"VSAQ"

① Write the conditions for the existence of Laplace Transform of a function.

Existence of Laplace Transform conditions:

$L[f(t)]$ exists if 1. $f(t)$ is a continuous function

2. $\lim_{t \to \infty} e^{-st} \cdot f(t)$ is a Finite.

② State 1. First shifting property
2. Second shifting property
3. change of scale property

1. First shifting property:

If $L\{f(t)\} = \bar{f}(s)$ then $L[e^{at} \cdot f(t)] = \bar{f}(s-a)$

2. Second shifting property:

If $L\{f(t)\} = \bar{f}(s)$ then $L[f(t-a)\,u(t-a)] = e^{-as} \cdot \bar{f}(s)$

3. change of scale property:

If $L[f(t)] = \bar{f}(s)$ then $L[f(at)] = \frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$

③ State convolution theorem?

Convolution theorem:

If $L^{-1}[\bar{f}(s)] = f(t)$ and $L^{-1}[\bar{g}(s)] = g(t)$ Then

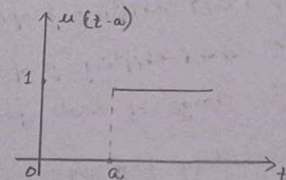
$$L^{-1}[\bar{f}(s) \cdot \bar{g}(s)] = \int_0^t f(u) \cdot g(t-u) \cdot du$$

(4) Define 1. unit step function
2. unit impulse function.

1. unit step function:

It is defined as $u(t-a) = \begin{cases} 0 & \text{For } t < a \\ 1 & \text{For } t \geq a \end{cases}$

where $a \in z^{+ve}$ and denoted as $u(t-a)$



2. unit Impulse function:

It is defined as

$$\delta(t-a) = \begin{cases} \infty & \text{for } t = a \\ 0 & \text{for } t \neq a \end{cases} \ni \int_0^\infty \delta(t-a)\, dt = 1, \ a \geq 0$$

(5) write the area enclosed by the curves (i) $y_1 = f_1(x)$ & $y_2 = f_2(x)$ between $x = x_1$ & $x = x_2$ (ii) volume as Triple Integral in Rectangular coordinates.

(i) Area enclosed by curves $y_1 = f_1(x)$ & $y_2 = f_2(x)$

B/n, $x = x_1$, $x = x_2$

$$\text{Area} = \int_{x=x_1}^{x=x_2} \int_{y=f_1(x)}^{y=f_2(x)} f(x,y)\, dy\, dx$$

(ii) volume as Triple Integral in Rectangular coordinates

$$\text{volume} = \iiint f(x, y, z)\, dx\, dy\, dz$$

⑥ write the Area in polar co-ordinates ?

Area for polar co-ordinates :

$$\text{Area} = \int_{\theta=\theta_1}^{\theta=\theta_2} \int_{r=f_1(\theta)}^{r=f_2(\theta)} r \cdot dr \cdot d\theta$$

⑦ write the volume as Triple Integral in spherical polar co-ordinates.

Volume for a Triple Integral in spherical polar co-ordinates:

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ and

$$\text{Jacobian } (J) = \frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$$

$$\therefore \text{volume} = \iiint_V r^2 \sin\theta\, dr\, d\theta\, d\phi$$

⑧ write the volume as Triple Integral in cylindrical polar co-ordinates.

cylindrical polar co-ordinates are $x = r\cos\theta$, $y = r\sin\theta$, $z = z$

$$\text{and Jacobian } (J) = \frac{\partial(x, y, z)}{\partial(r, \theta, z)} = r$$

$\therefore$ Volume $= \iiint_V r\, dr\, d\theta\, dz$

(9) Evalute the following 1. $\int_0^2\int_1^3\int_1^2 yz^2\, dz\, dy\, dx$

2. $\int_0^2\int_0^{x^2} e^{y/x}\, dy\, dx$; 3. $\iint xy\, dx\, dy$ over the Region

$R : \{0 \le x \le 1 \;\&\; 0 \le y \le 1\}$

(1) Given, $\int_0^2\int_1^3\int_1^2 yz^2\, dz\, dy\, dx$

$= \int_0^2\int_1^3 y \cdot z^2 [z]_1^2\, dy\, dx \Rightarrow \int_0^2\int_1^3 yz^2 (2-1)\, dy\, dz$

$\Rightarrow \int_0^2 \frac{z^2}{2} \cdot (y^2)_1^3\, dz \Rightarrow 4 \cdot \frac{(z^3)_0^2}{3} \Rightarrow \frac{32}{3}$

$\therefore = \frac{32}{3}$

(2) $\int_0^2\int_0^{x^2} e^{y/x}\, dy\, dx \Rightarrow \int_0^2 \frac{(e^{y/x})_0^{x^2}}{\frac{1}{x}}\, dx$

$\Rightarrow \int_0^2 x\,[e^{x^2/x} - e^0]\, dx$

$\Rightarrow \int_0^2 (xe^x - x)\, dx$

$$= \left((xe^{x} - x) - \frac{x^2}{2} \right)_0^2$$

$$= e^2 - 1$$

(10) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ Then Find Jacobian of Transformation from (x, y, z) to (r, θ, ϕ)

