Probabilistic model of user behavior:

- *The user encounters a list of ranked items in order* $I_1, I_2, \ldots, I_n$.
- *When they encounter item* I_j *in the list:*
  - *They find* I_j *interesting with probability* p_j*, independently of their decisions about previous items.*
  - *If they find* I_j *interesting, they consume it and leave the platform.*
  - *If they don't find* I_j *interesting, they leave the platform anyway with probability* q *(due to their impatience).*
  - *Otherwise they move on to consider item* I_{j+1}.

For simplicity, we'll focus here on the case in which there are only two items being ranked, since that already highlights the key behaviors of the model, and it is possible to extend the calculations we describe to handle any larger number of items.

Thus, let's suppose that n users arrive at the site and begin encountering the content in ranked order. We analyze the decisions that these users make as they encounter the ranked list, by asking and answering a sequence of questions. The calculations used to answer these questions are described below and also summarized in Figure 1.

- First, what is the expected number of users who consume item I_1? For each user, I_1 is the the first item they encounter in the ranked list, and they consume it with probability p_1. The expected number of users who consume I_1 is simply n (the number of users) times this probability, and hence np_1.

- What is the expected number of users who reject I_1 and then leave the site without encountering any other items? With probability $1 - p_1$ a user chooses not to consume I_1 (i.e. they reject I_1), and then with probabiity q they choose to leave the site without encountering any other items. These two events are independent, so the probability that both events happen is equal to the product of their probabilities. So the probability that a user leaves the site immediately after rejecting I_1 is

  $$(1 - p_1)q \tag{1}$$

  and the expected number of users who do this is again n times this probability, or $n(1 - p_1)q$.

- What is the expected number of users who consume item I_2? Three things have to happen in succession for a user to consume I_2. First, they have to decide not to consume I_1; this happens with probability $1 - p_1$. Second, they have to decide not to

