 1 - P(X \le k) = (1-p)^k$

- What is the probability that someone won with 20 points?
  - X = time until game ends, k = ?

How likely is a winning result?

Shifted Geometric Distribution Cumulative Distribution Function	$P(X \le k) = 1 - (1-p)^k$ $P(X \ge k) = (1-p)^{k-1}$ $P(X > k) = 1 - P(X \le k) = (1-p)^k$

- What is the probability that someone won with 20 points?
  - X = time until game ends, k = 21 (number of "independent trials"; we have to roll the 6 *after* the winner sits down in round 20, so k = 21)

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/

How likely is a winning result?

Shifted Geometric Distribution Cumulative Distribution Function	$P(X \le k) = 1 - (1-p)^k$ $P(X \ge k) = (1-p)^{k-1}$ $P(X > k) = 1 - P(X \le k) = (1-p)^k$

- What is the probability that someone won with 20 points?
    - X = time until game ends, k = 21 (number of “independent trials”; we have to roll the 6 *after* the winner sits down in round 20, so k = 21)
    - What is *p*?

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/

How likely is a winning result?

Shifted Geometric Distribution Cumulative Distribution Function	$P(X \leq k) = 1 - (1-p)^k$ $P(X \geq k) = (1-p)^{k-1}$ $P(X > k) = 1 - P(X \leq k) = (1-p)^k$

- What is the probability that someone won with 20 points?
    - X = time until game ends, k = 21
    - What is *p*? *p* = probability of "success" (game ending) each trial = ⅙

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/

How likely is a winning result?

Shifted Geometric Distribution Cumulative Distribution Function	$P(X \le k) = 1-(1-p)^k$ $\boxed{P(X \ge k) = (1-p)^{k-1}}$ $P(X > k) = 1-P(X \le k) = (1-p)^k$

- What is the probability that someone won with 20 points?
    - X = time until game ends, k = 21
    - *p* = ⅙

How likely is a winning result?

Shifted Geometric Distribution Cumulative Distribution Function	$P(X \le k) = 1-(1-p)^k$ $\boxed{P(X \ge k) = (1-p)^{k-1}}$ $P(X > k) = 1 - P(X \le k) = (1-p)^k$

- What is the probability that someone won with 20 points?
  - The probability that the first "success" (i.e., the game ending) takes at least 21 trials = $P(X \ge 21) = (1-p)^{k-1} = (1-\frac{1}{6})^{20}$ = 0.026. Seems unlikely (magic dice?)!

How likely is a winning result?

Shifted Geometric Distribution Cumulative Distribution Function	$P(X \le k) = 1-(1-p)^k$ $\boxed{P(X \ge k) = (1-p)^{k-1}}$ $P(X > k) = 1 - P(X \le k) = (1-p)^k$

- What is the probability that someone won with [winning #] points?

How likely is a winning result?

Shifted Geometric Distribution Cumulative Distribution Function	$P(X \le k) = 1-(1-p)^k$ $\boxed{P(X \ge k) = (1-p)^{k-1}}$ $P(X > k) = 1 - P(X \le k) = (1-p)^k$

- What is the probability that someone won with [winning # = n] points?
- The probability that the first "success" (i.e., the game ending) takes at least *n+1* trials = $P(X \geq n+1) = (1-p)^n = (1-⅙)^n$ =?

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/

Using your intuition, on average, how many rolls do we think it *should* have taken to roll a 6?

Geometric distribution expectation

- We can model this with a geometric distribution, too!
- The **expected value** of the distribution = on average, how many trials do I have to experience before getting my first "success" ***(here "success" = rolling a 6)***?

(Shifted) Geometric Distribution

Probability mass function	$P(X=k)=p(1-p)^{k-1}$
Cumulative Distribution Function	$P(X\le k)=1-(1-p)^{k}$ $P(X\ge k)=(1-p)^{k-1}$ $P(X>k)=1-P(X\le k)=(1-p)^{k}$
Mean:	$\mu=E(X)=\dfrac{1}{p}$
Variance:	$\sigma^2=V(X)=\dfrac{(1-p)}{p^2}$

$p = probability\ of\ success$

$k = \#\ of\ trials$

1 / (⅙) = 6 rolls

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/

On average, how many rolls do we think it should take to roll a 20 when using a 20-sided die?

(Shifted) Geometric Distribution

Probability mass function	$P(X=k)=p(1-p)^{k-1}$
Cumulative Distribution Function	$P(X\le k)=1-(1-p)^k$ $P(X\ge k)=(1-p)^{k-1}$ $P(X>k)=1-P(X\le k)=(1-p)^k$
Mean:	$\mu=E(X)=\frac{1}{p}$
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$p = probability\ of\ success$

$k = \#\ of\ trials$

1 / (1/20) = 20 rolls

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/

On average, how many rolls do we think it should take to roll a 6 when using a 20-sided die?

(Shifted) Geometric Distribution

Probability mass function	$P(X=k)=p(1-p)^{k-1}$
Cumulative Distribution Function	$P(X\le k)=1-(1-p)^{k}$ $P(X\ge k)=(1-p)^{k-1}$ $P(X>k)=1-P(X\le k)=(1-p)^{k}$
Mean:	$\mu=E(X)=\frac{1}{p}$
Variance:	$\sigma^2=V(X)=\frac{(1-p)}{p^2}$

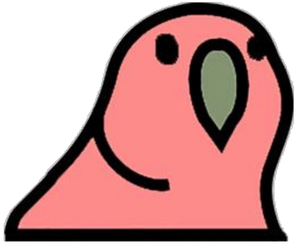
$p = probability\ of\ success$

$k = \#\ of\ trials$

Still 1 / (1/20) = 20 rolls

The probability of rolling a specific number (6) out of the total # sides of the die (20) is still 1 number / 20 options

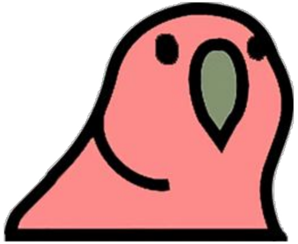
https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/



Geometric distribution

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability that we find the first parrot-loving student as the 6th student we ask?

P(X=k) or P(X≥k)?



Geometric distribution

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability that we find the first parrot-loving student as the 6th student we ask?

$$p = \square$$

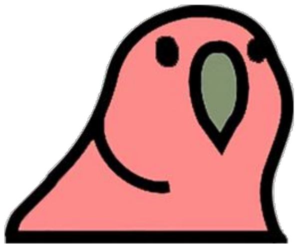
$$k = \square$$

$$\boxed{P(X = k)} = p(1-p)^{k-1}$$

$$P(X = \square) = \square$$

https://www.producthunt.com/products/party-parrots

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/



Geometric distribution

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability that we find the first parrot-loving student as the 6th student we ask?

$$p = \boxed{}$$

$$k = \boxed{}$$

$$P(X = k) = p(1-p)^{k-1}$$

$$P(X = \boxed{}) = \boxed{}$$

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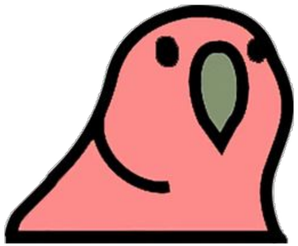

Geometric distribution

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability that we find the first parrot-loving student as the 6th student we ask?

$$p = \frac{3}{75} = 0.04$$

$$k = 6$$

$$P(X = k) = p(1 - p)^{k-1}$$

$$P(X = 6) = 0.04(1 - 0.04)^{6-1}$$

$$P(X = 6) = \boxed{0.04(0.96)^5} = 0.0326$$

https://www.producthunt.com/products/party-parrots

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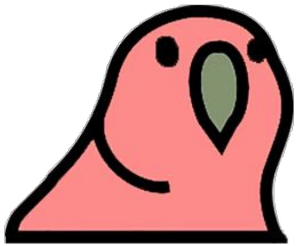

Geometric distribution

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **at least 6 students** to find the first parrot-loving student?

$P(X=k)$ or $P(X \geq k)$?

