Wake-based estimation of Mach number for supersonic plasma flows

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Abstract

Purpose - This study investigates the formation of Mach cones in supersonic plasma flows around various structures. The goal is to examine how these structures interact with plasma flows and to provide insights into the Mach number estimation process.

Methodology - The simulations were performed using a 3D Particle-in-Cell (PIC) code, EMSES, which allows for the calculation of plasma dynamics around objects. The study utilized grid-based plasma flow simulations in the positive x-direction, with the plasma density, ion and electron temperatures, and ion-to-electron mass ratio set. Structures were introduced into the flow to study the ion and electron density fields, and the Mach number was estimated based on the cone boundary derived from the density slopes.

Findings - The results did show that our proposed method is still insufficient and require further work. There is still a broad open field for improvement and we list further points for potential development.

Originality/Value - This research explores possibilities of simplifying the plasma flow model to a simple supersonic wave propagation with linear relations.

Keywords: particle-in-cell; plasma; simulation; Mach cone; supersonic flow

1 Introduction

It is necessary for humanity to understand the environment of space, as many spacecrafts are being used for the observation. Normally satellites fly on supersonic, and when objects are supersonic in plasma, Mach cones are generated as we see the same in the atmosphere. However, due to the collision-less environment in plasma, its behaviour differs from what we assume in the air. There is an interesting case reported which second Mach cone is generated behind the first Mach cone(Miloch, 2010), especially when the object is insulator. Also, the higher ratio of the electron temperature to the ion temperature is, the more outstanding the second Mach cone is. This is because of the ion focusing caused by the object charged negative which plasma is absorbed. We only have poor understanding about Mach cone in plasma, so further research is expected.
Mach number M is defined from the equation below.

$$M = \frac{1}{\sin \mu} = \frac{v}{a} \tag{1}$$

μ is Mach angle, a is a speed of sound, and v is the velocity. The relation between these parameters are shown in Figure 1.

3D PIC simulation method to place objects with multiple shape patterns in a supersonic flowing plasma and show the results of mach cones formed behind them.

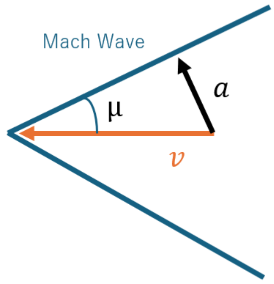

Figure 1: Mach angle

2 Methods

2.1 Numerical Code

The simulations are performed with a plasma particle code called EMSES, 3D Particle-in-cell(PIC) simulation code developed by *Miyake* and *Usui*(Miyake and Usui, 2009). PIC is useful when calculating the system of charged particles such as plasma, by solving the motion equation and Maxwell equations repeatedly. By dividing the area into grids, EMSES is capable of reproducing spatial distributions and time evolution. Based on the electromagnetic PIC method, therefore the code can perform the situation of plasma flows or magnetic field aligned in any directions. EMSES can include the conductors in use of capacitance matrix method (Hockney and Eastwood, 1981), which is helpful for thinking various shapes of the satellites. *Camphor*, the supercomputer on Kyoto university were used in the simulation.
The simulation box is $160 \times 160 \times 156$ grid in xyz direction. Plasma flow is in positive x-axis direction, speed being changed by the cases. Conductor objects are located around (60,80,78), having offset to left at x-axis in order to see the effect of the plasma flow easily. The boundary condition is free for the particles, periodic for the field, and Neumann condition for the potential in any directions. The basic simulation parameters used are shown in Table I.

Plasma Density	$10^5\,\mathrm{cm}^{-3}$
Electron temperature	1000 K
Ion temperature	750 K
Ion to electron mass ratio	1000
Magnetic field **B**	0

Table I: Parameters

2.2 Cone boundary detection

Numerical simulations produce several snapshots of the density fields of ions and electrons. Based on those, we can try to detect the cone boundary and thus estimate the Mach number of the flow relative to the acoustic wave speed. We estimate the boundary points of the cones for both ions and electrons, calculate the slopes in positive and negative y-direction, and finally obtain from that Mach number of the flow.

Figure 2 shows the example ion density data sampled from above the satellite. The field is quite fluctuating, thus we smooth it by averaging the values across nearest neighbourhood in z-direction. In the example window of 40 points was used. Later, unless stated otherwise, window of 21 points is used.

