AM modulation & Demodulation

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Previous Lecture

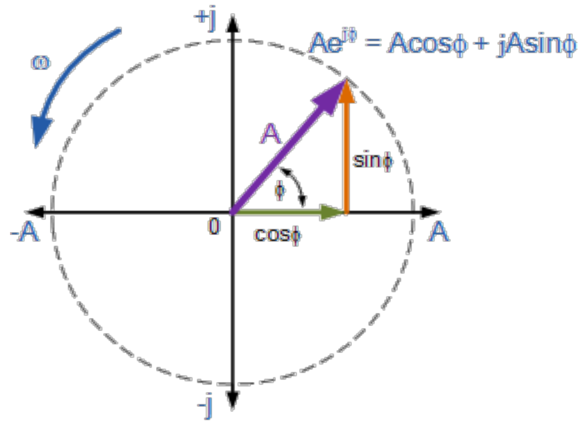
- Modulation and modulation types
- Amplitude modulation – basics
- Spectrum of AM signals
- Bandwidth requirements
- Power in AM signal

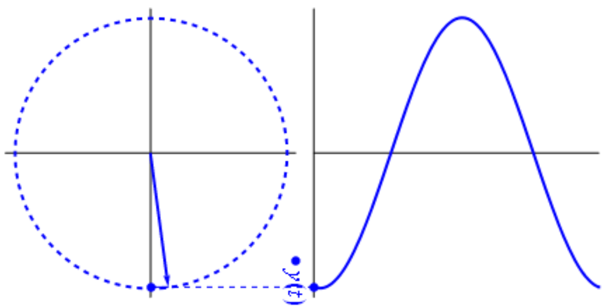
Today’s lecture

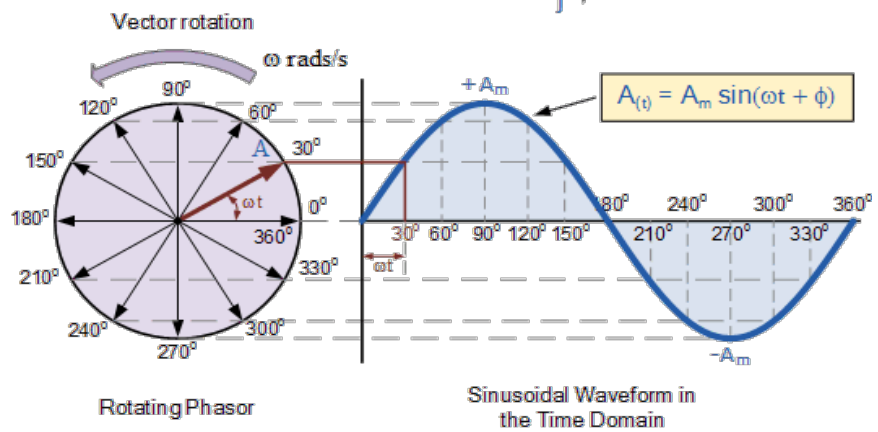
- Phasor representation of signals
- AM modulator
- Demodulation of AM signals
- Advantages and disadvantages of AM
- Double sideband – suppressed carrier modulation
- Costas receiver and quadrature carrier multiplexing

Phasor Representation of Signals

- A signal can be represented in a Cartesian form as:
  $z(t) = Ae^{(j\omega t+\varphi)} = A\cos(\omega t + \varphi) + jA\sin(\omega t + \varphi)$
- From the above we have:
  $\Re e\{Ae^{(j\omega t+\varphi)}\} = A\cos(\omega t + \varphi)$
  $\Im m\{Ae^{(j\omega t+\varphi)}\} = A\sin(\omega t + \varphi)$
- We can plot z(t) as a rotating phasor

