

Loan Types ○○○	Fixed, 25%, 5yrs ○○○	Fixed, 10%, 5yrs ○○○	Fixed, 10%, 30yrs ○○○	Var, 25%, 5yrs ○○○	Var, 10%, 5yrs ○○○	Var, 10%, 30yrs ○○○
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Discounted Cash Flow Valuation III

AEM 2241 - Finance

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Outline

Loan Types

Fixed-Payment Amortized Loans

Variable-Payment Amortized Loans

Bullet Loans

Amortization Tables

Fixed Payments, Short Maturity, High Interest

Fixed Payments, Short Maturity, Lower Interest

Fixed Payments, Long Maturity, Lower Interest

Variable Payments, Short Maturity, High Interest

Variable Payments, Short Maturity, Lower Interest

Variable Payments, Long Maturity, Lower Interest



Fixed-Payment Amortized Loans

- A fixed-payment annuity loan can be interpreted as being just a regular annuity. The yearly (more generally: per-period) payment will then be $C = \frac{r}{1 - (\frac{1}{1+r})^t} PV$.
- While payments are fixed in magnitude their structure is variable, as they consist of variable “mixtures” of interest and (repaid) principal. This is illustrated by analyzing the loan’s amortization table. We discussed this matter in class.
- This the structure of a row in the loan’s amortization table:

Year	Beg. Balance	Payment	Interest	Principal	End Balance
...
k	BB	C	$BB \cdot r$	$C - BB \cdot r$	$BB - (C - BB \cdot r) = (1 + r) BB - C$
...

Variable-Payment Amortized Loans

- Loan payments do not have to be constant. An important type of loan arises when at the end of each year (period) we pay back a fraction of $\frac{1}{t}$ of the principal, i.e. an amount of $\frac{PV}{t}$.
- In this case interest payments are variable, and each chunk of repaid principal is constant. This makes for a variable total yearly payment.
- This the structure of a row in the loan’s amortization table:

Year	Beg. Balance	Payment	Interest	Principal	End Balance
...
k	BB	$BB \cdot r + \frac{PV}{t}$	$BB \cdot r$	$\frac{PV}{t}$	$BB - \frac{PV}{t}$
...

Bullet Loans

- A bullet loan is set up as if it were amortized over a longer period of time (say, 15 years), but it matures (ends) after a shorter period (say, 5 years).
- Both fixed-payment and variable-payment loans can be structured as bullet loans.
- The amortization table of a bullet loan is the amortization table of the underlying loan (say, 15 years), but “cut off” at the maturity of the bullet loan (say, 5 years).
- The principal of a bullet loan is not already repaid by the time the loan matures, so the borrower will have a (still-outstanding) principal balance. This is either paid in full in a lump sum, or - more commonly - refinanced using another bullet loan.

Amortization Tables

- For a regular (i.e., non-bullet) loan, the ending balance of the principal should be 0 (if we ignore rounding errors).
- It is possible to write formulas that directly provide the values in a loan's amortization table. For example, consider the beginning balance of a variable-payment loan with principal PV , interest rate r , and maturity t , in year k :

$$BB = \frac{t-k+1}{t} PV \quad (1 \leq k \leq t).$$
 Can you explain why this formula is correct? Formulas for fixed-payment loans are more complicated.
- The terms of loans can be modified to suit particular clients and situations. It is possible to accelerate or delay the repayment of the principal outstanding, either by design, or on a contingent basis (if certain events occur). An amortization table derived from the terms of the loan can then be constructed.



