Sample Problems
AEM 2241 - Finance

1 Finance Matters

1.1. Some of these problems are more complex that the problems you may be expected to solve during the exam. Nevertheless, you should attempt to solve them as **sub-parts** of more complex questions may be posed as exam questions. You will benefit most if you work on these problems before you look at the solutions. The course staff can answer any questions that you may have in relation to these problems.

1.2. The problem set is issued before all the material testable on the exam was taught. It may happen that some topics may not have been covered (yet) in class before the exam. If so, just skip the relevant parts.

1.3. All assumptions, conventions, and notations that we normally use can be relied without further explanations. If you use non-standard notations, explain what they mean.

1.4. Unless stated otherwise, we ask that dollar amounts be rounded to two decimals, and interest rates be rounded to four decimals. For example: $156,798.38, $9.75, 0.0315 = 3.15%, 0.1425=14.25%.

1.5. Be careful to distinguish between per-period quantities, such as per-period coupon payments and interest rates, and their annualized versions, which are the ones that must be typically provided as results.

1.6. [When working on paper] Unless we tell you otherwise, you may use either formulas or financial calculators to solve a problem. Whether you are using formulas or calculators, you must show what you did (e.g. what formulas you used, what values you replaced, or what calculator buttons you pushed and what values you entered), and **you must briefly explain the logic of your solutions**.

1.7. [When working on paper] To eliminate any possible ambiguity, whenever using the calculator, you must indicate values for all 5 TVM buttons; except for the value that you compute. Indicate explicitly what buttons you pushed to get a result, and what the result is.

1.8. Whenever possible, interpret the meaning of the results in terms consistent with the problem.

Good Luck!

2 Present Value Calculations

Each row of the table below represents two cash flows, one being the present value of the other when viewed as an investment over a time horizon t, at an interest rate r. One or two numbers are missing in each row.

2.1. For rows (a) and (c) compute the value of the missing number using the formulas given in class. Write down the suitable general formulas first, then replace letter symbols with known numerical values; finally, compute the respective results.

2.2. For rows (b) and (d), use a financial calculator. Similar to what we did in class, show the values that you would set up for the TVM variables, then show what key combinations you would press to get to the solution; also, provide the calculator's answer.

2.3. Is there anything special about the problem in row (e)? If so, what is it? Explain, in no more than two sentences, how a situation like the one shown may arise. For this part you can use formulas or the calculator, but you must state what you did and show the steps that lead to the solution.

2.4. For part (f), where there are two missing numbers, determine, using formulas, **two** combinations of values for PV and r that would make the connection between the four variables in row (e) correct. Note: You need to provide two pairs of numbers, (PV_1, r_1) and (PV_2, r_2), which are both consistent with the data in row (f). You may be able to determine part of the answer by making a choice, as the problem is not fully determined.

Part	**PV [$]**	**t [years]**	**r [%]**	**FV [$]**
(a)	7,513	7	9	
(b)		29	13	48,318
(c)	48,000	15		185,000
(d)	18,400		9	289,715
(e)	200,000	5		175,415
(f)		8		89,980

3 Simple vs. Compound Interest Rates

Consider an investment of $10,000 that you make at the end of year 0. You are guaranteed an interest rate of 7.5% per annum for 10 years, compounded annually. We say that two interest rates are equivalent over a given time horizon t if the total interest earned by time t is the same in both cases.

3.1. Given a time horizon of $t = 5$ years, what is the simple interest rate that is equivalent to the compound interest rate described above?

3.2. Given a time horizon of $t = 10$ years, what is the simple interest rate that is equivalent to the compound interest rate described above?

3.3. Do the answers to the two questions above depend on your initial investment, assuming that the terms of the investment remain otherwise the same? Why?

3.4. Is it possible to have the same simple interest rate be equivalent to a given positive compound interest rate **simultaneously** over **several** investment periods longer than one year?

4 Discounted Cash Flow Valuation

The present value (at time 0) of the cash flow stream below is $7,500 when discounted at 9% annually. What is the value of the missing cash flow? What would be the missing cash flow if the present value were $4,500? Beside magnitude (the amount of dollars), is there anything different between the unknown cash flows in these two situations? Interpret this difference. Use formulas.

Year	**Cash Flow [$]**
1	1,700
2	x
3	2,450
4	2,980

5 Time Value of Money

Use formulas to solve this problem, except, possibly, for the first and last sub-part.

The 6-month interest rate is 3.5%. Consider two successive six-month periods.

5.1. Fill out the statement below relying on our usual conventions, so that it is consistent with the problem text.

"The interest rate is _____% per year, compounded semi-annually."

5.2. Compute the EAR.

5.3. What is the present value at time 0 of $500 to be paid at the end of the second 6-month period?

5.4. Now consider two equal payments made at the end of the first and second six-month periods, respectively. How big should these payments be so that their total present value at time 0 is equal to the value obtained in part (5.3) above?

Note: If you need the result of part (5.3), but you cannot calculate it, then use $400 for the needed total present value.

5.5. More abstractly, consider a payment P made at the end of the second 6-month period and its time-0 present value PV_0. Separately, consider two payments of size P/2 made at the end of the first and second six-month period, respectively, as well as their total present value $PV_0^{'}$. Can you determine, relying either on a qualitative argument **or** on formulas, whether PV_0 or $PV_0^{'}$ is bigger under ordinary circumstances, when interest rates are positive?

