Network Diffusion & Cascades (2)

NETWORKS INFO 2040 / CS 2850 / ECON 2040 / SOC 2090

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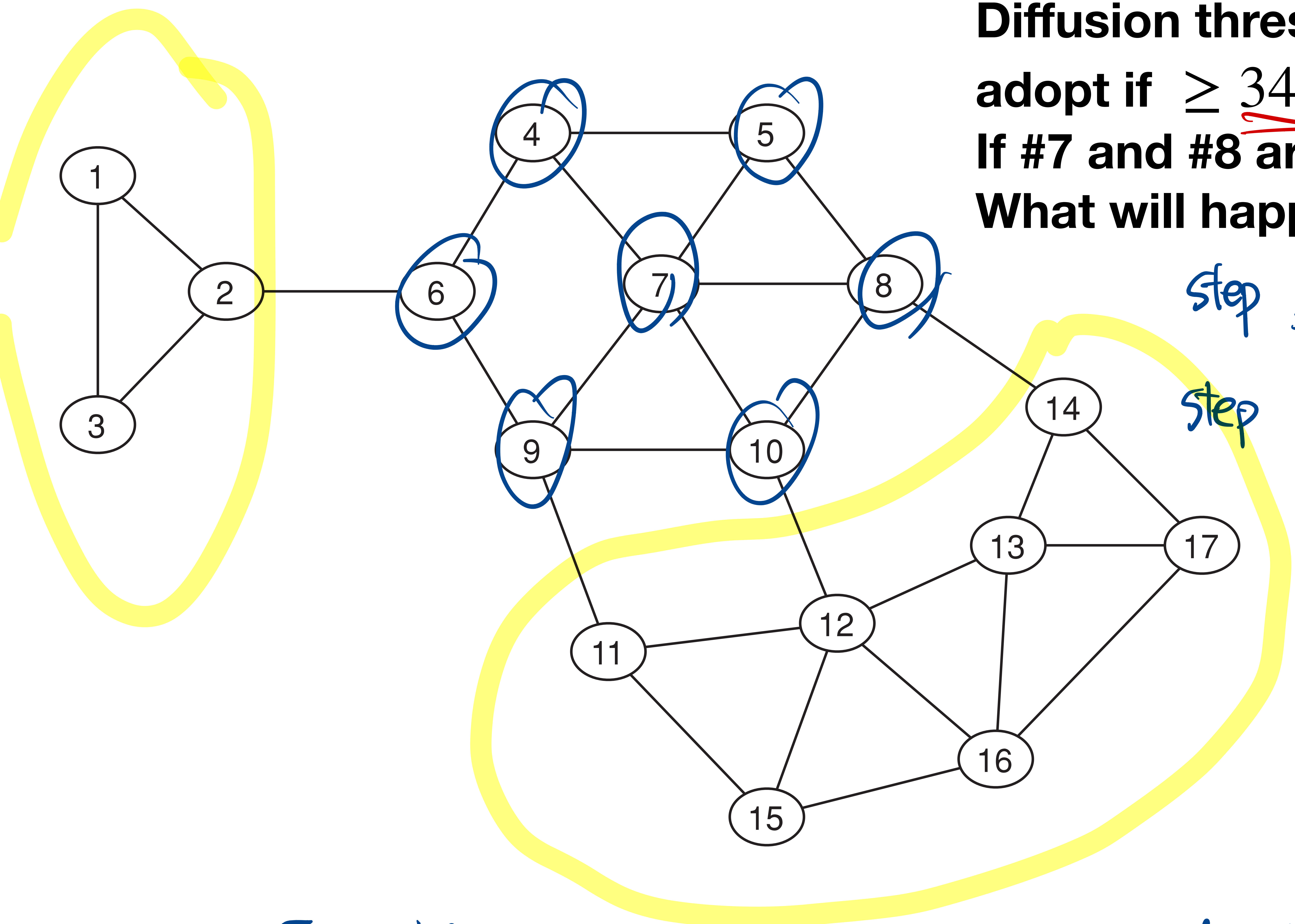
- Read Chapters 19 (Mon & Wed) & 22 (Fri)
- PS 7 out (due on Thur): answers typed, assign page numbers
  - Q4c-d: It's fine to just calculate "the number of" (instead of fraction)

(c) Power law distribution has a nice property that it is scale-free – if we upscale or downscale a power law distribution by some magnitude, we will still observe a similar distribution. As an illustrative example, let us consider a new index – the *popularity index* (p) of news article, which is approximately half of the raw view counts v. The exact relationship between popularity p and view count v is defined as

$$p = \begin{cases} v/2, & v = 0, 2, 4, \cdots \\ (v-1)/2, & v = 1, 3, 5, \cdots \end{cases} \tag{1}$$

Now let us consider $h(p)$, which is defined as the fraction of articles with a popularity index value p. Given that the number of articles each day that receive k views is c/k^α, can you write $h(p)$? You may express your answer in terms of c and α.

