Important Formulas in Finance

For use during Prelim II

1 General Comments

Unless we state otherwise, all cash flows occur at the end of a period and all interest is compound interest. Typically $r > 0$, but interest rates could also be negative or zero. Periods could be days, weeks, months, years. We ignore calendar idiosyncrasies related to months and years of variable length.

Not all formulas, not all versions of formulas, and not all interpretations of formulas are given. This is just an aide-mémoire, not a full reference. Some observations may be beyond the scope of the course (e.g., root finding methods for determining the implied interest rate for an annuity).

2 Time-Value of Money

PV = present value at time 0; FV = future value at time t; r = interest rate; t = number of periods.

- One period case: $FV = PV \cdot (1 + r)$.
- Multi-period case: $FV = PV \cdot (1 + r)^t$; $PV = \frac{FV}{(1+r)^t}$; $t = \frac{\ln \frac{FV}{PV}}{\ln(1+r)}$; $r = \left(\frac{FV}{PV}\right)^{\frac{1}{t}} - 1$.
- Doubling time of an investment: t$=\frac{\ln 2}{\ln(1+r)} \approx \frac{\ln 2}{r}$ (if r is small). If r is expressed in percents, then $t \approx \frac{72}{r}$ periods.
- Future value factor $= (1 + r)^t$; present value (or discount) factor $= \frac{1}{(1+r)^t}$.

3 Annuities

PV = present value of an annuity; FV = future value of an annuity; C = the constant payment of an ordinary annuity; r = interest rate; t = number of periods; g = growth rate of payouts.

- $PV = C \cdot \frac{1-\left(\frac{1}{1+r}\right)^t}{r}$; $C = \frac{r}{1-\left(\frac{1}{1+r}\right)^t} \cdot PV$; $t = -\frac{\ln\left(1-\frac{PV}{C}\cdot r\right)}{\ln(1+r)}$. There is no general formula for determining r - you can use a calculator, trial and error, or a systematic root finding method.
- Present value interest factor for annuities: $PFIVA(r, t) = \frac{1-\left(\frac{1}{1+r}\right)^t}{r}$, thus $PV = C \cdot PFIVA(r, t)$.
- $FV = C \cdot \frac{(1+r)^t-1}{r}$; $C = \frac{r}{(1+r)^t-1} \cdot FV$; $t = \frac{\ln\left(1+\frac{FV}{C}\cdot r\right)}{\ln(1+r)}$. There is no general formula for determining r.
- Annuity FV factor $= \frac{(1+r)^t-1}{r}$, thus $FV = C \cdot (Annuity\, FV\, factor)$.
- Perpetuity: $PV = \frac{C}{r}$.
- Annuity due: $PV = C \cdot \frac{1+r}{r} \cdot \left[1 - \frac{1}{(1+r)^t}\right]$.
- Growth annuity: $PV = C \cdot \frac{1-\left(\frac{1+g}{1+r}\right)^t}{r-g}$, if $r > g$. C is the payment at the end of period 1 (i.e., the first payment).
- Growth perpetuity: $PV{=}\frac{C}{r-g}$, if $r > g$.

4 Compounding Conventions; EAR

- If the nominal annual interest rate is r, and it compounds m times per year, then $FV = PV \cdot \left(1 + \frac{r}{m}\right)^m$.
- The effective annual rate is $EAR = \left(1 + \frac{r}{m}\right)^m - 1$.
- In the limit, when m tends to infinity, $FV = PV \cdot e^r$, $EAR = e^r - 1$.

5 Bonds

P = face value; C = yearly coupon (in dollars); t or T = maturity in years; y = yield; B = price. Note that the formulas depend on per-compounding-period interest rates and coupons. Formulas below are for semi-annual compounding; they may need to be adjusted.

- Bond price: $B = \frac{C}{2} \cdot \frac{1-\left(\frac{1}{1+\frac{y}{2}}\right)^{2t}}{\frac{y}{2}} + P \cdot \left(\frac{1}{1+\frac{y}{2}}\right)^{2t}$.
- Bond yields can be computed using the interval bisection method.

6 Inflation

R = nominal interest rate; h = inflation rate; r = real interest rate.

- $1 + R = (1 + r) \cdot (1 + h)$; $r = \frac{R-h}{1+h} \approx R - h$.

