Stock Valuation

AEM 2241 - Finance

Tibor Jánosi

Cornell University - Fall 2023

Outline

Common Stock Valuation

- Next to bonds, stocks are the second major way in which corporations can raise money.
- Shares of common stock are really "slices" of ownership.
- Common stock is harder to value than bonds because ...
  - Stocks have no maturity, even though they have finite life;
  - There are no promised cash flows; neither the size or even their timing is known;
  - The return that the market requires cannot be observed easily;
  - In case something goes wrong (i.e., if default occurs), stock owners will get the residual value of the firm after liquidation. It is not easy to foresee when or whether the company will go bankrupt, and what residual value will be left.

Stocks as Cash Flows

- Most stocks pay dividends; and many do so at predictable intervals, and in predictable amounts.
- If we a share of stock as a stream of dividends of known sizes and known payment times, we can value the share. We'll assume that a dividend D_i will be paid at time t_i in the future, such that times t_i are equally spaced. Further, we assume that the interest rate associated with the per inter-dividend time interval is constant and equal to R.
- But where do we stop? Do the dividends go on forever?

Stocks as Cash Flows (2)

- Assume that you only know about the first dividend. What about all the other dividends? Their aggregate present value computed at time 1 must be equal to the value of the stock at that future time. If P_0 and P_1 are stock prices at times 0 and 1, respectively, we have:

$$P_0 = \frac{D_1}{1+R} + \frac{P_1}{1+R} = \frac{D_1 + P_1}{1+R}.$$

- A similar reasoning applies if we are at time 1 and try to determine the price at time 2 :

$$P_1 = \frac{D_2 + P_2}{1+R} \Rightarrow P_0 = \frac{D_1}{1+R} + \frac{D_2}{(1+R)^2} + \frac{P_2}{(1+R)^2}.$$

- Iterating up to time t we get:

$$P_0 = \sum_{i=1}^{t} \frac{D_i}{(1+R)^i} + \frac{P_t}{(1+R)^t}.$$

Stocks as Cash Flows (3)

- Let us choose a time t far into the future (i.e., a large t). The ratio $\frac{P_t}{(1+R)^t}$ will then be very small; and as t increases, the ratio decreases toward 0. In the limit, we get:

$$P_0 = \sum_{i=1}^{\infty} \frac{D_i}{(1+R)^i}.$$

- **Theoretically**, the potentially infinite stream of dividends is all you need to value the stock, there is no need to know future stock prices.
- Of course, knowing when, and in what amounts a stock will pay dividends in the distant future is not trivial, either.
- Because of the rapidly decreasing discount factors $\frac{1}{(1+R)^i}$, the dividends from the distant future count for very little.

Dividends: Zero Growth

- The formulas are easy to evaluate if dividends stay constant: $D_1 = D_2 = \cdots = D_t = \cdots$.
- We have

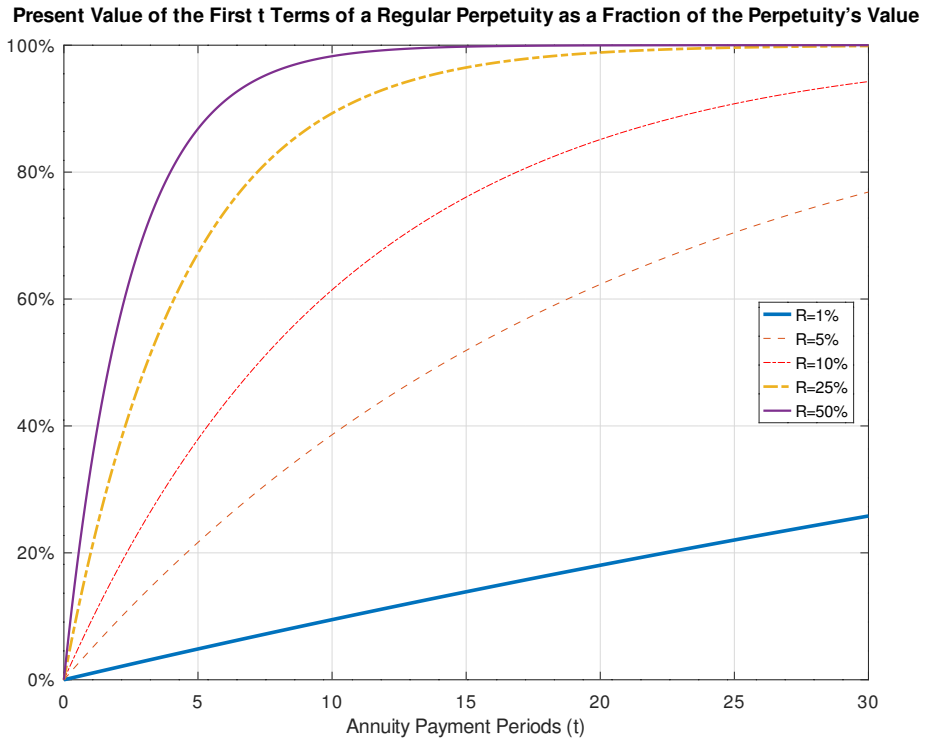
$$P_0 = D \cdot \sum_{i=1}^{t} \frac{1}{(1+R)^i} + \frac{P_t}{(1+R)^t} = D \cdot \frac{1 - \left(\frac{1}{1+R}\right)^t}{R} + \frac{P_t}{(1+R)^t}$$