Cumulative: Fixed Payments, Short Maturity, High Interest

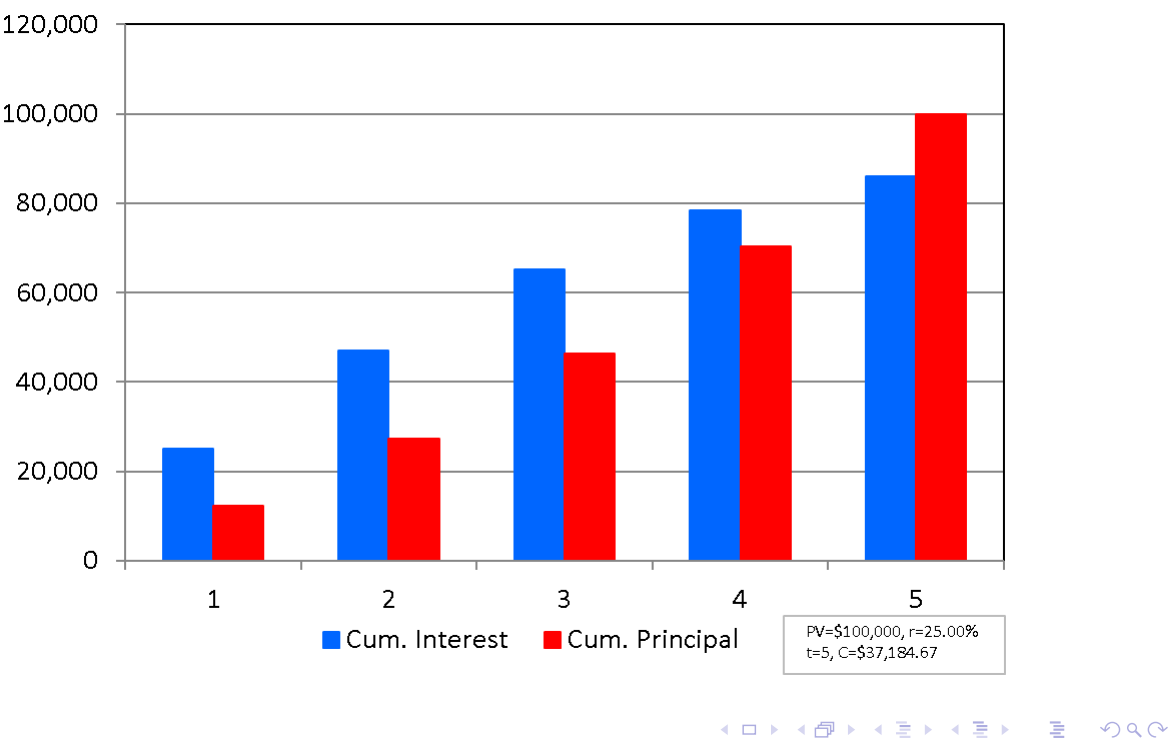
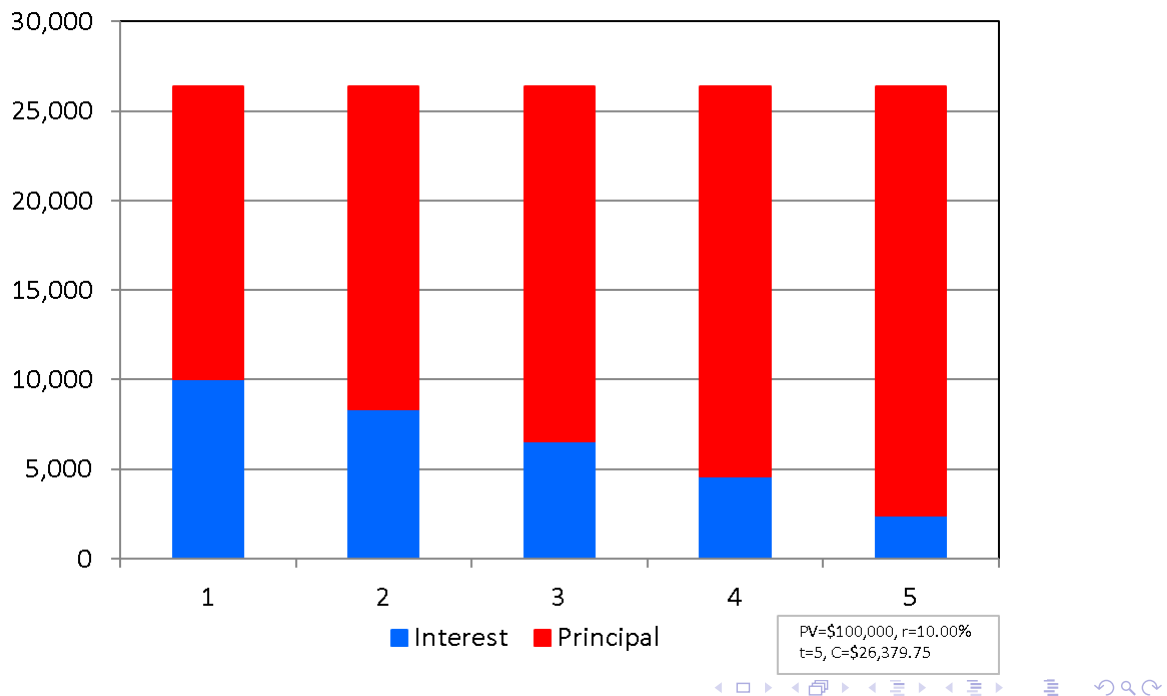


