

Calculating polarized scattering

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7th August 2025

In the Dirac representation, the Gamma matrices have the following form:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (0.1)$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad (0.2)$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad (0.3)$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad (0.4)$$

and the Dirac spinors look like:

$$u_+(E, \vec{p}) = \sqrt{E+m} \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2) \\ \frac{|\vec{p}|}{E+m} \cos(\theta/2) \\ \frac{|\vec{p}|}{E+m} \sin(\theta/2) \end{pmatrix} \quad (0.5)$$

$$u_-(E, \vec{p}) = \sqrt{E+m} \begin{pmatrix} -\sin(\theta/2) \\ \cos(\theta/2) \\ \frac{|\vec{p}|}{E+m} \sin(\theta/2) \\ -\frac{|\vec{p}|}{E+m} \cos(\theta/2) \end{pmatrix} \quad (0.6)$$

$$v_+(E, \vec{p}) = \sqrt{E+m} \begin{pmatrix} \frac{|\vec{p}|}{E+m} \sin(\theta/2) \\ -\frac{|\vec{p}|}{E+m} \cos(\theta/2) \\ -\sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix} \quad (0.7)$$

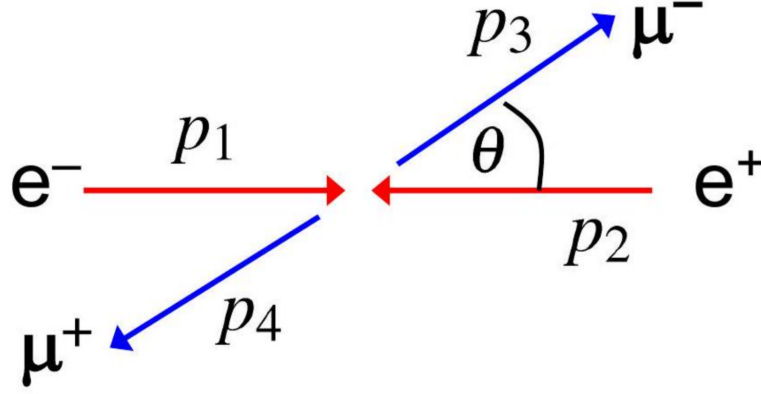


Figure 1: A much simpler version of the diagram you need for Exercise 1 – talk to us for all the details

$$v_-(E, \vec{p}) = \sqrt{E+m} \begin{pmatrix} \frac{|\vec{p}|}{E+m} \cos(\theta/2) \\ \frac{|\vec{p}|}{E+m} \sin(\theta/2) \\ \cos(\theta/2) \\ \sin(\theta/2) \end{pmatrix} \quad (0.8)$$

where $E^2 = |\vec{p}|^2 + m^2$, and $\vec{p} = (p_x, p_y, p_z) = (|\vec{p}| \sin \theta, 0, |\vec{p}| \cos \theta)$

Bonus exercise 1: Show that $(u_+ u_+^\dagger + u_- u_-^\dagger) \gamma^0 = (E \gamma^0 - \sum_{i=1}^3 p_i \gamma^i + m \mathbb{1}_{4 \times 4})$

Bonus exercise 2: Show that $(v_+ v_+^\dagger + v_- v_-^\dagger) \gamma^0 = (E \gamma^0 - \sum_{i=1}^3 p_i \gamma^i - m \mathbb{1}_{4 \times 4})$

Exercise 1: Write down the amplitude \mathcal{M} for $e^- e^+ \rightarrow \mu^- \mu^+$, assuming the initial electron and final muon both have “+” spin, and the initial positron and final state muon have “-” spin, just in terms of spinors and the gamma matrices. For this you can assume that both the electron and the muon have zero mass, so that $E = |\vec{p}|$.

Exercise 2: Using the representation of the spinors and gamma matrices given above, work out an explicit result for the amplitude \mathcal{M} .

Exercise 3: Using your previous answer, work out the differential cross-section

$$\frac{d\sigma}{d(\cos \theta)} = \frac{1}{32\pi s} |\mathcal{M}|^2 \quad (0.9)$$

where $s = 4E^2$.

Bonus exercise 3: Do exercises 1–3 again, but now for $e^- e^+ \rightarrow e^- e^+$. Similarly assume the initial and final electron both have “+” spin, and the initial and final positron have “-” spin. There is a second diagram now!