Dyson
Cornell
SC Johnson College of Business

AEM 2241
Finance
Tibor Jánosi
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AEM 2241 - Finance - Prelim I

Different exams had similar problems, but the details (e.g., the numerical values) may have been slightly different. We do not publish to all versions of all problems. If your problem is not directly listed, please solve it by analogy to the problems listed below. If you have any questions, the course staff will be happy to help.

1 General Instructions

This exam contains 11 pages, and 16 questions. Please check now that you have a complete exam. If not, ask for another copy.

This exam is "closed books, closed notes." However, a collection of useful formulas has been made available to you. You can use one of the **approved** financial calculators. Under no circumstances can you use a programmable device, including, but not limited to, calculators, tablets, phones, or laptops.

You are not allowed to cooperate with anybody else on this exam. All work submitted must be exclusively yours. Do not accept any assistance from, and do not provide any assistance to anybody.

We strongly suggest that you do **not** take your exam apart. If you do, you **must** staple the pages together securely before you hand in your exam. You are responsible for lost exam pages.

You have 75 minutes to work on the exam. Please plan to finish on time.

2 Finance Matters

1. All assumptions, conventions, and notations that we normally use can be relied without further explanations. If you use non-standard notations, explain what they mean.
2. Unless stated otherwise, we ask that in final answers dollar amounts be rounded to two decimals, and interest rates be rounded to four decimals. For example: \$156,798.38, \$9.75, 0.0315 = 3.15%, 0.1425=14.25%. For best results, consider using 4-5 decimals for intermediate answers.
3. Be careful to distinguish between per-period quantities, such as per-period coupon payments and interest rates, and their annualized versions, which are the ones that must be typically provided as results.
4. Unless we tell you otherwise, you may use either formulas or financial calculators to solve a problem.
5. Whenever possible, interpret the meaning of the results in terms consistent with the problem.

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- When faced with a multiple choice question, circle the answer you believe to be correct.
- If you believe NONE of the numerical answers suggested is close enough (within about 1-2% or less), write down your own result, circle it, and indicate unambiguously that that is the result you want us to consider.

Good Luck!

1. [5 points] Beatrice invests $1,360 in an account that pays 3 percent simple interest. How much more could she have earned over a 4-year period if the interest had been compounded annually?

   - $29.97
   - $7.49
   - $16.63
   - $26.79
   - $20.09

   Solution: We answer this question by taking the difference between the total amounts that would be/are in the accounts at the end of the 4 years: $1,360\,(1+0.03)^{\,4} - 1,360\,(1+0.03\cdot 4) =$ $7.4920.

2. [5 points] Miles invested $5,000 ten years ago and expected to have $10,000 today. He has neither added nor withdrawn any money since his initial investment. All interest was reinvested and compounded annually. As it turns out, he only has $8,400 in his account today. Which one of the following statements must be true?

   - He earned simple interest rather than compound interest.
   - He earned a lower interest rate than he expected.
   - He did not earn any interest on interest.
   - He ignored the Rule of 72, which caused his account to decrease in value.
   - The future value interest factor turned out to be higher than he expected.

   Solution: The second choice is the second answer.

3. [5 points] Your cousin deposited $2,500 today at 6.5 percent interest for 15 years. However, you can only earn 6.25 percent interest. How much more money must you deposit today than your cousin invested if you are to have the same amount saved at the end of the 15 years? (Assume annual compounding on both accounts.)

   - $92.19
   - $89.70

- \$88.78
- \$90.21
- \$93.39

Solution: Let x be the additional amount that must be deposited today so that you can earn as much as your cousin. The following equation expresses the fact that both you and your cousin end up with the same amount at the end of the 15-year investment period:

$$(2,500 + x) \cdot (1 + 0.0625)^{15} = 2,500 \cdot (1 + 0.0650)^{15}$$

We solve this equation to get $x = 2,500 \cdot \left[\left(\frac{1+0.0650}{1+0.0625} \right)^{15} - 1 \right] = \$89.7035$.

A simple solution using the calculator would have been possible: compute the amount the cousin has at the end of 15 years. Then, compute how much you had to deposit into the account at the beginning to earn the same amount. Finally, subtract \$2,500 from this amount to get the extra amount to be invested.

