Cost of Capital

AEM 2241 - Finance

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Outline

Preliminaries

- Until now, we analyzed risk and return from the perspective of an investor, who, say, buys shares. After studying the SML, we now understand how the expected return of the investment relates to its systematic risk. We also understand that idiosyncratic risk is not rewarded.
- In this lecture, we consider the problem from the perspective of the firm that issues the stock. The return the investor earns is the cost of that security to the company that issued it.

Cost of Capital Equity Debt WACC To Do!

Required Return vs. Cost of Capital

- If the required return on a project (an investment) is r, then the firm must earn at least r on that investment to have a positive NPV.
- The return r investors earn compensates them for the risk they undertook by investing in the project. In other words, r is the return that compensates investors for allocating capital to the respective project.
- Under these circumstances, we call r the **cost of capital** for the risky project.
- Hypothetically, if a project had zero risk, then it would be equivalent to the risk-free asset. Correspondingly, the appropriate return for investing in it would be the risk-free rate, and the same rate would also be the project's cost of capital.

Required Return vs. Cost of Capital (2)

- If the investment is risky, but all other terms are the same as for the earlier risk-free asset, the required return must be higher than the risk-free rate.
- What, from the investor's side is the **required return** or **(appropriate) discount rate**, from the firm's perspective is the **cost of capital**. We'll use these terms interchangeably.
- **The true cost of capital depends primarily on the use of the funds, not the source of funds.** The cost of capital is thus not directly tied to the source of financing, i.e., it is not a function of how capital is raised.

Financial Policy and Cost of Capital

- Firms raise financing through a mixture of debt and equity. This mixture determines the firm's **capital structure**. Management controls the firm's capital structure (but does not have complete freedom to modify it).
- A firm's capital structure may differ from what its management considers ideal, i.e., from the so-called **target capital structure**.
- In this chapter we assume that the capital structure is determined by the ratio between debt and equity. This debt-to-equity ratio will be fixed (i.e., set by management and unchanging).
- Given that a firm will generally use both equity and debt to raise financing, it will have to pay appropriate returns to both bond and equity holders (i.e., to creditors and stockholders).
- The capital structure and the cost of debt and equity capital will determine the firm's **overall cost of capital**.

The Cost of Equity

- Like many important quantities in finance, the required return on a stock is not directly observable.
- This required return on the stock is the firm's **cost of equity**.
- We must determine the cost of equity indirectly, by estimating it using one of the two methods that we studied:
  - The constant dividend growth model;
  - The SML.

Constant Dividend Growth Model

- Using our earlier notations, we revisit the relationship that describes the constant dividend growth model:

$$P_0 = \frac{D_0 \times (1+g)}{r_E - g} = \frac{D_1}{r_E - g}.$$

- From here, we get that

$$r_E = \frac{D_1}{P_0} + g.$$

- Since r_E is the return that investors require on the stock, it can be identified with the firm's cost of equity capital.

Estimating the Growth Rate

- D_0 and P_0 can be observed in the market. D_1 can be predicted either based on company policy and statements, or on the basis of D_0 and g.
- The real problem is estimating g.
- g can be estimated by averaging the growth in dividends over the recent past, or from analyst projections.
- Averaging can be done arithmetically or geometrically.
- Given historical yearly growth rates g_1, g_2, g_3, $\ldots$, g_N, their arithmetic average is $g_A = \frac{1}{N}(g_1 + g_2 + \cdots + g_N)$, while the geometric average is $g_G = \sqrt[N]{(1+g_1)(1+g_2)\ldots(1+g_N)} - 1$.
- The arithmetic average is easier to understand, but the geometric average growth rate, when compounded over N periods, matches the total growth of the dividend; this is not true for the arithmetic average growth rate.

Example: Constant Dividend Growth

- A firm paid a dividend of \$4 last year; the dividend is expected to grow by 6% per year indefinitely. The stock sells for \$60. What is the cost of equity capital for this firm?

$$D_1 = D_0 \times (1+g) = 4 \times 1.06 = \$4.24$$

$$r_E = \frac{D_1}{P_0} + g = \frac{4.24}{60} + 0.06 = 0.1307 = 13.07\%$$

- Similar to many basic financial models, e.g., NPV, the calculations are straightforward; however, getting realistic estimates for parameters is a complex problem often requiring advanced mathematics and access to hard-to-get data.

