The National Resident Matching Program

INFO 4220 – Networks II: Market Design

Adapted from Haeringer Ch.10

Today and next lecture

We will examine the case of the National Medical Residence Program:

- Description and history
- Equivalence (of the original) NRMP to the deferred acceptance algorithm
- Problems with the original NRMP and experimentation to improve algorithm
- The rural hospital theorem

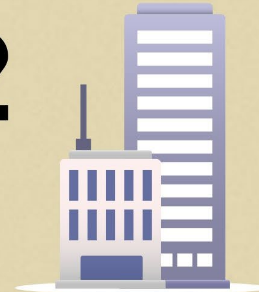
- Probably the longest running and most successful use of matching theory for a real-life problem
- After completing medical school, students must spend a few years working in hospitals as residents
- Today, the US NRMP Involves 30,000+ candidates, and 3,800 residency programs

Medical Match Process

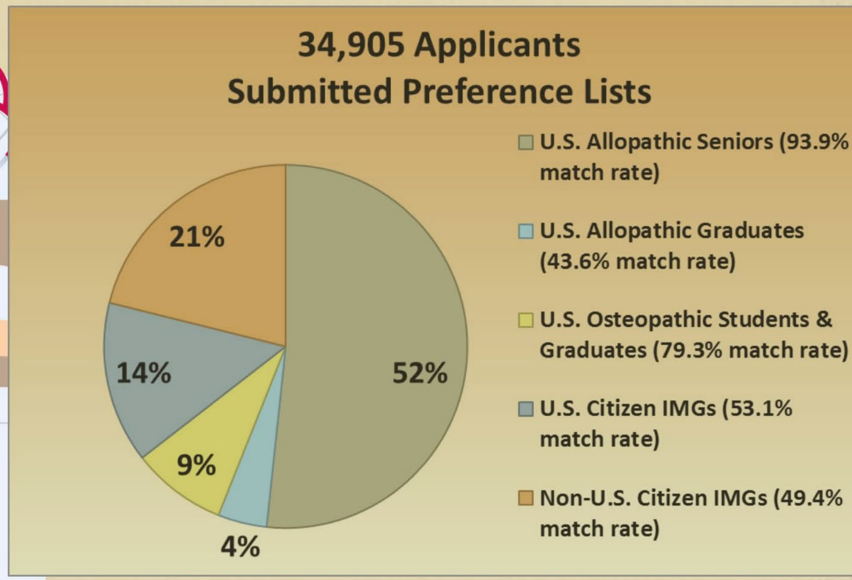
- Students submit application to hospitals
- Hospitals invite the students they like for interviews
- After the interviews happen:
  - Hospitals submit their preferences over the doctors they interviewed to a centralized clearinghouse
  - Students submit their preferences over hospitals with which they interviewed to the same clearinghouse
  - An algorithm matches hospitals and students
- There are rules: Hospitals cannot ask student what other hospitals they have interviewed with, and hospitals-doctors cannot communicate after the interviews

2015 Main Residency Match®

30,212
Positions



99.4%
Position Fill Rate



U.S. Allopathic Seniors:

93.9%
Match rate for seniors

78%
% of matched seniors who got one of top 3 choices

Residence Matching in the 1900s

Gilbert's Studio, Medical School Class and Staff (with Cadaver), ca. 1900, gelatin silver print, Smithsonian American Art Museum

Residence matching 1900-1945

- Matching was decentralized:
  - Candidates had to apply separately for positions
  - Hospitals decided who to hire
- Competition between hospitals to get best candidates lead to **unravelling**:
  - Candidates hired several years before graduation
- What’s the problem with unravelling?
  - Those who already secured a job have less incentives to study and perform
  - Likely that students and hospitals that matched early did not get the best match they could

Residence Matching 1945-1952

- In 1945 medical schools agreed not to disclose information about students before a certain date
  - Idea was to avoid early competition
- This didn’t solve anything. Matching mechanism is the same, just changed how much time people had to match. It created a bottleneck
- Matching is slow:
  - Contacting candidates and making offers take time
  - Candidates can take their time to respond (specially if they are hoping for a better offer)

Residence Matching 1945-1952

- Between 1945 and 1950 the time given to candidates to respond decreased:
  - 10 days in 1945
  - 12 hours in 1950
- It didn’t help improving the market
- Mismatches still were happening:
  - Pessimistic students would accept a bad offer (it is risky to say no if you don’t think you will get a better offer)
  - Optimistic students would end up with a bad match or unmatched (they refuse a good offer because they are hoping for a better offer that never arrives)

The National Resident Matching Program

- In 1952, several American medical associations agreed to create a centralized matching mechanism
- Students and hospitals submit (simultaneously) a ranked order preference list
- An algorithm finds the matches
- Matching is announced on what is come to be known as “Match Day”

Image from: https://uofuhealth.utah.edu/notes/postings/2017/03/match-day-2017.php

