Networks: Fall 2023	**Homework 9**
David Easley and Yian Yin	**Due 3:30pm, Thursday, November 30, 2023**

Homework solutions should be submitted by upload to Gradescope, which can be accessed via Canvas. The file you upload **must be typed and submitted in PDF format**. Handwritten assignments will not be graded. However, you can draw graphs and insert them into your pdf. You can create a separate file with the solutions (you don't need to repeat the questions); it is fine to create the homework in any format provided it's typed and handed in as a single PDF file. When you upload your pdf to Gradescope be sure to assign your answers to the correct question.

To be eligible for full credit, your homework must come in by 3:30pm Thursday. We will also accept late homeworks after 3:30pm Thursday until 3:30pm Friday for a deduction of 10% of the total number of points available. Gradescope will stop accepting homework uploads after 3:30pm Friday; after this point we can only accept late homework when it is accompanied by a University-approved reason that is conveyed to and **approved by the TA in charge of this homework prior to the due date of the homework**. (These include illness, family emergencies, SDS accommodations and travel associated with university activities.)

The TA in charge of this homework is Jonathan Aimuyo (oja7@cornell.edu).

Reading: The questions below are primarily based on the material in Chapters 7, 21 and 22.

---

(1) [5 points]

Consider a betting market with two horses A and B and three bettors 1, 2 and 3, as in Chapter 22. Bettor 1 has wealth \$3,000 and believes there is a probability 1/3 that horse A will win, bettor 2 has wealth \$4,000 and believes there is a probability 1/2 that horse A will win, bettor 3 has wealth \$9,000 and believes there is a probability 2/3 that horse A will win. All bettors have logarithmic utility for wealth, and they each choose bets to maximize the expected utility of wealth given their beliefs.

(a) How much money should each bettor bet on horse A?

(b) Find the equilibrium inverse odds on horse A. Provide a brief explanation of the difference (if any) between the average of the bettor's beliefs and the inverse odds on horse A.

---

(2) [6 points]

Suppose that there are two types of used cars – High quality ones and Low quality ones – and that sellers know which type of car they have. Buyers do not know which type of car a seller has, and sellers of High quality cars have no way of proving what kind of car they have.

Buyers know that the fraction of used cars in the total population of used cars that are High quality is 1/2 and the fraction of used cars that are Low quality is 1/2 (but as is typical in these situations, it is not necessarily the case that all the owners of these used cars will choose to sell them). The following table summarizes the values of the different cars for sellers and buyers in thousands of dollars

	Value for High quality car	Value for Low quality car
buyer	20	6
seller	12	4

We assume that buyers are risk-neutral; that is, they are willing to pay their expected value of a car.

(a) Find all of the self-fulling expectations equilibria in this market.

(b) Now some totally worthless cars are introduced into the market. These cars are so bad that their value to either buyers or sellers is 0. As with the other cars in the market the seller knows the quality of the car, but the buyer does not know the quality of the car. Let the fraction of these cars be p; so the sum of the fractions of High and low quality cars is now $1 - p$. Suppose that everyone knows this fraction p. Is there a critical fraction, p^*, of worthless cars such that for any $p > p^*$ there is not a self-fulling expectations equilibrium in the market for used cars in which High quality cars are sold? If so, what is p^*. If not, explain why not. Show your work.

(c) Is there a critical fraction, $\bar{p}$, of worthless cars such that for any $p > \bar{p}$ there is not a self-fulling expectations equilibrium in the market for used cars in which Low quality cars are sold? If so, what is $\bar{p}$. If not, explain why not. Show your work.

---

(3) [6 points]

Every day 200 sellers consider selling their goods using a trading platform and there are also 200 buyers who come to the platform and want to buy the seller's goods. Each seller has one unit of the good, but different sellers have different qualities of the good. Fifty of them have a unit of the good that is excellent, 50 have a unit that is very-good, 50 have a unit that is fair and 50 have a unit that is poor. Sellers know the quality of their unit of the good; buyers do not know the quality of any particular seller's unit of the good. But buyers do know the numbers of each type of potential seller.

The table below provides values for buyers and sellers for each quality of good.

	Value for excellent	Value for very-good	Value for fair	Value for poor
buyer	30	25	20	15
seller	25	20	16	10

Assume that buyers each want at most one unit of the good and are risk neutral—if they don't know which quality of good they are buying they are willing to pay up to its expected value. Assume that any buyers and sellers who do not transact on any day, as well as all who do transact, leave and are replaced the next day by identical buyers and sellers.

(a) Find all of the self-fulling expectations equilibria in this market. Show your work.