Table: Fixed Payments, Short Maturity, Lower Interest

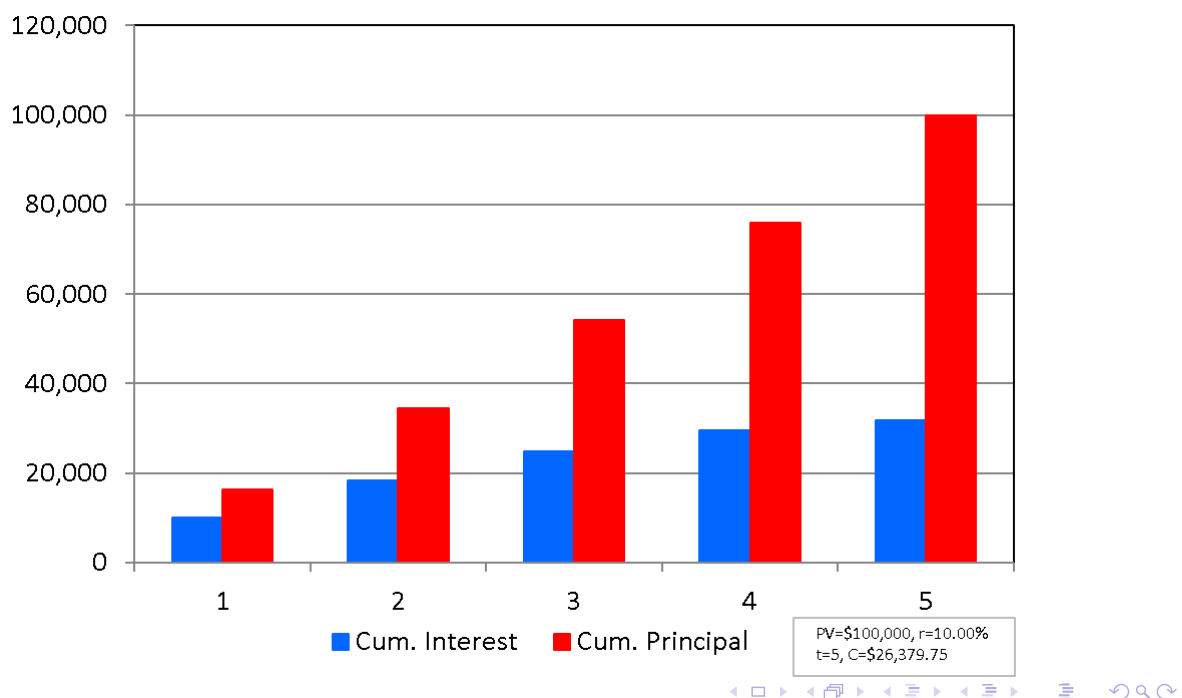
r	10.00%	C	26,379.75
t	5	Total int.:	31,898.74
PV	100,000.00		