6 Mystery Financial Product

You purchased (invested in) a financial product that will pay you $500,000 every six months for 4 years, starting exactly six months from today. The relevant interest rate is 12% per year, compounded quarterly.

6.1. Given the frequency of payments and the quarterly compounding interest, which of the following interest rates is closest to the rate you must use to discount the first cash flow you will receive?

(6.1.1) 3%

(6.1.2) 4%

(6.1.3) 6%

(6.1.4) 12%

6.2. The financial product that you purchased will be phased out (eliminated) and replaced with a similar product which has only 4 annual payments of $1,000,000, with the first payment arriving exactly one year from now. From your perspective, is the new contract more, or less valuable than the old contract?

(6.2.1) The new contract is more valuable.

(6.2.2) The two contracts are equally valuable.

(6.2.3) The new contract is less valuable.

6.3. In a different scenario, the financial product you invested in initially will be eliminated, and will be replaced with a similar contract. This time, however, you will receive four $1,000,000 payments, with the first one due in exactly 9 months. The second, third, and fourth payments will each be made exactly one, two, and three years after the first payment, respectively (i.e., payments following the first one will be made at the ends of successive one-year intervals following the first payment).

To make the exchange fair, if the value of the new contract to you is greater than that of the old contract, you will have to pay the difference in contract values to your counterparty. Should the value of the new contract be less than the value of the old contract, you will receive the difference in contract values from your counterparty. If the old and new contract have the same value, no money exchanges hands.

What will happen?

(6.3.1) The answer cannot be determined.

(6.3.2) No money will exchange hands.

(6.3.3) You will receive a payment from your counterparty.

(6.3.4) You will make a payment to your counterparty.

7 You Won the Lottery!

You have just won the lottery and will receive 10 yearly payments, as follows: you get $1,500,000 in the first year, after which yearly payments will increase by 2.7% per year. A company specializing in purchasing annuities (yes, they do exist!) offers you instant $14,000,000 in cash to purchase the right to receive your winnings. The relevant interest rate is 3% per year. Will you take the offer?

8 Annuity, But You Have to Wait

Consider an annuity with a yearly payment of $\$75,000$ that makes its first payment at the end of year 5, and consists of 10 payments. Assume that the appropriate interest rate is 11% per year.

8.1. Using formulas for annuities, value this annuity (compute its present value) as of the end of year 4 (i.e. as if time had passed, and "now" were at the end of year 4); name this value PV_4.

8.2. Using formulas for annuities, value this annuity as of the end of year 5; name this value PV_5. What kind of annuity is this when viewed from the end of year 5?

8.3. Compute the present value of this annuity at time 0 in two different ways, starting separately from PV_4 and PV_5, respectively. Compare and comment very briefly on the two answers that you get.

9 One or Several?

Consider an annuity payment that will pay 75,000 every six months for 5 full years. Afterwards, the payment will increase to 100,000 every six months, for 5 more years. The relevant interest rate is 7.5% per annum, compounded semi-annually.

9.1. Explain how this annuity is equivalent to the difference of **two** regular annuities, each starting at time 0, one with a maturity of 10 years, one with a maturity of 5 years. Specify in full the details of the two component annuities.

9.2. Using the observation made in the previous part, value the component annuities, and combine these values to get the value of the composite annuity. If you could not identify the two annuities into which the initial annuity can be decomposed, feel free to use any other method to value the initial annuity.

Note: Many complex financial instruments can be seen as collections of simpler instruments that can be understood and/or valued in isolation. In such cases the value of the complex financial instrument emerges by the suitable aggregation of the values of its respective components.

10 Amortizing loans with variable payments

Consider an amortizing loan with **variable** payments. The initial loan amount is \$10,000,000, while the initial loan maturity is 30 years. The borrower must make monthly payments, with payments due at the end of each month. The borrower's total payment on this loan that is due at the end of the 73^{rd} month equals $87,777.78.

10.1. What is the monthly principal repayment on this loan?

10.2. What is the balance outstanding on this loan at the beginning of the 73^{rd} month?

10.3. What is the interest rate that would have been quoted on this loan when it was issued?

Note: If you need to use results from either parts (10.1) or (10.2) above, or from both, but you were not able to determine the respective values, you may use $50,000 for the answer to part (10.1), and $5,000,000 for the answer to part (10.2), respectively.

11 Balloons

You are an important local real estate investor; you just got a $10,000,000 balloon loan to buy a new office building in your home town. The nominal maturity of the loan is 30 years, but the loan has a 10-year balloon payment. In other words, the loan will end at the end of the 10th year, and the outstanding balance will be paid off in a lump sum at that time. The interest on the loan is 6.7% per annum, compounded monthly.

11.1. Assume that the loan has fixed payments.

(11.1.1) What is the monthly fixed payment that you have to make?

(11.1.2) Let PV be the present value at time 0 of the fixed payments made over the 10-year life of the loan. What is PV?

(11.1.3) Can you find a connection between the $10,000,000 principal, PV, and the balloon payment at the end of year 10? Compute, using this connection, the size of the balloon payment at the end of year 10. If you cannot find the connection, a more work-intensive approach is to use an amortization table to compute the answer (use Excel!).

