NETWORKS (FALL 2023) – MIDTERM WITH BRIEF ANSWERS

This is a closed-book exam. No laptops or cell phones allowed. There are five questions, worth a total of 40 points. You can use any results proved in class or in the text without reproving them, but you must state explicitly which result(s) you are using. **Put your name and netid on this cover page; print them as clearly as possible as they will be scanned by Gradescope to assign your exam to you.** Put your name on **each** page of this exam. Answer each question (or part of a question) in the box provided below the question. **Good luck!**

Name:____

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Name:____

(1) [8 points] In the payoff matrix below, the rows correspond to player A's strategies and the columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

		Player B		
		L	M	R
Player A	U	$2,2$	$3,0$	$2,1$
	D	$1,4$	$5,6$	$6,2$

(1a) *(4 points)* Find all pure strategy Nash equilibria of this game.

Solution: (U,L) and (D,M).

(1b) *(4 points)* Does this game have any mixed strategy equilibria? If so find one; if not explain why not.

Solution: Let p be the probability on U, 1-p on D. Let q be the probability on L and 1-q on M. We can ignore R as it is strictly dominated. The mixed strategy Nash equilibrium is $p = 1/2$ and $q = 2/3$.

(2) [8 points]

Consider a sealed-bid auction with one seller auctioning off one unit of a good. There are three bidders, $1, 2, 3$, who have independent private values for the good, v_1, v_2, v_3, respectively. Each of these values is a whole number between 1 and 10, inclusive.

(2a) *(4 points)* Suppose that the seller runs a second-price auction for the good. What should bidder 1 bid?

Solution: v_1

(2b) *(4 points)* Suppose instead that the seller runs a third-price auction. That is, the good is sold to the highest bidder who pays the third highest bid. In a third-price auction bidding your value is not a dominant strategy. Suppose bidder 1's value is 5. Describe at least one set of possible bids for bidders 2 and 3 for which bidder 1 is better off bidding something other than the true value of 5. Note that this question asks what bidder 1 should do if 1 knew the bids of bidders 2 and 3.

Solution: If $\min\{b_2, b_3\} < 5$ then 1 should bid more than $\max\{b_2, b_3\}$. If this max is greater than or equal to 5 then 1 should bid more than 5. Any bids for 2 and 3 that satify these conditions are OK.

(3) [8 points] Three-hundred (300) travelers begin in city A and must travel to city B. There are two routes between A and B. Route I begins with a local street from city A to city C which requires a travel time per traveler in hours equal to the number of travelers on the street — let's call this number x — divided by 20. It ends with a highway from city C to city B which requires ten hours of travel time per traveler regardless of the number of travelers who use this highway. Route II begins with a highway from city A to city D which requires ten hours of travel time per traveler regardless of the number of travelers who use this highway. It ends with a local street from city D to city B which requires a travel time per traveler in hours equal to the number of travelers on the street — let's call this number y — divided by 40. All roads are one-way roads.

(3a) *(4 points)* Travelers simultaneously chose which route to use. Find Nash equilibrium values of x and y.

Solution: x=100 and y=200

(3b) *(4 points)* Now the government adds a new highway connecting cities C and D (the streets and highways described above are unchanged). This is a two-way highway; travelers can go from C to D or from D to C. Travel time on this highway is zero. Let z be the number of travelers who use this new high to travel from C to D and let w be the number of travelers who use this new highway to travel from D to C. Travelers simultaneously chose which route to use. Find Nash equilibrium values of w, x, y and z.

Solution: w=0, x=z=200 and y=300.

(4) [8 points]

For each statement provided below, assess its validity as either TRUE or FALSE. Provide a brief explanation to support your answer. Note that answers lacking a substantive explanation will not receive credit.

(4a) *(4 points)* We can find a balanced complete graph with 4 nodes such that there are 2 negative edges and 4 positive edges.

Solution: FALSE. Suppose AB is a negative edge. One of the edges AC and BC has to be a negative edge. Similarly, one of the edges AD and BD has to be a negative edge. Hence there will be at least 3 negative edges in the network.

(4b) *(4 points)* If a node A in a network satisfies the Strong Triadic Closure property and has at least two strong ties, then any weak tie it is involved in must be a local bridge.

Solution: FALSE. Let us consider a complete graph with 4 nodes, A, B, C, D, where AB is a weak tie and all other edges are strong ties. A satisfies the Strong Triadic Closure property and is involved in two strong ties (AC and AD), but AB – the weak tie – is not a local bridge.

(5) [8 points] Three buyers x, y and z are considering buying one of three houses A, B and C. The valuation that each buyer has for each house is given in the table below.

Buyer	Value for house A	Value for house B	Value for house C
x	5	2	2
y	4	3	8
z	6	4	3

Assume the price of house A is always twice that of house B, and the price of house C is fixed at 2. Specifically, the price vector $(p_A, p_B, p_C) = (2p_B, p_B, 2)$ where p_B, an integer greater than or equal to 0, is the price of house B.

(5a) *(2 points)* Calculate the payoff matrix (you can include p_B in the answer).

Solution: Payoff is $\begin{bmatrix} 5-2p_B & 2-p_B & 0 \\ 4-2p_B & 3-p_B & 6 \\ 6-2p_B & 4-p_B & 1 \end{bmatrix}$.

(5b) *(2 points)* Determine the prefered seller of buyer y and briefly explain why.

Solution: The prefered seller of buyer y is always C as

$$4 - 2p_B < 6, \quad 3 - p_B < 6.$$

(5c) *(4 points)* Determine the range of p_B for which the price is market clearing. [Remember that p_B is an integer.]

Solution: Market-clearing price needs to satisfy

$$5 - 2p_B \geq \max(2 - p_B, 0) \text{ and } 4 - p_B \geq \max(6 - 2p_B, 1) \quad (\text{if } x - A, y - C, z - B)$$

$$\text{i.e. } p_B \leq 2.5 \text{ and } 2 \leq p_B \leq 3 \Rightarrow 2 \leq p_B \leq 2.5$$

OR

$$2 - p_B \geq \max(5 - 2p_B, 0) \text{ and } 6 - 2p_B \geq \max(4 - p_B, 1) \quad (\text{if } x - B, y - C, z - A)$$

$$\text{i.e. } p_B \geq 3 \text{ and } p_B \leq 2 \Rightarrow \text{no feasible solution}$$

Hence $2 \leq p_B \leq 2.5$ is the only possible range and the only integer value is 2.