$$\approx D \cdot \frac{1 - \left(\frac{1}{1+R}\right)^t}{R} \ (\textit{if } t \textit{ is large});$$

$$P_0 = D \cdot \sum_{i=1}^{\infty} \frac{1}{(1+R)^i} = \frac{D}{R}.$$

- It is easy to compute these sums, since dividends are assumed to be constant and paid at constant intervals; hence, they form a regular annuity.

How Far Is Infinity?



Dividends: Constant Growth

- We assume that dividends increase at a constant rate of growth $g < R$ (what if $g \geq R$?).
- Since our discounting applies to whole multiples of inter-dividend periods; it is convenient to assume that we just after the dividend due at time 0, D_0, was paid. Then we have:

$$D_1 = D_0 \cdot (1+g)\,;\; D_2 = D_0 \cdot (1+g)^2\,;\; \ldots;\; D_i = D_0 \cdot (1+g)^i\,.$$

- This time we are dealing with a growth annuity:

$$P_0 = D_1 \cdot \sum_{i=1}^{t} \left(\frac{1+g}{1+R}\right)^i + \frac{P_t}{(1+R)^t} = D_1 \cdot \frac{1-\left(\frac{1+g}{1+R}\right)^t}{R-g} + \frac{P_t}{(1+R)^t}$$

$$\approx D_1 \cdot \frac{1-\left(\frac{1+g}{1+R}\right)^t}{R-g} = D_0 \cdot \frac{1+g}{R-g} \cdot \left[1-\left(\frac{1+g}{1+R}\right)^t\right] \text{ (if } t \text{ is large)};$$

$$P_0 = D_1 \cdot \sum_{i=1}^{\infty} \left(\frac{1+g}{1+R}\right)^i = \frac{D_1}{R-g} = \frac{D_0 \cdot (1+g)}{R-g}.$$

Example: Zero vs. Constant Growth

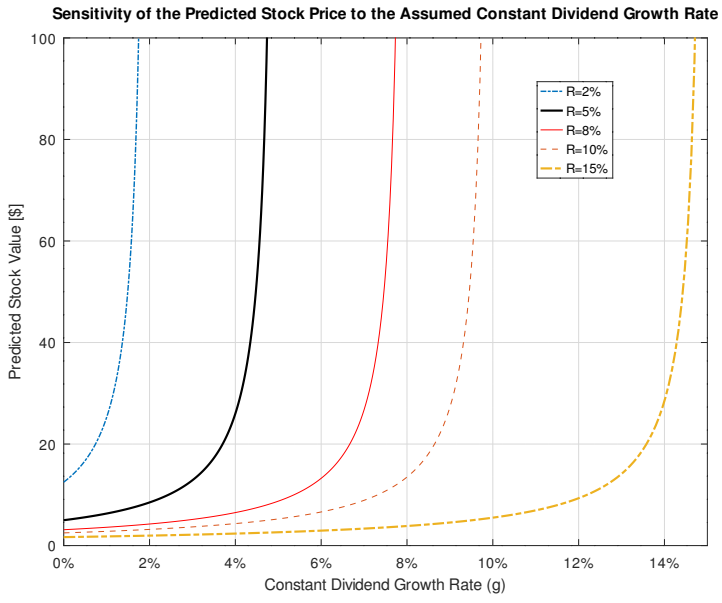
- Suppose that a stock just paid a dividend D_0 of \$2.30, $R = 13\%$. All rates are per year.
- Assuming (for now) that the stock does not grow, the price of the stock is $P_0 = \frac{D}{R} = \frac{2.30}{0.13} = \$17.69$. If we assume that neither the stock price, nor the dividend changes, this price will be the same in later years as well: $P_0 = P_1 = P_2 = \ldots$ What happens to the future price of the stock in terms of present values at time 0?
- Now assume that the dividend of the stock grows at a constant rate of $g = 5\%$. Then we have $P_0 = \frac{D_1}{R-g} = \frac{D_0 \cdot (1+g)}{R-g} = \frac{2.30 \cdot (1+0.05)}{0.13-0.05} = \$30.19$. In the future, the price of the stock will continue to increase, e.g. $P_1 = \frac{D_2}{R-g} = \frac{D_0 \cdot (1+g)^2}{R-g} = \frac{2.30 \cdot (1+0.05)^2}{0.13-0.05} = \$31.70$.

