공학수학 5-6 주차 과제 60242053 임세빈

3.1-3.

$y = C_1x + C_2 x\ln x$. $(0,\infty)$; $x^2y'' - xy' + y = 0$. $y(1)=3$, $y'(1) = -1$.

$y' = c_1 + C_2(1+\ln x) = C_1 + C_2 + C_2\ln x$

$y'' = \frac{C_2}{x}$. $x^2\cdot\frac{C_2}{x} - x(C_1 + C_2 + C_2\ln x) + y = 0$.

$C_2x - xC_1 - xC_2 - xC_2\ln x + y = y - xC_1 - xC_2\ln x = 0$.

$y = xC_1 + xC_2\ln x$.

$C_1 = 3$. $3 + C_2 = -1$ $C_2 = -4$. $y = 3x - 4x\ln x$.

3-1. 13.

$y = C_1e^x\cos x + C_2e^x\sin x$. $y'' - 2y' + 2y = 0$.

(a) $y = e^x(C_1\cos x + C_2\sin x)$. $y' = e^x(C_1\cos x + C_2\sin x) + e^x(C_2\cos x - C_1\sin x)$

$y'(\pi) = e^\pi(C_1(\cos\pi - \sin\pi) + C_2(\sin\pi + \cos\pi)) = -e^\pi(C_1 + C_2) = 0$. $C_1 + C_2 = 0$.

$y(0) = e^0(C_1) = 1$. $C_1 = 1$. $C_2 = -1$. → 가능.

(b) $C_1 = 1$. $y(\pi) = e^\pi(C_1\cos\pi + C_2\sin\pi) = e^\pi\cdot -C_1 = -e^\pi$ → 불가능.

(c) $C_1 = 1$. $y(\frac{\pi}{2}) = e^{\frac{\pi}{2}}(C_1\cos\frac{\pi}{2} + C_2\sin\frac{\pi}{2}) = e^{\frac{\pi}{2}}(0 + C_2) = e^{\frac{\pi}{2}}\cdot C_2$

$C_2 = \frac{1}{e^{\frac{\pi}{2}}}$ → 가능

(d) $C_1 = 0$. $y(\pi) = e^\pi(\cos\pi + C_2\sin\pi) = e^\pi C_2 \cdot 0 = 0$. → 가능.

3-1. 17.

$f_1(x) = 5$, $f_2(x) = \cos^2 x$, $f_3(x) = \sin^2 x$.

$5(\cos^2 x + \sin^2 x) = f_1(x)$. $f_1(x) = 5f_2(x) + 5f_3(x)$ → 일차종속

3-1. 25.

$y'' - 2y' + 5y = 0$. $e^x\cos 2x$, $e^x\sin 2x$, $(-\infty, \infty)$

$r^2 - 2r + 5 = 0$. $1 \pm 2i = r$. $e^{ix} = \cos x + i\sin x$. $y(x) = C_1e^{(1+2i)x} + C_2e^{(1-2i)x}$

$e^{(1+2i)x} = e^x(\cos 2x + i\sin 2x)$. $e^{(1-2i)x} = e^x(\cos 2x - i\sin 2x)$

$y(x) = C_1e^x\cos 2x + C_2e^x\sin 2x$

3-1. 37.

$y'' - 4y' + 4y = 2e^{2x} + 4x - 12$; $y = C_1e^{2x} + C_2xe^{2x} + x^2e^{2x} + x - 2$.

$r^2 - 4r + 4 = 0$. $r = 2$. $y_h(x) = C_1e^{2x} + C_2xe^{2x}$. $y_p(x) = Ax^2e^{2x} + Bx + C$.

$\therefore y = C_1e^{2x} + C_2xe^{2x} + x^2e^{2x} + x - 2$.

3-2. 1.

$r^2 - 4r + 4 = 0$. $r = 2$. $y = C_1e^{2x} + C_2xe^{2x}$. $y_2 = xe^{2x}$.

3-2. 7.

