NPV and Other Investment Criteria

AEM 2241 - Finance

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Decisions NPV IRR NPV vs. IRR Other Criteria To Do!

Outline

Investment Decisions

- In the time-value of money problems we discussed earlier, present values were always exactly equivalent to their future values, but for the shift in time, and the fair compensation (i.e., interest) that this shift entailed. As stated informally before, interest can be thought of as the "price of money," or the cost to "rent" (borrow) money.
- With stocks, and even with bonds, we talked about the demanded (required) rate (or yield), which embedded compensation for various risks born by investors, in addition to the fair compensation due to the time-shift of cash flows.
- With investments, we explicitly aim for an excess of inflows over outflows of cash.

Investment Decisions (2)

- From an abstract financial perspective, each investment is a set of positive and negative cash flows, each potentially different in size and timing. In addition, there are uncertainties related to these cash flows.
- We are not looking for our cash outflows to be exactly compensated by the project inflows. This may be "fair," but we did not earn anything, so we worked for nothing and did not generate extra value.
- How can we think about these cash flows in a way that allows us to select "good" investments? What, precisely, does "good" mean in this context?
- Because of the inherent imprecision of the data involving the future behavior of investments, quantitative methods are of limited use and have to be used with care.

Net Present Value

- A project's cash flows must be predicted/estimated, both with respect to their size and timing. Only cash flows that would not occur, but for the existence of the project, should be considered.
- Each cash flow is discounted back to time 0 (the reference time); all present values are added together to obtain the net present value (NPV).
- The NPV captures in present value terms the excess or deficit of inflows versus outflows. **If NPV>0, then a project generates more revenue than expenses (in present value terms), and should be undertaken.**
- Most financial experts agree that the NPV criterion is the best to use, assuming the data it relies on is reasonably accurate.

Net Present Value Example

Time [Yrs]	0	1	2	3	4	5	6	7	8
Cash Flow ['000s]	-$**30**	$6	$6	$6	$6	$6	$6	$6	$**8**

- The cash flows above can be decomposed into a time-0 outflow of $30,000, an 8-year regular annuity of $6,000, and a time-8 extra cash flow of $2,000.
- Let R=15% be the discount rate that accurately reflects the time value of money and the riskiness of the project.

$$\begin{aligned} NPV &= -30,000 + 6,000 \cdot \frac{1-\left(\frac{1}{1+0.15}\right)^8}{0.15} + \frac{2,000}{(1+0.15)^8} \\ &= -30,000 + 26,923.93 + 653.80 \\ &= -\$2,422.27. \end{aligned}$$

NPV and IRR: Using Calculators

- Your calculator can be used to calculate NPV's and IRR's (to be discussed later).
- BA II Plus:
  NPV and IRR instructions:
  https://education.ti.com/en/customer-support/knowledge-base/financial-calculators/product-usage/11241
- HP 12 C
  NPV instructions: https://education.ti.com/en/customer-support/knowledge-base/financial-calculators/product-usage/11241||https://education.ti.com/en/customer-support/knowledge-base/financial-calculators/product-usage/11241
  IRR instructions: https://hpofficesupply.com/wp-content/uploads/2021/10/HP-12c-Financial-Calculator-Internal-Rate-of-Return.pdf

NPV Limitations

- As an academic exercise, NPV is simple and straightforward. However, in practice it is non-trivial to use.
- There are many uncertainties related to NPV, including:
  - The time horizon to consider for the inclusion of cash flows (and any liquidation or final value in case the project has a finite horizon);
  - The size of cash flows and their timing;
  - The impact of contingent decisions (a project could evolve in different directions depending on future unknowable events);
  - The discount (demanded) rate to use, which should compensate both for the time value of money and all the additional risks that the investors bear when they finance the project under consideration.