To estimate the cone boundary, we choose points with density between 0.9 and 0.97. Figure 3 presents the region of interest. Fitting a linear function to those points with least-square method, we can extrapolate, at what y coordinate the slope would reach threshold of 1, or the original density. Taking several such measurements along the cone, we can trail the boundary and fit it with another linear function. The slope of the function can be used to estimate the ratio of the wave velocity (perpendicular to the boundary) and the flow velocity (along x-axis).

Based on eq. (1), as shown in Figure 1, we can use the linear component of the fitted function and obtain Mach number of the flow. As a mean of error estimation, we perform such analysis for all the snapshots produced by the simulations and provide the resultant Mach number standard

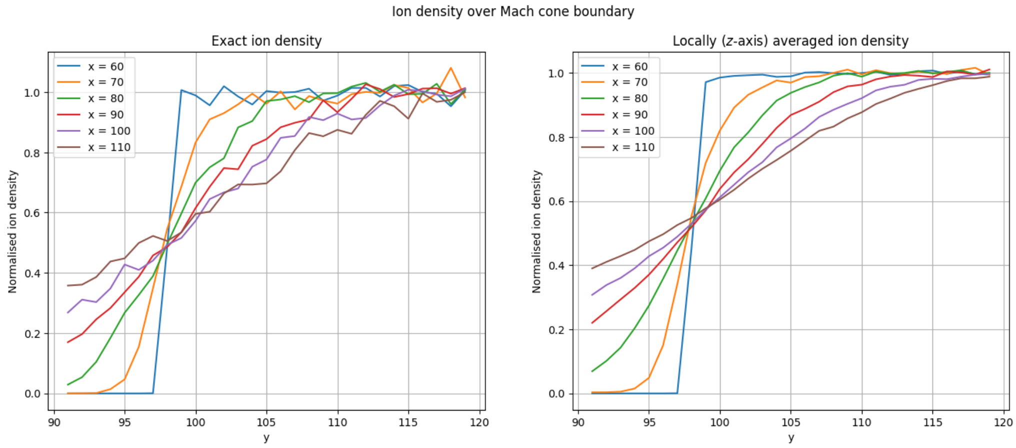

Figure 2: Ion densities sampled along y-axis from above of the satellite at z coordinate being the middle plane of the object. On the left the original data is shown. On the right averaged data along z-axis at ± 20 region around the symmetry plane.

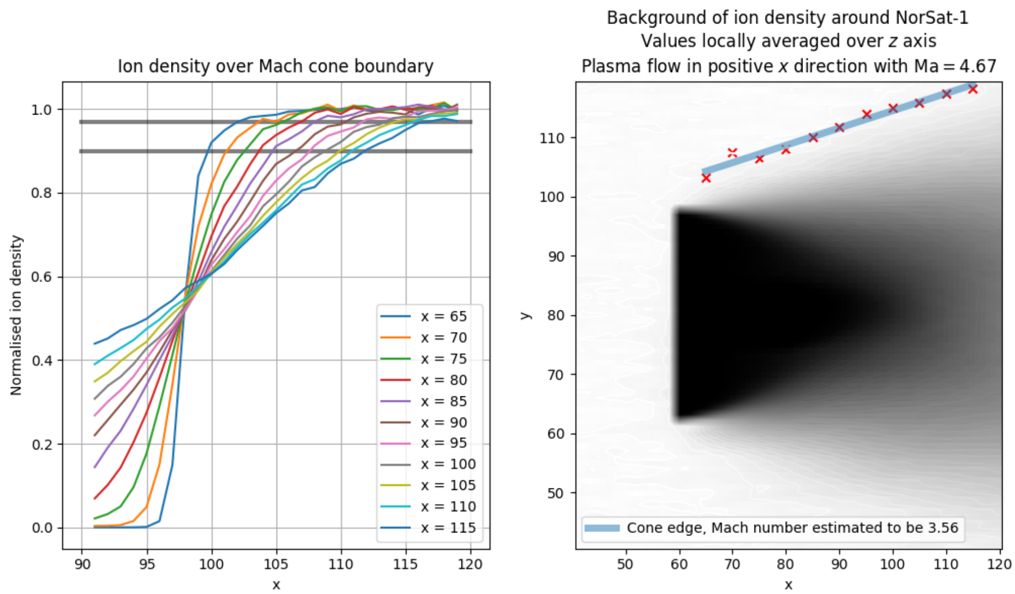

Figure 3: Mach number estimation method: the field density (here ions) is smoothed and for each sample along y-direction points between 0.9 and 0.97 of original density are chosen. Based on those, we estimate the cone boundary by fitting a straight line and searching for the crossing of density threshold 1. Those points are put together and used for another line fitting to estimate the cone boundary. Based one the slope, we can calculate Mach number of the flow. Derived Mach number is 3.56, significantly lower than expected 4.67.

