Κεφάλαιο 3

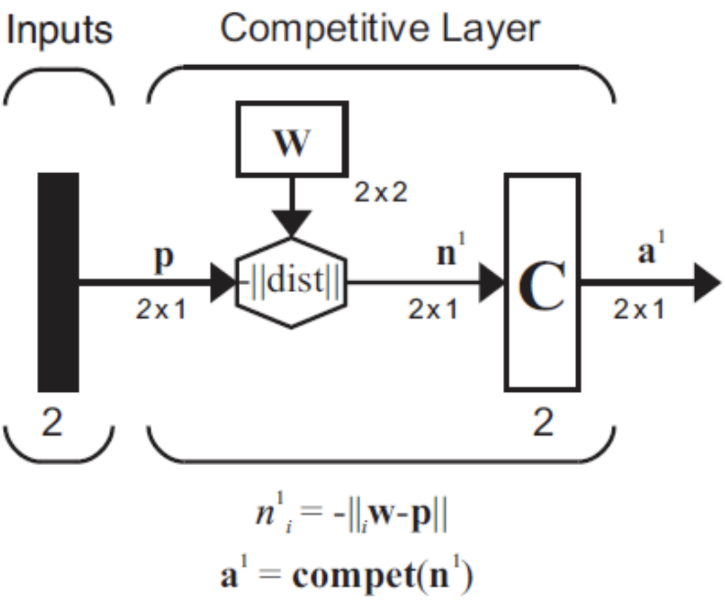
Πρόβλημα 3

The net input expression for LVQ networks calculates the distance between the input and each weight vector directly, instead of using the inner product. The result is that the LVQ network does not require normalized input vectors. This technique can also be used to allow a competitive layer to classify non-normalized vectors. Such a network is illustrated at image 3.1. Use this technique to train a two-neuron competitive layer on the (non-normalized) vectors below, using a learning rate, a, of 0.5.

$p_1 = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$, $p_2 = \begin{vmatrix} -1 \\ 2 \end{vmatrix}$, $p_3 = \begin{vmatrix} -2 \\ -2 \end{vmatrix}$.

Present the vectors in the following order: $p_1, p_2, p_3, p_2, p_3, p_1$. Here are the initial weights of the network:

$W_1 = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$, $W_2 = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$,



Εικόνα 3.1: *Problem 3 Network*

Έχουμε 2 κλάσεις , 1 υποκλάση για την κάθε κλάση

$$W^2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \tag{3.1}$$

$\mathbf{p}_1$:

$$a^1 = compet(n^1) = compet \begin{vmatrix} ||w_1 - p_1|| \\ ||w_2 - p_1|| \end{vmatrix} = compet \begin{vmatrix} || \begin{vmatrix} 0 & 1 \end{vmatrix}^T - \begin{vmatrix} 1 & 1 \end{vmatrix}^T || \\ || \begin{vmatrix} 1 & 0 \end{vmatrix}^T - \begin{vmatrix} 1 & 1 \end{vmatrix}^T || \end{vmatrix} = compet(\begin{vmatrix} 1 \\ 1 \end{vmatrix}) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}) \tag{3.2}$$

$$a^2 = W^2 \cdot a^1 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix} \tag{3.3}$$

$$W_1(1) = W_1(0) + \alpha \cdot (p_1 - W_1(0)) = \begin{vmatrix} 0 \\ 1 \end{vmatrix} + 0.5 \cdot (\begin{vmatrix} 1 \\ 1 \end{vmatrix} - \begin{vmatrix} 0 \\ 1 \end{vmatrix}) = \begin{vmatrix} 0 \\ 1 \end{vmatrix} + \begin{vmatrix} 0.5 \\ 0 \end{vmatrix} = \begin{vmatrix} 0.5 \\ 1 \end{vmatrix} \tag{3.4}$$

$\mathbf{p}_2$:

$$a^1 = compet(n^1) = compet \begin{vmatrix} ||w_1 - p_2|| \\ ||w_2 - p_2|| \end{vmatrix} = compet \begin{vmatrix} || \begin{vmatrix} 0.5 & 1 \end{vmatrix}^T - \begin{vmatrix} 1 & 2 \end{vmatrix}^T || \\ || \begin{vmatrix} 1 & 0 \end{vmatrix}^T - \begin{vmatrix} 1 & 2 \end{vmatrix}^T || \end{vmatrix} = compet(\begin{vmatrix} 1.8027756377 \\ 2.8284271247 \end{vmatrix}) = \begin{vmatrix} 0 \\ 1 \end{vmatrix}) \tag{3.5}$$

$$a^2 = W^2 \cdot a^1 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \tag{3.6}$$