N=15; I/Y=6.5; PV=-2,500; PMT=0; CPT FV - this computes the future value of your cousin's account.

You only need to update I/Y=6.25, then you hit CPT PV, followed by +/- (to make PV positive), then - (minus) 2,500, then = (equal). The answer is \$89.7035.

4. [5 points] An amortized loan:

- requires the principal amount to be repaid in even increments over the life of the loan.
- may have equal or increasing amounts of principal repaid from each loan payment.
- requires that all interest be repaid on a monthly basis while the principal is repaid at the end of the loan term.
- requires that all payments be equal in amount and include both principal and interest.
- repays both the principal and the interest in one lump sum at the end of the loan term.

Solution: From the discussion on loans, it should be clear that the correct answer is the second one.

5. [5 points] All else constant, which one of the following will result in the lowest present value of a lump sum?

   - 6 percent interest for 5 years
   - 6 percent interest for 8 years
   - 6 percent interest for 10 years
   - 8 percent interest for 5 years
   - 8 percent interest for 10 years

   Solution: All else being equal, longer investment periods yield a lower present value for the same (lump sum) future value. So option 3 yields the lowest present value among the choices that involve 6% interest. Similarly, option 5 yields the lowest present value among the choices that involve 8% interest.

   Now we must choose between option 3 and 5. But all else being equal, higher interest rates yield lower present values if the investment period is the same. So option 5 is the answer.

6. [5 points] Consider a plain-vanilla coupon bond A, as well as bond B, identical in all respects to bond A, but for the fact that bond B's coupons are double the coupons of bond A. Under these circumstances, which statement below must be true, assuming that ordinary economic conditions prevail?

   - The price of bond A is the same as the price of bond B.
   - The price of bond A is always greater than the price of bond B, but not as large as twice the price of bond B.
   - The price of bond B is always greater than that of bond A, but not as large as twice the price of bond A.
   - The price of bond A is at least twice as large as the price of bond B.
   - The price of bond B is at least TWICE as large as the price of bond A.
   - We cannot definitely tell the relationship between the price of bond A and B.

   Solution: If all cash flows (both coupons and principal) had doubled, then their aggregate present value (the price of the bond) would have doubled as well. But the principal stayed the same - so the present value of bond's B cash flows, i.e., bond B's price, will be less that twice the price of bond A. On the other hand, bond B sometimes pays more than bond A (on coupon payment dates), which means that the present value of bond B's cash flows will be greater than the present value of bond A's cash flows. Consequently, the price of bond B is above the price of bond A, but below twice the price of bond A.

A word ("twice") was left out of the fifth proposed answer when exams were printed. As a consequence, this choice of answer became acceptable (and was accepted as correct).

7. [5 points] Loan A is a fixed-payment loan, while loan B is a variable-payment loan; both of the kind discussed in class. Assume that both loans have the same payment time periods and that they also have the same per-period interest rate. Given that the very last payment amount on both these loans is identical, which of the statements below is true **for the last loan period**?

   - The beginning balance of both loans is the same.
   - Loan A has a higher beginning balance than loan B.
   - Loan B has a higher beginning balance than loan A.
   - We cannot determine the answer from the information provided.

   Solution: We re-create the last line of the amortization table for both loans:

BB	Payment	Interest	Principal	EB
X	$X \cdot (1+r)$	$X \cdot r$	X	0

   Let the beginning balance be X. Since this is the last line of the amortization table, the principal paydown must also be X, and the ending balance of the loan must be 0 (if we ignore small rounding errors). Now, interest for the loans that we discussed is always paid off at the end of the period; it is computed as simple interest at a rate r (same for both loans) applied to the beginning balance. Thus, the interest paid must be $B \cdot r$. The payment is always the sum of the interest paid and the principal paydown, thus the payment equals $X \cdot (1+r)$. Let the payment amount be P; then $P = X \cdot (1+r)$ and $X = \frac{P}{1+r}$. We note that the reasoning above applies to both types of loans (for the situation described here). Since we were told that the payment (P) was the same for both loans, and the interest rate (r) was the same, it follows that the beginning balance (X) must also have been the same for both loans.