11.2. Assume now that this loan has variable payments. At the end of each month fixed, equal portions of the loan's principal are paid down, such that the principal would be fully amortized at the end of the hypothetical 30-year loan period. What is the size of the balloon payment at the end of year 10 in this case?

12 Better Late Than Never

You work for a bank. Exactly five years ago, you helped Al Kapon, a well-known local businessperson, to get a \$15,000,000, 20-year variable-payment amortizing loan in order to build a "soft drink bottling facility." The loan carries an interest of 7% per annum, has monthly payments, and is structured like similar loans discussed in class; in particular, Kapon is expected to pay down the same amount of principal every month for the duration of the loan, in addition to the interest due monthly.

12.1. What is the payment due at the end of the very first month of this loan?

12.2. On the fifth anniversary of the loan Kapon comes to your office unexpectedly, and states that his business is in trouble. However, he hopes that difficulties are temporary, and that his business and finances will recover within one year. After some back and forth, you agree on behalf of the bank to forsake principal payments due for the next 12 months. However, Kapon still must pay in full the interest due at the end of each month.

(12.2.1) What is the loan balance at the end of 5 years, when Kapon asks for the modification of the loan?

(12.2.2) What payments will be made at the end of each month for the duration of the year when principal payments are suspended?

12.3. Principal repayments resume after the 12 months elapse. For the remainder of the loan's original term, the same amount of principal will be repaid every month, so that by the loan's original maturity date the principal is fully paid off.

(12.3.1) What will be the monthly principal payments due after the end of the principal repayment suspension?

(12.3.2) Provide the row of the loan's updated amortization table corresponding to the first month in which after principal payments have resumed.

13 Beginning Balance at the End...

Assume that you have a fixed-payment amortized loan with a principal of \$8,000,000, a yearly interest rate of 9% compounded monthly, and a maturity of 10 years.

To the closest thousand, what is the beginning balance of the loan at the start of the last month (i.e., at the beginning of the month at the end of which the very last payment due on the loan is made)?

13.1. $100,000

13.2. $101,000

13.3. $102,000

13.4. None of the numbers above can be the answer.

14 Collecting Coupons

Assume that a bond sells for $948; it has semi-annual coupons, a maturity of 8 years, yields 5.1%, and has a face value of $1,000. What is the coupon rate of this bond?

15 But How Much Do I Make?

Assume that a bond with a maturity of 10 years, face value of $1,000, coupon rate of 5%, with semi-annual coupons, has a market price of $903.25. You have already determined that the yield of the bond is between 6% and 7%. Show yields as percentages with two decimals precision.

15.1. Assuming that the yield of the bond were 6.5%, show how you would set up your calculator to compute the implied bond value and provide this respective value.

15.2. Set up a table similar to that given in class to determine an approximate value for the yield of the bond. Use your calculator to compute bond values, but do not show the details of your calculator operations. Stop when the mid-yield produces a price within $0.50 of the bond's true price.

Hints: You only need to compute the total value of the bond for various yields, not also the part attributable to coupons or principal. Also, since only the yield changes, you do not have to re-enter all the values into the TVM worksheet - just change the yield and recompute the value.

16 Treasuries

On February 28, 2023, the financial press announced that yields for US Treasury bonds with 10-year maturities were at record low levels, never before seen. Indeed, early in the day, 10-year Treasury yields were as low as 1.18%, corresponding to a price of $1,029.50 per $1,000 face value. Typical Treasury bonds pay semi-annual coupons.

16.1. Without performing any computations, can you provide a **lower bound** (lower limit) for the yearly coupon rate of this 10-year US Treasury bond? If yes, state what this lower bound is, and how you know it is correct. A trivial lower bound of 0 is not an acceptable answer.

16.2. What is the implied annual coupon rate for this bond?

17 Bond Decomposition

You are studying a bond that has a leftover maturity of 7 years, has a face value of $1,000, and a stated coupon rate of 5% per year, payable semiannually. The yield of the bond is 4.7% per annum; further, you may assume that per-period rates are constant (they do not depend on the time horizon).

The bond contract includes covenants (agreements) meant to protect bond investors' interest. In the recent past the company breached one of these covenants. Even though some improvements did occur since then, the breach is expected to persist for the next two years, after which it is expected to be cured (eliminated). While the breach persists, the coupon rate will be double the normal (stated) one. If default were to occur, such an accelerated payment schedule assures that investors get more of their money back; if default does not occur, then the increased coupons act as a penalty for the breach of the covenant.

17.1. Decompose this bond into simpler financial instruments that were studied in class. Specify what these simpler instruments are, what their parameters are, and how do you know your decomposition is correct.

17.2. Determine the current price of the bond.

18 NPV Mystery I

The NPV of a regular coupon bond's cash flows, when the discount rate is equal to the bond's yield, is equal to the bond's price (value). This statement is...

18.1. True

18.2. False

19 NPV Mystery II

For the types of bonds discussed in lectures, the NPV of the bond's principal (face value) always exceeds half of the bond's current price (value). This statement is...