Period	Beg. Balance	Payment	Interest	Principal	End Balance	Cum. Interest	Cum. Principal
0					100,000.00		
1	100,000.00	26,379.75	10,000.00	16,379.75	83,620.25	10,000.00	16,379.75
2	83,620.25	26,379.75	8,362.03	18,017.72	65,602.53	18,362.03	34,397.47
3	65,602.53	26,379.75	6,560.25	19,819.50	45,783.03	24,922.28	54,216.97
4	45,783.03	26,379.75	4,578.30	21,801.44	23,981.59	29,500.58	76,018.41
5	23,981.59	26,379.75	2,398.16	23,981.59	-	31,898.74	100,000.00

Interest vs. Principal: Fixed Payments, Short Maturity, Lower Interest



Cumulative: Fixed Payments, Short Maturity, Lower Interest



Cumulative: Fixed Payments, Long Maturity, Lower Interest

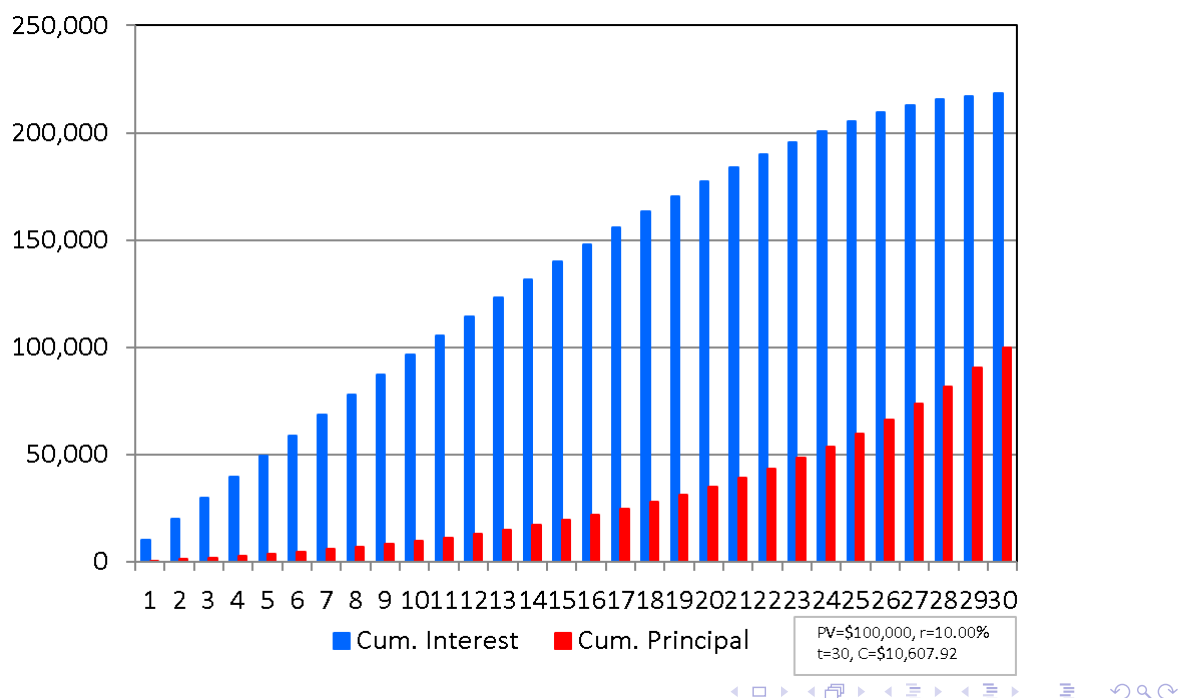
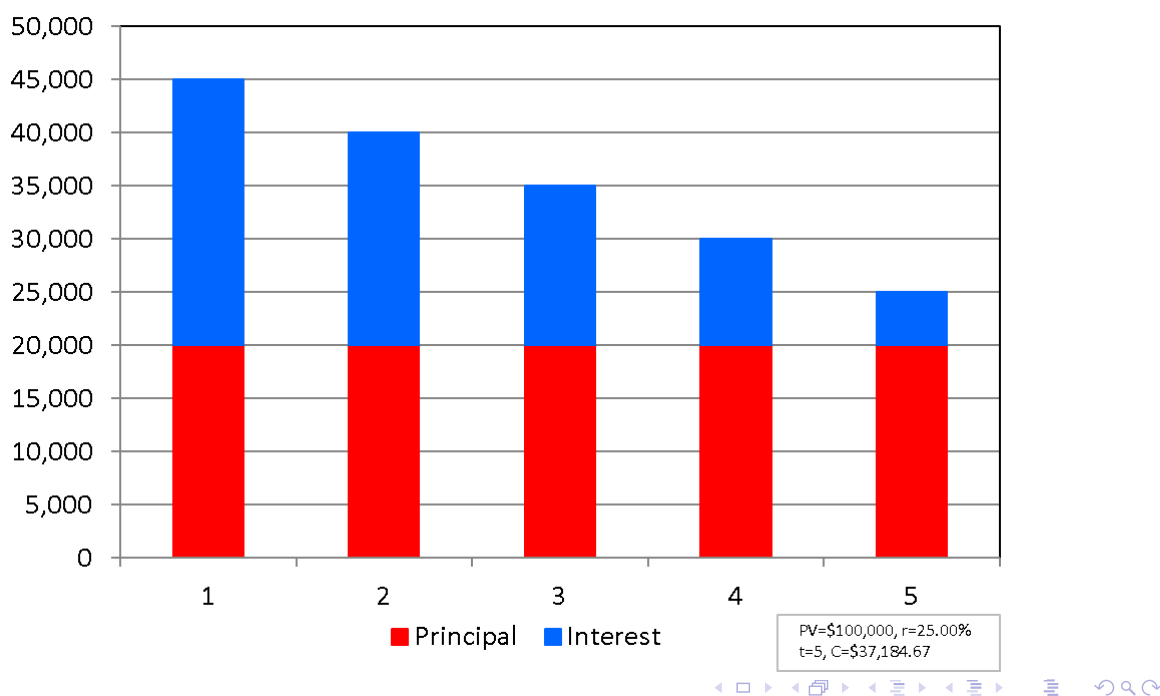


