Problem Set 3 – Deviations from Expected Utility

Econ 4310

Due: 11:59pm, 10/10/2025

Please turn in a single copy of the problem set on Canvas. Each of you must independently write your own solutions to the homework. You should feel free to discuss the questions with each other, with me, or with our graduate AIs. However, the work you produce must, in the end, represent your own thoughts and understanding of the problems. In particular, it would not be appropriate to study someone else's answer to a problem and reproduce it as your own.

1. **Independence Axiom** Consider a decision maker (DM) in the expected utility framework.

   (a) Suppose the DM prefers lottery A to lottery B. Now take any probability p and any third lottery C. State what the Independence Axiom requires for this DM.

   Take A = {50% of 0, 50% of 100} and B = {100% of 40}. Assume the DM prefers A to B. Now take lottery C = {40% of 0, 50% of 100, 10% of 200} and lottery D = {80% of 40, 10% of 100, 10% of 200}.

   b) Find a lottery E and a probability p so that you can (i) write lottery C as a mixture of A and E, of the form C = p of lottery A, 1-p of lottery E, and (ii) write lottery D as a mixture of B and E of the form D = p of lottery B, 1-p of lottery E.

   c) Assuming (only) that the DM obeys EU theory (and still prefers A to B), does she prefer C to D, D to C, or can it not be determined? Provide a one sentence explanation.

2. **Risk attitudes** Consider the following two lotteries. L1 pays \$100 with probability 1. L2 pays \$200 with probability 60% and pays \$0 with probability 40%.

   (a) What is the expected value of each lottery?

   For parts b-d, your answer should be L1, L2, or "needs more information to determine." Provide a one-sentence explanation with each answer.

   b) Which lottery would a risk-neutral DM prefer?

   c) Which lottery would a risk-loving DM prefer?

   d) Which lottery would a risk-averse DM prefer?

3. **Risk-averse utility function** Consider again L2 from the previous question. Draw a graph representing the utility function of a risk-averse decision maker (DM). Mark the points on the x-axis that correspond to the possible outcomes under L2. Label the utilities that the DM would obtain from these outcomes, u(200) and u(0), on the y-axis. Label the point on the graph that shows the DM's expected utility of L2. Now find the expected value of L2 and label the DM's utility of the sure amount EV(L2). Which is greater: E[u(L2)] or u(EV(L2))? Why?

4. **Uncertainty and Risk.** A DM is presented with two jars. Jar 1 has 67 red and 33 blue balls. Jar 2 consists of 100 total balls each of which is either red or blue but the colors are in an unknown proportion. An experiment consists of drawing a single ball from each jar. The DM faces the following two choices.

   The first choice is between option 1R which pays \$100 if the Jar 1 ball is red, and option 2R which pays \$100 if the Jar 2 ball is red (and \$0 otherwise). The second choice is between option 1B which pays \$100 if the Jar 1 ball is blue, and option 2B which pays \$100 if the Jar 2 ball is blue (and \$0 otherwise). Suppose the DM chooses 1R over 2R and 1B over 2B (and has a strict preference in each case).

   Are these choices consistent with subjective EU? In other words, does there exist a probability distribution over the contents of Jar 2 (that is, a belief that the proportion of Red balls is r and of Blue balls is 1-r) such that, given these beliefs, the choices of the DM can be rationalized by expected utility? If so, provide the subjective probabilities that rationalize the choices. If not, argue that there are no such probabilities. (Note: Because the outcomes are only \$0 and \$100, risk preferences play no role here. That is, all utility functions for which u(\$100) > u(\$0) are observationally equivalent on these choices. Notice that in order for the curvature of the utility function to be relevant, one would need to consider at least three wealth levels. This is the reason that the question asks only about probabilities and not also about the utility function.)

5. **Ellsberg Paradox.** Consider the following variation of the Ellsberg Paradox presented in class. (This variation is also contained in the original paper.) There is one box containing 90 balls. Exactly 30 of these balls are White. Each of the remaining 60 balls is either Green or Yellow. Nothing more is known about the box. One ball will be drawn at random from the box. Consider the following pair of bets:

   - Bet White: Wins \$10 if the ball drawn is White (and zero otherwise)
   - Bet Green: Wins \$10 if the ball drawn is Green (and zero otherwise).

   And consider this additional pair of bets:

   - Bet White/Yellow: Wins \$10 if the ball drawn is White or Yellow (and zero if Green)
   - Bet Green/Yellow: Wins \$10 if the ball drawn is Green or Yellow (and zero if White).

Take a decision maker who strictly prefers Bet White over Bet Green, and who also strictly prefers Bet Green/Yellow over Bet White/Yellow.

(a) For each of the pairs of bets, determine if it can be characterized purely by objective risk or not.

(b) Provide an intuitive justification for the provided preferences.

(c) Argue that this combination of preferences is incompatible with SEU. (Hint: an SEU decision maker must assign a belief to the expected number of green balls. Let us call that number g. Write down inequalities in terms of g that are implied by the preferences and show they lead to a contradiction.)

6. **Dynamic Consistency.** Consider a DM who is an exponential time discounter with discount factor δ. Let us consider a 3-period model in which the DM consumes c_t in period t, for $t = 1, 2, 3$. Instantaneous utility of consumption is given by $u(c) = c$.

(a) Write down an algebraic expression for total utility $U(c_1, c_2, c_3)$ in terms of δ.

(b) Consider the two consumption streams $A = (3, 0, 0)$ and $B = (0, 4, 0)$. For which values of δ does the DM prefer A over B?

(c) Consider the two consumption streams $C = (0, 3, 0)$ and $D = (0, 0, 4)$. If the DM is time consistent and prefers A over B, what can you conclude about her preference between C and D?

(d) For which values of δ does the DM prefer C over D?

(e) Based on this example, what do you conclude about exponential time discounting and time consistency?

7. **Dynamic Inconsistency** Now consider a DM who is a quasi-hyperbolic discounter with parameters β and δ. Let us consider a 3-period model in which the DM consumes c_t in period t, for $t = 1, 2, 3$. Instantaneous utility of consumption is given by a function $u(c)$.

(a) Write down an algebraic expression for total utility $U(c_1, c_2, c_3)$ in terms of u, β and δ.

(b) Set $\delta = 1$ and $u(c) = c$. What is $U(3, 0, 0)$? What is $U(0, 4, 0)$? For which values of β does the DM prefer $(3, 0, 0)$ to $(0, 4, 0)$?

(c) What is $U(0, 3, 0)$? What is $U(0, 0, 4)$? For which values of β does the DM prefer $(0, 3, 0)$ to $(0, 0, 4)$?

(d) For which values of β does the DM exhibit a preference reversal?

(e) Suppose $\beta = \frac{1}{2}$. If the DM is naïve, will she exhibit a preference for commitment?

(f) If the DM is sophisticated, will she exhibit a preference for commitment? How much would she be willing to pay in order to dictate now the choice at time 2?