Example: Estimating Dividend Growth

Year	Dividend	Dollar Change	Percentage Change
0	\$1.10		
1	\$1.20	\$0.10	$\frac{1.20}{1.10} - 1 = 9.09\%$
2	\$1.35	\$0.15	$\frac{1.35}{1.20} - 1 = 12.50\%$
3	\$1.40	\$0.05	$\frac{1.40}{1.35} - 1 = 3.70\%$
4	\$1.55	\$0.15	$\frac{1.55}{1.40} - 1 = 10.71\%$

- Arithmetic mean dividend growth:
  $g_A = \frac{1}{4}(9.09 + 12.50 + 3.70 + 10.71) = 9.00\%$.
- Geometric mean dividend growth:
  $g_G = \sqrt[4]{(1 + 0.0909)(1 + 0.1250)(1 + 0.0370)(1 + 0.1071)} - 1 = 8.95\%$.
- The geometric mean growth rate will be smaller than the arithmetic mean growth rate.

Discussion: Constant Dividend Growth

- Advantage: The constant dividend growth model is simple to implement.
- Disadvantages:
  - Some companies do not pay dividends, and some of these are important for financial markets.
  - Dividend growth is typically not constant, and even if it were in the short term, we rely on the assumption that the rate will be constant in perpetuity.
  - The sensitivity to the estimated value of g is high.
  - Risk estimates are not used explicitly in the model.

Using the SML

- This approach explicitly used the stock's riskiness, expressed in terms relative to the market, i.e., using β:

$$Er_E = r_F + \beta_E \times (Er_M - r_F)\,,$$

where Er_E is the expected return on equity, r_F is the risk-free rate, β_E is the relative risk of equity, Er_M is the expected return on the market, and $Er_M - r_F$ is the market premium.

- Consistent with the textbook, in order to simplify notation, we drop the expectations, and we interpret r_E as the expected (demanded) return on equity:

$$r_E = r_F + \beta_E \times (r_M - r_F)\,.$$

SML Example

- The market premium has historically been estimated to be approximately 7%. If on relies on this estimate, one only needs estimates of r_F and β_E to compute the required return on the given stock, i.e. the cost of equity.
- Consider a stock that has an estimated beta of 1.19, while the risk-free rate is 0.40%. Using the historical market premium estimate given above, determine the cost of equity.

$$\begin{aligned} r_E &= r_F + \beta_E \times (r_M - r_F) \\ &= 0.004 + 1.19 \times 0.07 \\ &= 8.43\%. \end{aligned}$$

Discussion: SML

- Advantages:
  - Risk is explicitly accounted for (there is an explicit adjustment for risk).
  - Companies do not have to pay constant ($g = 0$), or constantly growing dividends for the model to be applicable. In fact, no dividend payment is required at all.
- Disadvantages:
  - The dividend growth model requires one estimate only (g); using the SML requires two (β_E, market premium) or three estimates (r_F, β_E, and market premium), depending on how accurately one wants to measure the risk-free rate.
  - Estimates are based on past data, perhaps adjusted to reflected expected further conditions. These procedures may introduce large uncertainties.

The Cost of Debt

- Besides equity, firms can also issue debt. In fact, for large firms, issuing debt (i.e., borrowing) is easy and relatively cheap, as they can tap the financial markets directly.
- The cost of debt is the return that creditors demand on borrowing.
- For firms whose debt (bonds) trade in the market, the cost of debt can be determined directly, i.e., by computing the bonds' yield.
- Bond ratings can also be used to estimate the cost of debt - if a firm's bonds have (or are anticipated to have) a certain rating, then the demanded yield will be comparable to that demanded of bonds with the same rating.
- Notation for the cost of debt: r_D.

