K. Prakash Baby

UNIT – IV

(b) Output per hour = Unit capacity × % utilization × efficiency, where unit capacity = 3600 sec/100 sec per mould = 36 moulds per hour.

Output per hour = 36 × 0.6 × 0.9 = 19.44 moulds = 20 moulds (Approx.)

(c) Number of moulding machines needed = System capacity/output per hour
= 100/20 = 5 machines.

KPB

3.2 INVESTMENT DECISIONS

3.2.1 Interest Formulas

In the area of investment decision making, computations will be done in many ways. To simplify all these computations, it is highly necessary to know how to use interest formulas more effectively. Before discussing the effective application of the interest formulas for investment decision making, the various interest formulas are presented first.

Interest rate is the rental value of money. This represents the growth of capital per unit period. The period may be a month, quarter, semiannual or year. Interest rate can be classified into simple interest rate and compound interest rate.

In simple interest, the interest is computed based on the initial deposit for every interest period. In this case, calculation of interest on interest earned per unit period is not applied.

In compound interest, the interest for the current period is computed based on the amount (Principle Plus interest up to the previous period) at the beginning of the current period.

The notations which are used in various interest formulas are summarized below.

P – Principle amount

n – Number of interest periods

i – Interest rate (It may be compounded monthly, quarterly, semiannually or annually)

F – Future amount at the end of the year, n.

A – Equal amount deposited at the end of every interest period.

G – Uniform amount which will be added/subtracted period after period to/from the amount of deposit A_1 at the end of period 1.

Single payment compound amount. Here, the objective is to find the single future sum (F) of the initial payment (P) made at time 0 after n periods at an interest rate, i compounded every period. The cash flow diagram of this situation is shown in Fig. 3.1.

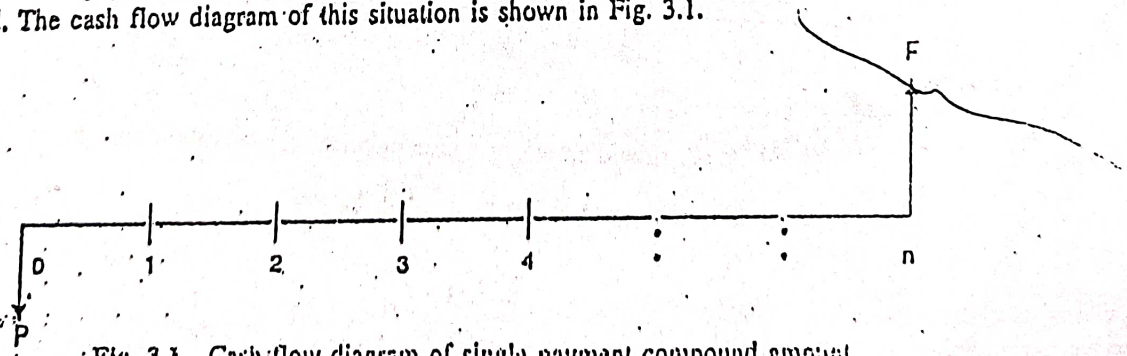


Fig. 3.1 Cash flow diagram of single payment compound amount.

The formula to obtain the single payment compound amount is given below.

$$F = P\,(1 + i)^n$$
$$= P\,(F/P, i, n)$$

where, $(F/P, i, n)$ is called as single payment compound amount factor.

Example 3.3. A person deposits a sum of Rs. 15,000 at the interest rate of 18% compounded annually for 5 years. Find the maturity value after 5 years.

Solution.

P = Rs. 15,000
i = 18% compounded annually
n = 5 years
$F = P\,(1 + i)^n$
$= P\,(F/P, i, n)$
$= 15{,}000\,(F/P, 18\%, 5)$
$= 15{,}000 \times 2.2878 = \text{Rs. } 34{,}317$

The maturity value of Rs. 15,000 invested now at 18% compounded yearly is equal to Rs. 34,317 after 5 years.

Single payment present worth amount. Here, the objective is to find the present worth amount (P) of a single future sum which will be received after n periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.2.

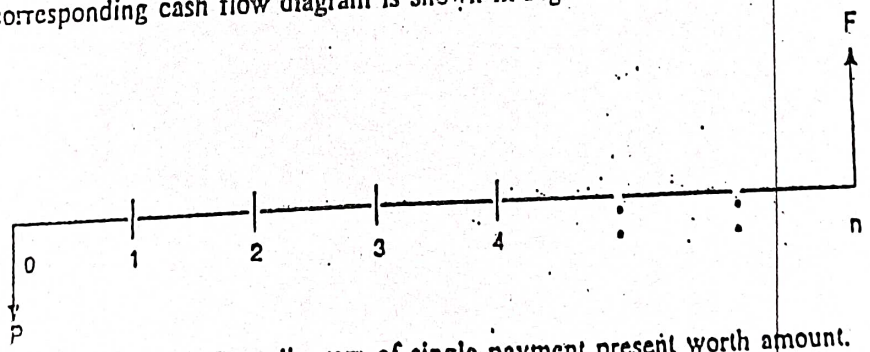


Fig. 3.2 Cash flow diagram of single payment present worth amount.

The formula to obtain the present worth is given below.

$$P = \frac{F}{(1 + i)^n}$$
$$= F\,(P/F, i, n)$$

where, $(P/F, i, n)$ is called as single payment present worth factor.

Example 3.4. A person wishes to have a future sum of Rs. 50,000 for his son's education 10 years from now. What is the single payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Solution.

F = Rs. 50,000

i = 15% compounded annually

n = 10 years

$P = F/(1+i)^n$

$= F(P/F, i, n)$

$= 50{,}000\ (P/F, 15\%, 10)$

$= 50{,}000 \times 0.2472$

$=$ Rs. 12,360

The person has to invest Rs. 12,360 now, so that he will get a sum of Rs. 50,000 after 10 years at 15% interest rate compounded annually.

Equal payment series compound amount. In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of nth interest period at an interest rate of i compounded at the end of each interest period.

The corresponding cash flow diagram is shown in Fig. 3.3.

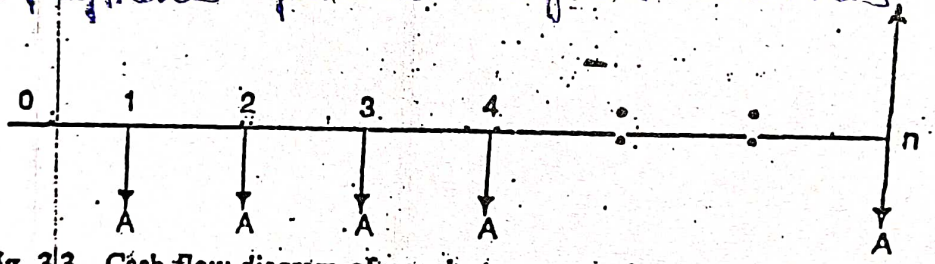


Fig. 3.3 Cash flow diagram of equal payment series compound amount.

Here,

A – Equal amount deposited at the end of each interest period.

n – Number of interest periods.

i – Rate of interest.

F – Single future amount.

The formula to get F is shown below.

$$F = A\frac{(1+i)^n - 1}{i} = A(F/A, i, n)$$

where $(F/A, i, n)$ is called as equal payment series compound amount factor.