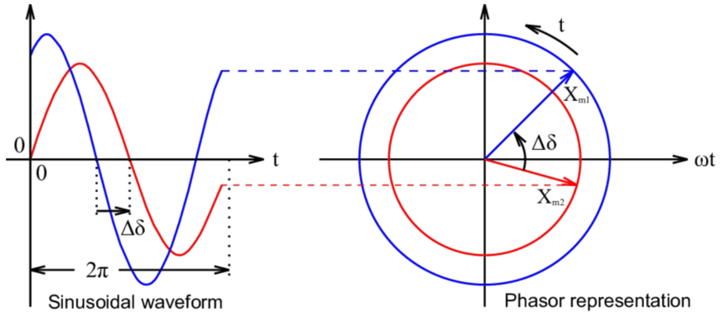




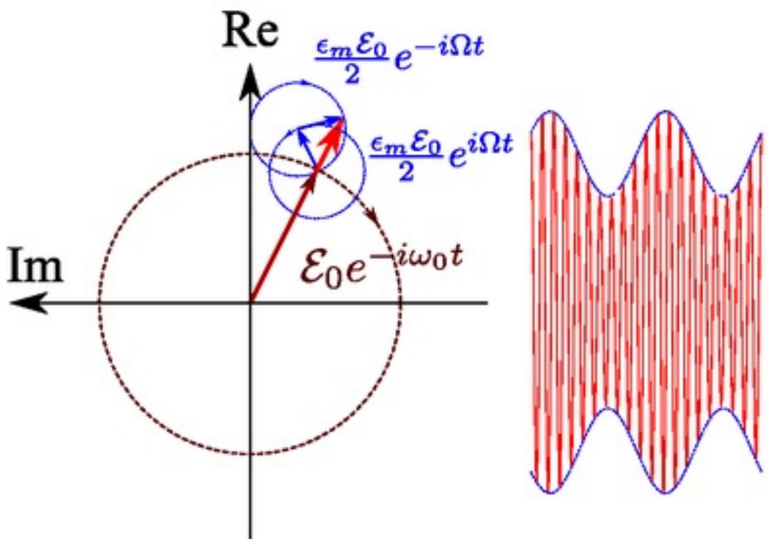


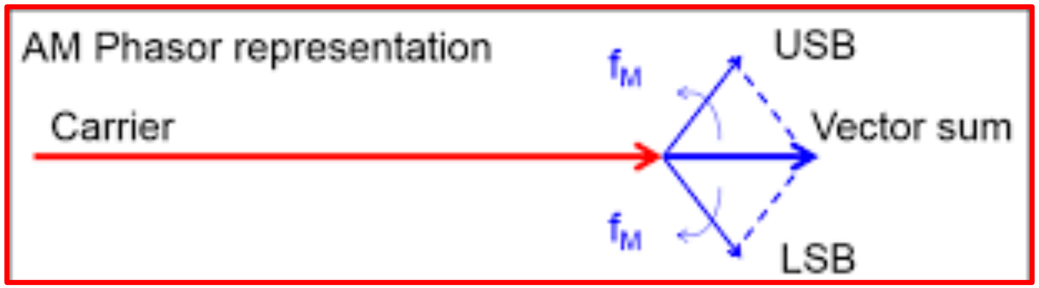
Phasor Representation of Signals

- The length of the vector represents the amplitude of the signal



- Phasor representation of AM signal:



