Team G20

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Solution 1:

a) $P(D, B \mid A) = P(D|B, A) * P(B|A) = P(D|B) * P(B) = 0.2 * 0.75 \; = 0.15$

b) $P(C) = P(C|A) * P(A) + P(C|{\sim}A) * P({\sim}A) = 0.3 * 0.5 + 0.5 * 0.6 = 0.45$

c) $P(F) = P(F|C) * P(C|A) * P(A) + P(F|C) * P(C|{\sim}A) * P({\sim}A) + P(F|{\sim}C) * P({\sim}C|A) * P(A) + P(F|{\sim}C) * P({\sim}C|{\sim}A) * P({\sim}A)$
$= (0.4 * 0.3 * 0.5) + (0.4 * 0.6 * 0.5) + (0.5 * 0.7 * 0.5) + (0.5 * 0.4 * 0.5)$
$= (0.060 + 0.120 + 0.175 + .100) = \; 0.455$

d) $P(B, {\sim}C, D, E, F) = P(F, E, {\sim}C, D, B)$
$= P(F|E, {\sim}C, D, B) * P(E|{\sim}C, D, B) * P({\sim}C|D, B) * P(D|B) * P(B)$
$= P(F|{\sim}C) * P(E|{\sim}C, D) * P({\sim}C) * P(D|B) * P(B)$
$= 0.5 * 0.2 * (1 - 0.45) * 0.2 * 0.75$
$= 0.00825$

Solution 2:

2)

a)

(i) $\vec{w} = w_1 x_1 + w_2 x_2$

$w_1 = w_2 \rightarrow \vec{w}$ is parallel to $x_1 + x_2 = 0$.

$\Rightarrow$ the support vectors are $(2,3)$ & $(4,5)$.

Let $A = (2,3) \rightarrow +$

$B = (4,5) \rightarrow -$

$\sum \alpha_i y_i = 0 \Rightarrow \alpha_A - \alpha_B = 0 \Rightarrow \boxed{\alpha_A = \alpha_B}$

Calculating the Kernel function Values,

Since this is linearly separable,

$K(A,A) = 2^2 + 3^2 = 13$

$K(B,A) = 2 \cdot 4 + 3 \cdot 5 = 23$

$K(A,B) = 2 \cdot 4 + 3 \cdot 5 = 23$

$K(B,B) = 4^2 + 5^2 = 41$

Writing out the system of eqns using the constraints,

We have,

$$13\alpha_A - 23\alpha_B + b = 1$$

$$23\alpha_A - 41\alpha_B + b = -1$$

But; $\alpha_A = \alpha_B$

$$\Rightarrow \quad -10\alpha_A + b = 1$$

$$-18\alpha_A + b = -1$$

Solving, the eqns,

$$8\alpha_A = 2 \Rightarrow \boxed{\alpha_A = 1/4}$$

$$\therefore \boxed{\alpha_B = 1/4} \quad \boxed{b = 7/2}$$

Now,

$$\vec{w} = \sum \alpha_i y_i x_i$$

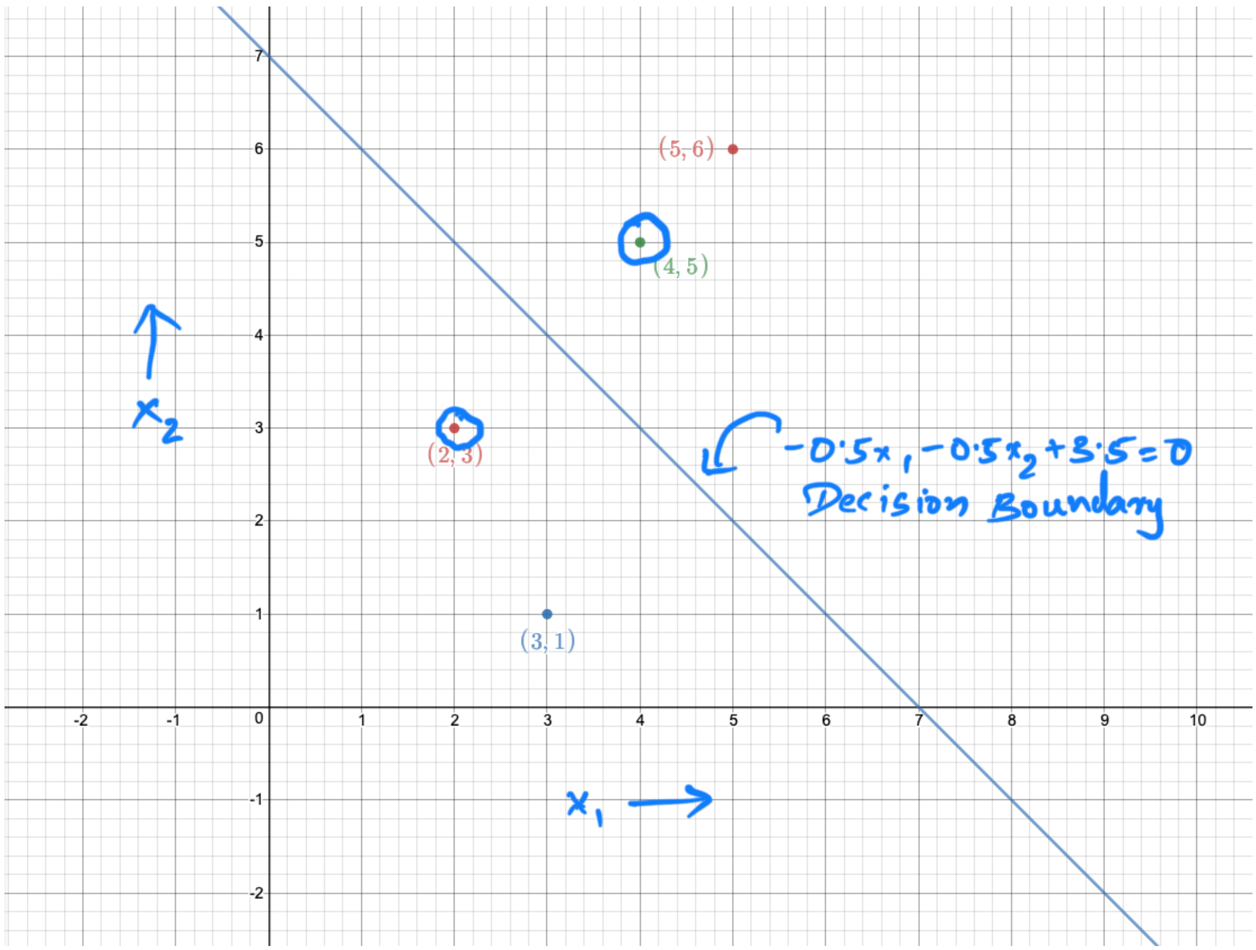
$$\vec{w} = \frac{1}{4}(2, 3) - \frac{1}{4}(4, 5)$$

