Information and time and frequency representation of signals

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WHAT HAVE WE LEARNT ?

- What is information and how we measure it
- Entropy
- Components of a communication system
- Signals: time and frequency domain
- Fourier coefficients: constructing a square wave
- Spectrum
- Aperiodic signals
- Energy and power signals

IMPORTANCE OF TELECOMS

- Telecommunications is the exchange of information over large distances…
- Information is the corner stone on our social and political life, economy etc.
- Access to it helps us safeguard our freedom (reason why dictatorships often block the citizens' access to Internet)
- In last year, telecommunication systems allowed us to continue to work, learn and meet with people, despite the pandemic

WHAT IS INFORMATION?

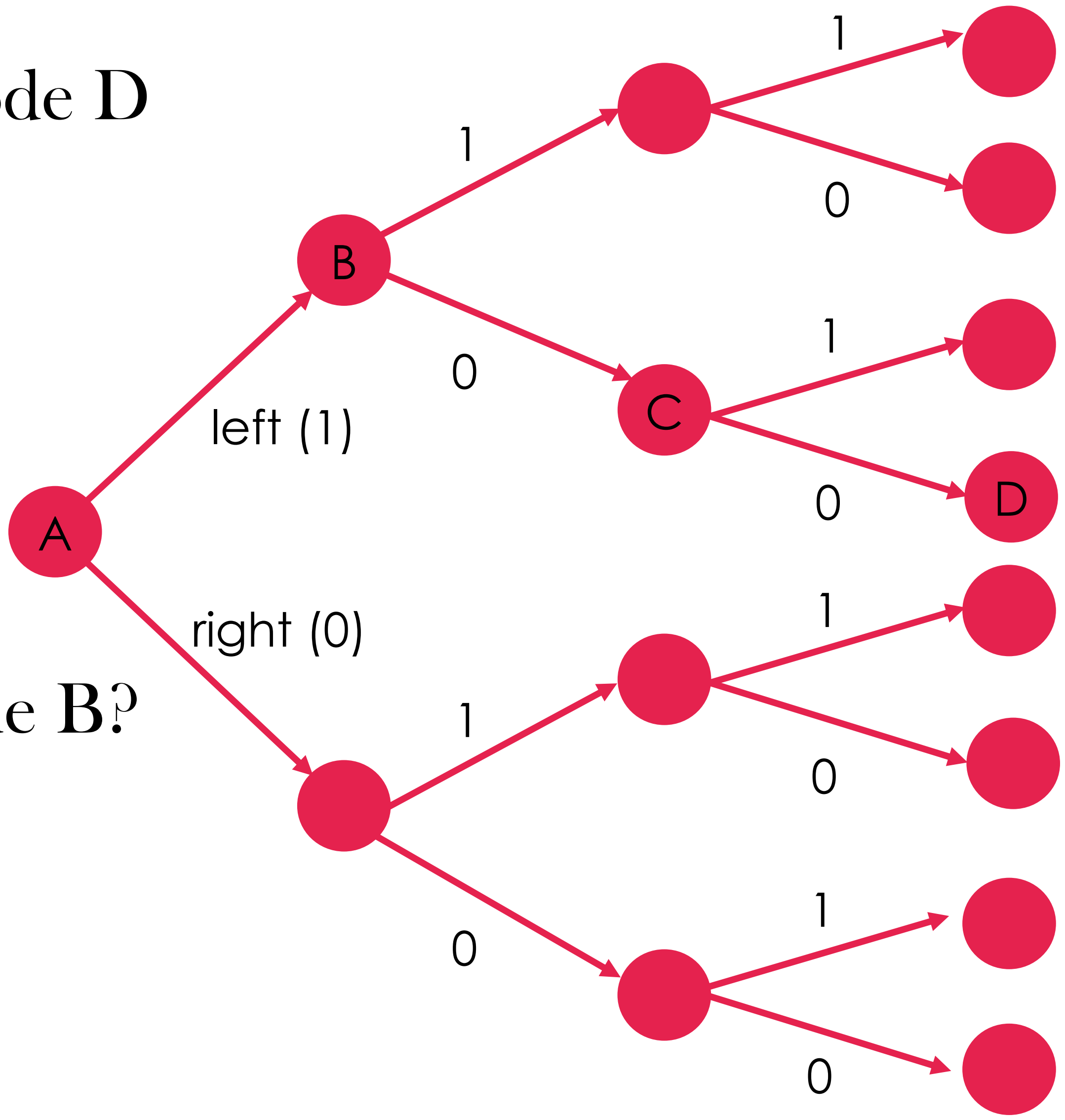
- Wikipedia: ”**Information is any entity or form that provides the answer to a <u>question</u> of some kind or resolves <u>uncertainty”</u>**
- It is thus related to <u>data</u> and <u>knowledge</u>:
  - ✓ Data represents values attributed to parameters (e.g. water temperature)
  - ✓ Knowledge signifies understanding of real things or abstract concepts
  - ✓ Data consists of signal (useful) and noise (useless)
- Information is extracted from data by separating signal from noise
  - ✓ Eyes and ears (e.g. Where is Wally?)
  - ✓ In telecommunications e.g. wireless communications…

WHAT IS INFORMATION?

