Market Clearing Price

NETWORKS INFO 2040 / CS 2850 / ECON 2040 / SOC 2090

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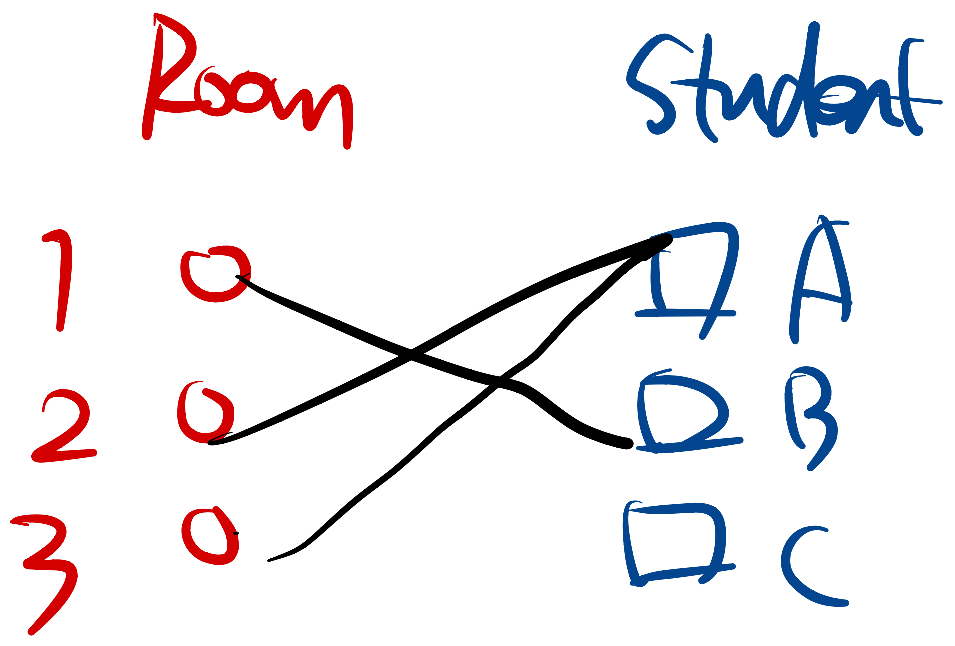
PS 2. Due tmr 3:30 PM

Bipartite Graph

2 types of nodes



Assignment Problem

Perfect Matching (P.M.)

Neighbor set N(S): {set of nodes that connect to at least one node in S}

$f(x) = 2x$ number → number

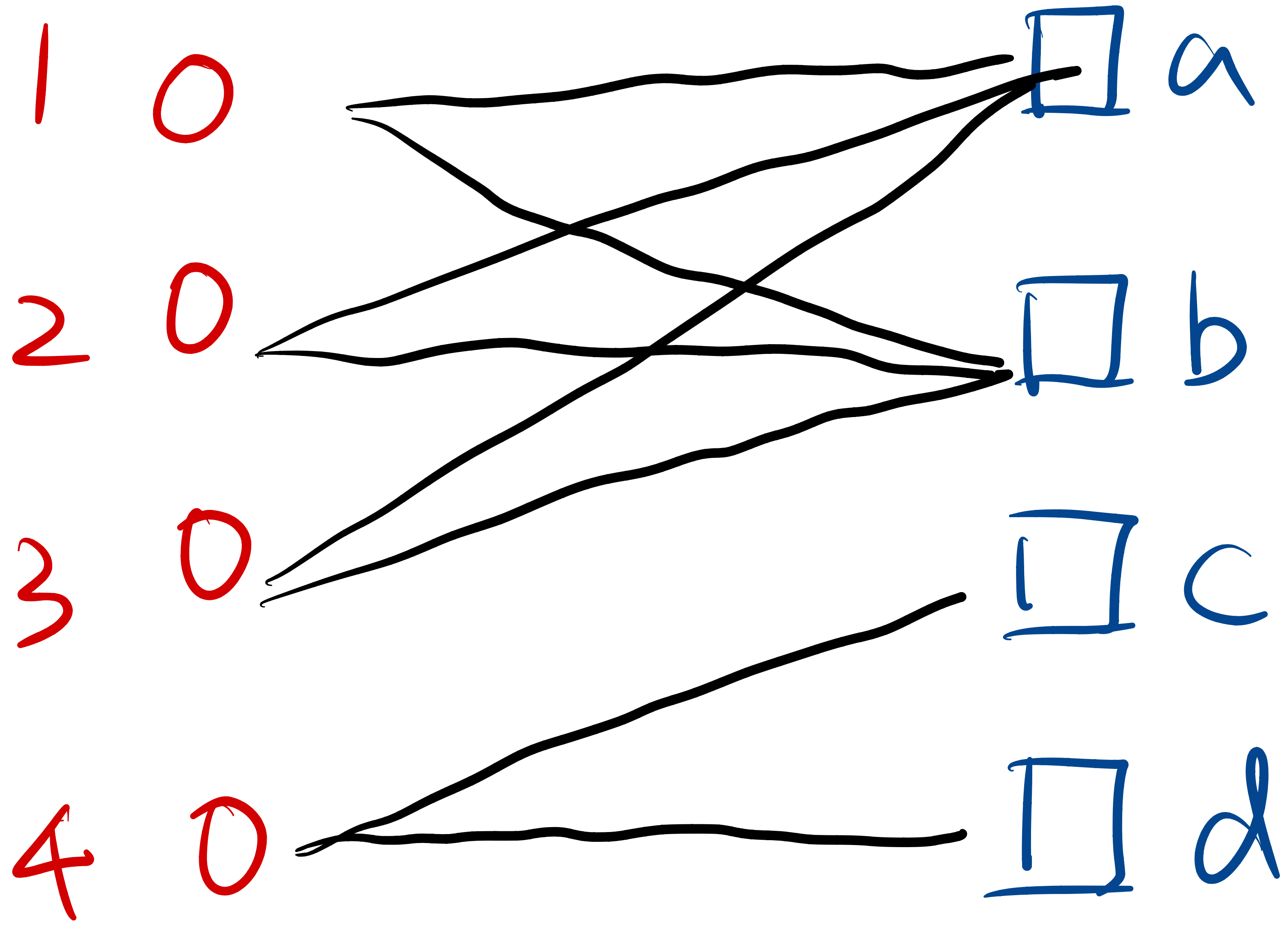
Function / Mapping on sets: $S = \{1, 2\}$ $N(S) = \{A, B\}$

