Κεφάλαιο 8

Πρόβλημα 8

Considering quadratic function 8.1, we want to use the steepest descent algorithm with momentum to minimize this function .

$$F(x) = \frac{1}{2}x^T \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} x + \begin{vmatrix} 1 & 2 \end{vmatrix} x + 2 = 1.5x_1^2 + x_1 x_2 + 1.5x_2^2 + x_1 + 2x_2 + 2 \tag{8.1}$$

$$A = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} \tag{8.2}$$

8.1 Υπολογισμός χαρακτηριστικού πολυωνύμου

$$P(\lambda) = det(A - \lambda \cdot I_2) = \begin{vmatrix} 3-\lambda & 1 \\ 3 & 1-\lambda \end{vmatrix} = (3-\lambda)\cdot(3-\lambda) - 1\cdot 3 = (3-\lambda)^2 - 3 = \lambda^2 - 6\lambda + 9 - 1 = \lambda^2 - 6\lambda + 8 \tag{8.3}$$

8.2 Υπολογισμός Ιδιοτιμών

P $(\lambda) = 0 \Longleftrightarrow \lambda^2 - 6\lambda + 8 = (\lambda - 4)(\lambda - 2) = 0$

$\lambda_1 = 4$ και $\lambda_2 = 2$, $\lambda_{max} = \max(\lambda_1, \lambda_2) =$ μαξ$(4, 2) = 4$

Συνεπώς οι ιδιοτιμές του δοθέντος πίνακα είναι :

- $\lambda_1 = 4$
- $\lambda_2 = 2$

8.3 Συνθήκη σταθερότητας αλγορίθμου

Ο αλγόριθμος είναι σταθερός εφόσον ισχύει η ακόλουθη συνθήκη :

$$|(1+\gamma) - (1-\gamma)\cdot \alpha \cdot \lambda_i| < 2\sqrt{\gamma} \tag{8.4}$$

8.4 Learning rate $\alpha = 1$ and momentum coefficient $\gamma = 0$

Algorithm is **not stable** for both cases ($\lambda_1 = 4$, $\lambda_2 = 2$) supposing $\alpha = 1$ and momentum coefficient $\gamma = 0$.

- $\lambda = 4$

$$|(1+0)-(1-0)\cdot 1\cdot 4| < 2\sqrt{0} \Longrightarrow |1-4| < 2\cdot 0 \Longrightarrow |-3| < 0 \Longrightarrow 3 < 0 \tag{8.5}$$

- $\lambda = 2$

$$|(1+0)-(1-0)\cdot 1\cdot 2| < 2\sqrt{0} \Longrightarrow |1-2| < 2\cdot 0 \Longrightarrow |-1| < 0 \Longrightarrow 1 < 0 \tag{8.6}$$

8.5 Learning rate $\alpha = 1$ and momentum coefficient $\gamma = 0.6$

Algorithm is **stable** for both cases ($\lambda_1 = 4$, $\lambda_2 = 2$) supposing $\alpha = 1$ and momentum coefficient $\gamma = 0.6$.

- $\lambda = 4$

$$|(1+0.6)-(1-0.6)\cdot 1\cdot 4| < 2\sqrt{0.6} \Longrightarrow |1.6-1.6| < 2\cdot 0.7745966 \Longrightarrow |0| < 0 \Longrightarrow 0 < 1.549193 \tag{8.7}$$

- $\lambda = 2$

$$|(1+0.6)-(1-0.6)\cdot 1\cdot 2| < 2\sqrt{0.6} \Longrightarrow |1.6-0.8| < 2\cdot 0.7745966 \Longrightarrow |0.8| < 0 \Longrightarrow 0.8 < 1.549193 \tag{8.8}$$

8.6 Python Code

```
import numpy as np

def stability(alpha, gama, lamda) :

    if abs((1 + gama) - (1 - gama)*alpha*lamda) < 2*np.sqrt(gama) :
        print("For learning rate a =", alpha, ", g =", gama,", l =", lamda, "algorithn is stable .\n")
    else :
        print("For learning rate a =", alpha, ", g =", gama,", l =", lamda, "algorithn is not stable .\n"
    )

A = np.matrix([[3, 1],
               [1, 3]])

l = np.linalg.eigvals(A)
print("Eigenvalues of A :", l, "\n")

```

```
l1 = l[0]
l2 = l[1]

a = 1
g1 = 0
g2 = 0.6

print("A | g = 0\n")
stability(a, g1, l1)
stability(a, g1, l2)

print("B | g = 0.6\n")
stability(a, g2, l1)
stability(a, g2, l2)
```