- TechTarget: **"Information is stimuli that has meaning in some context for its receiver."**
- Information is not "1010110" or "words". That is how we express it, information is the concept, value behind the words.
- The aim of a communication system is to transfer information efficiently & reliably.
- The amount of "information" in a message can be quantified.
- The measure of information is a bit – smallest unit allowing you to chose between 2 equally probable alternatives:
  - ✓ yes/no
  - ✓ left/right
  - ✓ up/down…

FINDING A ROUTE: BIT BY BIT

- You are travelling from node A to some node D
- You have no knowledge of the route
- At each fork you arrive at you are given direction: left or right - equally probable
- Assign binary digits: left=1, right=0
- How many digits do you need to get to node B?
- How many routes can you choose from?
- What about node C?
- D?



FINDING A ROUTE: BIT BY BIT

- To choose from 2 equally probable alternatives we need 1 bit of information:

1 or 0 => $\mathbf{2 = 2^1}$

- To choose from 4 equally probable alternatives we need 2 bits:

00 or 01 or 10 or 11 => $\mathbf{2 \cdot 2 = 2^2 = 4}$

- To choose from 8 equally probable alternatives we need 3 bits:

000, 001, 010, 011, 100, 101, 110, 111 => $\mathbf{2 \cdot 2 \cdot 2 = 2^3 = 8}$

- Or more general: given <u>n bits of information</u> you can choose from <u>m different routes</u>:

$$\underline{\boldsymbol{m = 2^n}}$$

EXAMPLE

Question: What is the age of this person?

1. Possible answer: ≤40 or >40?
   - ✓ Only 2 choices, thus the answer gives little information
2. Possible answers:

[0:10]	[11:20]	[21: 30]	[31:40]
[41:50]	[51:60]	[61:70]	[>71]

   - ✓ Now there are 8 age brackets, giving you much more information!

If you use binary numbers to represent the answers, how many bits do you need to encode the message in each case?

INFORMATION AND ENTROPY

- So, how can we define and quantify information?
- According to Claude Shannon (1916-2001), the father of information theory:

if

Entropy is a measure of the uncertainty associated with a random variable

then

Information is a measure of a reduction of the entropy of a random variable

WHAT DOES IT MEAN???

- Weaver explains Shannon's information:
  - ✓ Information is a measure of one's freedom of choice in selecting a message.
  - ✓ The greater this freedom of choice, the greater the information, the greater is the uncertainty that the message actually selected is some particular one.
  - ✓ <u>Greater freedom of choice, greater uncertainty & greater information go hand in hand</u>.[1]

[1] C.E.Shannon, W.Weaver, "The Mathematical Theory of Communication"