National Resident Matching Program

- The first job of medical students after they graduate is called the “residence” and is a continuation of their education
- When the market is not centralized, the hospital-doctor matching problem unravels
- Around 1945 medical associations tried multiple fix to the unravelling problem without success
- In 1952 they introduced a centralized clearinghouse that solved the issue and still operates today

The NRMP Algorithm

- Alvin Roth studied the NRMP algorithm (being used at the time) in 1984
- He demonstrated that algorithm was equivalent to the Deferred Acceptance Algorithm (with hospitals proposing)
- Main difference with what we did last class is that now we are talking about a many-to-one matching
  - Residents choose one hospitals
  - Hospitals takes many residents

Many-to-one matching model

The medical match problem starts with:

- A set of doctors: $D = \{d_1, d_2 \dots\}$
- S set of hospitals: $H = \{h_1, h_2, \dots\}$

The problem is:

- Each doctor wants to be hired by ONE hospital
- Each hospital can hire SEVERAL doctors

Each hospital $\boldsymbol{h} \in \boldsymbol{H}$ has a capacity $\boldsymbol{q_h}$ that specifies the maximum number of doctors it can hire

Preferences

- Doctor's preferences over hospitals are just like in the basic one-to-one matching model we saw:
  - Each doctor $d \in D$ has a strict preference relation P_d over hospitals and the option of not getting hired by any hospital
- Since hospitals $h \in H$ can hire several doctors, we need to define **preferences $\boldsymbol{P_h^\#}$ over sets of doctors**
  - $\{d_1, d_2\}P_h^\#\{d_3, d_4\}$ means that hospital h prefers to hire d_1 and d_2 rather than hiring d_3 and d_4
  - We could also have things like: $\{d_5\}P_h^\#\{d_1, d_2\}$

Responsive Preferences

- Things can get messy when working with preferences over sets of doctors
- We can assume that a preference over doctors (i.e. not over sets) is enough:
  - We assume that hospitals' preferences are responsive
- If we assume that each hospital $h \in H$ has a preference relation P_h over doctors

  $\Rightarrow$ Then, the preference $P_h^\#$ will be (partially) deduced from P_h

Responsive Preferences

We can build $P_h^{\#}$ by comparing sets of doctors that differ only by one doctor:

- Suppose that a hospital already hired Dr. X and Dr. Y
- Now the hospital has the choice between Dr. A and Dr. B
- The hospital should compare: {A, X, Y} and {B, X, Y}
- The responsive preferences hypothesis implies that it is sufficient to compare Dr. A and Dr. B:

$$\{A, X, Y\} P_h^{\#} \{B, X, Y\} \Leftrightarrow A \, P_h \, B$$

Definition: Responsive Preference

A preference $P_h^{\#}$ (over sets of doctors) is responsive if for any set of doctors S and two doctors d and d' such that:

- $d \in S$
- $d' \notin S$

We have

$$S \; P_h^{\#} \; S \cup \{d'\} \backslash \{d\} \Leftrightarrow \; d \; P_h \; d'$$

d' added to S
and d withdrawn from S
(Or we are swapping d for d' in S)

PollEV Question

Let $P_h = d_1, d_2, d_3, d_4$. Assuming responsive preferences, which groups of doctors would hospital h prefer (PollEv.com/info4220):

A) $\{d_1, d_3, d_4\}$ or B) $\{d_1, d_2, d_4\}$

PollEV Question

Let $P_h = d_1, d_2, d_3, d_4$. Assuming responsive preferences, which group of doctors should hospital h prefer? (PollEv.com/info4220)

A) $\{d_1, d_3\}$

B) $\{d_2, d_4\}$

C) Impossible to tell

PollEV Question

Let $P_h = d_1, d_2, d_3, d_4$. Assuming responsive preferences, which group of doctors should hospital h prefer.

A) $\{d_2\}$

B) $\{d_1, d_3\}$

C) Impossible to tell

Example

Let $P_h = d_1, d_2, d_3, d_4$. Assuming responsive preferences, which group of doctors should hospital h prefer?

A) $\{d_1, d_4\}$

B) $\{d_2, d_3\}$

C) Impossible to tell

Matching

- Similar to what we defined for one-to-one matching models, but with some changes:
  - Hospitals can be matched to more than one doctor
  - Hospitals have a maximum capacity

Definition:

A **matching** is a function $\mu: H \cup D \rightarrow H \cup D$ such that:

- For each doctor $d \in D, \mu(d) \in H \cup \{d\}$
  - **i.e. a doctor is matched to a hospital or himself**
- For each hospital $h \in H$:
  - $|\mu(h)| \leq q_h$
  - If $|\mu(h)| \geq 1$ then $\mu(h) \subseteq D$
- $\mu(d) = h$ if, and only if $d \in \mu(h)$

A hospital's match cannot exceed its capacity and a hospital is matched to doctors