19.1. True

19.2. False

20 Congratulations, You're an Analyst

You have just been hired as a junior analyst working for a bond trader. Your first assignment is to value a corporate bond paying semi-annual coupons at an annual rate of 9.5%, with a maturity of exactly 2 years. The bond has a face value of $1,000. A senior analyst has already processed the current Treasury price data and provided you with an up-to-date term structure chart, shown in Figure 1. You are told to treat this corporate bond similarly to government bonds; i.e. you will ignore all default, liquidity, and similar risks, which will be analyzed by more experienced colleagues.

20.1. The term structure of interest rates chart has a horizontal axis labeled "Maturity," and a vertical axis labeled "[Annual[1]] Yield." What kind of government bonds have their yields and maturities plotted on this chart?

20.2. From earlier problems you solved while in college, you learned to decompose more complex financial instruments into sums - or differences - of simpler instruments. Explain how you can decompose this corporate bond into a collection of zero-coupon bonds of different maturities, perhaps having atypical face values. Provide a brief statement explaining the decomposition and show, in a table, what would be the bond's maturities and face values, respectively.

20.3. You learned that in realistic settings cash flows that arrive later must be discounted at (usually) higher per-period interest rates. For each zero-coupon bond listed in part 20.2 above, use the term-structure chart and look up its corresponding yield. Next, compute the present value of each zero-coupon bond. Using these zero-coupon bond prices, and also relying your earlier insights, provide a computed (theoretical) price for your corporate bond.

20.4. You now have a price for your corporate bond - what is its yield?

20.5. You provide the result computed in item 20.4 to one of your colleagues, who explains that in the practice of your firm, in order to adjust for the risks that a bond like yours bears in addition to government bonds, its yield must be changed by 0.50% per year. The colleague did not say

[1] As you will note, the chart only uses "Yield" as a label for the vertical axis. The usual bond terminology expresses yields in annualized terms, and you should do the same in this class, as well as in other finance-related work that you do. We provide a reminder here, but you should **no**t assume that similar reminders will also be present when taking an exam.

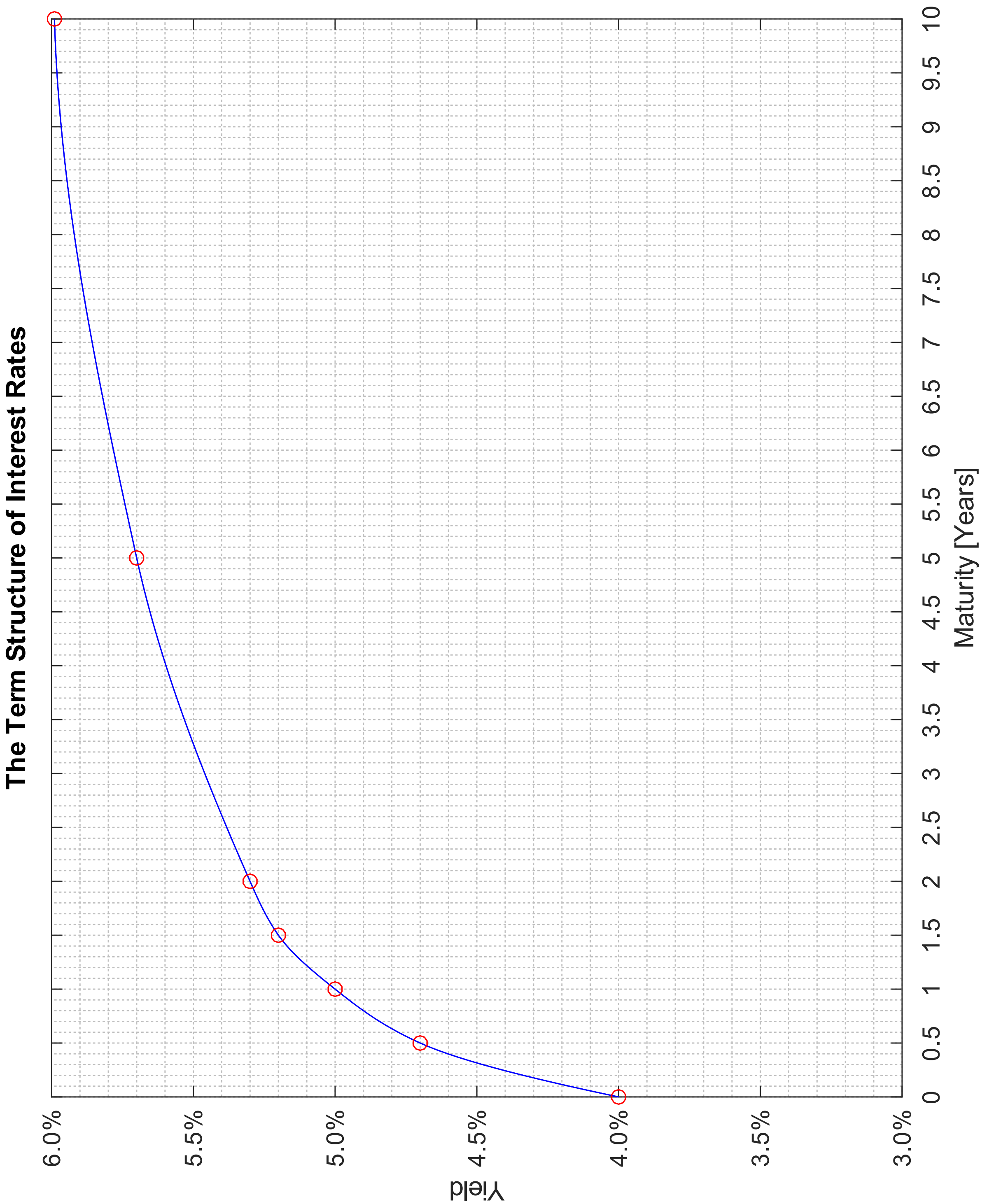

Figure 1: Term structure of interest rates on your first day as an analyst, as determined by a senior analyst using current government bond data.