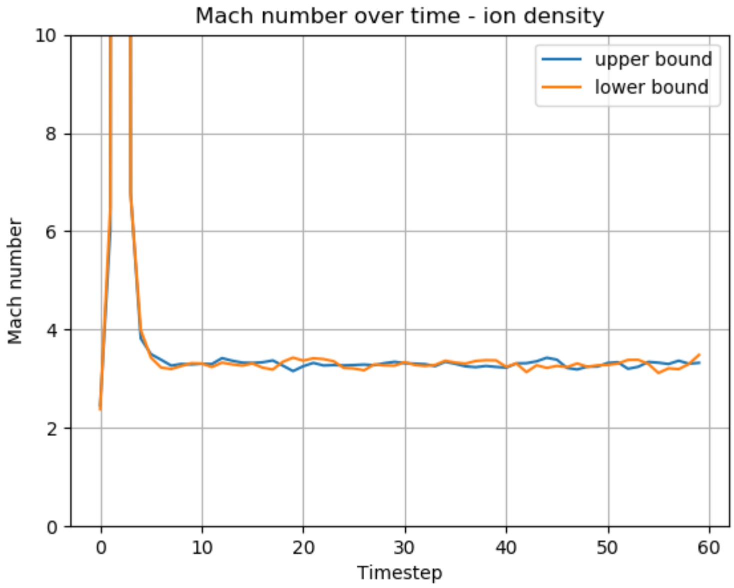

Figure 4: Mach number calculated based on the cone slope for ion density. Timesteps used here are every 500th simulation frame. Resulting Mach numbers are 3.3 ± 0.06 and 3.29 ± 0.07 for upper and lower bound respectively.

deviation as the error. Figure 4 displays an example result, where the first few frames are before simulation achieves stable state. Mach number is calculated by skipping first 10 snapshots, and calculating the average of the remainder.

Computed Mach number is compared with the theoretical Mach number for warm plasma. As shown by (G., 2003), ion acoustic waves travel at

$$c_s = \sqrt{\frac{\gamma_e k_B T_e + \gamma_i k_B T_i}{m_i}}. \tag{2}$$

This calculation, for $\gamma_{e,i} = 5/3$, is included in Emses plasma parameter sheet. The sheet provides two values, for cold and warm plasma. As ion temperature is similar to electron temperature ($T_i = 750\,\mathrm{K} \sim T_e = 1000\,\mathrm{K}$) we should consider warm plasma behaviour (Allen, 2013). All our simulations we refer to in this paper had their parameters tuned for theoretical Mach number to be either 1.33, 2.67 or 4.67. First value was the default simulation value, second its double, and the third one resulted from setting all the parameters to real life values for LEO.

2.3 Simulation cases

In the context of space exploration and satellite operations, understanding how various structures interact with plasma flows is critical for the design and protection of spacecraft. The structures examined in this study—sphere, NorSat-1(satellite model), and solar panel—were chosen based on their relevance to real-world spacecraft design and their varying geometrical properties.

2.3.1 Sphere

The spherical structure provides a symmetric geometry for analysing Mach cone formation in plasma flow. Due to its uniform curvature, the sphere is an ideal candidate for studying how plasma flow behaves when interacting with an isotropic object. This section examines the behaviour of the ion density and Mach cone formation around the sphere, highlighting its symmetric influence on plasma flow.

2.3.2 NorSat-1

Unlike the spherical structure, the satellite introduces a more complex, asymmetrical geometry that reflects real-world spacecraft designs. The interaction between plasma flow and the protruding components of the satellite is expected to cause variations in Mach cone formation.

2.3.3 Solar panel

The solar panel in this study consists of a single flat panel, offering a relatively simple and symmetric geometry. While the panel lacks complex or highly asymmetrical features, its broad and flat surface introduces unique interactions with the plasma flow. In addition, for some simulations the potential of the panel is fixed to 0 V to investigate the influence by floating potential of the satellite.

