
INFO 2950: Intro to Data Science

Lecture 26
2023-11-29



fall 2023 symposium

Join us for an exciting showcase of Cornell Data Journal's semester-long projects. **Food will be provided!**

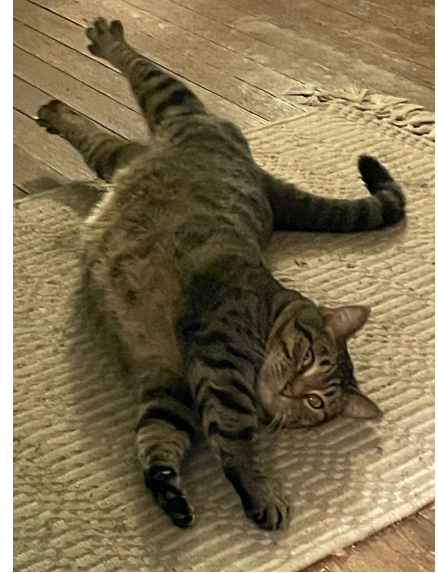
Friday, December 1st | 6:30 – 8:30 PM
Biotechnology Building G10

Funded by SAFC



Agenda

1. Python Review
2. Midterm Review
(Part 1)



When to use what languages?

- **Python**
 - Data manipulation
 - ML models

When to use what languages?

- **Python**
 - Data manipulation
 - ML models
- **SQL**
 - Merging data, group by's

In Python, (with documentation/Googling) you should now be able to do the following with dataframes:

In Python, (with documentation/Googling) you should now be able to do the following with dataframes:

- select rows, columns
- do arithmetic transforms of a column
- generate new rows / columns
- merges, sorting, data transformations
- aggregate statistics / group by
- run linear and logistic regressions
- run hypothesis tests / generate distributions
- run advanced models: NB, SVD, neural nets...

Python refresher / R intro



- We'll review basic syntax from the first few lectures
- ...and introduce how to do the same things in R!

Python refresher / R intro



You won't need to know R for the final exam, but we want to show you that much of what you learned is *transferable knowledge*!

R



- Designed by statisticians, not CS
- Harder to use as general-purpose programming language, but easier for stats-oriented work
- Lagging Python for recent NNs
- Base R is ok, "tidyverse" packages are 🐱

Python : Loading libraries

```
import numpy as _  
import pandas as _  
import seaborn
```

Python vs R: Loading libraries

```
import numpy as np
import pandas as pd
import seaborn
```

```
library(tidyverse)
```

— Attaching packages — tidyverse 1.3.1 —

✓ ggplot2 3.3.6 ✓ purrr 0.3.4

✓ tibble 3.1.7 ✓ dplyr 1.0.9

✓ tidyr 1.2.0 ✓ stringr 1.4.0

✓ readr 2.1.2 ✓ forcats 0.5.1

— Conflicts — tidyverse_conflicts() —

✗ dplyr::filter() masks stats::filter()

✗ dplyr::lag() masks stats::lag()

Python : Reading a file

```
countries =  
    pd.____("countries.tsv",  
    delimiter="____")
```

Python vs R: Reading a file

```
countries =  
    pd.read_csv("countries.tsv",  
                delimiter="\t")
```

```
countries <- read_tsv("countries.tsv")
```

Rows: 230 Columns: 63

— Column specification—

Delimiter: "\t"

chr (34): Country, Continent, Region, Location, Highest
Point, Langua...

dbl (28): Area, Borders, Length, Coastline, Height,
Temperature, Popu...

i Use `spec()` to retrieve the full column specification
for this data.

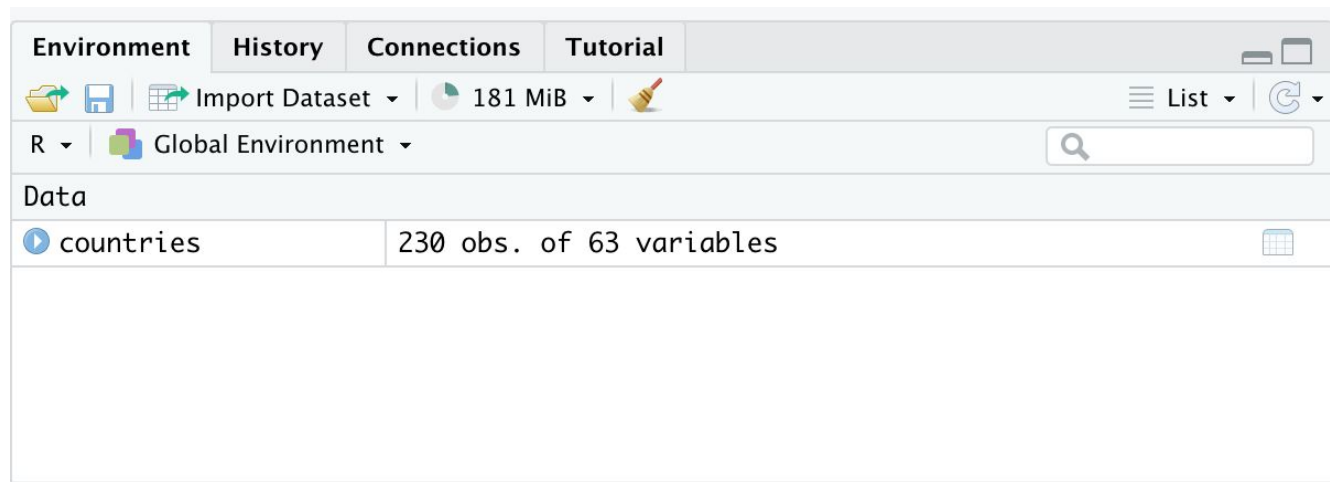
i Specify the column types or set `show_col_types =
FALSE` to quiet this message.

Python vs R: Reading a file

```
countries =  
    pd.read_csv("countries.tsv",  
                delimiter="\t")
```

```
countries <- read_tsv("countries.tsv")
```

RStudio keeps track
of defined variables
and their sizes



Python : Show the first 3 rows

countries.____(3)

Python vs R: Show the first rows

`countries.head(3)`

`head(countries, 3)`

or

`countries %>% head(3)`

Python vs R: Show the first rows

`countries.head(3)`

`head(countries, 3)`

or

`countries %>% head(3)`

"pipe" operator

value on the left becomes
input to the right

Python : Show two columns

```
countries__"Country",  
          "Population"__
```

Python vs R: Show two columns

```
countries[["Country",  
          "Population"]]
```

```
countries %>% select(Country,  
                    Population)
```

Python vs R: Show two columns

```
countries[["Country",  
          "Population"]]
```

```
countries %>% select(Country,  
                    Population)
```

R can figure out that
"Population" is a column
name, so it doesn't need
quotes

Python : Select rows by condition

```
countries._____  
    countries["Population"] >  
    100000000 _____
```

Python vs R: Select rows by condition

```
countries.loc[  
    countries["Population"] >  
    100000000 ]
```

```
countries %>%  
    filter(Population >  
    100000000)
```

**Research Question:
do more urbanized countries
have larger populations?**

Python : Sort rows by a column

```
countries.sort  
    (by="Urban")[[ "Country",  
                  "Urban"]]
```

Python vs R: Sort rows by a column

```
countries.sort_values  
(by="Urban")["Country",  
"Urban"]]
```

```
countries %>% arrange(Urban)  
%>% select(Country, Urban)
```

Pipes allow us to
combine multiple
operations

Python vs R: Sort rows by a column

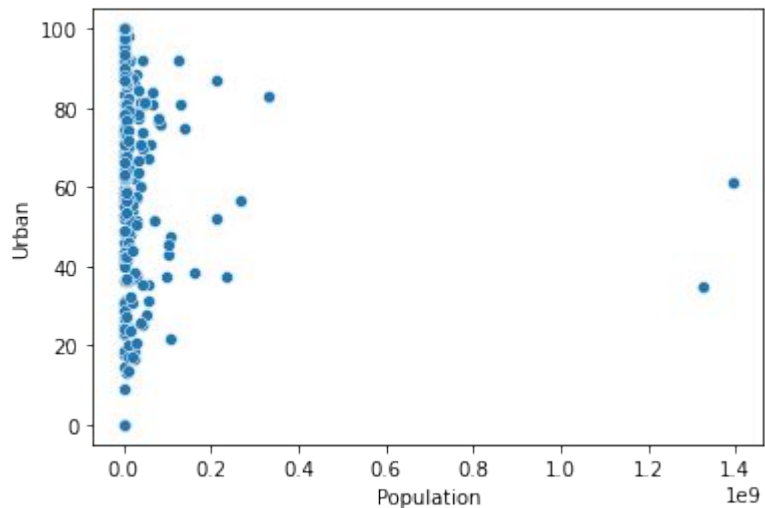
```
countries.sort_values  
(by="Urban")["Country",  
"Urban"]]
```

