**Lab 1: Linear Controls Analysis of a 1-DOF 3U CubeSat**

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**1 Introduction:**

A CubeSat is a small satellite on the order of 10 centimeters along each axis. A 1U satellite is a small cube with 10 cm sides. These satellites are used for a variety of missions and created by a variety of different organizations. When deployed from a rocket, a CubeSat may obtain a large angular velocity which must be reduced before most science missions or communications can take place. Maximizing solar energy charging also involves better pointing accuracy. To control the attitude of these small satellites, reaction wheels, magnetorquers and even the gravity gradient are used in low earth orbit (LEO) while reaction control thrusters are typically used in deep space. On a standard LEO CubeSat, 3 reaction wheels are used as well as 3 magnetorquers. In the initial phase of the CubeSat mission, the magnetorquers are used to reduce the angular velocity of the satellite down to a manageable level. Once the norm of the angular velocity is low enough, the reaction wheels can spin up reducing the angular velocity to zero. This paper investigates the necessary mathematics to understand the control architecture necessary for a simple CubeSat.

**2 Math Model:**

**2.1 Reaction Wheel Model:**

The reaction wheel model must be included before the attitude dynamics because they directly affect the inertia of the satellite. There are three reaction wheels on this satellite but to simplify the dynamics a 1-D system will be used. Thus, one reaction wheel will be modeled. This reaction wheel has an angular velocity and angular acceleration . The inertia of the reaction wheel is first written about the center of mass of the reaction wheel and is given by the equation below where the reaction wheel is modeled as a disk with finite radius () and height (). The subscript R is used to denote that this inertia matrix is about the center of mass of the reaction wheel while the super script R is used to denote the frame of reference in this case the body frame of the CubeSat.

(1)

The parallel axis theorem can then be used to shift the inertias to the center of mass of the satellite where the subscript RB denotes the reaction wheel inertia taken about the center of mass of the satellite.

(2)

Where , are the distances from the center of mass of the satellite to the center of mass of the reaction wheel in the satellite body reference frame. The total inertia of the entire satellite-reaction wheel system is then just a sum of the satellite and the reaction wheels.

(3)

The total angular momentum of the satellite is then equal to the following equation where is the angular velocity of the satellite.

(4)

In a similar fashion, the total torque placed on the satellite is equal to the following:

(5)

The reaction wheel acceleration is controlled by an input torque from a motor. However, the input torque is not constant since the reaction wheel cannot apply anymore torque once the reaction wheel has reached its maximum angular velocity. Thus, the equations of motion for the reaction wheel are given by:

(6)

Where .

**2.2 Rotational Equation of Motion in 1-D:**

The equations of motion of a satellite in free space can be given using the equation below assuming the inertia matrix is constant.

(7)

**2.3 Control Schemes for Reaction Wheels:**

Assuming each reaction is aligned with a principal axis of inertia the control scheme is extremely simple. The derivation here will just be for the aligned case. In this analysis it is assumed that a torque can be applied to the reaction wheel and thus the angular acceleration of the reaction wheel can be directly controlled through the torque input T. Assuming this, a simple PD control law can be used to orient the satellite at any desired orientation.

(8)

Note that is used here since the system cannot generate torque from nowhere. As such, the torque must first pass through a first order filter.

(9)

where a is the cutoff frequency of the first order filter. Furthermore is used since the sensor is not perfect or immediate and introduces some delay such that:

(10)

where is the cutoff frequency of the sensor. In order to design and select reaction wheels the maximum angular momentum of the satellite must be obtained by assuming the worst-case angular velocity times the moment of inertia of the satellite.

(11)

With this reaction wheels can be selected based on this worst-case scenario plus a safety factor of which is typically set to 2 in spacecraft operations. If reaction wheels cannot be used in the event of saturation or other issues, reaction control thrusters can be used. Typically, a value of is used.

