(1a) (4 points) Bidder 2 will win and will pay 5. The explanations is: Their dominant strategies are to bid their values and in a a second price auction the highest bidder wins and pays the second highest bid. The highest bidder will be bidder 2 and the second highest bid will be 5.

(1b) (4 points) Bidder 1 should bid 4.5. The explanation is that by bidding 4.5 bidder 1 is effectively offering to pay up to 4.5+0.5=5 which is his true value, and we know that bidding truthfully is a dominant strategy in second price auctions.

(2.a)

The pure Nash equilibria are (U, L) and (D, R). (U, L) is a Nash equilibrium because U is the best response for A when B is playing L (otherwise A would get 3 rather than 5) and L is the best response for B when A is playing U (otherwise B would get 2 rather than 5). (D, R) is a Nash equilibrium because D is the best response for A when B is playing R (otherwise A would get 2 rather than 4) and R is the best response for B when A is playing D (otherwise B would get 1 rather than 4). (D, L) and (U, R) are not Nash because both players are willing to change their actions in both of them.

(2.b)

To find the mixed Nash equilibria, let p be the probability that A will play U and let q be the probability that B will play L. We get p by solving $5p + 1(1-p) = 2p + 4(1-p)$ where we get $p = 0.5$ and get q by solving $5q + 2(1-q) = 3q + 4(1-q)$ where we get $q = 0.5$. So $(p = 0.5, q = 0.5)$ is the mixed Nash equilibrium of the game.

(3.a) See figure ignoring the dashed line.

(3.b) In an equilibrium both routes must be used as otherwise the unused route is cheaper. $x/100 + 1 = y/100 + 2$ and $x + y = 400$, which solves to $x = 250$ and $y = 150$

(3.c) The new route takes 3 hours (2+0+1), which is less than the travel time if only routes I and II are used (that time is 3.5 hours). So the new route must be used in an equilibrium and that means that travel time on all used routes must be 3. So we solve $x/100 + 1 = y/100 + 2 = 3$ and which solves to $x = 200$, $y = 100$, and the other travelers take the new route giving $z = 100$.

(3.d) travel time for (b) was 3.5 hours. So as long as the segment C to D is half an hour or longer, the new option does not change the Nash equilibrium in (b).

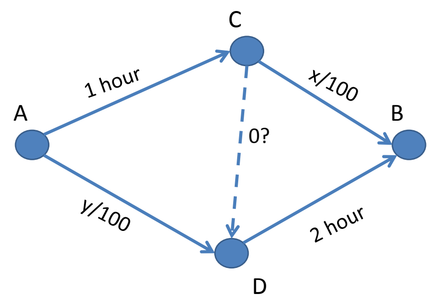

Figure 1: Picture for problem 3

(4)

With A and B priced at 2, z and x prefer C unless C has a price of at least 2 (if they both want C the market doesn't clear). With C priced at 2, the market clears (x gets B, y gets A and z gets C). This is the lowest price for C. The highest C's price can get is limited by making sure z still wants C. If C is priced at 7 the above is still market clearing, but for a price above 7, y and z both prefer A.

(5.a)

The answer is 'Yes', i.e. strong triadic closure is satisfied. To prove this, consider a node A who has strong ties with two people B and C. Then two cases are possible:

1. Both B and C are in the same grade, and so there is a strong tie between B and C,

   or

2. B and C are not in the same grade, and so at least one of the pairs (A,B) or (A,C) must be a sibling pair. Without loss of generality, suppose (A,C) is a sibling pair. Following the fact that there are no other siblings, one can deduce that A and B are from the same class. Moreover, either A is a 7th grader (and so B is also a 7th grader) and C is a 8th grader, or A is 8th grader (and so B is also a 8th grader) and C is a 7th grader. Hence B and C are either in grades 7 and 8, grades or 8 and 7 respectively, and in either cases they have weak ties.

(5.b)

The same line of thoughts works for the case when siblings are from grades 6 and 7. If (A,B), and (A,C) have strong ties then either all are in the same grade, and hence B and C have strong tie; or else one is the sibling pair and hence B and C are in grades 6 and 7 and have weak ties.

However, triadic closure fails for siblings in grades 6 and 8. If (A,B) are siblings and (A,C) are from the same class then B and C will be in grades 6 and 8, and there will be no edge between them.

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(6)

The network is not structurally balanced: for example, a village at the point (2,2) is allies with the villages at (0,2) and (4,2), but the villages at (0,2) and (4,2) are enemies.

(7.a)

B, C, D are in a strong position as they all have a very weak neighbor. A, E, F are weak as they only have one neighbor each and that neighbor is strong..

(7.b)

The outcome is *Not* consistent with Nash bargaining. In fact, A has no outside option, while B has an outside option of $12 - 9 = 3$. So the surplus is $s = 9$. As a result, 4.5 and 7.5 would be Nash bargaining solution, and not 3 and 9.

(7.c)

Yes, the outcome is stable and consistent with Nash bargaining (its balanced). In fact, A has no outside option, while B has an outside option of $12 - 8 = 4$, so the surplus is $s = 8$. Following this, $0 + 4 = 4$ and $4 + 4 = 8$ is the Nash bargaining solution.

(7.d)

B gets very strong having 3 very weak neighbors. A, E and F are weaker all being dependent on B. Moreover, C and D get a bit weaker as B got stronger. The balanced outcome has B getting 12, C and D getting 6 each and A,E and F each getting 0.

(8.a)

No, it is not a dominant strategy to bid truthfully. This is a first price auction; not a second price one. Bidding truthfully guarantees a 0 payoff; either you don't win or you do win but pay your full value. There is not enough information given to determine how you should bid. There is no dominant strategy; your optimal strategy depends on what others do.

(8.b)

Yes. This is now a second price auction in which truthful bidding is a dominant strategy.

(8.c)

No. The winning bidder pays an amount that depends on the amount bid and that is sufficient to make truthful bidding not a dominant strategy. In a bit more detail–if you win with a bid of f you pay $(1 - d)f$ so a slightly lower bid would reduce your probability of winning but it would also reduce the price you pay contingent on winning. A slightly higher bid would increase your probability of winning. Trading off these opposing factors would determine the optimal bid, but without knowing more about the environment it's impossible to determine an equilibrium. There is no dominant strategy; your optimal strategy depends on what others do.