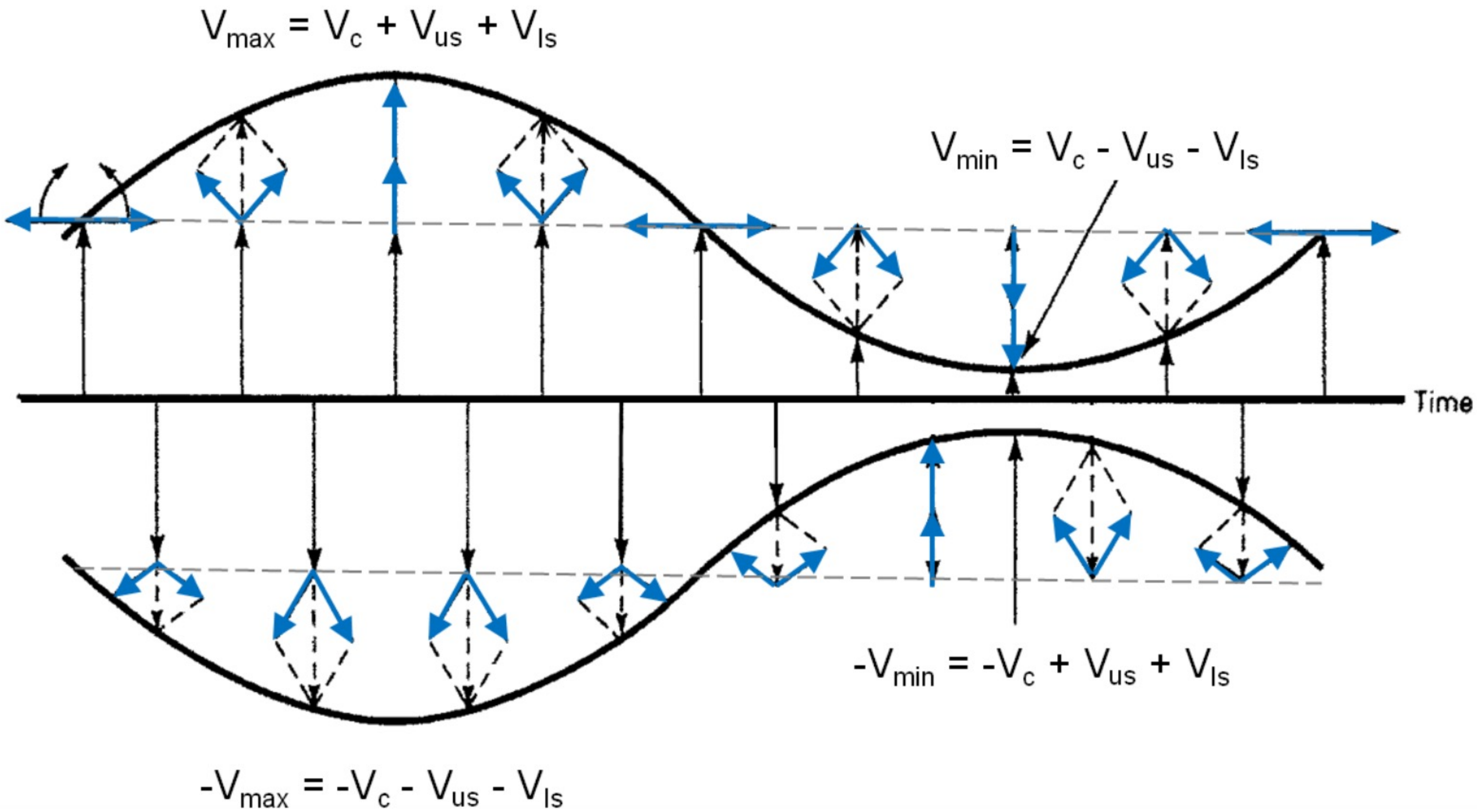


Phasor Representation of AM Signal

V_c- carrier amplitude

V_{us}- USB amplitude

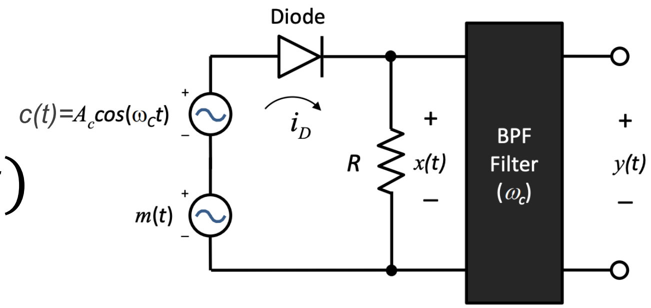
V_{ls}- LSB amplitude

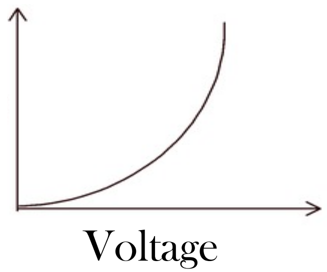


AM modulator - Square Law Modulation

- Makes use of non-linear current-voltage characteristics of diode
- Diode current can be expressed using Taylor series (first 2 terms):

$$i_D(t) = b_1 v_D(t) + b_2 v_D^2(t)$$





current -voltage characteristics

$$x(t) = Ri_D(t) = Rb_1 v_D(t) + Rb_2 v_D^2(t)$$

$$x(t) = a_1 v_D(t) + a_2 v_D^2(t)$$

$$\boxed{\begin{aligned} a_1 &= Rb_1 \\ a_2 &= Rb_2 \end{aligned}}$$

$$v_D(t) = c(t) + m(t) = A_c cos(2\pi f_c t) + \mathrm{m(t)}$$

$$x(t) = a_1(A_c cos(2\pi f_c t) + m(t)) + a_2(A_c cos(2\pi f_c t) + m(t))^2$$

AM modulator - Square Law Modulation

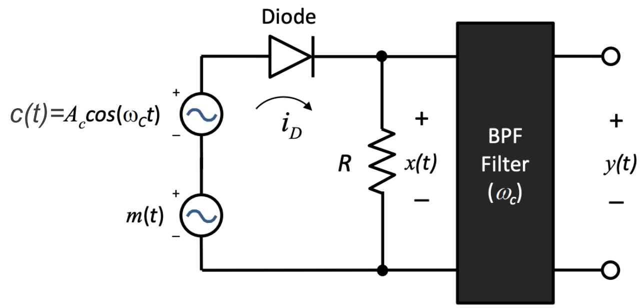
$$x(t) = a_1(A_c cos(2\pi f_c t) + m(t)) + a_2(A_c cos(2\pi f_c t) + m(t))^2$$

$$x(t) = a_1 A_c cos(2\pi f_c t) + a_1 m(t) + a_2 {A_c}^2 cos^2(2\pi f_c t) + a_2 m(t)^2 + 2a_2 A_c m(t) cos(2\pi f_c t)$$

$$\boxed{cos^2(\alpha) = \frac{1 + \cos(2\alpha)}{2}}$$

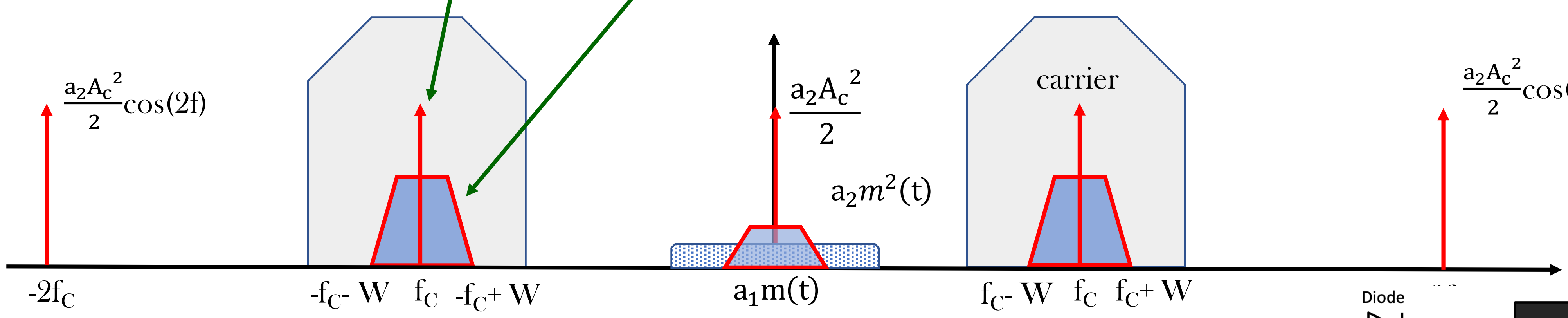
$$x(t) = a_1 A_c cos(2\pi f_c t) + a_1 m(t) + \frac{a_2 {A_c}^2}{2}\left(1 + cos\left(2\pi(2f_c)t\right)\right)$$

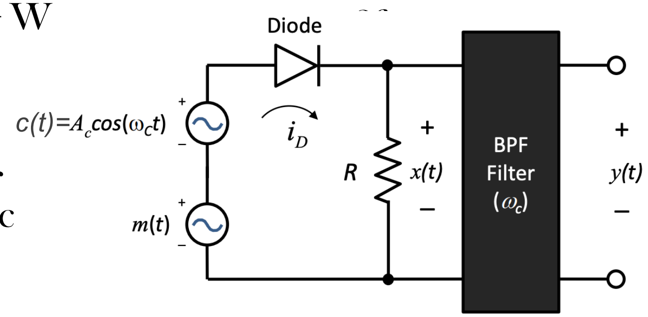
$$+a_2 m^2(t) + 2a_2 A_c m(t) cos(2\pi f_c t)$$