$9r^2 - 12r + 4 = 0$ $r = \frac{2}{3}$. $y = C_1e^{\frac{2}{3}x} + C_2xe^{\frac{2}{3}x}$. $y_2 = xe^{\frac{2}{3}x}$.

3 -2
3 -2

3-3. 1.

$4r^2 + r = 0$. $r(4r+1)$ $r = 0, -\frac{1}{4}$

$y(x) = C_1 + C_2e^{-\frac{1}{4}x}$

3-3. 5.

$r^2 - 8r + 16 = 0$ $r = 4$. $y(x) = C_1e^{4x} + C_2xe^{4x}$.

3-3. 11.

$r^2 - 4r + 5 = 0$ $r = -5, 1$. $y(x) = C_1e^x + C_2e^{-5x}$

5
-1

3-3. 15.

$r(r^2 - 4r - 5) = 0$. $r = 0, -1, 5$ $y(x) = C_1 + C_2e^{-x} + C_3e^{5x}$

-5
1

3-3. 19.

$r^3 + r^2 - 2r = 0$. $r(r^2 + r - 2) = 0$. $r = 0, 1, -2$.

2
-1

$y(x) = C_1 + C_2e^{-x} + C_3e^{-2x}$.

3-3. 31.

$y'' + 16y = 0$. $y(0) = 2$, $y'(0) = -2$.

$r^2 + 16r = 0$. $r = 0, -16$ $y(x) = C_1 + C_2e^{-16x}$. $C_1 + C_2 = 2$. $y'(x) = -16C_2e^{-16x}$

$-16C_2 = -2$ $C_2 = \frac{1}{8}$. $C_1 = \frac{15}{8}$. $y(x) = \frac{15}{8} + \frac{1}{8}e^{-16x}$.

3-3. 37.

$r^3 + 12r^2 + 36r = 0$. $r(r^2 + 12r + 36) = 0$ $r = 0, -6$ 중근.

$y(x) = C_1 + C_2e^{-6x} + C_3xe^{-6x}$. $y'(x) = -6C_2e^{-6x} + C_3e^{-6x} - 6C_3xe^{-6x}$.

$y''(x) = 36C_2e^{-6x} - 12C_3e^{-6x} + 36C_3xe^{-6x}$. $C_1 = -C_2$. $-6C_2 + C_3 = 1$.

$36C_2 - 12C_3 = -7$. $36C_2 - 12(6C_2 + 1) = -7$. $36C_2 - 72C_2 - 12 = -7$.

$-36C_2 = 5$ $C_2 = -\frac{5}{36}$. $C_3 = 6\times -\frac{5}{36} + 1 = \frac{1}{6}$.

$y(x) = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{1}{6}xe^{-6x}$.

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3-4.3.

$y''-10y'+25y=30x+3$. $r^2-10r+25=0$ $r=5$ (중)

$y_h(x)=C_1e^{5x}+C_2xe^{5x}$ $y_p(x)=Ax+B$. $y'_p=A$. $y''_p=0$.

$0-10A+25(Ax+B)=30x+3$. $25A=30$. $A=\frac{6}{5}$. $B=\frac{3}{5}$.

$y(x)=C_1e^{5x}+C_2xe^{5x}+\frac{6}{5}x+\frac{3}{5}$.

3-4.13.

$y''+4y=3\sin 2x$. $r^2+4r=0$. ~~$r=0,-4$~~.

~~$y_h(x)=C_1+C_2xe^{-4x}$~~. (x) $r=\pm 2i$. $y_h(x)=C_1\cos 2x+C_2\sin 2x$.

$y_p=x(A\cos 2x+B\sin 2x)$.

$y_p'=A\cos 2x+B\sin 2x+x(-2A\sin 2x+2B\cos 2x)$.

$=A\cos 2x+B\sin 2x-2Ax\sin 2x+2Bx\cos 2x$.

$y_p''=(-2A\sin 2x+2B\cos 2x)+(-2A\sin 2x-4Ax\cos 2x)+(2B\cos 2x-4Bx\sin 2x)$.