Example 3.5. A person who is just 40 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for next 20 years starting from the end of next year. The bank gives 18% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

Solution.

A = Rs. 10,000
n = 20 years
i = 18%
F = ?

The corresponding cash flow diagram is shown in Fig. 3.4.

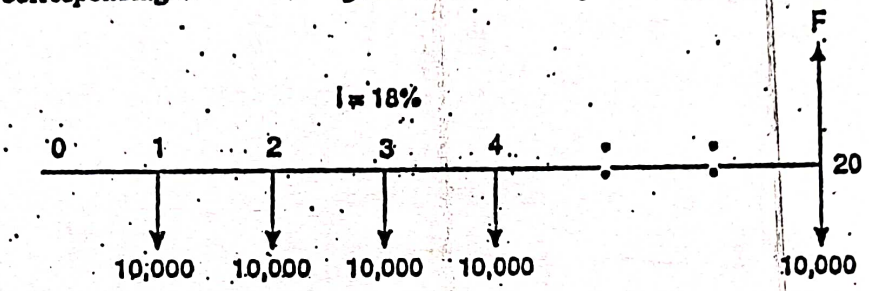


Fig. 3.4 Cash flow diagram of equal payment series compound amount.

$$F = A\frac{(1+i)^n - 1}{i}$$
$$= A.(F/A, i, n)$$
$$= 10{,}000(F/A, 18\%, 20)$$
$$= 10{,}000 \times 146.6280$$
$$= \text{Rs. } 1{,}466{,}280$$

The future sum after 20 years is equal to Rs. 1,466,280.

Equal payment series sinking fund. In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the nth interest period at an interest rate of i.

The corresponding cash flow diagram is shown in Fig. 3.5.

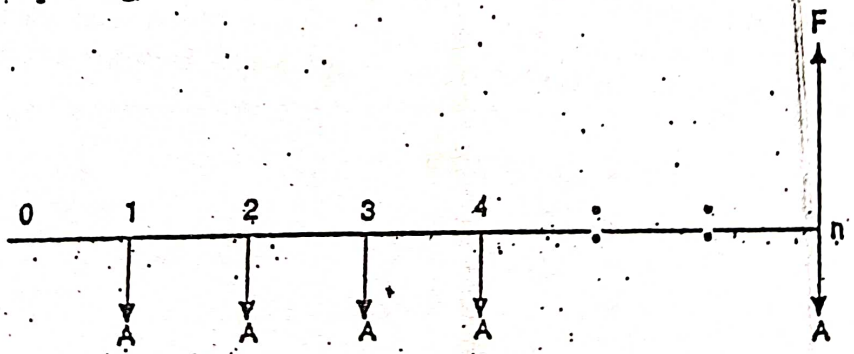


Fig. 3.5 Cash flow diagram of equal payment series sinking fund.

Here,

A – Equal amount deposited at the end of each interest period.

n – Number of interest periods.

i – Rate of interest.

F – Single future amount.

The formula to get A is shown below.

$$A = F\frac{i}{(1+i)^n - 1}$$

$$= F\,(A/F, i, n)$$

where $(A/F, i, n)$ is called as equal payment series sinking fund factor.

Example 3.6. A company has to replace a present facility after 10 years at an outlay of Rs. 500,000. It plans to deposit an equal fixed amount at the end of every year for the next 10 years at an interest rate of 18% compounded annually. Find the amount that must be deposited at the end of every year for the next 10 years.

Solution.

F = Rs. 500,000

n = 10 years

i = 18%

A = ?

The corresponding cash flow diagram is shown in Fig. 3.6.

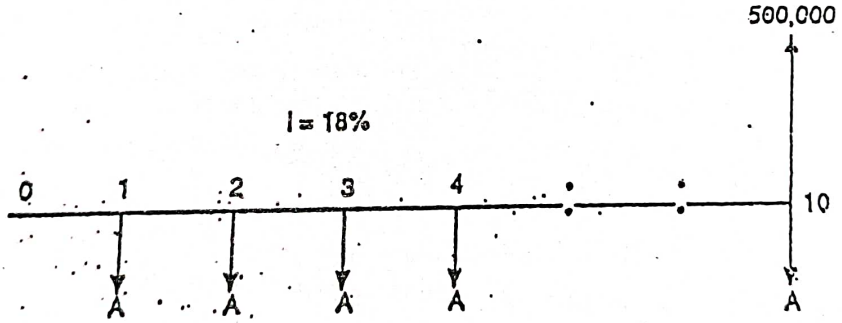


Fig. 3.6 Cash flow diagram of equal payment series sinking fund.

$$A = F\frac{i}{(1+i)^n - 1}$$

$$= F\,(A/F, i, n)$$

$$= 500{,}000\,(A/F, 18\%, 10)$$

$$= 500{,}000 \times 0.0425$$

$$= \text{Rs. } 21{,}250.$$

The annual equal amount which must be deposited is Rs. 21,250.

Equal payment series present worth amount. The objective of this mode of investment is to find the present worth of an equal payments made at the end of every interest period for n interest periods at an interest rate of i compounded periodically at the end of every interest period.

The cash flow diagram is as shown in Fig. 3.7.

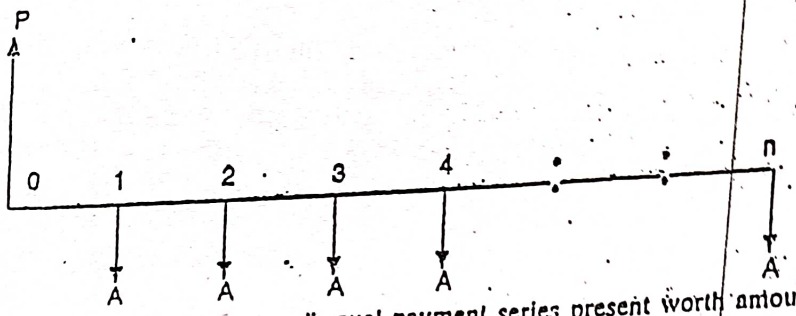


Fig. 3.7 Cash flow diagram of equal payment series present worth amount.

Here,

P – Present worth

A – Annual equivalent payment

i – Interest rate

n – number of periods of deposit

The formula to compute P is as given below.

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$= A\ (P/A,\ i,\ n)$$

where $(P/A, i, n)$ is called as equal payment series present worth factor.

Example 3.7. A company wants to set up a reserve which will help the company to have an amount of Rs. 1,000,000 for the next 25 years towards welfare measures of its employees. The reserve is assumed to grow at the rate of 18% annually. Find the single payment that must be made as the reserve amount.

Solution.

A = Rs. 1,000,000

i = 18%

n = 25 years

P = ?

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$= A\ (P/A,\ i,\ n) = 1{,}000{,}000 \times (P/A,\ 18\%,\ 25)$$

$$= 1{,}000{,}000 \times (5.467)$$

$$= \text{Rs. } 5{,}467{,}000$$

The amount of reserve which must be set up is equal to Rs. 5,467,000.

Equal payment series capital recovery amount. The objective of this mode of investment is to find the equal annual amount (A) which is to be recovered at the end of every interest period for n interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded periodically at the end of every interest period.

The cash flow diagram is as shown in Fig. 3.8.