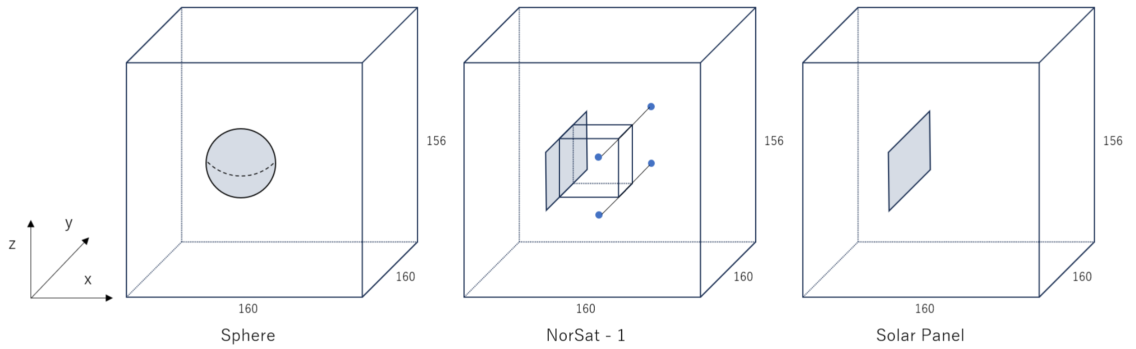

Figure 5: Simulation cases

By simulating the plasma flow around these distinct geometries, this study aims to provide a comprehensive comparison of how structural differences affect the formation of Mach cones and the resulting plasma dynamics. This analysis is not only fundamental for theoretical plasma physics but also has practical implications for spacecraft design, particularly in minimizing potential damage caused by plasma interactions.

3 Results

3.1 Simulation results

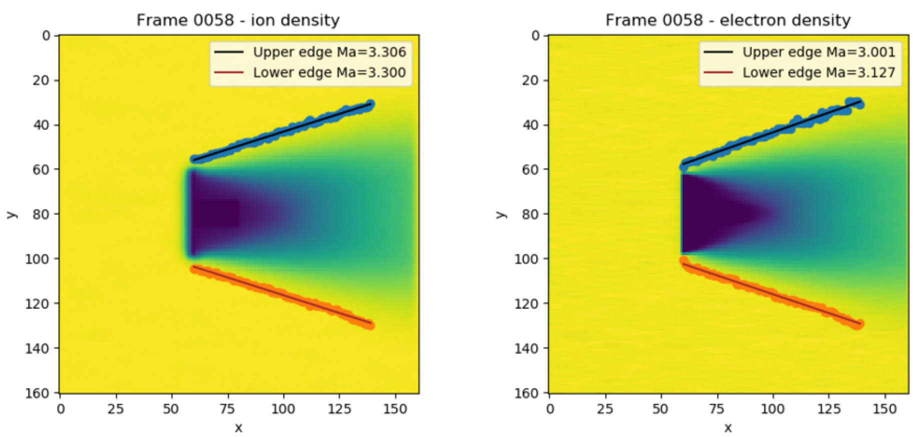

Figure 6: Ion and Electron Densities and Estimated Mach Number in Plasma Flowing at Mach 4.67(NorSat-1)

The result for NorSat-1 with 4.67 Mach number is discussed here. As shown in Figure 6, a Mach cone is formed, with a low-density region extending conically behind the satellite. The density remains consistently low inside the Mach cone, with no phenomena such as ion focusing observed(Miloch, 2010). This is because ion focusing are only observed under low temperature plasma. As there is no ion focusing, second Mach cone is not observed as well. Furthermore, regarding the estimated Mach cone edges, the ion edges display nearly identical Mach numbers for the upper and lower boundaries (3.306 and 3.300, respectively). In contrast, the electron edges show a slight discrepancy, with the upper edge having a Mach number of 3.001 and the lower edge 3.127. However, in both electrons and ion density, large errors are confirmed compared to the theoretical Mach number 4.67.

