

" $r\%$ per year, compounded m times"

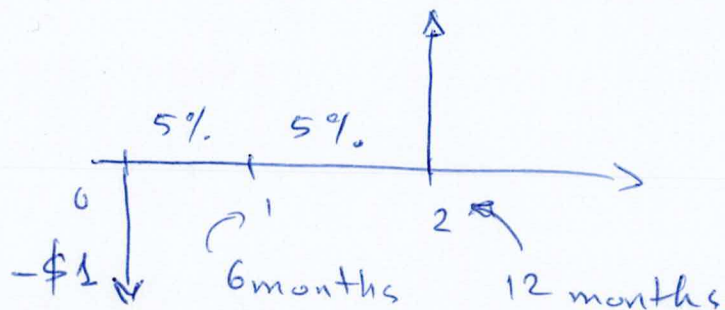
- period is $\frac{1}{m}$ years

- per-period interest rate is $\frac{r}{m}$

Example:

10% per year, compounded $\begin{cases} 2 \text{ times per year} \\ \text{semi-annually} \end{cases}$

$\rightarrow 5\%$ per $\frac{1}{2}$ year (6 months)



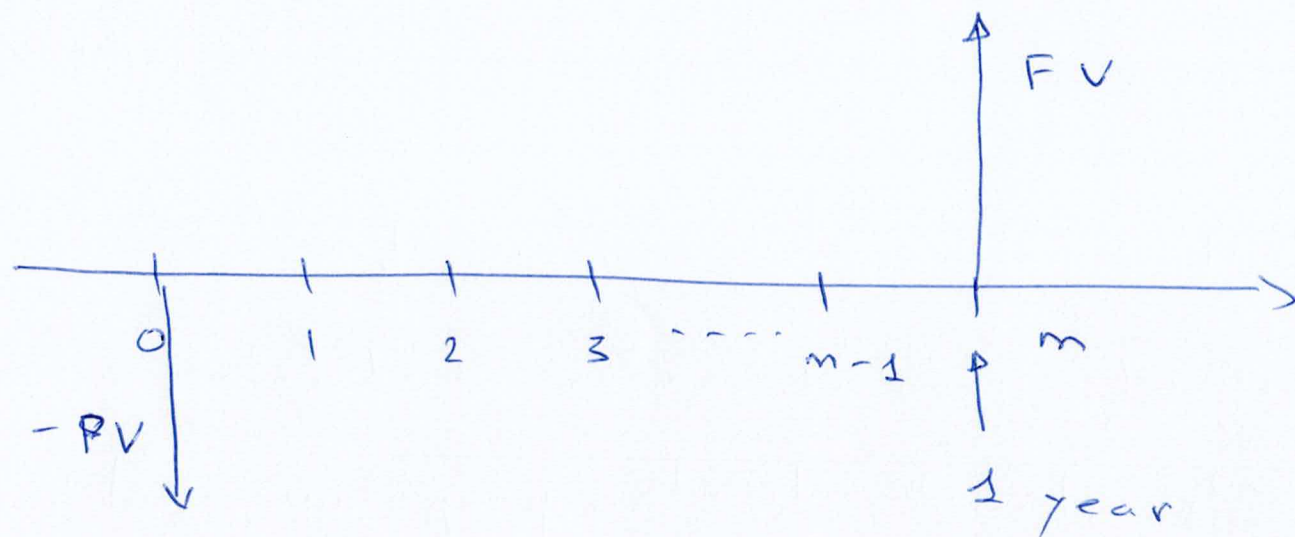
$$FV = \$1 \cdot \left(1 + \frac{10\%}{2}\right)^2$$

$$= 1.05^2 \approx 1.1025$$

earned 10.25%

Why?

- ① marketing = interest rates appear lower
- ② can calculate interest over shorter periods
w/o worrying about fractional periods.
 - ↳ daily compounding - compound 365 times per year.



$$FV = PV \cdot \left(1 + \left(\frac{r}{m}\right)^m\right) \quad \leftarrow \text{per-period interest rate}$$

$$\text{interest earned} = FV - PV$$

$$= PV \cdot \left[\left(1 + \frac{r}{m}\right)^m - 1 \right]$$

$$FV = PV \cdot (1 + \text{EAR}) \rightarrow \text{want same FV}$$

$$\text{EAR} = \left(1 + \frac{r}{m}\right)^m - 1$$

Monthly compounding:

$$r = 10\%$$

$$m = 12$$

$$EAR = \left(1 + \frac{10\%}{12}\right)^{12} - 1.$$

$$= \left(1 + \frac{0.10}{12}\right)^{12} - 1.$$

$$= 0.10471307$$

$$\approx 10.47\%$$

Calculator:

$$N = 12$$

$$i/Y = \frac{10}{12} \rightarrow \frac{10}{12} = 0.833\%$$

$$PV = -\$1.$$

$$PMT = 0$$

$$FV = \boxed{CPT}$$

$$FV = 1.1047...$$

$$EAR = .1047 = 10.47\%$$

EAR = 25% per year

→ use a large n (many compounding periods)

typically: $m = 365$

what is r ? Stated interest rate.

$$\text{EAR} = \left(1 + \frac{r}{m} \right)^m - 1$$

\uparrow 25% \uparrow 365

$$r = m \left[(1 + EAR)^{\frac{1}{m}} - 1 \right] = 365 \cdot \left[(1 + .25)^{\frac{1}{365}} - 1 \right] \approx 22.32\%$$

Just using the calculator:

$$n = 365, PV = -\$1, FV = \$1.25, PMT = 0$$

CPT i/Y

$$r = 12\%$$

$$m = 4$$

$$EAR = \left(1 + \frac{.12}{4}\right)^4 - 1$$

$$= 0.125508$$

$$\approx 12.55\%$$

quarterly
compounding

$$r = 12\%$$

$$m = \infty$$

$$EAR = e^{.12} - 1$$

$$= 0.127496$$

$$\approx 12.75\%$$

continuous
compounding.