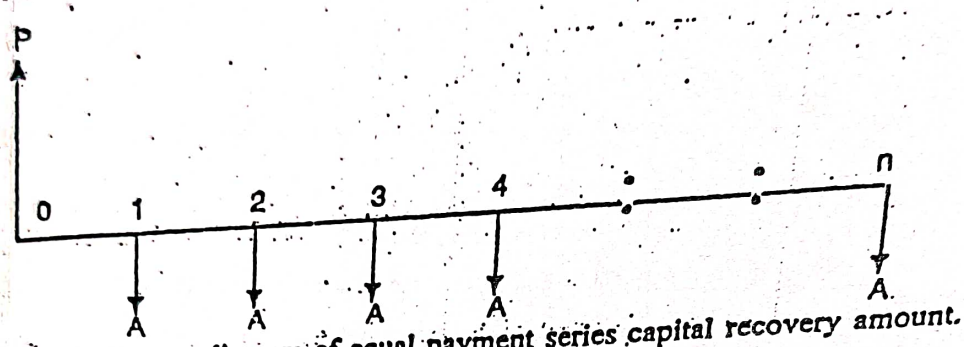


Fig. 3.8 Cash flow diagram of equal payment series capital recovery amount.

Here,

P – Present worth (Loan amount)

A – Annual equivalent payment (Recovery amount)

i – Interest rate

n – Number of periods of deposit

The formula to compute A is as given below.

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P\,(A/P, i, n)$$

where $(A/P, i, n)$ is called as equal payment series capital recovery factor.

Example 3.8. A bank gives a loan to a company to purchase an equipment which is worth of Rs. 1,000,000 at an interest rate, 15% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.

Solution.

P = Rs. 1,000,000

i = 15%

n = 15 years

A = ?

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= P\,(A/P, i, n) = 1{,}000{,}000 \times (A/P, 15\%, 15)$$

$$= 1{,}000{,}000 \times (0.1710)$$

$$= \text{Rs. } 171{,}000$$

The annual equivalent installment to be paid by the company to the bank is Rs. 171,000.

Uniform gradient series annual equivalent amount. The objective is to find the annual equivalent amount of a series with an amount $A1$ at the end of the first year and with an equal increment (G) at the end of each of the following $n-1$ years with an interest rate, i compounded annually.

The corresponding cash flow diagram is shown in Fig. 3.9.

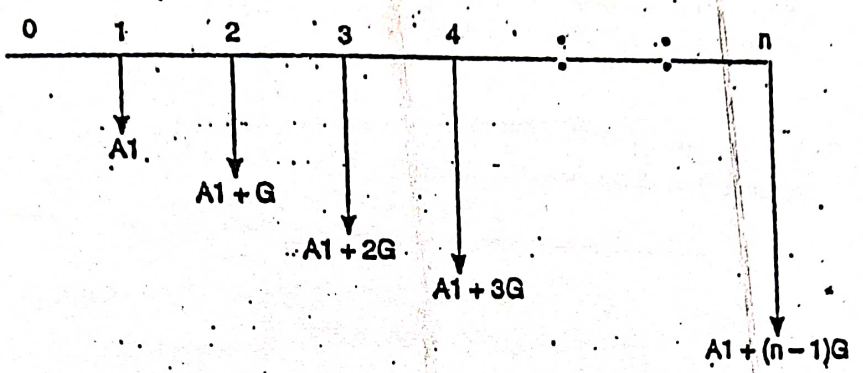


Fig. 3.9 Cash flow diagram of uniform gradient series annual equivalent amount.

Here, the formula for this situation is given below.

$$A = A1 + G\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A1 + G\,(A/G, i, n)$$

where $(A/G, i, n)$ is called as Uniform gradient series factor.

Example 3.9. A person is planning for his retired life. He has 10 more years of service. He would like to deposit 10% of his salary which is Rs. 2000 at the end of the first year and thereafter he wishes to deposit the same amount (Rs. 2000) with an annual increase of Rs. 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th year of the above series.

Solution. The cash flow diagram is shown in Fig. 3.10.

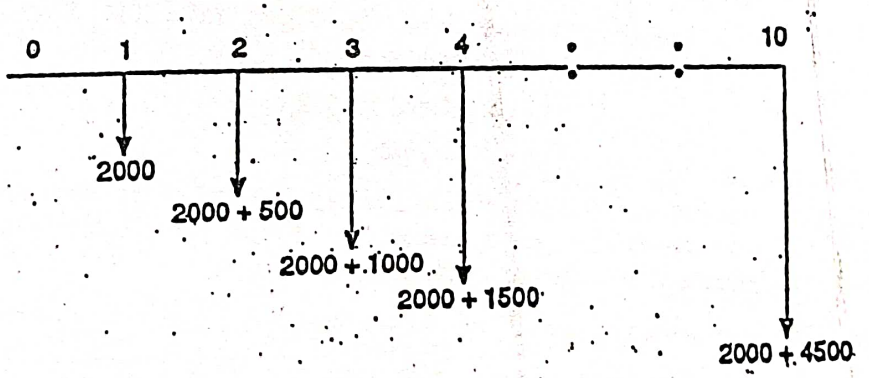


Fig. 3.10 Cash flow diagram of uniform gradient series annual equivalent amount.

Here,

$A1$ = Rs. 2000
G = Rs. 500
i = 15%
n = 10 years

$$A = A1 + G\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A1 + G\,(A/G, i, n)$$
$$= 2000 + 500\,(A/G, 15\%, 10)$$
$$= 2000 + 500 \times 3.3832$$
$$= \text{Rs. } 3691.60$$

This is equivalent to paying an equal amount of Rs. 3691.60 to be invested at the end of every year for the next 10 years. The future worth of this revised series at the end of the 10th year is obtained as follows.

$$F = A\,(F/A, i, n)$$
$$= A\,(F/A, 15\%, 10)$$
$$= 3691.60\,(20.304)$$
$$= \text{Rs. } 74,954.25$$

The compound amount at the end of the 10th year is Rs. 74,954.25.

3.2.2 Bases for Comparison of Alternatives

In most of the practical decision environments, executives will be forced to select the best alternative from a set of competing alternatives. Let us assume that an organization has huge sum of money for potential investment. There are three different projects whose initial outlay and annual revenues during their life are known. The executive has to select the best alternative among these three competing projects.

There are several bases for comparing the worthiness of the projects which are mentioned below.

(a) Present Worth Method
(b) Annual Equivalent Method
(c) Future Worth Method
(d) Rate of Return Method

Present worth method. In this method of comparison, the cash flows of each alternative will be reduced to time zero by assuming an interest rate. Then, depending on the type of decision, the best alternative will be selected by comparing the present worth amounts of the alternatives. In case the decision is to select the alternative which will result in minimum cost, then the alternative with the least present worth amount will be selected. On the other hand, if the decision is to select the alternative with the maximum profit, then the alternative with the maximum present worth will be selected. Here, the sign of various amounts at different points in time in a cash flow diagram is to be decided based on the type of the decision problem.

Example 3.10. Alpha Industry is planning for expanding its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues with respect to each of the technologies are summarized in the following table. Suggest the best technology which is to be implemented based on present worth method of comparison assuming 20% interest rate compounded annually.

