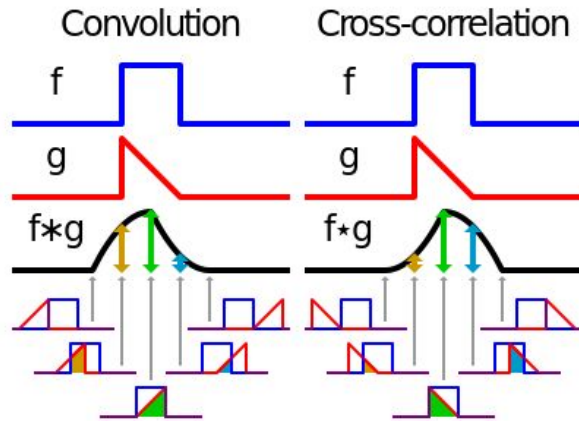




Origins of Convolution & Kernels

Convolution: A Signal Processing Perspective



- Correlation:
 - Given a noisy signal $y(t)$, is the signal $x(t)$ somehow present in $y(t)$?
- Convolution:
 - What is the output of this filter when its input is $x(t)$?

Convolution: Image Processing Perspective

- Convolutional Neural Network is a misnomer

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1																									
2																									
3																									
4																									
5																									
6																									
7																									
8																									
9																									
10																									
11																									
12																									
13																									
14																									
15																									
16																									
17																									
18																									
19																									
20																									
21																									

Input

1	0	1	0	2
1	1	3	2	1
1	1	0	1	1
2	3	2	1	3
0	2	0	1	0

Kernel

0	1	0
0	0	2
0	1	0

Intermediate Output

7	5	3
4	7	5
7	2	8

Output

19	13	15
28	16	20
23	18	25



What is a kernel?

- Nomenclature Problem
- Neural Networks:
 - Kernel = Filter = Weights
- Kernel Functions:
 - Efficient computation of higher dimensional dot products
 - Ex.) Polynomial kernel and RBF kernel

$$\mathbf{x} = (x_1, x_2, x_3)^T$$

$$\mathbf{y} = (y_1, y_2, y_3)^T$$

$$\phi(\mathbf{x}) = (x_1^2, x_1x_2, x_1x_3, x_2x_1, x_2^2, x_2x_3, x_3x_1, x_3x_2, x_3^2)^T$$

$$\phi(\mathbf{y}) = (y_1^2, y_1y_2, y_1y_3, y_2y_1, y_2^2, y_2y_3, y_3y_1, y_3y_2, y_3^2)^T$$

$$\phi(\mathbf{x})^T \phi(\mathbf{y}) = \sum_{i,j=1}^3 x_i x_j y_i y_j$$

$$\begin{aligned} k(\mathbf{x}, \mathbf{y}) &= (\mathbf{x}^T \mathbf{y})^2 \\ &= (x_1y_1 + x_2y_2 + x_3y_3)^2 \\ &= \sum_{i,j=1}^3 x_i x_j y_i y_j \end{aligned}$$