**3 Simulation Results:**

To begin the analysis, reaction wheels parameters were found from Blue Canyon Technologies. The parameters taken were momentum p, mass m, volume V, and max torque T where p was found to be 0.050 Nms, m was found to be 0.24kg, V was found to be 58x58x25 mm, and T was found to be 0.007 Nm. Length, width and thickness of the reaction wheel were also found to be 0.1 m each [1]. Mass of the satellite was taken found from NASA’s CubeSat Overview and was assumed to be around 1.33 kg [3]. Using Eqn. 2, the inertia of the satellite was found to be 0.0022 kgm^2, and the reaction wheel inertia was found to be 0.00062 kgm^2. Using Eqn. 3, the total inertia was calculated to be 0.00282 kgm^2. The delay for the attitude stabilization of the reaction wheel was given as 760 ms [1]. By solving for the time constant, a value of 1.3 was calculated. Values for the saturation block limits were taken from the max torque values found on Blue Canyon Technologies [1].

The purpose of this project was to stabilize 1-DOF 3U CubeSat with the calculated parameters. In order to stabilize this system, two lead compensators were used. A lead compensator helps to improve an undesirable frequency response in a feedback control system. This lead compensator added zeros to the system which helped to drive the poles to the left-hand plane and stabilize the system [2].

MATLAB and Simulink were used to simulate the satellite and determine if the system was stable. From Fig. 1, the system was determined to be stable when two lead compensators were applied. This is because the oscillations the satellite experiences from its initial launch are shown to decay. This shows that the satellite will come under our control after 80 seconds. After plotting the root locus, Fig. 2 shows the system will remain stable until the gain in increased to a certain value. At this value, the system will become unstable and begin to oscillate uncontrollably. To find how much gain can be added to the system, a bode plot for both the open loop system and the closed loop system were plotted. Fig. 3 and Fig. 4 show the bode plots of the open and closed loop system. For the open loop system (Fig. 4), the gain margin is negatively infinite, and the phase margin is -80.5 degrees. Negative margins whether phase or gain indicate instability for the system, so since both the gain and phase margins are negative, the open loop system is unstable. For the closed loop system (Fig. 3), the gain margin was found to be 107dB meaning we could have a gain of 106 dB before the system becomes unstable. Likewise, the phase margin was found to be 2.47 degrees meaning the system is stable. Finally, a pole-zero map was plotted in Fig. 5 to show the additional poles and zeros added by the lead compensator in the closed loop system.

With Simulink, a block diagram was created to show the gain, lead compensators, actuator dynamics, saturation block, reaction wheel dynamics, satellite dynamics, and sensor dynamics and is shown in Fig. 6. With this block diagram, the torque signal both before and after the saturation block, the angular velocity of the reaction wheel, and the angular displacement of the satellite were plotted to see the system dynamics over time. From Fig. 7, the measured angular displacement response of the system was plotted versus time to show the initial oscillations will decay with the lead compensators. From Fig. 8, the angular velocity of the reaction wheel is plotted versus time to show how the reaction wheel spin cancels the satellites oscillations. A saturation block was applied to the block diagram to limit the torque output signal coming from the actuator dynamics to constrain the output signal to the max torque. Fig. 9 shows the torque coming from the actuator dynamics before and after the saturation block.

**4 Conclusion:**

From the simulation results, the satellite is shown to be stabilized by using two lead compensators. These lead compensators add zeros to the system to drive the poles to the left-hand plan and make the system stable. Both a root locus and bode plot analysis in MATLAB were used to prove this stability as well as Simulink to create and simulate the system by using a block diagram. With the system stabilized, a linear controller can be derived and tested in order to apply full control on the system for a multitude of missions and maneuvers.

**5 Acknowledgements:**

We would like to thank James Helton and Slater Dozier. Both students helped us with the MATLAB and Simulink Code as well as understanding how to stabilize the system. Without their help, this project would not have been finished.