	Initial Outlay (Rs.)	Annual Revenue (Rs.)	Life (Yrs.)
Technology 1	1,200,000	400,000	15
Technology 2	2,000,000	600,000	15
Technology 3	1,800,000	500,000	15

Technology 1: The present worth expression for this technology is shown below.

$$PW(20\%)_1 = -1{,}200{,}000 + 400{,}000 \times (P/A, 20\%, 15)$$
$$= -1{,}200{,}000 + 400{,}000 \times (4.6755)$$
$$= -1{,}200{,}000 + 1{,}870{,}200$$
$$= 670{,}200$$

Technology 2: The present worth expression is as follows.

$$PW(20\%)_2 = -2{,}000{,}000 + 600{,}000 \times (P/A, 20\%, 15)$$
$$= -2{,}000{,}000 + 600{,}000 \times (4.6755)$$
$$= -2{,}000{,}000 + 2{,}805{,}300$$
$$= 805{,}300$$

Technology 3: The following is the present worth expression for this technology.

$$PW(20\%)_3 = -1{,}800{,}000 + 500{,}000 \times (P/A, 20\%, 15)$$
$$= -1{,}800{,}000 + 500{,}000 \times (4.6755)$$
$$= -1{,}800{,}000 + 2{,}337{,}750$$
$$= 537{,}750$$

From the above calculations, it is clear that the present worth of the Technology 2 is the highest among all the technologies. Hence, it is recommended for implementation to expand the production.

Example 3.11. An engineer has two bids for an elevator to be installed in a new building. The bids and his evaluation of the elevators are as follows:

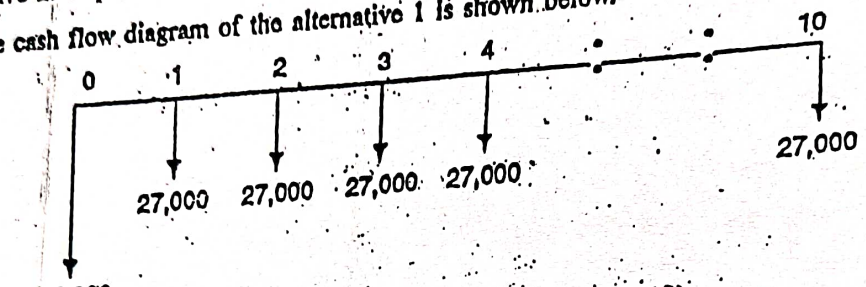
Alternative	Bids Initial Cost (Rs.)	Engineer's Estimates: Service Life (Yrs.)	Engineer's Estimates: Annual Operation and Maintenance Cost (Rs.)
Alpha Elevator Inc.	450,000	10	27,000
Beta Elevator Inc.	540,000	10	28,500

The engineer will make a present worth analysis using a 15% interest rate. Prepare the analysis and determine which bid should be accepted.

Solution.

Alternative 1: Alpha Elevator Inc.

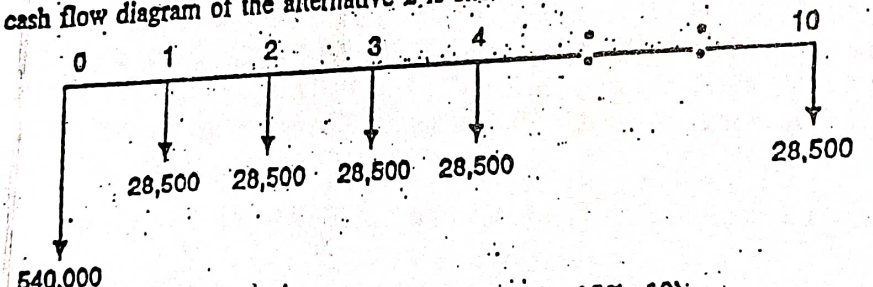
The cash flow diagram of the alternative 1 is shown below.



$$PW(15\%) = 450,000 + 27,000\,(P/A, 15\%, 10)$$
$$= 450,000 + 27,000 \times 5.019$$
$$= 450,000 + 135,513$$
$$= \text{Rs. } 585,513$$

Alternative 2: Beta Elevator Inc.

The cash flow diagram of the alternative 2 is shown below.



$$PW(15\%) = 540,000 + 28,500\,(P/A, 15\%, 10)$$
$$= 540,000 + 28,500 \times 5.019$$
$$= 540,000 + 143,042$$
$$= \text{Rs. } 683,042$$

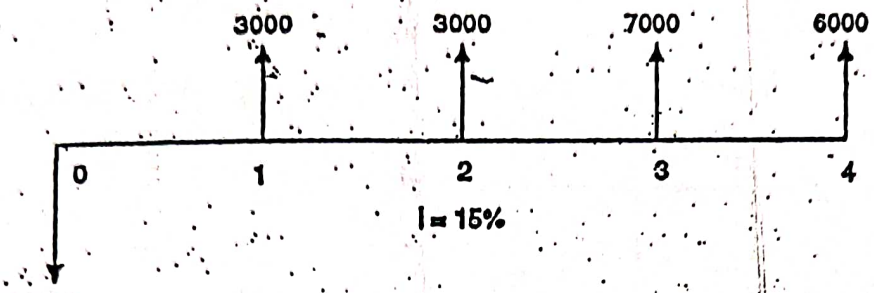
The total present worth of the alternative 1 is less than that of the alternative 2. Hence, the alternative 1 is to be selected for implementation. That means, the elevator from Alpha Elevator Limited is to be purchased and installed in the new building.

Example 3.12. Investment proposals *A* and *B* have the net cash flows as shown below.

Proposal	End of Years				
	0	1	2	3	4
A (Rs.)	– 10,000	3000	3000	7000	6000
B (Rs.)	– 10,000	6000	6000	3000	3000

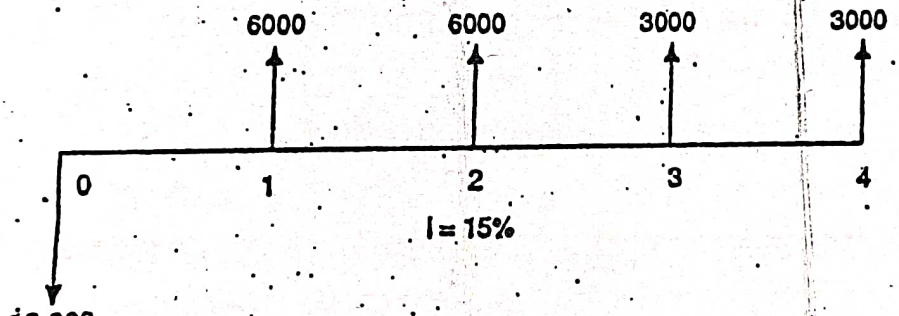
Compare the present worth of A with the present worth of B for $i = 15\%$. Which has the higher value?

Solution. The present worth of A at $i = 15\%$. The cash flow diagram of the proposal A is shown below.



$$PW_A(15\%) = -10{,}000 + 3000\ (P/F, 15\%, 1) + 3000\ (P/F, 15\%, 2) + 7000\ (P/F, 15\%, 3) + 6000\ (P/F, 15\%, 4)$$
$$= -10{,}000 + 3000\ (.8696) + 3000\ (.7562) + 7000\ (.6572) + 6000\ (.5718)$$
$$= 2908.60$$

The present worth of B at $i = 15\%$. The cash flow diagram of the proposal B is shown below.



$$PW_B(15\%) = -10{,}000 + 6000\ (P/F, 15\%, 1) + 6000\ (P/F, 15\%, 2) + 3000\ (P/F, 15\%, 3) + 3000\ (P/F, 15\%, 4)$$
$$= -10{,}000 + 6000\ (.8696) + 6000\ (.7562) + 3000\ (.6572) + 3000\ (.5718)$$
$$= 3442.70$$

At $i = 15\%$, the present worth of the proposal B is higher than that of the proposal A. Hence select the proposal B.