Example: Zero vs. Constant Growth (2)

- Consider again the information given on the prior slide.
- From the general formulas, we notice that $P_1 = P_0 \cdot (1+g)$.
- In general, we have $P_i = P_0 \cdot (1+g)^i$; in other words, the future price of the stocks also increases at the constant rate of g.
- More, the dividend is a constant fraction of the share price: $\frac{D_i}{P_i} = \frac{D_0 \cdot (1+g)^i}{P_0 \cdot (1+g)^i} = \frac{D_0}{P_0}$. One possible interpretation is that if cash flows (dividends) from an investment (buying the share) grow at a constant rate, then the investment's value also increases at a constant rate.
- Now assume that $g = 6\%$; then $P_0' = \frac{D_0 \cdot (1+g)}{R-g} = \frac{2.30 \cdot (1+0.06)}{0.13-0.06} = \$34.83$. A 1% variation in the **estimated** growth induced a relative change of price of $\frac{P_0'}{P_0} = \frac{34.83}{30.19} = 1.15$. Estimates are hard to get right, so model stock prices may be highly uncertain.

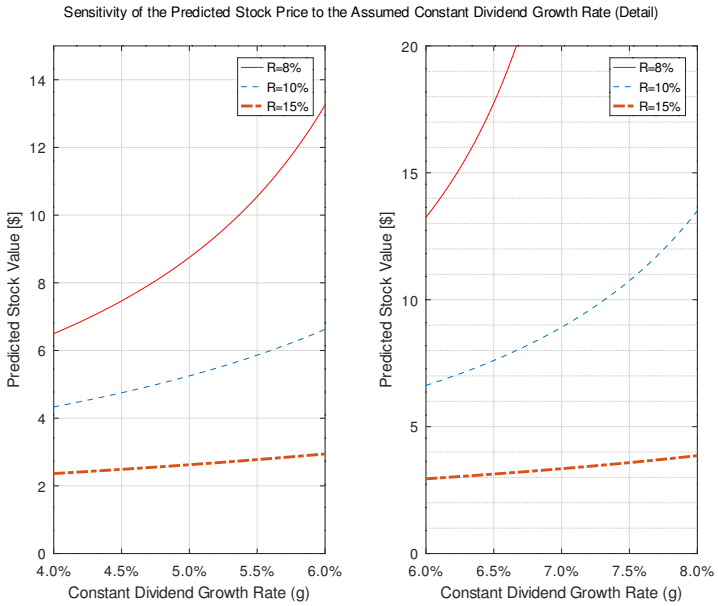
Sensitivity to Growth Rates

- Small changes in g can induce big swings in (model) stock prices as g approaches R from below. The rates shown are per year.



Sensitivity to Growth Rates: Detail

- Note how big the relative price changes can be even within a small estimated range for g. Estimating g with high precision is hard, perhaps outright impossible in many cases.



Dividends: Two Stage Growth

- Assume that dividends growth at a rate g_1 over the first t dividend periods, then they grow at a rate g_2 indefinitely after.
- Such a model allows for rapid growth for a while, followed by more typical, less rapid growth after the company matures.
- The dividend at time 1 is $D_1 = D_0 \cdot (1 + g_1)$, while the dividend at time t is $D_t = D_0 \cdot (1 + g_1)^t$.
- This is a regular growth annuity starting at time 0 and ending at time t, and a growth perpetuity starting at time t:

$$P_0 = D_1 \cdot \frac{1 - \left(\frac{1+g}{1+R}\right)^t}{R - g} + \frac{P_t}{(1+R)^t}$$
$$= D_0 \cdot \frac{1+g_1}{R-g} \cdot \left[1 - \left(\frac{1+g}{1+R}\right)^t\right] + \frac{P_t}{(1+R)^t},$$
$$P_t = \frac{D_{t+1}}{R - g_2} = \frac{D_t \cdot (1+g_2)}{R-g_2} = \frac{D_0 \cdot (1+g_1)^t (1+g_2)}{R-g_2}.$$