Table: Variable Payments, Short Maturity, High Interest

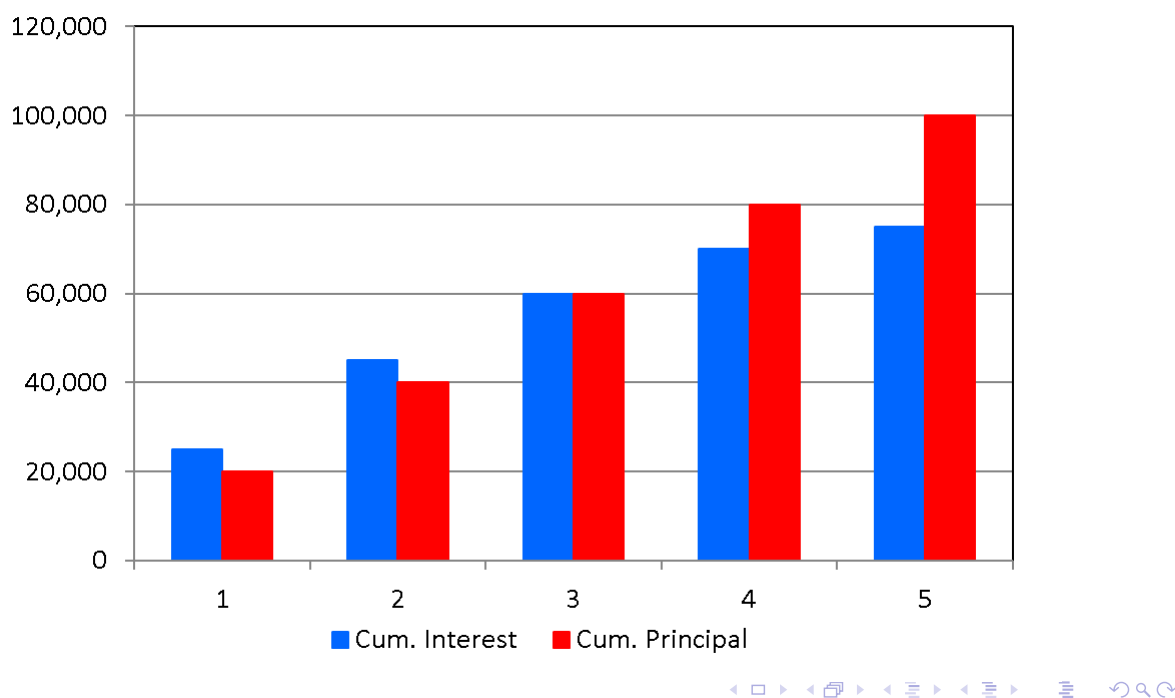
r	25.00%	C	37,184.67
t	5	Total interest:	75,000.00
PV	100,000.00	Principal pymt	20,000.00