AM modulator - Square Law Modulation

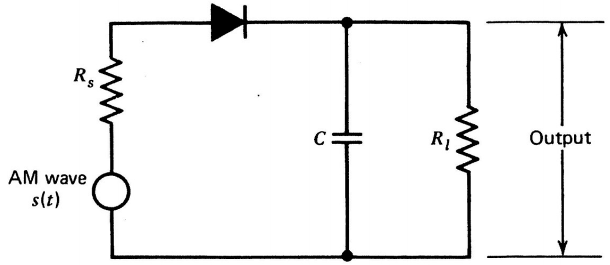
$$x(t) = a_1 m(t) \boxed{+ a_1 A_c \cos(2\pi f_c t) + 2a_2 A_c m(t)\cos(2\pi f_c t)} + a_2 m^2(t) + \frac{a_2 A_c{}^2}{2}\left(1 + \cos(2\pi(2f_c)t)\right)$$

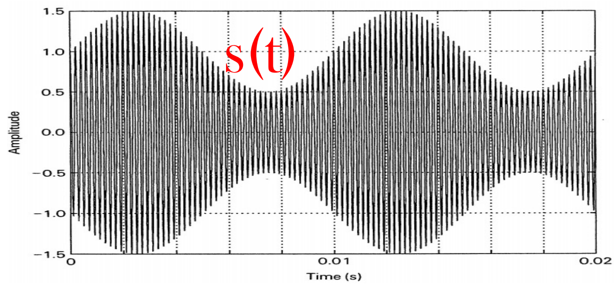


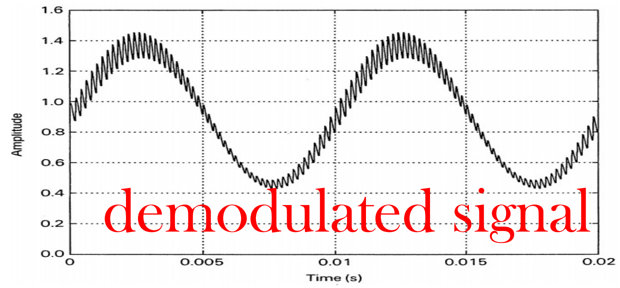


- ➢ Only 2 components around the frequency of interest i.e.: f_c
- ➢ Use band-pass filter to eliminate all other components

AM Demodulation: Envelop Detector







- Positive half-cycle: carrier charges the capacitor (charging time const. short in comparison to carrier period: $(r_d + R_s)C \ll \frac{1}{f_c}$)

➢ Negative half-cycle: the capacitor discharges slowly through the load resistance R_L

➢ Load resistance needs to be:

- ✓ large enough for the voltage on the capacitor not to follow the carrier,
- ✓ small enough not to distort the modulation signal: $\frac{1}{f_c} \ll R_L C \ll \frac{1}{W}$,

where W – information signal bandwidth

Characteristics of Modulation Formats

- Different modulation schemes have different characteristics e.g.:
  - Power efficiency - how much power per bit of information is required
  - Spectral efficiency - how much bandwidth is required for a given data rate
  - Ease of implementation - how easy is it to build Tx/Rx
  - Resistance to noise - how vulnerable is the modulation scheme to noise

Advantages of AM

- Simplicity:
  - Easy to modulated
  - Easy to demodulated – in time domain using envelop detector
- Well understood:
  - The oldest scheme and most intuitive
  - The waveform is also visually intuitive
- Well defined spectrum: the bandwidth required is twice the maximum frequency component in the signal and is located above/below the carrier.

Disadvantages of AM

- Requires a large bandwidth compared to other schemes: twice the bandwidth of the original signal

- Power hungry: two sidebands and the carrier to transmit, the carrier power does not convey any information.

- Any noise or interference that adds to the amplitude of the signal will directly affect the information signal.

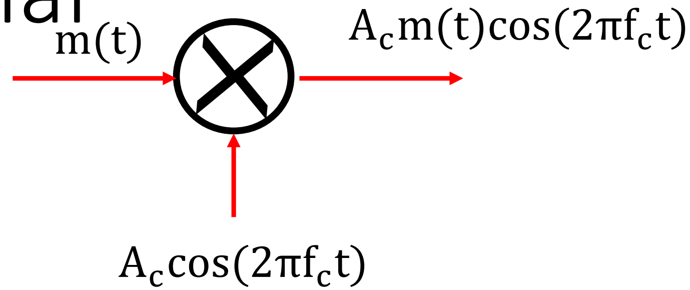
Double-sideband suppressed carrier – DSB-SC