explicitly whether the yield should be increased or decreased. State whether the yield must be increased or decreased, explain why, and then compute the new bond price. Determine what is the percentage change in the bond price when comparing prices before and after the risk adjustment, respectively.

20.6. Now adjust the yield in the **opposite direction** to that you decided was necessary in part 20.5 above. Compute the new price and the percentage price change when comparing the original (riskless) price and the price after the newest change in yield. Compare the percentage changes in the price of the corporate bond when the yield has been increased and decreased by the same amount, respectively. Which relative change is bigger? Could you have predicted which change is bigger by examining any of the slides discussed in class, without resorting to formulas?

21 We Have No Money, We're a Startup

Next Wonder, Inc. is a startup that cannot afford to pay dividends, since it has to finance its rapid growth, as it aims to take over the technology world. The company will pay no dividends for 5 years, but then it plans to pay $2.5 per share, per year for the next 5 years. After these 10 years, the stock dividend will jump to $5 in year 11, and will keep increasing by 3% per year indefinitely. The required return is 8% per year.

21.1. Assume that you are computing the price of the stock, P_{10}, at the end of year 10, just after the dividend due at the end of year 10 has been paid. What is P_{10}?

21.2. Assume that you are computing the price of the stock, P_5, at the end of year 5. What is P_5?

21.3. What is the price of the stock at time 0, P_0?

22 Stock price

You are a new analyst following the stock of company Big Break, Inc., valued at $15 per share on the morning of March 31, 2023. The company has been paying dividends of 35 cents on the very last day of each calendar quarter, and this was expected to continue indefinitely.

22.1. Assuming that the dividend model is reasonably accurate for this stock, what is the implied return rate demanded by investors in this stock, expressed in "per annum, compounded quarterly" terms?

22.2. In a news conference late in the day on March 31, 2023, the company's CEO announces "temporary difficulties related to external funding needs" that will force the company to cancel the dividend for four successive quarters, "in order to conserve cash." The dividend due on June 30 will be paid, however, as it had already been declared, so the first missed payment will occur on September 30, 2023. The CEO, a highly credible, seasoned industry veteran, states that the dividend policy will be reinstated in its current form after the temporary suspension ends. Note:

If you need to use the value that was requested in part (22.1) above, but you were not able to compute it, you may use 5.5% for the "per annum, compounded quarterly" rate.

(22.2.1) Estimate the price of stock as of March 31, 2024.

(22.2.2) Estimate the price of the stock as of March 31, 2023, just after the announcement.

23 Dividends

Fly Over Airlines just paid an annual dividend of 80 cents, and a new analysis revealed that dividends will grow long term at a rate of 4% per year. The company's stock price is $70. Estimate, to the closest percent, the rate of return demanded by investors who buy Fly Over's shares.

23.1. 5%

23.2. 6%

23.3. 7%

23.4. None of the above is the correct answer.

24 What's an Option Worth?

Over the last few weeks we got into the habit of describing more complex financial instruments as collections of simpler instruments. For example, we can decompose a typical bond into an annuity (the coupon payments), and a separate single payment (the principal). Similarly, we have seen atypical annuities described as sums or differences of regular annuities. In this problem you will build a synthetic bond (see below), by suitably combining fractional amounts of two other bonds.

Consider the following information about three Treasury bonds:

Maturity Date	Coupon Rate	Price	Bond Type
05/15/2024	6.500	106.31250	regular
05/15/2024	8.250	103.43750	callable
05/15/2024	12.000	134.78125	regular

All the bonds above have a face value of $1,000. Prices are expressed as percentages of face value. We do not know the precise date when these prices were quoted; as such, it is possible that we are not exactly at the beginning of a coupon period. We do know, however, that all coupon payment dates for all these bonds are the same; the coupons, of course, are not. The bond in the middle is a so-called callable bond, while the other two are regular Treasuries.

24.1. Read the textbook and/or research the web to understand what callable Treasuries are. Summarize your findings in a **brief** paragraph.

24.2. Given what you learned about callable bonds, under what conditions would the option to call the callable Treasury be exercised and by whom (the bond holder, or the bond's issuer)? In other words, when would the callable Treasury be called and by whom?
Hint: Assume that the call decision is rational; consider changes in interest rate levels as time passes.

24.3. From the **holder**'s perspective, would a callable Treasury bond be more, or less valuable than an otherwise identical non-callable, i.e., regular Treasury bond? Briefly state your opinion and justify it qualitatively.

24.4. Now consider the pricing information provided in the table above. Use the information on the regular bonds to construct a bond that is identical to the the callable bond in all respects, but for the callable feature. Use this artificial bond to compute the cost of the call option. From the perspective of the bond holder (the lender), is this cost positive or negative?