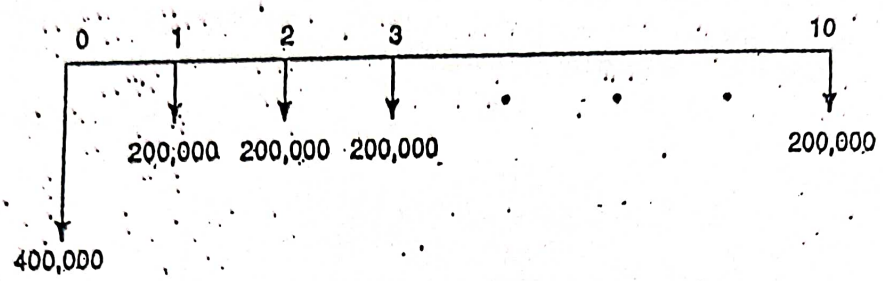
Example 3.13. A granite company is planning to buy fully automated granite cutting machine. If it is purchased under down payment, the cost of the machine is Rs. 1,600,000. If it is purchased on installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of Rs. 200,000 each. Suggest the best alternative for the company using present worth basis at $i = 20\%$.

Solution. There are two alternatives available for the company, which are mentioned below.

1. Down payment of Rs. 1,600,000
2. Down payment of Rs. 400,000 and 10 annual equal installments of Rs. 200,000 each.

The present worth calculation of the second alternative:

The cash flow diagram of the second alternative is shown below.



$$PW(20\%) = 400{,}000 + 200{,}000\,(P/A, 20\%, 10)$$
$$= 400{,}000 + 200{,}000 \times 4.1925$$
$$= \text{Rs. } 1{,}238{,}500$$

The present worth of this option is Rs. 1,238,500 which is less than the first option of complete down payment of Rs. 1,600,000. Hence, the company should select the second alternative.

Annual equivalent method: In this method of comparison, first the annual equivalent net cost or net revenue of each alternative will be evaluated. Then the alternative with the minimum annual equivalent net cost in the case of cost based comparison or with the maximum annual equivalent net revenue in the case of revenue based comparison will be selected as the best alternative.

Example 3.14. A company provides car to its chief executive. The owner of the company is concerned about the increasing cost of petrol. He feels that the cost of petrol will be increasing by Re. 1 every year. His experience with his company car indicates that it averages 9 kilometres per litre of petrol. The executive expects to drive an average of 20,000 kilometres each year for the next four years. What is the annual equivalent cost of fuel over this period of time? If he is offered similar service with the same quality on rental basis at Rs. 60,000 per year, whether the owner should continue to provide his own car for his executive or alternatively provide a rental car to his executive? Assume $i = 15\%$. If the rental car is preferred, then the company car will find some other alternate use within the company.

Solution. The average number of kilometre run/year = 20,000 km

Number of kilometres/litre of petrol = 9 km

Therefore, petrol consumption/year = 20,000/9 = 2222.2 litre

The cost/litre of petrol for the first year = Rs. 20.

The cost/litre of petrol for the second year = Rs. 20.00 + Rs. 1.00 = Rs. 21.00

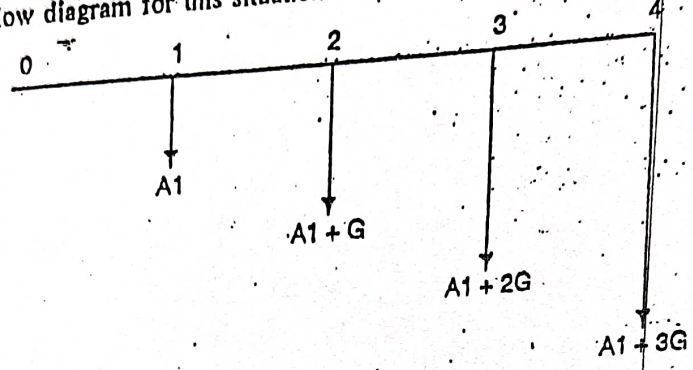
The cost/litre of petrol for the third year = Rs. 21.00 + Rs. 1.00 = Rs. 22.00

The cost/litre of petrol for the fourth year = Rs. 22.00 + Rs. 1.00 = Rs. 23.00

The fuel expenditure for 1st year = 2222.2 × 20 = Rs. 44,444.00

The fuel expenditure for 2nd year = 2222.2 × 21 = Rs. 46,666.20

The fuel expenditure for 3rd year = 2222.2 × 22 = Rs. 48,888.40.
The fuel expenditure for 4th year = 2222.2 × 23 = Rs. 51,110.60
The annual equal increment of the above expenditures is Rs. 2222.20 (G).
The cash flow diagram for this situation is shown below.



$A1$ = Rs. 44,444
G = Rs. 2222.20
$A = A1 + G\ (A/G, 15\%, 4)$
= 44,444 + 2222.2 (1.3263)
= Rs. 47,391.30

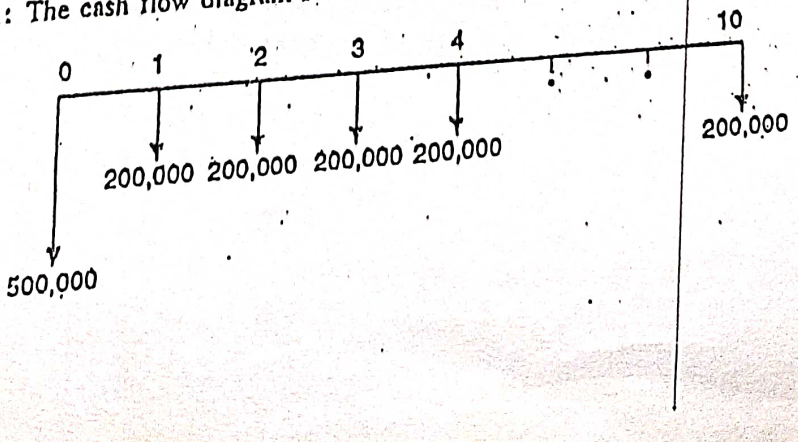
The proposal of using the company car by spending for petrol by the company will cost an annual equivalent amount of Rs. 47,391.30 for four years. This amount is lesser than the annual rental value of Rs. 60,000. Hence, the company should continue to provide its own car to the executive.

Example 3.15. A company is planning to purchase advanced machine centre. Three original manufacturers have responded to its tender whose particulars are shown below.