Shape	Theoretical	Electron 1	Electron 2	Ion 1	Ion 2
0.5 cm sphere	1.33	N/A	N/A	7.89 ± 7.54	10.86 ± 14
0.5 cm sphere	4.67	N/A	N/A	12.33 ± 5.61	11.46 ± 4.79
1.5 cm sphere	1.33	1.81 ± 2.44	1.32 ± 0.52	1.67 ± 0.48	1.61 ± 0.39
1.5 cm sphere	4.67	4.31 ± 0.54	2.44 ± 0.49	3.91 ± 0.62	4 ± 0.54
2.5 cm sphere	1.33	4.89 ± 11.06	3.83 ± 6.61	1.57 ± 0.13	1.55 ± 0.17
2.5 cm sphere	4.67	3.25 ± 1.89	3.01 ± 0.39	3.48 ± 0.26	3.42 ± 0.24
5 cm sphere	1.33	2.37 ± 0.86	2.04 ± 0.53	1.62 ± 0.11	1.63 ± 0.1
5 cm sphere	4.67	3.33 ± 0.35	3.33 ± 0.31	3.53 ± 0.08	3.55 ± 0.09
Grounded panel	1.33	1.9 ± 0.28	2.06 ± 0.41	1.56 ± 0.12	1.55 ± 0.11
Grounded panel	2.67	2.02 ± 0.15	2.27 ± 0.4	2.18 ± 0.03	2.19 ± 0.04
Single panel	1.33	2 ± 0.23	2.2 ± 0.48	1.62 ± 0.12	1.59 ± 0.1
Single panel	2.67	2.11 ± 0.16	2.3 ± 0.29	2.23 ± 0.05	2.24 ± 0.03
Single panel	4.67	3.45 ± 0.95	3.12 ± 0.07	3.41 ± 0.05	3.34 ± 0.05
NorSat-1	1.33	2.12 ± 0.4	2.14 ± 0.42	1.72 ± 0.13	1.73 ± 0.11
NorSat-1	2.67	2.22 ± 0.22	2.31 ± 0.42	2.25 ± 0.04	2.24 ± 0.04
NorSat-1	4.67	3.09 ± 0.18	3.05 ± 0.17	3.3 ± 0.06	3.29 ± 0.07

Table II: Mach numbers estimated from the simulation results compared to the theoretical Mach numbers. Estimations base on the density drop boundary for electrons in positive (Electron 1) and negative (Electron 2) y direction, and for ions in positive (Ion 1) and negative (Ion 2) y direction. N/A means the method failed to produce any result. Red colour means the relative error is more than 0.2 of the value suggesting low reliability of the result.

We are considering results of 16 simulations here. Table II shows the calculated Mach numbers for two edges of the cones for each of the species. Results which have high standard deviation from theoretical Mach number are marked with red to emphasise where the method is unstable. We can completely disregard sphere of 0.5 cm radius, as it is comparable with Debye length $\lambda \approx 0.69$ cm. The method failed to produce any result for electron density field, and highly erroneous results for ion density field.

It is appropriate to estimate the Mach number from ions, considering the fact huge deviations are less than that of electrons. Thinking of Mach number 1.33 of sphere, unless the radius is larger than 2.5 cm, the size of the sphere doesn't affect Mach number. Comparing with the other shapes, no difference is confirmed as well.For Mach number 2.67 and 4.67, no difference by shape is identified similarly. There are some of improvement for calculated Mach number by fixing the potential of the solar panel.

Figure 7 compares our results with the theory, showing high misalignment. Cones obtained from electron density fields are less stable and less linear. In case of ion density fields, results are consistent between the sides. With exception of 1.5 cm sphere for $Ma = 4.67$, all valid results focus around the same regions, that is 3.4 for theoretical $Ma = 4.67$, 2.2 for theoretical $Ma = 2.67$, and 1.6 for theoretical $Ma = 1.33$. Small spheres result in slightly sharper cones for high Ma. Satellite, single panel and grounded panel produce similar results.

4 Discussion

Simulation results align with theory to some degree. All the flow considered here was set up to be supersonic. The simulation is collisionless, thus we do not observe any pressure-based phenomena like shock waves and true Mach cones (A., 2020). We observed a weak discontinuity as mentioned by (Allen, 2013). (Miloch, 2010) observed a secondary cone forming behind the object in a supersonic flow, however, (Allen, 2013) points to the phenomenon to be only present in cold plasma ($T_i/T_e < 0.5$). All our cases are for warm plasma ($T_i/T_e = 0.75$), thus it is correct that we do not observe any here.