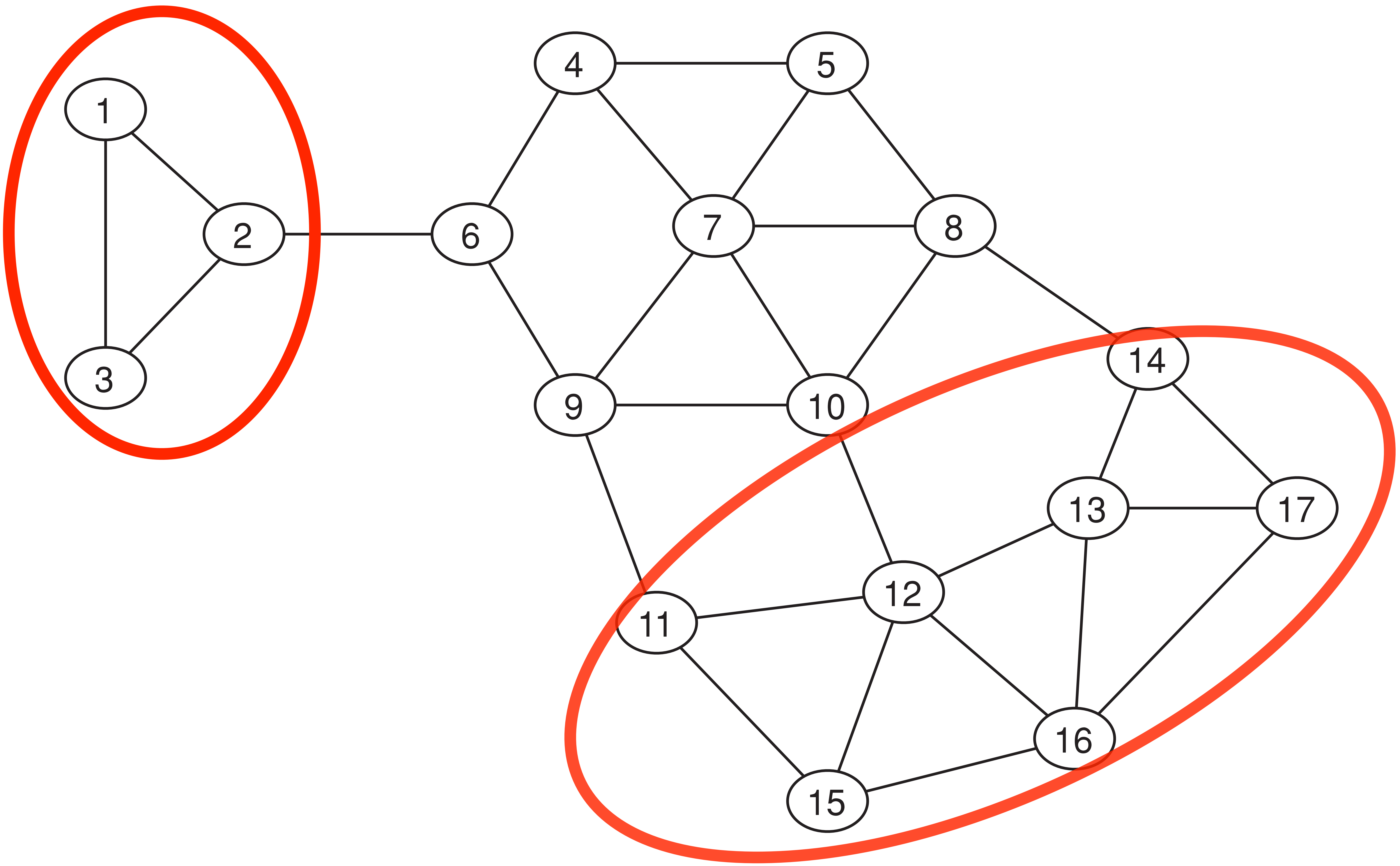
(d) Given that $h(p)$ approximately follows another power law d/p^β, can you determine the parameters d and β? You may express your answer in terms of c and α.
Hint: For question (d) we are primarily interested in popular articles with high k, where you can use the approximation $x^{-\alpha} \approx (x+1)^{-\alpha}$.



Diffusion threshold 0.34 (a person will adopt if $\geq 34\ \%$ of their friends have done so)
If #7 and #8 are the initial adopters ("seeds"), What will happen?

step 0 : #7 #8
step 1 : #5 #10
step 2 : #4 #9
step 3 : #6

Incomplete cascade : Some nodes eventually did NOT adopt


1
2
3
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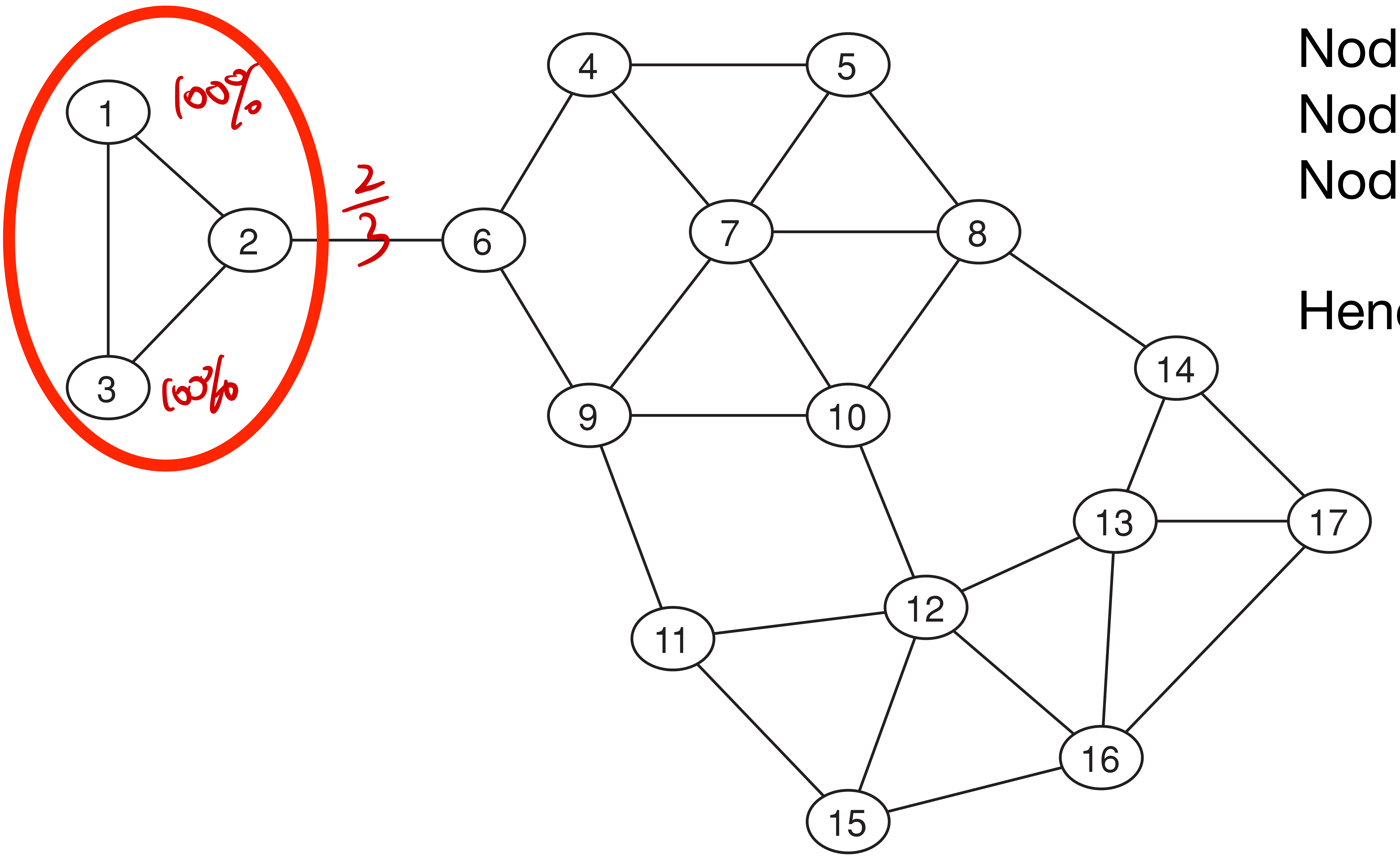
$\text{Min}\{1, 1, \frac{2}{3}\} = \frac{2}{3}$