We call a set S is **constricted** if S contains strictly more nodes than N(S)

$S = \{2, 3\}$ $N(S) = \{A\}$ $|S| = 2 > |N(S)| = 1$

Having a constricted set $\Leftrightarrow$ Not having a P.M.

iClicker Question



$S = \{c, d\}$

$N(S) = \{4\}$

$|S| = 2 > |N(S)| = 1$

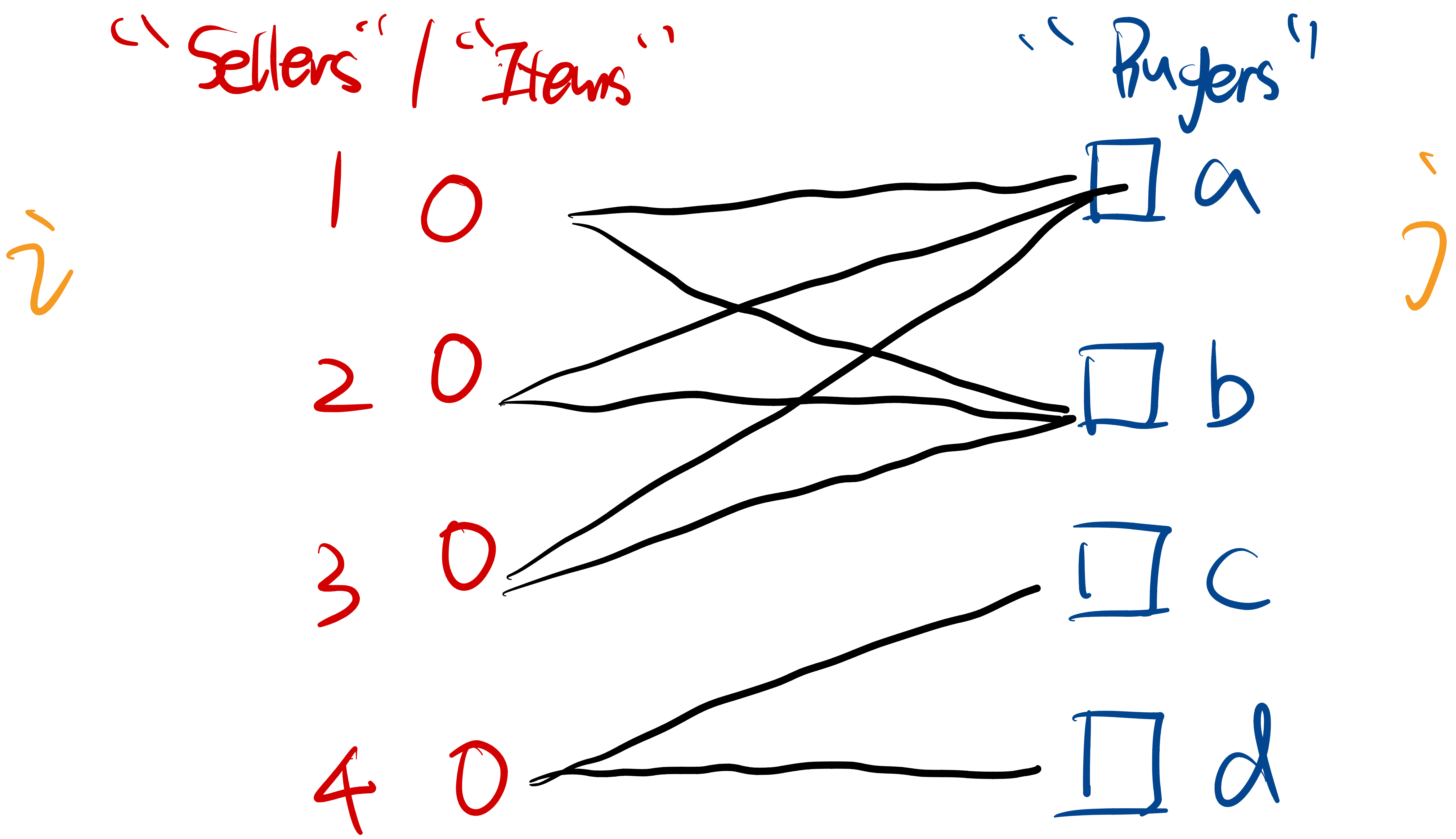
so S is constricted set

$\Rightarrow$ There's no P.M.

Does the graph has a perfect matching?

A Yes

B No



"Y/N"

Numerical ratings

Valuation

Each buyer can have a **valuation** for each item (seller)

$v_{ij} \geq 0$

How much it worth

$i \in \{1, 2, 3, 4\}$

$j \in \{a, b, c, d\}$

Valuation

Sellers / Items

1 0

2 0

3 0

Buyers

A

B

C

$V_{1A}=10$ $V_{2A}=3$ $V_{3A}=8$

$V_{1B}=0$ $V_{2B}=5$ $V_{3B}=6$

$V_{1C}=12$ $V_{2C}=2$ $V_{3C}=1$

10, 3, 8

0, 5, 6

12, 2, 1

What if each buyer simply maximizes their own valuation?

Buyer A: Choose item #1

B: item #3

C: item #1

Valuation under some price?

"Price"

[redacted]

	Sellers / Items	
P_1	1	0
P_2	2	0
