Instruction: submit your work as a pdf file on Canvas. The code and outputs must be included in the pdf file if Python is used in the problems. The runnable Python code in the format of Jupiter notebook file must also be uploaded on Canvas separately.

Problem 1 (**Use Python numpy package to finish this**)

Consider the following two real vectors:

$$\boldsymbol{x} = [-2.4 \quad 1.5 \quad 0.0 \quad 3.2]^T \; and \; \boldsymbol{y} = [1.3 \quad 4.2 \quad 3.0 \quad -2.5]^T$$

a) Find the 1-norm, 2-norm, 3-norm and infinity-norm of $\boldsymbol{x}$**,** then sort them in a **descending order.**

b) Find the distance between $\boldsymbol{x}$ *and* $\boldsymbol{y}$, $d(\boldsymbol{x},\boldsymbol{y})=\|\boldsymbol{x}-\boldsymbol{y}\|$, using 1-norm,2-norm, and infinity-norm, then sort them in an **ascending order**.

Problem 2 (**don't use Python in this problem**)

Consider the following matrix:

$$\boldsymbol{A} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

a) Find the eigenvalues and corresponding eigenvectors of matrix $\boldsymbol{A}$

b) Find the eigendecomposition of matrix $\boldsymbol{A}$ as $\boldsymbol{A}=\boldsymbol{Q\Lambda Q^T}$using the results from a)

c) What is the definiteness of the matrix?

Problem 3 (**use Numpy package in this problem**)

Consider the following matrix:

$$\boldsymbol{B} = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 4 & 7 \\ 2 & 5 & 4 \end{bmatrix}$$

Find the eigenvalues and corresponding eigenvectors of matrix $\boldsymbol{B}$ and then find the eigendecomposition of matrix $\boldsymbol{B}$**.** Verify that $\boldsymbol{B}=\boldsymbol{Q\Lambda Q^{-1}}$

Problem 4

Use the definition of a convex function to show that *affine functions* $f(\boldsymbol{x})=\boldsymbol{a}^T\boldsymbol{x}+b$ is convex, where $\boldsymbol{a}, \boldsymbol{x}\in\mathcal{R}^n, b\in\mathcal{R}$.

Problem 5

Consider the following objective function:

$$f(x,y) = x^2 + 2y^2 - 2xy$$

a) Compute the ***Hessian*** matrix ***H*** of function *f*, and find the definiteness of matrix ***H.***

b) Use calculus method to find the minimum value of $f(x,y)$

c) Use **Python** to implement the gradient descent search method to find the global minimum value of function *f*, suppose the initial values of [*x*, *y*] are [1,1]. Show the convergence curve of $x\ and\ y$ with learning rate: 0.1, 0.01, and 0.001. (**Don't use sk-learn package**)