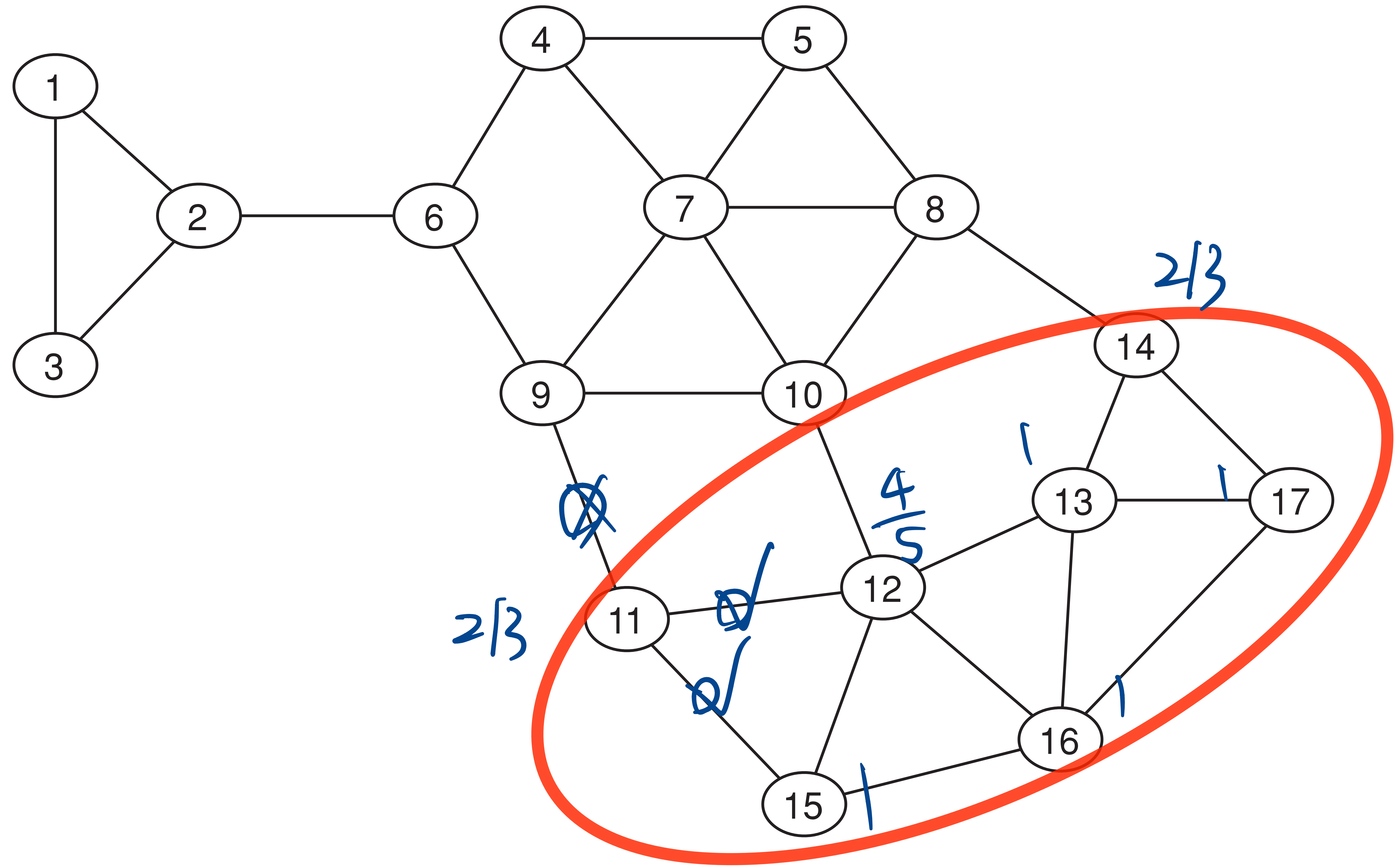


Node 1 has **2/2** neighbors in the subset
Node 2 has **2/3** neighbors in the subset
Node 3 has **2/2** neighbors in the subset

Hence {1,2,3} is a cluster of density **2/3**

We say a set of nodes *X* is a cluster of density *p* if each node in *X* has at least fraction *p* of its network neighbors in *X*

iclicker question



Subset {11,12,13,14,15,16,17}, is a cluster of density …?

