HOME ASSIGNMENT-2

1. Find univariate regression equation in the format $y = w * x + b$ based on the below data.

S.no	Feature x	target y	x-mean(x)	y-mean(y)	x-mean(x) y-mean(y)	$(x-\text{mean}(x))^2$
1	4	4	3.75	2.375	8.91	14.06
2	3	6.	2.75	4.375	12.03	7.56
3	1	8	0.75	6.375	4.78	0.56
4	1	2	-0.75	0.375	0.28	0.56
5	-4	3	-4.25	1.375	-5.84	18.06
6	-2	-2	-2.25	-3.625	8.16	5.06
7	2	-4	1.75	-5.625	-9.84	3.06
8	-3	-4	-3.25	-5.625	18.28	10.56
Mean	0.25	1.625			Sum = 36.76	59.48

Univariate Linear Regression equation.

$y = b + wx$

$$\hat{w} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{36.76}{59.48} = 0.618$$

$$b = \bar{y} - \hat{w}\bar{x} = 1.625 - 0.618 \times 0.25 = 1.4705$$

Univariate Linear Regression equation

$$y = 1.47 + 0.62 \times x$$

4)

SNO	Attributes x_1	Attributes x_2	y
1	4	4	18
2	6	3	12
3	8	1	25
4	2	1	24
5	3	-4	8
6	-2	-2	20
7	-4	2	17
8	-4	-3	15
Mean	1.625	0.25	17.375
Sum	13	2	139.

Find Multivariate Regression equation in the format $Y = w_0 + w_1 x_1 + w_2 x_2$ based on above data.

X_1^2	X_2^2	X_1Y	X_2Y	X_1X_2
165	60	248	74	15148
16	16	72	72	5184
36	9	72	36	2592
64	1	200	25	5000
4	1	48	24	1152
9	16	24	-32	-768
4	4	-40	-40	1600
16	4	-68	34	-2312
16	9	60	-45	2700

$\Sigma x_1^2 = \Sigma x_1^2 - (\Sigma x_1)^2/n$

$= 165 - (2.6406)/2$

$\Sigma x_1^2 = 164.6699$

$\Sigma x_2^2 = \Sigma x_2^2 = (\Sigma x_2)^2/n$

$= 60 - (0.25)^2/8$

$= 59.9921$

$\Sigma x_1 y = \Sigma x_1 y - (\Sigma x_1 \Sigma y_1)/n$

$= 248 - (1.625 \times 17.375)/8$

$= 248 - 3.5292$

$= 244.4708$

$\Sigma x_2 y = \Sigma x_2 y - (\Sigma x_2 \Sigma y)/n$

$= 74 - 0.5429$

$= 73.4571$

$\Sigma x_1 y_2 = \Sigma x_1 x_2 - (\Sigma x_1 5 x_2)/n$

$= 15148 - (0.0507)$

$= 15147.9493$

Now Calculate b_1, b_2, b_0.

$$b_1 = \frac{[(\Sigma x_2^2)(\Sigma x_1 y) - (\Sigma x_1 y_2)(\Sigma x_2 y)]}{[(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2]}$$

$$= \frac{(59.9021)(244.4708) - (15147.9493)(73.434)}{(164.6699)(59.994) - (15147.9493)^2}$$

$$= \frac{-1098058.10984}{-229,450489.102}$$

$$\boxed{b_1 = 0.00478}$$

$$b_2 = \frac{(\Sigma x_1^2)(\Sigma x_2 Y) - (\Sigma x_1 x_2)(\Sigma x_1 Y)}{(\Sigma x_1^2)(\Sigma x_2^2) - (\Sigma x_1 x_2)^2}$$

$$= \frac{(164.6699)(73.4571) - (15147.9493)(244.4759)}{-229450489.102}$$

$$= \frac{-3691135.11041}{-229450484.102}$$

$$\boxed{b_2 = 0.01608}$$

$$b_0 = \bar{Y} - b_1 \bar{X}_1 - b_2 \bar{X}_2$$

$$= 17.375 - (0.00478)(1.625) - (0.01608)(0.25)$$

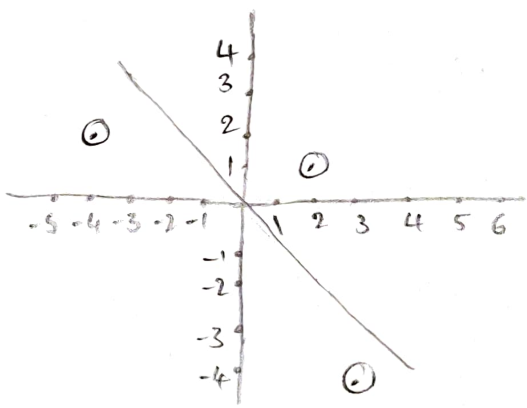
$$\boxed{b_0 = 17.36322}$$

$$Y = W_0 + W_1 X_1 + W_2 X_2$$

$$\boxed{Y = 17.36322 + 0.00478 X_1 + 0.01608 X_2}$$

7) Identify the separating boundary of positive and Negative samples using svm algorithm.

