Testing and Verification of VLSI
EE709

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1 Student Details

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2 Question 1

2.1 Calculating the product of sums formula (CNF) which describes this network.

CNF (product of sums formula)form is given below :

$$K = (\overline{\overline{AB} + \overline{BC}}) \oplus (\overline{\overline{BC} + \overline{CD}})$$

As it is XOR, we can also write it as-

$$(\overline{AB} + \overline{BC}) \oplus (\overline{BC} + \overline{CD})$$

$$((\overline{A} + \overline{B}) + (\overline{B} + \overline{C})) \oplus ((\overline{B} + \overline{C}) + (\overline{C} + \overline{D}))$$

$$(\overline{A} + \overline{B} + \overline{C}) \oplus (\overline{B} + \overline{C} + \overline{D})$$

$$\overline{A}BCD + ABC\overline{D}$$

$$BC(\overline{A}D + A\overline{D})$$

$$B.C.(A + D).(\overline{A} + \overline{D})$$

CNF is -

$$K = B.C.(A + D).(\overline{A} + \overline{D})$$

2.2 Using the minisat solver, find an input assignment such that the output K is 0

To solve the above, we will find the negation CNF form of the above to get

$$\overline{K} = \overline{\overline{A}BCD + ABC\overline{D}}$$

$$\overline{K} = (A + \overline{B} + \overline{C} + \overline{D}).(\overline{A} + \overline{B} + \overline{C} + D)$$

This CNF form in the minisat solver with the following code:

```
c
c   nvars nclauses
p cnf 4 2
c
c   -2 means (~x2), 2 means x2
c   A = x1, B = x2, C = x3, D = x4
```

```
c
c   x1 + x2 + x3 + ~x4
1 2 3 -4 0
c
c   ~x1 + x2 + x3 + x4
-1 2 3 4 0
```

Result -
SAT
-1 -2 -3 -4 0

The equation K = 0 can be satisfied using -

$$\overline{A}, \overline{B}, \overline{C}, \overline{D}$$

which is, as given by SAT solver

$$A = 0, B = 0, C = 0, D = 0$$

2.3 Using the minisat solver, find an input assignment such that the output K is 1.

We insert the above CNF form of K in the minisat solver with the following code:

$$K = B.C.(A + D).(\overline{A} + \overline{D})$$

```
c
c   nvars nclauses
p cnf 4 4
c.
c   -2 means (~x2), 2 means x2
c   A = x1, B = x2, C = x3, D = x4
c
c   x2
2 0
c
c   x3
3 0
c
```

```
c   x1 + x4
1 4 0
c
c   (~x1) + (~x4)
-1 -4 0
```

Result - SAT
-1 2 3 4 0

The equation K = 0 can be satisfied using SAT solver -

$$\overline{A}, B, C, D$$

which is,

$$A = 0, B = 1, C = 1, D = 1$$

3 Question 2

3.1 Method

Suppose the machine starts in the state $P = Q = R = S = 1$. Using minisat, finding a sequence of input values at X which will take the machine to the state $P = Q = R = S = 0$.

Let $P(k), Q(k), R(k), S(k)$ be the state variables at time k. Also, let $X(k)$ be the input at time k. Note that $Y(k) = P(k) \oplus S(k)$.

To solve the problem, we observe:

1. At $k = 0, P(k) = Q(k) = R(k) = S(k) = 1$. That is, at $k = 0$, the following function is true.

$$C_0 = P(0).Q(0).R(0).S(0)$$

2. At some time N, we want $P(N) = Q(N) = R(N) = S(N) = 0$. That is, at time N, the following function is true:

$$D_N = \overline{P(N)} \cdot \overline{Q(N)} \cdot \overline{R(N)} \cdot \overline{S(N)}$$

Further, we have -

$$\begin{aligned}
P(k+1) &= X(k) \oplus P(k) \oplus S(k) \\
Q(k+1) &= P(k) \\
R(k+1) &= Q(k) \\
S(k+1) &= R(k)
\end{aligned}$$

To find the solution, we proceed as follows. At any $k > 0$, the relation between the state variables at time k and those at time $k-1$ is described by the function

$$\begin{aligned}
\phi_k = &(P(k) = X(k-1) \oplus P(k-1) \oplus S(k-1)) \\
&\cdot (Q(k) = P(k-1)) \\
&\cdot (R(k) = Q(k-1)) \\
&\cdot (S(k) = R(k-1))
\end{aligned}$$

Thus, to see if it possible to find an input sequence which takes us to state $P(N) = Q(N) = R(N) = S(N) = 0$ at some time $N > 0$, we build the following function (for $N = 1, 2, \ldots, 15$)[1].

$$\Theta_N = C_0 \cdot \phi_1 \cdot \phi_2 \ldots \phi_N \cdot D_N$$

and check if it is satisfiable.

3.2 Solution

We define the above-mentioned variables at different N and find the CNF forms of each of the above. Next, we keep on adding our code for N = 1,2,3,4... and check the resultant output for each case until we get our desired output as P(N)=Q(N)=R(N)=S(N)=0.

3.3 Code

```
c   nvars nclauses
p cnf 10 22
c   -2 means (~x2), 2 means x2
c   X(0)=1, P(0)=2, Q(0)=3, R(0)=4, S(0)=5
c   X(1)=6, P(1)=7, Q(1)=8, R(1)=9, S(1)=10
```

```
c   At time N=0 (phi_0)
c   P(0)
2 0
c   Q(0)
3 0
c   R(0)
4 0
c   S(0)
5 0
c   At time N=1 (phi_1)
c   ~X(0) ~P(0) ~S(0) P(1)
-1 -2 -5 7 0
c   X(0) P(0) ~S(0) P1
1 2 -5 7 0
c   X(0) ~P(0) S(0) P(1)
1 -2 5 7 0
c   ~X(0) P(0) S(0) P(1)
-1 2 5 7 0
c   X(0) ~P(0) ~S(0) ~P(1)
1 -2 -5 -7 0
c   ~X(0) P(0) ~S(0) ~P1(1)
-1 2 -5 -7 0
c   ~X(0) ~P(0) S(0) ~P(1)
-1 -2 5 -7 0
c   X(0) P(0) S(0) ~P(1)
1 2 5 -7 0
c   Q(1) + ~P(0)
8 -2 0
c   ~Q(1) + P(0)
-8 2 0
c   R(1) + ~Q(0)
9 -3 0
c   ~R(1) + Q(0)
-9 3 0
c   S(1) + ~R(0)
10 -4 0
c   ~S(1) R(0)
-10 4 0
```

```
c   n=N=1 (D_n)
c   ~P(1)
-7 0
c   ~Q(1)
-8 0
c   ~R(1)
-9 0
c   ~S(1)
-10 0
```

There is no values satisfying the above equation as we get its output as **Output for N = 1** UNSAT
Similarly, we write code for N = 2,3,4 and observe the outputs for then:
Output for N = 2 UNSAT
Output for N = 3 UNSAT

For N=4, the following values are satisfying the conditions P(N)=Q(N)=R(N)=S(N)=0.

```
SAT
-1 2 3 4 5 6 -7 8 9 10 11 -12 -13 14 15 16 -17 -18 -19 20 -21 -22 -23 -24 -25 0
```

4 Conclusion

Hereby the assignment is completed using the minisat solver.