A. 1/2
B. 1/3
C. 2/3
D. 3/4

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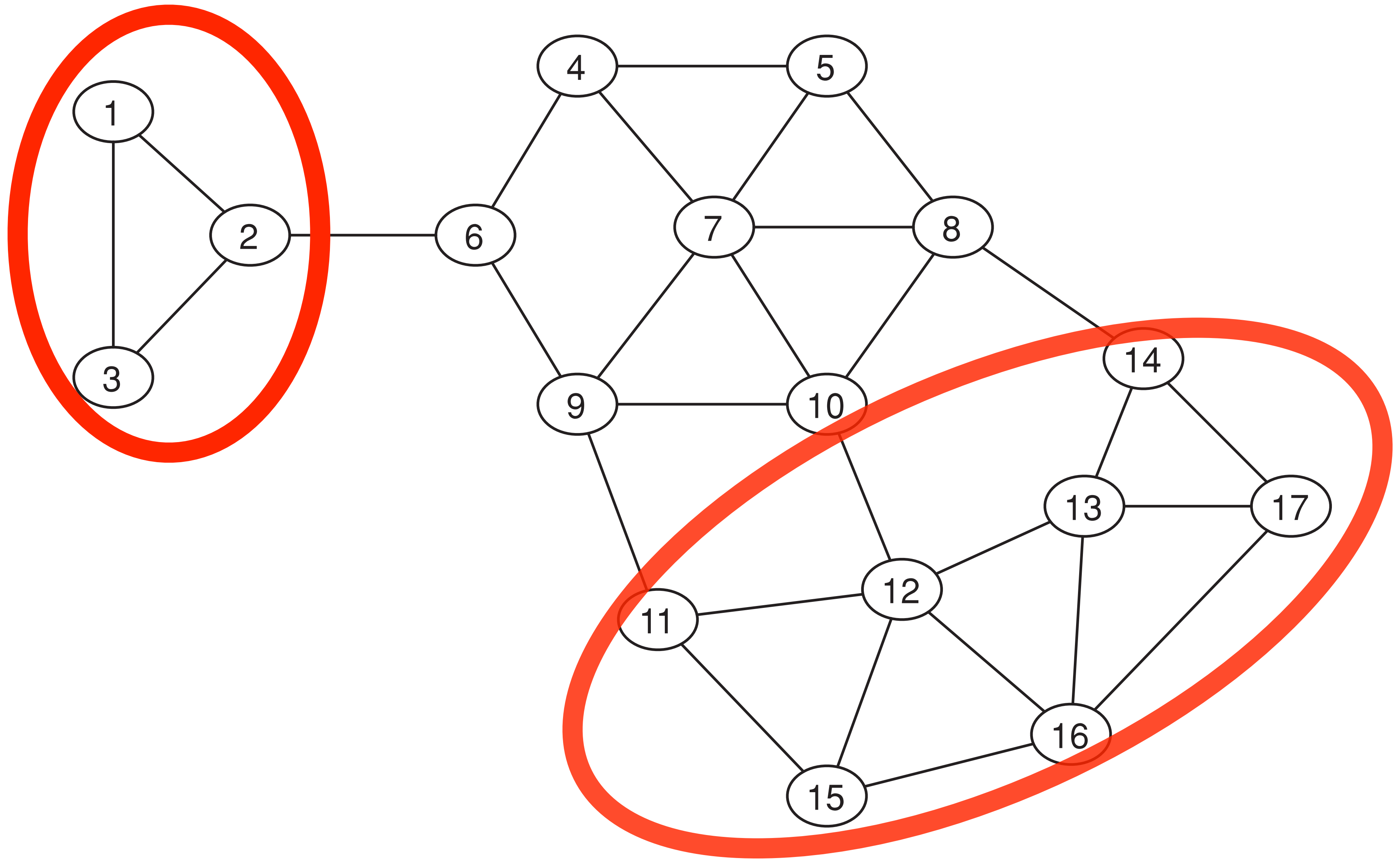
Cluster and cascades



Cluster with high density d (w.o. seeds)

≈ 0.667

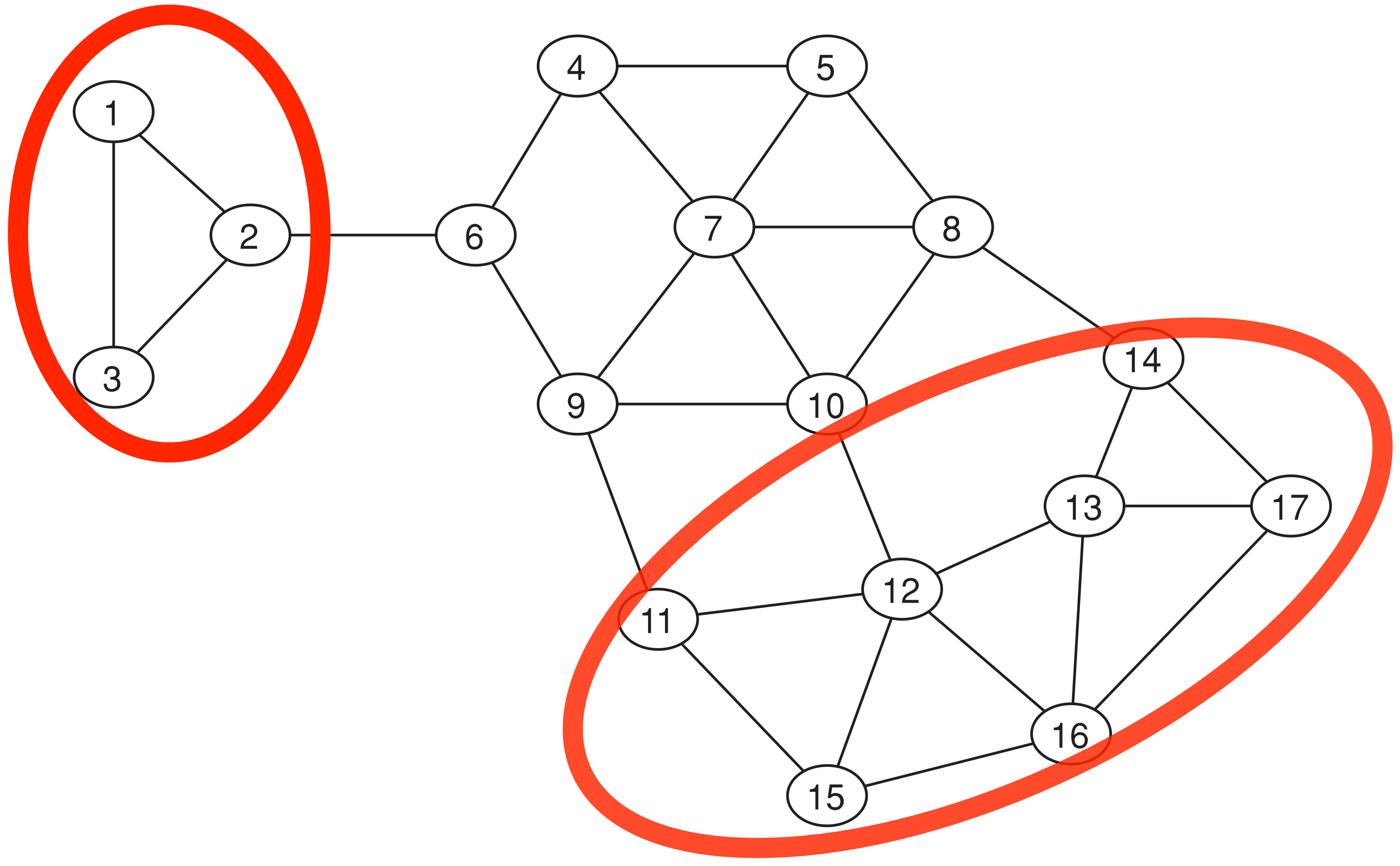
Complete cascade with high threshold q

≈ 0.34

Consider a set of initial adopters of behavior A, with a threshold of q for nodes in the remaining network to adopt behavior A.


