Using the cumulative distribution (continuous RVs)

Recall that the cumulative distribution, F_X of a continuous random variable X with density f_X is the function

$$s \to F_X(s) = P(X \leq s) = \int_{-\infty}^{s} f_X(x)dx.$$

It is a very convenient tool to deal with continuous random variables, often easier to handle than the density itself. If we have the cumulative distribution, we can typically recover the density as $f_X(x) = F'_X(x)$ (this requires that f_X be continuous a x but discontinuities can be ignored in most examples). READ Fact 3.13 in the book.

Examples:

- If X is Exp(λ), $f_X(x) = \lambda e^{-\lambda x}\mathbf{1}_{(0,+\infty)}(x)$ and $F_X(x) = (1 - e^{-\lambda x})\mathbf{1}_{(0,+\infty)}(x).$ There is a discontinuity at 0 but we do not care about the value of f_X at one particular single point.
- Suppose X is Exp(λ) and $Y = \ln(X)$. This is well defined because $X > 0$. The cumulative function for Y is, for $s > 0$,

  $$F_Y(s) = P(Y \leq s) = P(\ln X \leq s) = P(X \leq e^s) = 1 - e^{-\lambda e^s}.$$

  Taking derivative for $y > 0$ gives $f_Y(y) = \lambda e^y e^{-\lambda e^y}$. Of course, we know that the density f_Y is 0 on the entire negative semi-axis.

Question: What is the best way to see that the function $s \to \lambda e^s e^{-\lambda e^s}$ is integrable on $(0, +\infty)$?