**6 Appendix:**

**6.1 Code:**

clear

clc

close all

A = tf([1.3],[1,1.3])

G = tf([1],[.00303,0,0])

H = tf([1000],[1,1000])

GOL = A\*G\*H;

bode(GOL)

margin(GOL)

figure()

rlocus(GOL)

C1 = tf([1,1],[1,650])

C2 = tf([1,5],[1,300])

C = C1\*C2;

CGOL = C\*GOL

bode(CGOL)

margin(CGOL)

figure()

rlocus(CGOL)

K = 5;

GCL = K\*CGOL/(1+K\*CGOL);

figure()

step(GCL)

figure()

pzmap(GCL)

**6.2 Figures:**



Figure : Amplitude vs Time of the Satellite When Hit with a Step Function



Figure : A Root Locus Analysis of the Satellite Determining Stability



Figure : A Bode Plot of the Closed Loop System Determining Gain and Phase Margin



Figure : A Bode Plot of the Open Loop System Determining Gain and Phase Margin



Figure : PZ Map of the Satellite System to Show the Poles and Zeros of the System

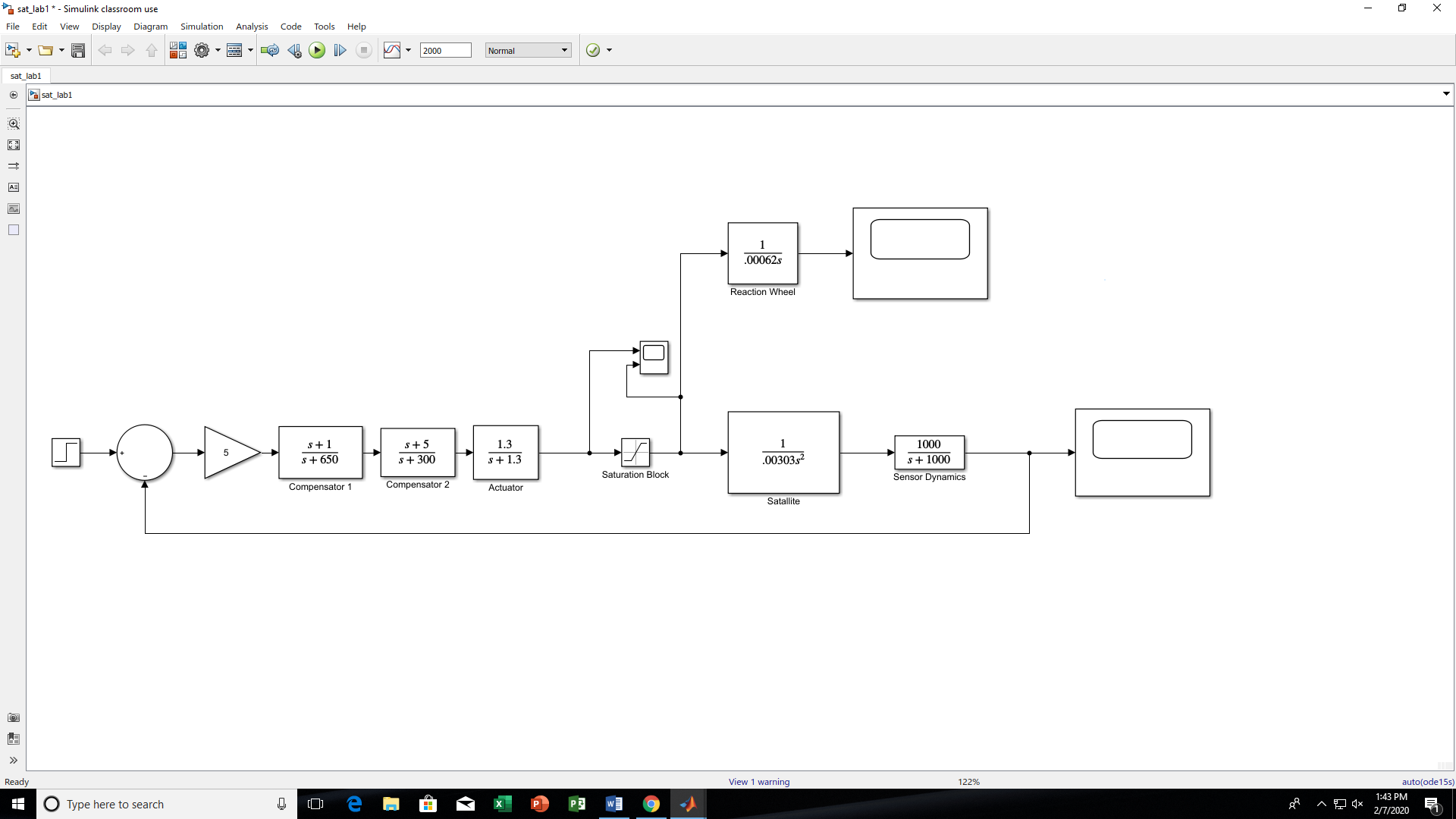


Figure : A Control Block Diagram of the Satellite System for Measuring Outputs of the Reaction Wheel, Torque, and Angle of the Satellite

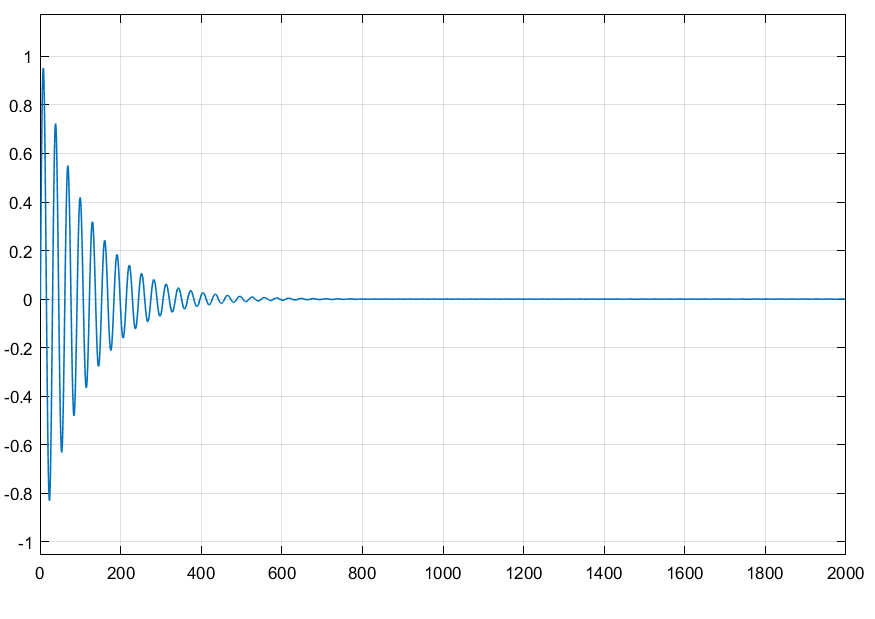


Figure : The Measured Theta of the Satellite with Theta along the X-axis and Time along the Y-axis

