1. $\dot{y} + y = x \qquad y(t) = (1 - e^{-t})u(t)$

$sY + Y = X$

$X \to \boxed{H(s)} \to Y$

$x \to \boxed{h(t)} \to y$

$H(s) = \frac{Y}{X} = \frac{1}{s+1} \qquad H(s) = \frac{A_1}{s+p_1}$

$h(t) = A_1 e^{-p_1 t} u(t) \qquad x = u(t) \qquad X = \frac{1}{s}$

$X \cdot H(s) = Y$

$\frac{1}{s}\left(\frac{1}{s+1}\right) = Y \qquad \frac{1}{s^2+s} \qquad \frac{1}{s(s+1)}$

$\frac{A}{s} + \frac{B}{s+1} = \frac{1}{s^2+s}$

$A(s+1) + B(s) = 1$

$As + A + Bs = 1$

$s(A+B) + A = 1$

$A = 1$

$B = -1$

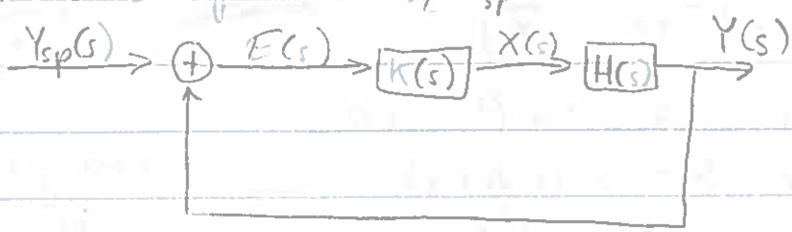
$\frac{s(s+1)}{s(s+1)}\left(\frac{1}{s} + \frac{-1}{s+1}\right) = \frac{s+1-s}{s(s+1)}$

$Y = \frac{1}{s} - \frac{1}{s+1}$

$y = u(t) - e^{-at} u(t)$

$\boxed{y = (1 - e^{-at})u(t)}$

2. Find DC gain $Y(s)/Y_{sp}(s)$

a. 

$$\frac{Y}{Y_{sp}} = \frac{kH}{1+kH}$$

step response $u(t) \cdot h(t) = \mathcal{L}^{-1}\{H(s)\}$

$$\frac{Y}{Y_{sp}} = \frac{k_1 H}{1+kH} \qquad \frac{Y}{Y_{sp}} = \frac{\frac{k_1 H}{s}}{\frac{s+k_1 H}{s}}$$

$$\text{DC gain} = \frac{Y}{Y_{sp}} = \frac{k_1 H}{s} \cdot \frac{s}{s+k_1 H} = \frac{k_1 H}{s+k_1 H}$$

DC gain approaches 1.

b: $H(s) = \frac{1/\tau}{s+1/\tau}$ Find $Y(s)/Y_{sp}(s)$

$$\frac{Y}{Y_{sp}} = \frac{\frac{k_1}{s}\left(\frac{1/\tau}{s+1/\tau}\right)}{s) + \frac{k_1}{s}\left(\frac{1/\tau}{s+1/\tau}\right)}$$

$$\frac{\frac{k_1/\tau}{s^2+s/\tau}}{\frac{s^2+sk_1/\tau}{s^2+s/\tau}} \qquad \frac{\frac{k/\tau}{s+1/\tau}}{s^2+\frac{k/\tau}{s+1/\tau}} \times \frac{(s+1/\tau)}{(s+1/\tau)} = \frac{\frac{k(s+1/\tau)}{\tau}}{(s^3+1/\tau)+\frac{k(s+1/\tau)}{\tau}}$$

$$\frac{k(s+1/\tau)}{\tau(s^3+1/\tau)+k(s+1/\tau)} = \frac{\tau k(s+1)}{\tau^2(s^3+1)+\tau k(s+1)} = \frac{k(s+1)}{\tau(s^3+1)+k(s+1)}$$

$$H(s) = \frac{1/\tau}{s+1/\tau} \qquad \frac{Y}{Y_{sp}} = \frac{kH}{1+kH} = \frac{\frac{k}{s}H}{1+\frac{k}{s}H} =$$

$$\frac{KH}{1+KH} = \frac{\frac{k}{s}\cdot\frac{1/\tau}{s+1/\tau}}{1+\frac{k}{s}\cdot\frac{1/\tau}{s+1/\tau}} \cdot \frac{s(s+1/\tau)}{s(s+1/\tau)} = \frac{k/\tau}{s^2+s/\tau+k/\tau}\cdot\frac{\tau}{\tau} = \frac{k}{s^2\tau+s+k}$$

