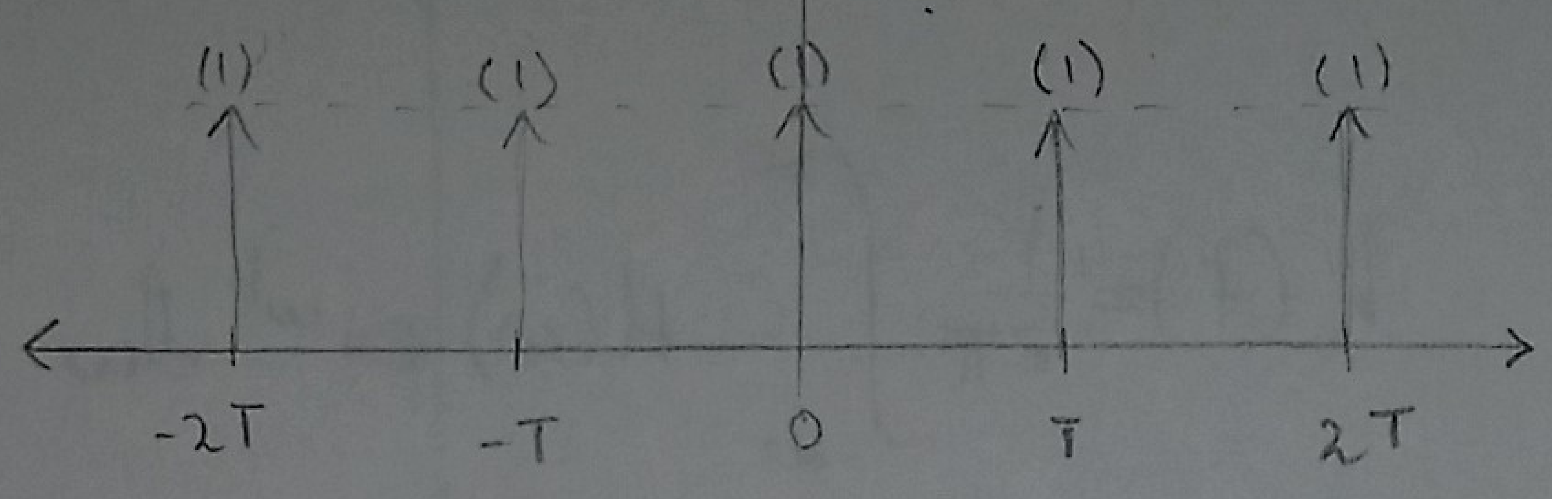
PS07

1.

a.



b. $X(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$ $\qquad C_k = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$

$$P(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T}\, e^{j\frac{2\pi}{T}kt}$$

c. $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$ $\qquad X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$$C_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-j\frac{2\pi}{T}kt}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}\, e^{-j\omega t}\, dt$$

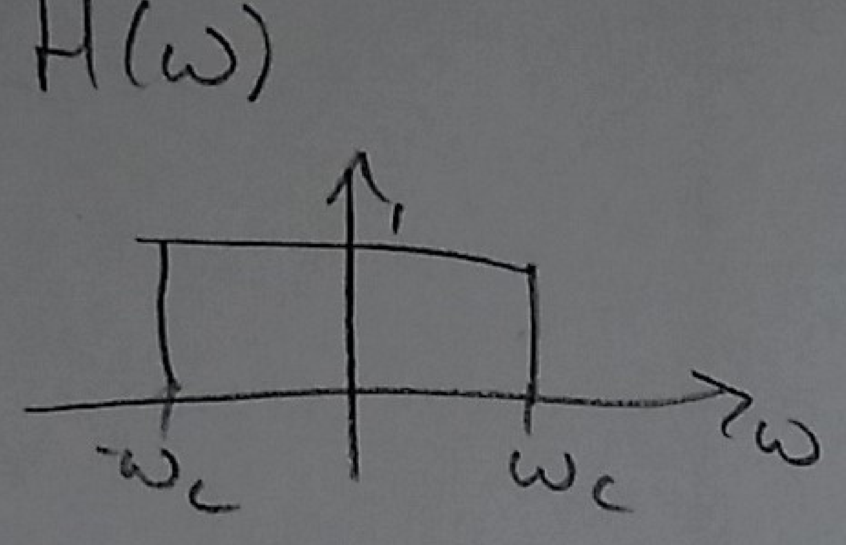
$$X(\omega) = \int \sum C_k e^{j\frac{2\pi}{T}kt - j\omega t}\, dt$$