	Country	Urban
227	Wallis and Futuna (France)	0.0
226	Tokelau (New Zealand)	0.0
224	Montserrat (United Kingdom)	9.1
68	Papua New Guinea	13.3
202	Burundi	13.7
...
175	Macau (China)	100.0
182	Sint Maarten (Netherlands)	100.0
7	Singapore	100.0
229	Holy See	100.0
194	Kosovo	NaN

230 rows × 2 columns

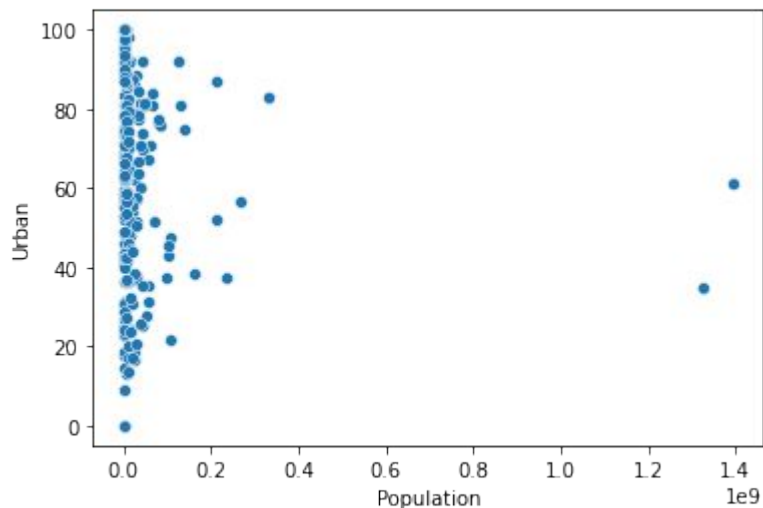
Python : plot data as points

```
seaborn._____ (data=countries,  
x="_____", y="_____")
```

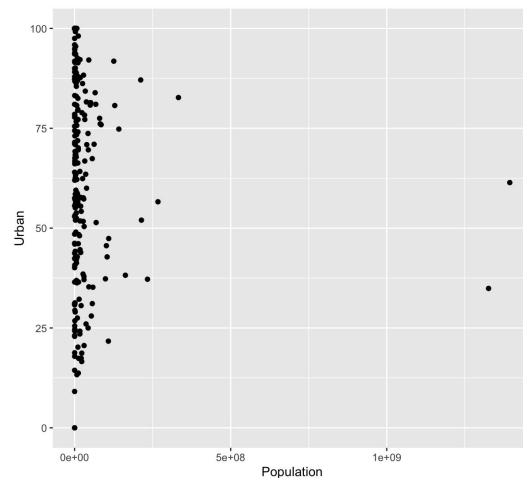


Python vs R: plot data as points

```
seaborn.scatterplot(data=countries,  
                    x="Population", y="Urban")
```



```
ggplot(data=countries,  
       aes(Population, Urban)) +  
  geom_point()
```

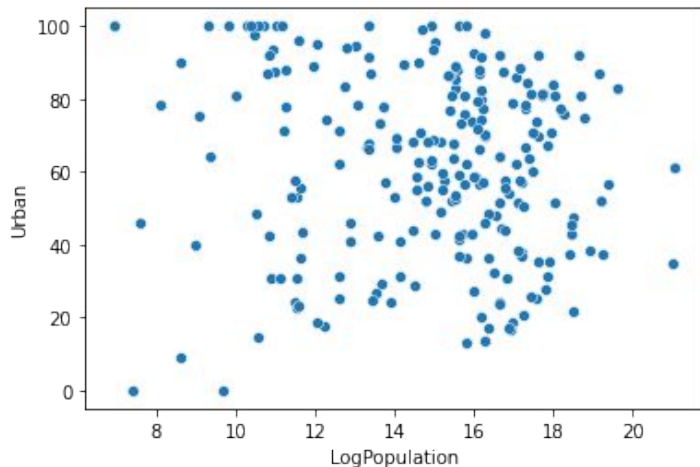


Python : Add a "LogPopulation" column

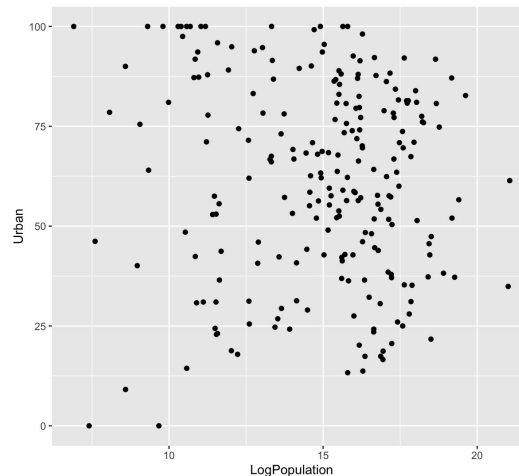
```
countries[_____] =  
    np._____(_____[_____])
```

Python vs R: Add a "LogPopulation" column

```
countries["LogPopulation"] =  
    np.log(countries["Populat  
ion"])
```



```
countries <- countries %>%  
    mutate(LogPopulation =  
        log(Population))
```



Python vs R: Add a "LogPopulation" column

```
countries["LogPopulation"] =  
    np.log(countries["Populat  
ion"])
```

```
countries <- countries %>%  
    mutate(LogPopulation =  
        log(Population))
```

```
countries <- countries %>% mutate
```

RStudio helps with
API hints (also Google
Colab and VSCode for
Python)

◆ mutate	{dplyr}
◆ mutate_	{dplyr}
◆ mutate_all	{dplyr}
◆ mutate_at	{dplyr}
◆ mutate_each	{dplyr}
◆ mutate_each_	{dplyr}
◆ mutate_if	{dplyr}

mutate(.data, ...)

Create, modify, and delete columns

mutate() adds new variables and
transmute() adds new variables
variables overwrite existing variables
variables can be removed by setting their
value to NULL

Press F1 for additional help

Files

Python: Predict population from Urban%

```
urban_pop_model = LinearRegression()  
    .____(countries[["Urban"]], countries["LogPopulation"])  
  
urban_pop_model.____, urban_pop_model.____  
  
(array([-0.0051409]), 16.475302332927527)
```

Python: Predict population from Urban%

```
urban_pop_model = LinearRegression()  
    .fit(countries[["Urban"]], countries["LogPopulation"])  
  
urban_pop_model.coef_, urban_pop_model.intercept_  
  
(array([-0.0051409]), 16.475302332927527)
```

R: Predict population from Urban%

```
urban_pop_model <- lm(LogPopulation ~ Urban, data=countries)
summary(urban_pop_model)
```

Residuals:

Min	1Q	Median	3Q	Max
-6.8100	-0.9732	0.1155	1.2519	4.8958

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	16.475302	0.479951	34.327	<2e-16 ***
Urban	-0.005141	0.007324	-0.702	0.484

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.97 on 142 degrees of freedom

Multiple R-squared: 0.003458, Adjusted R-squared: -0.00356

F-statistic: 0.4927 on 1 and 142 DF, p-value: 0.4839

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R uses "~" to build a
formula for linear models

Coefficients:

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Is our coefficient >
or < 0?

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Urban coefficient
slightly < 0

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Is our coefficient
“statistically
significant”?