P_3	3	0

Buyers
□ A
□ B
□ C

$V_{1A}=10$ $V_{2A}=3$ $V_{3A}=8$
$V_{1B}=0$ $V_{2B}=5$ $V_{3B}=6$
$V_{1C}=12$ $V_{2C}=2$ $V_{3C}=1$

ach buyer simply maximizes their own valuation?

What if each buyer simply maximizes their own ~~valuation~~?

Payoff := Valuation − price

(i, j) $V_{ij} - P_i$

"
A

Buyer A

Used to if no price

$\max\{V_{1A}, V_{2A}, V_{3A}\}$

Now: price

$y = \max\{V_{1A} - P_1, V_{2A} - P_2, V_{3A} - P_3\}$

if $y<0$, buy nothing

if $y \geq 0$ buy item(s) that let me achieve y

Valuation under some price?

Price

$P_1 = 9$

$P_2 = 1$

$P_3 = 2$

	Sellers / Items		Buyers	
P_1	1	0	☐	A
P_2	2	0	☐	B
P_3	3	0	☐	C

Valuation

$V_{1A} = 10$	$V_{2A} = 3$	$V_{3A} = 8$
$V_{1B} = 0$	$V_{2B} = 5$	$V_{3B} = 6$
$V_{1C} = 12$	$V_{2C} = 2$	$V_{3C} = 1$

Payoff = Val − Pri

$$\begin{bmatrix} 10-9 & 3-1 & 8-2 \\ 0-9 & 5-1 & 6-2 \\ 12-9 & 2-1 & 1-2 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 2 & 6 \\ -9 & 4 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

