Networks: Fall 2023	**Practice Midterm**
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The midterm is **Monday, October 2, in class**. It will be a closed-book, closed-notes exam, and calculators are not allowed. The best guide to the coverage of the midterm is the contents of the course lectures up through the end of the network exchange theory topic covered in class (Chapter 12). (The material in the chapters and sections that we skipped, such as Chapters 4 and 7, will not be on the midterm. And the trading networks topic from Chapter 11 that we will cover next week will not be on the midterm.) It is also useful to review the homeworks, and the exercises in the online course (on EdX or in Canvas) provide a good source of additional questions (with answers) for review.

To help in studying, we are providing the following practice midterm questions below. The style of questions approximately resembles the real midterm, although of course the actual questions on the real midterm may cover topics from the course that are not explicitly the focus of any question here. A typical midterm would consist of roughly five or six questions in the style of those given below; we are including eight questions here so as to provide one question based on each of Chapters 3, 5, 6, 8, 9, 10, and 12 and one question on auctions applied to crypto. If there is a question on crypto it will consist of proposed auction formats for miners to charge fees to customers who want to have their transactions recorded in a block; we would expect you to analyze these auctions using principles about auctions from class, but you will not need any knowledge specific to the operation of blockchains or cryptocurrencies. The practice midterm questions are not meant to be handed in; rather, we will discuss them at the midterm review session in class on Friday, September 29, and we will post solutions then. You can also discuss the questions with us in office hours.

(1) Consider a second-price, sealed-bid auction with one seller who has one unit of the object and two bidders $1, 2$ who have values of $v_1 \in [1, 10]$ and $v_2 \in [1, 10]$, respectively, for the object. The values v_1, v_2 are independent, private values.

(a) Suppose that both bidders follow their dominant strategies and the values are actually $v_1 = 5$ and $v_2 = 7$. Which bidder will win the auction and how much will that bidder pay for the good? Explain briefly.

(b) Now suppose that the winner of the auction has to pay the second highest bid plus an "auction fee" of 0.5. The loser still doesn't pay anything. That is, if bidder i wins and the second highest bid is x then bidder i actually pays $x + 0.5$. Suppose that bidder 1's value is $v_1 = 5$ and that bidder 1 does not know bidder 2's value or bidder 2's actual bid. How much should bidder 1 bid in this auction? (Recall that if a bidder wins the auction then he or she pays the second highest bid plus the "auction fee" of 0.5.) Explain briefly.

(2) In the payoff matrix below the rows correspond to player A's strategies and the columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

		Player B	
		L	R
Player A	U	$5, 5$	$2, 2$
	D	$3, 1$	$4, 4$

(2a) Find all pure strategy Nash equilibria of this game.

(2b) Does this game have any mixed strategy equilibria? If so find one; if not explain why not.

(3) Four hundred travelers begin in city A and must travel to city B. There are two routes between A and B.

Route I starts with a highway from A to C that takes one hour, and then is followed by a local street from city C to city B, which requires a travel time in hours equal to the number of travelers on the street divided by 100.

Route II begins with a local street from city A to city D, which requires a travel time per traveler in hours equal to the number of travelers on the street divided by 100, and ends with a highway from city D to city B which requires two hours.

All roads are one-way roads.

(a) Draw the network described above and label the edges with the travel time needed to move along the edge. Let x be the number of travelers who use Route I and let y be the number of travelers who use Route II.

(b) Travelers simultaneously choose which route to use. Find Nash equilibrium values of x and y, and explain your answer.

(c) Recent construction added a new option, an edge connecting C to D, which facilitates a new Route III: A to C to D to B. Let z be the number of travelers using Route III (and x and y be the number of travelers who use Route I and II as before). Suppose that travel on the C to D segment is instantaneous; it takes no time. Find Nash equilibrium values of x, y and z, and briefly explain your answer.

(d) What are possible travel times for the C to D segment that keeps your solution to part (b) the Nash equilibrium even with the new option added?

(4) Three buyers x, y and z and considering buying one of three houses A, B and C. Their valuations for the houses are given in the table below.

Buyer	Value for house A	Value for house B	Value for house C
x	8	10	10
y	7	3	2
z	3	2	8

You also know that the market cleared and houses A and B both sold for 2. You'd like to determine the possible range of prices that could be assigned to house C, so that together with prices of 2 for A and B we have a set of market-clearing prices. In particular, here is the question: what are the lowest and highest possible values for the price of C under these conditions? Explain your answer.

(5) A set of child-development researchers are studying the social network in a small middle school consisting grades 6, 7, and 8. The school has at least 10 students in each grade. From their observations, they have identified the following facts about the social ties, all of which we assume to be symmetric. (We'll simplify for the sake of asking a cleaner question.)

They observed that all students have strong ties to all other students in their own grade, and weak ties to students in neighboring grades, but 6th graders typically don't have any ties to 8th graders. They found one exception to this rule: siblings have strong ties no matter what pairs of grades they attend.

(5a) Suppose the school has a single pair of siblings. These siblings are in grades 7 and 8 respectively. Does the resulting social network satisfy the Strong Triadic Closure Property? Give a brief explanation for your answer.