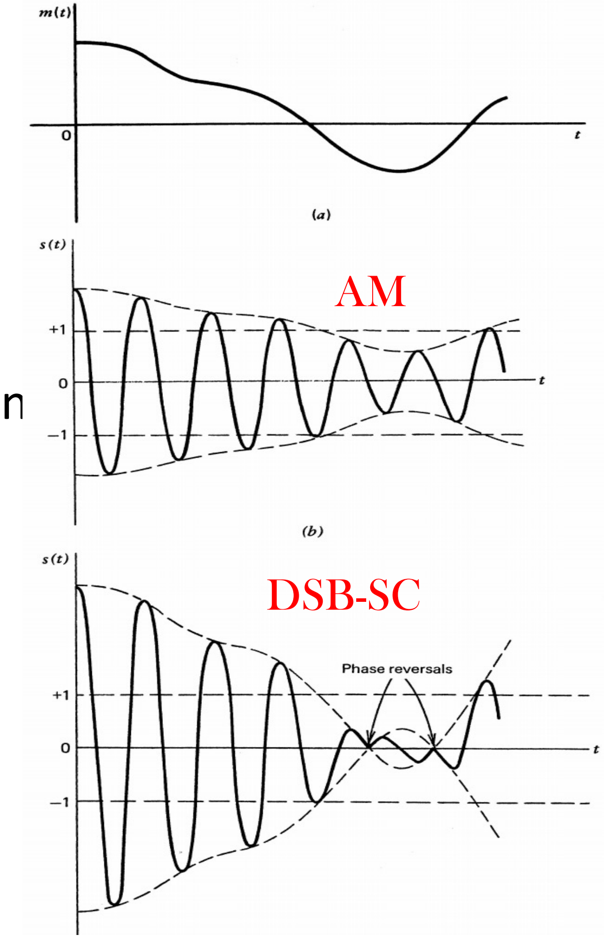
- Several modifications of the basic AM exist, which aim to improve the bandwidth and power efficiency of the scheme
- However, they also increase the system complexity
- DSB-SC modulation eliminates the carrier – improves power efficiency
- DSB-SC signal is generated by multiplying the carrier and information:

DSB-SC signal

$$y(t) = A_c\, m(t) \cos(2\pi f_c t)$$

DSB-SC signal

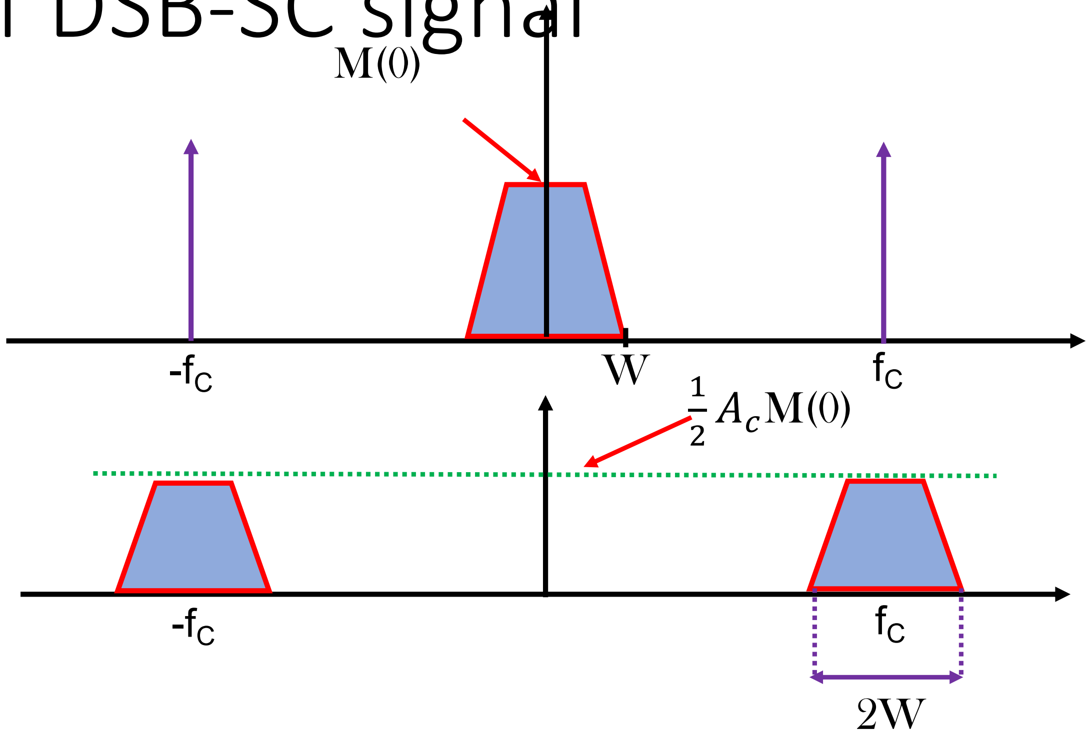




- DSB-SC can be generated using simple product modulator
- DSB-SC carrier undergoes a phase reversal when the information sign
  m(t) crosses zero
- Its envelop is thus different than the modulating signal
- The Fourier transform of $y(t) = A_c\, m(t) \cos(2\pi f_c t)$ is:

$$Y(f) = \frac{1}{2} A_c [M(f - f_c) + M(f + f_c)]$$

Spectrum of DSB-SC signal



- Modulation shifts baseband spectrum by $\pm f_c$
- There is amplitude scaling
- Bandwidth requirement is the same as for AM i.e.: 2W