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F-statistic: 0.4927 on 1 and 142 DF, p-value: 0.4839

Nope!
The intercept is
definitely not zero,
but Urban% looks
random

R: Predict population from Urban% + Area

```
ur_ar_pop_model <- lm(LogPopulation ~ Urban + Area, data=countries)  
summary(ur_ar_pop_model)
```

Residuals:

Min	1Q	Median	3Q	Max
-6.4715	-0.9001	0.2607	1.1208	3.6736

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.644e+01	4.378e-01	37.544	< 2e-16 ***
Urban	-9.844e-03	6.736e-03	-1.462	0.146
Area	3.765e-07	6.911e-08	5.448	2.21e-07 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.797 on 141 degrees of freedom

Multiple R-squared: 0.1768, Adjusted R-squared: 0.1651

F-statistic: 15.14 on 2 and 141 DF, p-value: 1.108e-06

Include multiple
predictors with +

R: Predict population from Urban% + Area

```
ur_ar_pop_model <- lm(LogPopulation ~ Urban + Area, data=countries)
summary(ur_ar_pop_model)
```

Residuals:

Min	1Q	Median	3Q	Max
-6.4715	-0.9001	0.2607	1.1208	3.6736

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
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F-statistic: 15.14 on 2 and 141 DF, p-value: 1.108e-06

1. Is Urban coefficient statistically significant at the 0.001 level?
2. Is Area coefficient statistically significant at the 0.001 level?

R: Predict population from Urban% + Area

```
ur_ar_pop_model <- lm(LogPopulation ~ Urban + Area, data=countries)
summary(ur_ar_pop_model)
```

Residuals:

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1. No 2. Yes

Urban% is still not significant, but lower p value.
Land area is highly significant!

R: Predict population from Urban% + Area

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ur_ar_pop_model <- lm(LogPopulation ~ Urban + Area, data=countries)
summary(ur_ar_pop_model)
```

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F-statistic: 15.14 on 2 and 141 DF, p-value: 1.108e-06

Which coefficient has
larger magnitude, Urban
or Area?

R: Predict population from Urban% + Area

```
ur_ar_pop_model <- lm(LogPopulation ~ Urban + Area, data=countries)
summary(ur_ar_pop_model)
```

Residuals:

Min	1Q	Median	3Q	Max
-6.4715	-0.9001	0.2607	1.1208	3.6736

Coefficients:

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Magnitude means absolute value; Urban's is bigger:
 $0.009844 > 0.0000003765$

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Multiple R-squared: 0.1768, Adjusted R-squared: 0.1651

F-statistic: 15.14 on 2 and 141 DF, p-value: 1.108e-06

1 unit (percentage point) increase in Urban% corresponds to a _____
[increase/decrease] in log population.

1 unit (square miles) increase in Area corresponds to a _____
[increase/decrease] in log population.

R: Predict population from Urban% + Area

```
ur_ar_pop_model <- lm(LogPopulation ~ Urban + Area, data=countries)
summary(ur_ar_pop_model)
```

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(Intercept)	1.644e+01	4.378e-01	37.544	< 2e-16 ***
Urban	-9.844e-03	6.736e-03	-1.462	0.146
Area	3.765e-07	6.911e-08	5.448	2.21e-07 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

1 unit (percentage point) increase in Urban% corresponds to a .0098 decrease in log population.

1 unit (square miles) increase in Area corresponds to a 0.0000003765 increase in log population. Small but significant!

Residual standard error: 1.797 on 141 degrees of freedom

Multiple R-squared: 0.1768, Adjusted R-squared: 0.1651

F-statistic: 15.14 on 2 and 141 DF, p-value: 1.108e-06

Python: Predict population from Urban%

```
outputs, predictors = patsy.dmatrices("LogPopulation ~ Urban",  
    data=countries, return_type="dataframe")  
sm_urban_pop_model = sm.OLS(outputs, predictors).fit()  
sm_urban_pop_model.summary()
```

OLS Regression Results

Dep. Variable:	LogPopulation	R-squared:	0.003
Model:	OLS	Adj. R-squared:	-0.004
Method:	Least Squares	F-statistic:	0.4927
Date:	Wed, 30 Nov 2022	Prob (F-statistic):	0.484
Time:	10:56:43	Log-Likelihood:	-300.98
No. Observations:	144	AIC:	606.0
Df Residuals:	142	BIC:	611.9
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
Intercept	16.4753	0.480	34.327	0.000	15.527	17.424
Urban	-0.0051	0.007	-0.702	0.484	-0.020	0.009

Omnibus:	16.672	Durbin-Watson:	1.709
Prob(Omnibus):	0.000	Jarque-Bera (JB):	21.589
Skew:	-0.684	Prob(JB):	2.05e-05
Kurtosis:	4.315	Cond. No.	192.

Using statsmodels
package

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Python: Predict population from Urban%

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                                       data=countries, return_type="dataframe")  
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Urban	-0.0051	0.007	-0.702	0.484	-0.020	0.009

**dmatrixes uses the same
formula syntax as R**

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Df Residuals:	142	BIC:	611.9			
Df Model:	1					
Covariance Type:	nonrobust					
	coef	std err	t	P> t	[0.025	0.975]
Intercept	16.4753	0.480	34.327	0.000	15.527	17.424
Urban	-0.0051	0.007	-0.702	0.484	-0.020	0.009

statsmodels does not
include intercept by
default, dmatrices
function adds an
intercept

Python: Predict population from Urban%

```
outputs, predictors = patsy.dmatrices("LogPopulation ~ Urban",  
    data=countries, return_type="dataframe")  
sm_urban_pop_model = sm.OLS(outputs, predictors).fit()  
sm_urban_pop_model.summary()
```

OLS Regression Results

Dep. Variable:	LogPopulation	R-squared:	0.003
Model:	OLS	Adj. R-squared:	-0.004
Method:	Least Squares	F-statistic:	0.4927
Date:	Wed, 30 Nov 2022	Prob (F-statistic):	0.484
Time:	10:56:43	Log-Likelihood:	-300.98
No. Observations:	144	AIC:	606.0
Df Residuals:	142	BIC:	611.9
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
Intercept	16.4753	0.480	34.327	0.000	15.527	17.424
Urban	-0.0051	0.007	-0.702	0.484	-0.020	0.009
Omnibus:	16.672	Durbin-Watson:	1.709			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	21.589			
Skew:	-0.684	Prob(JB):	2.05e-05			
Kurtosis:	4.315	Cond. No.	192.			

Same output!

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Python & R: data reshaping

	Python	SQL	R
Wide to long	melt(), stack(), ...	CROSS APPLY(), UNPIVOT()	melt(), pivot_longer(), gather(), ...
Long to wide	pivot(), pivot_table(), unstack(), ...	PIVOT()	pivot_wider(), dcast(), spread(), ...

1 minute break & attendance!



tinyurl.com/ey7w79k5

Python : inner join of df1 and df2 (on column 'id')

(Using Pandas merge!)

```
____.merge(____, how=____,  
           on=____)
```

Python : inner join of df1 and df2 (on column 'id')

(Using Pandas merge!)

```
df1.merge(df2, how='inner',  
          on='id')
```

SQL : inner join of df1 and df2 (on column 'id')

(Using Pandas merge!)

```
df1.merge(df2, how='inner',  
          on='id')
```

(Using SQL merge!)

```
SELECT *
```

```
FROM df1
```

```
_____
```

```
_____ df1.id _____ df2.id
```

SQL : inner join of df1 and df2 (on column 'id')

```
df1.merge(df2, how='inner',  
          on='id')
```

```
SELECT *  
  FROM df1  
  INNER JOIN df2  
  ON df1.id = df2.id
```

Python & SQL & R: inner join of df1 and df2 (on column 'id')

```
df1.merge(df2, how='inner',  
          on='id')
```

```
SELECT *  
  FROM df1  
 INNER JOIN df2  
ON df1.id = df2.id
```

Using Base R:
`merge(df1, df2, by='id')`

Using dplyr package:
`df1 %>% inner_join(df2,by='id')`

Python : get average 'price' grouped by 'product' from dataframe 'df'

(Using Pandas!)

____.groupby(____)[____].____()

Python : get average 'price' grouped by 'product' from dataframe 'df'

(Using Pandas!)

```
df.groupby('product')['price'].mean()
```

SQL : get average 'price' grouped by 'product' from dataframe 'df'

(Using Pandas!)

```
df.groupby('product')['price'].mean()
```

(Using SQL!)

```
SELECT _____  
____ df  
_____  
_____  
_____
```

SQL : get average 'price' grouped by 'product' from dataframe 'df'

(Using Pandas!)

```
df.groupby('product')['price'].mean()
```

(Using SQL!)

```
SELECT AVG(price)
FROM df
GROUP BY product
```

Python & SQL & R: get average 'price' grouped by 'product' from dataframe 'df'

(Using Pandas!)

```
df.groupby('product')['price'].  
    mean()
```

(Using SQL!)

