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1. Find the values of p for which the integral converges

    (a) $\displaystyle\int_1^2 \frac{1}{x\,(\ln x)^p}\,dx$

    (b) $\displaystyle\int_2^\infty \frac{1}{x\,(\ln x)^p}\,dx$
    (Ans: (a) $p < 1$, (b) $p > 1$)

2. Estimate each of the following definite integrals using the Trapezoidal Rule with $n = 4$.

    (a) $\displaystyle\int_1^2 x\,dx$ (b) $\displaystyle\int_1^3 (2x - 1)\,dx$

    (Ans: (a) $\frac{3}{2}$, (b) 6)

3. Estimate each of the following definite integrals using Simpson's Rule with $n = 4$.

    (a) $\displaystyle\int_{-1}^1 \left(x^2 + 1\right)\,dx$ (b) $\displaystyle\int_{-2}^0 \left(x^2 - 1\right)\,dx$

    (Ans: (a) $\frac{8}{3}$, (b) $\frac{2}{3}$)

4. Prove that the volume of the cone with height h and radius r is $\frac{1}{3}\pi r^2 h$.

5. (a) The equation of a circle with center at the origin and radius r is described by the equation $x^2 + y^2 = r^2$. Use integration to prove that the area of the circle is πr^2.

    (b) When the region bounded by the x-axis and the curve $y = \sqrt{r^2 - x^2}$ for $-r \le x \le r$ is rotated about the x-axis, a sphere with radius r is obtained. Use integration to prove that the volume of the sphere is given by $\frac{4}{3}\pi r^3$.

6. Use integration by substitution to prove the following.

    (a) $\displaystyle\int \tan x\,dx = \ln|\sec x| + C$

    (b) $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$

    (c) $\displaystyle\int \sin^3 x \cos^8 x\,dx = -\frac{\cos^9 x}{9} + \frac{\cos^{11} x}{11} + C$