$$\boxed{\vec{w} = -\frac{1}{2}x_1 - \frac{1}{2}x_2} \quad \text{or} \quad \vec{w} = (-0.5, -0.5)$$

$\therefore$ Decision Boundary $\Rightarrow$ $\vec{w} \cdot \vec{x} + b = 0$

$$\boxed{\text{Decision Boundary} \Rightarrow \quad -0.5x_1 - 0.5x_2 + 3.5 = 0}$$

ii)



(2)

b)

(i) $K(\vec{x},\vec{z}) = (1 + 2 \cdot \vec{x} \cdot \vec{z})^2$

Let $\vec{x} = (x_1, x_2)$

Let $\vec{z} = (z_1, z_2)$

$\therefore K(\vec{x},\vec{z}) = (1 + 2(x_1 z_1 + x_2 z_2))^2$

$\Rightarrow K(\vec{x},\vec{z}) = 1 + 4(x_1 z_1 + x_2 z_2) + 4(x_1 z_1 + x_2 z_2)^2$

$= 1 + 4x_1 z_1 + 4x_2 z_2 + 4\left[x_1^2 z_1^2 + 2x_1 x_2 z_1 z_2 + x_2 z_2^2\right]$

$= 1 + 4x_1 z_1 + 4x_2 z_2 + 4x_1^2 z_1^2 + 8x_1 x_2 z_1 z_2 + 4x_2 z_2^2$

$= (1, 2x_1^2, 2x_2^2, 2x_1, 2x_2, 2\sqrt{2}x_1 x_2) \cdot (1, 2z_1^2, 2z_2^2,$

$2z_1, 2z_2, 2\sqrt{2}z_1 z_2)$

$= \phi(\vec{x}) \cdot \phi(\vec{z})$

$\therefore \phi(\vec{x}) = (1, 2x_1^2, 2x_2^2, 2x_1, 2x_2, 2\sqrt{2}x_1 x_2)$

ii)

X1 = (0,1)	1	0	2	0	2	0
X2 = (0,-1)	1	0	2	0	-2	0
X3 = (1,0)	1	2	0	2	0	0
X4 = (-1,0)	1	2	0	-2	0	0

iii)

Let,

$A = (0,1) \rightarrow -$
$B = (0,-1) \rightarrow -$
$C = (1,0) \rightarrow +$
$D = (-1,0) \rightarrow +$

} Support Vectors

We have,

$$\sum \alpha_i y_i = 0 \Rightarrow \alpha_C + \alpha_D - \alpha_A - \alpha_B = 0.$$

$$\Rightarrow \boxed{\alpha_B = \alpha_C + \alpha_D - \alpha_A} \quad -(1)$$

Calculating the Kernel function values,

Since $K(x \cdot z) = (1 + 2 \cdot x \cdot z)^2$

$K(A,A) = (1 + 2 \times 1)^2 = 9$
$K(B,A) = (1 + 2 \times -1)^2 = 1$
$K(C,A) = (1 + 2 \times 0)^2 = 1$
$K(D,A) = (1 + 2 \times 0)^2 = 1$

$K(A,B) = (1 + 2 \times -1)^2 = 1$
$K(B,B) = (1 + 2 \times 1)^2 = 9$
$K(C,B) = (1 + 2 \times 0)^2 = 1$
$K(D,B) = (1 + 2 \times 0)^2 = 1$