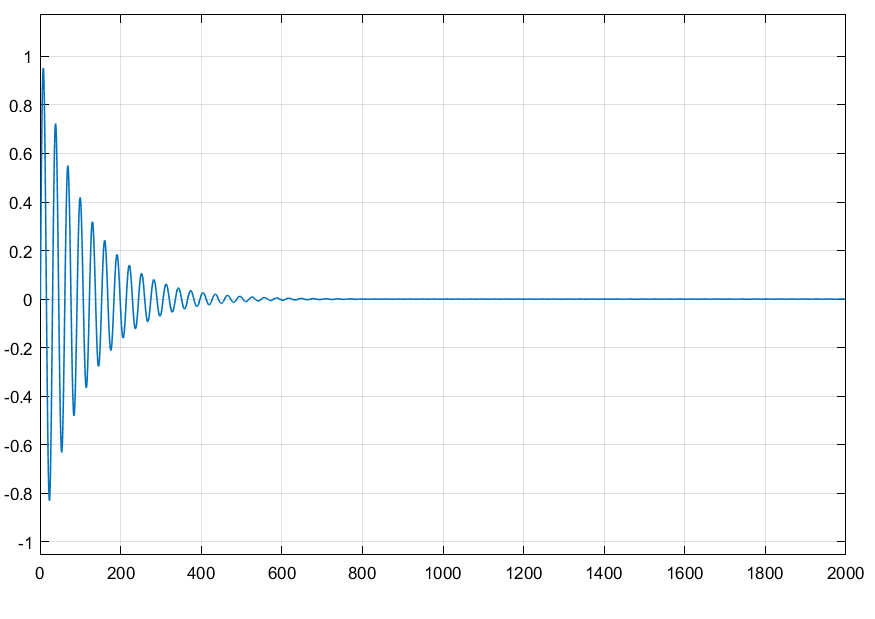


Figure : The Angular Velocity Measured from the Reaction Wheel with Angular Velocity along the X-axis and Time along the Y-axis

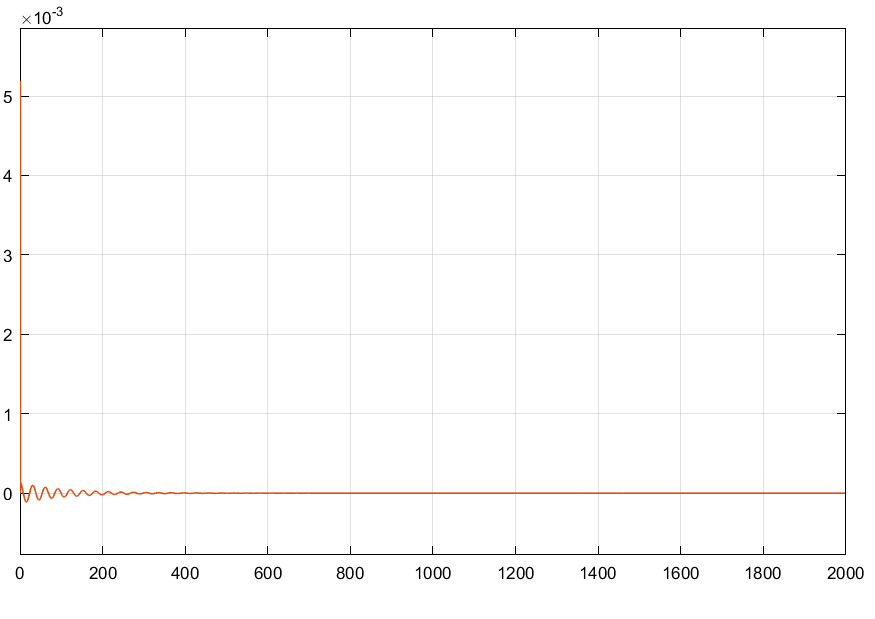


Figure : Torque Coming from the Actuator Dynamics Measured Before and After the Saturation Block with Torque along the X-axis, Time along the Y-axis, the Blue Line Pre-Saturation Torque, and the Orange Line Post-Saturation Torque

**6.3 References:**

1. Blue Canyon Technologies, “Blue Canyon Technologies Components,” *Blue Canyon Technologies* Available: <https://www.bluecanyontech.com/components>.
2. Franklin, G. F., Powell, J. D., and Emami-Naeini, A., *Feedback control of dynamic systems*, Upper Saddle River, NJ: Pearson, 2020.
3. Loff, S., “CubeSats Overview,” *NASA* Available: <https://www.nasa.gov/mission_pages/cubesats/overview>.