(b) You are the manager of the platform and your objective is to run it to maximize the total amount of money that is paid each day by buyers to sellers (the price times the number of transactions). If multiple prices are possible, and multiple equilibria are possible, you may assume that the price and equilibrium that has the highest amount of payments is what occurs. You know the numbers of each type of seller and the values for buyers and sellers, and unlike buyers you do know each seller's quality. You can't convince buyers of the quality of any seller. However, if you decide to exclude some group of sellers, say all with poor quality or fair quality items, you can convince buyers that you have succeeded in doing this.

Do you want to exclude any type of sellers? If so, which one(s) and what is the total amount of money paid per day both before and after your restriction? Show your work.

---

(4) [6 points]
Consider the game in Figure 1:

		Player B X	Player B Y
Player A	X	$1, 1$	$1, 0$
Player A	Y	$0, 1$	$2, 2$

Figure 1: The payoff matrix for Question (4).

(a) Find all of the pure strategy Nash equilibria for this game.

(b) Is every strategy used in any pure strategy Nash equilibrium of this game an evolutionarily stable strategy? Explain briefly.

(c) Suppose we change the 0s in the payoff matrix to 1s. Would this change your answer about the relationship between the Nash equilibria and the evolutionarily stable strategies in the game? Explain briefly.

---

(5) [6 points] A large farm is trying to decide how to reduce the spread of a virus that is spreading contagiously among its animals. Currently each animal comes into contact with 20 others and any infected animal has a .08 probability of infecting each other animal it comes in contact with.

They have allocated a budget of $200,000 to take measures against the virus, and they are considering the following two kinds of interventions.

(i) The farm could reorganize the layout of its operations to have fewer animals come in contact with each other; this would be expensive, since it would use more space, but if they spend x dollars, they can add enough space that each animal comes in contact with only

$$20 - \frac{x}{20,000}$$

others.

(ii) The farm could install machines that emit ultraviolet light to reduce the amount of virus in the air; the more money they spend on this, the more of these machine they can install. If they spend y dollars, they can install enough machines to reduce the probability that each infected animal spreads the virus to each animal it comes in contact with; the new probability, if they spend y dollars, would be

$$.08 - \frac{y}{10,000,000}.$$

The farm is currently unsure how to divide its budget of \$200,000 among these two interventions, so they're planning to spend equally on the two of them: \$100,000 on each.

They ask you whether you think this is the right plan, or whether they should do something different. If you think they're doing the right thing, explain why. If you think there's a better way they could divide the budget between these two options, then describe a different way of dividing the budget, and explain why you think this different allocation of the budget would be a better idea than their current plan.

---

(6) [6 points] The structure of contact network plays an important role in determining the rate and range of epidemic spreading. To see this, let us consider a group of 4 people, connected following two possible types of contact networks (instead of the 'tree structure' we discussed in class). The process starts with a person $x \in \{A, B, C, D\}$ getting infected. In subsequent waves, each newly infected individual (if any) may transmit the disease to each of their network neighbors, independently with probability $p = 0.5$. Our primary focus in this scenario is to determine q, the probability that the disease will spread to at least one additional person beyond the initially infected individual x.

(a) Suppose we are interested in the *expected outcome*. In particular, x is randomly selected from $\{A, B, C, D\}$. What is the average probability that the disease will spread to at least one additional person beyond x? Calculate the probability for Networks 1 and 2 respectively.

(b) Suppose we are interested in the *worst outcome*. In particular, x is selected from $\{A, B, C, D\}$ to maximize the probability of interest q (the choice of x might vary for Networks 1 and 2). What is the highest possible probability that the disease will spread to at least one additional person beyond x? Calculate the probability for Networks 1 and 2 respectively.

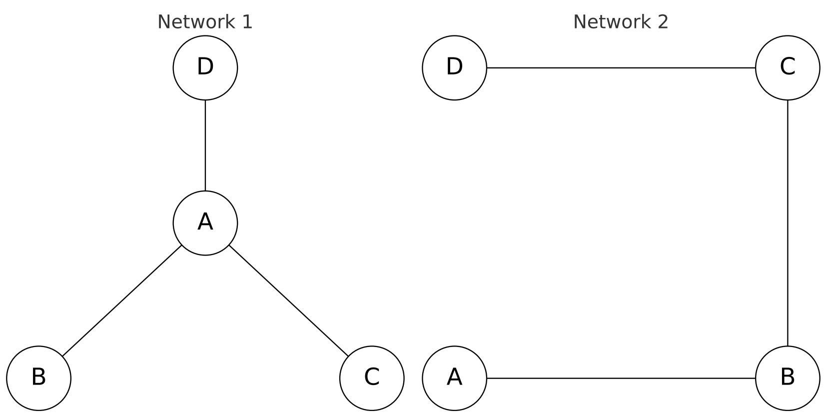

Figure 2: The Networks for problem 6