SigSys Final Report

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1 Introduction

As we reached the end of our Signals and Systems course, it was time to choose a project to apply what we have learned during this semester. The direction we decided to go in for our project deals with near-field wireless power transmission.

Wireless Power Transfer (WPT) has been a dream of scientists and engineers since Nikola Tesla. Nikola Tesla pioneered ideas in WPT and used his "tesla coils" to radiate energy to nearby fluorescent light bulbs. Due to energy losses that dissipate at a distance of r^2, his ideas seemed too inefficient for practical uses. Focus shifted from transferring power to transferring information, leading to radio and cellular communications. During the 1950's, wireless power transfer was again examined to get power to flying vehicles and for other purposes.

Technically, WPT is any system that transmits power via time-varying electromagnetic fields. In our setup, we used a pair of closely matched copper wire coils (courtesy of our professor, José Oscar Mur-Miranda) to transmit and receive power. A voltage across an antenna that is time-varying induces a current in a receiving antenna.

2 System Characterization

2.1 Circuit Diagram

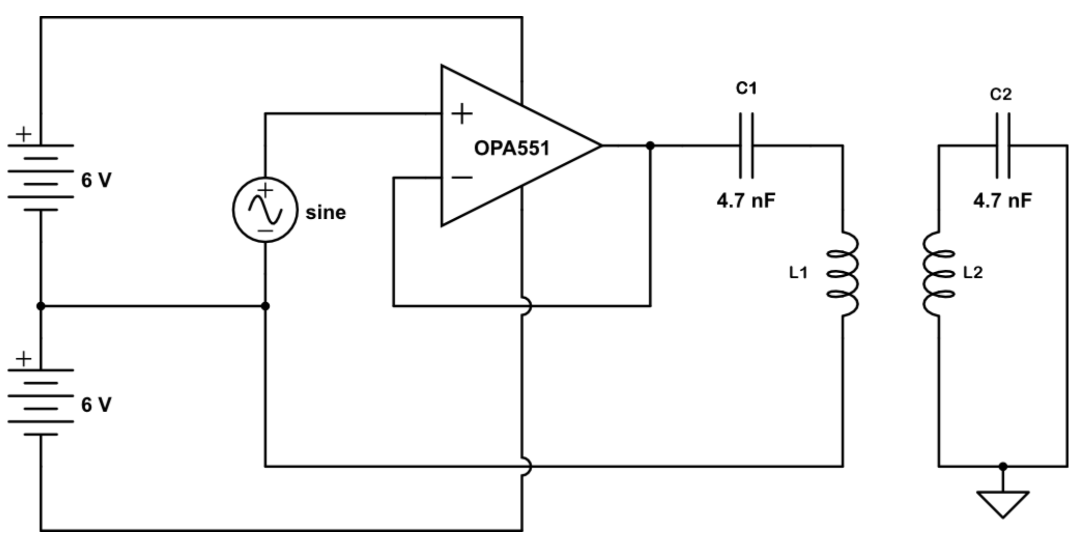

Figure 1: Circuit Diagram. Sending coil loop (L1) and receiving coil loop (L2).

In the Figure 1, our circuit diagram uses an operational amplifier, in order to amplify the input waveform. There are also two capacitors (C1 and C2), one for the transmitting end and one for the receiving end. These capacitors cancel out the self-inductance created by the coils, only when the system operates at its resonant frequency.

2.2 System Diagram

Figure 2: System Diagram

The system is described by the transfer function of series RC or series RLC circuit:

$$H(s) = \frac{1/RC}{s + 1/RC} \qquad \text{or} \qquad H(s) = \frac{1/LC}{s^2 + (R/L)s + 1/LC}$$

The transfer function was derived by simplifying the transmitting circuit. Figure 3 show a resistor, capacitor and inductor in series. The capacitor is the same as shown above that cancels out the self-inductance of the coil. The resistor (R) represents the resistance of the coil and the coil has its own inductance (L) and capacitance (C) values.