Why don’t they “Drop the gun” (behavior A)

Cluster and cascades



Cluster with density d>1-q (w.o. seeds)

$\Downarrow$

X **Complete cascade with threshold q**

Consider a set of initial adopters of behavior A, with a threshold of q for nodes in the remaining network to adopt behavior A.

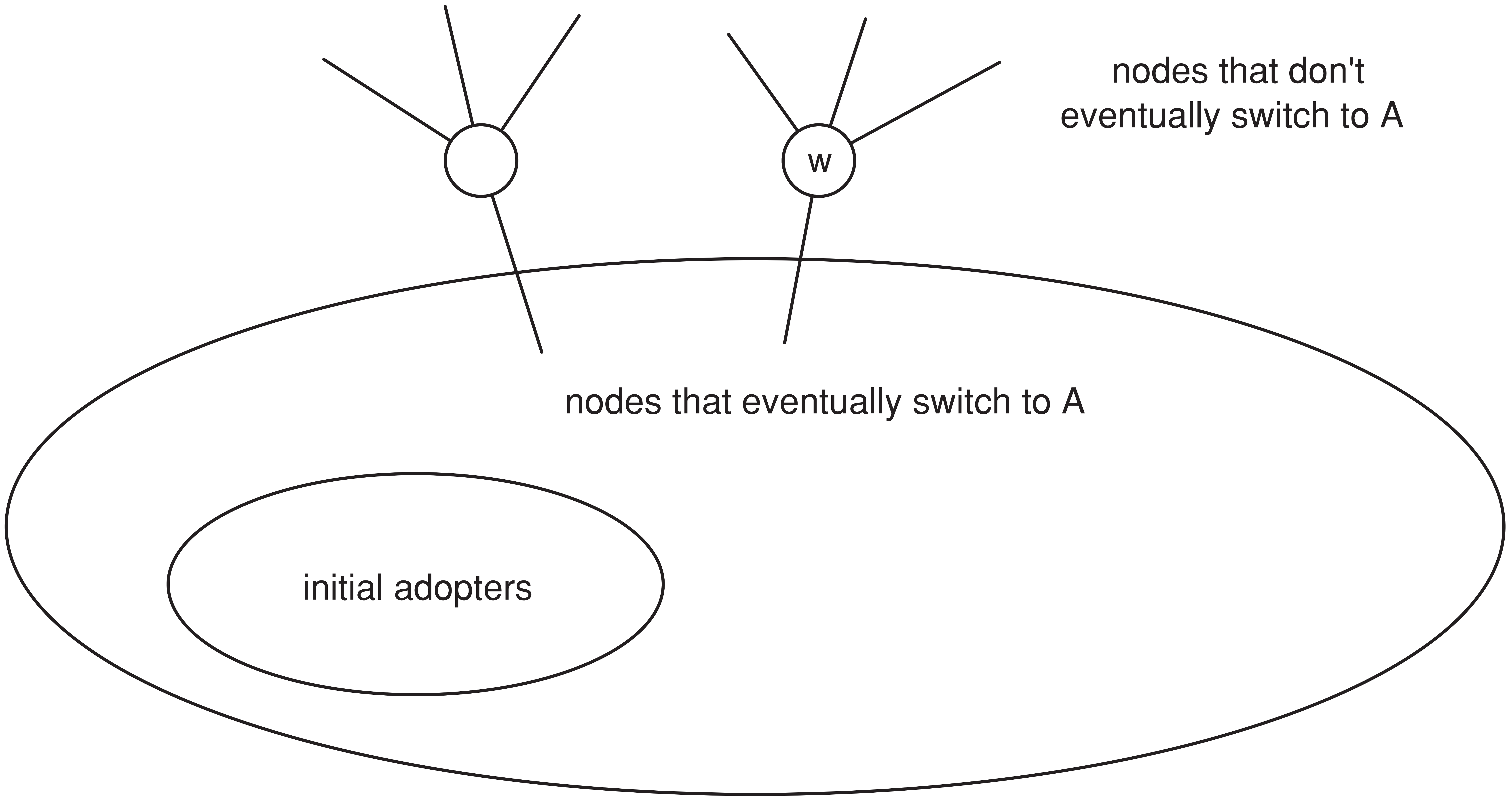
(i) If the remaining network contains a cluster of density greater than 1-q, then the set of initial adopters will not cause a complete cascade.

