Solution Sketch for PS3

1. **Solution**

You should disagree with the recommendation of $b = (N-1)/N100$. This is an ascending bid auction and the optimal strategy in this type of auciton is the same as in a second price auction—bid truthfully. Giving the proposed bidding agent the recommended value of b is equivalent to bidding less than your true value. This only affects your payoff if you would have won with a bid of your true value, 100, but lose with the lower recommended bid. In this case your payoff with the lower bid is 0 while your payoff with a bid of 100 would have been positive.

2. **Solution**

(a.1) 3/4

(a.2) $(3/4)100 + (1/4)50 = 87.5$

(b.1) In a second price auction each bidder bids truthfully. So the bids in the four possible states, $((100, 100), (100, 80), (80, 100), (80, 80))$, will be $((100, 100), (100, 80), (80, 100), (80, 80))$.

(b.2) Expected revenue is the expectation of the second highest bid which is $(1/4)100 + (3/4)80 = 85$.

3. **Solution**

See Figure 1 below.

(a) Nodes are defined as the four sessions and four TAs. We draw a link between a TA and a session if the TA can possibly server in the session. We can find a perfect assignment: Adam Tue in-person; Brian Mon in-person; Jemma Wed Zoom, Karen Mon Zoom.

(b) Nodes are defined as the light and dark squares. We draw a link between two squares if they share a common edge (i.e. they can be combined as a domino). We cannot find a perfect assignment as we have different number of nodes in two sides of the bipartite graph.

(c) Nodes are defined as the light and dark squares. We draw a link between two squares if they share a common edge (i.e. they can be combined as a domino). We cannot find a perfect assignment as there is a constricted set $S = \{A, E\}$ (or $S = \{B, G, H\}$) where $|S| > |N(S)|$.

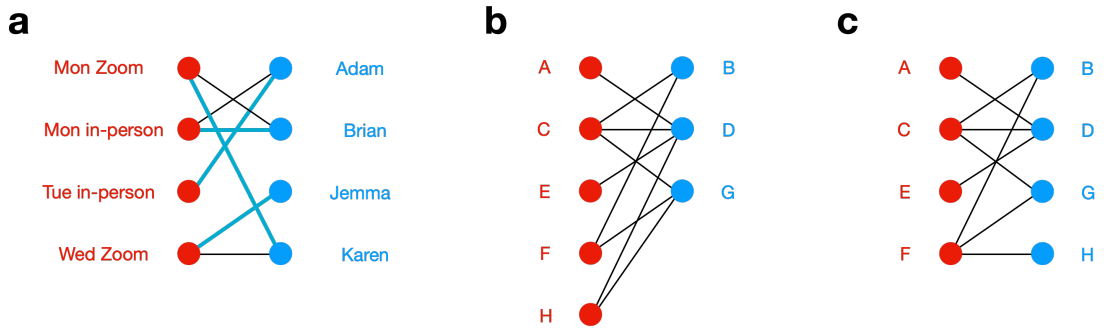

Figure 1: Q3 answer

4. **Solution**

(a) (p_a, p_b, p_c) in Round 1: $(0, 0, 0)$; Round 2: $(0, 0, 1)$; Round 3: $(0, 0, 2)$.

(b) Under the given price, the resulting preferred seller graph has a perfect matching: $x \to a, \quad y \to b, \quad z \to c$.

The updated valuation table is $\begin{bmatrix} 6 & 4 & 6 \\ 2+2t & 10 & 12 \\ 6+t & 2 & 16 \end{bmatrix}$

To make sure the price remains market clearing, we need $0 \leq t \leq 4$.

5. **Solution**

(a) The diagram should cover all nodes (x, y) satisfying $x - y \in \{-2, -1, 0, 1\}$.

(b) Any pair of prices above the upper diagonal line cannot be market-clearing since item b will be too expensive (at least 3 units higher than a) such that no buyer will prefer b; Any pair of prices below the lower diagonal line cannot be market-clearing since item a will be too expensive (at least 2 units higher than b) such that no buyer will prefer a.