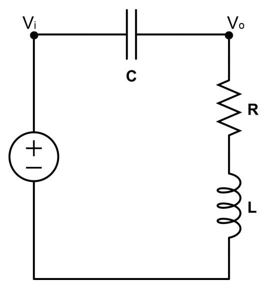

Figure 3: Simplified Transmitting Coil Diagram

The derivation for the circuit is as follows:

$$\begin{aligned}
H(\omega) &= \frac{V_o}{V_i} \\
z_R &= R \\
z_L &= j\omega L \\
z_C &= \frac{1}{j\omega C} \\
H(\omega) &= z_C/(z_L + z_R + z_C) \\
&= \frac{\frac{1}{j\omega C}}{j\omega L + R + \frac{1}{j\omega C}} \\
&= \frac{\frac{1}{LC}}{(j\omega)^2 + j\omega(\frac{R}{L}) + \frac{1}{LC}} \\
s &= j\omega \\
H(s) &= \frac{1/LC}{s^2 + (R/L)s + 1/LC}
\end{aligned}$$

This transfer function also applies to the receiving coil, as the circuit is basically the same.

2.3 Sending Coil Characterization

The sending coil has RLC values of $R = 393\omega, L = .745H$, and $C = 4.7nF$.
The receiving coil has RLC values of $R = 395\omega, L = .740H$, and $C = 4.7nF$.
The Bode plots of the two coil systems are given in Figure 4.

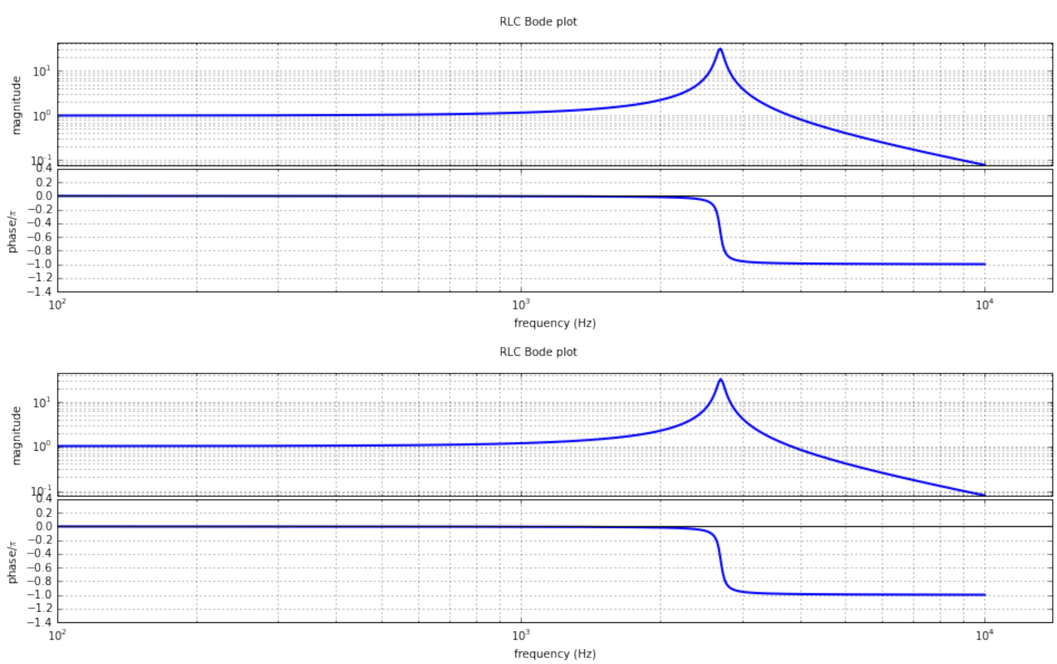

Figure 4: Bode plot of the sending coil (top) and receiving coil (bottom)

There is a peak in magnitude at the resonant frequency of both systems. Ideally, we want the two systems to have an almost identical frequency response, with matching resonant frequencies. In this case, the two systems are very close, with the sending coil with a resonant frequency of $r_f = 2690Hz$ and the receiving frequency with a resonant frequency of $r_f = 2700Hz$.

