1 Installation

Install gsl library which can be found on `http://www.gnu.org/software/gsl/`. Thereafter install the optimization package from `http://ool.sourceforge.net/`. Finally, compile contin.c within the matlab-shell

```
>> mex -lool -lgsl -lgslcblas -lm contin.c
```

2 Description

Contin based on an algorithm originally developed by by Stephen W. Provencher to compute numerically the inverse of an integral kernel operation. For given Kernel $K(t,s)$ and given function $y(t)$ we want to know the function $g(s)$ defined by

$$y(t) = \int_{-\infty}^{\infty} K(t,s) g(s) ds + b \tag{1}$$

For a given arbitrary y it is not certain to solve the equation in an exact manner, however, the best possible solution can be found by a minimization of the following expression

$$\min_g \left\{ \| y - (Kg + b) \| \right\}^2 + \alpha^2 \| g'' \| \tag{2}$$

involving the the second derivative of g allows to adjust the smoothness of g. A Small values of α generates a spiky g. Whereas a large value of α causes a smooth g. Discretizing the above minimization gives

$$\min_g \left\{ \sum_i w_i \left[y_i - \left(\sum_j c_j K_{i,j} g_j + b \right) \right]^2 + \alpha^2 \sum_i (g_i'')^2 \right\} \tag{3}$$

Due to the fact that y_i is a measured quantity it has a variance σ_i^2. Therefore, we use weights $w_i = 1/\sigma_i^2$. The second derivative can be approximated up to second order by the discretization $g_i'' = g_{i-1} - 2g_i + g_{i+1}$.

3 Usage

Within Matlab the algorithm can be invoked by

```
>> [s, g, b] = contin(t, y, var, s0, s1, m, alpha, kernel);
```

t and **y** are the observed data points and **var** is the variance of y. $[\mathbf{s_0}, \mathbf{s_1}]$ is the interval within we try to find $\mathbf{g}(\mathbf{s})$. We take **m** sampling points for the quadrature of the integration. The parameter α reflects the strength of the regularizer. For **kernel** there are the following choices

$$
\begin{aligned}
0 : K(t,s) &= e^{-t/s} && (4)\\
1 : K(t,s) &= \frac{1}{\pi}\frac{s}{t^2+s^2} && (5)
\end{aligned}
$$

4 Application

One application is to find the half-widths σ_n of a multi-lorentzian signal

$$y(t) = \sum_n g_n \cdot \frac{1}{\pi}\frac{\sigma_n}{t^2+\sigma_n^2} \tag{6}$$

a more general form of the above sum expression is to write it as a kernel integration

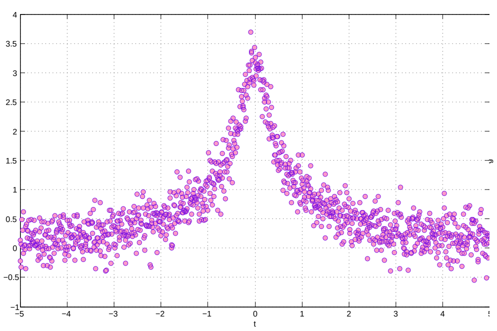
$$y(t) = \int g(s)\frac{1}{\pi}\frac{\sigma}{t^2+\sigma^2}d\sigma \tag{7}$$

As an example we look at a signal consisting of two lorentz profiles

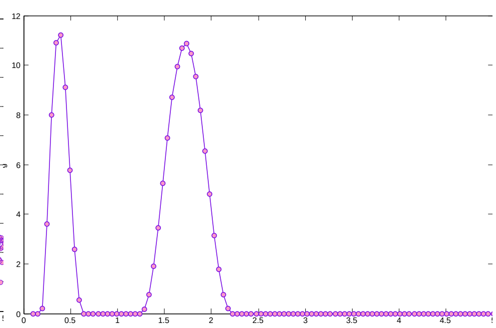
$$y(t) = 3 \cdot \frac{1}{\pi}\frac{0.4}{t^2+0.4^2} + 5 \cdot \frac{1}{\pi}\frac{2}{t^2+2^2} \tag{8}$$

Matlab code

```
>> x = -5 : 0.01 : 5;
>> y = 3*1/pi*0.4./(x.^2 + 0.4^2) + 5*1/pi*2./(x.^2 + 2^2);
>> dy = 0.25 * randn(1, length(y));
>> var = 0.25^2*ones(1, length(y));
>> [s, g, b] = contin(x, y+dy, var, 0.1, 5, 50, 0.1, 1);
```



(a) Noisy lorentz signal



(b) HWFM spectrum

Figure 1: (a) Signal consisting of two lorentz functions with HWFM of $\sigma_1 = 0.4$ and $\sigma_2 = 2.0$ and intensities $g_1 = 3.0$ and $g_2 = 5.0$ respectively. Furthermore, a gaussian noise of variance $\sigma^2 = 0.125$ was added to the signal. (b) Applying contin algorithm we obtain a HWFM spectrum of the signal revealing the two lorentzians and the HWFM at $\sigma = 5$ can be attributed to a constant background.