



IIT ROORKEE



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CERTIFICATION COURSE

# LOGISTIC REGRESSION PART-4

## LECTURE 49

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# LOGISTIC REGRESSION

- Odds and logit can be written as a function of probability of class 1 membership
  - Open RStudio
- In logistic regression model, we predict the logit values and therefore corresponding probability of a categorical outcome
  - Predicted probabilities values become the basis for classification
  - A prediction model for classification task

# LOGISTIC REGRESSION

- Estimation Technique
  - Least squares method used in multiple linear regression cannot be used
    - Non-linear formulation of logistic regression
  - Maximum likelihood method is used
    - Estimates are optimized in order to maximize the likelihood of obtaining the observations used in training the model
    - Less robust than estimation techniques used in linear regression
    - Reliability of estimates
      - Outcome variable categories should have adequate proportion
      - Adequate sample size w.r.t no. of estimates



# LOGISTIC REGRESSION

- Estimation Technique
  - Maximum likelihood method is used
    - Collinearity issues similar to linear regression
- Interpretation of Results
  - Logit model
    - Additive factor ( $\beta$ )
      - If  $\beta < 0$ , increase in  $x \Rightarrow$  decrease in logit values
      - If  $\beta > 0$ , increase in  $x \Rightarrow$  increase in logit values
    - For any value of  $x$ , interpretative statements of results are same



# LOGISTIC REGRESSION

- Interpretation of Results
  - Odds model
    - Multiplicative factor ( $e^{\beta}$ )
      - If  $\beta < 0$ , increase in  $x \Rightarrow$  decrease in odds
      - If  $\beta > 0$ , increase in  $x \Rightarrow$  increase in odds
    - For any value of  $x$ , interpretative statements of results are same
  - Probability model
    - For a unit increase in a particular predictor, corresponding change in the probability value is not a constant, while holding all other predictors constant
      - Depends on the specific values of the predictor
    - Interpretative statements of results depend on specific values of  $x$

# LOGISTIC REGRESSION

- Odds and odds ratios
  - Odds is a ratio of two probability values (prob. of class 1/prob. Of Class 0)
  - Odds ratio is ratio of two odds (odds of class m1/odds of class m2)
    - Odds ratio  $> 1 \Rightarrow$  odds of class m1 are higher than class m2
- Open RStudio

# LOGISTIC REGRESSION

- Linear Regression for a categorical outcome variable?
  - Can be done by treating the outcome variable as continuous and coding it numerically
  - However, anomalies will lead to spurious modeling
    - Predictions can take any value, not just dummy values  $\{0,1\}$
    - Outcome variable or residuals don't follow normal distribution
      - binomial distribution
    - Variance of outcome variable is not constant across all records (violation of homoscedasticity)
      - $np(1-p)$



# LOGISTIC REGRESSION

- Logistic Regression for Profiling Task
  - Apart from model performance on validation partition
  - Model's fit to data is assessed on training partition
    - However, still avoid overfitting
    - Usefulness of predictors is examined
  - Goodness of fit metrics
    - Overall fit of the model
      - Deviance (equivalent to SSE in linear regression)
      - 1 - Deviance/Null Deviance (equivalent to multiple  $R^2$  in linear regression)
    - Single predictors





# LOGISTIC REGRESSION

- Outcome variable with  $m$  classes ( $m > 2$ )
  - Multinomial logistic regression
    - Separate binary logistic regression model for  $m-1$  classes (one class is treated as reference class)
  - Ordinal logistic regression
    - Large no. of ordinal classes: treat ordinal variable as continuous variable and apply multiple linear regression



# LOGISTIC REGRESSION

- Outcome variable with m classes ( $m > 2$ )
  - Ordinal logistic regression
    - Small no. of ordinal classes: Proportional odds or cumulative logit method
      - Separate binary logistic regression model for m-1 cumulative probabilities

For a three class case: C1, C2, and C3 and a single predictor  $x_1$

$$\begin{aligned} \text{logit}(C1) &= \alpha_0 + \beta_1 x_1 \\ \text{logit}(C1 \text{ or } C2) &= \beta_0 + \beta_1 x_1 \end{aligned}$$

- RStudio



# Key References

- Data Science and Big Data Analytics: Discovering, Analyzing, Visualizing and Presenting Data by EMC Education Services (2015)
- Data Mining for Business Intelligence: Concepts, Techniques, and Applications in Microsoft Office Excel with XLMiner by Shmueli, G., Patel, N. R., & Bruce, P. C. (2010)

# Thanks...