In addition to finding the bode plots, we characterized the step response of both systems. The step response of the sending coil is given in Figure 5.

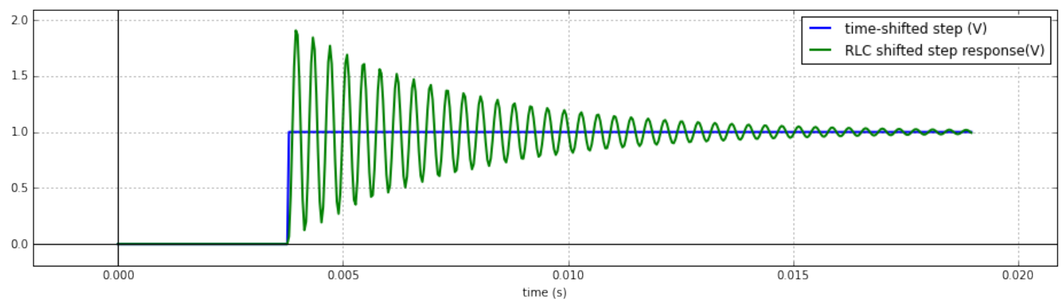

Figure 5: Step response of the sending coil. The step response for the receiving coil is nearly identical.

The step response oscillates at the resonant frequency, with damping. This means that there are two poles for the system, with a positive and negative imaginary component as well as a negative real component causing the damping.

3 Experimental Results

We analyzed the frequency response experimentaly of both coils which yeilded a peak magnitude in the approximate location we were expecting as seen in Figure 6. The receiving bode plot was extremely similar and as such was not included.

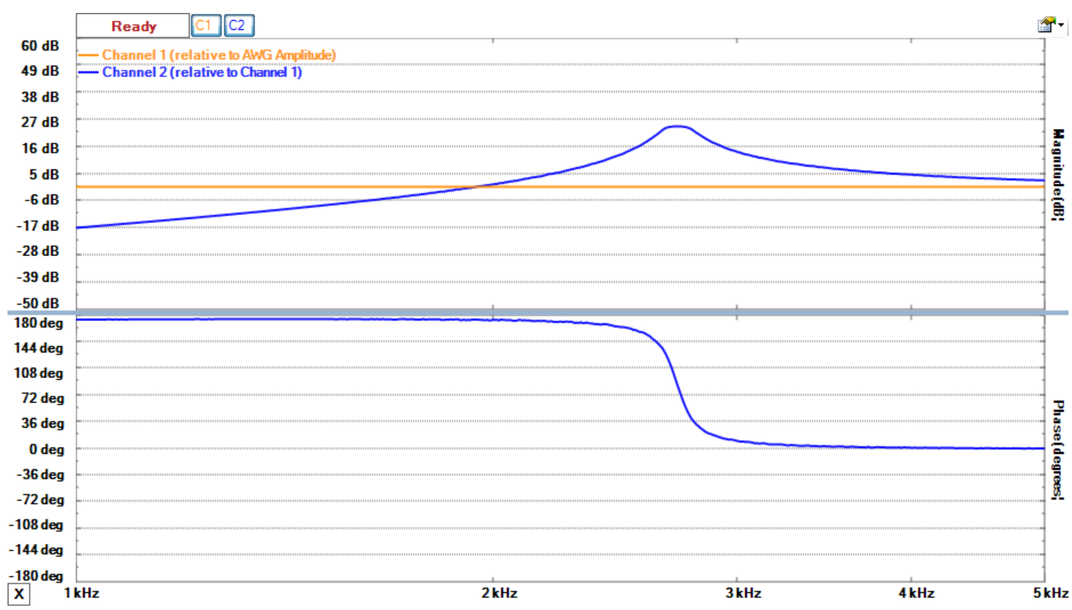

Figure 6: Experimental Bode plot of the transmitting coil

In order to confirm our analysis of the transmitting coil, we measured the output voltage of the system and compared it to the frequency of the input wave. Figure 7 shows the output voltage of the transmitting coil for three different input frequencies.

