Chapter 13: Generalization

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13.5

We are given the following density function with random variable t:

$$f(t|x) = \frac{t}{x^2}e^{-\frac{t}{x}}$$

To find the maximum likelihood, we first need to find the product of all the densities of the ditribution:

$$\prod^{q} \frac{t}{x^2}e^{-\frac{t}{x}} = (\frac{t}{x^2})^q e^{-\Sigma^q \frac{t}{x}}$$

Now, we find the natural logarithm:

$$\ln[(\frac{t}{x^2})^q e^{-\Sigma^q \frac{t}{x}}] = \ln(\frac{t}{x^2})^q + \ln[e^{-\Sigma^q \frac{t}{x}}] = q\ln(\frac{t}{x^2}) - [\Sigma^q \frac{t}{x}]$$

Note that the summation of t/x depends upon t, not x, so we can pull it out, to get $\frac{1}{x}\Sigma t$.

Now, we find the partial derivative with respect to x:

$$\frac{\partial}{\partial x}[q\ln(\frac{t}{x^2}) - [\frac{1}{x}\Sigma t]]$$

$$\frac{\partial}{\partial x}[q\ln(t) - q\ln(x^2) - [\frac{1}{x}\Sigma t]$$

$$0 - \frac{2q}{x} - \frac{\partial}{\partial x}[\frac{1}{x}\Sigma t]$$

$$-\frac{2q}{x} + \frac{2\Sigma t}{x^2}$$

Now, we set this equal to zero and solve for x:

$$-\frac{2q}{x} + \frac{2\Sigma t}{x^2} = 0$$

Which can be shown to be at $x = \frac{\Sigma t}{q}$.

The maximum likelihood estimate of x is:

$$x^{ML} = \frac{\Sigma t}{q}$$