$y_p''=-4A\sin 2x-4Bx\sin 2x+4B\cos 2x-4Ax\cos 2x$

$4y_p=4x(A\cos 2x+B\sin 2x)$. $y_p''+4y_p=-4A\sin 2x+4B\cos 2x$.

$3\sin 2x+0\cos 2x$. $A=-\frac{3}{4}$. $B=0$.

$y(x)=C_1\cos 2x+C_2\sin 2x-\frac{3}{4}x\cos 2x$.

3-4.17.

$r^2-2r+5=0$. $r=\frac{2\pm 4i}{2}=1\pm 2i$

$y_h=C_1e^x\cos 2x+C_2e^x\sin 2x$. $y_p=xe^x(A\cos 2x+B\sin 2x)$

$y_p'=e^x(A\cos 2x+B\sin 2x)+xe^x(A\cos 2x+B\sin 2x)+xe^x(-2A\sin 2x+2B\cos 2x)$

$y_p'=e^x(A\cos 2x+B\sin 2x)+xe^x[(A\cos 2x+B\sin 2x)+(2B\cos 2x-2A\sin 2x e^x)]$

$=e^x(A\cos 2x+B\sin 2x)+xe^x[(A+2B)\cos 2x+(B-2A)\sin 2x]$

$y''-2y'+5y_p=e^x[(C_1\cos 2x+C_2\sin 2x)]$ 2e $C_1=1$, $C_2=0$. $A=0$ $B=\frac{1}{10}$.

$\therefore y(x)=C_1e^x\cos 2x+C_2e^x\sin 2x+\frac{1}{10}xe^x\sin 2x$.

3-4.31.

$r^2+4r+5=0$. $r=\frac{-4\pm\sqrt{16-20}}{2}=-2\pm i$

$y_h(x)=e^{-2x}(C_1\cos x+C_2\sin x)$. $y_p=Ae^{-4x}$. $y_p'=-4Ae^{-4x}$.

$y_p''=16Ae^{-4x}$. $16Ae^{-4x}-16Ae^{-4x}+5Ae^{-4x}=5Ae^{-4x}$. $A=17$.

$y_p=17e^{-4x}$. $y(x)=e^{-2x}(C_1\cos x+C_2\sin x)+17e^{-4x}$. $y(0)=-3$.

$y(0)=C_1+C_2+17=C_1+17=-3$. $C_1=-10$.

$y'(x)=e^{-2x}((-2)(C_1\cos x+C_2\sin x)+(-C_1\sin x+C_2\cos x))$

$=e^{-2x}((-2C_1-C_2)\cos x+(-2C_2+C_1)\sin x)-28e^{-4x}$.

$y'(0)=(-2C_1-C_2+(-2C_2+C_1)\times 0)-28=-2C_1-C_2-28$.

$20-C_2-28=-C_2-8$. $C_2=-9$.

$\therefore y(x)=e^{-2x}(-10\cos x-9\sin x)+17e^{-4x}$.

3-4.37.

$y''+y=x^2+1$. $y(0)=5$, $y(1)=0$. $r^2+1=0$. $r=\pm i$.

$y_h=C_1\cos x+C_2\sin x$. $y_p=Ax^2+Bx+C$. $y_p''=2A$.

$2A+(Ax^2+Bx+C)=Ax^2+Bx+2A+C=x^2+1$. $A=1$, $B=0$. $2A+C=1$.

$C=-1$. $y_p=x^2-1$.

$y(x)=C_1\cos x+C_2\sin x+x^2-1$. $y(0)=5$. $C_1-1=5$. $C_1=6$.

$y(1)=C_1\cos 1+C_2\sin 1+1-1=0$. $6\cos 1+C_2\sin 1=0$. ..?

$C_2=\frac{-3.241}{0.8415}\approx -3.85\Rightarrow\frac{-6\cos 1}{\sin 1}$

$y(x)=6\cos x-3.85\sin x+x^2-1$.

3-5.1

$y''+y=\sec x$. $r^2+1=0$. $r=\pm i$ $y_h=C_1\cos x+C_2\sin x$.