Example: Two Stage Growth

- A company is undergoing fast growth. It can be predicted that its current dividend of \$2.5 will grow by 25% per year for 5 years, but then growth will stabilize and a more sedate dividend growth of 3% per year is expected. If $R = 10\%$, what is the price of the stock?
- The price of the stock in five years will be:

$$P_t = \frac{D_{t+1}}{R - g_2} = \frac{D_t \cdot (1 + g_2)}{R - g_2} = \frac{D_0 \cdot (1 + g_1)^t (1 + g_2)}{R - g_2}$$

$$P_5 = \frac{2.5 \cdot (1 + 0.25)^5 \cdot (1 + 0.03)}{0.10 - 0.03} = \$112.26,$$

$$\begin{aligned} P_0 &= D_0 \cdot \frac{1 + g_1}{R - g_1} \cdot \left[1 - \left(\frac{1 + g_1}{1 + R}\right)^t\right] + \frac{P_t}{(1 + R)^t} \\ &= 2.5 \cdot \frac{1 + 0.25}{0.10 - 0.25} \cdot \left[1 - \left(\frac{1 + 0.25}{1 + 0.10}\right)^5\right] + \frac{112.26}{(1 + 0.10)^5} \\ &= 18.64 + 69.70 = \$88.34. \end{aligned}$$

Dividends: Non-Constant Growth

- Sometimes a firm evolves so quickly in the short term that is impossible to fit its near-term dividend patterns to a model like the ones discussed above. Dividends may be neither constant, and they may not be growing at a constant rate.
- In this case we choose a time horizon t up to which we allow for each dividend to be potentially different, and use their estimated numerical values. Beyond the time horizon t we use a dividend model to estimate P_t, which we discount back to time 0.

$$P_0 = \sum_{i=1}^{t} \frac{D_i}{(1 + R)^i} + \frac{P_t}{(1 + R)^t}.$$

Example: Non-Constant Growth

- Assume that a new public company is doing well; it will start paying a dividend of \$1 next year, followed by dividends of \$2 and \$2.5 in the subsequent years. After that, dividends are expected to grow indefinitely at a rate of 3% per year. Assuming that $R = 10\%$, what is the value of the stock?
- First, we compute the value of the stock at the end of year 3:

$$P_3 = \frac{D_4}{R-g} = \frac{D_3 \cdot (1+g)}{R-g} = \frac{2.5 \cdot (1+0.03)}{0.10-0.03} = \$36.79$$

$$\begin{aligned} P_0 &= \sum_{i=1}^{t} \frac{D_i}{(1+R)^i} + \frac{P_t}{(1+R)^t} \\ &= \frac{1.00}{1+0.10} + \frac{2.00}{(1+0.10)^2} + \frac{2.50}{(1+0.10)^3} + \frac{36.79}{(1+0.10)^3} \\ &= 0.91 + 1.65 + 1.88 + 27.64 \\ &= \$32.08. \end{aligned}$$

No Dividends: Worthless Stocks?

- Some stocks do not pay dividends. Examples include famous companies like Google/Alphabet.
- A simplistic approach would imply that the price of such stocks must be 0 (see, for example, the constant dividend model, or the constant growth dividend model).
- No-dividend stocks are often highly valued, however. Where is the contradiction?
- Investors implicitly assume that these firms will start paying dividends at some future time t. If we assume that we value the firm at that time, we get a non-zero price, which we can discount back to time 0. Starting at time t any dividend model can be used, as discussed above.

No Dividends: Worthless Stocks? (2)

- Firms that are facing difficulties, including the possibility of default/bankruptcy, may also not pay dividends.
- These firms, however, do not have an infinite time horizon; quite contrary, they may be liquidated quickly.
- The stock of these companies may still be worth something, assuming that there is leftover residual value after the firm is liquidated and its debts are paid.
- Firms can also temporarily decrease or suspend dividends in order to conserve cash. Investors often attribute negative significance to decreasing or unpaid dividends; we say that dividends "have signaling value."

Required Return

- Consider the constant dividend growth model discussed above:

$$P_0 = \frac{D_1}{R-g} = \frac{D_0 \cdot (1+g)}{R-g},$$

where D_0 is the dividend that was just paid, and D_1 is the next dividend that will be paid at the end of the current dividend period.

- If we observe P_0, the market price of the stock, we know D_1, and we have a good estimate of g, we can compute the **required return** R as

$$R = \frac{D_1}{P_0} + g,$$

where $\frac{D_1}{P_0}$ is the **dividend yield** and g is the **capital gains yield** (or **stock price appreciation**).