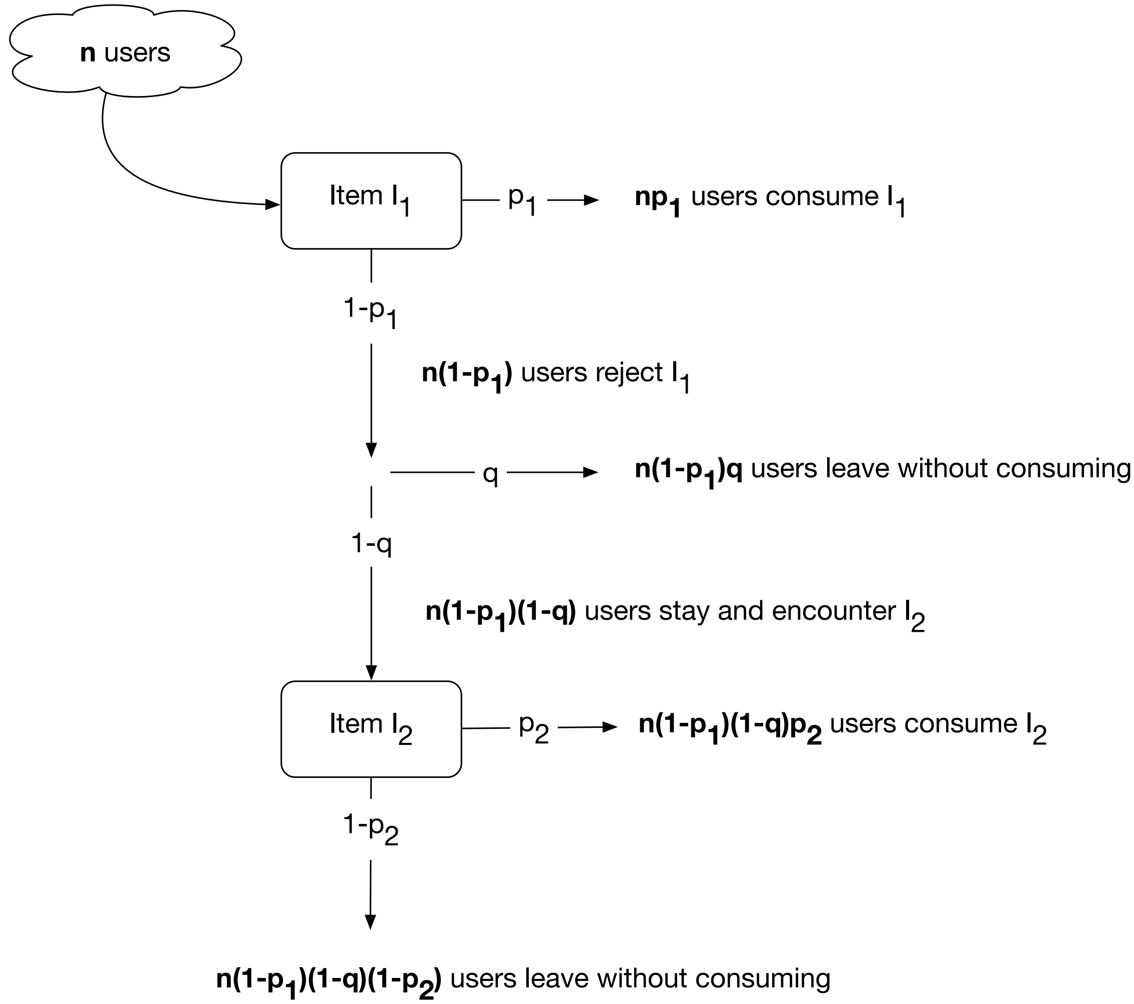

Figure 1: .

leave the site; since a user leaves the site with probability q after failing to consume something, they stay on the site with the complementary probability $1-q$. Third, the user has to decide to consume item I_2 after encountering it, which has probability p_2.

These are all independent events, so as before we multiply their probabilities together to get the probability that all three events happen; this is equal to

$$(1-p_1)(1-q)p_2. \tag{2}$$

This is the probability a user consumes I_2, so the expected number of users who consume I_2 is $n(1-p_1)(1-q)p_2$.

- Finally, what is the expected number of users who encounter both items but leave without consuming either of them? Here too, three things must happen in succession in order for this to occur: a user has to reject I_1 (with probability $1-p_1$), then choose

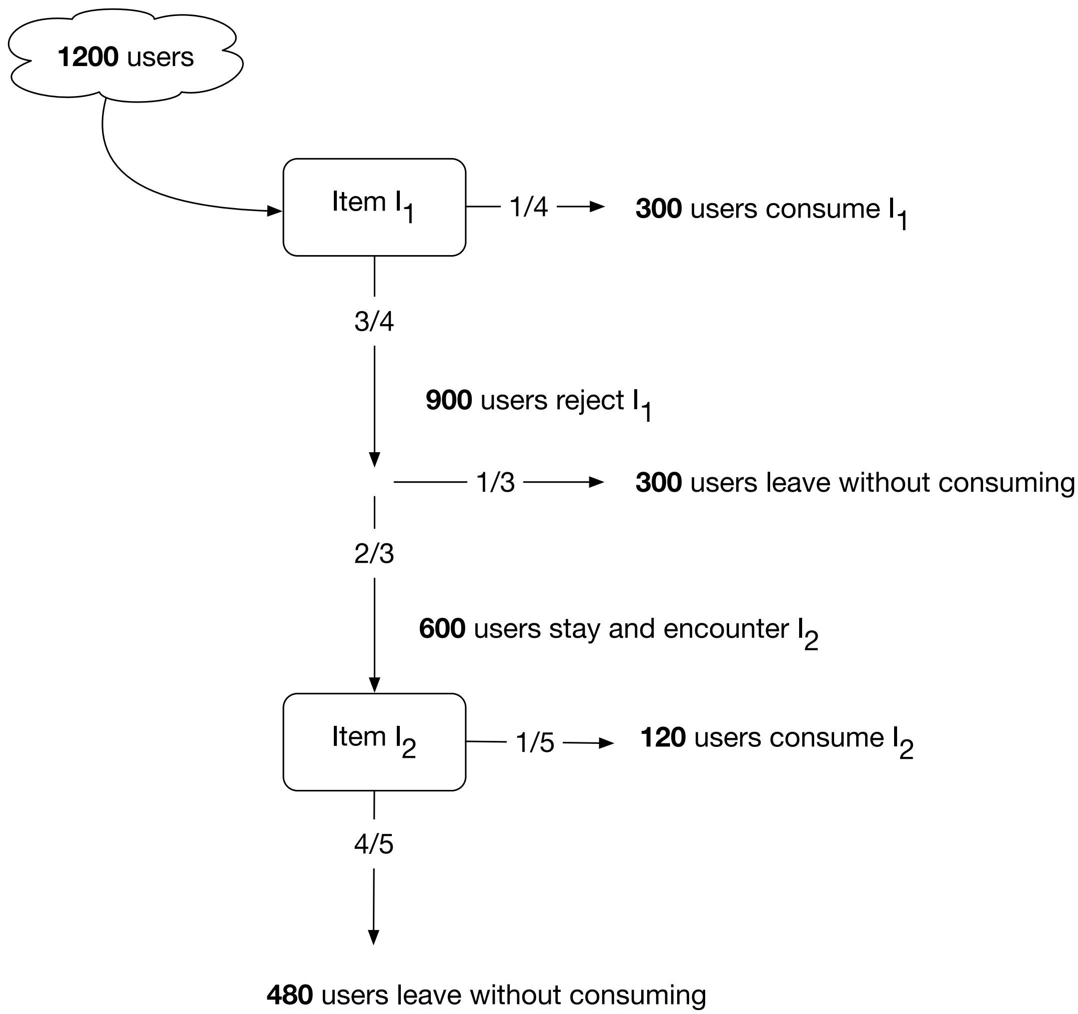

Figure 2: .