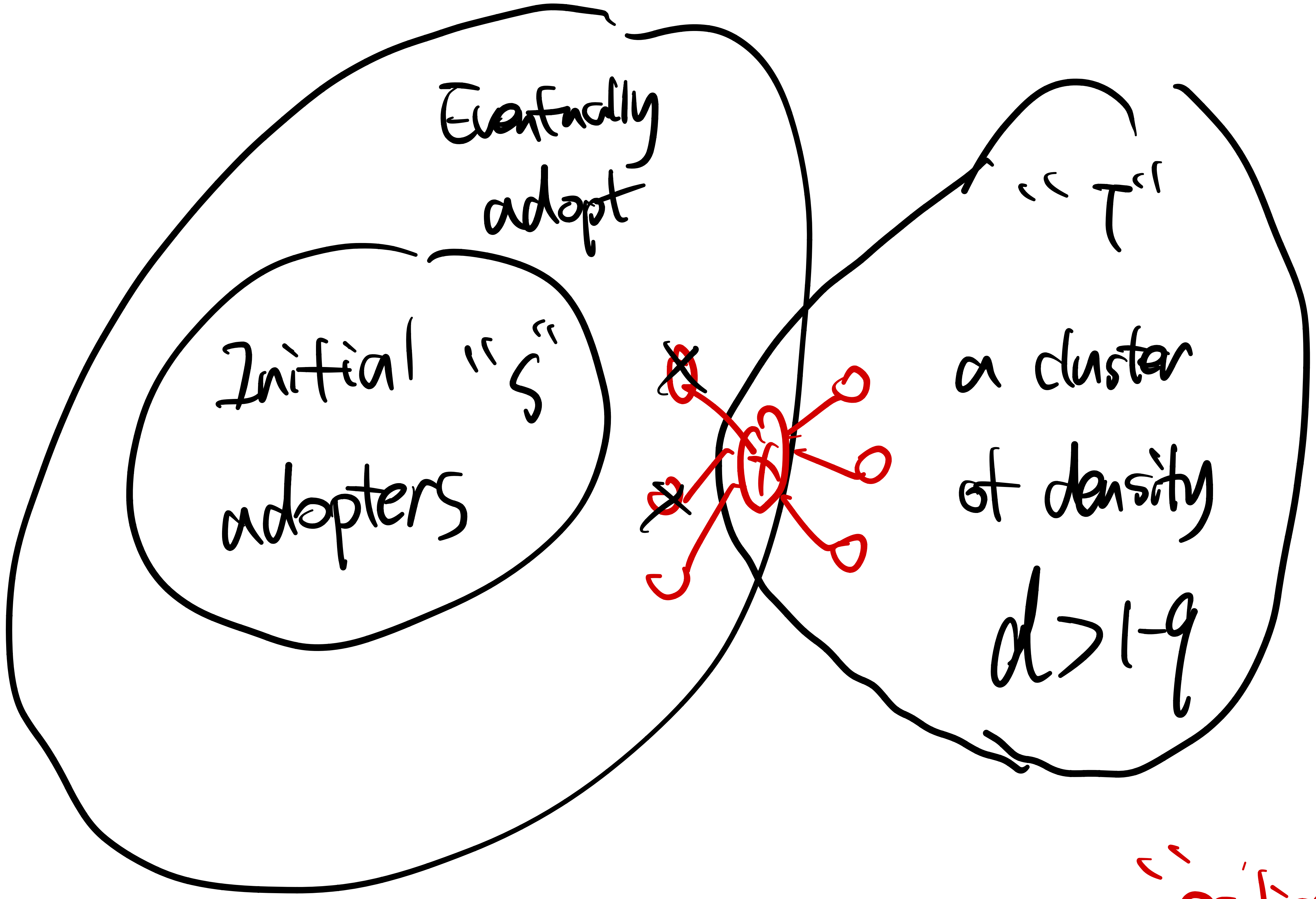
(ii) If a set of initial adopters does not cause a complete cascade with threshold q, the remaining network is a cluster of density greater than 1-q.


Why don’t they “Drop the gun” (behavior A)

Cluster and cascades

(i) If the remaining network contains a cluster of density greater than 1-q, then the set of initial adopters will not cause a complete cascade.





Claim: Nobody in T will eventually adopt

Prove by contradiction

Assume: if some people in T eventually adopt

Step 0: initial adopters. Nobody in T adopted

Step 1, 2, ..., n-1 Nobody in T adopted

"earliest" Step n: somebody in T adopted $\leftarrow x$

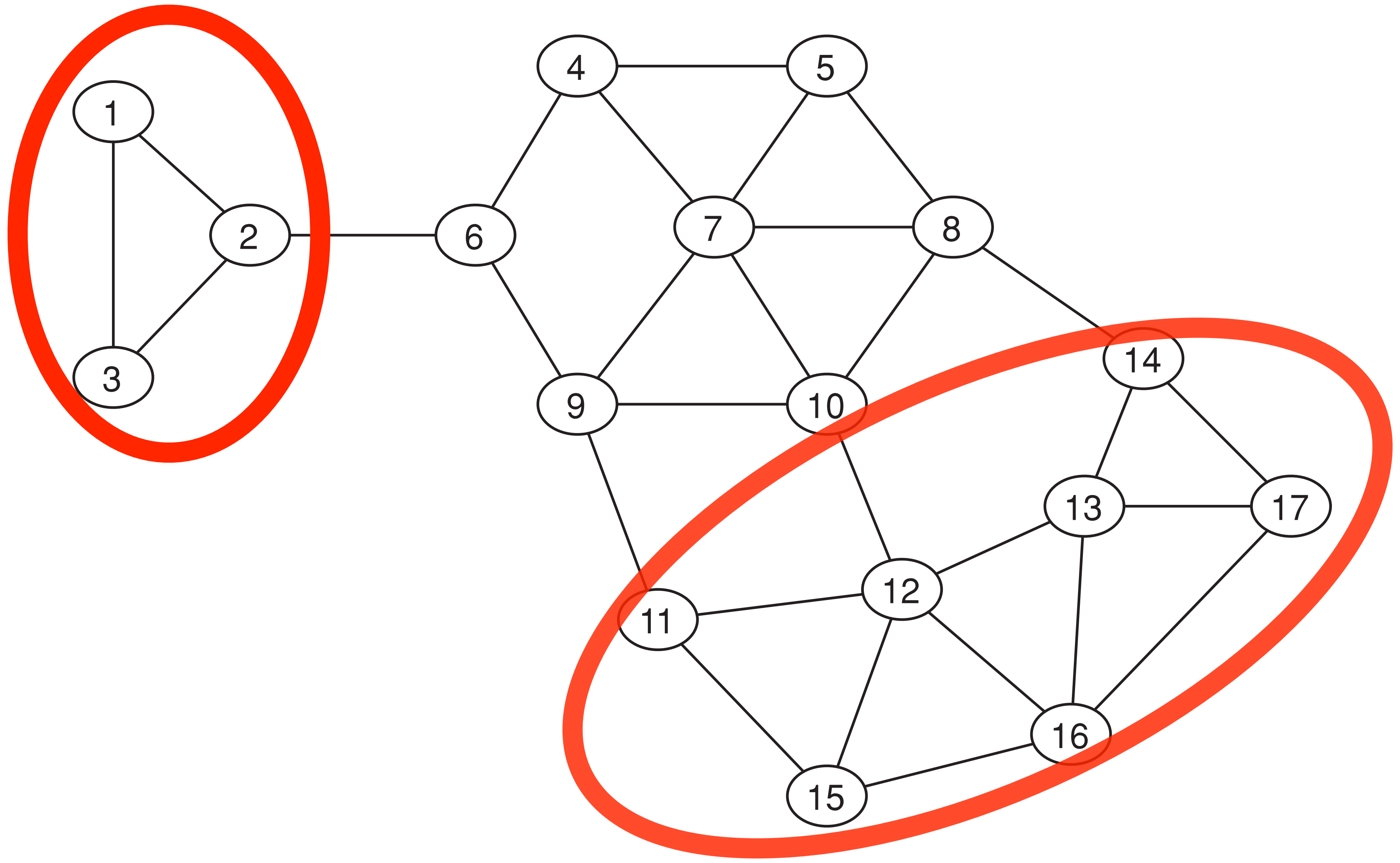
① x has at least q frac. of friends adopting before step n. $\rightarrow$ outside cluster T

$\Rightarrow$ x has at least q frac. of friends outside cluster T

② x has at least d frac of friend in cluster T

But we know $d > 1-q$

Cluster and cascades



Cluster with density d>1-q (w.o. seeds)

Complete cascade with threshold q

Consider a set of initial adopters of behavior A, with a threshold of q for nodes in the remaining network to adopt behavior A.