The Cost of Preferred Stock

- Preferred stock can also been seen as a form of debt.
- In the base case, when the preferred dividend is assumed to be paid out forever, the preferred stock is a perpetuity:

$$r_P = \frac{D}{P_0},$$

where r_P is the cost of preferred stock, D is the constant preferred dividend, and P_0 is the current price of the stock.
- What is the cost of preferred stock if the stock pays \$4.92 annually, and the stock trades at \$98.97 per share?

$$r_D = \frac{4.92}{98.97} = 4.97\%.$$

Weighted Average Cost of Capital (WACC)

- Let E be the **market** value of the firm's equity. Let D be the **market** value of the firm's debt.
  Shares often have par value, which in the US is both very small (typically) and practically irrelevant with respect to market value.
  Bonds have face value, which is distinct from market value (though face value impacts certain calculations, i.e., the coupon payment).
- The firm's total market value is $V = E + D$.
- The **capital structure weights** are defined as $w_E = \frac{E}{V} = \frac{E}{E+D}$ (the weight of equity), and $w_D = \frac{D}{V} = \frac{D}{E+D}$ (the weight of debt).
- $w_E + w_D = 1$.
- The shares and the debt (loans) of many smaller corporations do not trade. Since market values are not available, estimates must be used.

Debt and Taxes

- Companies can treat interest payments on their debt as expenses that are tax deductible. Depending on jurisdiction, in practice, there may be limits to this deductibility; here, however, we assume that interest can be deducted in full.
- Assume that the company must pay interest of I, and that the tax rate is flat and equal to T_c. Then the payment of I reduces the net income by the same amount, and taxes payable by $I \times T_c$. The net effect is that the firm pays $I - I \times T_C = (1 - T_c) \times I$. In effect, government subsidizes part of the interest payment.
- Example: Assume that a firm pays $r = 9\%$, and pays tax at a flat rate of 21%. What is the after-tax interest rate?
  Assume that the firm pays interest on N dollars: $I = r \times N$. The after-tax interest is $(1 - T_c) \times I$, while the after-tax interest rate is $\frac{(1-T_c)\times I}{N} = \frac{(1-T_c)\times r \times N}{N} = (1 - T_c)\times r = (1 - 0.21)\times 0.09 = 7.11\%$.

WACC

- The weighted average cost of capital is defined as

$$\begin{aligned} WACC &= \frac{E}{V} \times r_E + \frac{D}{V} \times r_D \times (1 - T_c) \\ &= w_E \times r_E + w_D \times r_D \times (1 - T_c) \end{aligned}$$

- If a firm also has preferred stock with a market value of P, then $V = E + P + D$, and we define $w_E = \frac{E}{V}$, $w_P = \frac{P}{V}$, $w_D = \frac{D}{V}$. We have

$$WACC = w_E \times r_E + w_P \times r_P + w_D \times r_D \times (1 - T_c) \,.$$

WACC Example

- A firm has 1.4 million shares outstanding. The stock sells for $20 per share and has an estimated β of 0.74. The firm has issued bonds with a total face value of $5M; these trade at 93% of face value and have a yield of 11%. The risk-free rate is 8%, the market risk premium is 7%. The tax rate is 21%, flat. What is the WACC?
- $r_E = r_F + \beta_E \times (r_M - r_F) = 0.08 + 0.74 \times 0.07 = 13.18\%$ (from SML).
- $r_D = 11\%$. The yield is the usual estimate of the cost of debt.
- $E = 1{,}400{,}000 \times \$20 = \$28{,}000{,}000$; $D = 0.93 \times \$5{,}000{,}000 = \$4{,}650{,}000$; $V = E + D = \$32{,}650{,}000$.
- $w_E = \frac{28}{32.650} = 85.76\%$; $w_D = \frac{4.650}{32.650} = 14.24\%$ $(w_E + w_D = 1)$.
- $WACC = w_E \times r_E + w_D \times r_D \times (1 - T_c) =$ $0.8576 \times 0.1318 + 0.1424 \times 0.1100 \times (1 - 0.21) = 12.54\%$.

The SML and the WACC

- The WACC captures the overall cost of capital.
- **A typical use for the WACC is as a discount rate for future cash flows**. If one discounts the firm's cash flows, this use is appropriate. If one discounts project cash flows, however, the use may not be appropriate.
- **Individual projects may not be close to the firm's average in terms of riskiness.** As such, WACC may not accurately reflect the cost of capital for the project at hand. This may lead to incorrect investment decisions.
- Consider the diagram below, which shows a company with $\beta = 1$, i.e. the company is as risky as the market overall. The firm has no debt. The market risk premium is 8%, and the risk-free rate is 7%. Then the WACC is equal to 15%.