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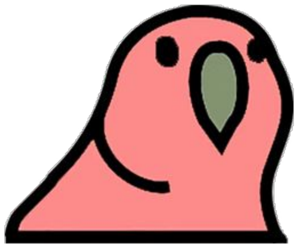

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$$p = \boxed{}$$

$$k \geq \boxed{}$$

$$\boxed{P(X \geq k)} = (1-p)^{k-1}$$

$$P(X \geq \boxed{}) = \boxed{}$$

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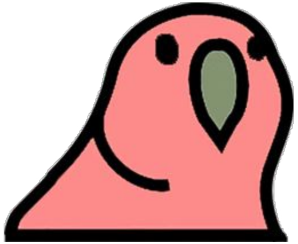


Geometric distribution

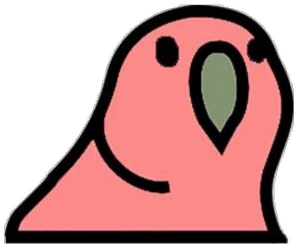
We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **at least 6 students** to find the first parrot-loving student?

$$p = \boxed{}$$

$$k \geq \boxed{}$$

$$P(X \geq k) = (1 - p)^{k-1}$$

$$P(X \geq \boxed{}) = \boxed{}$$



Geometric distribution

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **at least 6 students** to find the first parrot-loving student?

$$p = \frac{3}{75} = 0.04$$

$$k \geq 6$$

$$P(X \geq k) = (1 - p)^{k-1}$$

$$P(X \geq 6) = (1 - 0.04)^{6-1}$$

$$P(X \geq 6) = \boxed{(0.96)^{5}} = 0.815$$

https://www.producthunt.com/products/party-parrots

https://calcworkshop.com/discrete-probability-distribution/geometric-distribution/

Geometric Distribution in Python

1. **No packages:** calculate the pmf or cdf using the formulas we just used

2. **Numpy:** generate the geometric distribution

3. **Scipy:** use scipy.stats.geom to then call functions like pmf() and cdf()

Geometric Distribution in Python

No need for k because this just gives you a random draw from a geometric distribution

numpy.random.geometric

`random.geometric(p, size=None)`

How many draws you want from the distribution

Draw samples from the geometric distribution.

Bernoulli trials are experiments with one of two outcomes: success or failure (an example of such an experiment is flipping a coin). The geometric distribution models the number of trials that must be run in order to achieve success. It is therefore supported on the positive integers, `k = 1, 2, ...`.

The probability mass function of the geometric distribution is

$$f(k) = (1 - p)^{k-1} p$$

where p is the probability of success of an individual trial.

Geometric Distribution in Python

rvs(p, loc=0, size=1, random_state=None)	Random variates.
pmf(k, p, loc=0)	Probability mass function.
logpmf(k, p, loc=0)	Log of the probability mass function.
cdf(k, p, loc=0)	Cumulative distribution function.
logcdf(k, p, loc=0)	Log of the cumulative distribution function.
sf(k, p, loc=0)	Survival function (also defined as `1 - cdf`, but *sf* is sometimes more accurate).
logsf(k, p, loc=0)	Log of the survival function.
ppf(q, p, loc=0)	Percent point function (inverse of `cdf` — percentiles).
isf(q, p, loc=0)	Inverse survival function (inverse of `sf`).
stats(p, loc=0, moments='mv')	Mean('m'), variance('v'), skew('s'), and/or kurtosis('k').
entropy(p, loc=0)	(Differential) entropy of the RV.
expect(func, args=(p,), loc=0, lb=None, ub=None, conditional=False)	Expected value of a function (of one argument) with respect to the distribution.
median(p, loc=0)	Median of the distribution.
mean(p, loc=0)	Mean of the distribution.
var(p, loc=0)	Variance of the distribution.
std(p, loc=0)	Standard deviation of the distribution.
interval(confidence, p, loc=0)	Confidence interval with equal areas around the median.

scipy.stats.geom

`scipy.stats.geom = <scipy.stats._discrete_distns.geom_gen object>`

A geometric discrete random variable.

Recap on geometric distribution:

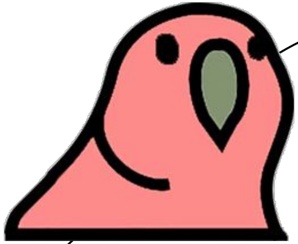
Use when…

1. Each trial has only 2 possible outcomes: "success" (1) or "failure" (0)
2. You conduct each "trial" exactly the same way and **count the # trials until the first success**
3. The probability of success p is the same for each trial
4. Trials are independent

1 min break

Draw your favorite emoji, if you'd like!

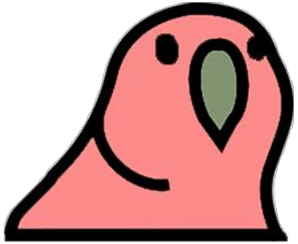
https://imgflip.com/i/1xtebc



Geometric Distribution

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the **first** parrot-loving student?

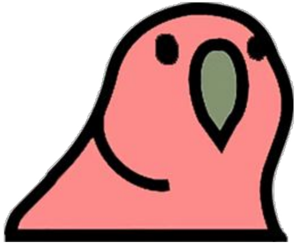
https://www.producthunt.com/products/party-parrots



Can we still use geometric distribution?

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **three** parrot-loving students?

https://www.producthunt.com/products/party-parrots



Can we still use geometric distribution?

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **three** parrot-loving students?

No, we need to introduce a new distribution!

Negative Binomial

Event space: integers **r** to infinity

Parameters: chance of "success" p

Story: how many trials before the ~~first~~ r^{th} success?

Negative Binomial

Event space: integers **r** to infinity

Parameters: chance of "success" p

Story: how many trials before the ~~first~~ r^{th} success?

$$f(x) = P(X = x) = \binom{x-1}{r-1} (1-p)^{x-r} p^r$$

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial

Event space: integers r to infinity

Parameters: chance of "success" p

Story: how many trials before the ~~first~~ r^{th} success?

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

Notation? Where'd *k* go?

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial

Event space: integers **r** to infinity

Parameters: chance of "success" p

Story: how many trials before the ~~first~~ rth success?

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

little x represents the same thing as k

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial

Event space: integers **r** to infinity

Parameters: chance of "success" p

Story: how many trials before the ~~first~~ rth success?

```
def P(X, r, p):
        return [some math]
```

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial

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```
def P(X, r, p):
        return [some math]
```

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

P(X=4, r=3, p=0.5)

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial

Event space: integers **r** to infinity

Parameters: chance of "success" p

Story: how many trials before the ~~first~~ rth success?

The probability of running 4 trials before reaching the 3rd success, if each success occurs with probability 0.5

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

P(X=4, r=3, p=0.5)

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial

Geometric distributions are just a special case of negative binomial when r = 1!

Geometric PMF →

$$P(X=k)=p(1-p)^{k-1}$$

Neg Bin PMF →

$$f(x)=P(X=x)=\binom{x-1}{r-1}(1-p)^{x-r}p^r$$

Negative Binomial

Geometric distributions are just a special case of negative binomial when r = 1!

Geometric PMF →

$$P(X = k) = p(1 - p)^{k-1}$$

Neg Bin PMF when r = 1 →

$$f(x) = P(X = x) = \boxed{\binom{x-1}{r-1}}(1-p)^{x-r}p^r$$

(x-1 choose 0) is 1 (there's only 1 way to pick nothing)

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial

Geometric distributions are just a special case of negative binomial when r = 1!

Geometric PMF →

$$P(X = k) = p(1 - p)^{k-1}$$

Neg Bin PMF when r = 1 →

$$f(x) = P(X = x) = \cancel{\boxed{}}\,(1 - p)^{x-1}p^{1}$$

Negative Binomial

Geometric distributions are just a special case of negative binomial when r = 1!

Geometric PMF →

$$P(X = k) = p(1 - p)^{k-1}$$

Neg Bin PMF when r = 1 →

$$f(x) = P(X = x) = \cancel{\boxed{}}\,(1 - p)^{x-\cancel{r}\,1}\,p^{\cancel{r}\,1}$$

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial

Event space: integers **r** to infinity

Parameters: chance of "success" p

Story: how many trials before the **r**$^{\text{th}}$ success?

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial

Event space: integers **r** to infinity

Parameters: chance of "success" p

Story: how many trials before the r^{th} success?

If r = 1, then this is just the # trials before the first success (geometric distribution)

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial Properties

$$\mu = E(X) = \frac{r}{p}$$

$$\sigma^2 = Var(x) = \frac{r(1-p)}{p^2}$$

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial Properties

At what value of *r* does the expected value of Negative Binomial dist equal the expected value of Geometric distribution?

$$\mu = E(X) = \frac{r}{p}$$

$$\sigma^2 = Var(x) = \frac{r(1-p)}{p^2}$$

https://online.stat.psu.edu/stat414/lesson/11/11.5

Negative Binomial Properties

When r=1, we get E(X) = 1/p, which is what we had with the geometric distribution expected value!

$$\mu = E(X) = \frac{r}{p}$$

$$\sigma^2 = Var(x) = \frac{r(1-p)}{p^2}$$

Negative Binomial in Python

numpy.random.negative_binomial