λάθος κλάση

$$W_2(1) = W_2(0) - \alpha \cdot (p_2 - W_2(0)) = \begin{vmatrix} 1 \\ 0 \end{vmatrix} - 0.5 \cdot (\begin{vmatrix} -2 \\ 2 \end{vmatrix} = \begin{vmatrix} 2 \\ -1 \end{vmatrix} \tag{3.7}$$

$\mathbf{p}_3$:

$$a^1 = compet \begin{vmatrix} || \begin{vmatrix} 0.5 & 1 \end{vmatrix}^T - \begin{vmatrix} -2 & 2 \end{vmatrix}^T || \\ || \begin{vmatrix} 2 & -1 \end{vmatrix}^T - \begin{vmatrix} -2 & 2 \end{vmatrix}^T || \end{vmatrix} = compet(\begin{vmatrix} 70 \\ 5 \end{vmatrix}) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}) \tag{3.8}$$

$$a^2 = W^2 \cdot a^1 \begin{vmatrix} 1 \\ 0 \end{vmatrix} \tag{3.9}$$

λάθος κλάση

$$W_1(2) = W_1(1) - \alpha \cdot (p_3 - W_1(1)) = \begin{vmatrix} 0.5 \\ 0 \end{vmatrix} - 0.5 \cdot (\begin{vmatrix} -2.5 \\ 1 \end{vmatrix} = \begin{vmatrix} 1.75 \\ 0.5 \end{vmatrix} \tag{3.10}$$

$\mathbf{p}_2$:

$$a^1 = compet(n^1) = compet\begin{vmatrix} \| \begin{vmatrix} 1.75 & 0.5 \end{vmatrix}^T - \begin{vmatrix} -1 & 2 \end{vmatrix}^T \| \\ \| \begin{vmatrix} 2 & -1 \end{vmatrix}^T - \begin{vmatrix} -1 & 2 \end{vmatrix}^T \| \end{vmatrix} = compet(\begin{vmatrix} 3.13 \\ 4.24 \end{vmatrix}) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}) \quad (3.11)$$

$$a^2 = W^2 \cdot a^1 \begin{vmatrix} 1 \\ 0 \end{vmatrix} \quad (3.12)$$

$$W_1(3) = W_1(2) - a \cdot (p_2 - W_1(2)) = \begin{vmatrix} 1.75 \\ 0.5 \end{vmatrix} + 0.5 \cdot (\begin{vmatrix} -2.75 \\ 1.5 \end{vmatrix} = \begin{vmatrix} 0.375 \\ 1.25 \end{vmatrix} \quad (3.13)$$

$\mathbf{p}_3$:

$$a^1 = compet(n^1) = compet\begin{vmatrix} \| \begin{vmatrix} 0.375 & 1.25 \end{vmatrix}^T - \begin{vmatrix} -2 & 2 \end{vmatrix}^T \| \\ \| \begin{vmatrix} 2 & -1 \end{vmatrix}^T - \begin{vmatrix} -2 & 2 \end{vmatrix}^T \| \end{vmatrix} = compet(\begin{vmatrix} 2.95 \\ 5 \end{vmatrix}) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}) \quad (3.14)$$

$$a^2 = W^2 \cdot a^1 \begin{vmatrix} 1 \\ 0 \end{vmatrix} \quad (3.15)$$

λάθος κλάση

$$W_1(4) = W_1(1) - a \cdot (p_3 - W_1(3)) = \begin{vmatrix} 0.375 \\ 1.25 \end{vmatrix} - 0.5 \cdot (\begin{vmatrix} -2.375 \\ 0.75 \end{vmatrix} = \begin{vmatrix} 1.5625 & 0.875 \end{vmatrix} \quad (3.16)$$

$\mathbf{p}_1$:

$$a^1 = compet(n^1) = compet\begin{vmatrix} \| \begin{vmatrix} 1.5625 & 0.875 \end{vmatrix}^T - \begin{vmatrix} 1 & 1 \end{vmatrix}^T \| \\ \| \begin{vmatrix} 2 & -1 \end{vmatrix}^T - \begin{vmatrix} 1 & 1 \end{vmatrix}^T \| \end{vmatrix} = compet(\begin{vmatrix} 0.57 \\ 2.23 \end{vmatrix}) = \begin{vmatrix} 1 \\ 0 \end{vmatrix}) \quad (3.17)$$

$$a^2 = W^2 \cdot a^1 \begin{vmatrix} 1 \\ 0 \end{vmatrix} \quad (3.18)$$

$$W_1(5) = W_1(4) - a \cdot (p_1 - W_1(4)) = \begin{vmatrix} 1.5625 \\ 0.875 \end{vmatrix} + (\begin{vmatrix} -0.28125 \\ 0.0625 \end{vmatrix} = \begin{vmatrix} 1.28125 & 0.9375 \end{vmatrix} \quad (3.19)$$