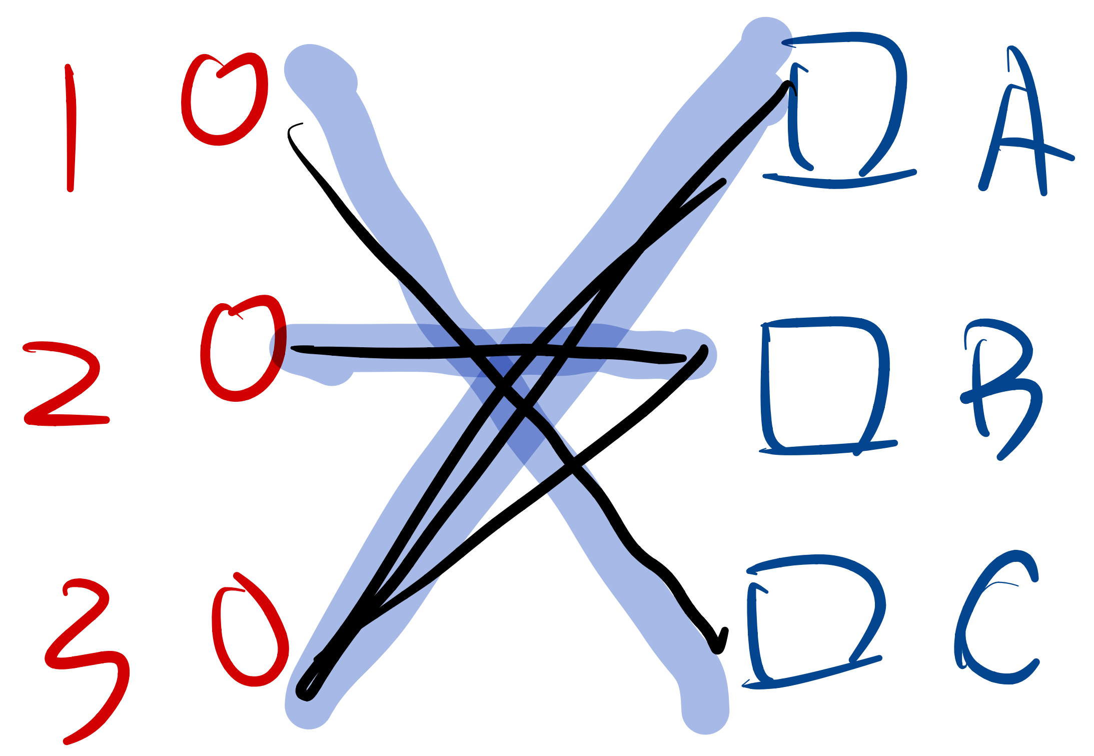
ach buyer simply maximizes their own valuation?

Buyer A: 3

Buyer B: 2 3

Buyer C: 1

"Preferred sellers"



"Perferred seller Graph"

Market-clearing Price (MCP)

For a set of prices, we define the **preferred-seller graph** on buyers and sellers by simply constructing an edge between each buyer and her preferred seller or sellers.

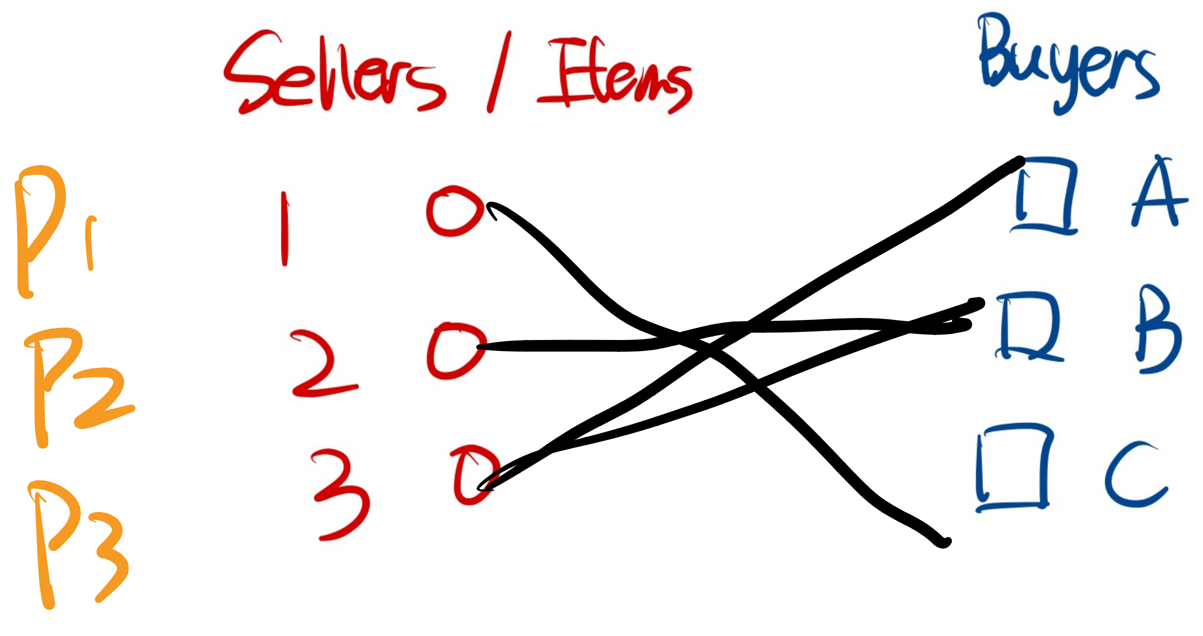
A set of prices is **market clearing** if the resulting preferred-seller graph has a perfect matching.