Alternative	Down Payment	Yearly Equal Installment (Rs.)	Number of Installments
1	500,000	200,000	10
2	400,000	300,000	10
3	600,000	150,000	10

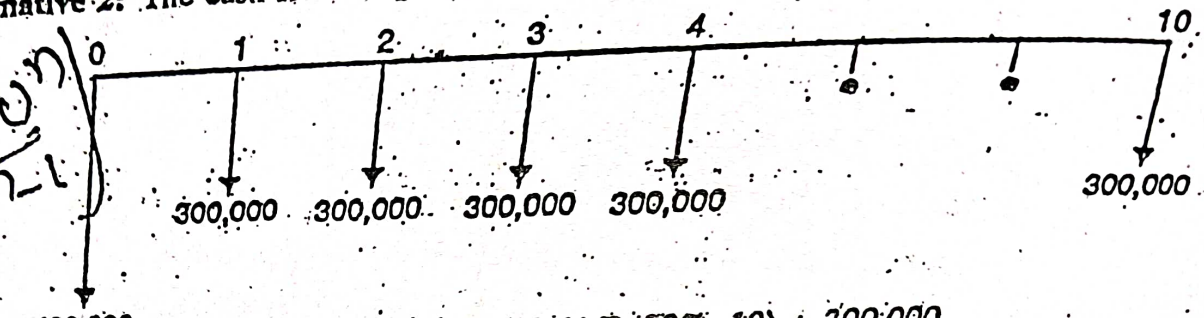
Determine the best alternative based on the annual equivalent method by assuming i = 20%.

Alternative 1: The cash flow diagram for the manufacturer 1 is shown below.



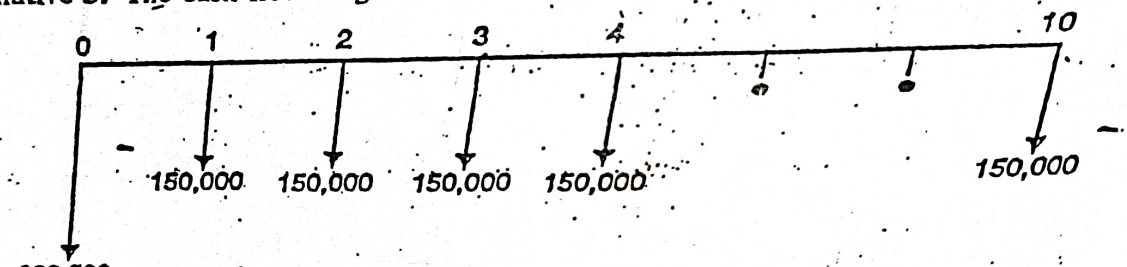
$$AE_1(20\%) = 500{,}000\,(A/P, 20\%, 10) + 200{,}000$$
$$= 500{,}000\,(.2385) + 200{,}000$$
$$= 319{,}250$$

Alternative 2: The cash flow diagram for the manufacturer 2 is shown below.



$$AE_2(20\%) = 400{,}000\,(A/P, 20\%, 10) + 300{,}000$$
$$= 400{,}000\,(.2385) + 300{,}000$$
$$= Rs.\ 395{,}400.$$

Alternative 3: The cash flow diagram for the manufacturer 3 is shown below.



$$AE_3(20\%) = 600{,}000\,(A/P, 20\%, 10) + 150{,}000$$
$$= 600{,}000\,(.2385) + 150{,}000$$
$$= Rs.\ 293{,}100.$$

The annual equivalent cost of the alternative 3 (machine centre from manufacturer 3) is lesser than that of the alternative 1 and the alternative 2. Hence, the company should buy the advanced machine centre from the manufacturer 3.

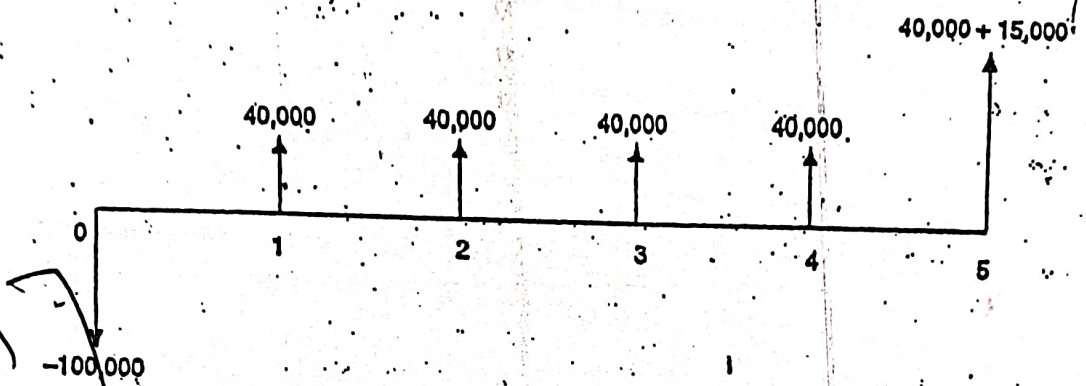
Example 3.16. A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following investments, annual returns and salvage values.

	Alternative	
	A	B
Investment (Rs.)	− 100,000	− 120,000
Annual equal return (Rs.)	+ 40,000	+ 50,000
Salvage value (Rs.)	+ 15,000	+ 35,000

Determine the best alternative based on the annual equivalent method by assuming $i = 25\%$.

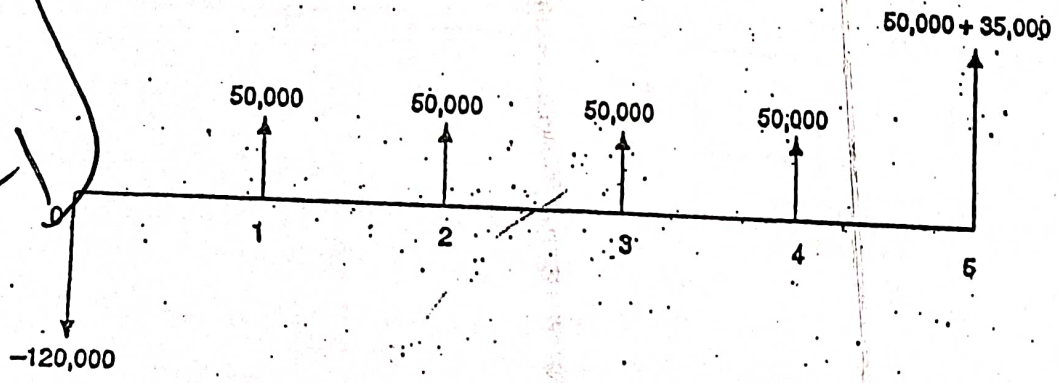
Solution.

Alternative A: The cash flow diagram for the alternative *A* is shown below.



$$AE_A(25\%) = -100{,}000\,(A/P, 25\%, 5) + 15{,}000\,(A/F, 25\%, 5) + 40{,}000$$
$$= -100{,}000\,(.3719) + 15{,}000\,(.1219) + 40{,}000$$
$$= \text{Rs. } 4638.50$$

Alternative B: The cash flow diagram for the alternative *B* is shown below.



$$AE_B(25\%) = -120{,}000\,(A/P, 25\%, 5) + 35{,}000\,(A/F, 25\%, 5) + 50{,}000$$
$$= -120{,}000\,(.3719) + 35{,}000\,(.1219) + 50{,}000$$
$$= \text{Rs. } 9638.50$$

The annual equivalent net return of the alternative *B* is more than that of the alternative *A*. Hence, the company should select the alternative *B*.

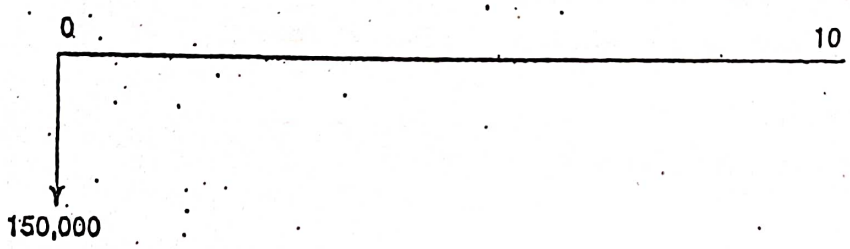
Example 3.17. A certain individual firm desires an economic analysis to determine which of the two machines is attractive in a given amount of time. The minimum attractive rate of return for the firm is 12%. The following data are to be used in the analysis.