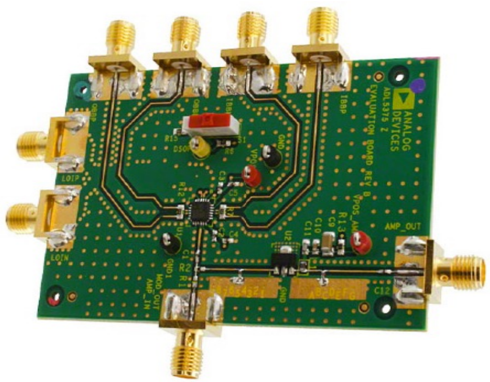
Generation of the DSB-SC Signal

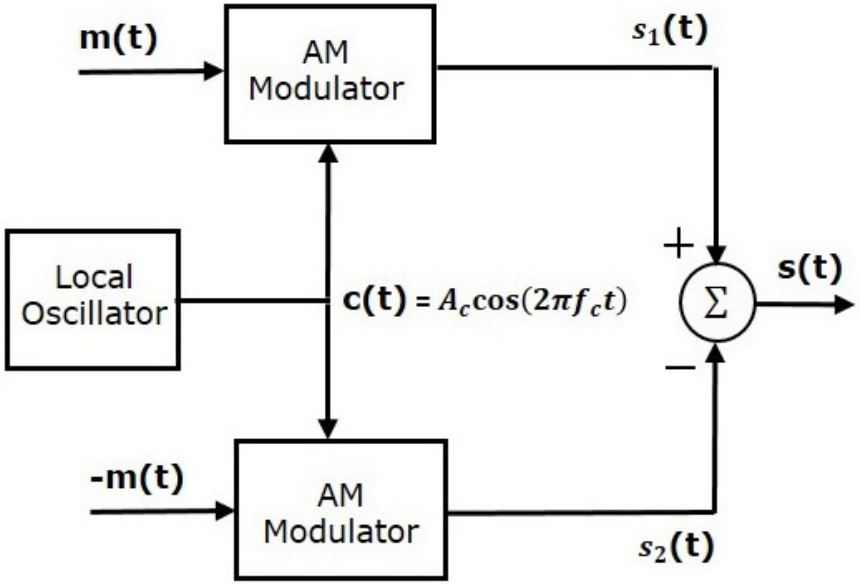
- DSB-SC can also be generated using 2 identical AM modulators in balanced configuration

➢ Summing the outputs of each supresses the carrier

➢ Mathematically, identical to a single product modulator

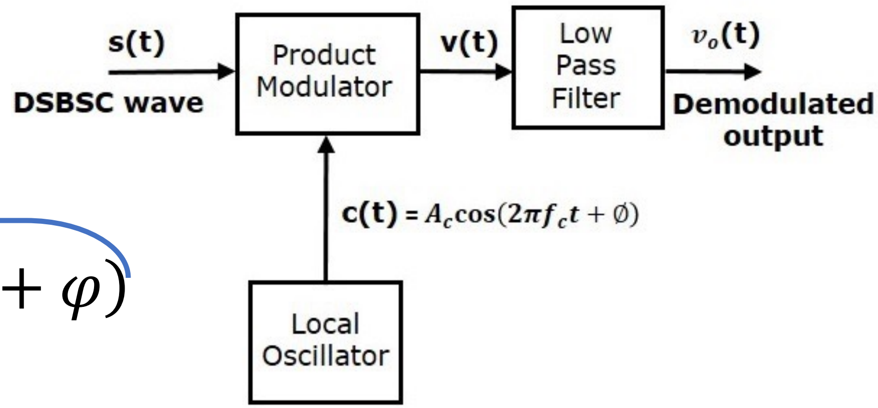
➢ However, it proves to be more stable in practice





Demodulation of DSB-SC signal

- The baseband signal m(t) can be recovered by multiplying the DSC-SC wave by a local sinusoidal signal and passing the product through a low-pass filter
- This process is called a **coherent demodulation**



$$v(t) = A'_c\, cos(2\pi f_c t + \varphi) \cdot \; s(t)$$

$$= A'_c \cdot cos(2\pi f_c t) \cdot A_c \cdot m(t) \cdot cos(2\pi f_c t + \varphi)$$

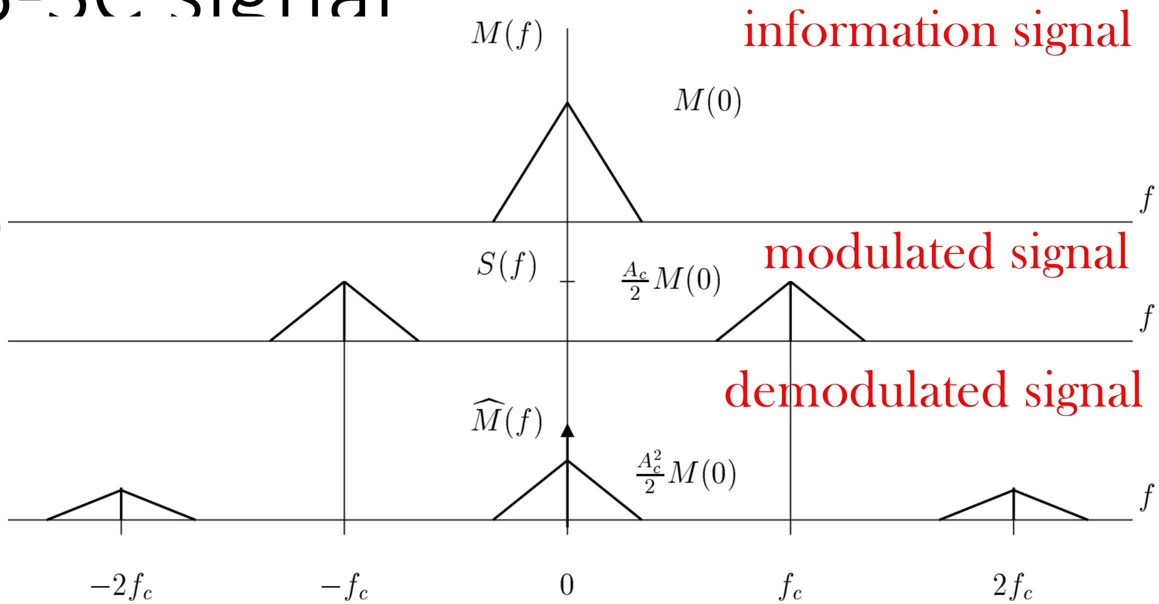
$$= \frac{1}{2} A'_c A_c m(t) \cdot cos(4\pi f_c t + \varphi) + \frac{1}{2} A'_c A_c m(t) \cdot cos(\varphi)$$