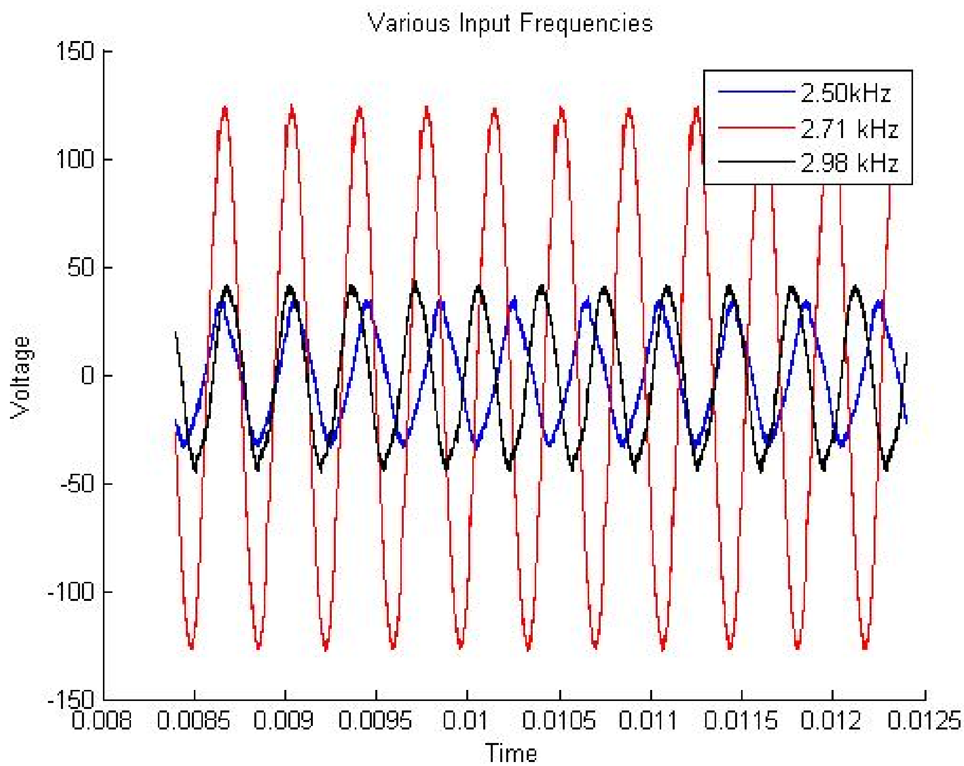

Figure 7: Sending coil for different input frequencies

As expected, the output voltage was much higher for the case where the input frequency $\approx$ 2.7 kHz, the resonant frequency we calculated for the system.

We tested the step response of our transmitting circuit to again validate the resonant frequency we received from our calculations. We input a low frequency square wave and tested the response of the wave, which can be seen in Figure 8. The measured peak-to-peak frequency on the oscilloscope was 2.717 kHz, verifying our calculated value and the value we observed by adjusting the frequency of the input wave.