The SML and the WACC (2)

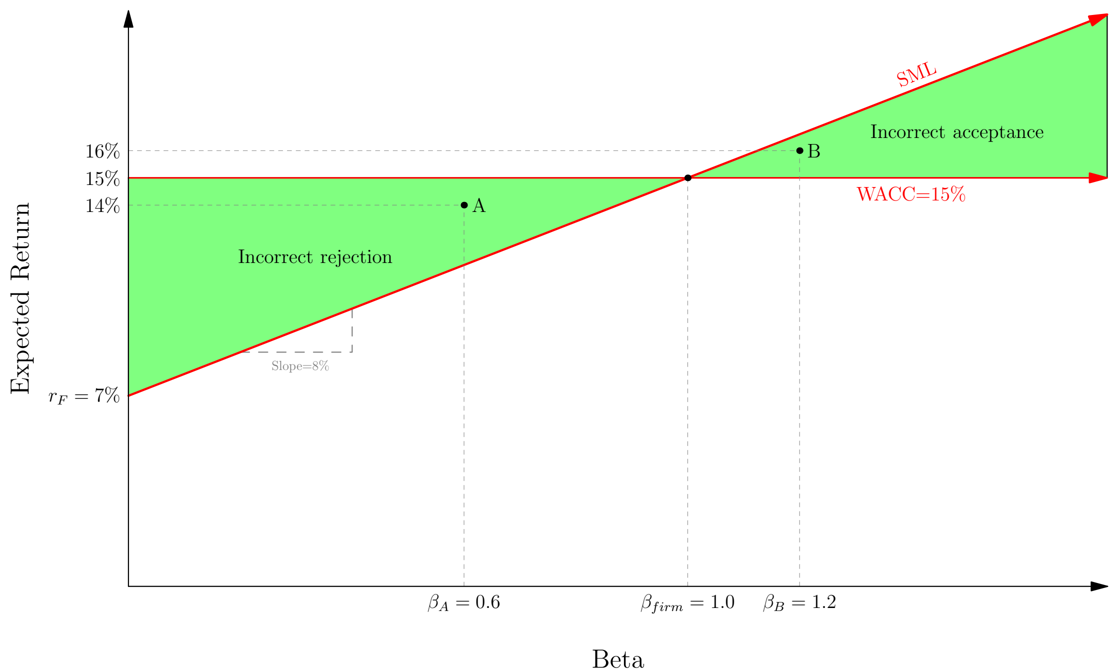
- Consider project A in the diagram, which is above the SML, but below the WACC.
- According to the SML, project A is desirable, since it pays more than the market requires for its level of risk.
- However, project A does not earn a return greater than the WACC, so it will be rejected.
- Project A is less risky than the firm overall, since $\beta_A = 0.6 < \beta_{firm} = 1$, so the WACC **overestimates** the cost of capital for this project. Hence, rejecting project A **is an incorrect decision**.

Cost of Capital Equity Debt WACC To Do!

The SML and the WACC (3)

- Now consider project B in the diagram, which is below the SML, but above the WACC.
- According to the SML, project B is undesirable, since it pays less than the market requires for its level of risk.
- However, project B earns a return greater than the WACC, so it will be accepted.
- Project B is more risky than the firm overall, since $\beta_B = 1.2 > \beta_{firm} = 1$, so the WACC **underestimates** the cost of capital for this project. Hence, accepting project B **is an incorrect decision.**

The SML and the WACC (4)



Divisional Cost of Capital

- It is clear that there are situations when the riskiness (and thus the cost of capital) of a project differ markedly from the firm's analogous parameter.
- WACC is not a one-size-fits-all measure.
- If the risk of projects can broadly be associated with various divisions of the firm, then one can compute (estimate) WACC per-division, not per firm.
- Example: Assume has a division that manages oil pipelines that connect big oil fields with ports and oil refineries. The fields are young, and expected to produce over the long term. A lot of the pipeline capacity is tied up by long-term, favorable contracts. Now consider a second division that engages in oil prospecting, often in regions not known for existing oil production. Clearly, the former division is much less risky than the latter, and the divisional WACC rates should reflect this.

