ECON 4390

Subsidized Loans

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Classical Education Model

Recall the classical education model:

$$W_0 = aw + \frac{aw}{1+r}$$

$$W_1 = -e + \frac{ahw}{1+r}$$

Government Subsidized Loans

The government often subsidizes college tuition via loans at a rate, r_G, which is less than the market rate, r (i.e. $r_G < r$).

Hence, for students using government subsidized loans:

$$W_0 = aw + \frac{aw}{1+r}$$

$$W_G = -e + \frac{ahw}{1+r} + e - e \cdot \frac{1+r_G}{1+r}$$

$$= \frac{ahw}{1+r} - e \cdot \frac{1+r_G}{1+r}$$

Government Subsidized Loans

To prefer government subsidized loans, (i.e. $W_G > W_1$), what must be true?

$$W_G = \frac{ahw}{1+r} - e \cdot \frac{1+r_G}{1+r} > W_1 = -e + \frac{ahw}{1+r}$$

$$-e \cdot \frac{1+r_G}{1+r} > -e$$

$$\boxed{e \cdot \frac{1+r_G}{1+r} < e}$$

$$\frac{1+r_G}{1+r} < 1$$

$$1 + r_G < 1 + r$$

$$\boxed{r_G < r}$$

- which holds as we defined $r_G < r$

Government Subsidized Loans

What is the rate of return, $\rho_G(a)$, on the government subsidized loan?

$$\rho_G(a) = \frac{(awh - aw) - (aw + e \cdot \frac{1+r_G}{1+r})}{aw + e \cdot \frac{1+r_G}{1+r}}$$

$$\rho_G(a) = \frac{aw(h-1)}{aw + e \cdot \frac{1+r_G}{1+r}} - 1$$

$$\rho_G(a) + 1 = \frac{aw(h-1)}{aw + e \cdot \frac{1+r_G}{1+r}} = \frac{h-1}{1 + \frac{e}{aw} \cdot \frac{1+r_G}{1+r}}$$

Government Subsidized Loans

Statics for subsidized & non-subsidized loans:

$$\rho_G(a) + 1 = \frac{h-1}{1 + \frac{e}{aw} \cdot \frac{1+r_G}{1+r}} \quad \text{and} \quad \rho(a) + 1 = \frac{h-1}{1 + \frac{e}{aw}}$$

	$h \uparrow$	$e \uparrow$	$r \uparrow$	$r_G \uparrow$	$w \uparrow$	$a \uparrow$
$\rho_G(a)$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\rho(a)$	$\uparrow$	$\downarrow$	—	—	$\uparrow$	$\uparrow$

- Why is $\rho_G(a)$ conditional on r_G and r?

How do we fund government subsidized loans?

- Taxation
- Borrowing (i.e. deficit spending)

Funding: Taxation

Suppose the government funded subsidized loans via a proportional tax, τ, on college-educated workers.

Without tax:

$$W_G = -e \cdot \frac{1 + r_G}{1 + r} + \frac{ahw}{1 + r}$$

With tax:

$$W_G = -e \cdot \frac{1 + r_G}{1 + r} + \frac{ahw(1 - \tau)}{1 + r}$$

Funding: Taxation

If $W_G(a^*) = W_0(a^*)$, then $a^* = ???$

$$W_0(a^*) = a^* w + \frac{a^* w}{1+r} = W_G(a^*) = -e \cdot \frac{1+r_G}{1+r} + \frac{a^* hw(1-\tau)}{1+r}$$

$$(1+r) \cdot a^* w \left(1 + \frac{1}{1+r} - \frac{h(1-\tau)}{1+r}\right) = -e \cdot \frac{1+r_G}{1+r} \cdot (1+r)$$

$$a^* w \left((1+r) + (1) - [h(1-\tau)]\right) = -e \cdot (1+r_G)$$

$$\boxed{a^* = \frac{e}{w} \cdot \frac{1+r_G}{(h(1-\tau)-1) - (1+r)}}$$

Funding: Taxation

What is the rate of return, $\rho_G(a)$, on the government subsidized loan?

$$\rho_G(a) = \frac{(awh - aw) - (aw + e \cdot \frac{1+r_G}{1+r})}{aw + e \cdot \frac{1+r_G}{1+r}}$$

Funding: Taxation

So what is the government's budget, B, for this program?

- **Assumption:** Only funded by proportional tax on college-educated workers
- **Recall:** $a \sim U(a_m, a_M)$ (i.e. $\min\{a\} = a_m$ and $\max\{a\} = a_M$)

$$B = \int_{a^*}^{a_M} \frac{e}{a_M - a_m} da \quad \leftarrow \quad \text{total program cost}$$

$$B = \int_{a^*}^{a_M} \frac{\tau awh}{a_M - a_m} da \quad \leftarrow \quad \text{total revenue for program}$$

$$\text{Total Cost} = \text{Total Revenue}$$

$$B = \int_{a^*}^{a_M} \frac{e}{a_M - a_m} da = \int_{a^*}^{a_M} \frac{\tau awh}{a_M - a_m} da$$

Funding: Taxation

$$B = \int_{a^*}^{a_M} \frac{e}{a_M - a_m} da = \int_{a^*}^{a_M} \frac{\tau awh}{a_M - a_m} da$$

$$\frac{e}{a_M - a_m} \int_{a^*}^{a_M} 1 \; da = \frac{\tau wh}{a_M - a_m} \int_{a^*}^{a_M} a \; da$$

$$\left(\frac{e}{a_M - a_m} \right) \left(a \Big|_{a^*}^{a_M} \right) = \left(\frac{\tau wh}{a_M - a_m} \right) \left(\frac{a^2}{2} \Big|_{a^*}^{a_M} \right)$$