Hint: Let B_1, B_2, and B_3 be the three bonds shown in the table above, starting with the bond at the top. If we ignore the call feature, the listed Treasuries are characterized by a small number of parameters: their coupon sizes, coupon payment times, and face values. You should build a portfolio (combination) of the two regular bonds B_1 and B_3, such that their combination produces B_2 (again, except for the call feature). Let B_{13} be an artificial (often called "synthetic") bond that you can create from a combination of B_1 and B_3, and assume that B_{13} consists of a fraction f of B_1 and a fraction $1-f$ of bond B_2, where $0 \leq f \leq 1$. Write down equations so that the resulting coupon and face value of B_{13} (seen as a mix/combination/portfolio of B_1 and B_3) matches the respective parameters of B_2. Use these equations to determine f, and then use f to determine an appropriate price for B_{13}.

25 Computing IRRs

Consider a set of **signed** cash flows CF_i that arrive at times t_i, where $i = 1, 2, \cdots, N$, and $t_1 \leq t_2 \leq \cdots \leq t_N$. The (per-period) internal rate of return (IRR) is the constant interest rate that makes the net present value (NPV) of this series of cash flows to be equal to 0:

$$\sum_{i=1}^{N} \frac{CF_i}{(1+IRR)^{t_i}} = 0.$$

Clearly, this equality is only possible if some cash flows are positive, and some cash flows are negative. As mentioned in class, we have already encountered IRRs (in a hidden form) when we spoke about bond yields. Using, for now, **unsigned** cash flows, let B be the price of the bond, C the per-period coupon, and P the principal (face value) of the bond; further, assume that there are exactly N time periods to maturity. We can immediately write:

$$B = \sum_{i=1}^{N} \frac{C_i}{(1+y)^i} + \frac{P}{(1+y)^N},$$

or, equivalently,

$$-B + \sum_{i=1}^{N} \frac{C_i}{(1+y)^i} + \frac{P}{(1+y)^N} = 0.$$

Now, if we consider signed cash flows, and we assume that **B** is the purchase price of the bond, then **B** is a cash outflow, and thus **B** is a negative cash flow. Keeping this in mind, we can rewrite the equation above for **signed** cash flows:

$$\boldsymbol{B} + \sum_{i=1}^{N} \frac{C_i}{(1+y)^i} + \frac{P}{(1+y)^N} = 0.$$

To avoid confusion between the two interpretations of the bond price, we denoted the unsigned bond price by B, and the signed bond price by **B** (bold B). B and **B** are equal in magnitude, i.e., $B = |\boldsymbol{B}|$; only their signs are different (B>0, **B**<0).

In the last formula, it is easy to recognize that the yield of the bond is, in fact, the bond's IRR. You already know how to compute the yield of bonds - this problem will teach you how to compute IRRs. Given that we did not specify what the compounding period is in this problem, all interest rates will be expressed simply as "per-period" rates, consequently, you will not have to worry about annualizing the rates.

Consider the cash flows shown below:

Time	**0**	**1**	**2**	**3**	**4**
Cash Flow [$]	1,000,000	-1,525,000	947,000	-1,826,000	1,420,500

25.1. Using a suitable tool (such as Excel), compute the net present value of the cash flows shown above for interest rates ranging from 0% to 30%, by incrementing interest rates by 1% at each step. Complete a table like the one below:

Interest Rate	0%	1%	2%	...	30%
NPV	...	...	...	...	...

You may format the table vertically, or you may create multiple columns, if you find it necessary or useful.

25.2. Plot the values of the NPV as a function of the interest rate. Make sure that the plot axes are labeled, and that enough values are shown for the reader to understand your plot in detail. Make the plot big enough to be easily legible.

25.3. Study the plot and the table above, and answer the following questions:

(25.3.1) How many IRRs does this series of cash flows have?

(25.3.2) For each IRR whose existence you ascertained, provide an interest rate interval that contains the respective IRR. This step brackets each IRR.

Do not necessarily strive to provide very narrow intervals, as the procedure described below will quickly reduce their size. However, make sure that **each bracketing interval contains exactly one IRR**, and that **the sign of the NPV is different at the two ends of the interval,** i.e., to the left, and to the right of the IRR. In other words, each interval that you pick will contain exactly one IRR value, and the NPVs computed at the of the respective interval will have opposite signs.

25.4. Consider now a presumptive IRR, let it be irr, and the corresponding bracketing interval $[low, high]$; i.e., $low < irr < high$. Like with bond yields, we must reduce the width of the interval so that we can find the approximate value of the IRR. We start each step by computing the midpoint of the interval: $mid = \frac{1}{2}\,(low{+}high)$. Now we compute the NPV of the given cash flows rates are equal to low, $high$, and mid, respectively. We must have one of the following situations:

NPV@low	**NPV@mid**	**NPV@high**	**New Bracketing Interval**
positive	**positive**	**negative**	[mid,high]
positive	**negative**	negative	[low,mid]
negative	**positive**	positive	[low,mid]
negative	**negative**	**positive**	[mid,high]

The rules[2] from the table above make sure that the updated, narrower bracketing interval for the IRR always contains the IRR we are chasing. This is because the NPV has different signs at the ends of the updated interval; this, combined with the fact that the bigger bracketing interval contains only one IRR, means that the NPV, seen as a function of the interest rate, will cross 0 somewhere within the interval, as it goes from positive to negative values, or from negative to positive values. This crossing identifies the location of the IRR.