$$\int \sum C_k e^{j\omega kt - j\omega t}\, dt$$

$$X(\omega) = \int \sum C_k e^{j\omega t(k-1)}\, dt$$

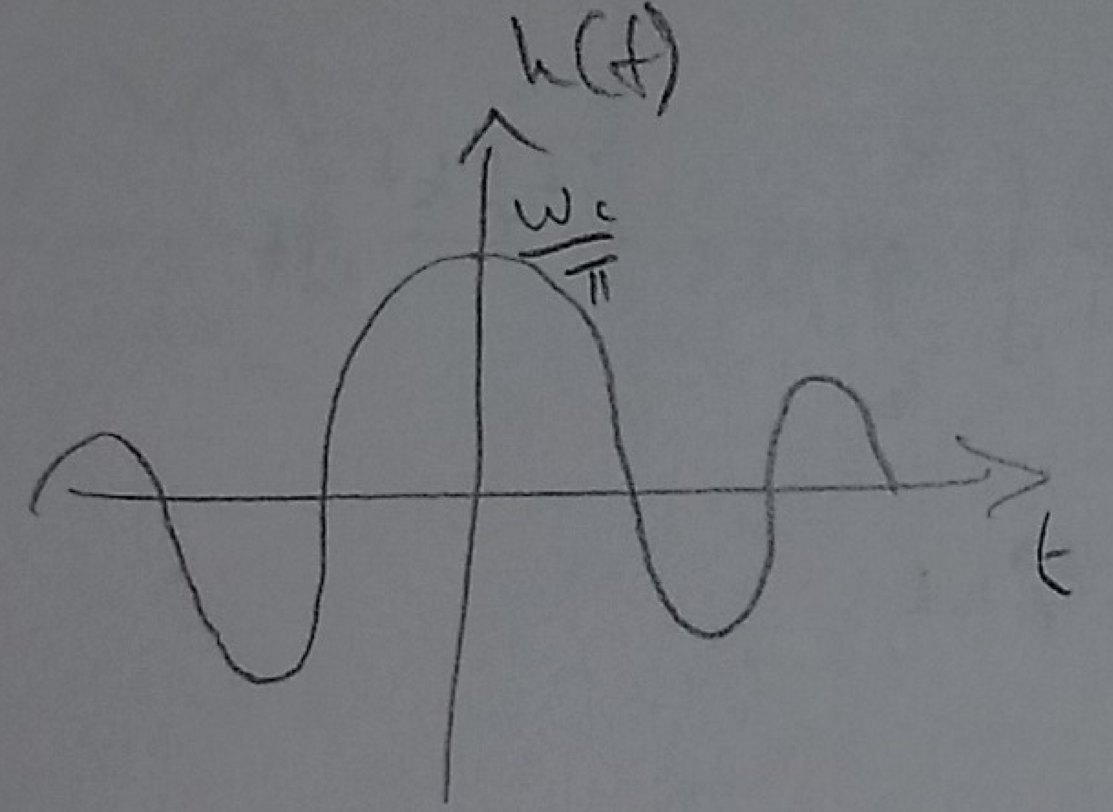
2.

a.