NPV Limitations

- Project proponents tend to have optimistic views of (positive) cash flows; opponents may underestimate them.
- A discount rate that is too aggressive (large) may unfairly penalize good projects, falsely showing them to be under-performing. Conversely, a low discount rate may overestimate the NPV of the project.
- Projects may have consequences that may be impossible to capture realistically, e.g. reputation risk.
- Many corporate financial models provide one value and one associated time for each projected cash flow. More advanced approaches involve the use of probabilities to show distributions of possible outcomes, including possible cash flow amounts and times.

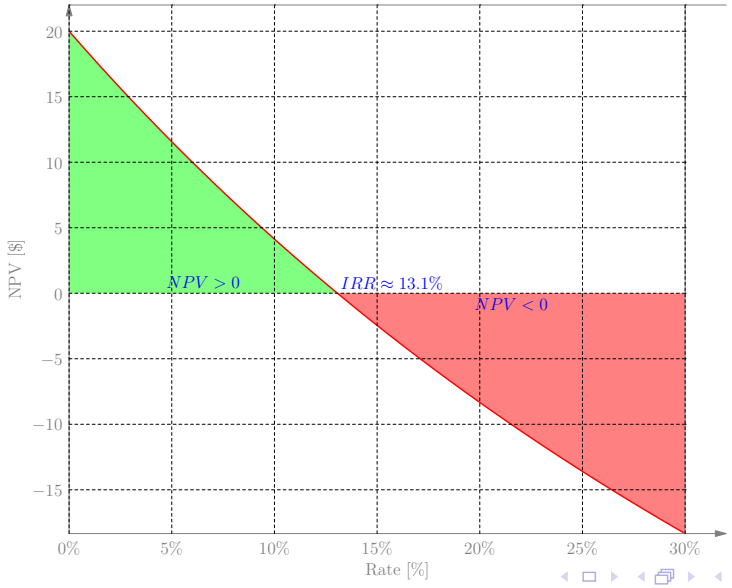
Internal Rate of Return

- The internal rate of return (IRR) is the interest rate (demanded return, discount rate) which makes the NPV of a series of cash flows equal to zero.
- A necessary condition for the IRR to be defined is to have both positive and negative cash flows.
- We have already encountered the IRR in a hidden form: if you consider the (purchase) price of a bond as a negative cash flow, and the bond's payments as positive cash flows, the IRR of the bond is its yield.
- Calculating IRRs is more challenging that computing bond yields, but not by much. With some precautions, the bisection method can still be used.

IRR Example

Time	0	1	2
Cash Flow	-$100	$60	$60

- The NPV of the cash flows above depends on the discount rate as shown in the diagram below. The IRR is equal to 13.1%.



IRR Investment Rule

- Take a look again at the chart showing the NPV as a function of the discount rate. Note that the NPV is positive as long as the discount rate is less than the IRR.
- This immediately suggests the following IRR investment criterion: **An investment should be undertaken if the demanded return is less than the investment's IRR; otherwise, it should be rejected**.
- This is a simple, straightforward criterion, but in practice it often leads to problems that make it ultimately less useful than the NPV-based approach. Some - but not all - of the problems will be described below, **including an exception**.
- A major concern related to the IRR is the relative ease of manipulation of its numerical value. This is a particular concern in investment management, when performance fees may be tied to IRRs, and not NPVs.

IRRs Are Not Unique

- Consider the entire series of signed cash flows associated with the project; sort these in the order of their arrival, starting with the initial cash flow.
- In all realistic cases, there will be at least one sign change in this series. For example, the first cash flow will be negative, the other ones may all be positive.
- If there is one cash flow sign change, the IRR will be unique; if there are several, however, there may be multiple IRRs.
- There can as many IRRs as many sign changes there are in the series of cash flows.
- This raises a major problem: if there are multiple IRRs, which one is the real one?