For **each** IRR, and its corresponding initial bracketing interval, set up a table like the one below and fill out the rows by computing narrower and narrower intervals that contain the IRR. Stop when the computed NPV of the given cash flows is close to 0, e.g., it is within the interval [-\$0.10,\$0.10].

low		**high**		**mid**	
Per-Period	**NPV**	**Per-Period**	**NPV**	**Per-Period**	**NPV**
...	...	...	...	...	...

[2] We note that the rules for computing bond yields were derived from these more general rules, and were particularized for the special case we were then examining. For example, for bonds, the cash flows represented by coupon payments and the principal payment are all positive, and thus a bond's NPV always decreases as yields increase. We commented on this effect when we plotted the bond price against bold yields earlier in the course. We can thus eliminate the last two rows from the more general IRR rule table, given that the NPV of a bond can never increase from negative to positive values as the rate increases from *low* to *high*. Can you show that the first two rows of the IRR rule table lead to the rules we provided for bond yield computations? Hint: You may wish to examine the bond formulas discussed at the beginning of this problem.

Analogously to the computation of bond yields, **after** you set up the first row, you only need to compute one new set of values for each table row, i.e., you must compute the new mid-point and its associated NPV. The other needed values are already available in the immediately prior row.

For each table you construct, state what the best estimate of the corresponding IRR is.

26 IRR

Consider the IRR calculation algorithm that uses the interval bisection method you learned in this course. You are given the table below, which shows part of such a calculation. Rely on our usual conventions to interpret the meaning of the various columns shown. Assume that there is exactly one IRR in the interval 18.00% to 21.50%.

Low		**High**		**Mid**	
Per-Year	**NPV**	**Per-Year**	**NPV**	**Per-Year**	**NPV**
...	...	...	...	...	...
18.00%	70,096.86	21.50%	**X**	19.75%	23,065.09
Y	23,065.09	**Z**	**Q**	20.63%	655.92
U	...	**V**	...	...	...
...	...	...	...	...	...

26.1. What is the value of Y in the table above?

(26.1.1) 18.00%

(26.1.2) 19.75%

(26.1.3) 20.63%

(26.1.4) 21.50%

(26.1.5) None of the above values is a correct value for Y.

26.2. What can we say for sure about Q in the table above?

(26.2.1) Q is negative.

(26.2.2) Q is very close to 0.

(26.2.3) Q is positive.

(26.2.4) None of the statements above are necessarily true.

26.3. What is the value of V-U in the table above?

(26.3.1) V-U is greater than 2%.

(26.3.2) V-U is between 1% and 2%.

(26.3.3) V-U is between 0% and 1%.

(26.3.4) V-U is negative.

27 Can You Trust Your Co-Worker?

After modeling, the cash flows for a project are estimated to be -\$50M, \$40M, and \$15M; they will occur at the end of year 0, year 1, and year 2, respectively. You overheard a colleague stating that the IRR of these cash flows is approximately 7.82%. You suspect that this would be a rate that compounds annually. Is your colleague right?

27.1. Yes

27.2. No

28 Best or Worst?

You are creating a model for a project, and are currently working on scenarios, specifically, on developing a "best case" scenario. You have chosen (fixed) projected values for all but one input of your scenario. The only parameter of your scenario that is not yet fully determined by the choices you already made is the aggregate level of short-term credit that you may have to extend to your clients (by allowing them to pay for their purchases within 30 days from delivery). Given the remaining uncertainty, you believe that the total dollar value of this credit can range from a low value of L to a high value of H (L < H). The level of the credit will otherwise stay constant over the lifetime of the project.

Which value will you include in a best-case scenario?

28.1. L would be included.

28.2. H would be included.

28.3. The answer cannot be determined based on the information given.

29 I Have a Project For You...

Estimates for a new manufacturing project include the following: Initial investment in manufacturing equipment is $14,000. The equipment will be depreciated linearly over four years, which is also the lifetime of the project. The equipment has no salvage value. Fixed yearly costs are $11,000, variable costs per unit are $11, while the per-unit sale price is $25. The working capital needs of this project are negligible.

29.1. Given a production level of 2,000 units per year, what is the average manufacturing cost?

(29.1.1) $5.5

(29.1.2) $11.0

(29.1.3) $6.5

(29.1.4) None of the above.

29.2. Given a constant per-year production level of 3,500 units and a tax rate of 25%, what number below is within $1,000 of the yearly OCF?

(29.2.1) $10,000

(29.2.2) $20,000

(29.2.3) $30,000

(29.2.4) No value above is within $1,000 of the yearly OCF.

29.3. Ignoring taxes, which of the numbers below is within 100 units of the yearly production level that is needed to achieve accounting break-even?

(29.3.1) 1,000

(29.3.2) 2,000

(29.3.3) 3,000

(29.3.4) None of the above are within 100 units of the accounting break-even production level.