Given,

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

$$\text{Jacobian } (J) = \frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{vmatrix}$$

$$\Rightarrow \cos\theta\left[r^2\sin\theta\cos\theta\cos^2\phi + r^2\sin\theta\cos\theta\sin^2\phi\right] +$$
$$r\sin\theta\left[r\sin^2\theta\cos^2\phi + r\sin^2\theta\sin^2\phi\right.$$

$$\Rightarrow \cos\theta\left[r^2\sin\theta\cos\theta\right] + r\sin\theta\left[r\sin^2\theta\right]$$

$\therefore \; J = r^2 \sin\theta$

(11) If $u = x+y$, $v = x-2y$ then find $\frac{\partial(x,y)}{\partial(u,v)}$

Given,

$$\begin{aligned} x - 2y &= v \\ -x + y &= -u \\ \hline -3y &= v - u \end{aligned}$$

$$y = \frac{u-v}{3}$$

$$x = 2y + v$$

$$x = 2\left(\frac{u-v}{3}\right) + v$$

$$x = \frac{2u+v}{3}$$

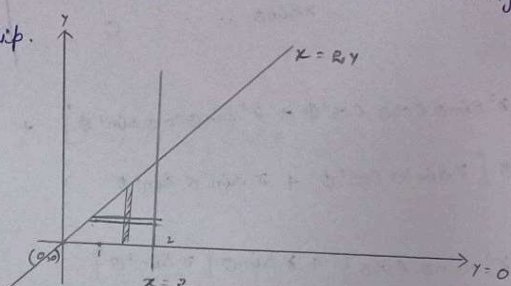
$$\therefore \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \Rightarrow \begin{vmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-1}{3} \end{vmatrix} \Rightarrow -\frac{1}{3}$$

(12) Evaluate $\iint e^{x^2}\,dx\,dy$ over the Region $R : \{2y \le x \le 2 \;\&\; 0 \le y \le 1\}$

Given,

$$\int_{y=0}^{y=1} \int_{x=2y}^{x=2} e^{x^2}\,dx\,dy$$

Given strip is a horizontal strip and it is changing to the vertical strip.



$\therefore$ Vertical strip limits : $y = 0$ TO $y = \frac{x}{2}$

$x = 0$ TO $x = 2$

$$\int_{x=0}^{x=2}\int_{y=0}^{y=\frac{x}{2}} e^{x^2}\, dy\, dx \Rightarrow \int_0^2 e^{x^2}\cdot [y]_0^{x/2}\, dx$$

$$\Rightarrow \int_0^2 e^{x^2}\cdot\left[\frac{x}{2}\right]\cdot dx$$

$x^2 = t$

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$$\Rightarrow \frac{1}{2}\int_0^2 x\, e^{x^2} dx \Rightarrow \frac{1}{2}\frac{\left(e^{x^2}\right)_0^2}{2}$$

$\int x e^{x^2} dx$

$= \frac{1}{2}\int e^t dt = \frac{e^{x^2}}{2}$

$$\Rightarrow \frac{e^4 - 1}{4}$$

(13) How can you convert $= \iiint_V f(x, y, z)\, dx\, dy\, dz$ into cylindrical polar co-ordinates.

In cylindrical polar co-ordinates : $x = r\cos\theta_1$, $y = r\sin\theta_1$, $z = z$

$$J = \frac{\partial(x, y, z)}{\partial(r, \theta_1, z)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial z} \end{vmatrix}$$

$$J = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\therefore J = r\cos^2\theta + r\sin^2\theta = r$$

$$\therefore J = r$$

$$\therefore \iiint_V f(x,y,z)\,dx\,dy\,dz = \iiint_V f(r\cos\theta, r\sin\theta, z)\,|J|\,dr\,d\theta\,dz$$

$$= \iiint_V f(r\cos\theta, r\sin\theta, z)\, r\,dr\,d\theta\,dz$$

(14) Define 1. Double Integration ; 2. Triple Integration

1. Double Integration :

Let, $f(x,y)$ be a positive function of 2 Independent variables x & y defined over a closed Region R in xy plane. Divide into Sub Divisions, Regions "R" into Sub Regions by drawing the lines parallel to co-ordinate axes.

Let, (x_r, y_r) be any point inside Region r^{th} rectangle whose is δA_r

consider sum,

$$\sum_{r=1}^{n} f(x_r, y_r)\,\delta A_r = f(x_1, y_1)\,\delta A_1 + f(x_2, y_2)\,\delta A_2 + \cdots + f(x_n, y_n)\,\delta A_n$$

If we increase these sub regions indefinitely such that the largest linear dimension $\delta A_r \longrightarrow 0$, the limit of sum if it exists is called "DOUBLE INTERAL" of $f(x, y)$ over region R & denoted by $\iint_R f(x,y)\,dx\,dy$

2. Triple Integral :

consider, function $f(x, y, z)$ defined at every point of 3-D finite region "V". (Dif) Divide V into n elementary volumes, $\delta V_1, \delta V_2, \cdots \delta V_n$.

Let (x_r, y_r, z_r) be any point within r^{th} sub division δV_r

consider sum, $\sum_{r=1}^{\infty} f(x_r, y_r, z_r)\,\delta V_r$

The limit of this sum, If it exists as $n \longrightarrow \infty$ & $V_r \longrightarrow 0$ is called "TRIPLE INTEGRAL" of $f(x, y, z)$ over Region V & is denoted by $\iiint_V f(x, y, z)\,dx\,dy\,dz$