Stability

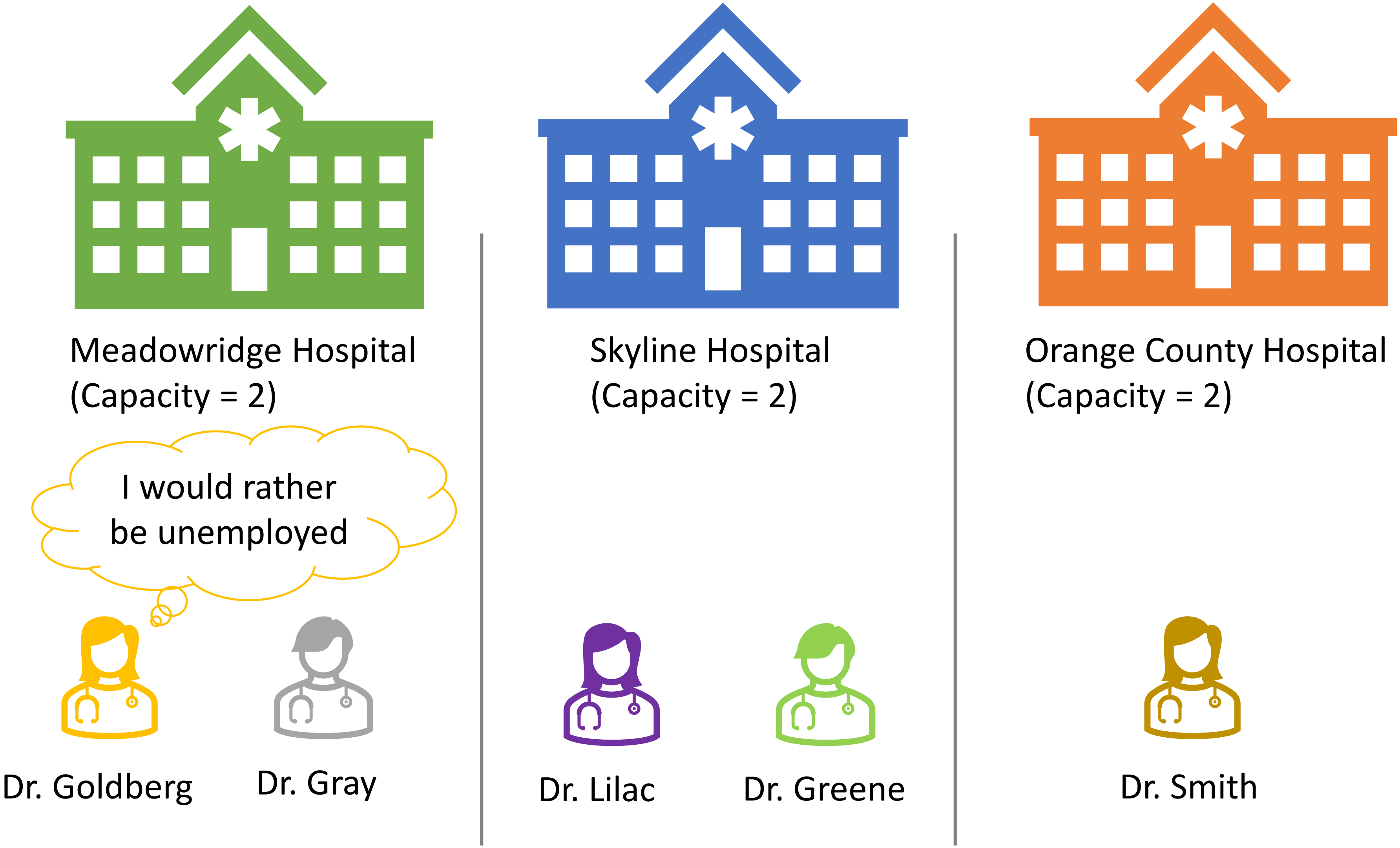
In a many-to-one matching problem, stability is defined by three requirements:

- Individual rationality
- Absence of blocking pairs
- **Non-wastefulness**

Individual Rationality

A matching μ is **individually rational** if:

- For each doctor $d \in D, \mu(d) R_d d$
  - Each doctor weakly prefers the match she is assigned to being unmatched
- For each hospital $h \in H$, there is no doctor $d \in \mu(h)$ such that $\emptyset P_h d$
  - For each hospital h there isn't any doctor in its match that is unacceptable (i.e. that the hospital would rather not have)



Not individually rational: Dr. Goldberg would prefer to be unmatched rather than working at Meadowridge