A Pathological Example

Time [Yrs]	0	1	2	3	4
Cash Flow	-1,000,000	4,500,000	-7,587,500	5,681,250	-1,593,900

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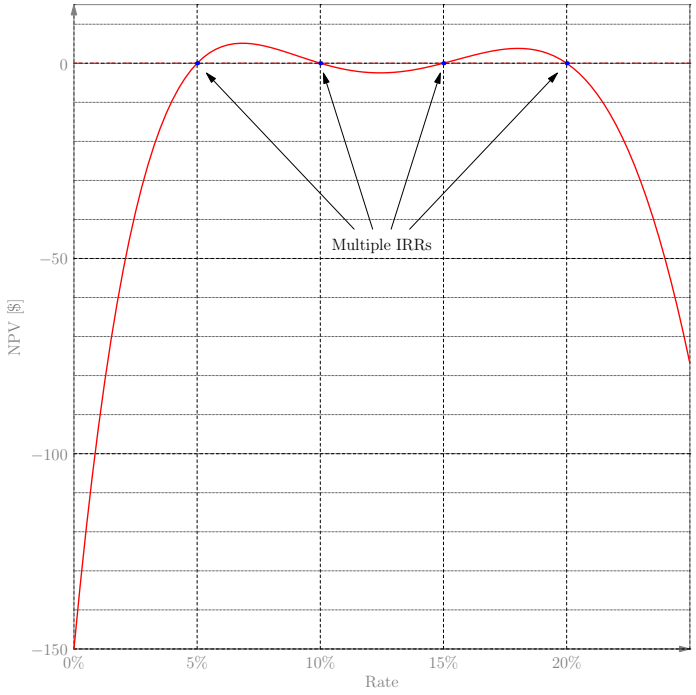
- Consider the investment cash flows shown above.
- Note that this series of 5 cash flows has 4 sign changes: we invest in years 0, 2, and 4, and we get money back in years 1 and 3.
- The simple sum of cash flows is -$150, so we are losing a small amount of money. The project is not a good business opportunity.
- The IRR would not change if we switched the signs of all cash flows; in that case, however, the project would generate a small nominal profit of $150.
- We can claim that we make 20% on this opportunity; detractors, however, say we earn only 5%. Both statements are (somewhat) right!

Found the IRR! Here's Another One! ... and Another One!



Analysis of the Example

- This series of cash flows has been designed to have IRRs of 5%, 10%, 15%, and 20%, respectively.
- In fact, any value between 5% and 20% leads to an NPV well within the range of -$10 to $10, which is much less than the magnitude of the original cash flows. Talking a bit loosely, any of these values is (a good approximation of) an IRR.
- Multiple IRRs do arise in practice, though you will typically not find this degree of multiplicity.
- Even specialized software (e.g. Excel) will often find one IRR, but will not warn you about the existence of others.

Calculating IRRs

- IRRs can be computed using the bisection method, analogously to the algorithm described for bond yields. There are two important adjustments, however:
  - We do not compare the computed price to the market price; instead, we compare the computed NPV of the investment cash flows to 0 (the target NPV for the IRR). Think of the NPV as the analogue of the bond market price, and of 0 as the bond's observed (market) price.
  - We must bracket the IRR, which is easy if we know that we have a single IRR.
  - If there may be several IRRs, we should plot the NPV against various discount rates. We then choose a relatively narrow interval of initial rates that bracket a region where the NPV reaches zero a single time. The bisection method can then be applied for each such region in turn to find each individual IRR.
  - Crossover rates are just the rates (IRRs) that force NPV(Investment A) - NPV(Investment B) to be equal to 0.

Modified IRR

- Sign changes in the sequence of investment cash flows may lead to non-unique IRRs, as discussed above.
- The non-uniqueness of the IRR can be eliminated through a mathematical artifice that changes cash flows so that there is only one sign change. The (modified) IRR is computed using these changed cash flows.
- While you may always get a unique MIRR, this value does not have a clean interpretation. Few experienced individuals I met believe that MIRRs are useful.
- Variants:
  - Discount all negative cash flows back to time 0 and add them to the initial outflow. Leave positive cash flows unchanged.
  - Compute the future value of all cash flows, except the very first one, at the end of the project, using a "suitable" reinvestment rate.
  - Discount negative cash flows back to time 0, and roll forward positive cash flows to the end date of the project.