	Machine X	Machine Y
First cost (Rs.)	150,000	240,000
Estimated life (Yrs.)	10.	10
Salvage value (Rs.)	0	60,000
Annual maintenance cost (Rs.)	0	4500

Which machine would you choose? Base your answer on annual cost.

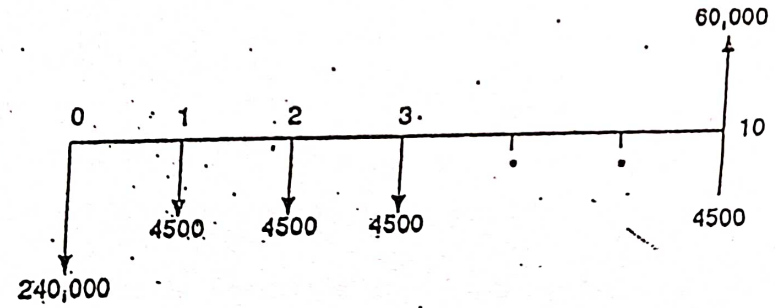
Solution.

Machine X: The cash flow diagram of the machine X is shown below.



$$AE_X\,(12\%) = 150{,}000\,(A/P,\ 12\%,\ 10)$$
$$= 150{,}000\ (0.1770)$$
$$= \text{Rs. } 26{,}550$$

Machine Y: The cash flow diagram of the machine Y is shown below.



$$AE_Y(12\%) = 240{,}000\,(A/P,\ 12\%,\ 10) + 4500 - 6000\,(A/F,\ 12\%,\ 10)$$
$$= 240{,}000\ (0.177) + 4500 - 6000\ (.0570)$$
$$= \text{Rs. } 46{,}638$$

The annual equivalent cost of the machine X is lesser than that of the machine Y. So, X is the attractive machine.

Future worth method. In this type of comparison of alternatives, future worth of various alternatives will be computed. Then, the alternative with the minimum future worth of net cost or with the maximum future worth of net revenue will be selected as the alternative for implementation.

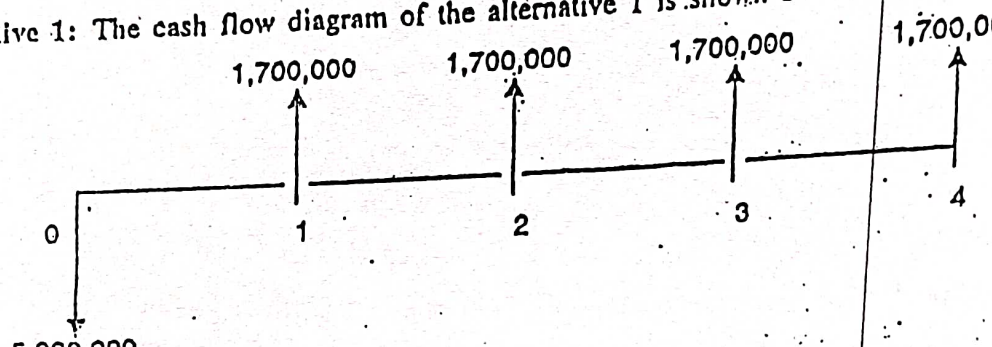
Example 3.18. Consider the following mutually exclusive alternatives.

	End of Year				
Alternative	0	1	2	3	4
1	–5,000,000	1,700,000	1,700,000	1,700,000	1,700,000
2	–5,300,000	1,800,000	1,800,000	1,800,000	1,800,000

At i = 12%, select the best alternative based on future worth method.

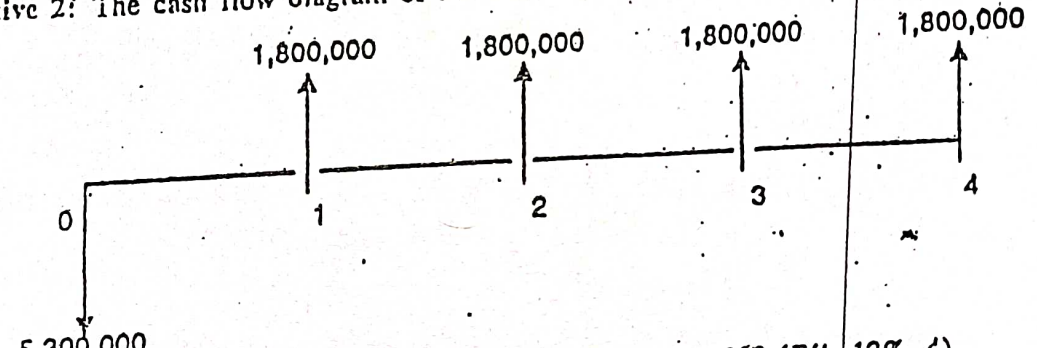
Solution.

Alternative 1: The cash flow diagram of the alternative 1 is shown below.



$$FW_A(12\%) = -5,000,000\ (F/P, 12\%, 4) + 1,700,000\ (F/A, 12\%, 4)$$
$$= -5,000,000\ (1.574) + 1,700,000\ (4.779)$$
$$= 254,300$$

Alternative 2: The cash flow diagram of the alternative 2 is shown below.



$$FW_B(12\%) = -5,300,000\ (F/P, 12\%, 4) + 1,800,000\ (F/A, 12\%, 4)$$
$$= -5,300,000\ (1.574) + 1,800,000\ (4.779)$$
$$= 260,000$$

The future worth of the net revenue of the alternative 2 is greater than that of the alternative 1. Hence, the alternative 2 should be selected.

Example 3.19. A man owns a corner plot. He must decide which of the several alternatives to select in trying to obtain a desirable return on his investment. After much study and calculation, he decides that the two best alternatives are as shown in the following table.

	Build Gas Station	Build Soft Ice Cream Stand
First cost (Rs.)	2,400,000	3,600,000
Annual property taxes (Rs.)	90,000	150,000
Annual income (Rs.)	830,000	980,000
Life of building (Yrs.)	20	20
Salvage value (Rs.)	0	0

Evaluate the alternatives based on future worth method at $i = 12\%$.

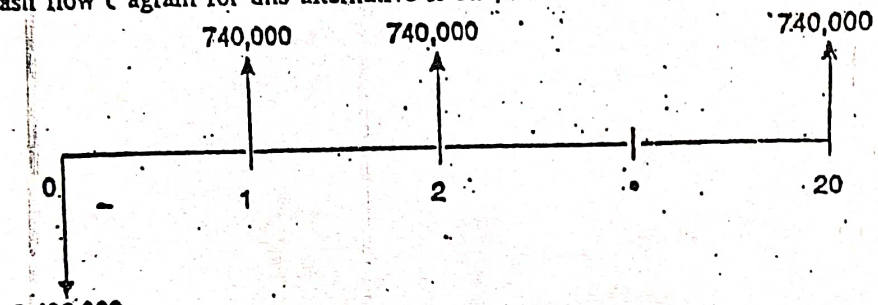
Alternative 1: Build Gas Station.