```
SELECT AVG(price)  
FROM df  
GROUP BY product
```

(Using R tidyverse)

```
df %>% group_by(product)  
    %>% summarise(mean(price))
```

Numpy stats in 1-D

```
>>> a = np.array([[1, 2], [3, 4]])
```

```
>>> _____._____ (2,2)
```

```
>>> np.mean(a, axis=0) ?
```

```
>>> np.median(a, axis=1) ?
```

Numpy stats in 1-D

```
>>> a = np.array([[1, 2], [3, 4]])
```

```
>>> np.shape                                (2,2)
```

```
>>> np.mean(a, axis=0)                    array([2., 3.])
```

```
>>> np.median(a, axis=1)                  array([1.5, 3.5])
```

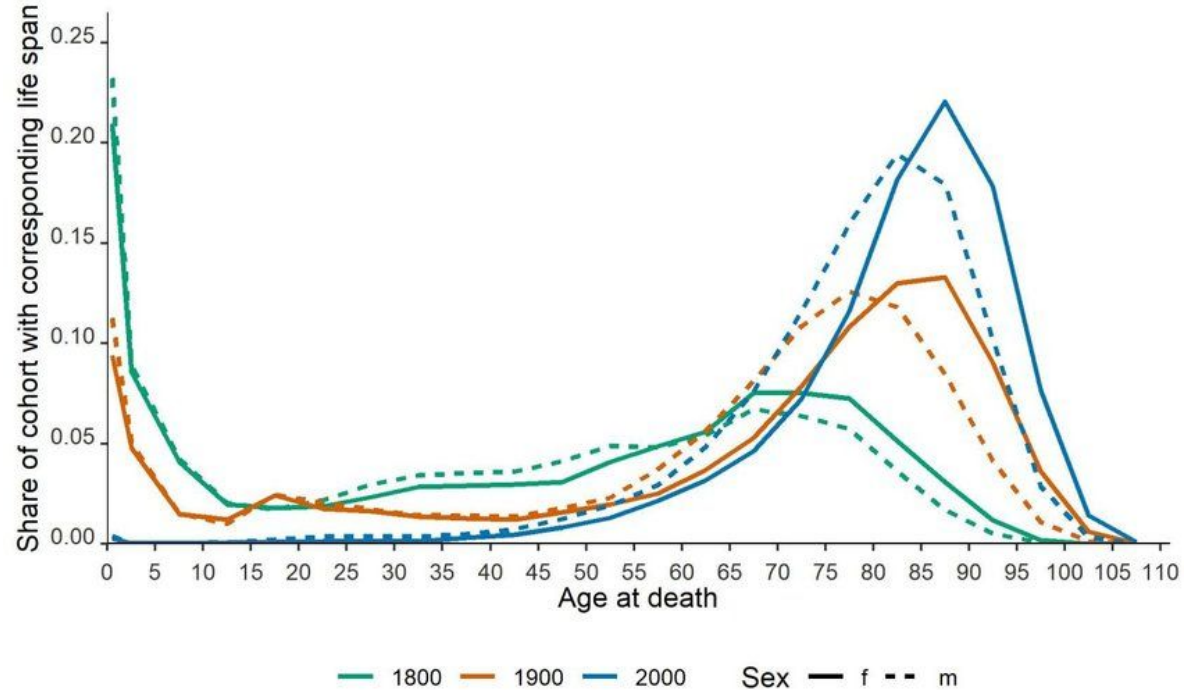
Numpy stats in 1-D

```
>>> a = np.array([[1, 2], [3, 4]])  
  
>>> __.____ (2,2)  
  
>>> np.mean(a, axis=0) array([2., 3.])  
  