8. [5 points] You need to have $31,000 in 11 years. You can earn an annual interest rate of 3 percent for the first 3 years, 3.6 percent for the next 2 years, and 4.3 percent for the final 6 years. How much do you have to deposit today?

- \$20,623.75
- \$22,395.06
- \$18,145.87
- \$19,766.43
- \$20,531.69

Solution: We present a calculator solution. We discount in reverse temporal order: final 6 years, intermediate 2 years, then first 3 years. We could have applied discounting in any other order and we would have gotten the same result (do you know why?). This order of operations, however, provides us with the present values of the \$31,000 at times t=5, t=3, and t=0, respectively.

N=6; I/Y=4.3; PMT=0; FV=31,000; CPT PV

Now hit +/- to switch the sign of the just computed PV (at t=5); then hit FV

N=2; I/Y=3.6; CPT PV (note that some TVM registers have already been set)

Hit +/- to switch the sign of the PV (at t=3); then hit FV

N=3; I/Y=3; CPT PV (note again that some TVM registers have already been set)

Ther answer (ignoring the negative sign) is \$20,531.6881.

The same answer could have been obtained by using discounting to compute the present value at time t=0:

$$\frac{31,000}{(1+0.03)^3\,(1+0.036)^2\,(1+0.043)^6} = \$20,531.6881.$$

9. [5 points] You made an investment of \$17,000 into an account that paid you an annual interest rate of 4 percent for the first 6 years and 8.4 percent for the next 9 years. What was your annual rate of return over the entire 15 years?

   - 6.62 %
   - 6.20 %
   - 5.29 %
   - 7.35 %
   - 5.96 %

Solution: Calculator solution:

N=6; I/Y=4; PV=-17,000; PMT=0; CPT FV (to get future value at t=6 years)

Hit +/- (to switch the sign of the future value), and hit PV

N=9; I/Y=8.4; CPT FV (some TVM registers were already set)

N=15; PV=-17,000; CPT I/Y to get 6.618%.

We computed the future value at time 15, after first obtaining the future value at time t=6. We then computed the yearly compounded interest that would have led to the same future value over 15 years, assuming the same initial investment of $17,000.

We can also use formulas to obtain the same result:

$$17,000\,(1+0.04)^6\,(1+0.084)^9 = 17,000\,(1+r)^{15}$$

Solving the equation yields: $r = \left(1.04^6 \cdot 1.084^9\right)^{\frac{1}{15}} - 1 = 6.618\%$.

10. [5 points] One year ago, the Jenkins Family Fun Center deposited $4,700 into an investment account for the purpose of buying new equipment four years from today. Today, they are adding another $6,500 to this account. They plan on making a final deposit of $8,700 to the account next year. How much will be available when they are ready to buy the equipment, assuming they earn a rate of return of 8 percent?

- $25,631.64
- $26,708.51
- $25,011.27
- $25,728.32
- $24,094.06

Solution: We can solve this problem in multiple ways, the following being just one of them:

Let "now" be t=0; then a payment was made at time t=-1 and a payment will be made at time t=1. The future value of the three payments at time t=1 will be $4,700 \cdot (1+0.08)^2 + 6,500 \cdot (1+0.08) + 8,700$. We then compute the future value of this amount at t=4 (3 years into the future versus t=1). We get

$$\left[4,700 \cdot (1+0.08)^2 + 6,500 \cdot (1+0.08) + 8,700\right] \cdot (1+0.08)^3$$

The final amount is $26,708.5146.

11. [5 points] You just won the $48 million Ultimate Lotto jackpot. Your winnings will be paid as $2,400,000 per year for the next 20 years. If the appropriate interest rate is 5.7 percent, what is the value of your windfall?

    - $29,151,329.65
    - $29,818,989.14
    - $28,210,964.18
    - $27,418,989.14
    - $26,800,415.97

    Solution: We need to compute the present value of an annuity. Lottery winnings are one of the situations when annuities appear unexpectedly. This was also mentioned in class. We use the calculator, but the standard formula would have worked equally well: N=20; I/Y=5.7%; PMT=2,400,000; FV=0. We hit CPT PV to get $28,210,964.18 (we ignored the negative sign). The big lottery winnings are much smaller when accounted for in present-value terms!