(5b) Would your answer change depending on the grades the siblings attend? What would be your answer for grades 6 and 7? What would be your answer for grades 6 and 8? (In all these cases, you should assume this is the only pair of siblings in the school.) Give a brief explanation for your answer.

(6) Together with some anthropologists, you're studying a set of small villages in a square-shape region of rain forest. They've imposed a two-dimensional coordinate system on the region of rain forest: the anthropologists are based in a village that they label with the point $(0, 0)$, and then (x, y) in their coordinate system (with $x \geq 0$ and $y \geq 0$) refers to a point that is x miles east and y miles north of their special designated village.

Suppose for simplicity in this story, there is a village each point of the form (x, y) where x and y are each whole numbers between 0 and 5 inclusive.

Now, suppose that two villages view each other as allies if they are at most two miles apart, and they view each other as enemies otherwise.

To study these relationships, the anthropologists build a signed complete graph on the set of villages, with a "+" edge between each pair of villages that are allies and a "-" edge between each pair of villages that are enemies. Does this graph satisfy the Structural Balance property? Provide an explanation for your answer.

(7) Suppose a network exchange theory experiment is run on the graph shown in Figure 1. The experiment uses the one-exchange rule with \$12 placed on each edge.

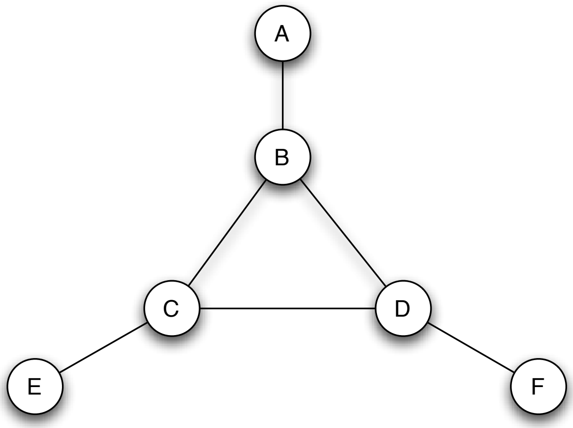

Figure 1: A graph for a network exchange theory experiment.

(7a) Which nodes will have the most power in this experiment (i.e make the most money)? Which node will have the least? Give a brief (1-2 sentence) explanation for your answer.

(7b) Consider the outcome in which nodes A, E, and F each get \$3 and nodes B, C, and D each get \$9. Is this a stable outcome? Are the values the nodes get a Nash Bargaining Outcome? Briefly explain your answer.

(7c) Consider the outcome in which nodes A, E, and F each get \$4 and nodes B, C, and D each get \$8. Is this a stable outcome? Are the values the nodes get a Nash Bargaining Outcome? Briefly explain your answer.

(7d) Now the experimenters vary the network as follows. They break the edges connecting E to C and F to D. They then re-attach both E and F to node B by a single edge. (So in the new network, E and F still each have a single outgoing edge, but each of these edges goes to B.) They then run a new round of experiments with the same people, but in this reconfigured network.

Explain what you think will happen to the relative power of each of the nodes in this new network, compared to the power that each had in (a). Give a brief (1-3 sentence) explanation for your answer.

(8) Customers who want to have their BitCoin transaction recorded on the blockchain offer to pay a fee to the miner who builds the block. Let's suppose that the block has room for exactly one transaction. We can interpret this setting as an auction in which the miner is an auctioneer with one object to sell (the one slot on the block) and the customers are the bidders—their proposed fee is their bid. We will assume that the miner acts as a trusted auctioneer; the miner does not submit fake bids and follows the rules of the auction.

Suppose that customers have independent private values, v_i for customer i, for having their transaction be the one that is recorded on the current block. If customer i's transaction is selected to be placed on the current block then i has a payoff of v_i minus any fee that i pays; if it's not placed on the current block then i's payoff is 0. To simplify things a bit we will assume that each customer submits a bid (the fee they propose to pay), the miner sees all of the bids, and the miner selects the customer with the highest bid. Exactly how the payment from the selected customer to the miner is determined and what affect it has is considered below.

(a) Suppose that if a customer who bids a fee of f is selected by the miner then the customer pays the fee f, and this customer's transaction is placed on the block. Is it a dominant strategy for customers to bid truthfully, that is, for each customer i to bid a fee of v_i? Explain.

(b) Suppose instead that if a miner selects a costumer who bids f, the customer pays the second-highest bid. So if the selected customer had a bid of f and second-highest bid was g, then the selected customer would pay g. Is it a dominant strategy for customers to bid truthfully, that is, for each customer i to bid a fee of v_i? Explain.

(c) Unfortunately, it's difficult to verify bids that were submitted by customers whose transactions were not placed on the block. So charging the second-highest bid for the one slot on the block (as in part (b)) isn't really feasible. An alternative that has been proposed is to charge the selected customer the amount they bid less a small percentage discount. (For example, if the selected customer bids f then they pay $(1-d)f$, for some small d.) If this proposal is implemented would it be a dominant strategy for customers to bid truthfully? Explain. [You do not need to determine an optimal bid.]