>>> np.median(a, axis=1) array([1.5, 3.5])
```

Remember: median minimizes sum of “absolute distances”
but the Python default is the midpoint!

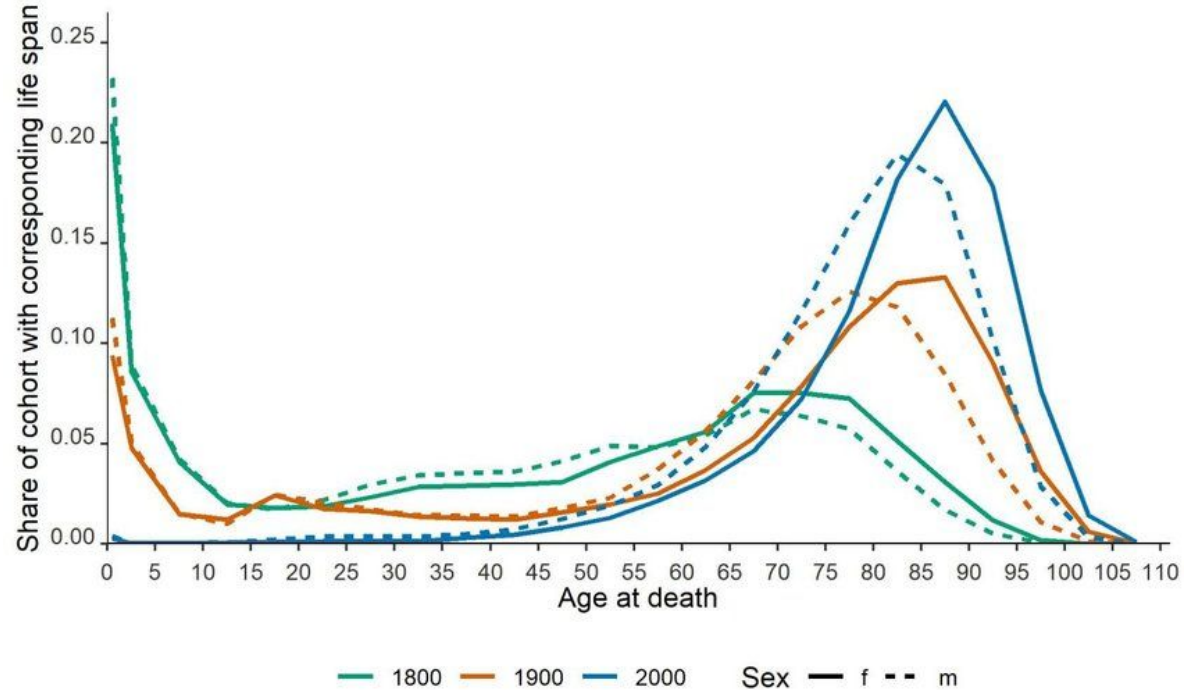
Swedish ages at death

1. Is mean meaningful?
2. Is median meaningful?



Swedish ages at death

Nope & nope,
consider using a
metric like average
adult age of death for
bimodal data →



What are these called?

$$\frac{\sum_i (X_i - \bar{X})^2}{N}$$

1.

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

2.

What are these called?

$$\frac{\sum_i (X_i - \bar{X})^2}{N}$$

1. Variance

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

2. Covariance

Covariance

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

What signs will covariance take in each quadrant?



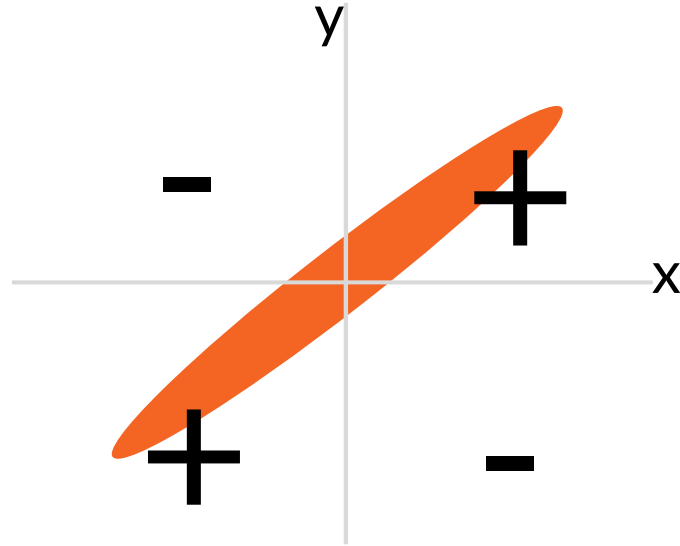
(Assume mean X and mean Y = 0)

Covariance

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

What signs will covariance take in each quadrant?

most positive
covariance:
close to the
diagonal, in the
positive
quadrants



Covariance

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

What is the covariance of x with itself?

Covariance

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

What is the covariance of x with itself? **Variance!**

$$\frac{\sum_i (X_i - \bar{X})^2}{N}$$

Normalization

What is this called? $(X_i - \bar{X})/\sigma_x$

Normalization

What is this called?

$$(X_i - \bar{X})/\sigma_x$$

“z-score”

Covariance calculated with z-scores?

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

$$(X_i - \bar{X})/\sigma_x$$

What is this called?

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})/(\sigma_x \sigma_y)}{N}$$

Covariance calculated with z-scores?

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

$$(X_i - \bar{X})/\sigma_x$$

(Pearson) Correlation:

$$\text{cov}(X, Y)$$

$$\sigma_x \sigma_y$$

What is correlation of X with itself?

$$\frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y}) / (\sigma_x \sigma_y)}{N}$$

What is correlation of X with itself?

$$\frac{\sum_i (X_i - \bar{X})(X_i - \bar{X}) / (\sigma_x \sigma_x)}{N}$$

$$= \text{Var}(X) / (\sigma_x \sigma_x)$$

$$= 1$$

Covariance, Correlation

- Correlation is normalized and measures both strength and direction of linear relationship between X and Y
- Covariance just measures direction of linear relationship between X and Y

**When is it useful to look at
correlation matrices?**

When is it useful to look at correlation matrices?

- Make a visual summary of lots of data to understand patterns
- Check for multicollinearity when deciding on regression inputs

Why is multicollinearity bad?

- Do you get similar coefficients on **ad** if you run...
 - $\text{sales} \sim \text{price} + \text{ad} + \text{loc} + \text{volume}$
 - $\text{sales} \sim \text{price} + \text{ad} + \text{loc}$

	sales	price	ad	loc	volume
sales	1.00	-0.70	0.12	0.01	0.39
price	-0.70	1.00	0.00	0.00	-0.18
ad	0.12	0.00	1.00	0.00	-0.74
loc	0.01	0.00	0.00	1.00	-0.04
volume	0.39	-0.18	-0.74	-0.04	1.00

Why is multicollinearity bad?

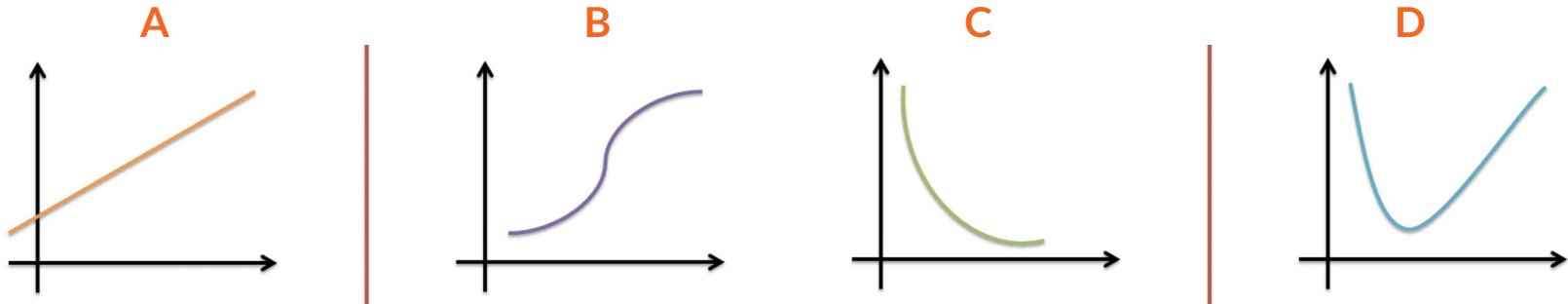
- Collinear inputs can make the regression coefficients very unstable