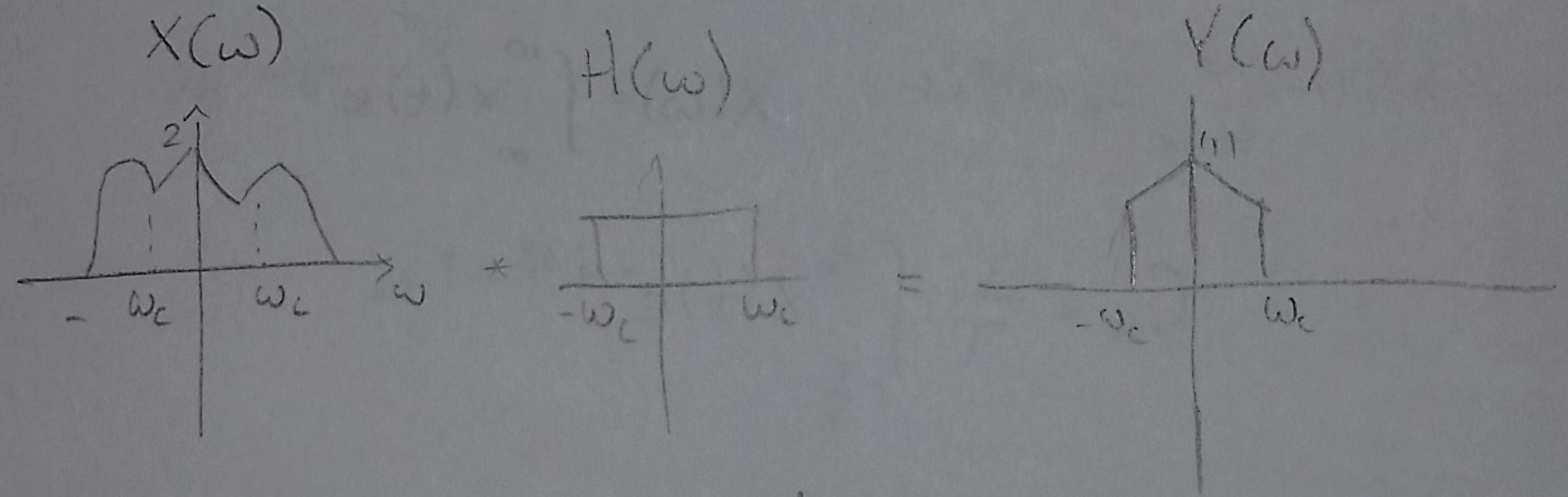


$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{\sin(\omega_c t)}{\pi t}$$

