CubeSAT Attitude Control

Abstract

A CubeSAT is a small satellite on the order of 10 centimeters along each axis. A 1U satellite is a small cube with 10 cm sides. These satellites are used for a variety of missions and created by a variety of different organizations. When deployed from a rocket, a CubeSAT may obtain a large angular velocity which must be reduced before most science missions or communications can take place. Maximizing solar energy charging also involves better pointing accuracy. To control the attitude of these small satellites, reaction wheels, magnetorquers and even the gravity gradient are used in low earth orbit (LEO) while reaction control thrusters are typically used in deep space. On a standard LEO CubeSAT, 3 reaction wheels are used as well as 3 magnetorquers. In the initial phase of the CubeSAT mission, the magnetorquers are used to reduce the angular velocity of the satellite down to a manageable level. Once the norm of the angular velocity is low enough, the reaction wheels can spin up reducing the angular velocity to zero. This paper investigates the necessary mathematics to understand the control architecture necessary for a simple CubeSAT.

x, y, z	components of the mass center position vector in the inertial frame (m)
ϕ, θ, ψ	Euler roll,pitch, and yaw angles (rad)
q_0, q_1, q_2, q_3	quaternions
u, v, w	components of the mass center velocity vector in the body frame (m/s)
p, q, r	components of the mass center angular velocity vector in the body frame (rad/s)
$\vec{\omega}_{B/I}$	angular velocity vector of the satellite in the body frame (rad/s)
$\mathbf{T}_{IB}$	rotation matrix from frame I to frame B
$\mathbf{H}$	relationship between angular velocity components in body frame and derivative of Euler angles
m	mass (kg)
I	mass moment inertia matrix about the mass center in the body frame ($kg - m^2$)
X, Y, Z	components of the total force applied to CubeSAT in body frame (N)
L, M, N	components of the total moment applied to CubeSAT in body frame (N-m)
$\vec{r}_{A \to B}$	position vector from a generic point A to a generic point B (m)
$\vec{V}_{A/B}$	velocity vector of a generic point A with respect to a generic frame B (m/s)
$\mathbf{S}(\vec{r})$	skew symmetric matrix operator on a vector. Multiplying this matrix by a vector is equivalent to a cross product

1 Reaction Wheel Model

The reaction wheel model must be included before the attitude dynamics beause they directly affect the inertia of the satellite. There are three reaction wheels on this satellite but to simplify the dynamics a 1D system will be used. Thus 1 reaction wheel will be modeled. This reaction wheel has an angular velocity $\dot{\theta}_R$ and angular acceleration $\ddot{\theta}_R$. The inertia of the reaction wheel is first written about the center of mass of the reaction wheel and is given by the equation below where the reaction wheel is modeled as a disk with finite radius (r_{RW}) and height (h_{RW}). The subscript R is used to denote that this inertia matrix is about the center of mass of the reaction wheel while the super script R is used to denote the frame of reference in this case the body frame of the CubeSAT

$$I_R^B = m_R r^2/2 \tag{1}$$

The parallel axis theorem can then be used to shift the inertias to the center of mass of the satellite where the subscript RB denotes the reaction wheel inertia taken about the center of mass of the satellite.

$$I_{RB}^B = I_R^B + m_R(x_R^2 + y_R^2) \tag{2}$$

Where x_R, y_R are the distances from the center of mass of the satellite to the center of mass of the reaction wheel in the satellite body reference frame. The total inertia of the entire satellite-reaction wheel system is then just a sum of the satellite and the reaction wheels

$$I_S = I_B + I_{RB}^B \tag{3}$$

The total angular momentum of the satellite is then equal to the following equation where $\dot{\theta}$ is the angular velocity of the satellite.

$$H_S = I_B\dot{\theta} + I_{RB}^B(\dot{\theta}_R + \dot{\theta}) \tag{4}$$

In a similar fashion, the total torque placed on the satellite is equal to the following

$$M_R = I_{RB}^B \ddot{\theta}_R \tag{5}$$

The reaction wheel acceleration is controlled by an input torque from a motor. However the input torque is not constant due to the fact that the reaction wheel cannot apply anymore torque once the reaction wheel has reached its maximum angular velocity. Thus the equations of motion for the reaction wheel are given by

$$\ddot{\theta}_R = T/I_{RB}^B \tag{6}$$

where $|T_{MAX}| = |T_0 - \frac{T_0}{\dot{\theta}_{R,max}}\dot{\theta}_R|$.

2 Rotational Equations of Motion in 3D

The equations of motion of a satellite in free space can be given using the equation below assuming the inertia matrix is constant.

$$M_R = \mathbf{I}_S \ddot{\theta} \tag{7}$$

3 Control Schemes for Reaction Wheels

Assuming each reaction is aligned with a principal axis of inertia the control scheme is extremely simple. The derivation here will just be for the aligned case. In this analysis it is assumed that a torque can be applied to the reaction wheel and thus the angular acceleration of the reaction wheel can be directly controlled through the torque input T. Assuming this, a simple PD control law can be used to orient the satellite at any desired orientation.

$$T_C = -k_p(\tilde{\theta} - \theta_C) - k_d(\dot{\tilde{\theta}} - \dot{\theta}_C) \tag{8}$$

Note that T_C is used here since the system cannot generate torque from nowhere. As such the torque must first pass through a first order filter.

$$\dot{T} + aT = T_C \tag{9}$$

where a is the cutoff frequency of the first order filter. Furthermore $\tilde{\theta}$ is used since the sensor is not perfect or immediate and also introduces some delay such that

$$\dot{\tilde{\theta}} + b\tilde{\theta} = \theta \tag{10}$$

where b is the cutoff frequency of the sensor. In order to design and select reaction wheels the maximum angular momentum of the satellite must be obtained by assuming the worst case angular velocity times the moment of inertia of the satellite.

$$H_{required} = n||I_s \dot{\theta}_{MAX}|| \tag{11}$$

With this reaction wheels can be selected based on this worst case scenario plus a safety factor of n which is typcially set to 2 in spacecraft operations. If reaction wheels cannot be used in the event of saturation or other issues reaction control thrusters can be used. Typically a value of $|\dot{\theta}_{MAX}| = 5^o/sec$ is used.