	Estimate
(Intercept)	125.931
price	-11.836
ad	131.283
loc	7.768
volume	11.870

	Estimate
(Intercept)	662.733
price	-15.100
ad	20.500
loc	1.833

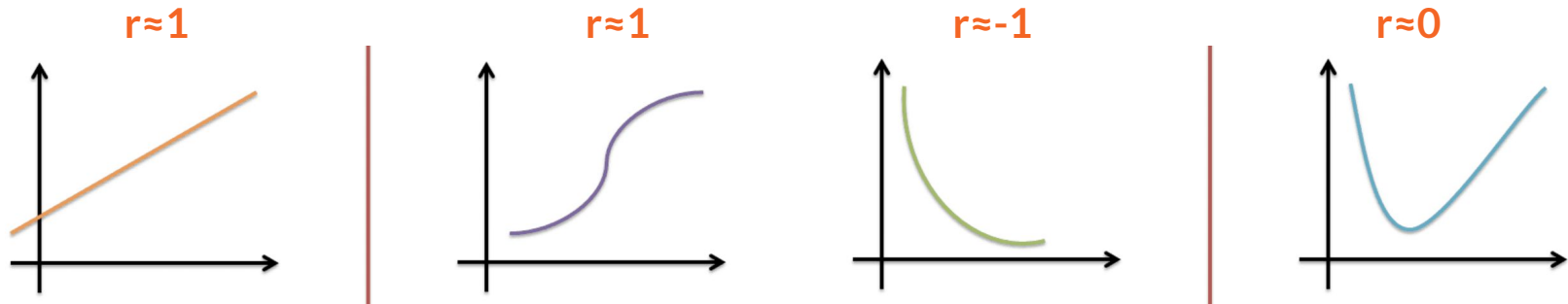
Rank (spearman) correlations

- Used to understand x and y monotonicity instead of linearity
- Which is/are non-monotone?



Rank (spearman) correlations

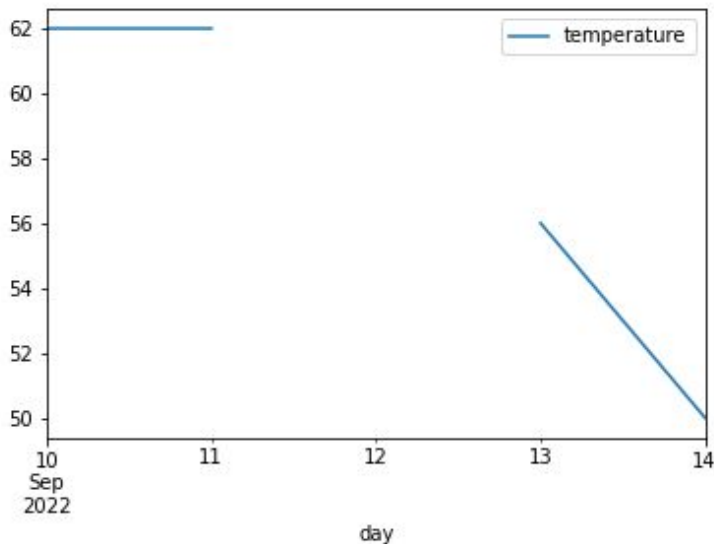
- D is non-monotone



Time Series

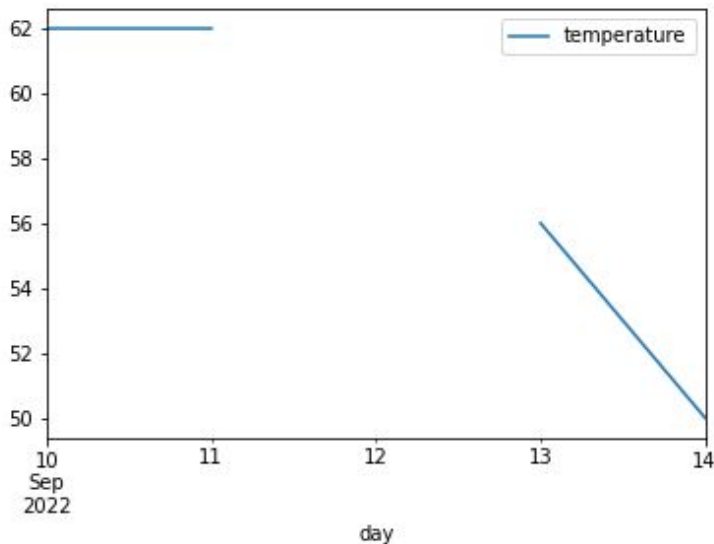
- Data where one column denotes time (*datetime* format)
- Most meaningful when data is aggregated so that each “time step”:
 - Is regularly spaced (e.g. daily, monthly, quarterly data) chronologically
 - Has corresponding data per time step
 - Is unique
 - Deals with missing values

Time Series: why is this happening?



Time Series: why is this happening?

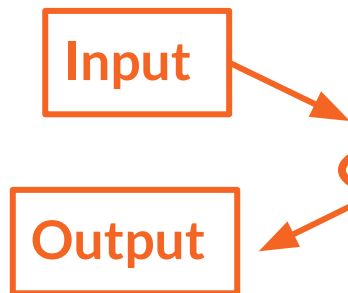
Missing data
(has NaNs)!



Regression motivations

1. Make predictions
2. Summarize relationship between variables
3. Inspect outliers and other oddities

Regression motivations



1. Make predictions
2. Summarize relationship between variables
3. Inspect outliers and other oddities

- $y = \alpha + \beta \cdot x$

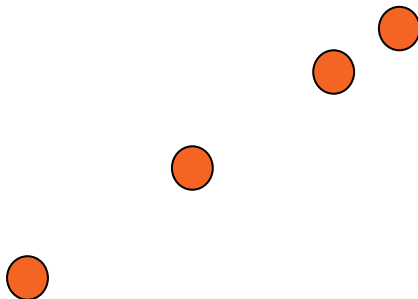
- $y \sim x$

Regression motivations

Does this regression equation $y = \alpha + \beta \cdot x$
pass through all input data?

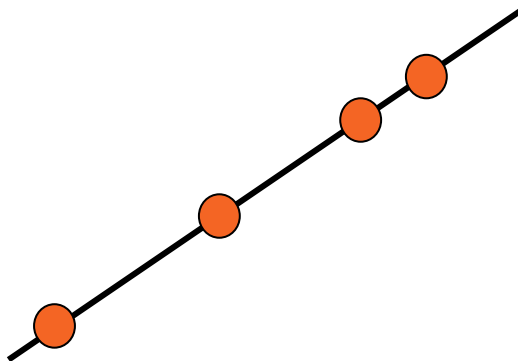
Regression motivations

Does this regression equation $y = \alpha + \beta \cdot x$
pass through all input data?



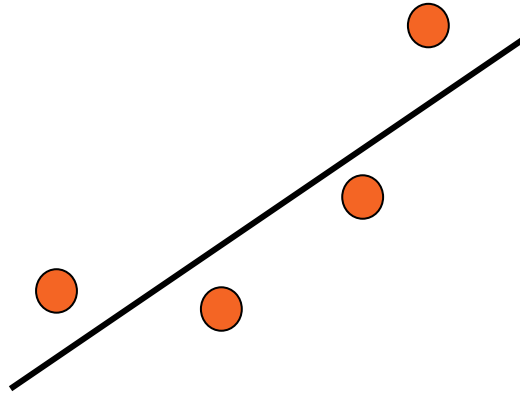
Regression motivations

Does this regression equation $y = \alpha + \beta \cdot x$
pass through all input data?



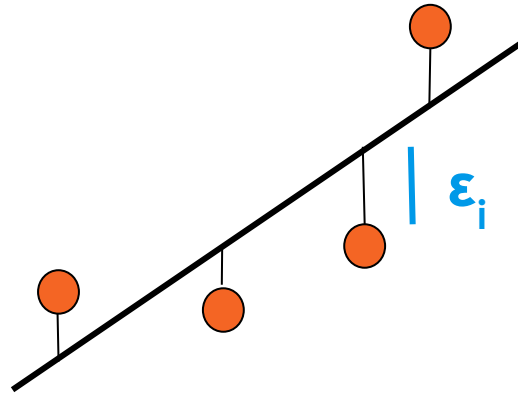
Regression motivations

Does this regression equation $y = \alpha + \beta \cdot x$
pass through all input data?



Regression motivations

Does this regression equation $y = \alpha + \beta \cdot x$ pass through all input data?



Regression motivations

Does this regression equation $y = \alpha + \beta \cdot x$
pass through all input data?

No, need error term (residual) corresponding
to each input i to represent each individual
dot: $y_i = \alpha + \beta x_i + \epsilon_i$

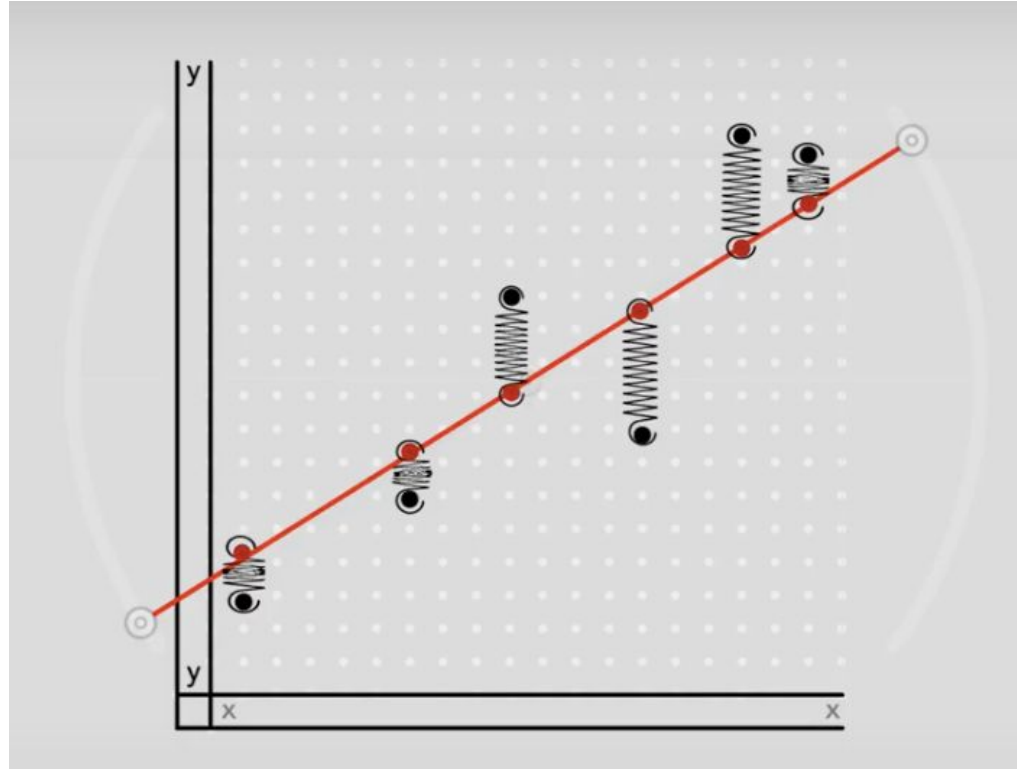
Regression (OLS)

Does this regression equation $y = \alpha + \beta \cdot x$ pass through all input data?

No, need error term (residual) corresponding to each input i to represent each individual

dot: $y_i = \alpha + \beta x_i + \epsilon_i$

KEY INSIGHT: we want to minimize this error across all of our points i



Comparing linear models

Is $y = \alpha$ a linear regression model?

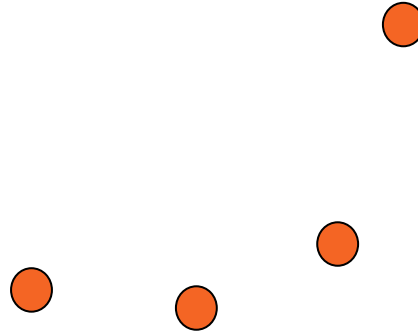
Comparing linear models

Is $y = \alpha$ a linear regression model? YES!

Comparing linear models

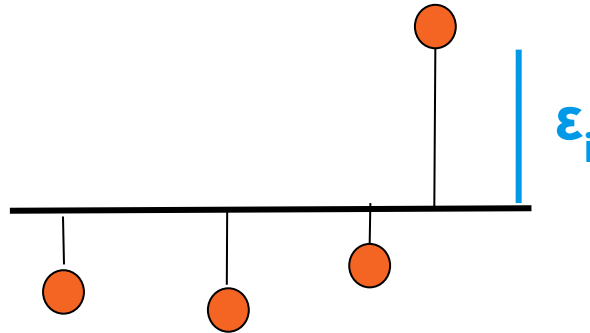
Is $y = \alpha$ a linear regression model? YES!

Draw the line for this model if $\alpha = \bar{y}$:



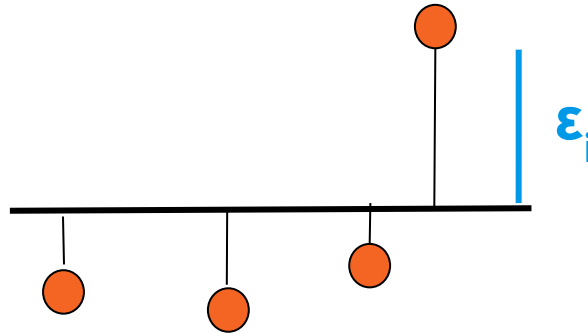
Comparing linear models

Draw the line for $y = a = \bar{y}$:



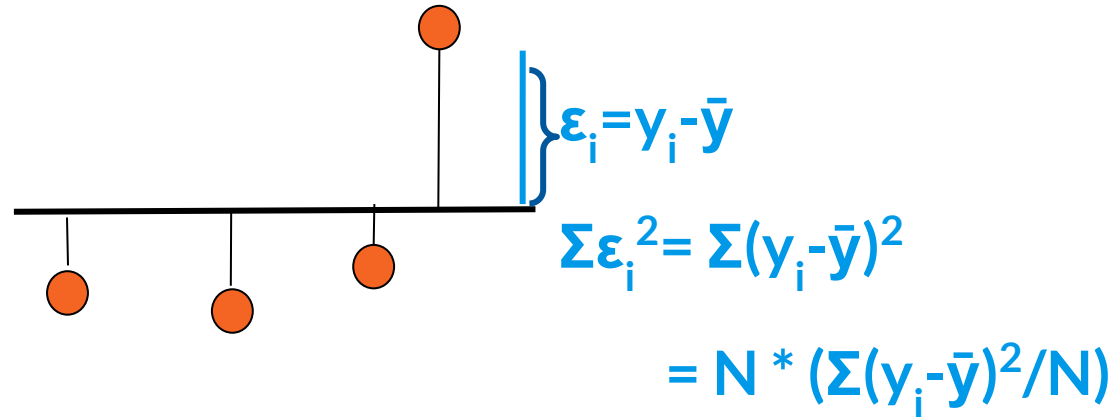
Comparing linear models

What statistic is the sum of squared residuals ϵ_i for $y = \alpha = \bar{y}$ equal to?



