ORIE 3510/5510
DISCUSSION 1

SPRING 2024

MONTY HALL: REVISED STRATEGIES

Monty takes you into a room with three closed doors A,B,C. Behind one of the doors is a car. Behind the other two doors is a goat. You are asked to pick a door and state which door you have chosen. Assume that you pick door C (*you have no insider information*).
Monty, knowing what's behind each door, picks a door that was not chosen by you and reveals that there is a goat behind that door (*the game show host will always choose to open a door with a goat*). You are then asked, "Would you like to switch from your chosen door?" You therefore want to know the probability that you win the car if you switch. But you quickly realize that this probability depends on the strategy Monty uses to open the door.

To make sense of this situation, let's model it with a probability space. Let $\mathcal{S} = \{A, B, C\}$ be the set of doors and $X \sim Unif\{A, B, C\}$ be a random variable denoting the location of the car. Your job is the following;

- Construct an appropriate and suitable sample/outcome space Ω such that $Y : \Omega \to \mathcal{S}$, where Y is a random variable denoting the door that Monty opens.
- Compute the probability of winning the car if the switch is made i.e. $\mathbb{P}\{X = B \mid Y = A, X \neq A\}$, under below mentioned conditions.

Compute $\mathbb{P}\{X = B \mid Y = A, X \neq A\}$ if

(a) Monty prefers door A over B.
(b) Monty prefers door B over A.
(c) If possible, Monty is equally likely to open A and B.
(d) If possible, the probability that Monty will open door A is $p \in [0, 1]$.

MONTY HALL: LAST GAME

Monty is retiring soon and it seems he does not care about the car.

(e) Compute $\mathbb{P}\{X = B \mid Y = A, X \neq A\}$, if Monty will never open the door that you chose but he might open the door with a car behind it.
(f) Compute $\mathbb{P}\{X = B \mid Y = A, X \neq A\}$, if Monty will never open the door with the car but he might open the door that you chose.
(g) Compute $\mathbb{P}\{X = B \mid Y = A, X \neq A\}$, if Monty is equally likely to open any one of the three doors.

MONTY HALL: CHALLENGE PROBLEM

(h) Suppose you are free to choose any one of the three doors. Once you have chosen a door, Monty uses some unknown strategy to open a door that was not chosen by you and does not have a car behind it. By constructing a suitable outcome space, come up with a strategy you would use to choose a door (irrespective of Monty's strategy) such that the probability of winning the car is 2/3, given that you made the switch.

(i) Assume that the room has 3000 doors and there is a car behind one of the doors. Behind other doors is a goat. Suppose Monty opens 2998 doors all with goats and does not opens your door as well. What is the probability of winning the car if you make the switch?