NPV vs. IRR Comparison

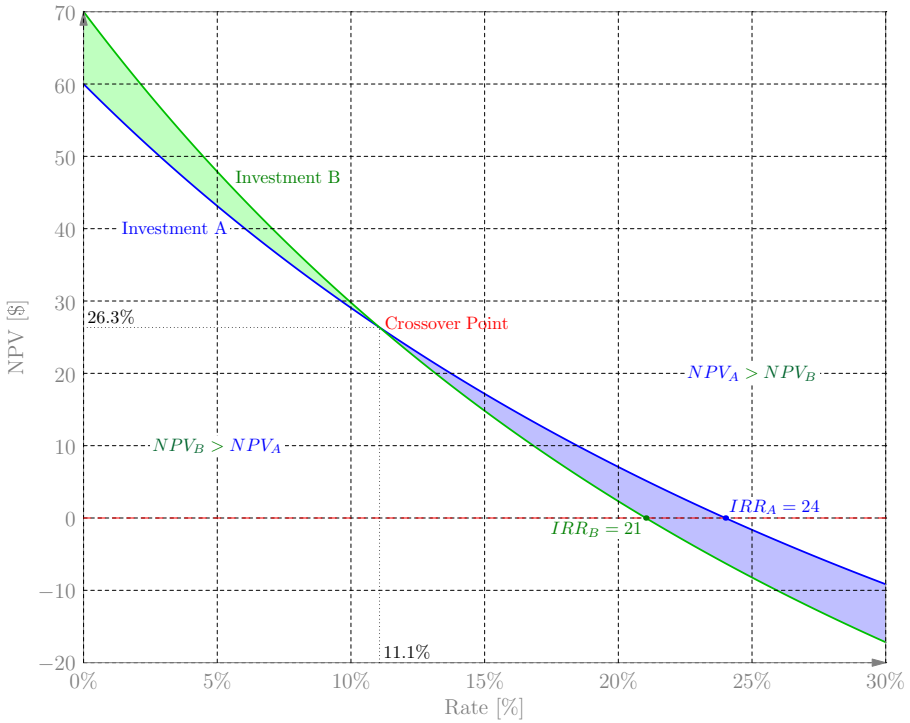
- The IRR criterion is closely related to the NPV criterion. Do they always yield the same decision? **Generally, no.** However, if projects are evaluated in isolation (independently of each other) **and** the cash flows are conventional (a negative cash flow is followed by positive cash flows), then the two criteria will produce the same answer.
- The NPV is the right criterion from a theoretical perspective. However, the IRR can be computed without the need to estimate a demanded rate (discount rate).
- Some people find the IRR more intuitive and make statements like "[w]e make [IRR]% on our money, so we are doing well."
- Non-uniqueness, as well as incorrect rankings when projects are mutually exclusive are major problems associated with IRRs.

Mutually Exclusive Investments

Time [Yrs]	0	1	2	3	4
Investment A	-$100	$50	$40	$40	$30
Investment B	-$100	$20	$40	$50	$60

- Consider the two investments specified above - which one is better?
- As the diagram on the next slide shows, the answer depends on what criterion we are using, NPV vs. IRR. Secondarily, the answer depends on the demanded rate that we are using for computing NPVs.
- If we use the IRR criterion, Investment A is better than Investment B, since $IRR_A = 24\% > IRR_B = 21\%$.
- Using NPVs, we first note that a rate of 11.1% implies $NPV_A = NPV_B$. This is a crossover point. Below this rate, Investment B dominates; above it, Investment A dominates.
- There is no unique answer, as it depends on the demanded rate, and NPV and IRR may not agree!