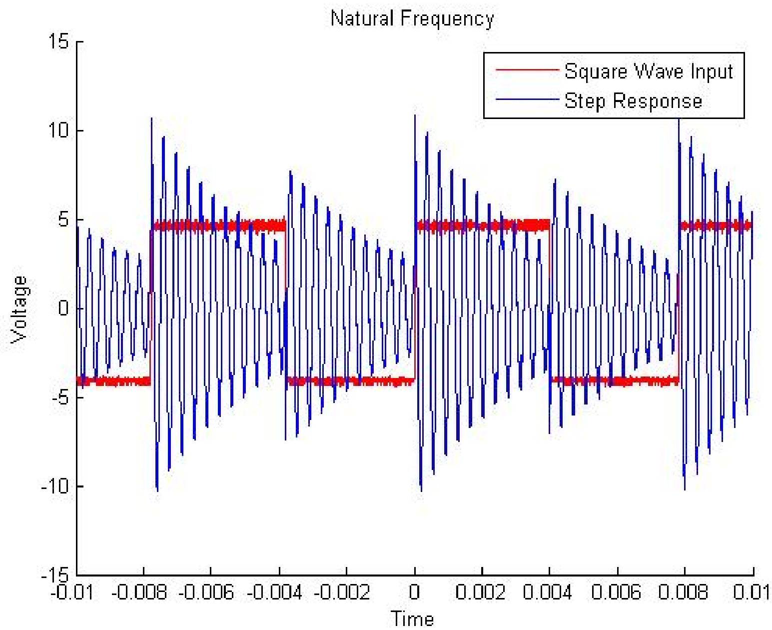

Figure 8: Step response of the sending coil

We measured both the sending voltage and receiving voltages, with an input frequency at resonance (Figure 9). We then adjusted the distance between the two coils, and observed where the receiving voltage was at a maximum. This occured at a distance of about 6cm. One important thing to note is that the receiving voltage is at a 90 degree phase offset from the transmitting voltage.

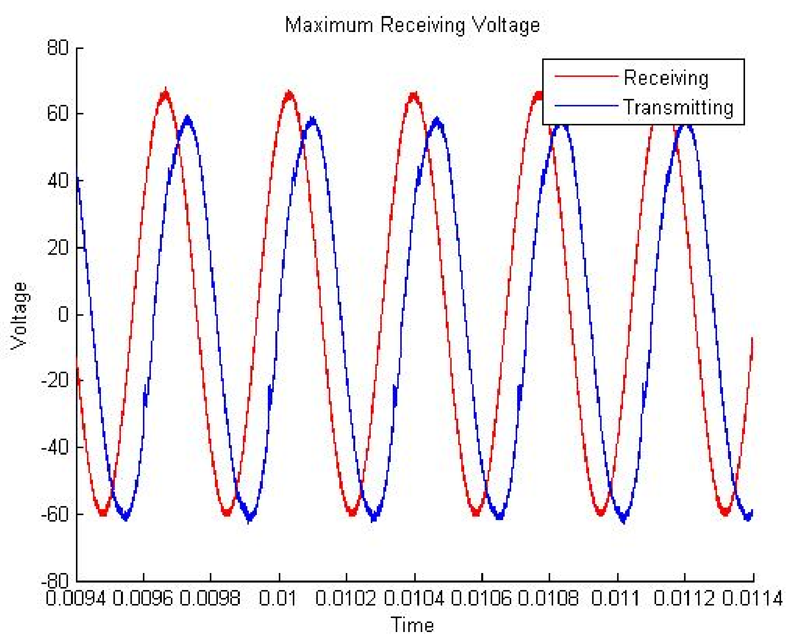

Figure 9: Receiving and transmitting voltages at a distance of 6cm.

The following graphs (Figures 10, 11 and 12) show the interference patterns of the voltages from the transmitting and receiving coils.

