ECON 4310 HW1

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(1) *Consider the set of alternatives $X = \{x, y, z\}$. A binary relation R on X is a set of ordered pairs from X. If $(a, b) \in R$, we write aRb and read "a is related to b". In the case of preferences, this means "a is at least as good as b".*

(a) *Provide an example of a binary relation on X that is complete and reflexive, but not transitive.*

Take $R_1 = \{(x, x), (y, y), (z, z), (x, y), (y, z), (z, x)\}$.

R_1 is reflexive: $(x, x), (y, y), (z, z) \in R_1$
R_1 is complete: $(x, y), (y, z), (z, x) \in R_1$
R_1 is not transitive: $(x, y), (y, z) \in R_1$ but $(x, z) \notin R_1$

(b) *Provide an example of a binary relation on X that is reflexive and transitive, but not complete.*

Take $R_2 = \{(x, x), (y, y), (z, z), (x, y)\}$.

R_2 is reflexive: $(x, x), (y, y), (z, z) \in R_2$
R_2 is not complete: $(z, y), (y, z) \notin R_2$
R_2 is transitive: the only nontrivial link is xRy, and there's no $yR\cdot$ to trigger a violation.

(2) *Provide an example from your life or experience where either the axiom of transitivity or completeness has been violated with respect to your preferences.*

Last weekend when I went to Katie's for dinner with some friends, my choices violated the axiom of transitivity. I had three options worth considering: pizza, salad, and ice cream. I preferred pizza over salad (because it's more filling), salad over ice cream (because it's healthier), and ice cream over pizza (because I love ice cream).

This violated the axiom of transitivity because:

$$\text{pizza} > \text{salad} > \text{ice cream} > \text{pizza}$$

(3) *Suppose a decision maker chooses $\{\text{cherries}\}$ from the set $\{\text{apples}, \text{bananas}, \text{cherries}\}$.*

(a) *The next day they choose from $\{\text{bananas}, \text{cherries}\}$. What, precisely, is required of their choice in order for the choice data to satisfy Property A (IIA)?*

They must choose $\{\text{cherries}\}$ as $\{\text{bananas}, \text{cherries}\} \subset \{\text{apples}, \text{bananas}, \text{cherries}\}$.

(b) *On the third day they choose from $\{\text{apples}, \text{bananas}\}$. What, precisely, is required of this choice in order for the choice data to satisfy Property A (IIA)?*

They may choose either $\{\text{apples}\}$ or $\{\text{bananas}\}$. For IIA to hold, for any $X \subset \{\text{apples}, \text{bananas}, \text{cherries}\}$ s.t. $\{\text{cherries}\} \in X$, the decision maker will choose $\{\text{cherries}\}$. However, $\{\text{cherries}\} \notin \{\text{apples}, \text{bananas}\}$.

(4) *Suppose a decision maker chooses* {bananas, cherries} *from the set* {bananas, cherries} *and chooses* {cherries} *from the set* {apples, bananas, cherries}.

  (a) *Does this data satisfy Property B? Why or why not?*

  No. If Property B were to be satisfied, the decision maker would choose {bananas, cherries}.

  (b) *Does this data satisfy Property A (IIA)? Why or why not?*

  Yes, because {cherries} is still being chosen.

(5) *Suppose a decision maker produces choice data in which a single alternative is chosen from every menu.*

  (a) *Does this data necessarily satisfy Property A (IIA)? Why or why not?*

  No. Choosing one alternative per menu does not guarantee IIA because it does not require that for any menu, M, containing an item, x, $x \in c(M)$.

  (b) *Does this data necessarily satisfy Property B? Why or why not?*

  Yes, but vacuously. If a *single* alternative is chosen from every menu, then it is impossible for $x, y \in c(M_i)$. Thus, the premise of Property B never holds, and the implication

  $$(x, y \in c(M_i) \ \wedge \ y \in c(M_j)) \ \Rightarrow \ x \in c(M_j) \quad \text{where} \quad M_i \subset M_j$$

  is satisfied vacuously.

(6) *Provide give an example from your life or experience where Property A (IIA) or Property B has been violated with respect to your choices.*

  One time at a team dinner, I was going to order a steak with a glass of red wine. Normally, I would not have chosen the wine since I don't like alcohol, but I was told it paired well with the steak and, since I wasn't paying, I considered it. However, they were sold out of the steak, so I dropped the wine as well and ordered the cacio e pepe instead.

  This choice violates IIA. In the initial menu, M_1, my choice was {steak, red wine}. But under the reduced menu $M_2 = M_1 \setminus$ {steak}, my choice became {cacio e pepe}. Thus, the red wine—originally part of the chosen bundle—was not selected once steak was removed, contradicting the independence of irrelevant alternatives.

(7) *Now that we have formally defined what it means to be a rational decision maker, do you think your choice on whether to sit in the computer section or not was rational?*

  No, my choice to sit in the computer section was not rational because new information—the computer section being in the back—arose after I made my choice. Because my vision is bad, I can't read the lecture slides in the back, so I moved to the front and switched to paper notes.