Period	Beg. Balance	Payment	Interest	Principal	End Balance	Cum. Interest	Cum. Principal
0					100,000.00		
1	100,000.00	45,000.00	25,000.00	20,000.00	80,000.00	25,000.00	20,000.00
2	80,000.00	40,000.00	20,000.00	20,000.00	60,000.00	45,000.00	40,000.00
3	60,000.00	35,000.00	15,000.00	20,000.00	40,000.00	60,000.00	60,000.00
4	40,000.00	30,000.00	10,000.00	20,000.00	20,000.00	70,000.00	80,000.00
5	20,000.00	25,000.00	5,000.00	20,000.00	-	75,000.00	100,000.00

Interest vs. Principal: Variable Payments, Short Maturity, High Interest



Cumulative: Variable Payments, Short Maturity, High Interest



Cumulative: Variable Payments, Short Maturity, Lower Interest

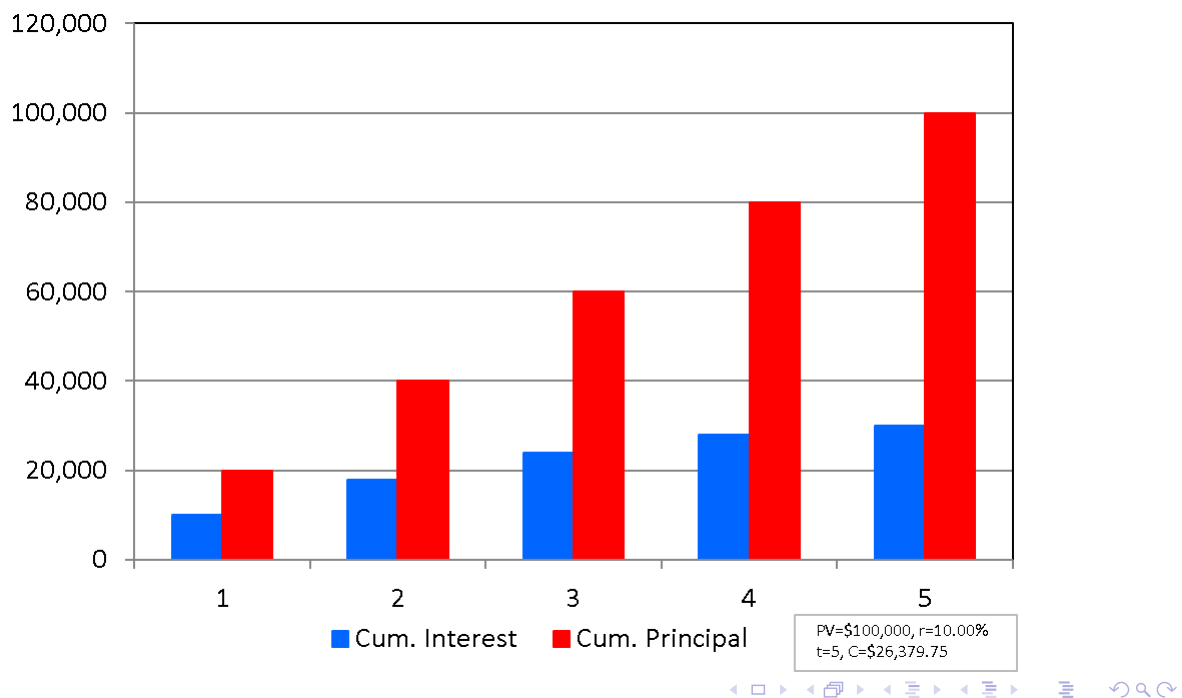


Table: Variable Payments, Long Maturity, Lower Interest

t PV	10.00%	C		10,607.92			
	30	Total interest:		155,000.00			
	100,000.00	Principal pymt		3,333.33			
Period	Beg. Balance	Payment	Interest	Principal	End Balance	Cum. Interest	Cum. Principal
0					100,000.00		
1	100,000.00	13,333.33	10,000.00	3,333.33	96,666.67	10,000.00	3,333.33
2	96,666.67	13,000.00	9,666.67	3,333.33	93,333.33	19,666.67	6,666.67
3	93,333.33	12,666.67	9,333.33	3,333.33	90,000.00	29,000.00	10,000.00
4	90,000.00	12,333.33	9,000.00	3,333.33	86,666.67	38,000.00	13,333.33
5	86,666.67	12,000.00	8,666.67	3,333.33	83,333.33	46,666.67	16,666.67
6	83,333.33	11,666.67	8,333.33	3,333.33	80,000.00	55,000.00	20,000.00
7	80,000.00	11,333.33	8,000.00	3,333.33	76,666.67	63,000.00	23,333.33
8	76,666.67	11,000.00	7,666.67	3,333.33	73,333.33	70,666.67	26,666.67