S.No	Attribute1	Attribute2	Class
1	4	4	+
2	6	3	+
3	8	1	+
4	2	1	+
5	3	-4	-
6	-2	-2	-
7	-4	2	-
8	-4	-3	-



Support Vector is +ve, -Ve instance which are near to the each other

$S_1 = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ $S_2 = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ $S_3 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Assume each Support Vector has 3 dimensions.

i.e $S_1 = (-4\ 2\ 1)$ $S_2 = (3\ -4\ 1)$ $S_3 = (2\ 1\ 1)$

$S_1 \cdot S_1 = (-4\ 2\ 1) \times (-4\ 2\ 1) = +16+4+1 = 21$

$S_1 \cdot S_2 = (-4\ 2\ 1) \times (3\ -4\ 1) = -12-8+1 = -19$

$S_1 \cdot S_3 = (-4\ 2\ 1) \times (2\ 1\ 1) = -8+2+1 = -5$

$S_2 \cdot S_2 = (3\ -4\ 1) \times (3\ -4\ 1) = 9+16+1 = 26$

$S_2 \cdot S_3 = (3\ -4\ 1) \times (2\ 1\ 1) = 6-4+1 = 3$

$S_3 \cdot S_3 = (2\ 1\ 1) \times (2\ 1\ 1) = 4+1+1 = 6.$

$\alpha_1 \bar{S}_1 \cdot \bar{S}_2 + \alpha_2 \bar{S}_2 \cdot \bar{S}_1 + \alpha_3 \bar{S}_3 \cdot \bar{S}_1 = -1$

$\alpha_1 \bar{S}_1 \cdot \bar{S}_2 + \alpha_2 \bar{S}_2 \cdot \bar{S}_2 + \alpha_3 \bar{S}_2 \bar{S}_3 = -1$

$\alpha_1 \bar{S}_1 \cdot \bar{S}_3 + \alpha_2 \bar{S}_2 \cdot \bar{S}_3 + \alpha_3 \bar{S}_3 \cdot \bar{S}_3 = +1$

$21\alpha_1 - 19\alpha_2 - 5\alpha_3 = -1 \longrightarrow (1)$

$-19\alpha_1 + 26\alpha_2 + 3\alpha_3 = -1 \longrightarrow (2)$

$-5\alpha_1 + 3\alpha_2 + 6\alpha_3 = 1 \longrightarrow (3)$

$(1) \times 3 \Rightarrow 63\alpha_1 - 57\alpha_2 - \cancel{15\alpha_3} = -3$

$2 \times 5 \Rightarrow -95\alpha_1 + 130\alpha_2 \cancel{+ 15\alpha_3} = -5$

$-32\alpha_1 + 73\alpha_2 = -8 \longrightarrow (4)$

$2\times 2 \Rightarrow -38\alpha_1 + 52\alpha_2 + 6\alpha_3 = -2$

$3\times 1 \Rightarrow -5\alpha_1 + 3\alpha_2 + 6\alpha_3 = 1$

$\quad + \qquad + \qquad - \qquad -$

$-33\alpha_1 + 49\alpha_1 = -3 \rightarrow (5)$

$\alpha_1 = \frac{-173}{841} \qquad \alpha_2 = \frac{-168}{841} \qquad \alpha_3 = \frac{-80}{841}$

Calculating the weights based on α_1, α_2 and α_3 ..

$$\bar{\omega} = \sum_i \alpha_i S_i$$

$$= \frac{-173}{841}\begin{pmatrix}-4\\2\\1\end{pmatrix} - \frac{168}{841}\begin{pmatrix}3\\-4\\1\end{pmatrix} + \frac{80}{841}\begin{pmatrix}2\\1\\1\end{pmatrix}$$

$$= \frac{1}{841}\begin{bmatrix}+692\\-346\\-173\end{bmatrix} + \frac{1}{841}\begin{bmatrix}-504\\+672\\-168\end{bmatrix} + \frac{1}{841}\begin{bmatrix}160\\80\\80\end{bmatrix}$$

$$= \frac{1}{841}\begin{bmatrix}692-504+160\\-346+672+80\\-173+168+80\end{bmatrix}$$

$$= \frac{1}{841}\begin{bmatrix}348\\406\\-261\end{bmatrix}$$

$$= \begin{bmatrix}0.41\\0.48\\0.310\end{bmatrix}$$

$$y = \omega x + b$$

$$\omega = \begin{pmatrix}0.41\\0.48\end{pmatrix} \qquad b = 0.310.$$