```
random.negative_binomial(n, p, size=None)
```

Draw samples from a negative binomial distribution.

Samples are drawn from a negative binomial distribution with specified parameters, *n* successes and *p* probability of success where *n* is > 0 and *p* is in the interval [0, 1].

https://numpy.org/doc/stable/reference/random/generated/numpy.random.negative_binomial.html

Negative Binomial

Event space: integers **r** to infinity

Parameters: chance of "success" p

Story: how many trials before the ~~first~~ r^{th} success?

Be careful: lots of different variants of the NB distribution!

Negative Binomial(ish)

Event space: integers **0** to infinity

Parameters: chance of "success" p

Story: how many ~~trials~~ before the ~~first~~ rth success?

failures

This is an 'alternative' formulation to the NB distribution, sometimes called the Pascal distribution

Neg Bin (Pascal) Distribution Properties

X = how many **failures** before the **rth** success?

$$\Pr(X = k) = \binom{k+r-1}{r-1}(1-p)^k p^r$$

the mean is $(1-p)r/p$ and the variance is $(1-p)r/p^2$

https://en.wikipedia.org/wiki/Negative_binomial_distribution

Neg Bin (Pascal) Distribution Properties

X = how many **failures** before the **rth** success?

$$\Pr(X = k) = \binom{k + r - 1}{r - 1}(1 - p)^k p^r$$

the mean is $(1 - p)r/p$ and the variance is $(1 - p)r/p^2$

Lots of programming languages (e.g. R) default to the Pascal distribution when you ask for a negative binomial distribution!

https://en.wikipedia.org/wiki/Negative_binomial_distribution

Neg Bin vs. Neg Bin (Pascal)

Neg Bin PMF →

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

Pascal PMF →

$$\Pr(X = k) = \binom{k+r-1}{r-1}(1-p)^k p^r$$

https://en.wikipedia.org/wiki/Negative_binomial_distribution
https://online.stat.psu.edu/stat414/lesson/11/11.5

Neg Bin vs. Neg Bin (Pascal)

Neg Bin PMF →

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

Pascal PMF →

$$\Pr(X = k) = \binom{k+r-1}{r-1}(1-p)^k p^r$$

If we let *x = k+r*, these are the exact same thing!

(Analogous to shifting by *r*, like between geometric dist and shifted geometric dist shifted by *r=1*)

https://en.wikipedia.org/wiki/Negative_binomial_distribution
https://online.stat.psu.edu/stat414/lesson/11/11.5

Neg Bin vs. Neg Bin (Pascal)

Counting total trials *x* to reach *r* successes →

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

Counting total failures *k* until *r* successes →

$$\Pr(X = k) = \binom{k+r-1}{r-1}(1-p)^k p^r$$

It is always true that *x* = *k*+*r*, since total trials *x* = total failures *k* + total successes *r*

https://en.wikipedia.org/wiki/Negative_binomial_distribution
https://online.stat.psu.edu/stat414/lesson/11/11.5

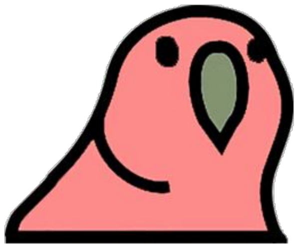
Tons of different formulations!

Be careful: always check documentation for distribution properties!

If you blindly copy numbers out of Python, you might be interpreting things incorrectly!

	X is counting...	Probability mass function	Formula	Alternate formula (using equivalent binomial)	Alternate formula (simplified using: $n = k + r$)	Support
1	*k* failures, given *r* successes	$f(k; r, p) \equiv \Pr(X = k) =$	$\binom{k+r-1}{k} p^r (1-p)^k$ [7][5][8]	$\binom{k+r-1}{r-1} p^r (1-p)^k$ [2] [9][10][11]	$\binom{n-1}{k} p^r (1-p)^k$	for $k = 0, 1, 2, \ldots$
2	*n* trials, given *r* successes	$f(n; r, p) \equiv \Pr(X = n) =$	$\binom{n-1}{r-1} p^r (1-p)^{n-r}$ [5][11][12][13][14]	$\binom{n-1}{n-r} p^r (1-p)^{n-r}$		for $n = r, r+1, r+2, \ldots$
3	*n* trials, given *r* failures	$f(n; r, p) \equiv \Pr(X = n) =$	$\binom{n-1}{r-1} p^{n-r} (1-p)^r$	$\binom{n-1}{n-r} p^{n-r} (1-p)^r$	$\binom{n-1}{k} p^k (1-p)^r$	
4	*k* successes, given *r* failures	$f(k; r, p) \equiv \Pr(X = k) =$	$\binom{k+r-1}{k} p^k (1-p)^r$	$\binom{k+r-1}{r-1} p^k (1-p)^r$		for $k = 0, 1, 2, \ldots$