Blocking Pairs

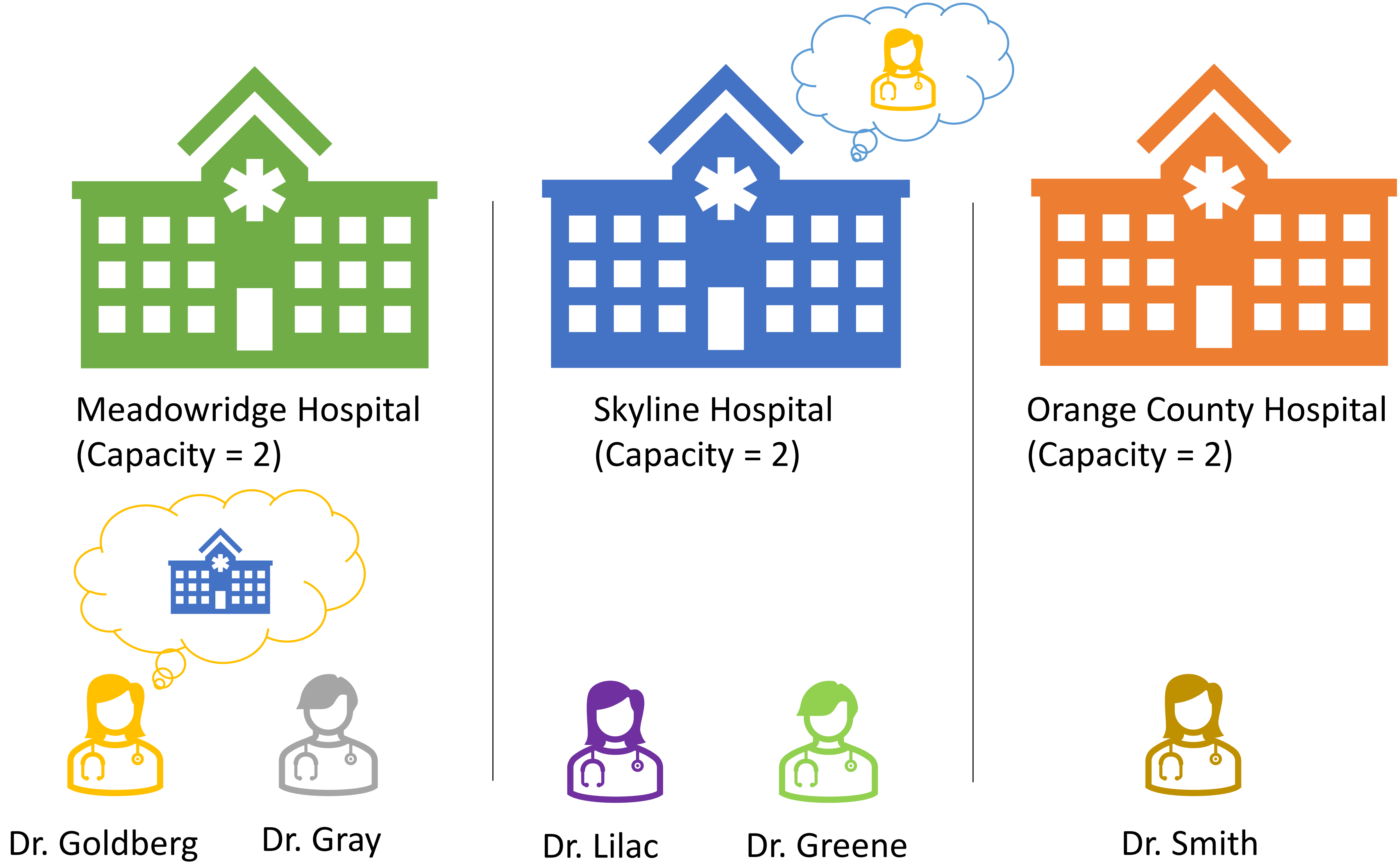
A **pair $(\boldsymbol{d}, \boldsymbol{h})$ blocks a matching** μ if:

- $\mu(d) \neq h$
  - Doctor d is not matched to h
- $h \; P_d \; \mu(d)$
  - Doctor d would like to be matched to h instead than being with her current match
- $d \; P_h \; d'$ for some doctor $d' \in \mu(h)$
  - Hospital h would rather change doctor d' from its current match for doctor d

With responsive preferences this is the same as:

$$\mu(h) \cup \{d\} \setminus \{d'\} P_h^{\#} \mu(h)$$

d added to $\mu(h)$
and d’ withdrawn from $\mu(h)$



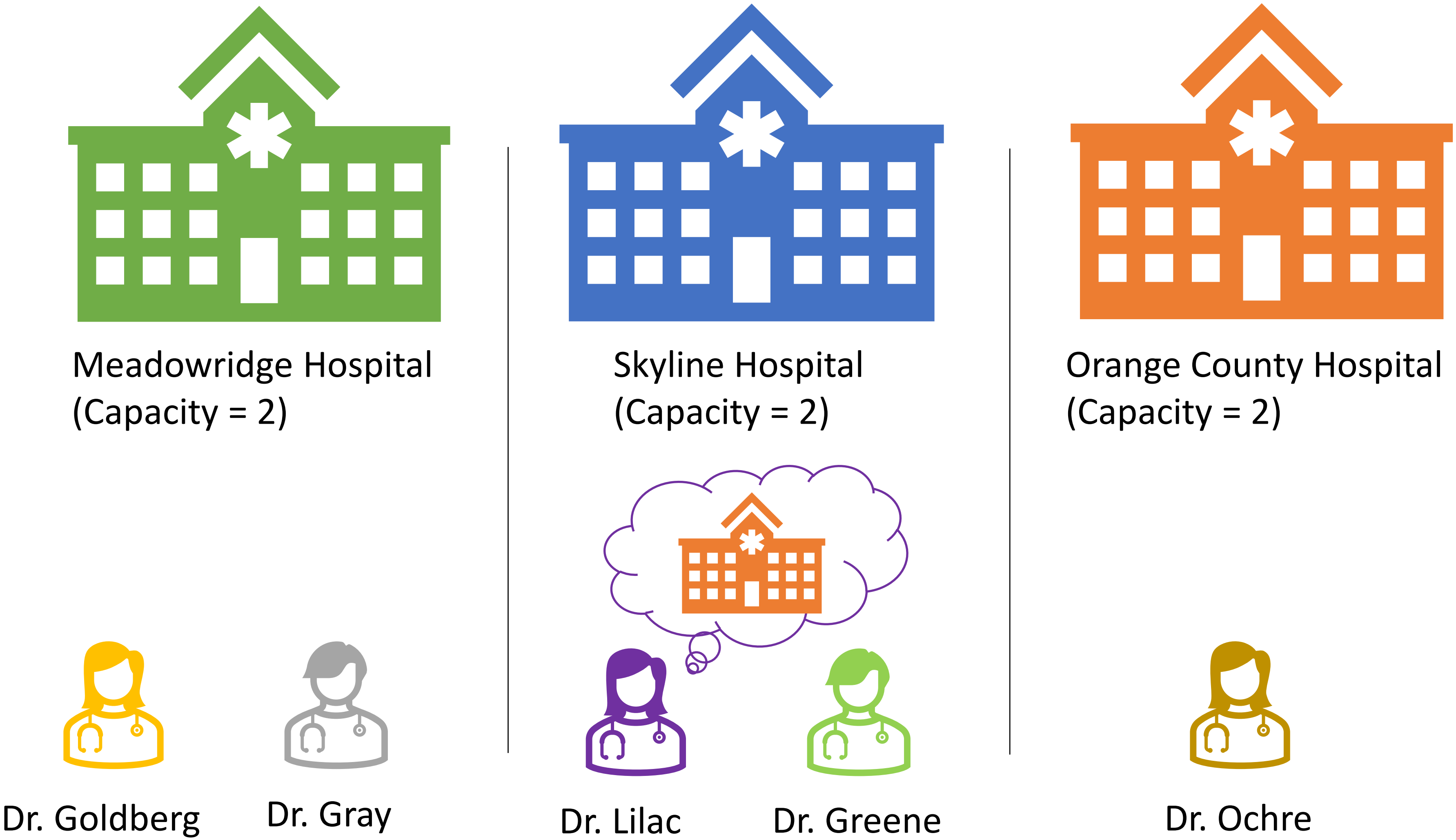
Dr. Goldberg and Skyline Hospital are a Blocking Pair: They are not matched to each other, and they would each prefer to be matched to each other rather than their current match (Let's say, for example, Skyline Hospital likes Dr. Goldberg better than Dr. Lilac)

Non-wastefulness

A matching μ is **non-wasteful** is for every doctor $d \in D$,

$$h \; P_d \; \mu(d) \; \Rightarrow |\mu(h)| = q_h$$

If d prefers hospital $\boldsymbol{h}$ to her match, then hospital $\boldsymbol{h}$ has filled its capacity



Wasteful: Dr. Lilac would prefer to work at the Orange County Hospital rather than at Skyline, and Orange County is not full and does not find Dr. Lilac unacceptable

PollEV Question

Hospital h_1 has a capacity of 2, and hospital h_2 has a capacity of 1

P_{d_1}	P_{d_2}	P_{d_3}
h_1	h_1	h_1
h_2	h_2	h_2

P_{h_1}	P_{h_2}
d_1	d_1
d_2	d_3
d_3	d_2

$$\mu(d_1) = h_1, \mu(d_2) = h_2, \mu(d_3) = h_1$$

What is wrong with this matching?

a) Nothing

b) No individually rational

c) Blocking pairs

d) It is wasteful

Example

Hospital h_1 has a capacity of 2, and hospital h_2 has a capacity of 1

P_{d_1}	P_{d_2}	P_{d_3}
h_1	h_1	h_1
h_2	h_2	h_2

P_{h_1}	P_{h_2}
d_1	d_1
d_2	d_3
d_3	d_2

$$\mu(d_1) = h_1, \mu(d_2) = h_1, \mu(d_3) = h_2$$

This matching is stable because: It is individually rational, there are no blocking pairs, and it is not wasteful.

