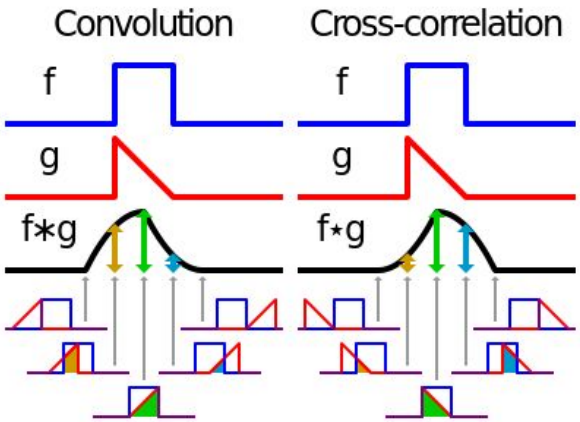
Origins of Convolution & Kernels

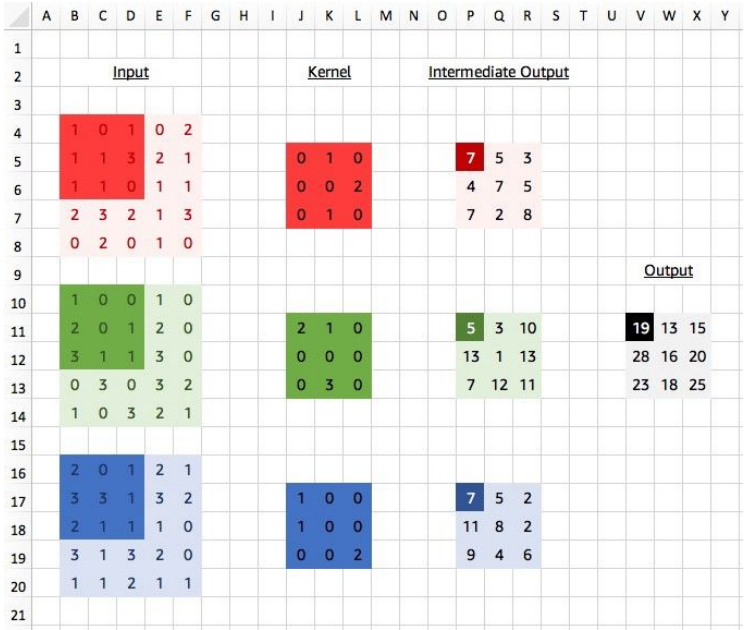
Convolution: A Signal Processing Perspective



- Correlation:
  - Given a noisy signal y(t), is the signal x(t) somehow present in y(t)?
- Convolution:
  - What is the output of this filter when its input is x(t)?

Convolution: Image Processing Perspective

- Convolutional Neural Network is a misnomer



What is a kernel?

- Nomenclature Problem
- Neural Networks:
  - Kernel = Filter = Weights
- Kernel Functions:
  - Efficient computation of higher dimensional dot products
  - Ex.) Polynomial kernel and RBF kernel

$$\mathbf{x} = (x_1, x_2, x_3)^T$$
$$\mathbf{y} = (y_1, y_2, y_3)^T$$

$$\phi(\mathbf{x}) = (x_1^2, x_1x_2, x_1x_3, x_2x_1, x_2^2, x_2x_3, x_3x_1, x_3x_2, x_3^2)^T$$
$$\phi(\mathbf{y}) = (y_1^2, y_1y_2, y_1y_3, y_2y_1, y_2^2, y_2y_3, y_3y_1, y_3y_2, y_3^2)^T$$

$$\phi(\mathbf{x})^T\phi(\mathbf{y}) = \sum_{i,j=1}^{3} x_i x_j y_i y_j$$

$$\begin{aligned} k(\mathbf{x}, \mathbf{y}) &= (\mathbf{x}^T\mathbf{y})^2 \\ &= (x_1y_1 + x_2y_2 + x_3y_3)^2 \\ &= \sum_{i,j=1}^{3} x_i x_j y_i y_j \end{aligned}$$