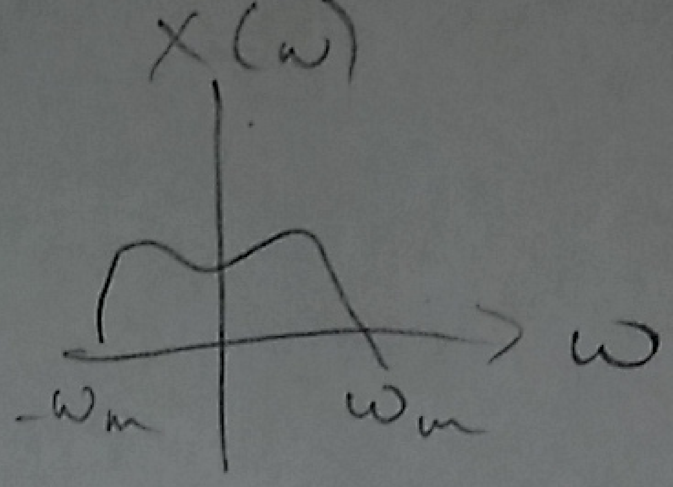


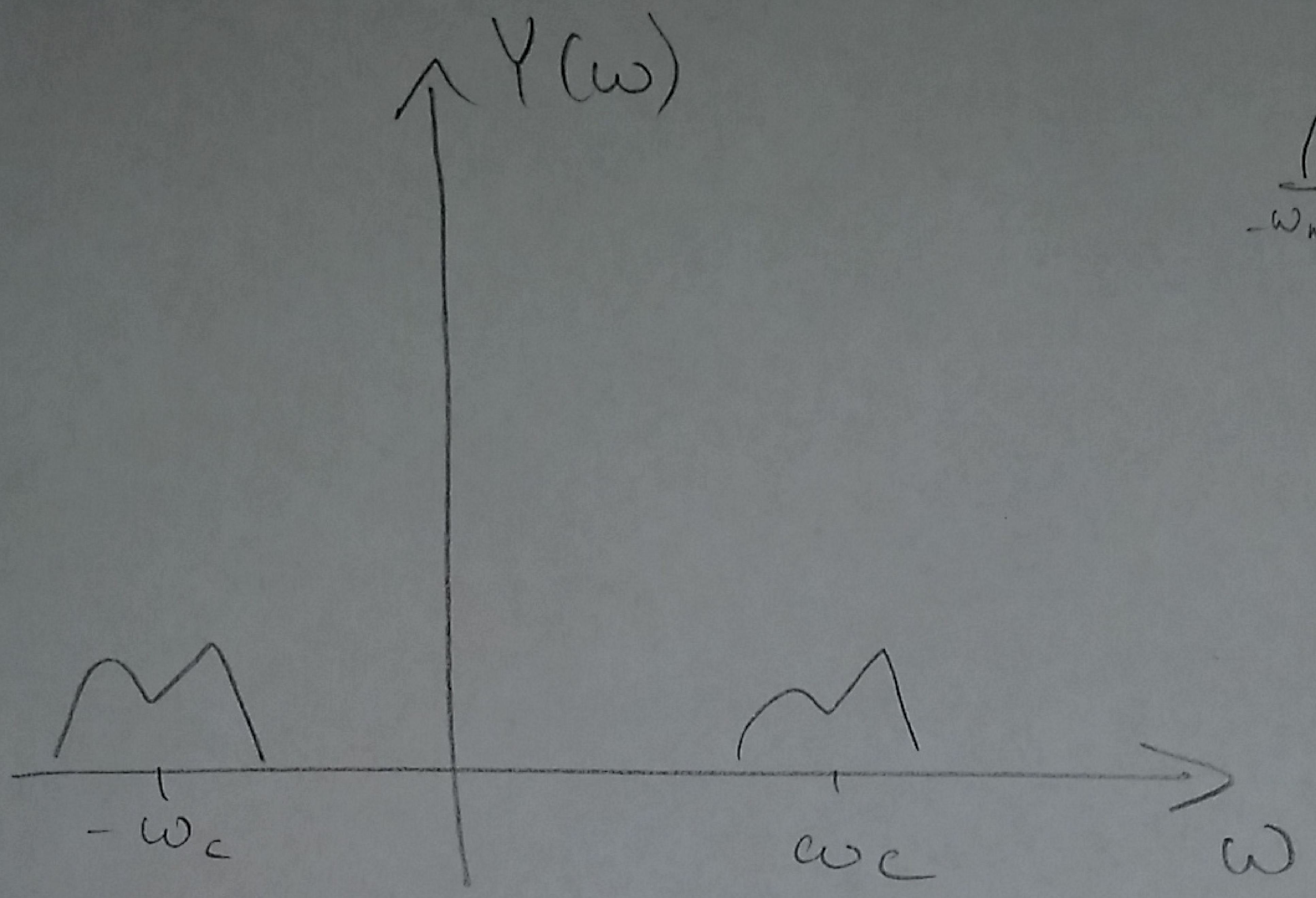
b.