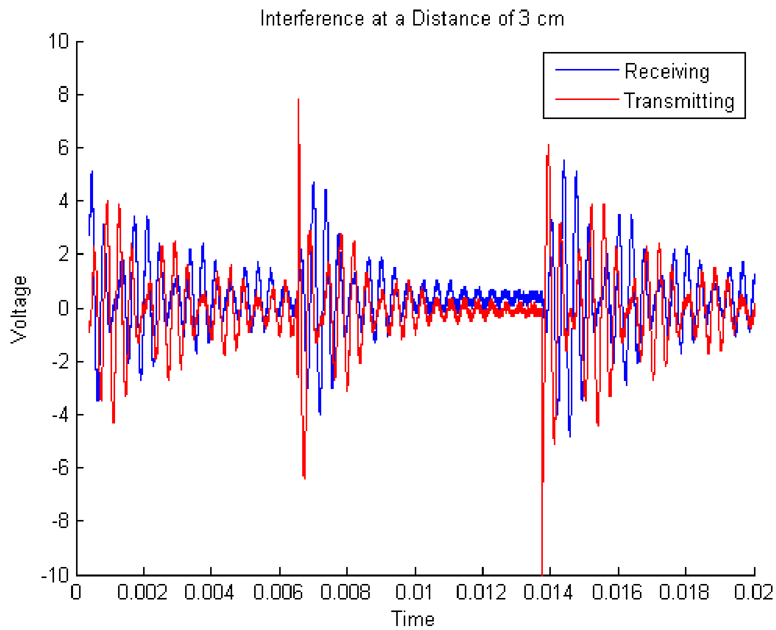

Figure 10: Interference Pattern, Distance = 3 cm

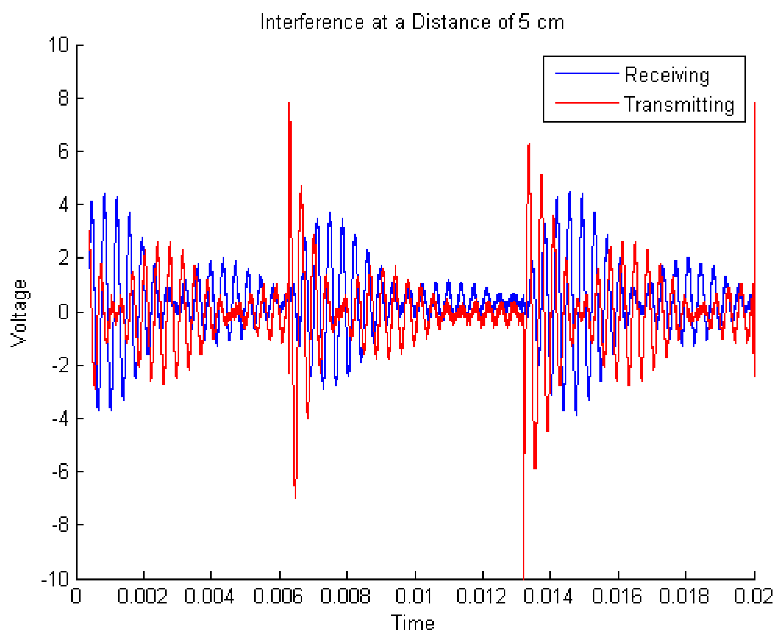

Figure 11: Interference Pattern, Distance = 5 cm

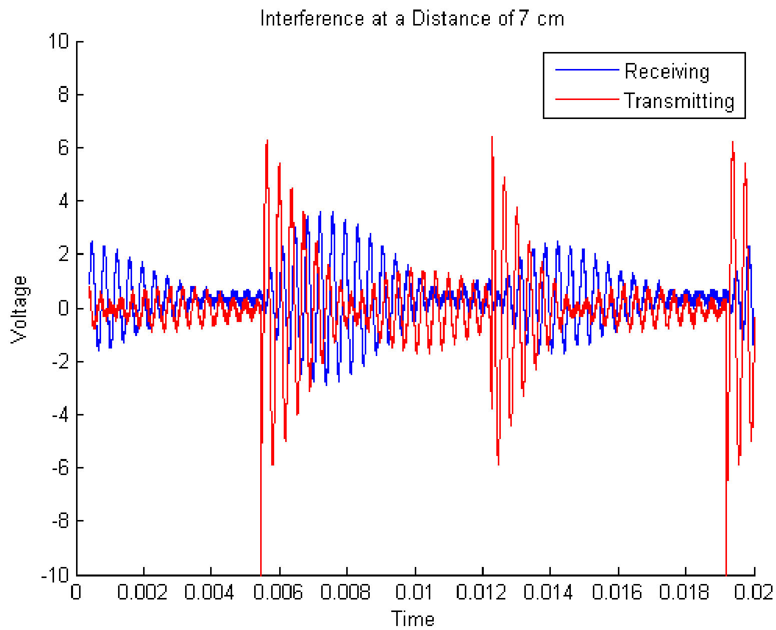

Figure 12: Interference Pattern, Distance = 7 cm

As can be seen in these figures, at a closer distance, both coils seem to resonate a similar frequency. However, as the coils are pulled apart, their magnetic fields seem to interact and cause these coils to resonate at different frequencies. This effect will oscillate with distance.

4 Efficiency

To calculate the power dissipation on the receiving end, we placed a load (resistor) in series with the capacitor and measured the voltage difference