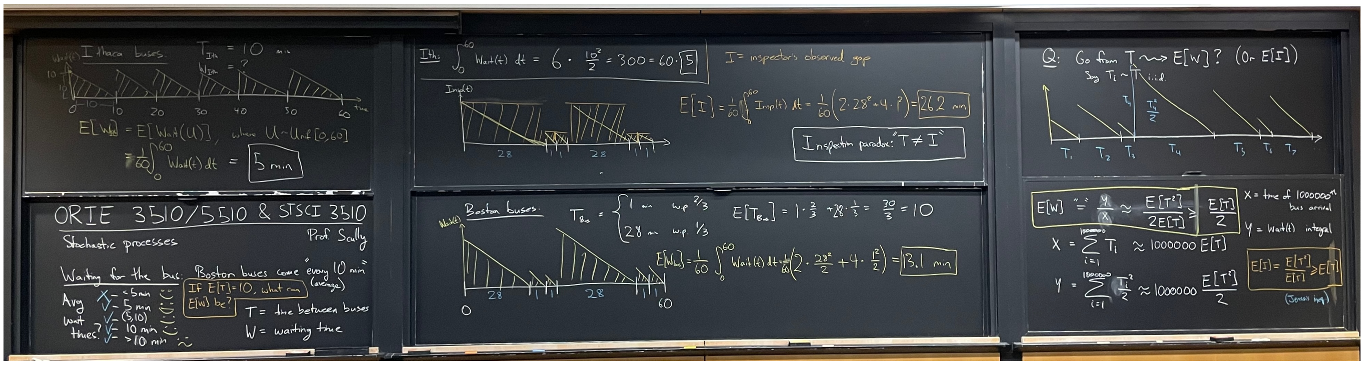
Lecture 1



Lecture 1 learning outcomes

- State a counterintuitive result about a stochastic process
- Compare averages taken from different perspectives by informal picture-based reasoning
- Name the document where course policies, logistics, etc. can be found
  - Starts with "s", ends with "yllabus.pdf on Canvas"

Waiting for the bus

- Today's lecture is all informal and non-rigorous
  - Pinning down many of these concepts more formally is a goal of the course
- The most stochastic process of them all: waiting for the bus
  - Suppose buses arrive randomly at a bus stop every 10 minutes on average
  - Suppose customers arrive at "uniformly random" times
  - **Question:** what's the average waiting time between arrival and the next bus?
    - Less than 5 minutes?
    - Exactly 5 minutes?
    - Between 5 and 10 minutes?
    - Exactly 10 minutes?
    - More than 10 minutes?
  - If the answer depends on the details, which of the above are possible answers?

Tale of two cities

- Ithaca vs. Boston
  - Ithaca: regular buses
  - Boston: very irregular buses

- Let T_c be the distribution of the amount of time (in minutes) between buses in city c
  - Time between buses in Ithaca:

$$T_{\text{Ithaca}} = 10$$

  - Time between buses in Boston:

$$T_{\text{Boston}} = \begin{cases} 1 & \text{with probability } \frac{2}{3} \\ 28 & \text{with probability } \frac{1}{3} \end{cases}$$

- What's the average time between buses?
  - $\mathbf{E}[T_{\text{Ithaca}}] = \mathbf{E}[T_{\text{Boston}}] = 10$
  - So it seems like "buses come every 10 minutes on average" in both places
- What's the average waiting time $\mathbf{E}[W_c]$ until the next bus in city c?
  - Ithaca
    - Draw timeline
    - Draw waiting-time-if-arriving-now as a function of time
      - Looks like triangles, or a sawtooth wave
    - Compute average by dividing integral of function by total time
    - Result: $\mathbf{E}[W_{\text{Ithaca}}] = 5$
  - **Activity:** Repeat for Boston
    - Difficulty: not all the between-bus intervals have the same length
    - Draw timeline with twice as many $T = 1$ intervals as $T = 28$ intervals
    - Draw triangles, compute integral
    - Result: $\mathbf{E}[W_{\text{Boston}}] = 13.1$
      - More than 10 minutes?!
- What happened here?
  - Two competing phenomena
    - Arrive after some of T has passed
      - So expect $\mathbf{E}[W]$ to be smaller
    - More likely to arrive during a large value of T
      - So expect $\mathbf{E}[W]$ to be larger
      - This is called the *inspection paradox*
  - Depending on T, either factor can be dominant
  - But how much impact does each factor have exactly?

General rule for average waiting time

- Now consider some generic nonnegative distribution T
- How do we compute average waiting time?

- Same main steps
  - Draw timeline
  - Draw triangles/sawtooth
  - Compute integral under triangles, divide by total length of timeline
- Result: $\mathbf{E}[W] = \frac{\mathbf{E}[T^2]}{2\mathbf{E}[T]}$
  - So $\mathbf{E}[W]$ has to do with the *variance* of T, not just its mean!

More inspection paradox: classroom sizes

- Classroom sizes from different perspectives
  - Cornell says its average class size is 30
  - Students say their average class size is more than 90
  - Can both be right?
- Maybe class size distribution C is

$$C = \begin{cases} 10 & \text{w.p. } \frac{5}{6} \\ 130 & \text{w.p. } \frac{1}{6} \end{cases}$$

  - Cornell's perspective: $\mathbf{E}[C] = 30$
  - Students' perspective:
    - Assume for simplicity that each student is in one class
      - But basically the same reasoning works as long as every student has the same number of classes
    - Draw row of 180 students
      - Analogous to timeline in bus example
      - Put students in the same class next to each other
    - Draw "class size as a function of student"
      - Looks like squares, e.g. a clump of 10 students in a row are in a class of size 10, giving a 10-by-10 square
    - Do sum of square areas and divide by total number of students
- Result: averaging over students, their class's average size is $\frac{\mathbf{E}[C^2]}{\mathbf{E}[C]}$
  - Just like waiting for the bus, but without the $\frac{1}{2}$
  - The difference is triangle areas (which have a $\frac{1}{2}$) vs. square areas (which don't)

Logistics

- Main thing: see the syllabus
  - Highlights below
- Textbook: Introduction to Probability for Computing

    - PDFs of all chapters available for free
- Homework
    - Basically every week
    - Mini-homework if after break or before prelim
    - Usually due Thursday at 10pm
        - Short grace period, but don't push it
    - This week's mini-homework due Friday
- Discussion
    - Starts Friday
    - Guided problem solving, but not the homework
    - Work independently or in groups for 60–75 min, then solutions
    - Feel free to arrive a little late (especially for the 8am sections!), though you will miss some solving time
- Office hours
    - Help with homework and course material
    - Office hours schedule will be announced soon
- Ed Discussions
    - Most questions about the course
        - Technical questions
        - Admin questions of general interest
    - Admin questions specific to you (e.g. test conflict): email ORIE-3510-SPRING-2024-STAFF-L@list.cornell.edu
- Course announcements
    - Big ones on Canvas
    - Minor ones on Ed Discussions
- 110% grading