Comparing linear models

What statistic is the sum of squared residuals ϵ_i for $y = a$ equal to? $N \cdot \text{Var}(Y)$



Regression (OLS)

Does this regression equation $y = \alpha + \beta \cdot x$ pass through all input data?

No, need error term (residual) corresponding to each input i to represent each individual

dot: $y_i = \alpha + \beta x_i + \epsilon_i$

KEY INSIGHT: we want to minimize this error across all of our points i

How to get α , β ?

- **Calculus** (minimize sum of squared error)
- **Stochastic gradient descent** (need this method for complicated models)
- **Use Python**
 - Fit linear regression
 - Call `intercept_` and `coef_`

How to get α , β ?

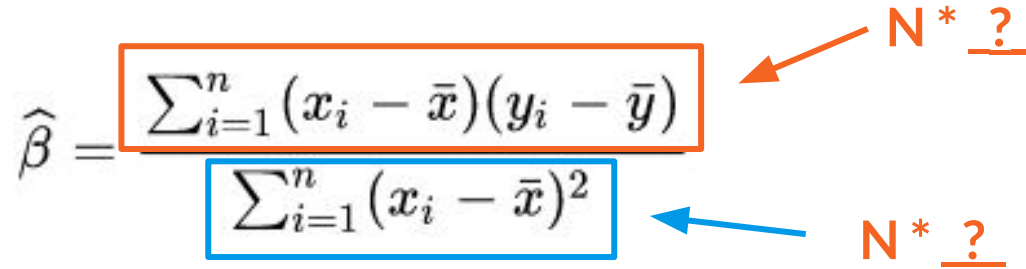
- **Calculus** (minimize sum of squared error)
- **Stochastic gradient descent** (need this method for complicated models)
- **Use Python**
 - Fit linear regression
 - Call `intercept_` and `coef_`

Linear Regression β formula

$$\hat{\beta} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

Linear Regression β formula

$$\hat{\beta} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

An orange box highlights the numerator $\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$. A blue box highlights the denominator $\sum_{i=1}^n (x_i - \bar{x})^2$. An orange arrow points from the text $N * \underline{?}$ to the orange box. A blue arrow points from the text $N * \underline{?}$ to the blue box.

Linear Regression β formula

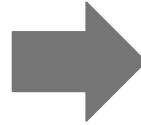
$$\hat{\beta} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$N * \text{cov}(x,y)$

$N * \text{var}(x)$

Which is tall, which is wide?

input_x	is_high	output_y
2022-09-19	high	77
2022-09-19	low	58
2022-09-20	high	73
2022-09-20	low	55
2022-09-21	high	80
2022-09-21	low	57



input_x	high	low
2022-09-19	77	58
2022-09-20	73	55
2022-09-21	80	57

Tall

input_x	is_high	output_y
2022-09-19	high	77
2022-09-19	low	58
2022-09-20	high	73
2022-09-20	low	55
2022-09-21	high	80
2022-09-21	low	57



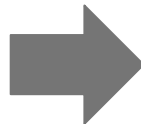
Wide

input_x	high	low
2022-09-19	77	58
2022-09-20	73	55
2022-09-21	80	57

- Which df would you use to run a regression?
- Write the model with ~ and variable names.

Tall

input_x	is_high	output_y
2022-09-19	high	77
2022-09-19	low	58
2022-09-20	high	73
2022-09-20	low	55
2022-09-21	high	80
2022-09-21	low	57



Wide

input_x	high	low
2022-09-19	77	58
2022-09-20	73	55
2022-09-21	80	57

- Tall
- $\text{output}_y \sim \text{input}_x + \text{is_high}$

Tall

input_x	is_high	output_y
2022-09-19	high	77
2022-09-19	low	58
2022-09-20	high	73
2022-09-20	low	55
2022-09-21	high	80
2022-09-21	low	57



Wide

input_x	high	low
2022-09-19	77	58
2022-09-20	73	55
2022-09-21	80	57

- What keyword do you use to go from tall to wide?

Tall

input_x	is_high	output_y
2022-09-19	high	77
2022-09-19	low	58
2022-09-20	high	73
2022-09-20	low	55
2022-09-21	high	80
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Wide

input_x	high	low
2022-09-19	77	58
2022-09-20	73	55
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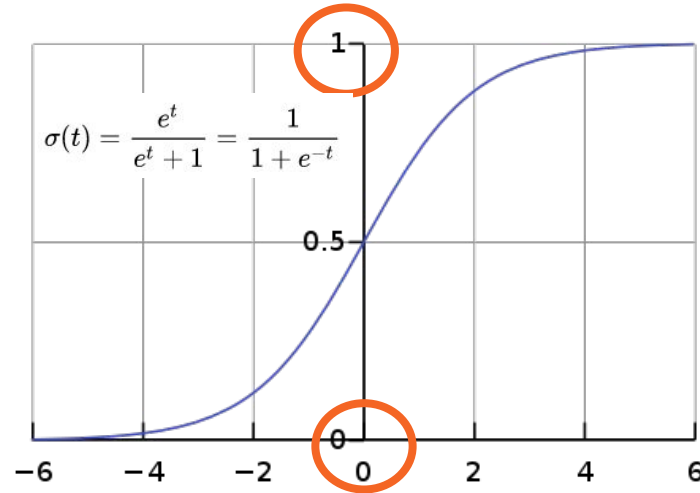
- What keyword do you use to go from tall to wide? **Pivot**