across the resistor, while changing the distance between the coils. Figure 13 shows the results we received from this experimental process.

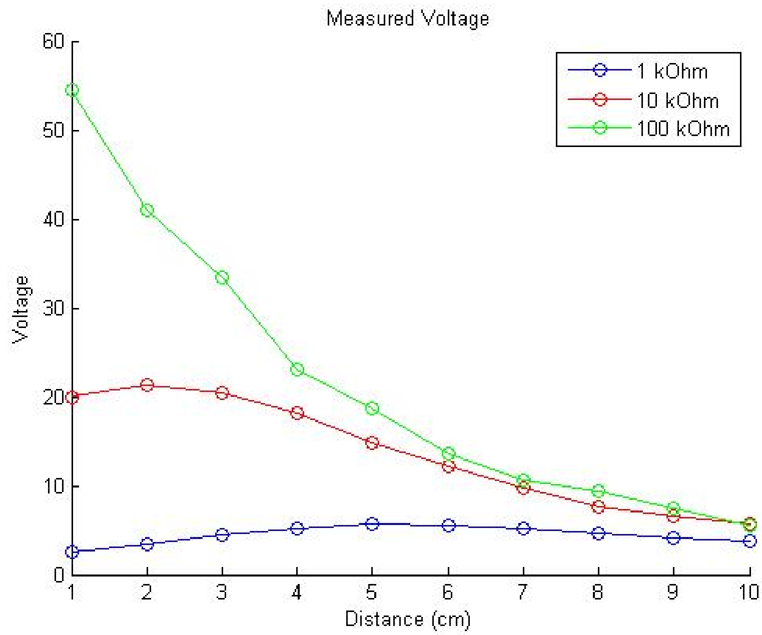

Figure 13: Voltage vs. Distance for Various Loads

These measured voltages illustrate the relationship between voltage and distance for different resistance values. Figure 13 specifically demonstrates how increasing the resistor value not only generates a higher voltage in the receiving coil, but also these higher voltages decrease significantly with a slight change in distance.

Since we mainly want to look at the power being transmitted between the coils, Figure 14 shows how the power varies with distance for each resistor.

