Important Formulas in Finance

Fall 2023 - For use during Prelim I.

1 General Comments

Unless we state otherwise, all cash flows occur at the end of a period and all interest is compound interest. Typically $r > 0$, but interest rates could also be negative or zero. Periods could be days, weeks, months, years. We ignore calendar idiosyncrasies related to months and years of variable length.

Not all formulas, not all versions of formulas, and not all interpretations of formulas are given. This is just an aide-mémoire, not a full reference. Some observations may be beyond the scope of the course (e.g., root finding methods for determining the implied interest rate for an annuity).

2 Time-Value of Money

PV = present value at time 0; FV = future value at time t; r = interest rate; t = number of periods.

- One period case: $FV = PV \cdot (1+r)$.
- Multi-period case: $FV = PV \cdot (1+r)^t$; $PV = \frac{FV}{(1+r)^t}$; $t = \frac{\ln \frac{FV}{PV}}{\ln(1+r)}$; $r = \left(\frac{FV}{PV}\right)^{\frac{1}{t}} - 1$.
- Doubling time of an investment: t$=\frac{\ln 2}{\ln(1+r)} \approx \frac{\ln 2}{r}$ (if r is small). If r is expressed in percents, then $t \approx \frac{72}{r}$ periods.
- Future value factor $= (1+r)^t$; present value (or discount) factor $= \frac{1}{(1+r)^t}$.

3 Annuities

PV = present value of an annuity; FV = future value of an annuity; C = the constant payment of an ordinary annuity; r = interest rate; t = number of periods; g = growth rate of payouts.

- $PV = C \cdot \frac{1-\left(\frac{1}{1+r}\right)^t}{r}$; $C = \frac{r}{1-\left(\frac{1}{1+r}\right)^t} \cdot PV$; $t = -\frac{\ln\left(1-\frac{PV}{C}\cdot r\right)}{\ln(1+r)}$. There is no general formula for determining r - you can use a calculator, trial and error, or a systematic root finding method.
- Present value interest factor for annuities: $PFIVA(r,t) = \frac{1-\left(\frac{1}{1+r}\right)^t}{r}$, thus $PV = C \cdot PFIVA(r,t)$.
- $FV = C \cdot \frac{(1+r)^t-1}{r}$; $C = \frac{r}{(1+r)^t-1} \cdot FV$; $t = \frac{\ln\left(1+\frac{FV}{C}\cdot r\right)}{\ln(1+r)}$. There is no general formula for determining r.
- Annuity FV factor $= \frac{(1+r)^t-1}{r}$, thus $FV = C \cdot (Annuity\, FV\, factor)$.
- Perpetuity: $PV = \frac{C}{r}$.
- Annuity due: $PV = C \cdot \frac{1+r}{r} \cdot \left[1 - \frac{1}{(1+r)^t}\right]$.
- Growth annuity: $PV = C \cdot \frac{1-\left(\frac{1+g}{1+r}\right)^t}{r-g}$, if $r > g$. C is the payment at the end of period 1 (i.e., the first payment).
- Growth perpetuity: $PV{=}\frac{C}{r-g}$, if $r > g$.

4 Compounding Conventions; EAR

- If the nominal annual interest rate is r, and it compounds m times per year, then $FV = PV \cdot \left(1 + \frac{r}{m}\right)^m$.
- The effective annual rate is $EAR = \left(1 + \frac{r}{m}\right)^m - 1$.
- In the limit, when m tends to infinity, $FV = PV \cdot e^r$, $EAR = e^r - 1$.

5 Bonds

P = face value; C = yearly coupon (in dollars); t or T = maturity in years; y = yield; B = price. Note that the formulas depend on per-compounding-period interest rates and coupons. Formulas below are for semi-annual compounding; they may need to be adjusted.

- Bond price: $B = \frac{C}{2} \cdot \frac{1 - \left(\frac{1}{1+\frac{y}{2}}\right)^{2t}}{\frac{y}{2}} + P \cdot \left(\frac{1}{1+\frac{y}{2}}\right)^{2t}$.
- Bond yields can be computed using the interval bisection method.

6 Inflation

R = nominal interest rate; h = inflation rate; r = real interest rate.

- $1 + R = (1 + r) \cdot (1 + h)$; $r = \frac{R-h}{1+h} \approx R - h$.

7 Stock Valuation

P_0 = time-0 price of the stock; D_i= dividend that will be paid at the end of period i; R = per-period interest rate; g, g_1 , g_2 = constant, per-period dividend growth rates.

- General formula, with dividends up to time t: $P_0 = \sum_{i=1}^{t} \frac{D_i}{(1+R)^i} + \frac{P_t}{(1+R)^t}$.
- $P_0 = \sum_{i=1}^{\infty} \frac{D_i}{(1+R)^i}$. In reality, there are no infinite streams of dividends.
- Zero growth dividends: $P_0 = \frac{D}{R}$.
- Dividend growth model: $P_t = \frac{D_{t+1}}{R-g}$, if $R > g$.
- Non-constant growth: $P_0 = \sum_{i=1}^{t} \frac{D_i}{(1+R)^i} + \frac{P_t}{(1+R)^t}$; $P_t = \frac{D_t \cdot (1+g)}{R-g}$ **if** we assume constant growth after t.
- Two-stage growth: $P_0 = \frac{D_1}{R-g_1} \cdot \left[1 - \left(\frac{1+g_1}{1+R}\right)^t\right] + \frac{P_t}{(1+R)^t}$; $P_t = \frac{D_{t+1}}{R-g_2} = \frac{D_0 \cdot (1+g_1)^t \cdot (1+g_2)}{R-g_2}$.
- Required return: $R = \frac{D_1}{P_0} + g$.
- Multiples: $P_t = (\text{benchmark PE ratio}) \cdot EPS_t$.