-	*k* successes, given *n* trials	$f(k; n, p) \equiv \Pr(X = k) =$	This is the binomial distribution not the negative binomial: $\binom{n}{k} p^k (1-p)^{n-k}$			for $k = 0, 1, 2, \ldots, n$

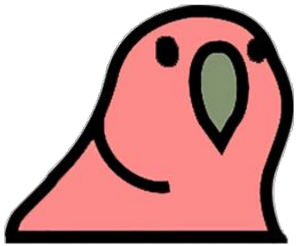
https://en.wikipedia.org/wiki/Negative_binomial_distribution



Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **3** parrot-loving students?

https://www.producthunt.com/products/party-parrots

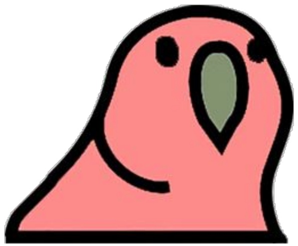


Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **3** parrot-loving students?

X = number of students to survey
x = ?
r = ?
p = ?
Want to calculate P(?)

https://www.producthunt.com/products/party-parrots

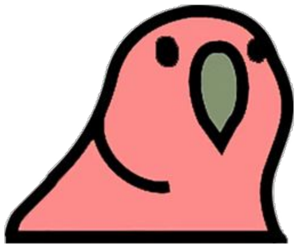


Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **3** parrot-loving students?

X = number of students to survey
x = 6 (?)
r = 3
p = 3/75 = 0.04
Want to calculate P(X=x)

https://www.producthunt.com/products/party-parrots



Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **3** parrot-loving students?

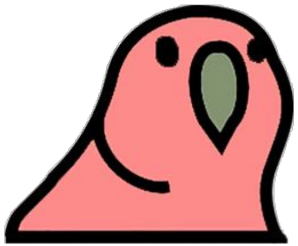
X = number of students to survey

x = 6, r = 3, p = 3/75, want to find P(X = x)

Formulation: do we want to count
total # trials or total # failures?

https://www.producthunt.com/products/party-parrots

https://online.stat.psu.edu/stat414/lesson/11/11.5



Negative Binomial example

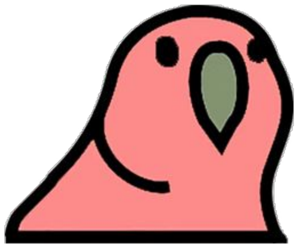
We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **3** parrot-loving students?

X = number of students to survey
x = 6, r = 3, p = 3/75, want to find P(X = x)
where x = **total number of trials**

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

https://www.producthunt.com/products/party-parrots

https://online.stat.psu.edu/stat414/lesson/11/11.5



Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **3** parrot-loving students?

X = number of students to survey
x = 6, r = 3, p = 3/75, want to find P(X = x)
where x = **total number of trials**

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

If x = total # failures, then we'd *instead* want to study P(X=3) using the Pascal dist.

https://www.producthunt.com/products/party-parrots

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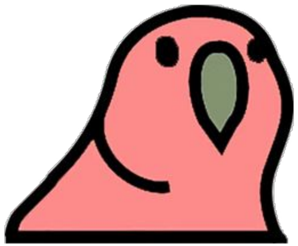

Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. **What is the probability it takes asking 6 students to find the first 3 parrot-loving students?**

X = number of students to survey
x = 6, r = 3, p = 3/75, want to find P(X = x)
where x = total number of trials

$$f(x) = P(X = x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$$

https://www.producthunt.com/products/party-parrots

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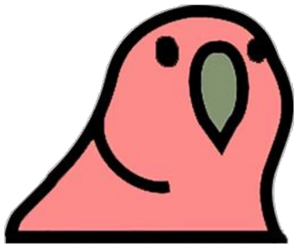


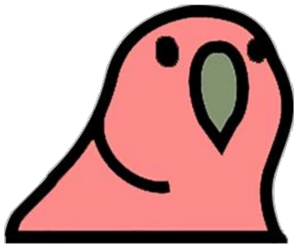
Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **3** parrot-loving students?

X = total number of trials = total number of students to survey (i.e. trials until 3 successes)

$P(X = k) = (5 \text{ choose } 2) * (1-(3/75))^{6-3} * (3/75))^{3} =$

$\mathbf{10*0.96^{3}*0.04^{3}} = 0.000566$



Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 students** to find the first **3** parrot-loving students?

X = total number of trials = total number of students to survey (i.e. trials until 3 successes)

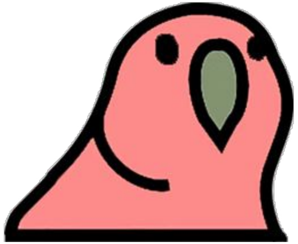
P(X = k) = (5 choose 2) * $(1-(3/75))^{6-3}$ * $(3/75))^{3}$ =

$10*0.96^{3}*0.04^{3}$= 0.000566

Unlikely it'd take only 6 trials to find 3 parrot lovers!

https://www.producthunt.com/products/party-parrots

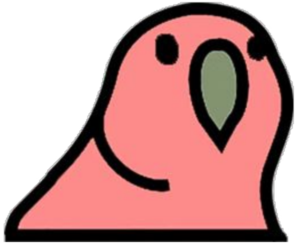
https://online.stat.psu.edu/stat414/lesson/11/11.5



Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 not-parrot-loving students** to find the first **3** parrot-loving students?

https://www.producthunt.com/products/party-parrots

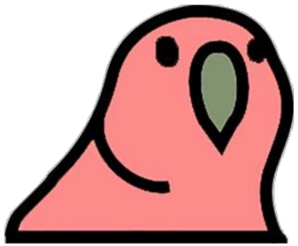


Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 not-parrot-loving students** to find the first **3** parrot-loving students?

Formulation: do we want to count **total # trials or total # failures?**

https://www.producthunt.com/products/party-parrots



Negative Binomial example

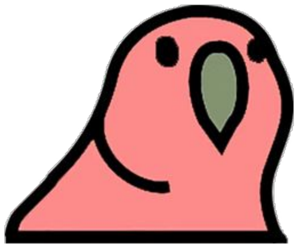
We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 not-parrot-loving students** to find the first **3** parrot-loving students?

$$\Pr(X = k) = \binom{k+r-1}{r-1}(1-p)^k p^r$$

(Now, k is counting **total # failures**)

https://www.producthunt.com/products/party-parrots

https://en.wikipedia.org/wiki/Negative_binomial_distribution



Negative Binomial example

We survey students, and we know that on average 3 out of every 75 students love parrots. What is the probability it takes asking **6 not-parrot-loving students** to find the first **3** parrot-loving students?

X = number of students to fail in survey
k = 6, r = 3, p = 3/75, want to find P(X = k)

$$\Pr(X = k) = \binom{k+r-1}{r-1}(1-p)^k p^r$$

https://www.producthunt.com/products/party-parrots

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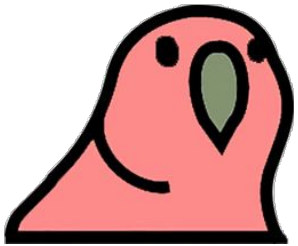

Negative Binomial example

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Alternatively, we could have X = total # surveyed using our Neg Bin formulation from before, but use Pr(X=9)

X = number of students to fail in survey
k = 6, r = 3, p = 3/75, want to find P(X = k)

$$\Pr(X = k) = \binom{k+r-1}{r-1}(1-p)^k p^r$$

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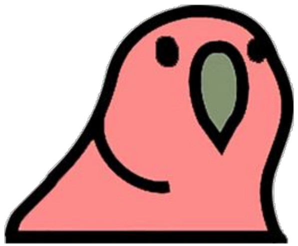

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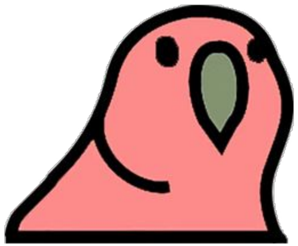
X = **number of students to fail in survey**

$P(X = k) = (8 \text{ choose } 2) * (1-(3/75))^6 * (3/75))^3 =$

$\mathbf{28*0.96^6*0.04^3} = 0.0014$



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P(X = k) = (8 choose 2) * $(1-(3/75))^6 * (3/75))^3$ =
$28*0.96^6*0.04^3$= 0.0014

More likely to find 3 parrot lovers when you have 6 failures as compared to having 6 total trials (0.000566)

https://www.producthunt.com/products/party-parrots

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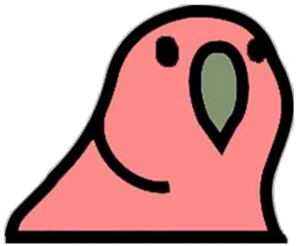
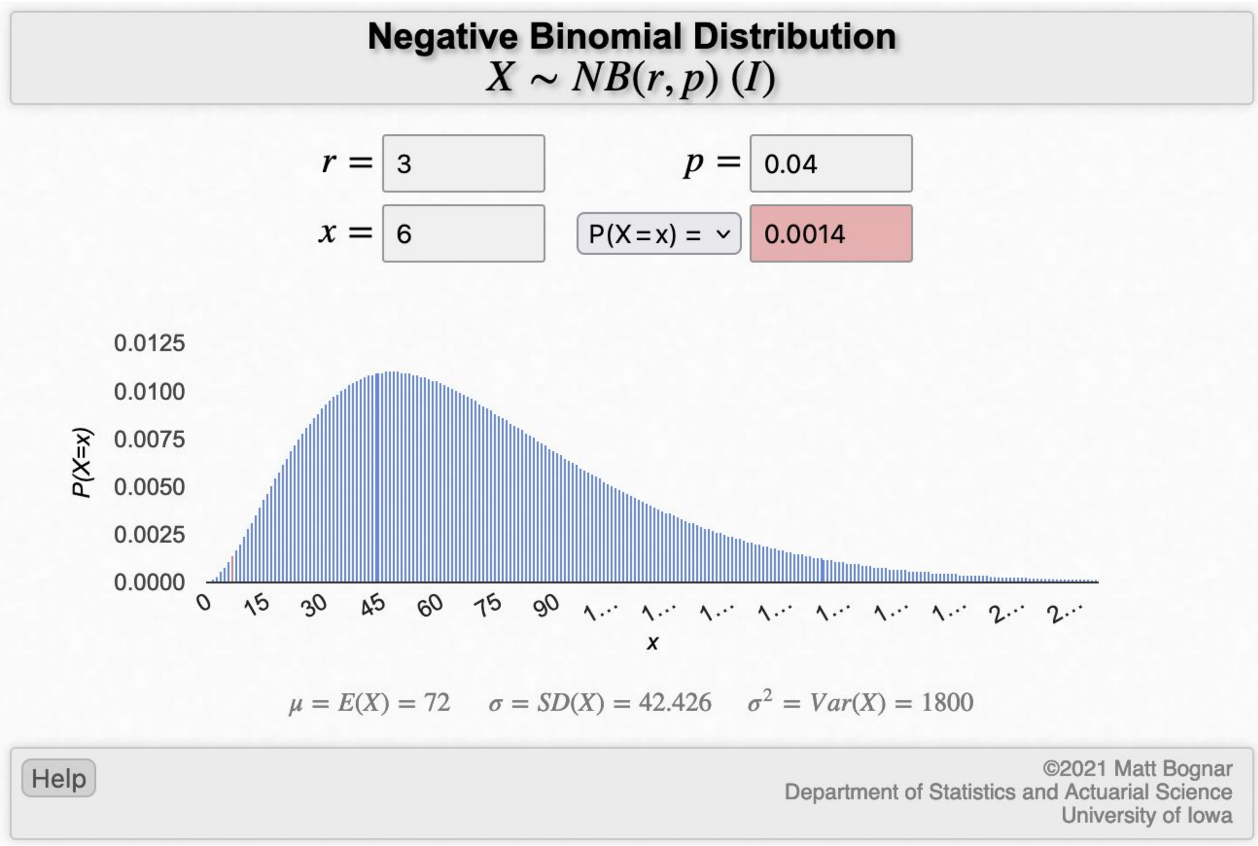


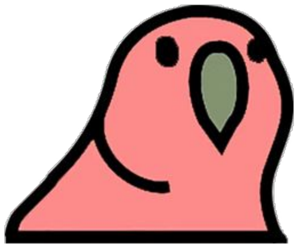
Which NB formulation is this? (**k = # trials or # failures?**)



https://www.producthunt.com/products/party-parrots

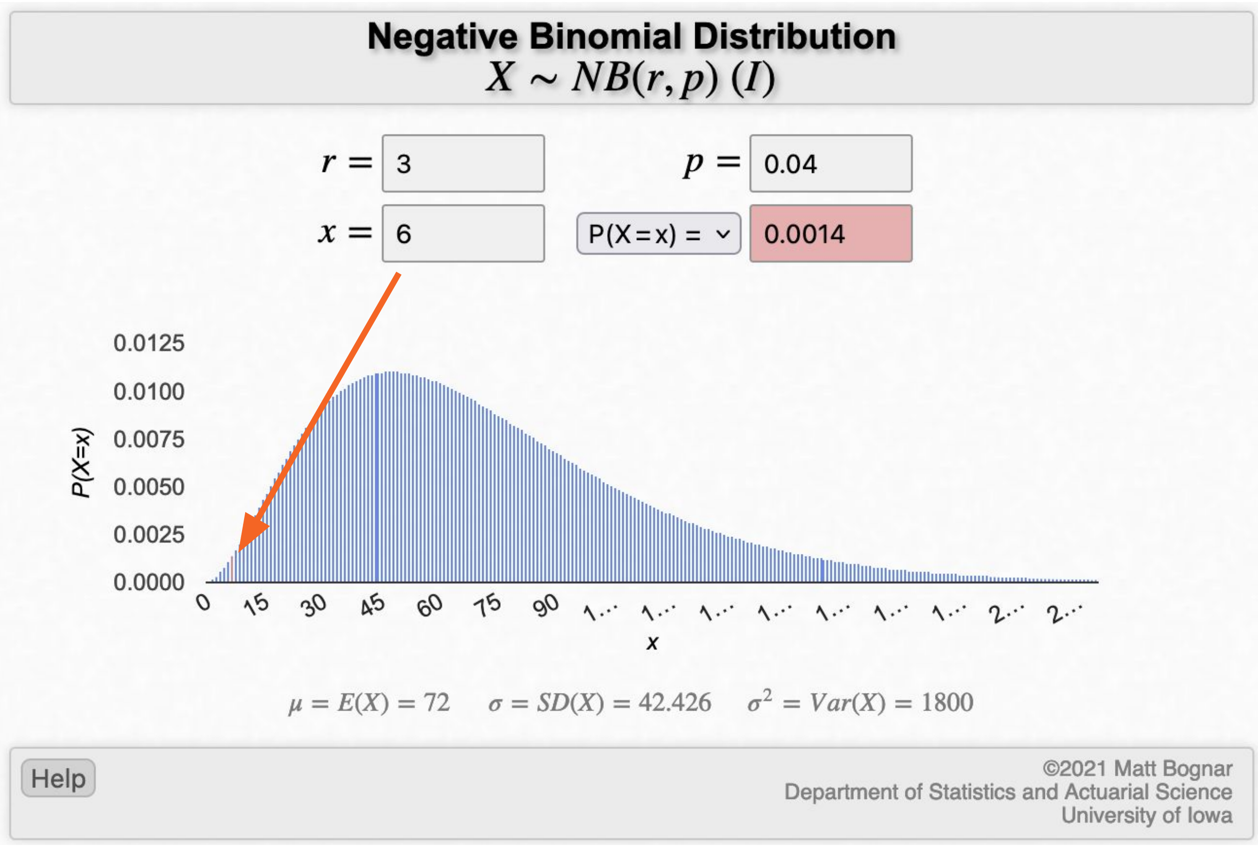
https://homepage.divms.uiowa.edu/~mbognar/applets/nb1.html

—



Pr(X=6) = 0.0014

for **X = # failures**



https://www.producthunt.com/products/party-parrots

https://homepage.divms.uiowa.edu/~mbognar/applets/nb1.html

1 min break & attendance

tinyurl.com/22wmzshh

Admin

- Phase 3 due Thursday 11/3
- HW5 due Tuesday 11/7
    - Problem C2 typo: should say “tick_label” instead of “tick_labels”

A new distribution!

Story: number of events that happen during an interval

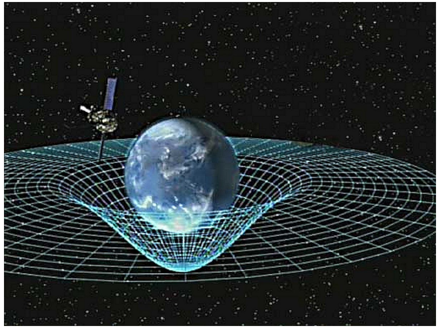
Poisson Distribution

Event space: integers 0 to infinity

Parameters: rate $\lambda > 0$

Story: number of events that happen during an interval

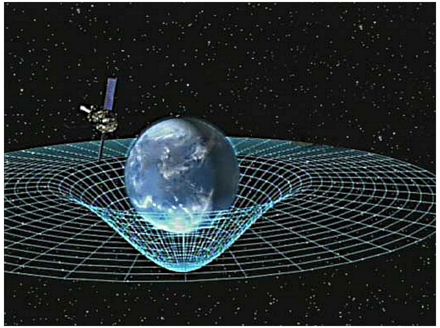
Poisson Examples



- **"Intervals" can occur in time or space**
- **Our story X can represent...**
  - The # students arriving to Instructor Thalken's OH
  - The # typos on a textbook page

https://en.wikipedia.org/wiki/Spacetime

Poisson Examples



- "Intervals" can occur in time or space
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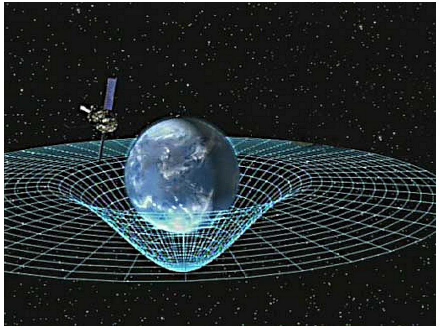