not to leave (with probability $1-q$), and then reject I_2 (with probability $1-p_2$). Here too, these are all independent events, and so the event we are asking about — that all three occur — has a probability equal to their product,

$$(1-p_1)(1-q)(1-p_2). \tag{3}$$

Hence the expected number of users who do this is $n(1-p_1)(1-q)(1-p_2)$.

Working out a specific example It can be useful to think about these calculations in the context of a specific example, as a way of viewing them more concretely. For our example, let's continue to think about the case in which we have two items; suppose that $p_1 = 1/4$, $p_2 = 1/5$, and $q = 1/3$. In particular, this means that item I_1 is more appealing to users, on its own, than item I_2: if a user is presented with item I_1, the probability that they consume it is $1/4$, while if the user is presented with item I_2, the probability they consume it is only $1/5$.

Now, let's suppose that 1200 users arrive at the site. Figure 2 shows the trajectory that they follow through the ranked list of items. We'll talk about the number of users who experience different outcomes, and we'll refer to these numbers as before *in expectation*, since they are expected values over the probabilistic outcomes.

First, all 1200 users encounter item I_1 at the top of the ranking, and in expectation, $1200 \cdot 1/4 = 300$ of them consume item I_1. Of the 900 users in expectation who reject item I_1, $900 \cdot 1/3 = 300$ in expectation leave the site just after. Of the remaining 600 users in expectation who go on to encounter item I_2, $600 \cdot 1/5 = 120$ users in expectation consume I_2. Finally, the remaining 480 users in expectation leave having encountered both items but consuming neither.

Implications of the Ranking Model

Having worked out how our model operates at a mathematical level, let's consider a few conclusions about rankings more generally that we can suggest by interpreting the model's behavior. We'll highlight two phenomena in particular, and we'll discuss them at two parallel levels — both as consequences of the model, and general observations about rankings in practice. In keeping with our earlier themes about the role of models, we'll argue that the simple way in which they emerge from the model points to something about their robustness in practice across different settings and specifics.

Rankings in order of appeal can maximize consumption Recall our argument from earlier that there's something natural about rankings, in the following sense. Human attention is linear — we can't pay attention to everything simultaneously and so must focus on specific things at any given time — and so if you have multiple items to display to your users, they're going consider them in *some* order. A ranking is simply a way of trying to shape the order in which they look at these things, rather than allowing this order to be determined accidentally or haphazardly.

Given this, here's an argument for the order that we chose in the previous section — ranking I_1 ahead of I_2 rather than vice versa. Suppose we are hoping that users will consume at least one of the items; presumably that's why we're displaying them. If we look at Figure 1, we see that a user can *fail* to consume an item in one of two ways: either by leaving after rejecting I_1, and or by encountering both items and rejecting both of them. As we determined in the previous section, the expected number of users who fail to consume an item in the first way is $n(1 - p_1)q$ and the expected number of who fail to consume an item in the second way is $n(1 - p_1)(1 - q)(1 - p_2)$, for total of

$$n(1 - p_1)q + n(1 - p_1)(1 - q)(1 - p_2). \tag{4}$$

Now, suppose we decided to rank I_2 ahead of I_1 instead. In this case, we can use exactly the same reasoning and calculations, with the one difference that p_2 would play the role of p_1 in the calculations, and vice versa. That is, the roles of p_1 and p_2 in all the formulas are simply swapped, and so the expected number of users who consume nothing in this ranking with I_2 first is

$$n(1-p_2)q + n(1-p_2)(1-q)(1-p_1). \tag{5}$$

Now let's compare which expectation is smaller between the expressions in (4) and (5). An interesting point is that they actually have the same second terms, so the comparison comes down to which first term is smaller. The first terms are $n(1-p_1)q$ and $n(1-p_2)q$ respectively, which share two of their factors; but since $1-p_1$ is smaller than $1-p_2$, it follows that the former expression is smaller, and therefore (4) is smaller than (5).

So what have we concluded? Since (4) is smaller than (5), there are fewer people who *don't* consume anything when we rank I_1 first, and therefore more people *do* consume something when we rank I_1 first. This shows that ranking two items in descending order of appeal, with I_1 ahead of I_2, maximizes the expected amount of consumption across the user population in our model in general.