7 Stock Valuation

P_0 = time-0 price of the stock; D_i= dividend that will be paid at the end of period i; R = per-period interest rate; g, g_1 , g_2 = constant, per-period dividend growth rates.

- General formula, with dividends up to time t: $P_0 = \sum_{i=1}^{t} \frac{D_i}{(1+R)^i} + \frac{P_t}{(1+R)^t}$.
- $P_0 = \sum_{i=1}^{\infty} \frac{D_i}{(1+R)^i}$. In reality, there are no infinite streams of dividends.
- Zero growth dividends: $P_0 = \frac{D}{R}$.
- Dividend growth model: $P_t = \frac{D_{t+1}}{R-g}$, if $R > g$.
- Non-constant growth: $P_0 = \sum_{i=1}^{t} \frac{D_i}{(1+R)^i} + \frac{P_t}{(1+R)^t}$; $P_t = \frac{D_t \cdot (1+g)}{R-g}$ **if** we assume constant growth after t.
- Two-stage growth: $P_0 = \frac{D_1}{R-g_1} \cdot \left[1 - \left(\frac{1+g_1}{1+R}\right)^t\right] + \frac{P_t}{(1+R)^t}$; $P_t = \frac{D_{t+1}}{R-g_2} = \frac{D_0 \cdot (1+g_1)^t \cdot (1+g_2)}{R-g_2}$.
- Required return: $R = \frac{D_1}{P_0} + g$.
- Multiples: P_t = (benchmark PE ratio) $\cdot EPS_t$.

8 NPV and IRR

- NPV = sum of discounted (project) cash flows. The discount (demanded, required) rate must be specified.
- NPV = 0 for discount rate r $\iff IRR = r$. The IRR may not be unique.
- NPV investment criterion: undertake the project if $NPV > 0$.
- Given projects A and B, a crossover point occurs when $NPV_A = NPV_B$ for a given discount rate. The discount rate can be determined by subtracting the cash flows of project B from those of project A, and computing the IRR of the resulting cash flows.
- IRR investment criterion: undertake the project if required return $< IRR$. The required return does not have to be made explicit, as long as one can argue that the IRR is high enough for the inequality to hold. Exception: if we have financing-type cash flows; then we undertake the project if required return $> IRR$.
- When comparing exclusive projects, NPV and IRR may disagree.
- IRR values can be computed using the interval bisection method.

9 Project Cash Flows

- Earnings before interest and taxes = EBIT = sales - costs - depreciation.
- Net income = NI = EBIT - taxes.
- Taxes = EBIT × tax rate. Tax rates may be constant, or they may depend on the magnitude of the EBIT. Taxes may apply to negative EBIT, case in which they are negative (a benefit).
- Operating cash flow = OCF = EBIT + depreciation − taxes.
  - OCF = net income + depreciation;
  - OCF = sales - costs - taxes;
  - OCF = (sales - costs) × $(1 - T_c)$ + depreciation × T_c, where T_c is the tax rate.
- Project cash flow = OCF − change in net working capital − project capital spending.

10 Costs and Break-Even Points

- TC = total cost; VC = total variable cost; v = variable cost per unit; FC = total fixed cost; AC = average cost; MC = marginal cost; q = production level (number of widgets produced, units of service delivered, etc); D = depreciation. Writing a quantity as a function of q, e.g., $VC(q)$, indicates that the respective quantity depends on the production level.
- $VC(q) = q \times v$.
- $TC(q) = FC + VC(q)$.
- $AC(q) = \frac{TC(q)}{q} = \frac{FC}{q} + v$.
- $MC = v < AC(q)$.

- Production levels derived from break-even calculations must typically be rounded up. Break-even calculations can be performed per-year, or over multiple years (project time horizon). Unless explicitly told otherwise, break-even calculation ignore taxes. Then $OCF = (P - v) \times q - FC$.
  - Production level that corresponds to a given OCF: $q = \frac{FC+OCF}{P-v}$.
  - Accounting break-even: OCF just equals depreciation; $q = \frac{FC+D}{P-v}$.
  - Cash break-even: OCF covers fixed costs, but not depreciation; $q = \frac{FC}{P-v}$.
  - Financial break-even is reached when the NPV of the operating cash flows is equal to the NPV of investments.