

Lecture 3

• Results
• Roll your name
• Events + res
• Side puzzles
• Distributions

Ex: if-7 series (first to 4 wins)
• A gets 4 wins, B gets 3
• How many wins up to 7?
• Why does schedule matter?

Ex: coin puzzle sketch
• Each coin is $U[0,1]$, value $U \sim \text{Unif}[0,1]$
• Flip 10 times, count $N = \# \text{ heads}$
• Q: Dist of N ? Ex: $P(N=5) = ?$

Ex: 2n flips a % coin twice (heads prob. 1/2, resp. 1/4)
 $\Omega_1 = \{00, 01, 10, 11\}$
 $P_1(\{00\}) = \frac{1}{4}$
 $P_1(\{01\}) = \frac{1}{4}$
 $P_1(\{10\}) = \frac{1}{4}$
 $P_1(\{11\}) = \frac{1}{4}$
 $U_1(0)=0$
 $U_1(1)=1$
 $N_1(0)=1$
 $N_1(1)=2$

Ex: 2n simulator flip a % coin twice
Source of randomness: RNG \rightarrow Samples from $\text{Unif}[0,1]$
 $\Omega_2 = [0,1]^2 = [0,1] \times [0,1]$
 $P_2(A) = \text{area of } A$
 $U_2 = \text{area}$
 $N_2(0)=1$
 $N_2(1)=2$

Random Variables
Random variable is a fn. $\Omega \rightarrow S$ for some set S
Ex: $N = \# \text{ heads}$
 $N_1(\omega) = \begin{cases} 0 & \text{if } \omega = 00 \\ 1 & \text{if } \omega = 01 \\ 2 & \text{if } \omega = 10 \\ 2 & \text{if } \omega = 11 \end{cases}$
realization of $N_1(\omega)$

Eric's coin puzzle sketch:

U_0, \dots, U_{10}
 δ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$ $\begin{bmatrix} \uparrow +1 \\ \downarrow -1 \end{bmatrix}$
 $N(\omega_0, \dots, \omega_{10}) = \# \text{ } \omega_i \text{'s } \omega_0 \text{ (} i \geq 1 \text{)}$