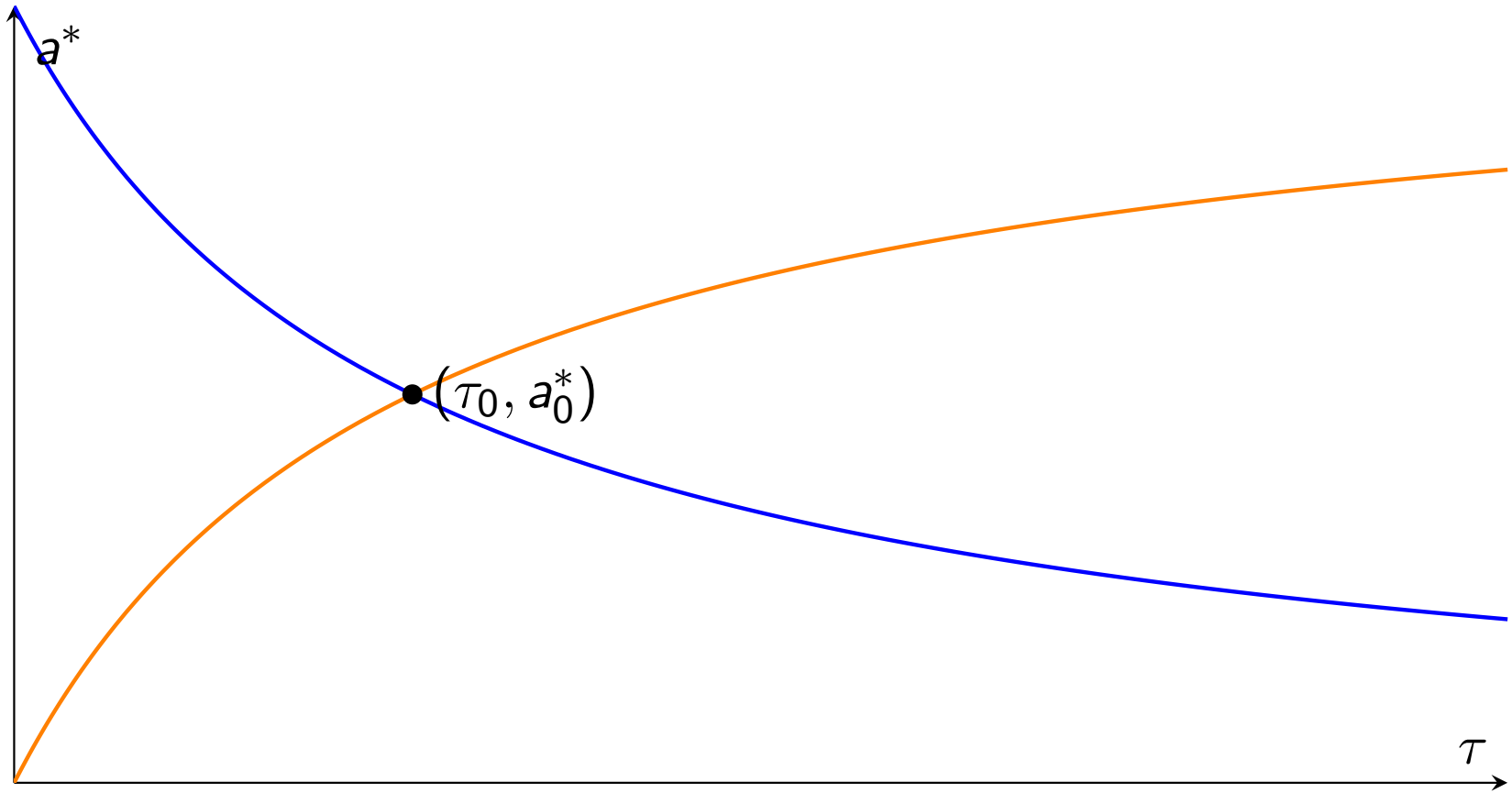
$$\frac{e(a_M - a^*)}{a_M - a_m} = \left(\frac{\tau wh}{a_M - a_m} \right) \left[\frac{(a_M)^2}{2} - \frac{(a^*)^2}{2} \right]$$

$$e(a_M - a^*) = \frac{\tau wh}{2} \cdot \left[(a_M)^2 - (a^*)^2 \right]$$

$$e(a_M - a^*) = \frac{\tau wh}{2} \cdot (a_M - a^*) \cdot (a_M + a^*)$$

$$e = \frac{\tau wh}{2} \cdot (a_M + a^*) \Rightarrow \boxed{a^* = \frac{e}{w} \cdot \frac{2}{\tau h} - a_M}$$

Funding: Taxation



SCRAPPED SLIDES

Content we don't want to cover goes after here.

Government Subsidized Loans

$$W_0(a^*) = W_G(a^*) \Rightarrow a^* = ???$$

$$W_0(a^*) = a^* w + \frac{a^* w}{1+r} = W_G(a^*) = \frac{a^* h w}{1+r} - e \cdot \frac{1+r_G}{1+r}$$

$$a^* w \left(1 + \frac{1}{1+r} - \frac{h}{1+r}\right) = -e \cdot \frac{1+r_G}{1+r}$$

$$a^* w \left(\frac{(1+r) - (h-1)}{1+r}\right) = -e \cdot \frac{1+r_G}{1+r}$$

$$\boxed{a^* = \frac{e}{w} \cdot \frac{1+r_G}{(h-1) - (1+r)}}$$