Deferred Acceptance Algorithm

- The DA can be used to obtain a stable matching
- Like in the one-to-one case, there are two versions:
  - Doctors propose, and hospitals accepts/rejects proposal
  - Hospitals propose, and doctors accepts/rejects proposal
- The version where doctors propose is similar to the one-to-one version, with the difference that hospitals can hold multiple offers (up to capacity)
- At any step in the algorithm, the hospitals evaluate:
  - The set of doctors it is holding from previous steps (if any)
  - The set of doctors who made an offer in current time (if any)
- From this set, the hospital accepts doctors up to its capacity, one at a time starting with the most preferred

Example: DA with Doctors Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 2$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_1	h_2	h_2	h_2
h_2	h_1	h_1	h_1

P_{h_1}	P_{h_2}
d_1	d_2
d_2	d_3
d_3	d_4
d_4	d_1

Step 1:
Each doctor proposes to their most preferred hospital

h_1	h_2
d_1	d_2, d_3, d_4

Example: DA with Doctors Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 2$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_1	h_2	h_2	h_2
h_2	h_1	h_1	h_1

P_{h_1}	P_{h_2}
d_1	d_2
d_2	d_3
d_3	d_4
d_4	d_1

Step 1:
Each doctor proposes to their most preferred hospital

h_1	h_2
d_1	d_2, d_3, d_4

Hospital 2 is over capacity and rejects d_4 to hold its 2 most preferred offers

Example: DA with Doctors Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 2$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_1	h_2	h_2	h_2
h_2	h_1	h_1	h_1

P_{h_1}	P_{h_2}
d_1	d_2
d_2	d_3
d_3	d_4
d_4	d_1

Step 2:
Doctors d_1, d_2, d_3 are temporarily matched, so they do nothing
Doctor d_4 proposes to its second most preferred hospital h_1

h_1	h_2
d_1	d_2, d_3, d_4
d_4	

Example: DA with Doctors Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 2$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_1	h_2	h_2	h_2
h_2	h_1	h_1	h_1

P_{h_1}	P_{h_2}
d_1	d_2
d_2	d_3
d_3	d_4
d_4	d_1

Step 2:
Doctors d_1, d_2, d_3 are temporarily matched, so they do nothing
Doctor d_4 proposes to its second most preferred hospital h_1

h_1	h_2
d_1	d_2, d_3, d_4
d_4	

No rejections. Algorithm ends in match: $\mu(d_1) = h_1, \mu(d_2) = h_2, \mu(d_3) = h_2, \mu(d_4) = h_1$

DA with Hospitals Proposing

We have to introduce more changes with respect to the one-to-one matching. Now hospitals can make multiple proposals per step:

Step 1:

- Each hospital proposes to its most preferred set of doctors
- Each doctor rejects all hospitals except for the most preferred acceptable hospital that proposed to her

DA with Hospitals Proposing

Step k, k $\geq$ 2:

- Hospitals that had one or more rejections in previous steps make offers to its most preferred set of doctors that satisfy the conditions:
  - The set must include all doctors the hospitals made offers to in previous steps and did not reject the offers
  - Any additional doctors added to the set must be a doctor the hospital has not proposed to yet
- Each doctor rejects all hospitals except for the most preferred acceptable hospital that proposed to her
- The algorithm ends if there were not rejections

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
h_1	h_1	h_3	h_2
h_2	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
d_1	d_1	d_3
d_2	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
h_1, h_2	h_1	h_3	

Step 1:
Hospitals
h_1 : Offers d_1, d_2
h_2 : Offers d_1
h_3 : Offers d_3

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
h_1	h_1	h_3	h_2
$\cancel{h_2}$	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
d_1	$\cancel{d_1}$	d_3
d_2	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
$h_1, \cancel{h_2}$	h_1	h_3	

Step 1:
Hospitals
h_1 : Offers d_1, d_2
h_2 : Offers d_1
h_3 : Offers d_3
Doctors:
d_1 : Rejects h_2

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
h_1	h_1	h_3	h_2
$\cancel{h_2}$	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
d_1	$\cancel{d_1}$	d_3
d_2	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
$h_1, \cancel{h_2}$	h_1	h_3	
	h_2		

Step 2:
Hospitals
h_2 : Offer d_2

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
h_1	$\cancel{h_1}$	h_3	h_2
$\cancel{h_2}$	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
d_1	$\cancel{d_1}$	d_3
$\cancel{d_2}$	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
$h_1, \cancel{h_2}$	$\cancel{h_1}$	h_3	
	h_2		

Step 2:
Hospitals
h_2 : Offer d_2

Doctors:
d_2: Rejects h_1

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
h_1	$\cancel{h_1}$	h_3	h_2
$\cancel{h_2}$	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
d_1	$\cancel{d_1}$	d_3
$\cancel{d_2}$	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
$h_1, \cancel{h_2}$	$\cancel{h_1}$	h_3	
	h_2		
		h_1	

Step 3:
Hospitals
h_1 : Offers d_3

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
h_1	$\cancel{h_1}$	$\cancel{h_3}$	h_2
$\cancel{h_2}$	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
d_1	$\cancel{d_1}$	$\cancel{d_3}$
$\cancel{d_2}$	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
$h_1, \cancel{h_2}$	$\cancel{h_1}$	$\cancel{h_3}$	
	h_2		
		h_1	