$$\boxed{\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}$$

Demodulation of DSB-SC signal

$$v(t) = \frac{1}{2}A'_c A_c m(t) \cdot cos(4\pi f_c t + \varphi)$$

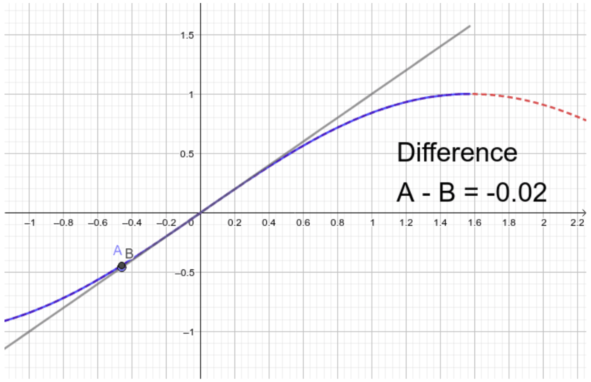
$$\boxed{+\frac{1}{2}A'_c A_c m(t) \cdot cos(\varphi)}$$

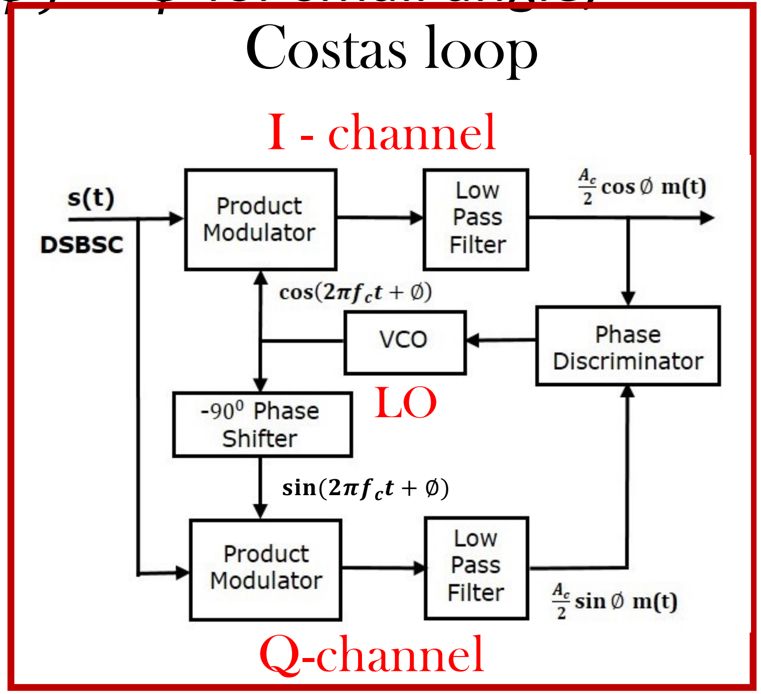


- The first term has a frequency of $2f_c$, which can be easily filtered out
- The second term is proportional to m(t), but depends on the phase error
- The power of m(t) is max. for $\varphi = 0$ and min. for $\varphi = \pm\frac{\pi}{2}$ (quadrature null effect)
- If φ is const. over time – there is no distortion to m(t)

DSB-SC Demodulation: Costas Receiver

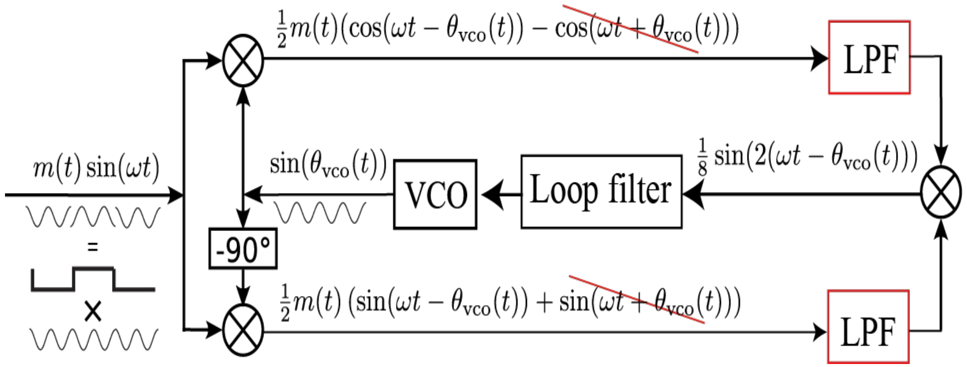
- DSB-SC signal split into 2, each mixed with a different LO signal
- Cosine path is the in-phase or I-channel, sine is quadrature or Q-channel
- Phase discriminator determines the phase drift: $\sin(\varphi) \approx \varphi$ for small angle, positive in 1 direction, negative in the other



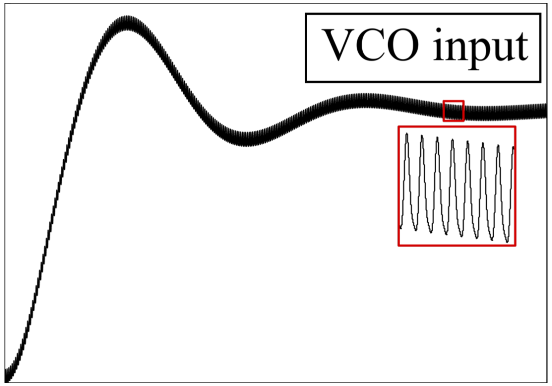


➢ Voltage controlled oscillator (VCO) driven by the output of phase discriminator