Just to verify this in our example, we can compare the effect of ranking I_2 before I_1 against the results in Figure 2, which shows the effect of ranking I_1 first. If we rank I_2 first, then the number of users consuming each item is given by Equation (5): in expectation, $1200p_2 = 1200 \cdot {}^1\!/_5 = 240$ users consume item I_2, and

$$1200(1-p_2)(1-q)p_1 = 1200 \cdot {}^4\!/_5 \cdot {}^2\!/_3 \cdot {}^1\!/_4 = 160$$

users consume I_1. This gives a total of 400 users in expectation consuming some item, in contrast to the expected 420 who consume an item if I_1 is ranked first.

Rankings can be self-fulfilling, even when they're wrong Let's go back to a question we were considering earlier — the contrast between the ranking that puts I_1 before I_2 and the ranking that puts I_2 before I_1. If we study this in the context of the example from Figure 2, then as we've observed earlier, ranking I_1 before I_2 results in 300 users consuming I_1 and 120 users consuming I_2 in expectation, whereas ranking I_2 before I_1 results in 240 users consuming I_2 and 180 users consuming I_1.

There's something striking about the contrast in this example, which is that if we place the item with lower probability first, it will still get consumed at a higher rate. It's clear why this is happening in the calculations for the model: with I_2 first, it gets consumed with probability p_2 while I_1 gets consumed with probability $(1-p_2)(1-q)p_1$, and even though $p_1 > p_2$, the two leading terms $(1-p_2)(1-q)$ pull the overall product down below p_2. It's also clear why it's happening at a more intuitive level: although I_1 is more appealing to users in isolation, the only users who get to see it are those who have rejected I_2 and then decided to remain on the platform. And this latter explanation corresponds to the way the effect manifests itself in practice; it's entirely possible for a more appealing item to get buried below worse items and therefore not get much exposure.

But what this means is that rankings can be *self-fulfilling*, in the following sense. Suppose that the people operating the platform believed that I_2 was the more appealing of the two items, and for that reason they decided to show it first. Then they'd observe I_2 gets 240 clicks for every 180 clicks by I_1, and they weren't attuned to the possibility of position bias, then they might imagine that this click data has confirmed their belief that I_2 is indeed more appealing to users than I_1.

It's worth pausing here to notice that this effect depends on the specific numbers in the example; it would be possible to create an example that's similar but with p_2 smaller where I_2 gets less consumption even when it's placed first. For example, if we set $p_2 = 1/10$ but kept the other values the same, then even in second place I_1 would get consumed with probability

$$(1 - p_2)(1 - q)p_1 = 9/10 \cdot 2/3 \cdot 1/4 = 3/20,$$

which is higher than $p_2 = 1/10$. So with this set of numbers, if the platform operators placed item I_2 first, then it would get fewer clicks than I_1 despite its advantage in position, and it would be clear that it can't be the more appealing item.

In addition to the way in which a ranking can be self-fulfilling — giving more consumption to the first-ranked item even when it's less appealing to users on its own — the model also shows how it can be difficult to tell which item is more appealing just from the evidence of a single fixed ranking. In particular, let's go back to the numbers from the example in Figure 2, and consider the numbers there as observations that the platform operators sees when they rank I_1 ahead of I_2: 300 users consume I_1, and 120 users consume I_2.

Now, suppose instead the underlying values were $p_1 = 1/4$, $q = 2/3$, and $p_2 = 2/5$; in other words, with these different values, the users are significantly more impatient (and hence more likely to leave the site after rejecting the first item they see) but also significantly more interested in I_2. In fact, with these new values, I_2 is more appealing to users on its own than I_1 is, since $2/5 > 1/4$.

With these new values, I_1 will of course get the same number of clicks as in Figure 2, since p_1 has remained the same. But when we discover is that the expected number of users who consume I_2 with these new values is

$$n(1 - p_1)(1 - q)p_2 = 1200 \cdot 3/4 \cdot 1/3 \cdot 2/5 = 120, \tag{6}$$

just as we had with the old values. The increased impatience of users has, numerically, exactly canceled out the increased appeal of I_2.

This result within our model surfaces a risk that applies in practice more generally: that simply observing the behavior of users on a platform might not be sufficient to tell us everything we need to know about the items they are consuming. In particular, there can be multiple *indistinguishable* scenarios that produce the same observable behavior, but where we might want to pursue different courses of action if only we knew which scenario we were in. In the case of our current example, we cannot whether or not I_2's lower rate of clicks is because it is genuinely less appealing than I_1, or because users are so impatient that too few of them ever encounter I_2. But if we knew which of these two cases we were in, it could tell us whether we might want to promote I_2 to the first position in the ranking.