Our method manages to trace the cone behind the objects in supersonic flows, however the results do not conform to the theory. As the boundary is weakly discontinuous, it is difficult to state, where it really lies. Our approach attempt to extrapolate the edge based on the region of stronger density variation. We only consider one set of specific thresholds (0.9 to 0.97). We also only use linear fitting. Prior attempts with fitting quadratic or cubic functions did not produce

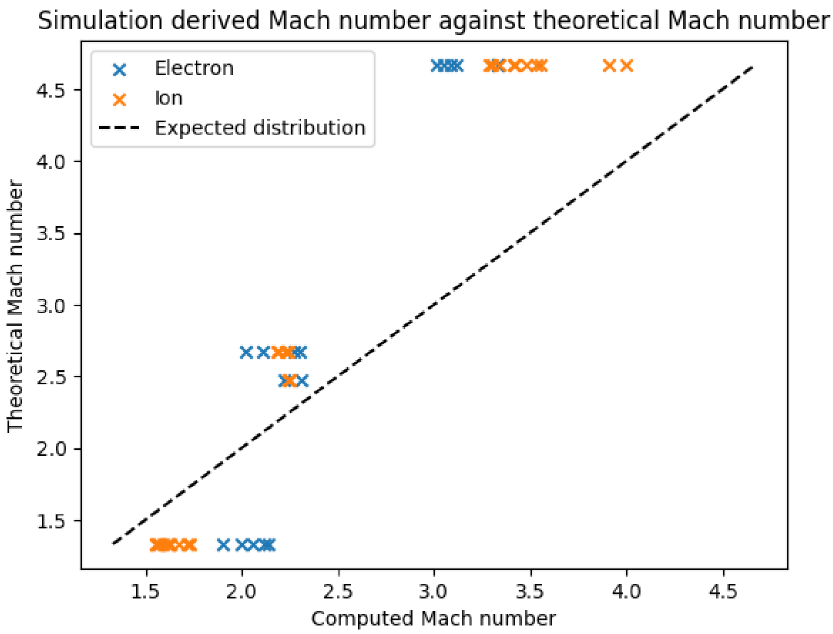

Figure 7: Calculated Mach numbers plotted against the expected values. All values marked red in Table II were removed. There is a visible misalignment with the theory. Ion density based derivation aligns more with the theory than electron density based.

satisfactory results and would require more work to be performed. We also do not consider fitting errors in further analysis and only limit ourselves to the variations over time.

Electrons seem to form a much less stable cone behind the object. We explored the possibility of negative potential on the probe influencing it, yet there is not significant difference between grounded and not-grounded panel which suggest it is not the case. It is noteworthy that the flow is not supersonic for electrons and the it they only follow ions neutralising the electric potential.

Ions do form a stable cone much faster, and we see much less variation in temporal observed Mach number for them. The values are misaligned with the theoretical ones. There is a slight bias for low Ma, and the response to rising theoretical Mach number is slower than expected. The theoretical values, however, are taken from the prepared parameter sheet for the simulation framework, and were not separately verified by us.

5 Conclusion

The simulations and analysis did show rather poor fit for our method. The results are strongly misaligned with the theory. We postulated possible impact of charge on the objects in plasma influencing the shape of the cone, however tests on a grounded panel did not show any significant change. We also suspected possibility of different values for γ in eq. (2) or different parameters, this, however, would result only in all values changing proportionally to the expected values, as we observe a bias for low Ma, it cannot be the only reason.

We suspect simplifying the flow to just a single wave propagation may be insufficient, requiring us to verify the models and to come up with new hypotheses for the flow mechanics. We should also perform similar simulations with different method using other tools, as it may be a problem specific to EMSES or to some of its tools used by the simulation. It may be also important to test different methods of trailing the cone's edge. Linear fit in the region, where density drop is visibly non-linear may be a reason for the errors. We should also include the errors of derivations at different levels, as the method may produce correct results, but with high error margins. Also, we should explore using different density regions, or even developing a method for finding an optimal one for the given case.

We hope the future work will explore the topic much more and will be able to improve the model and achieve satisfactory results. As of now, the method requires much improvement and further research. For any interested party, all the analysis code is published at the repository available under `https://github.com/MJOdorczuk/Aurora`.

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