9	73,333.33	10,666.67	7,333.33	3,333.33	70,000.00	78,000.00	30,000.00
10	70,000.00	10,333.33	7,000.00	3,333.33	66,666.67	85,000.00	33,333.33
11	66,666.67	10,000.00	6,666.67	3,333.33	63,333.33	91,666.67	36,666.67
12	63,333.33	9,666.67	6,333.33	3,333.33	60,000.00	98,000.00	40,000.00
13	60,000.00	9,333.33	6,000.00	3,333.33	56,666.67	104,000.00	43,333.33
14	56,666.67	9,000.00	5,666.67	3,333.33	53,333.33	109,666.67	46,666.67
15	53,333.33	8,666.67	5,333.33	3,333.33	50,000.00	115,000.00	50,000.00
16	50,000.00	8,333.33	5,000.00	3,333.33	46,666.67	120,000.00	53,333.33
17	46,666.67	8,000.00	4,666.67	3,333.33	43,333.33	124,666.67	56,666.67
18	43,333.33	7,666.67	4,333.33	3,333.33	40,000.00	129,000.00	60,000.00
19	40,000.00	7,333.33	4,000.00	3,333.33	36,666.67	133,000.00	63,333.33
20	36,666.67	7,000.00	3,666.67	3,333.33	33,333.33	136,666.67	66,666.67
21	33,333.33	6,666.67	3,333.33	3,333.33	30,000.00	140,000.00	70,000.00
22	30,000.00	6,333.33	3,000.00	3,333.33	26,666.67	143,000.00	73,333.33
23	26,666.67	6,000.00	2,666.67	3,333.33	23,333.33	145,666.67	76,666.67
24	23,333.33	5,666.67	2,333.33	3,333.33	20,000.00	148,000.00	80,000.00
25	20,000.00	5,333.33	2,000.00	3,333.33	16,666.67	150,000.00	83,333.33
26	16,666.67	5,000.00	1,666.67	3,333.33	13,333.33	151,666.67	86,666.67
27	13,333.33	4,666.67	1,333.33	3,333.33	10,000.00	153,000.00	90,000.00
28	10,000.00	4,333.33	1,000.00	3,333.33	6,666.67	154,000.00	93,333.33
29	6,666.67	4,000.00	666.67	3,333.33	3,333.33	154,666.67	96,666.67
30	3,333.33	3,666.67	333.33	3,333.33	0.00	155,000.00	100,000.00