WACC and the Pure Play Approach

- Divisions within a large firm often specialize in a small(er) number of relatively homogeneous activities/projects within the larger firm.
- Whenever this is true, the divisional WACC may be estimated by finding independent firms whose activity profile approximates well that of the division under consideration.
- If the independent firm's equity and debt trades publicly, then then WACC can be computed for the "analogous" firm, and can then be attributed, perhaps with some adjustments, to the division.
- In an idealized setting, both the division and the analogous firm would engage in one type of activity. This ideal justifies naming this method the "pure play" approach.

WACC and the Subjective Approach

- It may not be possible to identify divisional WACC using the pure play approach. This may arise when divisions have no close analogues that trade publicly (e.g. because of the mix of activities, size, location, or other factors).
- The firm can classify its projects into a small set of risk categories, e.g. "low risk," "medium risk," "high risk," and "mandatory." Each project category is then assigned a WACC that is derived from adjusting the firm-wide WACC to (subjectively) account for the project's risk.
- Many firms must engage in projects that are legally mandated, e.g. in measures of pollution control. Such projects must be undertaken irrespective of costs and profitability, so a WACC-type approach is not suitable for their analysis.

Example: Subjective Approach

- Assume that a firm's WACC is 14%.
- The firm may develop the following subjective approach to adjust WACC to account for its projects' riskiness:

Category	Example	WACC Adjustment	Project WACC
High risk	New products or markets	+6%	20%
Medium risk	Cost savings, expansion of existing lines	+0%	14%
Low risk	Equipment replacement	-4%	10%
Mandatory	Pollution control	N/A	N/A

WACC and Company Valuation

- Interest on debt is a financing cost, not an operating cost. As such, it should not be considered in the valuation of the firm (financing is, in principle, an independent decision).
- Interest does decrease taxes, however, so a firm's financial results must be adjusted to undo this effect.
- If one can reconstruct the relevant cash flows for the firm, then the firm itself can be valued as a project.
- We'll use our standard notation, but we'll add a starred superscript (*) to indicate quantities modified to remove the effect of interest payments. We denote the cash flow from assets (free cash flow) by CFA. We use the Greek capital letter Δ to denote change.

WACC and Company Valuation (2)

- $Taxes^* = EBIT \times T_c$
- $CFA^* =$
  $EBIT + Depreciation - Taxes - \Delta NWC - (capital\ spending)$
- $CFA^* =$
  $EBIT \times (1 - T_c) + Depreciation - \Delta NWC - (capital\ spending)$
- Both internal and external analysts can try to produce estimates for how CFA^* changes over time. If the firm is in a steady state, then one can assume that the CFA^* will stay approximately constant for a long time (for ever). If appropriate, one can also infer a constant growth rate g. In these simple cases, the firm's cash flows form a perpetuity and can be valued appropriately.

WACC and Company Valuation (3)

- Regular perpetuity: CFA^* stays constant. $Firm\ value_0 = \frac{CFA^*}{WACC}$.
- Growth perpetuity: CFA^* grows at a constant rate of g.
  $Firm\ value_0 = \frac{CFA_1^*}{WACC-g}$.
  Example: Consider a firm with CFA^* of \$120M **for the next year**, which will grow at a constant rate of 5% per year. The WACC is estimated to be 9%.

$$Firm\ value_0 = \frac{120}{0.09 - 0.05} = \$3,000M = \$3B.$$

- Irregular growth can be handled like we did with equity valuation: we produce estimates up to time t, after which we estimate the value of the firm V_t at time t (typically using a perpetuity of some kind). $Firm\ value_0 =$
  $\frac{CFA_1^*}{1+WACC} + \frac{CFA_2^*}{(1+WACC)^2} + \frac{CFA_3^*}{(1+WACC)^3} + \cdots + \frac{CFA_t^*}{(1+WACC)^t} + \frac{V_t}{(1+WACC)^t}$.

To Do!

- Read section 14.7 “Flotation Costs and the Average Cost of Capital.”