Calculator:

$$r = 12\%$$

$$m = 12$$

$$\frac{r}{m} = \frac{.12}{12} = .01 = 1\%$$

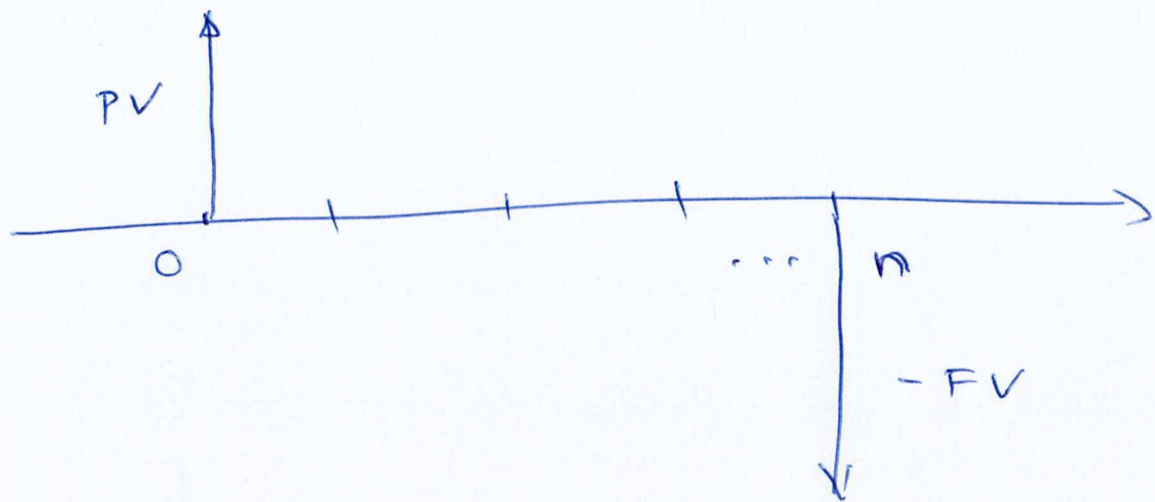
$$N = 12 \quad PV = -\$1 \quad \boxed{CPT \ FV}$$

$$i/y = 1 \quad PMT = 0 \quad FV = 1.126825$$

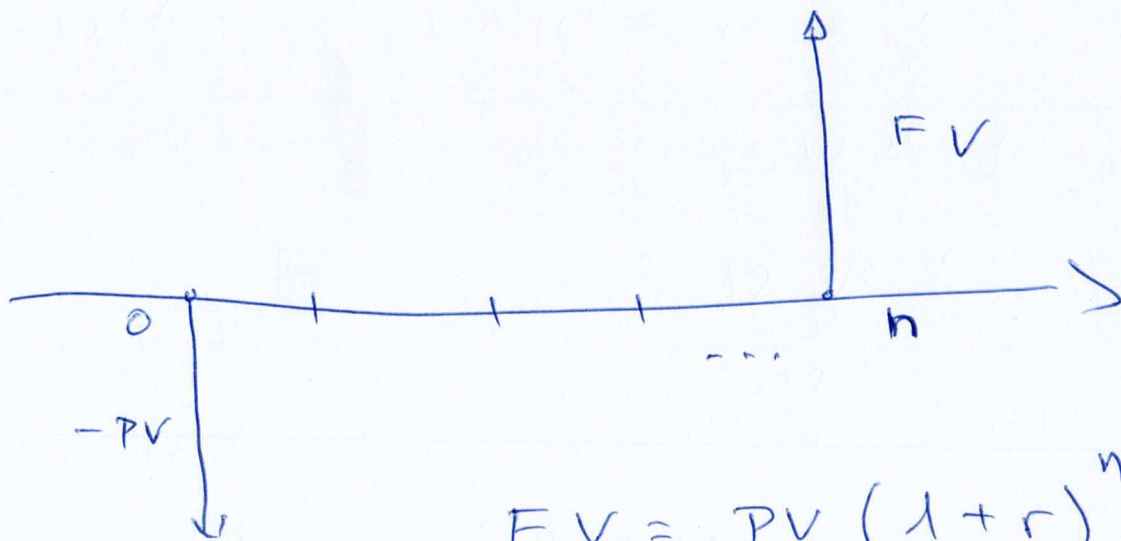
$$FV = \$1 \cdot (1 + EAR) \rightarrow EAR = 12.68\%$$

monthly
compounding
EAR obtained
using calculator

PURE DISCOUNT LOAN



"borrower"



"lender"

$$FV = PV(1+r)^n$$

$$PV = \$5,000$$

$$n = 5 \text{ years}$$

$$r = 9\% \text{ (compounded annually)}$$

$$C = ?$$

$$\textcircled{PV} = C \cdot \underbrace{\frac{1 - \left(\frac{1}{1+r}\right)^{\textcircled{n}}}{\textcircled{r}}}_{\text{PV factor}}$$

Calculator: $N = 5$ $PV = +\$5,000$
 $i/y = 9$ ~~PMT~~ $FV = 0$

~~CMT~~ CPT PMT \Rightarrow

$$\underline{\underline{PMT = -\$1,285.46}}$$