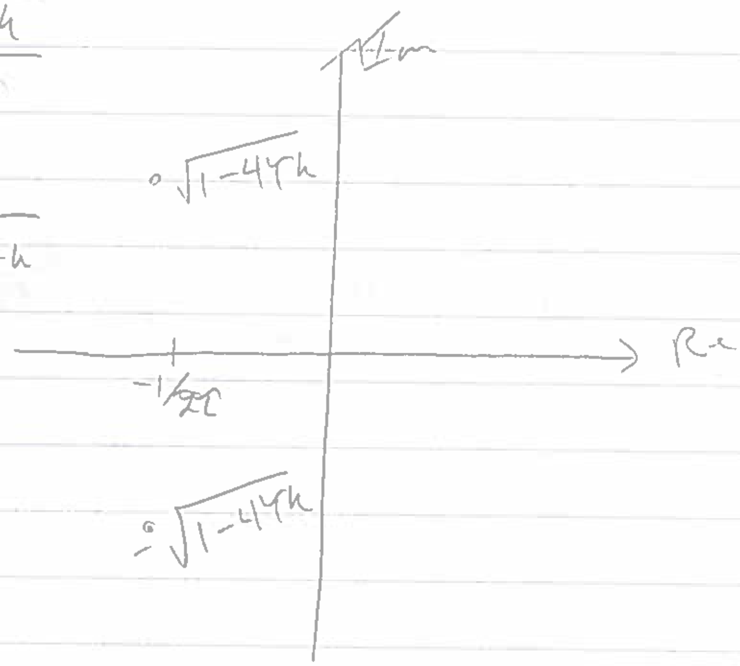
$$\frac{k}{s^2\tau+s+k}$$

$$(s^2\tau+s+k)=0$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-1\pm\sqrt{1^2-4(\tau)(k)}}{2(\tau)}$$

$$\frac{-1\pm\sqrt{1-4\tau k}}{2\tau}$$

$$\frac{Y}{Y_{sp}} = \frac{k}{s^2\tau+s+k}$$



Qs^3+bs^2+cs+d

4. $H(s) = \dfrac{1}{s^2 - 0.01s + 1}$

b. $k(Y_{sp} - HX) = X$

$$\frac{X}{Y_{sp}} = \frac{k}{1+kH} \qquad Y = HK$$

$$\frac{Y}{Y_{sp}} = \frac{kH}{1+kH} \qquad \frac{k\left(\frac{1}{s^2-0.01s+1}\right)}{1+k\left(\frac{1}{s^2-0.01s+1}\right)}$$

$k = 2$

$$\frac{\frac{k}{s^2-0.01s+1}}{1+\frac{k}{s^2-0.01s+1}} \cdot \frac{s^2-0.01s+1}{s^2-0.01s+1} \qquad \frac{Y}{Y_{sp}} = \frac{k}{s^2-0.01s+1+k}$$

$$\frac{k}{s^2-0.01s+1+k}$$

c. $\dfrac{\frac{k}{s}H}{1+\frac{k}{s}H} = \dfrac{\frac{k}{s}\left(\frac{1}{s^2-0.01s+1}\right)}{1+\frac{k}{s}\left(\frac{1}{s^2-0.01s+1}\right)} = \dfrac{\frac{k}{s}}{s^2-0.01s+1+\frac{k}{s}} \quad \dfrac{s}{s}$

$$\frac{k}{s^3-0.01s^2+s+k}$$

d. $\dfrac{ksH}{1+ksH} = \dfrac{ks\left(\frac{1}{s^2-0.01s+1}\right)}{1+ks\left(\frac{1}{s^2-0.01s+1}\right)} = \dfrac{ks}{s^2-0.01s+1+ks}$

$$\frac{ks}{s^2-0.01s+ks+1}$$

$$s^2+s(k-0.01)+1$$

$$k=0.01$$