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Cluster and cascades

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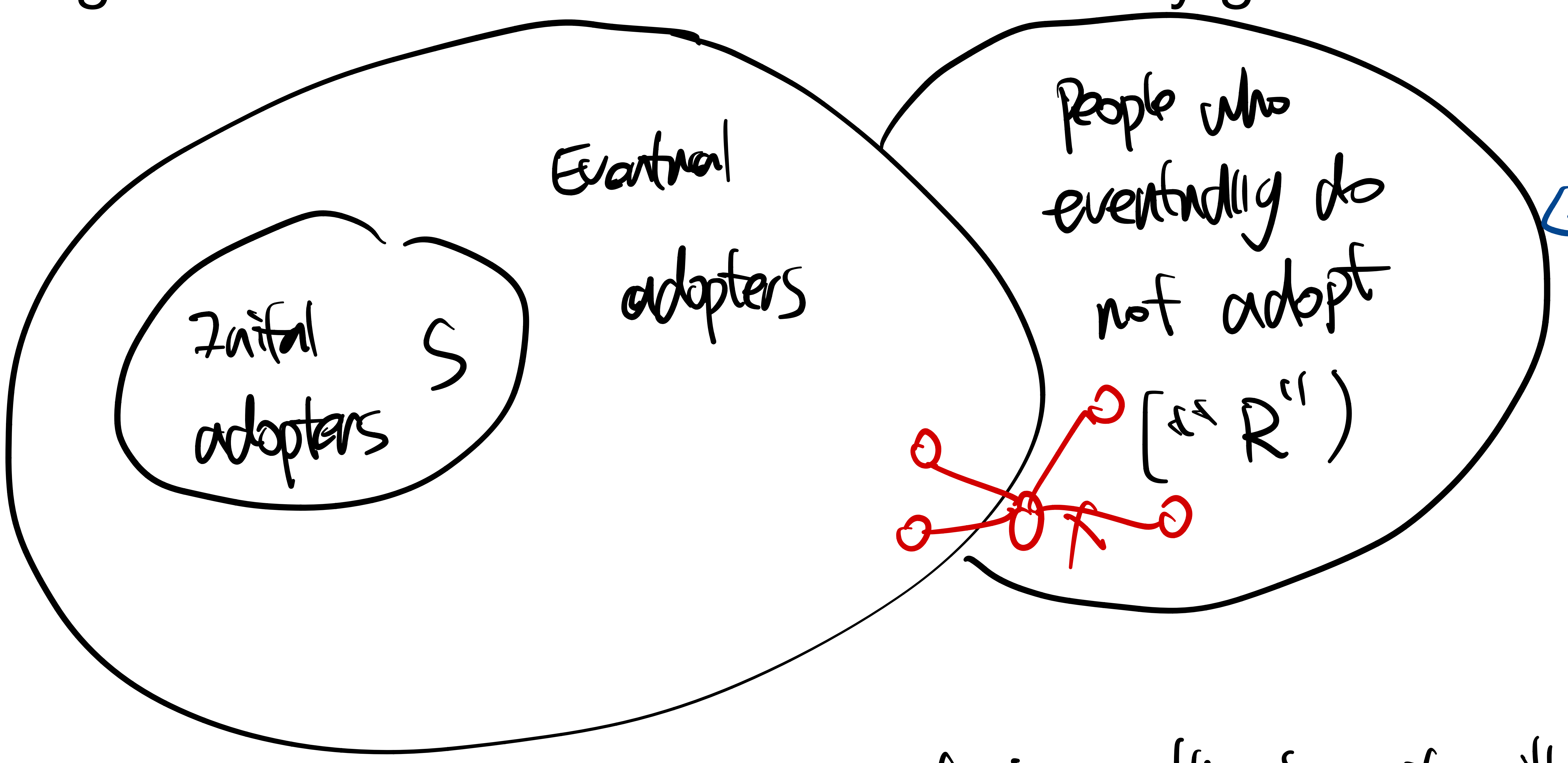
Claim: "R" is a cluster of density > 1-q

