Chapter 11: Backpropagation

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1/21/2021

E11.6

We are given the following initial weights and biases:

$$w^1(0) = 1,\ b^1(0) = -2,\ w^2(0) = 1,\ b^2(0) = 1$$

along with the following transfer functions:

$$f^1(n) = n^2,\ f^2(n) = \frac{1}{n}$$

We want to perform one iteration of backpropagation with $\alpha = 1$ on the input/target pair $p = 1, t = 1$. First, we need to propagate the input through the network

$$n^1 = w^1 p + b^1 = (1)(1) - 2 = -1$$

$$a^1 = f^1(n^1) = f^1(-1) = (-1)^2 = 1$$

$$n^2 = w^2 a^1 + b^2 = (1)(1) + 1 = 2$$

$$a^2 = f^2(n^2) = f^2(12) = \frac{1}{2} = 0.5$$

$$e = (t - a^2) = (1 - 0.5) = 0.5$$

Now we backpropagate the sensitivites:

$$s^2 = -2F^2(n^2)(t - a^2)$$

where $F^2(n^2)$ is the first derivative of the second function, $f' = -\frac{1}{n^2} = -\frac{1}{n}\frac{1}{n} = -f^2 = -a^2$ in our case. Then:

$$s^2 = -2(-(0.5)^2)(0.5) = 0.25$$

Next,

$$s^1 = F^1(n^1)(w^2)s^2$$

where F^1 is the first derivative of the first function, $f' = 2n$:

$$s^1 = 2(-1)(1)(0.25) = -0.5$$

Lastly, we update the weights and biases:

$$w^2(1) = w^2(0) - \alpha s^2(a^1) = 1 - 1(0.25)(1) = 0.75$$

$$w^1(1) = w^1(0) - \alpha s^1(a^1) = 1 - 1(-0.5)(1) = 1.5$$

$$b^2(1) = b^2(0) - \alpha s^2 = 1 - 1(0.25) = 0.75$$

$$b^1(1) = b^1(0) - \alpha s^1 = -2 - 1(-0.5) = -1.5$$