$K(A,C) = (1 + 2 \times 0)^2 = 1$
$K(B,C) = (1 + 2 \times 0)^2 = 1$
$K(C,C) = (1 + 2 \times 1)^2 = 9$
$K(D,C) = (1 + 2 \times -1)^2 = 1$

$K(A, D) = (1 + 2\times 0)^2 = 1$
$K(B, D) = (1 + 2\times 0)^2 = 1$
$K(C, D) = (1 + 2\times -1)^2 = 1$
$K(D, D) = (1 + 2\times 1)^2 = 9$

Equations using the SVM Constraints:

$\sum \alpha_i y_i K(x_i, x) + b = 1$ (for +ve gutter)

$\sum \alpha_i y_i K(x_i, x) + b = -1$ (for -ve gutter)

Using these Constraints:

$-9\alpha_A - \alpha_B + \alpha_C + \alpha_D + b = -1$ — (2)

$-\alpha_A - 9\alpha_B + \alpha_C + \alpha_D + b = -1$ — (3)

$-\alpha_A - \alpha_B + 9\alpha_C + \alpha_D + b = 1$ — (4)

$-\alpha_A - \alpha_B + \alpha_C + 9\alpha_D + b = 1$ — (5)

Solving (1), (2), (3), (4) & (5), we have.

$\alpha_A = 1/8$	$b = 0$
$\alpha_B = 1/8$	
$\alpha_C = 1/8$	
$\alpha_D = 1/8$	

Now,

$$\vec{w} = \sum \alpha_i y_i \phi(x_i)$$

$$= \frac{-1}{8}\left(1, 0, 2, 0, 2, 0\right) +$$

$$\frac{-1}{8}\left(1, 0, 2, 0, -2, 0\right) +$$

$$\frac{1}{8}\left(1, 2, 0, 2, 0, 0\right) +$$

$$\frac{1}{8}\left(1, 2, 0, -2, 0, 0\right)$$

$$\Rightarrow \vec{w} = \left(0, 1/2, -1/2, 0, 0, 0\right)$$

Decision Boundary $\Rightarrow \vec{w} \cdot \phi(\vec{x}) + b \geq 0$

$$\Rightarrow \boxed{D.B = (0, 1/2, -1/2, 0, 0, 0) \cdot \phi(\vec{x}) \geq 0}$$

Solution 3:

(i)

Linear regression form, $\mathbf{y = w_1x^2 + w_0}$

Using least square to determine w_0 and w_1

$$\sum_{i=1}^{n} \frac{\partial}{\partial W0}\left(\mathrm{Yi} - (\mathrm{W1}Xi^2 + \mathrm{W0})\right)^2 = 0$$

$$\sum_{i=1}^{n}(\mathrm{Yi} - (\mathrm{W1}Xi^2 + \mathrm{W0})) = 0$$

On taking summations inside and solving,

$$\mathrm{W0} = \frac{\sum_{i=1}^{n}(\mathrm{Yi} - \mathrm{W1}Xi^2)}{n} \quad \text{-----------------------} \ (1)$$

$$\sum_{i=1}^{n} \frac{\partial}{\partial W1}\left(\mathrm{Yi} - (\mathrm{W1}Xi^2 + \mathrm{W0})\right)^2 = 0$$

$$\sum_{i=1}^{n}(\mathrm{Yi} - (\mathrm{W1}Xi^2 + \mathrm{W0}))Xi^2 = 0$$

On taking summations inside and solving,

$$\mathrm{W1} = \frac{\sum_{i=1}^{n} Xi^2(\mathrm{Yi} - \mathrm{W0})}{\sum_{i=1}^{n} Xi^4} \quad \text{-----------------------} \ (2)$$

Use W0 in W1,

$$\mathrm{W1} = \frac{n \sum_{i=1}^{n} Xi^2\mathrm{Yi} - \sum_{i=1}^{n} Xi^2 \sum_{i=1}^{n} \mathrm{Yi}}{n \sum_{i=1}^{n} Xi^4 - (\sum_{i=1}^{n} Xi^2)^2}$$

On substituting values,

$$\mathrm{W1} = \frac{(3 * (-7)) - ((13) * (0))}{(3 * 97) - (169)}$$

$$\mathrm{W1} = \frac{(-21)}{(122)}$$

W1 = - 0.1721

W0 = (0 – (- 0.6889 – 1.5489) / 3

W0 = (2.2378) / 3

W0 = 0.7459

(ii) Calculate the training RMSE for the fitted linear regression.

Final equation,

Y = - 0.1721 X^2 + 0.7459

Given points,

(2,5), (0,−2), (3,−3)

For X = 2

Y = 0.0575

For X = 0

Y = 0.7459

For X = 3

Y = - 0.8030

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (Predicted_i - Actual_i)^2}{N}}$$

RMSE = Sqrt of ($\frac{24.4283 + 7.5399 + 4.8268}{3}$)

RMSE = Sqrt of ($\frac{36.7950}{3}$)

RMSE = Sqrt of (12.2650**)**

RMSE = 3.5021