⇔ for any node $x \in R$, frac. of x's friends in R is $> 1-q$

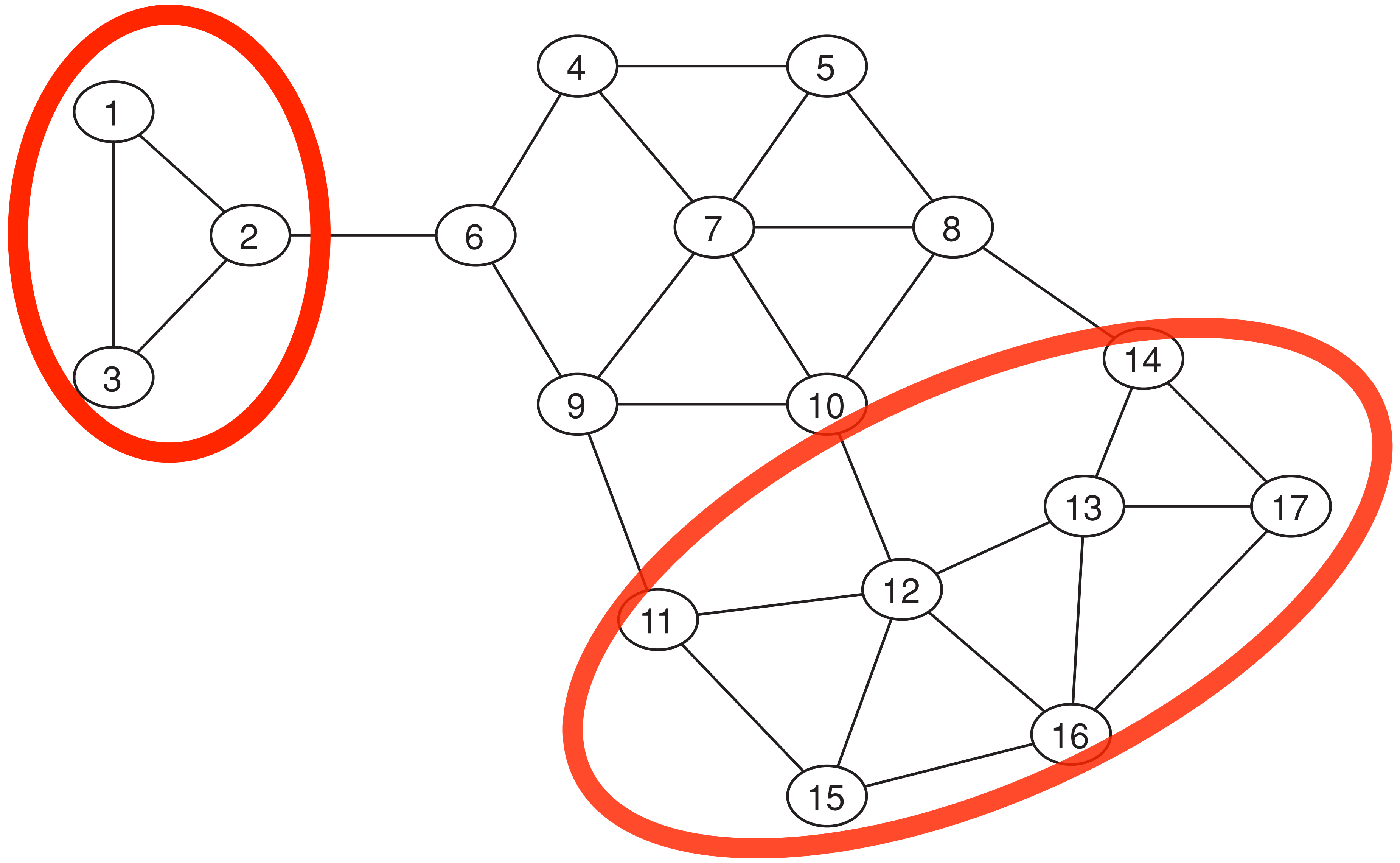


frac of x's friends outside R is $< q$ (since otherwise x will also adopt $\Rightarrow x \notin R$)

⇕

inside R $> 1-q$

Cluster and cascades



Cluster with density d>1-q (w.o. seeds)

Complete cascade with threshold q

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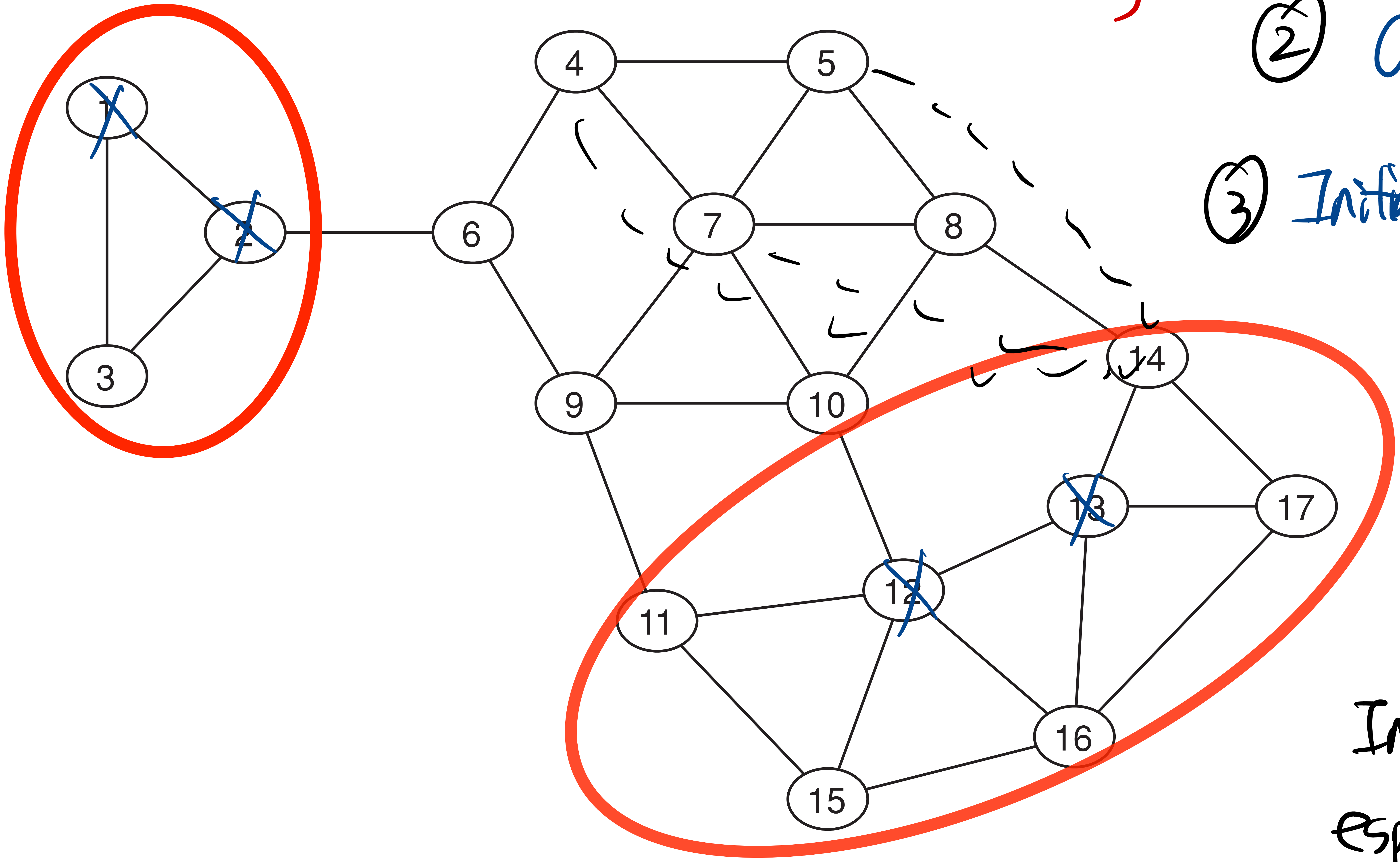
Diffusion of innovation: Implications

$q = 34\%$

$d = \frac{2}{3}$



① q: threshold   Lower my $q \downarrow$

② d: density of cluster   lower $d \downarrow$

③ Initial adopters.

2. introduce more links across cluster boundary

Increase the set of "seeds" especially target at someone in the cluster