Poisson Examples



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  - The # typos on a textbook page
  - The # mobile users seeing an ad each minute
  - The # Steph Curry 3pt shots per 36 minutes
  - The # customers at the Gates Gimme in 10 minute intervals

https://en.wikipedia.org/wiki/Spacetime

Poisson Properties

Probability that k events happened in an interval

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\lambda = \mathrm{E}(X) = \mathrm{Var}(X)$$

https://en.wikipedia.org/wiki/Poisson_distribution

Poisson Properties

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

"rate"
lambda

$$\lambda = \mathrm{E}(X) = \mathrm{Var}(X)$$

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How many 100-year floods?

If massive floods occur randomly with average 1 in 100 years, how many "100-year" floods do you expect within a 100 year period?

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Binomial model: **What is N? ____ What is p? ____**

(This isn't asking about *negative binomials*. Remember binomial models like coin flips?)

How many 100-year floods?

If massive floods occur randomly with **average 1 in 100 years**, how many "100-year" floods do you expect within a **100 year period**?

Binomial model: What is N? **100** What is p? **1/100**

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In Python (scipy.stats): For each number of floods *k*, the probability is **binom.pmf(k, 100, 0.01)**

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Binomial model: What is N? **100** What is p? **1/100**

	0 floods	**1**	**2**	**3**	**4**
Binomial	36.6%	36.9%	18.4%	6.0%	1.5%

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binom.pmf(2, 100, 0.01)

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If you wanted a cumulative probability, binom.cdf(2, 100, 0.01)

What if we use Poisson instead?

If massive floods occur randomly with average 1 in 100 years, how many "100-year" floods do you expect within a 100 year period?

Poisson model: Rate λ = __

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If massive floods occur randomly with **average 1** in 100 years, how many "100-year" floods do you expect within a 100 year period?

Poisson model: Rate λ = **1**

	0 floods	**1**	**2**	**3**	**4**
Poisson	36.8%	36.8%	18.3%	6.3%	1.5%

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In Python (scipy.stats): For each number of floods *k*, the probability is **poisson.pmf(k,1)**

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poisson.pmf(___)

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Poisson model: Rate λ = 1

	0 floods	1	2	3	4	
Poisson	36.8%	36.8%	18.3%	6.3%	1.5%	

If you want to go cumulative in the other direction, instead of using cdf, you can use poisson.sf(2,1)

What if we use Poisson instead?

If massive floods occur randomly with average 1 in 100 years, how many "100-year" floods do you expect within a 100 year period?

Poisson model: Rate $\lambda = 1$

These results are pretty similar, even though they use different distributions!

	0 floods	1	2	3	4
Binomial	36.6%	36.9%	18.4%	6.0%	1.5%
Poisson	36.8%	36.8%	18.3%	6.3%	1.5%

Why would you ever use Poisson instead of Binomial?

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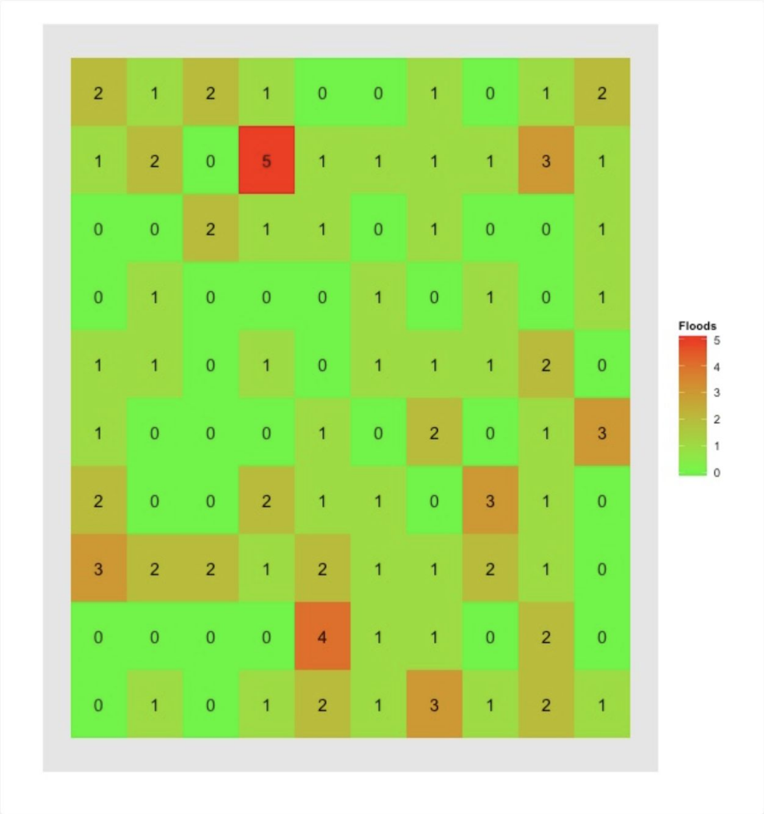
Use Poisson when you aren't sure what N and p are, but you can guess the mean # of occurrences

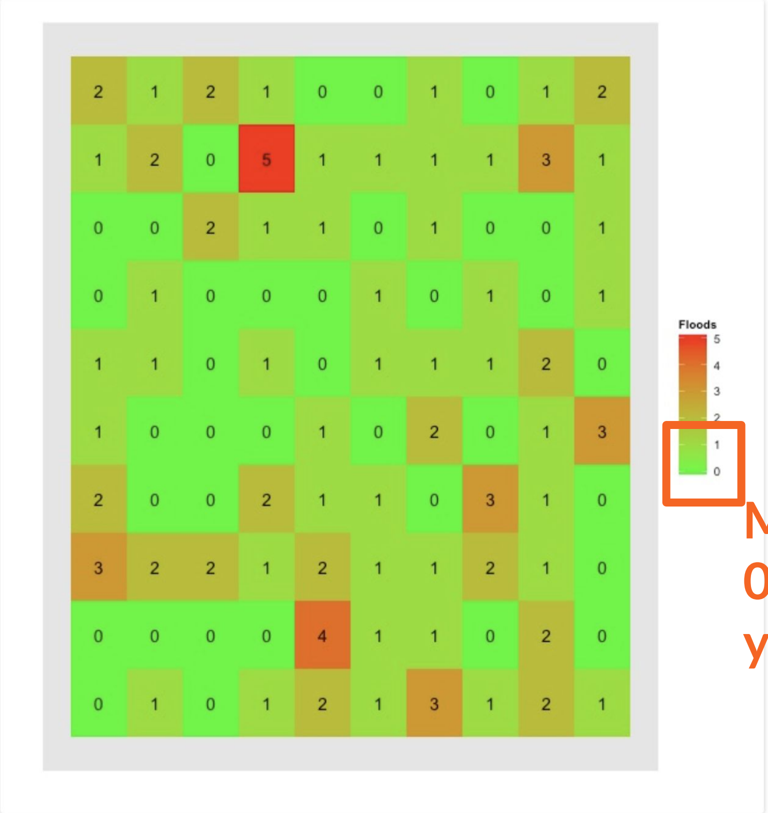
100 simulations from a Poisson distribution



Simulation of the number of 100-year floods occurring in 100 (10 x 10), 100-year sequences

https://tonyladson.wordpress.com/2015/06/29/100-year-flood-poisson-distribution/

100 simulations from a Poisson distribution