$y_p=u_1\cos x+u_2\sin x$.

$$\begin{cases} u_1'\cos x+u_2'\sin x=0. \\ -u_1'\sin x+u_2'\cos x=\sec x. \end{cases}$$ $u_1'=-u_2'\frac{\sin x}{\cos x}=-u_2'\tan x$.

$-(-u_2'\tan x)\sin x+u_2'\cos x=\sec x$.

$u_2'\tan x\sin x+u_2'\cos x=\sec x$. $u_2'\left(\frac{\sin^2 x}{\cos x}+\cos x\right)=\sec x$.

$u_2'\left(\frac{\sin^2 x+\cos^2 x}{\cos x}\right)=\sec x$. $u_2'=\cos x\sec x=1$. $u_1'=-\tan x$.

$u_1=\int u_1'\,dx=\int -\tan=\ln|\cos x|+C_1$.

$u_2=\int u_2'\,dx=x+C_2$. $y_p=u_1\cos x+u_2\sin x$

$=(\ln|\cos x|)\cos x+x\sin x$.

$y(x)=C_1\cos x+C_2\sin x+\ln|\cos x|\cos x+x\sin x$.

3-5.11.

$r^2-1=0$. $r=\pm 1$. $y_h=C_1e^x+C_2e^{-x}$

$y_p=u_1e^x+u_2e^{-x}$.

$$\begin{cases} u_1'e^x+u_2'e^{-x}=0. \\ u_1'e^x-u_2'e^{-x}=\cosh x. \end{cases}$$ $2u_1'e^x=\cosh x$. $u_1'=\frac{\cosh x}{2e^x}$.

$2u_2'e^{-x}=-\cosh x$. $u_2'=-\frac{\cosh x}{2e^{-x}}=-\frac{\cosh x\cdot e^x}{2}$.

$u_1=\frac{1}{2}\int\frac{e^x+e^{-x}}{2e^x}dx=\frac{1}{2}\int\left(\frac{1}{2}+\frac{e^{-2x}}{2}\right)dx=\frac{1}{4}x-\frac{1}{4}\times\frac{e^{-2x}}{2}+C=\frac{x}{4}-\frac{1}{8}e^{-2x}$.

$u_2=-\frac{1}{2}\int(e^x\cdot\cosh x)dx=-\frac{1}{2}\int\left(e^x\cdot\frac{e^x+e^{-x}}{2}\right)dx$

$=-\frac{1}{2}\int\left(\frac{e^{2x}+1}{2}\right)dx=-\frac{1}{2}\left(\frac{1}{2}\int e^{2x}dx+\frac{1}{2}\int 1\,dx\right)$

$=-\frac{1}{2}\left(\frac{e^{2x}}{4}+\frac{x}{2}\right)=-\frac{1}{8}e^{2x}-\frac{x}{4}$.

$y_p=\left(\frac{x}{4}-\frac{1}{8}e^{-2x}\right)e^x+\left(-\frac{1}{8}e^{2x}-\frac{x}{4}\right)e^{-x}$.

$=\frac{x}{4}(e^x-e^{-x})-\frac{1}{8}(e^x+e^{-x})$.

$y(x)=C_1e^x+C_2e^{-x}+\frac{x}{4}(e^x-e^{-x})-\frac{1}{8}(e^x+e^{-x})$.

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3-5.21.

$4y''-y=xe^{\frac{x}{2}}$. $y(0)=1,\ y'(0)=0$.

$y''-\frac{1}{4}y=\frac{1}{4}xe^{\frac{x}{2}}$. $r^2-\frac{1}{4}=0$. $r=\pm\frac{1}{2}$. $y_h=C_1e^{\frac{x}{2}}+C_2e^{-\frac{x}{2}}$.

$y_p=u_1e^{\frac{x}{2}}+u_2e^{-\frac{x}{2}}$.