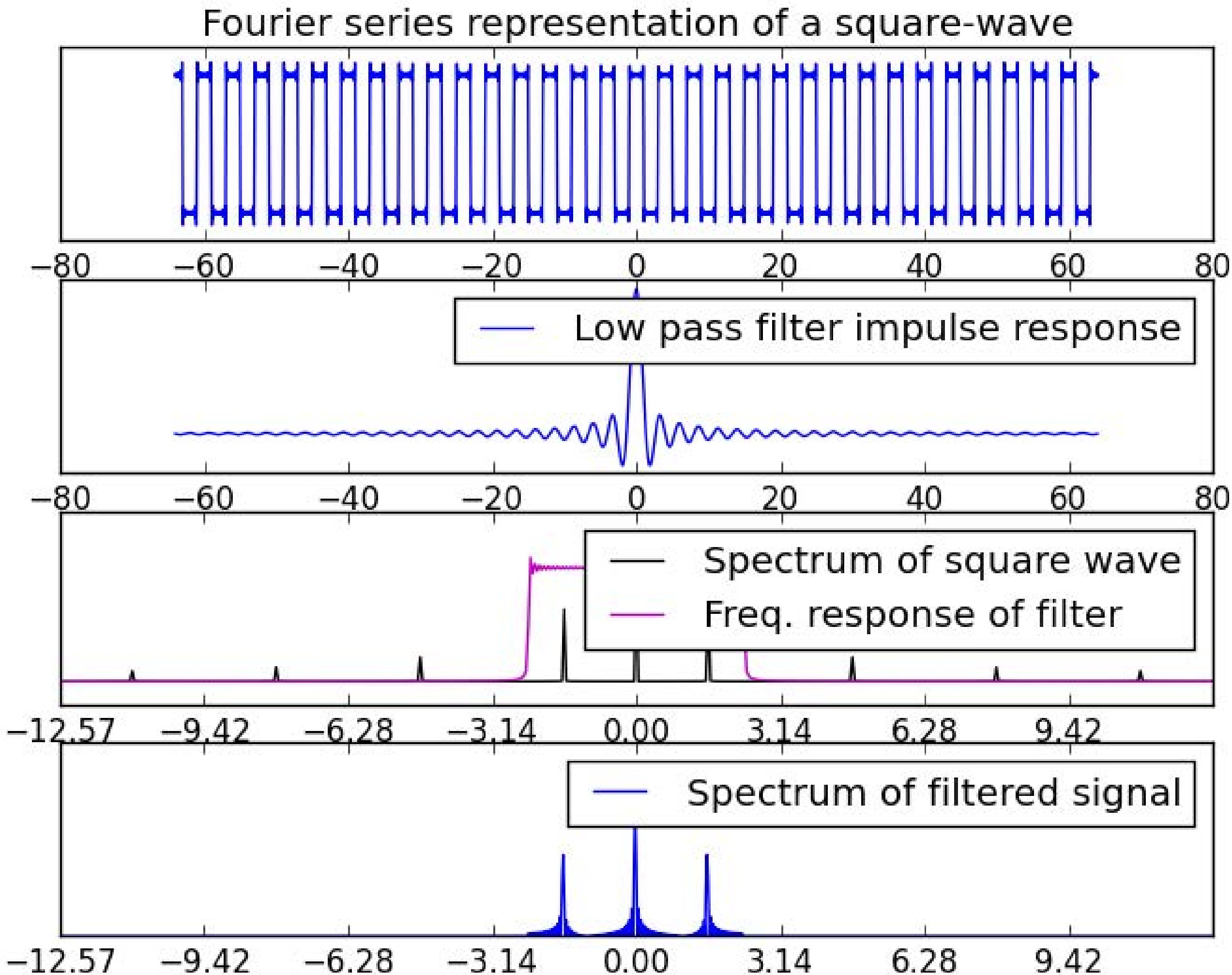


c. Since $H(\omega)$ has no value outside of $|\omega_c|$, there can be no value for $Y(\omega)$ outside that value. Hence this is an ideal low pass filter because only values less than ω_c are passed.