Logistic regression

- How do we “smoosh” $\alpha + \beta \cdot x$ predictions into a $[0,1]$ probability range?

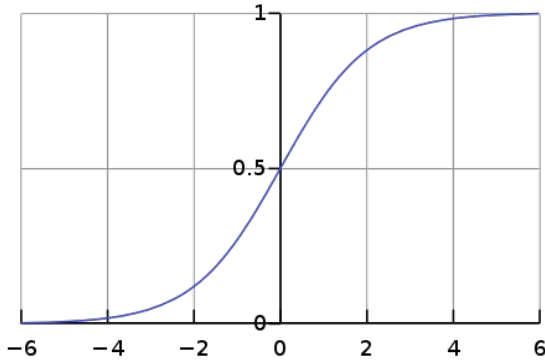
Logistic regression

- How do we “smoosh” $\alpha + \beta \cdot x$ predictions into a $[0,1]$ probability range?



**Logistic
(sigmoid)
function**

Logistic regression



- This allows us to predict the probability that $y=\text{TRUE}$

$$p(x) = \frac{1}{1+e^{-(\alpha+\beta \cdot x)}}$$

- So, interpret logistic regression differently than linear regression

What is this called?

Probability $y=1$:
$$p(x) = \frac{1}{1+e^{-(\alpha+\beta \cdot x)}}$$

Probability $y=0$:
$$1 - p(x)$$

$$\text{Pr(Magnus win)} / \text{Pr (Magnus lose)} = p(x) / (1-p(x))$$

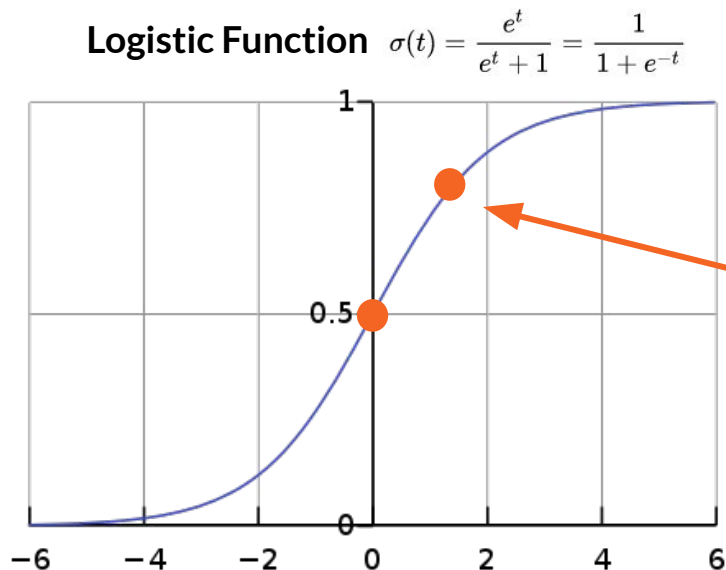
The Odds Ratio

Probability $y=1$:
$$p(x) = \frac{1}{1+e^{-(\alpha+\beta \cdot x)}}$$

Probability $y=0$:
$$1 - p(x)$$

$$\text{Pr(Magnus win)} / \text{Pr (Magnus lose)} = p(x) / (1-p(x))$$

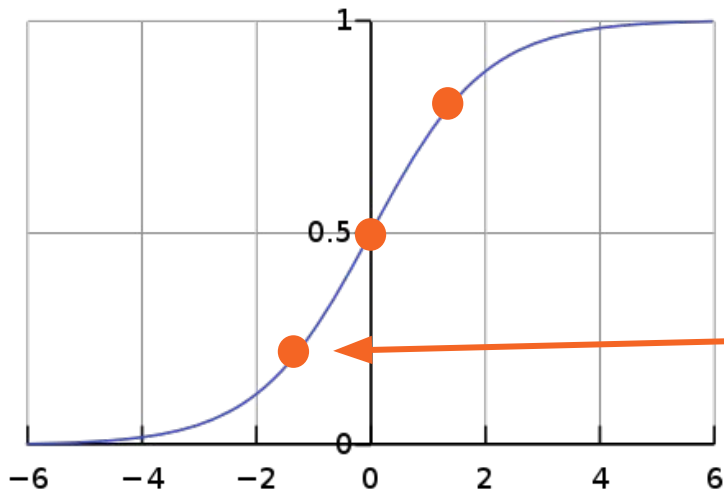
Intuition: log odds ratio



Log odds	Probability	Odds $e^{1.38} = 4$
0.0	0.5	1:1
1.38	0.8 $\sigma(1.38) = 0.8$	4:1 $0.8 = 4/(4+1)$
-1.38	?	1:4
-2.94	0.05	?
-4.59	0.01	?

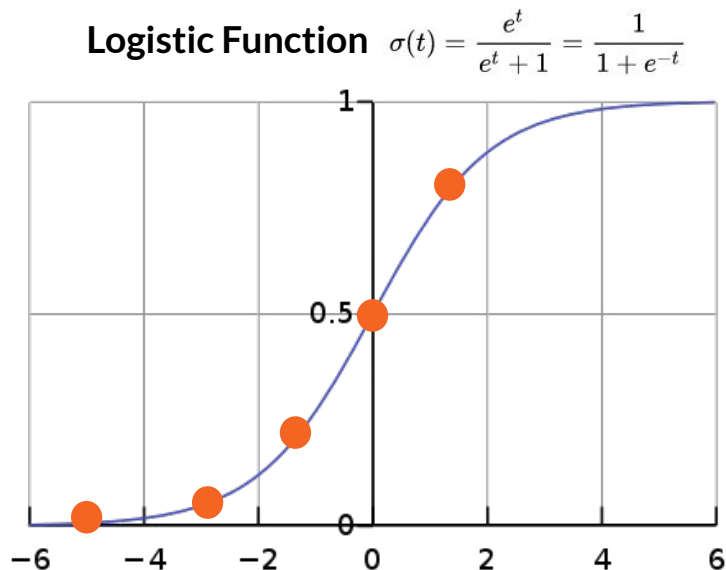
Intuition: log odds ratio

Logistic Function $\sigma(t) = \frac{e^t}{e^t + 1} = \frac{1}{1 + e^{-t}}$



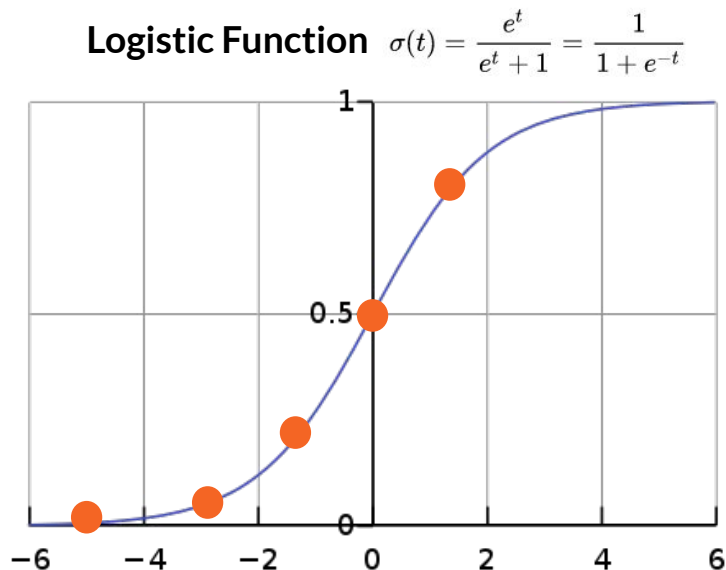
Log odds	Probability	Odds $e^{-1.38} = 0.25$
0.0	0.5	1:1
1.38	0.8	4:1
-1.38	0.2	1:4
-2.94	0.05	?
-4.59	0.01	?

Intuition: log odds ratio



Log odds	Probability	Odds
0.0	0.5	1:1
1.38	0.8	4:1
-1.38	0.2	1:4
-2.94	0.05	?
-4.59	0.01	?

Intuition: log odds ratio



Log odds	Probability	Odds
0.0	0.5	1:1
1.38	0.8	4:1
-1.38	0.2	1:4
-2.94	0.05	1:19
-4.59	0.01	1:99

Interpreting regressions: prediction (first line), summary (next lines)

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Canvas > Modules >
2. Regression and
Linear Models

Review how to
apply in examples &
derive the things in
this table!

Model	Regression Interpretation
Linear $y = \alpha + \beta x$	If $x=0$, $y = \alpha$ 1 unit change in x is associated with a β unit change in y
Linear-log $y = \alpha + \beta \ln(x)$	If $x=1$, $y = \alpha$ If x is multiplied by e , we expect a β unit change in y 1% change in x is associated with a $0.01 \cdot \beta$ unit change in y
Log-linear $\ln(y) = \alpha + \beta x$	If $x=0$, $y = e^\alpha$ For a 1 unit change in x , we expect y to be multiplied by e^β 1 unit change in x is associated with a $100 \cdot (\exp(\beta) - 1)\%$ change in y
Log-log	If $x=1$, $y = e^\alpha$

Admin

- Phase 5 due Dec 4th
- Final exam on Dec 10th at 2pm
 - You may bring one double-sided 8x11" cheat sheet
 - No calculators will be allowed

Admin

- Canvas > Modules
 - Discussion 13: Final exam from Fall 2022 for practice
 - 4. *Real World Applications*: List of topics in the 2nd half of the class to review
 - (in conjunction with the midterm review list, posted in 2. *Regression and Linear Models*)

Goodbye from Instructor Thalken!



- Used 2950 skills to get a new paper published
- Will be on a free trip to Singapore to go to a conference during next lecture!

Next Class

- Final exam review, continued!