$\begin{cases} u_1'e^{\frac{x}{2}}+u_2'e^{-\frac{x}{2}}=0 \\ u_1'\cdot\frac{1}{2}e^{\frac{x}{2}}-u_2'\frac{1}{2}e^{-\frac{x}{2}}=\frac{1}{4}xe^{\frac{x}{2}} \end{cases}$ $u_1'=-u_2'e^{-x}$.

$\frac{1}{2}(-u_2'e^{-x})e^{\frac{x}{2}}-\frac{1}{2}u_2'e^{-\frac{x}{2}}=\frac{1}{4}xe^{\frac{x}{2}}$. $-u_2'e^{-\frac{x}{2}}=\frac{1}{4}xe^{\frac{x}{2}}$. $u_2'=-\frac{1}{4}xe^{x}$.

$u_1'=-u_2'e^{-x}=(\frac{1}{4}xe^{x})e^{-x}=\frac{1}{4}x$.

$u_1=\frac{1}{8}x^2$. $u_2=-\frac{1}{4}\cdot(xe^x-\int e^x dx)=-\frac{1}{4}(xe^x-e^x)=-\frac{1}{4}e^x(x-1)$

$y_p=(\frac{1}{8}x^2)e^{\frac{x}{2}}+(-\frac{1}{4}e^x(x-1))e^{-\frac{x}{2}}=(\frac{1}{8}x^2-\frac{1}{4}x+\frac{1}{4})e^{\frac{x}{2}}$.

$y(x)=C_1e^{\frac{x}{2}}+C_2e^{-\frac{x}{2}}+(\frac{1}{8}x^2-\frac{1}{4}x+\frac{1}{4})e^{\frac{x}{2}}$. $y(0)=1$. $C_1+C_2+\frac{1}{4}=1$ $C_1+C_2=\frac{3}{4}$.

$y'(x)=f'e^{\frac{x}{2}}+f\cdot\frac{1}{2}e^{\frac{x}{2}}-\frac{1}{2}C_2e^{-\frac{x}{2}}$. $f'(0)=-\frac{1}{4}$. $f(0)=C_1+\frac{1}{4}$.

$y'(0)=-\frac{1}{4}+(C_1+\frac{1}{4})\frac{1}{2}-\frac{1}{2}C_2=\frac{C_1-C_2}{2}-\frac{1}{8}$. $C_1-C_2=\frac{1}{4}$.

$C_1=\frac{1}{2}$ $C_2=\frac{1}{4}$ $y(x)=(\frac{1}{2}+\frac{1}{8}x^2-\frac{1}{4}x+\frac{1}{4})e^{\frac{x}{2}}+\frac{1}{4}e^{-\frac{x}{2}}$

3-5.23.

$r^2-2r+1=0$. $r=1$ (중). $y_h=C_1e^x+C_2xe^x$. $y_p=u_1e^x+u_2xe^x$.

$\begin{cases} u_1'e^x+u_2'xe^x=0 \\ u_1'e^x+u_2'(x+1)e^x=e^x\sec^2x \end{cases}$ $u_1'=-u_2'x$.

$-u_2'x+u_2'(x+1)=\sec^2x$. $u_2'=\sec^2x$. $u_1'=-x\sec^2x$.

$u_2=\int\sec^2x\,dx=\tan x$. $u_1=\int -x\sec^2x\,dx$. $u_1=-(x\tan x-\int\tan x\,dx)=-x\tan x+\ln|\cos x|$

$y_p=u_1e^x+u_2xe^x=(-x\tan x+\ln|\cos x|)e^x+x\tan x e^x=\ln|\cos x|\cdot e^x$.

$y(x)=C_1e^x+C_2xe^x+\ln|\cos x|e^x$. $y(0)=1$. $C_1=1$.

$y'(x)=e^x(C_1+C_2x+\ln|\cos x|)'+e^x(C_1+C_2x+\ln|\cos x|)$

$=e^x(C_2-\tan x+C_1+C_2x+\ln|\cos x|)$. $x=0$. $\tan 0=0$. $\ln 1=0$.

$y'(0)=C_1+C_2=1+C_2=0$. $C_2=-1$. $y(x)=e^x(1-x+\ln|\cos x|)$.