Step 3:
Hospitals
h_1 : Offers d_3

Doctors:
d_3: Rejects h_3

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
h_1	$\cancel{h_1}$	$\cancel{h_3}$	h_2
$\cancel{h_2}$	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
d_1	$\cancel{d_1}$	$\cancel{d_3}$
$\cancel{d_2}$	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
$h_1, \cancel{h_2}$	$\cancel{h_1}$	$\cancel{h_3}$	
	h_2		
		h_1	
h_3			

Step 4: Hospitals h_3 : Offers d_1

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
$\cancel{h_1}$	$\cancel{h_1}$	$\cancel{h_3}$	h_2
$\cancel{h_2}$	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
$\cancel{d_1}$	$\cancel{d_1}$	$\cancel{d_3}$
$\cancel{d_2}$	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
$\cancel{h_1}, \cancel{h_2}$	$\cancel{h_1}$	$\cancel{h_3}$	
	h_2		
		h_1	
h_3			

Step 4:
Hospitals
h_3 : Offers d_1

Doctors:
d_1: Rejects h_1

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
$\cancel{h_1}$	$\cancel{h_1}$	$\cancel{h_3}$	h_2
$\cancel{h_2}$	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
$\cancel{d_1}$	$\cancel{d_1}$	$\cancel{d_3}$
$\cancel{d_2}$	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
$\cancel{h_1}, \cancel{h_2}$	$\cancel{h_1}$	$\cancel{h_3}$	
	h_2		
		h_1	
h_3			
			h_1

Step 5:
Hospitals
h_1 : Offers d_4

Exercise: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
$\cancel{h_1}$	$\cancel{h_1}$	$\cancel{h_3}$	h_2
$\cancel{h_2}$	h_3	h_2	h_3

P_{h_1}	P_{h_2}	P_{h_3}
$\cancel{d_1}$	$\cancel{d_1}$	$\cancel{d_3}$
$\cancel{d_2}$	d_2	d_1
d_3	d_3	d_2
d_4	d_4	d_4

d_1	d_2	d_3	d_4
$\cancel{h_1}, \cancel{h_2}$	$\cancel{h_1}$	$\cancel{h_3}$	
	h_2		
		h_1	
h_3			
			h_1

Step 5:
Hospitals
h_1 : Offers d_4

Doctors:
No Rejections
Algorithm ends

$$\mu(h_1) = \{d_3, d_4\}, \mu(h_2) = d_2, \mu(h_3) = d_1$$

Quiz: DA with Hospitals Proposing

Capacities: $q_{h_1} = 2, q_{h_2} = 2$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_1	h_2	h_2	h_2
h_2	h_1	h_1	h_1

P_{h_1}	P_{h_2}
d_1	d_2
d_2	d_3
d_3	d_4
d_4	d_1

One-to-one vs. Many-to-one

Many of the things we discussed in one-to-one matchings also happen in many-to-one matchings:

- DA will find a stable matching
- DA with doctors proposing will produce the **doctor-optimal matching.**
  - This is the most preferred matching for doctors
  - And the least preferred for hospitals
- DA with hospitals proposing will produce the **hospital-optimal matching**
  - This is the most preferred matching for hospitals
  - And the least preferred to doctors

One-to-one vs. Many-to-one

- DA with doctors proposing is strategyproof for doctors
- DA with hospitals proposing **IS NOT** strategyproof for hospitals

Capacities: $q_{h_1} = 2, q_{h_2} = 1, q_{h_3} = 1$

P_{d_1}	P_{d_2}	P_{d_3}	P_{d_4}
h_3	h_2	h_1	h_1
h_1	h_1	h_3	h_2
h_2	h_3	h_2	h_3

$\widehat{P_{h_1}}$	P_{h_1}	P_{h_2}	P_{h_3}
d_2	d_1	d_1	d_3
d_4	d_2	d_2	d_1
d_3	d_3	d_3	d_2
d_1	d_4	d_4	d_4

- Cosider that h_1 submits the following preferences: d_2, d_4, d_3, d_1
- We had found the matching:
  - $\mu_H(h_1) = \{d_3, d_4\}, \mu_H(h_2) = d_2, \mu_H(h_3) = d_1$
- The deviation is profitable for h_1 (and h_2) as the new matching is:
  - $\widehat{\mu_H}(h_1) = \{d_2, d_4\}, \widehat{\mu_H}(h_2) = d_1, \widehat{\mu_H}(h_3) = d_3$

Importance of Stability

- The case of the NRMP highlights the importance of having a stable matching algorithm through a centralized market
- Alvin Roth also studied the medical market in the UK:
  - Similar problem than in the US. Medical graduates need to find hospitals for their residency
  - Unlike the US, there is not centralized market. Market is split in regional markets
  - Not all markets use the same procedure