Example: Required Return

- In real life, stocks are not simple investments. In order to invest, one needs to assume many risks. This will only happen if investors are properly compensated. One way to think about this is to say that investors "demand" a certain return R on their investment.
- The market, in aggregate, determines R, but R is not directly observable. We infer R from stock information and market prices.
- A stock sells for \$20 a share; it will next pay a dividend of \$1. The dividend is expected to grow at a yearly rate of 4% forever. What is the implied required return R?

$$R = \frac{D_1}{P_0} + g = \frac{1}{20} + 0.04 = 9\%.$$

- In aggregate, investors in this stock demand a return of 9% per year.

Valuation Using Multiples

- Dividend models have obvious limits, including the fact that valuation is hard in the absence of dividends, or in case of uncertain long-term dividend policies.
- A common way to value stocks is based on the **PE ratio**, known formally as the **price-per-earnings** ratio.
- The formula typically used is

$$P_t = (\text{benchmark PE ratio}) \cdot EPS_t,$$

  where EPS_t are the **earnings per share** estimated for time t.
- The PE benchmark quantifies the answer to the question: How much is it worth paying for one dollar of earnings per share, for a company like this (at the time for which we estimate the price)?
- PE benchmarks are often based, at least in part, on comparisons with similar firms, or on comparisons with industry averages, both current and historical.

Valuation Using Multiples (2)

- Earnings per share from the past are known, in principle, from the company's financial reporting. EPS is an accounting measure, and as such it may be impacted by changes in accounting procedures and by the limits of accrual accounting to capture the true economic state of a firm. In addition there may be one-time positive and negative effects embedded in historical EPS numbers.
- Analysts spend a lot of time estimating future EPS, which involves, in part, undoing one-time effects in historical EPS values, undoing accounting artifacts (distortions), and accounting for future changes in the functioning of the firm and its economic environment.
- It is not atypical for an analyst to use several valuation models, and to produce multiple estimates of stock prices. These estimates may be close, or they may be wildly different.

Using Financial Models

- Since each model can misfire, analysts must understand the characteristics of their models in order to apply judgment to numerical results. One must not be lured into complacency just because one can produce numbers that appear precise.
- If models produce wildly different results, skepticism should be heightened, and the correctness of the models, implementation and data, should be re-examined.
- Results that contradict common sense should be treated with extreme care. Skepticism and human judgment are paramount when using quantitative models.
- Some models may perform better that others under certain economic or market conditions. It may be that using different models at different times provides a better insight into the underlying truth of economic phenomena.
- **Do no believe your own press releases!**

Characteristics of Common Stocks

- Common stock is stock that gives holders no preferences either in receiving dividends or in bankruptcy.
- Common stock typically gives holders the right to...
  - ... elect corporate directors;
  - ... share proportionally in dividends paid;
  - ... share proportionally in the proceeds after the company's creditors have paid off and the company was liquidated;
  - ... share proportionally in any new share issues (preemptive right);
  - ... vote on corporate matters of great importance (e.g. mergers).
- A corporation may have several classes of stock, not all of which may have the same rights. For example, certain classes may give diminished, or no voting rights to their holders. This is an important matter of corporate governance.

Stock Dividends

- Common stocks may pay dividends or not, at the discretion of the corporation's directors. There is no legal obligation to pay dividends, unless they have been "declared." Undeclared dividends are not a liability of the corporation.
- Dividends provide a signal about the financial health of a corporation. Reducing or suspending dividend payments is typically a sign of major corporate distress.
- One-time "special" or "extraordinary" dividends are possible.
- Dividends are not tax-deductible for the paying corporation. In addition, they are also taxed at the individual level. Consequently, corporations try to find more tax-favorable ways to return money to their investors (e.g., share repurchases).

Preferred Stock

- Preferred stock is treated preferentially when paying dividends and when distributing proceeds left over from the corporation's liquidation.
- The board of directors may decide not to pay preferred dividends; if so, dividends are not paid on common stock either.
- Unpaid dividends are typically cumulative (they are carried forward as arrearage); however, these obligations are not treated as corporate debt.
- Preferred stock holders may gain rights, such as voting rights, in case of dividend non-payment. This may allow them to influence the board of directors to pay off the dividends.
- Preferred dividends are taxed like regular dividends.

Important: To Do!

Read the following sections from chapter 8 of your textbook:

- 8.2: Some Features of Common and Preferred Stocks;
- 8.3: Stock Markets.