Net annual income = Annual income – Annual property tax

= Rs. 830,000 – Rs. 90,000

= Rs. 740,000

The cash flow diagram for this alternative is shown below.



$FW_1(12\%) = -2{,}400{,}000\ (F/P, 12\%, 20) + 7{,}400{,}000\ (F/A, 12\%, 20)$

$= -2{,}400{,}000\ (9.646) + 740{,}000\ (72.052)$

$= 30{,}168{,}080$

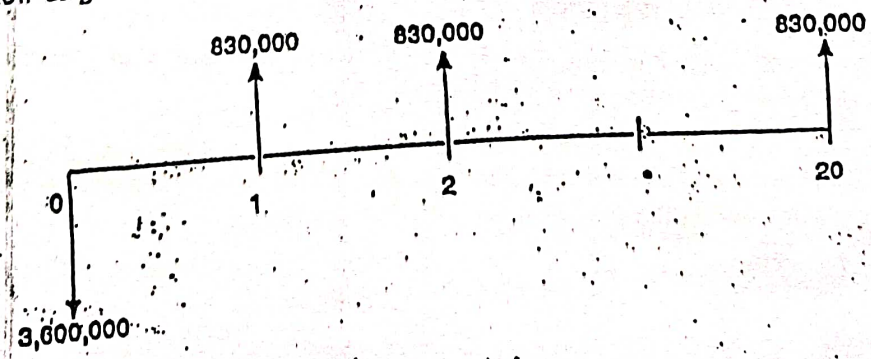
Alternative 2: Build Soft Ice-Cream Stand.

Net annual income = Annual income – Annual property tax

= Rs. 980,000 – Rs. 150,000

= Rs. 830,000

The cash flow diagram for this alternative is shown below.



$$FW_2(12\%) = -3,600,000\ (F/P, 12\%, 20) + 830,000\ (F/A, 12\%, 20)$$
$$= -3,600,000\ (9.646) + 830,000\ (72.052)$$
$$= \text{Rs. } 25,077,560$$

The future worth net revenue of the alternative 1 is greater than that of the alternative 2. Hence, building the gas station is the best alternative.

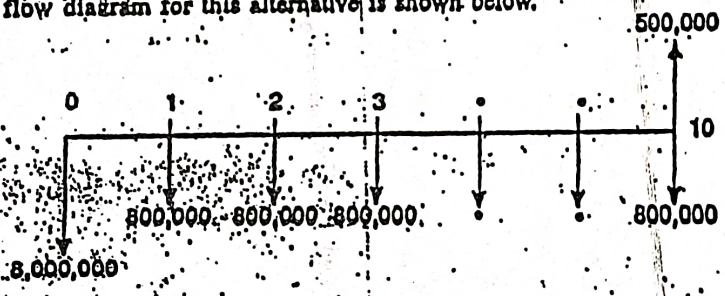
Example. 3.20. M/s. Krishna Castings Ltd is planning to replace its annealing furnace. It has received tenders from three different original manufacturers of annealing furnace. The details are as follows. Which is the best alternative based on future worth method at $I = 20\%$.

	Manufacturer		
	1	2	3
Initial cost (Rs.)	8,000,000	7,000,000	9,000,000
Life (years)	10	10	10
Annual operation and maintenance cost (Rs.)	800,000	900,000	850,000
Salvage value after 10 years	500,000	400,000	700,000

Solution.

Alternative 1: Manufacturer 1

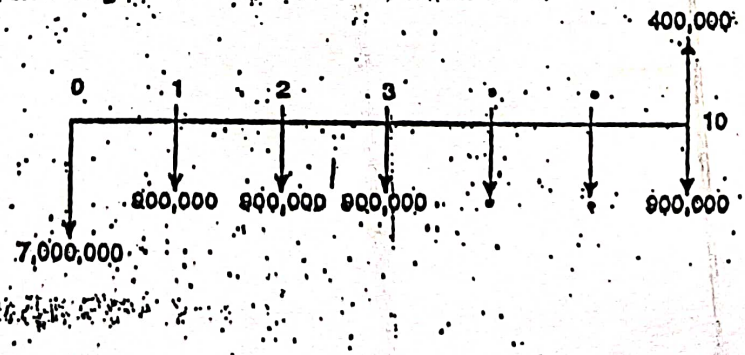
The cash flow diagram for this alternative is shown below.



$$FW_1(20\%) = 8,000,000\ (F/P, 20\%, 10) + 800,000\ (F/A, 20\%, 10) - 500,000$$
$$= 8,000,000\ (6.192) + 800,000\ (25.959) - 500,000$$
$$= \text{Rs. } 69,803,200$$

Alternative 2: Manufacturer 2

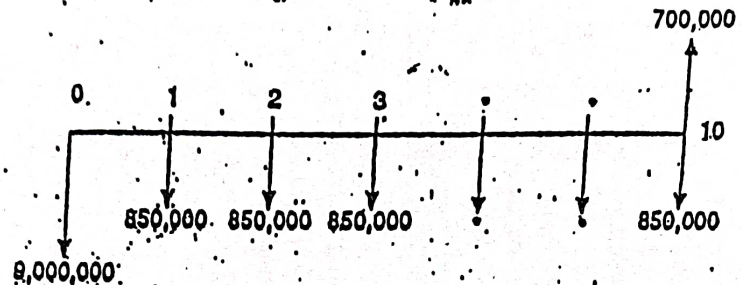
The cash flow diagram for this alternative is shown below.



$FW_2(20\%) = 7,000,000\ (F/P, 20\%, 10) + 900,000\ (F/A, 20\%, 10) - 400,000$
$= 7,200,000\ (6.192) + 900,000\ (25.959) - 400,000$
$= Rs.\ 66,307,100$

Alternative 3: Manufacturer 3

The cash flow diagram for this alternative is shown below.



$FW_3(20\%) = 9,000,000\ (F/P, 20\%, 10) + 850,000\ (F/A, 20\%, 10) - 700,000$
$= 9,000,000\ (6.192) + 850,000\ (25.959) - 700,000$
$= 77,093,150$

The future worth cost of the alternative 2 is lesser than that of the other two alternatives. Hence, M/s. Krishna castings should buy the annealing furnace from the manufacturer 2.

Rate of return method. Rate of return of a cash flow is the interest rate at which the present worth of that cash flow reduces to zero. In this method of comparison, the rate of return for each alternative is computed. Then the alternative which has the highest rate of return is selected as the best alternative.

Example 3.21. A company is planning to expand its present business activity. It has two alternatives for the expansion programme and corresponding cash flows are given in the following table. Each alternative has a life of 5 years and a negligible salvage value. Evaluate them based on the rate of return method and suggest the best alternative to the company.

	Initial Investment (Rs.)	Yearly Revenue (Rs.)
Alternative 1	500,000	150,000
Alternative 2	800,000	250,000

Solution.

Alternative 1: The present worth function of the alternative 1 is given as:

$$PW(i) = -500,000 + 150,000\ (P/A, i, 5)$$

The above function approximately reduces to zero at 15.24% interest rate. Hence, the rate of return of the expansion programme as per the alternative 1 is 15.24%.

Alternative 2: The present worth function of the alternative 2 is as follows:

$$PW(i) = -800,000 + 250,000\ (P/A, i, 5)$$