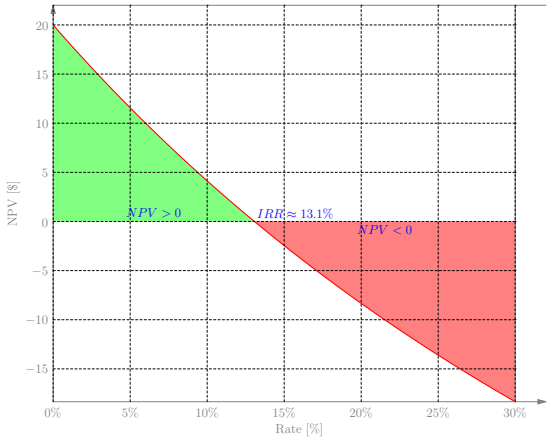
Mutually Exclusive Investments

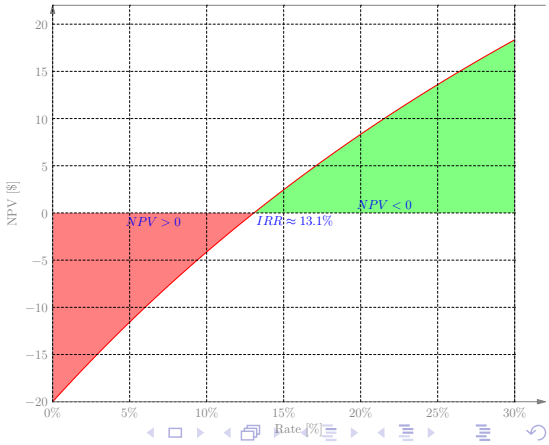


Investing vs. Financing

- Consider two investments identical in all respects, but for the signs of their cash flows, which are the opposites of each other, as shown below:

Time	0	1	2
Investment A	-$100	+$60	+$60
Investment B	+$100	-$60	-$60





Investing vs. Financing (2)

- Investment A is more typical: in this case we have an initial outflow that is (presumably) put to some productive use. The positive proceeds that flow back more than compensate for the initial outlay.
- Investment B is atypical: in this case, perhaps, customers have prepaid for certain products and services; later outflows represent the cost of providing these products and services.
- Note that the IRR is the same in both cases.
- Let us assume the demanded return on these investments is 10%. Based on the IRR rule we would accept both.
- At a rate of 10%, $NPV_A > 0 > NPV_B$, i.e., based on the NPV rule, we should undertake Investment A and not undertake investment B. However, if the demanded rate is above the IRR, then $NPV_B > 0 > NPV_A$, and the situation is reversed.

Investing vs. Financing (3)

- In the case of Investment A, the IRR may be thought of as a rate that you are earning on your money in a "fair world" with no extra profits. You can understand this by analogy with the simple time value of money example when you lend some money at time 0 (and thus have an initial negative cash flow), and then get back a bigger positive cash flow.
- In the case of Investment B, however, you are really **paying** the IRR rate, since you are "borrowing" the initial positive cash flow, which you are repaying later.
- In the case of Investment B, the IRR criterion must be modified: you must undertake the investment only if the demanded rate is **higher** than the IRR. The explanation for this is that your later "repayments" of the initial borrowing have low present value, since they have been discounted aggressively.

Other Investment Criteria

- **Payback rule**: ignores the time value of money, just sums the cash flows from time 0 on until the sum becomes positive (inflows exceed outflows in nominal terms). The time needed to achieve this is the "payback period (time)." An investment is undertaken if the payback period is under a pre-specified threshold.
- **Discount payback rule**: Similar, except that what are added are the present values of estimated future cash flows.
- Both these rules are simple and easy to use. For projects involving smaller amounts over short periods of time in times of low interest rates, they can be practical approaches to investment decisions.
- Other methods are also used; however, their importance has diminished, and as such they are not significant from the perspective of modern quantitative finance.

To Do!

Please read the following sections of your textbook to become acquainted with alternative, lesser used investment criteria:

- Section 9.2: The Payback Rule;
- Section 9.3: The Discounted Payback Rule;
- Section 9.4: The Average Accounting Return;
- Section 9.6: The Profitability Index.

These methods will not be a major focus of our course, as they are simple to understand and - importantly - are of limited practical use in sophisticated, finance-savvy corporate environments. Still, you should be broadly familiar with the terminology of the area so that you can meaningfully participate in conversation when these concepts might come up.