Loan Types
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Fixed, 25%, 5yrs
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Fixed, 10%, 5yrs
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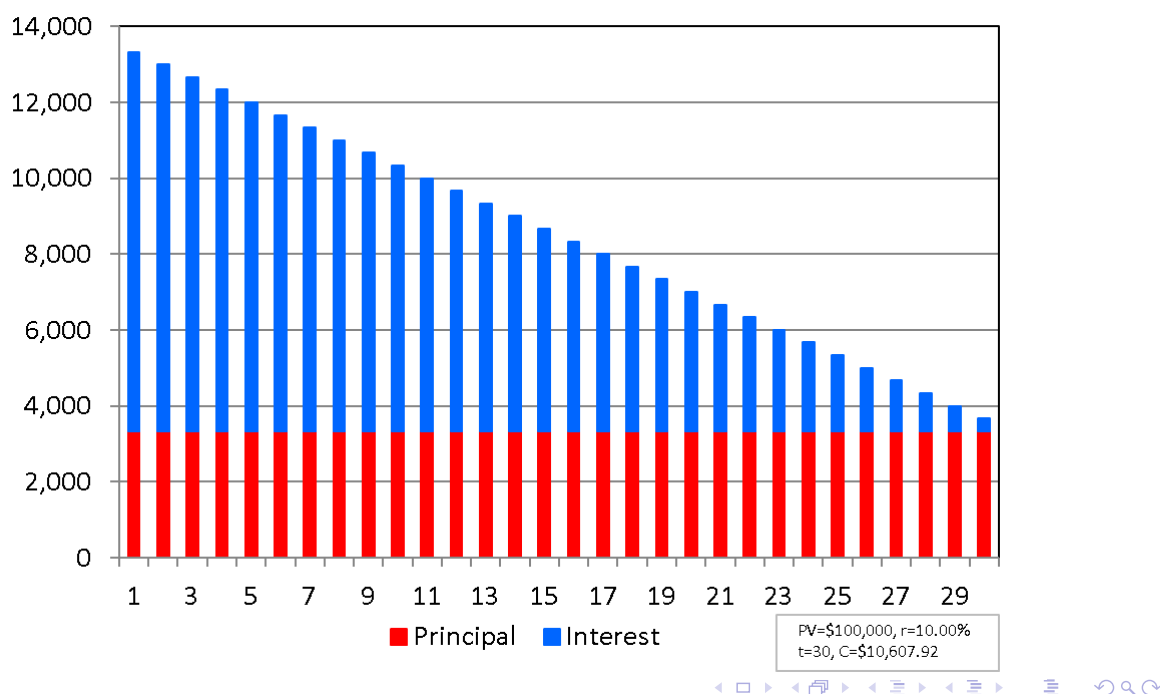
Fixed, 10%, 30yrs
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Var, 25%, 5yrs
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Var, 10%, 5yrs
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Var, 10%, 30yrs
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Interest vs. Principal: Variable Payments, Long Maturity, Lower Interest



Loan Types
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Fixed, 25%, 5yrs
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Fixed, 10%, 5yrs
○○○

Fixed, 10%, 30yrs
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Var, 25%, 5yrs
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Var, 10%, 5yrs
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Var, 10%, 30yrs
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Cumulative: Variable Payments, Long Maturity, Lower Interest