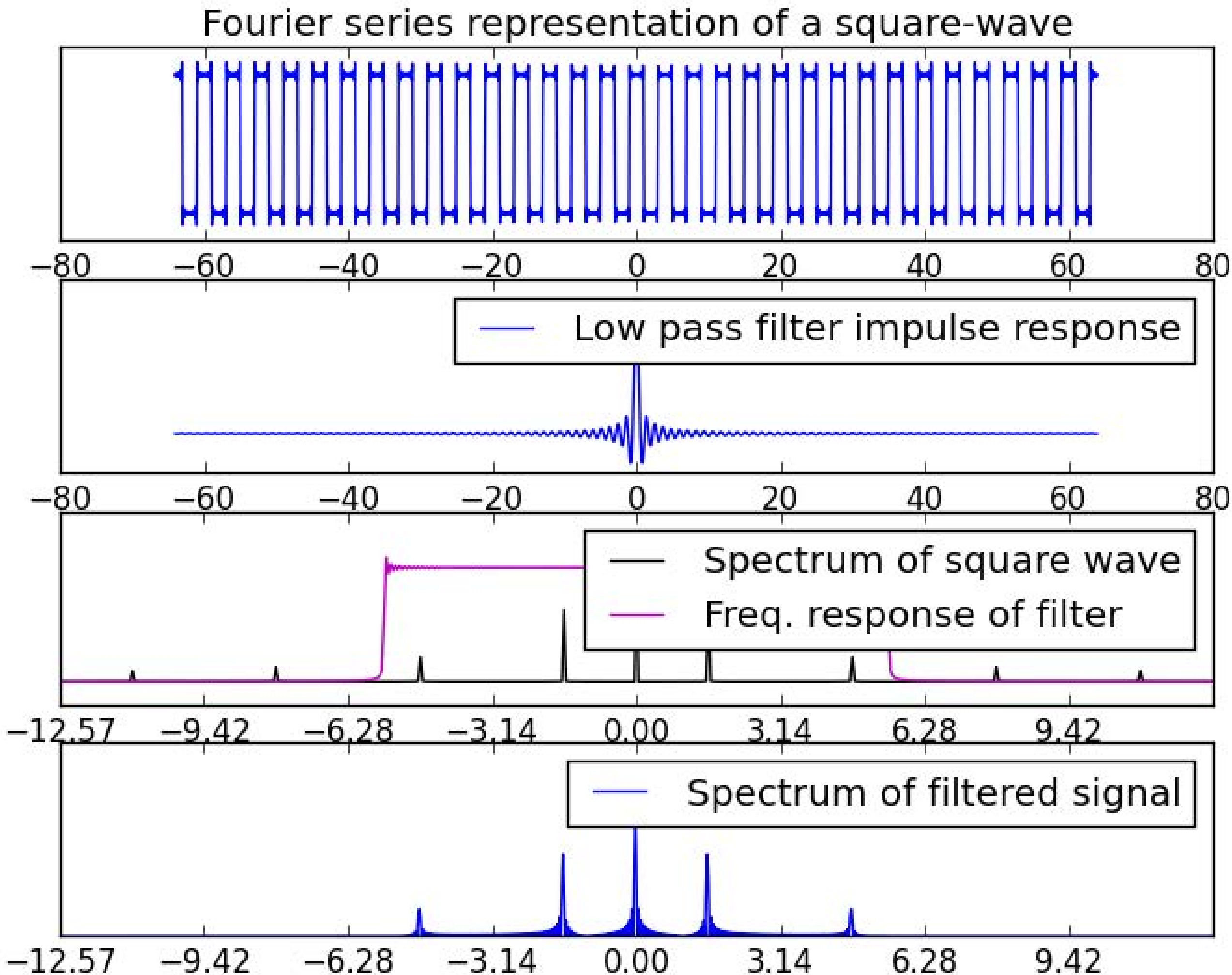
3.







Fourier series representation of a square-wave
−80
−60
−40
−20
0
20
40
60
80
Low pass filter impulse response
Spectrum of square wave
Freq. response of filter
−12.57
−9.42
−6.28
−3.14
0.00
3.14
6.28
9.42
Spectrum of filtered signal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