Importance of Stability

Market	Uses Stable Algorithm?	Still used in 1990?
Edinburgh (1969)	Yes	Yes
Cardiff	Yes	Yes
Cambridge	No	Yes
London Hospitals	No	Yes
Birmingham	No	No
Edinburgh (1967)	No	No
Newcastle	No	No
Sheffield	No	No

- Markets that use stable algorithm perform well, those that don't are abandoned
- **An unstable algorithm produces matchings with blocking pairs, or that are not individually rational. Individuals have incentives to abandon the match:**
  - Cambridge and London are exceptions because they have strong social pressures, which limits incentives to walk away from match

Rural Hospitals

- A problem that arose quickly in the design of the NRMP match was the issue of rural hospitals
  - Candidates tend to prefer hospitals in large urban areas
  - Hospitals in rural areas have a hard times filling their openings
- Can we find an algorithm that?
  - Always produces a stable matching
  - Enable rural hospitals to fill their openings?

Rural Hospitals Theorem

- For any preferences of doctors and hospitals, if at a stable matching a hospital does not fill all of its vacancies, then it does not fill all its vacancies at any stable matching
- Furthermore, **if a hospital does not fill its vacancies at some stable matching, it is matched to the same set of doctors at all stable matchings**

Couples in the NRMP

- In the 1970s there was a drop in participation in the NRMP. Couples were more common in medical school, and they abstained from participating in the matching
- Several fixed were attempted. The first try:
  - Each couple names a leading member
  - Once the leading member is matched, the preference list of partner is edited to remove distant positions
- The problem persisted because couples were not able to submit preferences over pairs of positions

Couples in the NRMP

- In the 1980's a new fix allowed preference over pairs, but the problem persisted

Albert & Alice	Bill	Carol
(h_1, h_2)	h_1	h_2
$(h_{3,} h_3)$	h_2	h_1
$(Albert, h_2)$	h_3	h_3

P_{h_1} (q=1)	P_{h_2} (q=1)	P_{h_3} (q=2)
Bill	Alice	
~~Albert~~	~~Carol~~	

- DA with doctors proposing:
  - Step 1 (doctors): Albert and Bill propose to h_1, Alice and Carol propose to h_2
  - Step 1 (hospitals): Albert and Carol are rejected

Couples in the NRMP

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Albert & Alice	Bill	Carol
(h_1, h_2)	h_1	h_2
(h_3, h_3)	h_2	h_1
$(Albert, h_2)$	h_3	h_3

P_{h_1} (q=1)	P_{h_2} (q=1)	P_{h_3} (q=2)
Bill	Alice	Albert
~~Albert~~	~~Carol~~	Alice

- DA with doctors proposing:
  - Step 1 (doctors): Albert and Bill propose to h_1, Alice and Carol propose to h_2
  - Step 1 (hospitals): Albert and Carol are rejected
  - Step 2: Alice and Albert have to be able to propose to h_3, but that would make h_2 regret rejecting Carol

Couples in the NRMP

Roth and Peranson proposed the following solution:

- Switch to an algorithm with doctors proposing
  - Original NRMP used hospitals proposing
  - Doctors proposing is fairer for candidates
- Process some proposals sequentially:
  - In DA proposals are made simultaneously
  - With sequential offers is easier to detect sources of instability and correct them along the way

Couples in the NRMP

The new algorithm works as follows:

1. Run DA with doctors proposing, excluding couples (only use single doctors' preferences)
2. One by one, match couples to pairs of hospitals (in order of their preferences). This matches can displace single doctors matched in step 1
3. For doctors displaced in step 2, match them one by one to a hospital (in order of their preferences)

Couples in the NRMP

- New algorithm first used in 1998
- Most problems were resolved, and participation went up again
- Highlights limited usefulness of theory. Roth and Peranson used simulation extensively to test various designs
- Theory predicts that stable matchings may not exist when there are couples
  - Preferences observed in real life generally allow for the existence of stable matchings

Take Away

- When the market is not centralized, the hospital-doctor matching problem unravels
- Around 1945 medical associations tried multiple fix to the unravelling problem without success
- In 1952 they introduced a centralized clearinghouse that solve the issue and still operates today
- Alvin Roth demonstrated that the NRMP clearinghouse is equivalent to the deferred acceptance algorithm

Take Away

- Solving a many-to-one matching problem is similar to a one-to-one matching, with some changes
- We need preferences over sets of doctors
- Responsive preferences allow us to infer preferences over sets starting from single preferences
- Matching is defined in a similar way as in one-to-one matching, but adding a condition for hospital capacity
- Stability is defined in a similar was as in one-to-one matchings, but adding a non-wastefulness condition

Take Away

- Most results of the one-to-one matching carry over to the many-to-one problem
  - Except for strategyproofness of DA with hospitals proposing
- From evidence in the UK, it seems that using a stable algorithm is critical for the survival of market
- Rural hospital theorem: All stable matching always match the same agents
- Stable matching is not guaranteed when there are couples in NRMP. Experimentation led to acceptable solution