12. [5 points] You want to buy a house and will need to borrow $240,000. The interest rate on your loan is 5.71 percent compounded monthly and the loan is for 25 years. What are your monthly mortgage payments?

    - $1,496.94
    - $1,504.06
    - $1,453.92
    - $1,579.26
    - $1,521.68

    Solution: We need to compute the payment (coupon) of an ordinary annuity. The standard formula can also be used.

    N=300 ($12 \cdot 25$); I/Y=0.47583 ($\frac{5.71}{12}$); PV=240,000; FV=0. We hit CPT PMT to get $1,504.0537 (we ignored the sign).

13. [5 points] What is the effective annual rate for an annual rate of 16.60 percent compounded monthly?

    - 17.66%
    - 17.79%
    - 18.82%
    - 16.78%
    - 17.92%

    Solution: We use the definition given in class: $EAR = \left(1 + \frac{0.1660}{12}\right)^{12} - 1 = 17.923\%$.

14. [5 points] A bond that pays interest annually yields a rate of return of 6.25 percent. The inflation rate for the same period is 4 percent. What is the real rate of return on this bond?

    - 10.25%
    - 2.16%
    - 1.02%
    - 1.56%
    - 4.00%

    Solution: $r = \frac{R-h}{1+h} = \frac{0.0625-0.04}{1+0.04} = 2.163\%$.

15. [5 points] Lincoln Park Company has a bond outstanding with a coupon rate of 5.87 percent and semiannual payments. The yield to maturity is 6.9 percent and the bond matures in 13 years. What is the market price if the bond has a par value of $2,000?

    - $1,826.85
    - $1,827.86
    - $1,861.55
    - $1,825.05
    - $1,830.27

Solution: We provide the calculator solution; the formula for the pricing of a bond could also be used.

N=26 ($13 \cdot 2$); I/Y=3.45 ($\frac{6.9}{2}$); PMT=58.70 ($\frac{5.87\% \cdot 2{,}000}{2}$); FV=2,000. We hit CPT PV to get -$1,825.0512.

16. [5 points] Your company, Something & Co (SCo) and Nothing, Inc (NInc) cooperated on the techology that, as it became clear by now, will surely supersede AI in the next decade. However, this was not so obvious in the past. At some point, you inadvertently allowed NInc to unilaterally license some of the patents whose ownership you shared to Other AG (OAG), a foreign firm. As a consequence, OAG has started making yearly $200,000,000 payments to NInc; the first payment having been made exactly 5 years ago. There will be a total of 10 such payments.

You believe that half of all these payments, past and future, should be paid to SCo. You have been suing NInc to recover these amounts. Unfortunately, you cannot break up the contract between NInc and OAG, nor can you get OAG to pay you directly.

Today, just after OAG made this year's payment, you sat down to estimate your actual losses, expressed in today's dollars. Assuming a constant yearly interest rate of 5.9% per year, what would be the amount that you would accept today as the fair value of all the license payments between NInc and OAG that form SCo's fair share?

Solution: Let us label "now" with t=0. Then the first payment was made "5 years" ago, at time t=-5. To date, including **today's** payment, there have been 6 payments. There will be 4 more, with the last one being made in 4 years, at time t=4. The simplest approach to this problem is to position ourselves at time t=-6, one year before the first payment was made, compute the present value PV_{-5} of the annuity at that time, then compute the future value (versus time t=-6) at time t=0. We do this using the calculator (formulas would work equally well):

N=10; I/Y=5.9; PMT=100,000,000; FV=0. Hit CPT PV to get PV_{-5}.

N=6; PMT=0. Hit CPT FV to get the final answer, which is $1,043,082,903.

The problem could have been solved in several other ways. For example, you could have computed the future value of the entire annuity at time t=4, which then would have been discounted back to time t=0. Alternatively, the future value at t=0 of all the payments made from t=-5 to t=0 (inclusive) could have been computed, then added to the present value at time t=0 of all future payments between t=1 and t=4.

End of Exam