Price

$P_1 = 9$

$P_2 = 1$

$P_3 = 2$



Valuation

$V_{1A} = 10$ $V_{2A} = 3$ $V_{3A} = 8$

$V_{1B} = 0$ $V_{2B} = 5$ $V_{3B} = 6$

$V_{1C} = 12$ $V_{2C} = 2$ $V_{3C} = 1$

Buyer A: 3

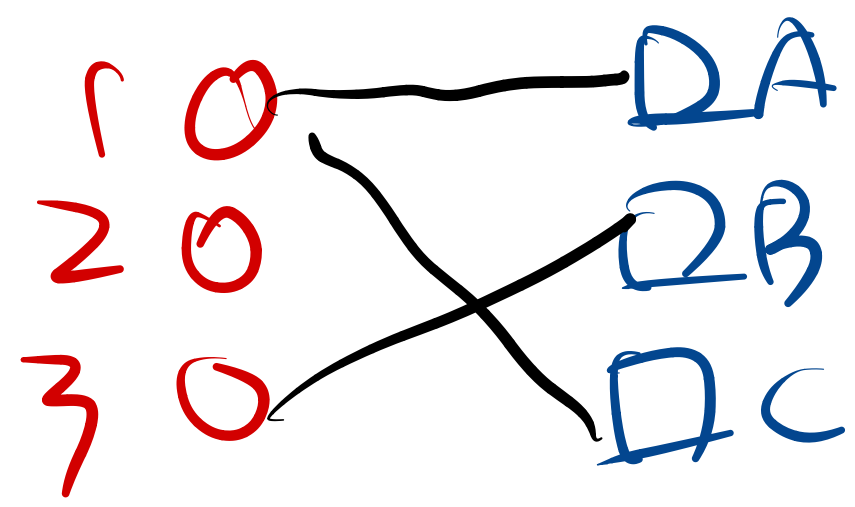
Buyer B: 2 3

Buyer C: 1

"Preferred sellers"

Price (9, 1, 2) is Market clearing

Price (0, 0, 0)



Market-clearing Price (MCP)

Two important properties:

1. MCP always exists Friday
2. MCP (and the resulting P.M on preferred seller graph) is "socially optimal"

"Prisoner's dilemma" Each person max. their own payoff. → total payoff is NOT max

MCP + PM total payoff is maximized

$$\begin{bmatrix} \boxed{1} & 2 & 6 \\ -9 & 4 & \boxed{4} \\ 3 & \boxed{1} & -1 \end{bmatrix}$$

Total payoff. 13

6

Market-clearing Price (MCP)

Two important properties:

1. MCP always exists
2. **MCP (and the resulting P.M on preferred seller graph) is "socially optimal"**

$$\begin{bmatrix} \boxed{1} & 2 & 6 \\ -9 & 4 & \boxed{4} \\ 3 & \boxed{1} & -1 \end{bmatrix}$$

Total payoff. 13

6

Buyer A

$\geq$

$\geq$

✓

True for Buyer B

$\Rightarrow$ Add up: $+$ $+$

$\geq$ $+$ $+$

Market-clearing Price (MCP)

Two important properties:

1. MCP always exists
2. **MCP (and the resulting P.M on preferred seller graph) is "socially optimal"**

Payoff

1	2	6
-9	4	4
3	1	-1

Valuation

10,	3,	8
0,	5,	6
12,	2,	1

Max total payoff ✓

Max total Valuation ?

$p_1 + p_2 + p_3$

$9 + 1 + 2 = 12$

Total Payoff = Total Valuation − Total Price

$p_1 + p_2 + \dots$

Total Payoff	Total Valuation	Total Price
13	25	12
6	18	12

→ "Constant"

Give a set of prices