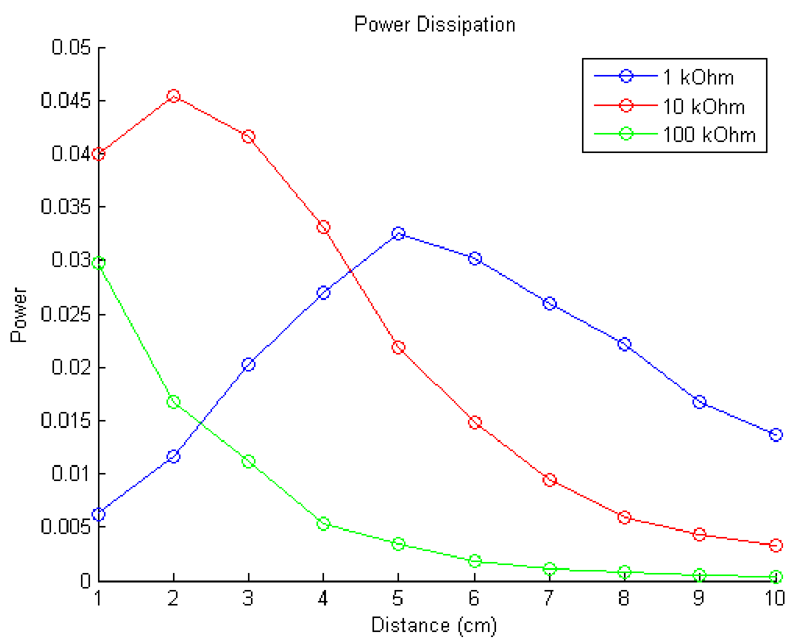

Figure 14: Power Dissipation Using Various Loads

Figure 14 shows a trend among the resistors. They generate a general looking inverted curve. These curves are simply shifted and compressed or stretched versions of one another. As a result, we can see that there is an optimal distance for each resistor value. This graph provides insight into what values are most efficient at specific distances. For instance, for any distance under 5 cm, the 10 kΩ resistor would be the best, but a 1 kΩ resistor would be much better for distances greater than 5 cm.

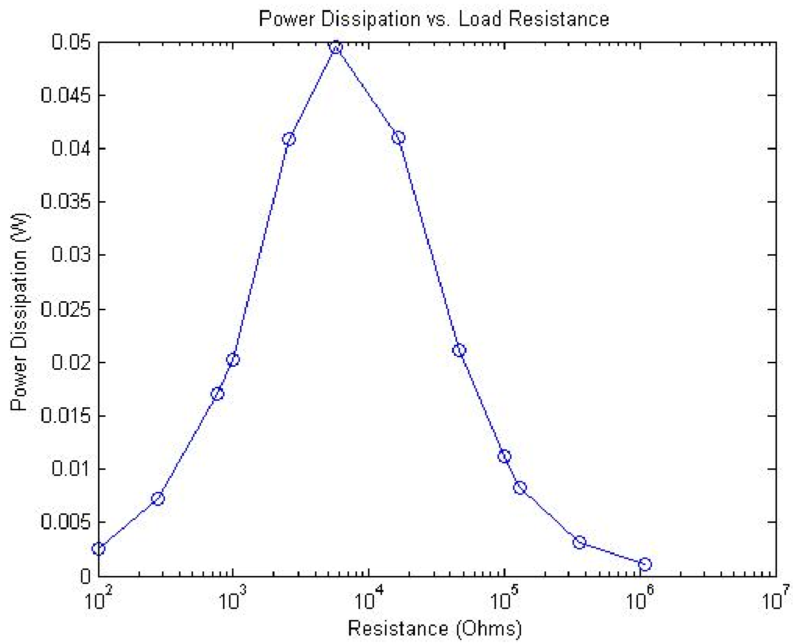

Figure 15: Power Dissipation Using Various Loads at a Distance of 3 cm

Figure 15 shows the power dissipation levels at resonant frequency for various resistors. The graph we were expecting was an inverted curve which is quite similar to our results. We expected to see this result because we considered the two extreme cases where:

1. $V \neq 0$; $I = 0$ and
2. $V = 0$; $I \neq 0$,

the first case results in an infinite resistance and the second case would result in a resistance of 0. In both of these cases, the powere dissipated must be 0. Therefore, there must be a resistor value between 0 and infinity for which the power dissipated is maximized. Although the curve does depend on the distance between the transmitting and receiving coils, there is a distance-resistance relationship which maximizes the power transferred. Power dissipation and resistance should always have an optimal value for the power dissipated. In our case, the 5.7 kΩ resistor was the optimal value at a distance of 3 cm.