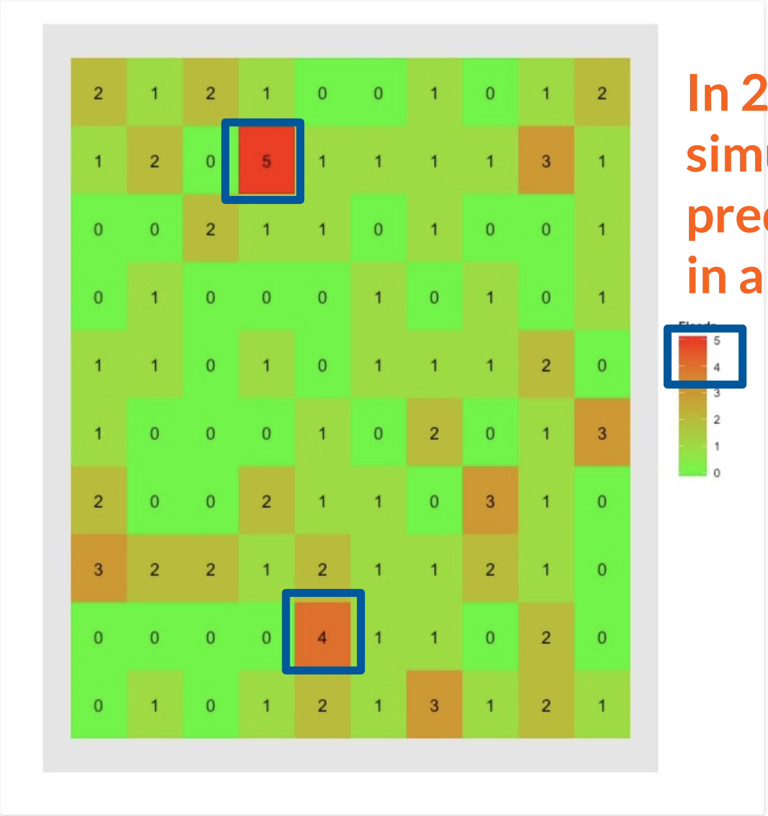


Simulation of the number of 100-year floods occurring in 100 (10 x 10), 100-year sequences

Mostly expecting 0-1 floods in 100 years

https://tonyladson.wordpress.com/2015/06/29/100-year-flood-poisson-distribution/

100 simulations from a Poisson distribution



Simulation of the number of 100-year floods occurring in 100 (10 x 10), 100-year sequences

In 2/100 simulations we predict 4-5 floods in a 100 year span

https://tonyladson.wordpress.com/2015/06/29/100-year-flood-poisson-distribution/

Poisson example

of Steph Curry 3P shots per game ~ Poisson(4.5)

# of Andrew Wiggins 3P per game ~ Poisson(2.5)

https://www.nba.com/warriors/news/curry-wiggins-2022-all-star-announcement-20220127

https://www.basketball-reference.com/

Poisson example

of Steph Curry 3P shots per game ~ Poisson(4.5)

# of Andrew Wiggins 3P per game ~ Poisson(2.5)

In Python, to get a random draw from these distributions:

np.random.poisson(4.5, size=None)

np.random.poisson(2.5, size=None)

https://www.nba.com/warriors/news/curry-wiggins-2022-all-star-announcement-20220127

https://www.basketball-reference.com/

Poisson is additive and divisible

of Steph Curry 3P shots per game ~ Poisson(4.5)

# of Steph Curry 3P shots per **1/2 game** ~ Poisson(?)

https://www.nba.com/warriors/news/curry-wiggins-2022-all-star-announcement-20220127

https://www.basketball-reference.com/

Poisson is additive and divisible

of Steph Curry 3P shots per game ~ Poisson$\boxed{(4.5)}$

Divide 4.5 by 2!

# of Steph Curry 3P shots per 1/2 game ~ Poisson(2.25)

https://www.nba.com/warriors/news/curry-wiggins-2022-all-star-announcement-20220127

https://www.basketball-reference.com/

Poisson is additive and divisible

of Steph Curry 3P shots per game ~ Poisson(4.5)

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Curry OR Wiggins 3P shots per game ~ Poisson(?)

https://www.nba.com/warriors/news/curry-wiggins-2022-all-star-announcement-20220127

https://www.basketball-reference.com/

Poisson is additive and divisible

of Steph Curry 3P shots per game ~ Poisson$\boxed{(4.5)}$

# of Andrew Wiggins 3P per game ~ Poisson$\boxed{(2.5)}$

We can just sum these!

Curry OR Wiggins 3P shots per game ~ Poisson(7)

https://www.nba.com/warriors/news/curry-wiggins-2022-all-star-announcement-20220127

https://www.basketball-reference.com/

Poisson Properties

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\lambda = \mathrm{E}(X) = \mathrm{Var}(X)$$

If rate λ = 4, what is the standard deviation of the Poisson distribution?

Poisson Properties

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\lambda = \mathrm{E}(X) = \mathrm{Var}(X)$$

Stdev = sqrt(Var(X)) = sqrt(λ) = sqrt(4) = 2

Poisson Properties

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\lambda = \mathrm{E}(X) = \mathrm{Var}(X)$$

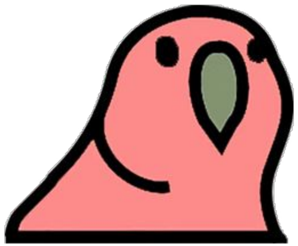
If rate λ = 400, what is the standard deviation of the Poisson distribution?

Poisson Properties

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\lambda = \mathrm{E}(X) = \mathrm{Var}(X)$$

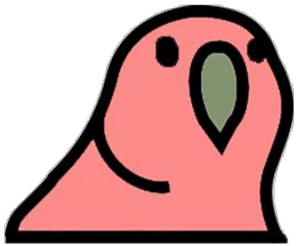
Stdev = sqrt(Var(X)) = sqrt(λ) = sqrt(400) = 20



Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has one parrot-lover?

https://www.producthunt.com/products/party-parrots



Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has one parrot-lover?

X = number of parrot lovers in a discussion section

λ = ?

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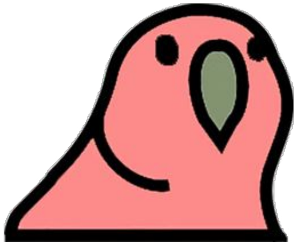


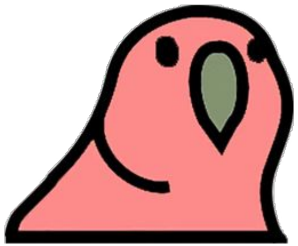
Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has one parrot-lover?

X = number of parrot lovers in a discussion section

λ = **mean = 3**

https://www.producthunt.com/products/party-parrots



Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has one parrot-lover?

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We are trying to find: P(?)

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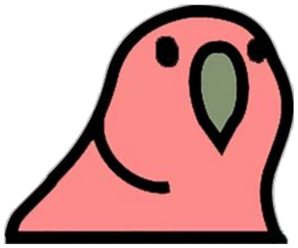

Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has **one parrot-lover**?

X = number of parrot lovers in a discussion section
λ = mean = 3

We are trying to find: P(**X=1**)

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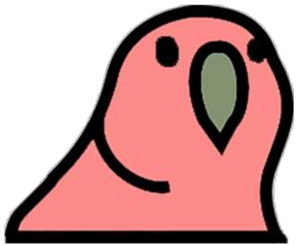

Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has **one** parrot-lover?

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\boxed{P(X=1) = 3^1 * e^{-3} / 1! = 3e^{-3} = 0.15}$$

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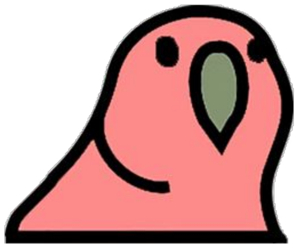

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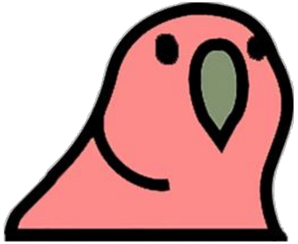

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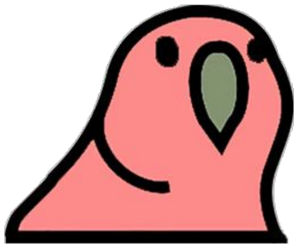

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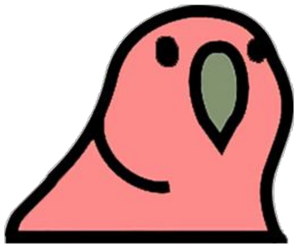

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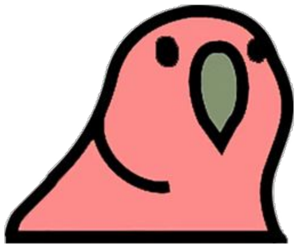


Poisson Example

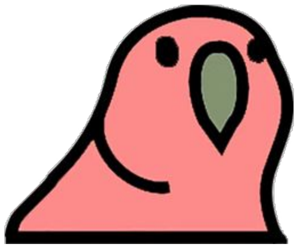
There is a mean of 3 parrot-lovers per discussion section. What is the **probability** that a randomly selected discussion section has one parrot-lover?

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$\boxed{P(X=1) = 3^1 * e^{-3} / 1! = 3e^{-3} = \mathbf{0.15}}$

Final answer: 15%

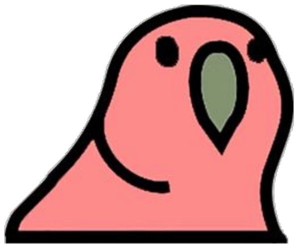
https://www.producthunt.com/products/party-parrots



Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has **at least one** parrot-lover?

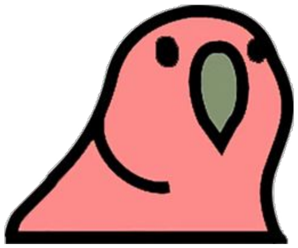
https://www.producthunt.com/products/party-parrots



Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has **at least one** parrot-lover?

We are trying to find: P(?)



Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has **at least one** parrot-lover?

We are trying to find: $P(X \geq 1)$

But, we only showed you the PMF and not the CDF

Can we still solve this?

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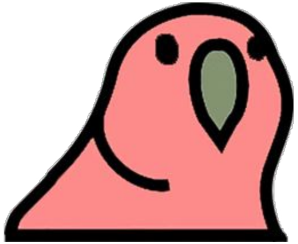

Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has **at least one** parrot-lover?

Fill in the blank:

$P(X \geq 1) = 1 -$ _____

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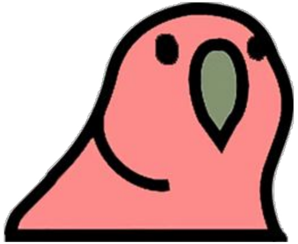

Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has **at least one** parrot-lover?

$P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0)$

Discrete numbers mean that we can add up our PMF values to get cumulative probabilities!

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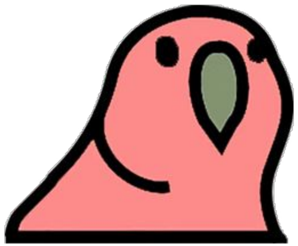

Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has **at least one** parrot-lover?

$P(X \geq 1) = 1 - P(X=0) = ?$

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

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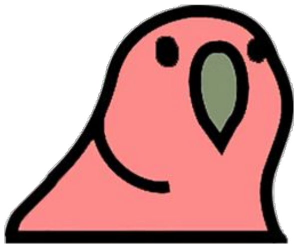

Poisson Example

There is a mean of 3 parrot-lovers per discussion section. What is the probability that a randomly selected discussion section has **at least one** parrot-lover?

$$P(X \geq 1) = 1 - P(X=0) = 1 - [3^0 * e^{-3} / 0!] = \mathbf{1 - e^{-3}} = 0.95$$

$$\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

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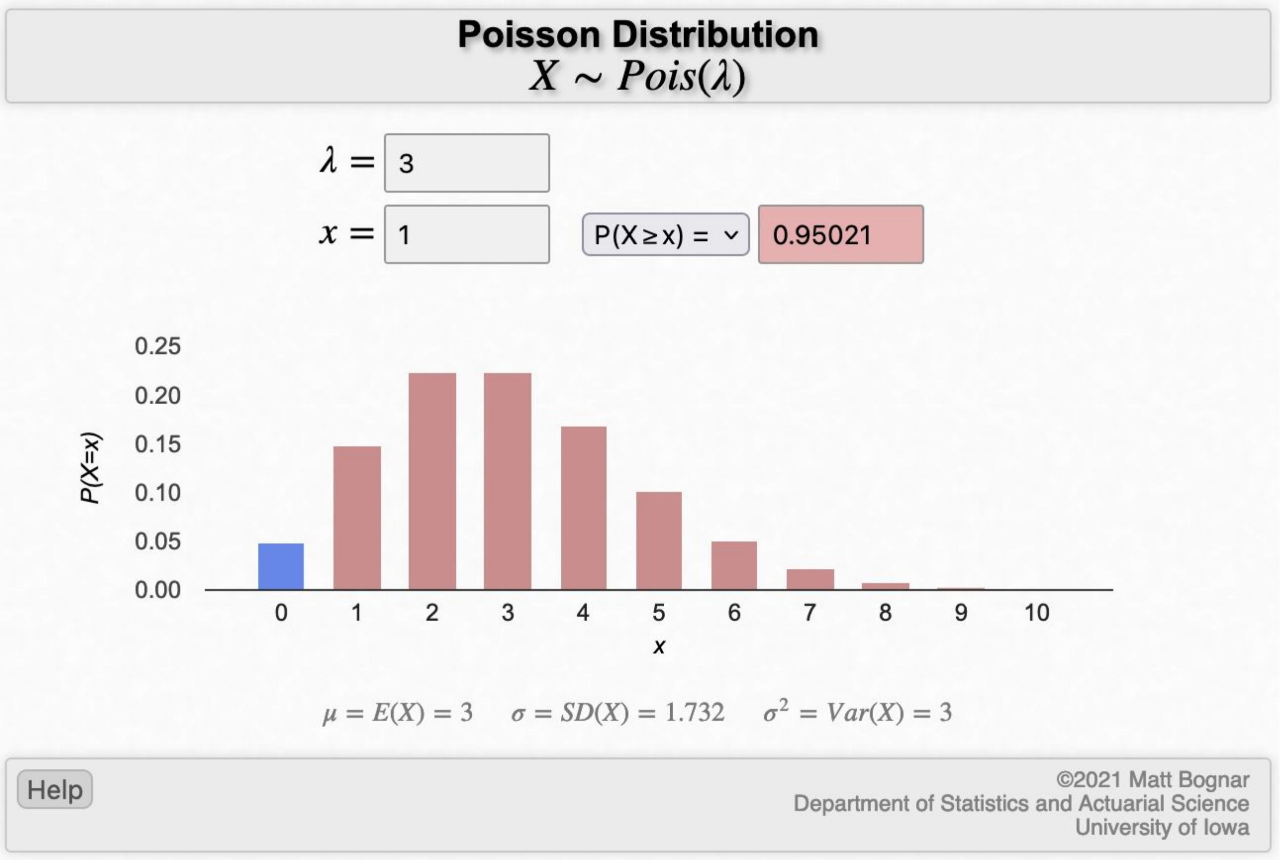
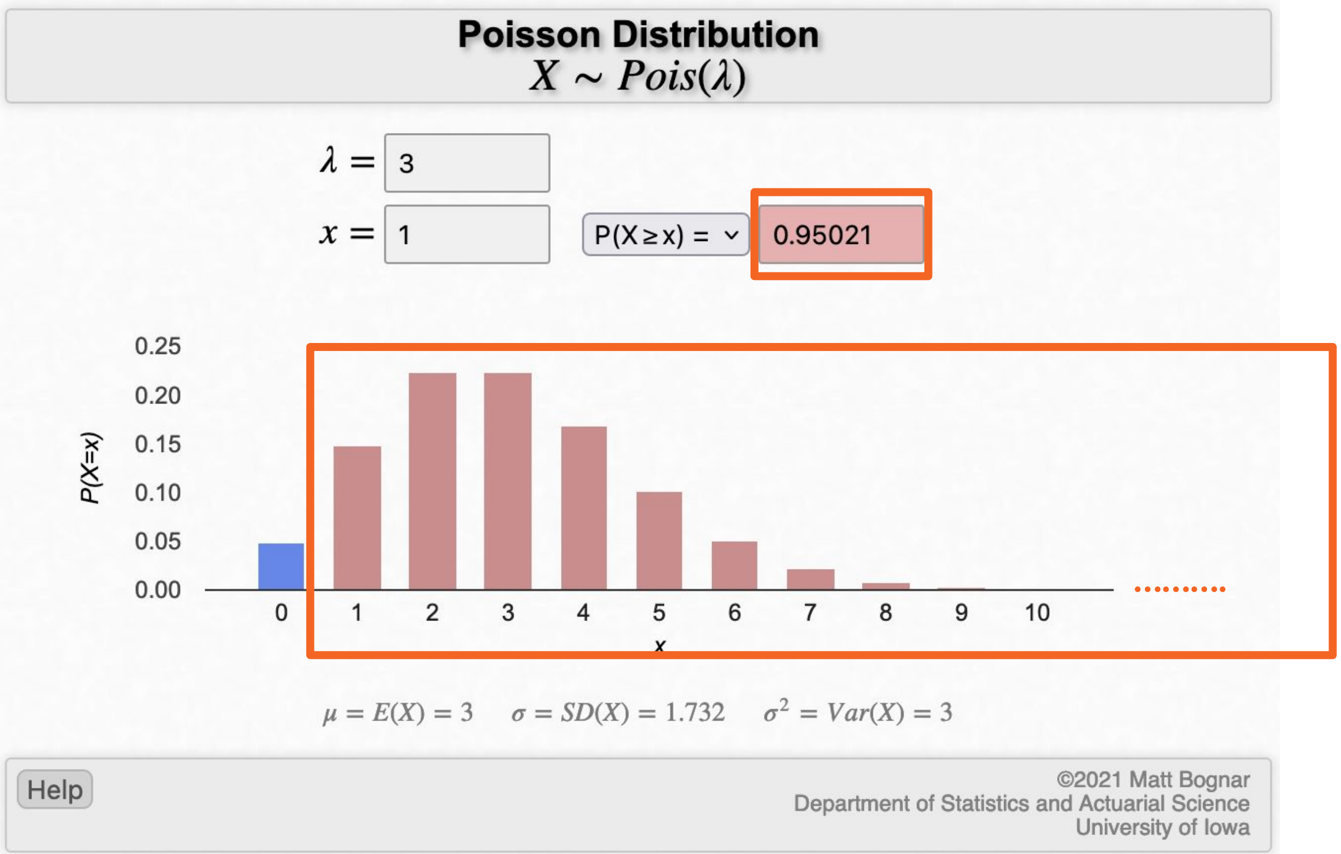


https://homepage.divms.uiowa.edu/~mbognar/applets/pois.html



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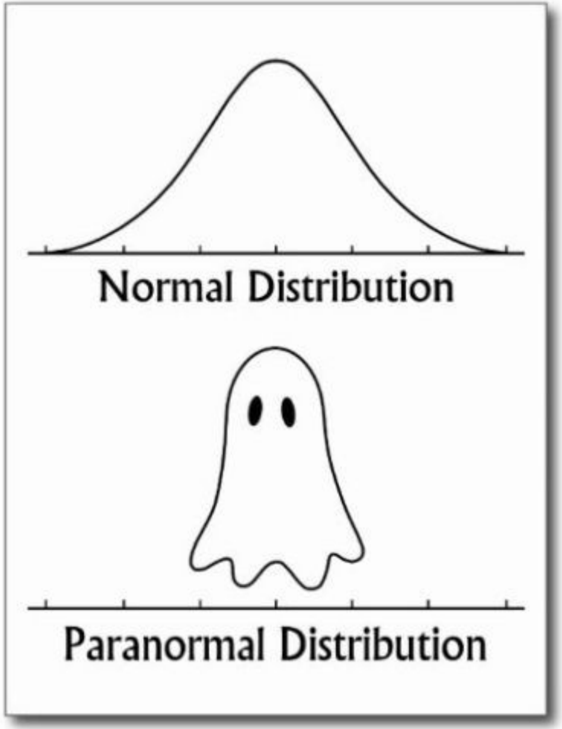

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