29.4. Assume that at a certain constant yearly production level L, the yearly OCF is $4,175 for each year over the lifetime of this project. If the required return is 14% per year, compounded annually, what can you say about the relationship of production level L and the financial break-even production level F?

(29.4.1) L if below F

(29.4.2) L is above F

(29.4.3) The relationship between L and F cannot be determined.

30 Equipment Sale

Your firm uses a complex manufacturing machine that, at the end of year 6 of its life, after the depreciation for the just-ending year has been applied, has a book value of $50,000. The machine breaks at the very end of year 6, in a way that makes it useless for your business. However, the machine does not need to be immediately replaced, as you have spare manufacturing capacity, and there is no expectation that demand for your products will increase. Your tax rate is 25%.

Note: Do not look for hidden connections to MACRS. Take the depreciation rules as given and use them, when appropriate, to answer the questions below.

30.1. You can sell the machine at the very end of year 6 for $60,000. If you sell the machine instantly for cash, which cash flows, if any, will change for year 6, compared to the case when you do not sell the machine at this time?

(30.1.1) OCF

(30.1.2) Cash flows related to capital spending.

(30.1.3) Both OCF and cash flows related to capital spending.

(30.1.4) Neither OCF, nor cash flows related to capital spending will change.

30.2. Assume that you put off selling the machine at the end of year 6, but you sell it at the very end of year 7. By the end of this final year, the machine is fully depreciated. You sell the machine for $60,000. Which year 7 cash flows, if any, will change compared to the situation when the machine is sold at the end of year 6?

(30.2.1) OCF

(30.2.2) Cash flows related to capital spending.

(30.2.3) Both OCF and cash flows related to capital spending.

(30.2.4) Neither OCF, nor cash flows related to capital spending will change.

30.3. Assuming ordinary economic conditions, as well as the facts stated for this problem, including in parts (a) and (b) above, when should you sell the machine - at the end of year 6, or at the end of year 7? Assume that except for the timing of this sale, all other aspects of the business are the same under both scenarios.
Hint: With the numerical information given you may perform calculations, and then you can use basic financial knowledge to provide an answer. You may be able to avoid calculations altogether if you reason carefully about the impact of depreciation on the scenarios outlined here.

(30.3.1) You should sell at the end of year 6.

(30.3.2) The year of the sale is irrelevant, as it has no impact on the project's finances.

(30.3.3) You should sell at the end of year 7.

(30.3.4) The question cannot be decided.

31 Cash Flows and NPVs

Quad Enterprises is considering a new three-year expansion project that requires an initial fixed asset investment of \$2.32 million. The fixed asset will be depreciated straight-line to zero over its three-year tax life, after which time it will be worthless. The project is estimated to generate \$1.735 million in annual sales, with costs of \$650,000. The tax rate is 21%.

31.1. What is the level of the OCF over the lifetime of this project?

31.2. What is the level of project cash flows over the lifetime of this project? If the required return (i.e., discount rate) is 12%, what is the project's NPV?

31.3. Assume, in addition to the above, that the project requires an initial investment in net working capital of \$250,000, and the fixed asset will have a market value of \$180,000 at the end of the project.

(31.3.1) What are the project's cash flows?

(31.3.2) What is the new NPV?

32 Cutting Costs

Masters Machine Shop is considering a four-year project to improve the efficiency of its production facilities. Buying a new machine press for \$385,000 is estimated to result in \$145,000 in annual pretax cost savings. The press falls in the five-year MACRS class, and it will have a salvage value of \$45,000 at the end of the project. The press also requires an initial investment in spare parts inventory of \$20,000, along with an additional \$3,100 in inventory at the end of each succeeding project year. At the end of the project all working capital is recovered.

If the shop's tax rate is 22% and its discount rate is 9%, should the company buy and install the machine press?

You may assume that machine press is bought on the last day of Year 0. Further, assume that MACRS rules allow you to take a half-year depreciation **in the year of acquisition**. The MACRS schedule given in class and in the textbook already contain the half-year adjustment for the depreciation percentage shown for the first year.

You may find that in certain years a project generates losses. If EBIT is negative the "tax" computed on this amount is also negative. This should be interpreted as a form of tax shield. A business could use this tax shield to "cancel out" positive taxes in other parts of the business. It is also possible for losses to be "carried forward" or to be "carried backward" and for taxes to be reduced for past or future years. For this problem you should assume that this tax shield is usable, but you do not have to decide how. Mathematically, it is just the negative of a regular tax.

33 Scenarios

You are considering a new product launch. The project will cost \$1,950,000, have a four-year life, and have no salvage value; depreciation is straight-line to zero. Sales are projected to be 210 units per year; price per unit will be \$17,500, variable costs per unit will be \$10,600, and fixed costs will be \$560,000 per year. The required return on the project is 12%, and the relevant tax rate is 21%.

33.1. What is the base-case NPV?

33.2. Based on your experience, you think the unit sales, variable cost, and fixed cost projections above are within $\pm 10\%$. What are the upper and lower bounds, respectively, for these projections? What are the best and worst case scenarios? You may assume that all these variables are independent. Note: Provide only the variables [inputs] that define these scenarios; do not set up the corresponding full models.

33.3. What is the cash break-even level of output for this project (ignoring taxes)?

33.4. What is the accounting break-even level of output for this project (ignoring taxes)?