Chapter 13: Generalization

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E13.14

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We are given the following input-output pairs:

$$p_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, t_1 = -2$$

$$p_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, t_2 = 2$$

$$p_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, t_3 = 4$$

Assuming equal probability of each input occuring, we can find the quadratic mean square error performance index:

$$c = E[t^2] = (-2)^2(0.33\bar{3} + (2)^2(0.33\bar{3}) + (4)^2(0.33\bar{3}) = 8$$

$$h = E[tz] = (0.33\bar{3})(-2)\begin{bmatrix} -1 \\ 1 \end{bmatrix} + (0.33\bar{3})(2)\begin{bmatrix} 1 \\ 2 \end{bmatrix} + (0.33\bar{3})(4)\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$d = -2h = -2\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -8 \\ -4 \end{bmatrix}$$

$$A = 2R = 2E(zz^T) = 2(0.33\bar{3})(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\begin{bmatrix} -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \end{bmatrix}) = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

$E_D = c + x^T d + \frac{1}{2}x^T A x$.

The minimum of the mean square occurs at $x^{ML} = -A^{-1}d = R^{-1}h$:

$$R^{-1}h = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

The Hessian matrix of the mean square error is $\nabla^2 E_D(x) = A = 2R$. Now we need to find the eigen values of the matrix:

```r
A = matrix(c(4, 2, 2, 4), ncol=2)
eigen(A)
```

```
## eigen() decomposition
## $values
## [1] 6 2
##
## $vectors
##           [,1]       [,2]
## [1,] 0.7071068 -0.7071068
## [2,] 0.7071068  0.7071068
```

Thus, we get two positive eigen values, $\lambda_1 = 6, \lambda_2 = 2$.

From Eq. (13.55), the effective number of parameters are:

$$\gamma = \Sigma \frac{\beta \lambda_i}{\beta \lambda_i + 2\alpha}$$

However, we were given that $\rho = 1 = \frac{\alpha}{\beta}$; thus:

$$\gamma = \Sigma \frac{\lambda_i}{\lambda_i + 2p} = \frac{6}{6+2} + \frac{2}{2+2} = \frac{5}{4}$$

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Not Available