Costas Loop



Costas loop before synchronization

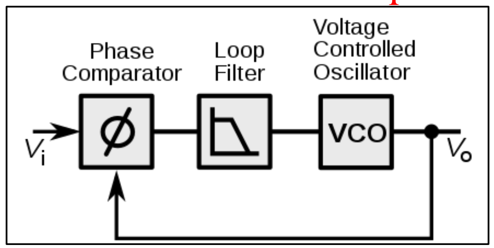


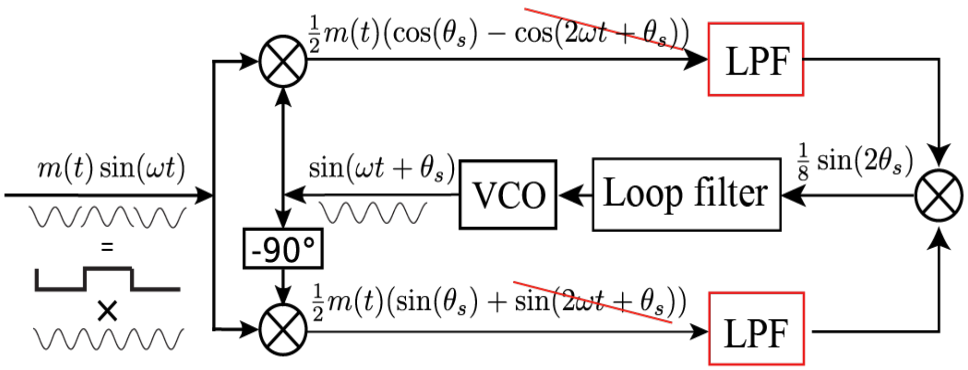
VCO input during synchronization

$$\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

Phase locked loop



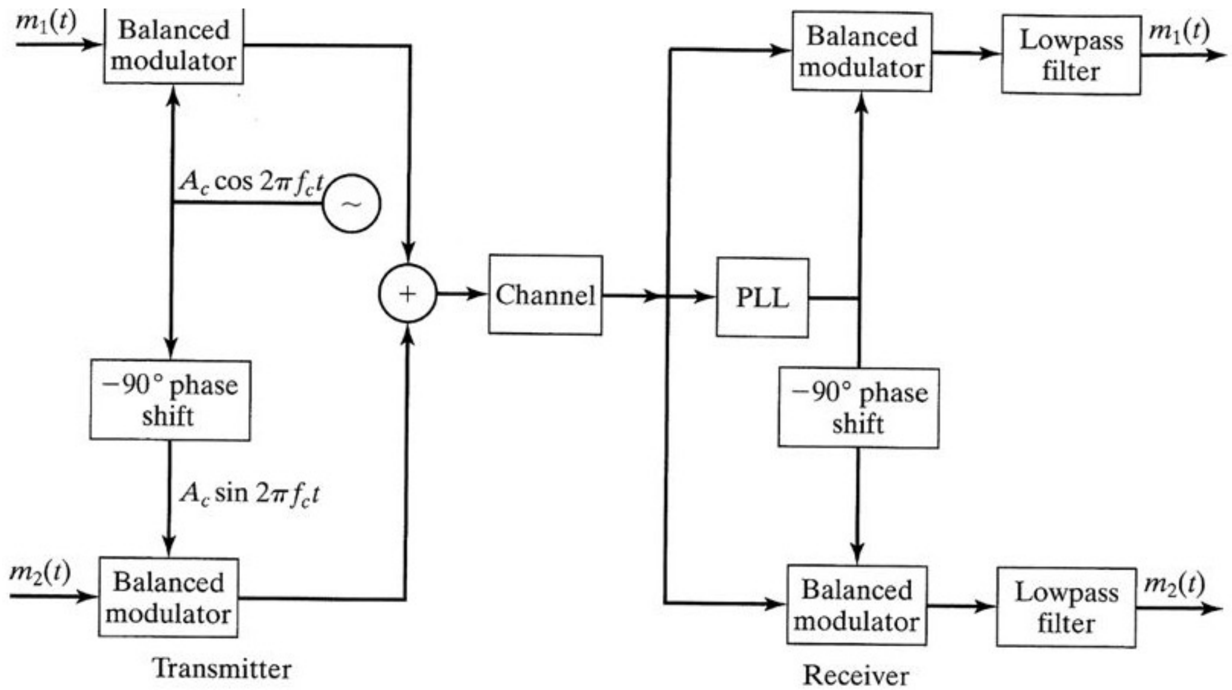


Costas loop after synchronization

- Costas loop error voltage is $\sin 2(\theta_s - \theta_{VCO})$ as compared to $\sin(\theta_s - \theta_{VCO})$ for PLLs
- This translates to double the sensitivity

Quadrature carrier multiplexing

- The quadrature null effect can be used to multiplex 2 different information signals on the same carrier (2x spectral utilisation)
- Phase and frequency need to be precisely controlled to avoid any cross-talk



What have we learnt?

- Phasor representation of signals
- AM modulator
- Demodulation of AM signals
- Advantages and disadvantages of AM
- Double sideband – suppressed carrier modulation
  - Spectrum
  - Generation
  - Coherent demodulation
- Costas receiver and quadrature carrier multiplexing