EXAMPLE REVISITED

If we know nothing about the age of a person

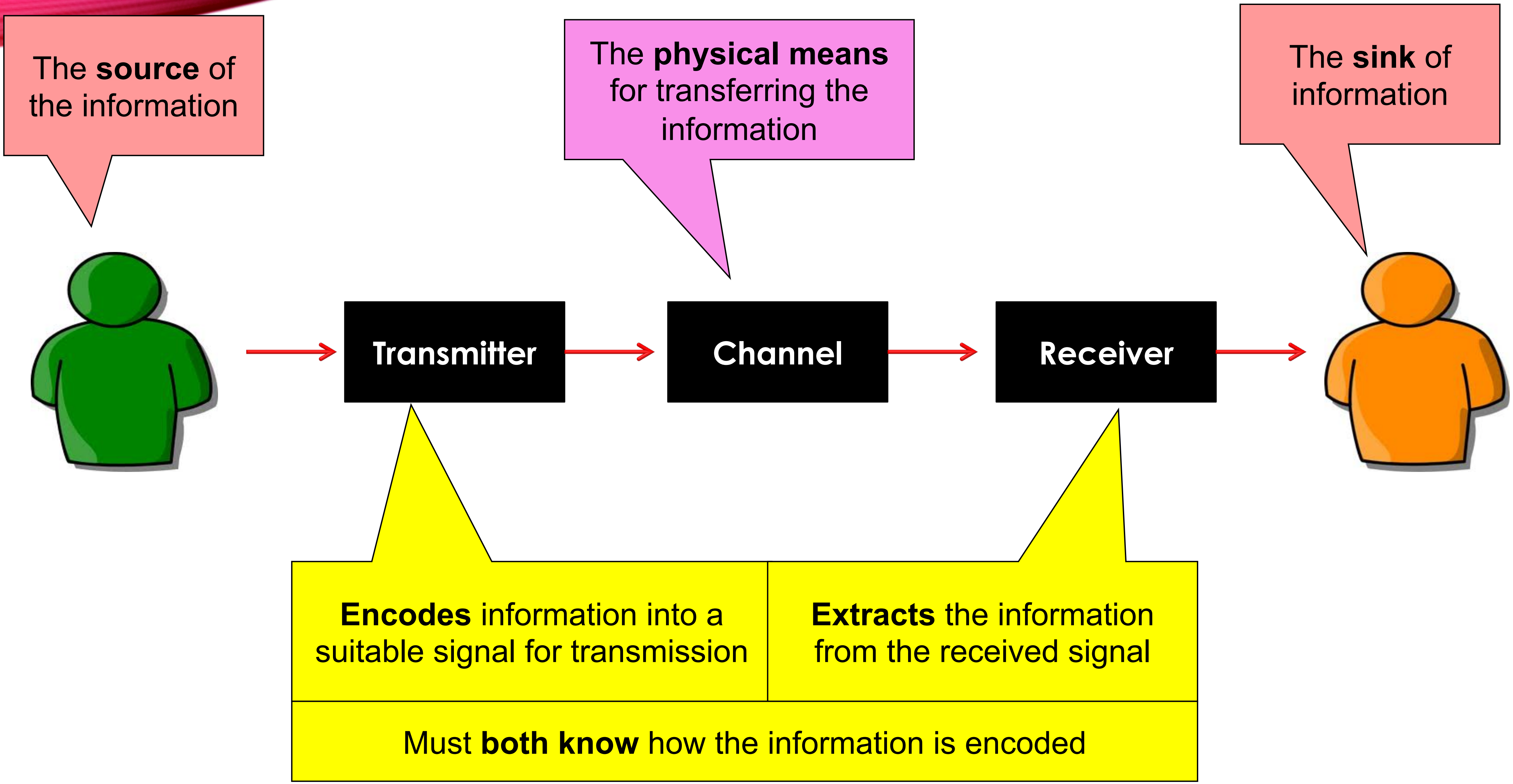
1. Being told that the person is older/younger than 40 could give us max.1 bit of information*

2. Identifying one of 8 brackets, could give us up to 3 bits of information*

Case 2 has a greater freedom of choice, thus answer provides greater information

* Actual amount of information depends on the probability that a person is of a particular age

A COMMUNICATION SYSTEM



TELECOMMUNICATION PROCESS

Telecommunication is the exchange of information over significant distance

Telecommunications requires:

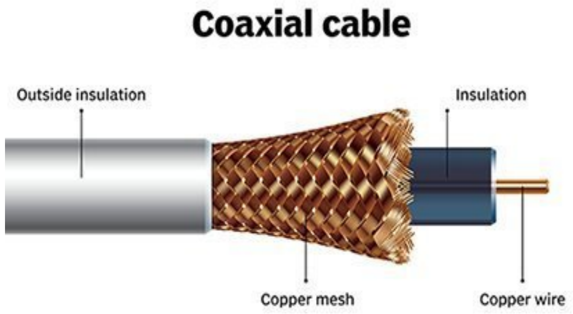
- Message composition
- Message encoding (e.g.: into digital data, written text, speech, pictures etc).
- Transmission of the encoded message using a specific channel or medium
- Reception of signals and reassembling of the encoded message
- Decoding of the reassembled encoded message.
- Interpretation of the presumed original message

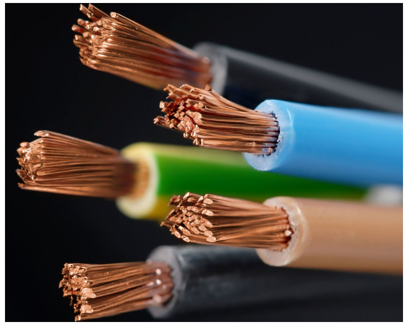
TELECOMMUNICATION CHANNEL

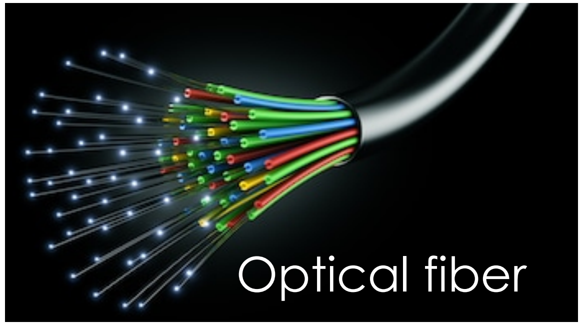
Use electrical signal or electromagnetic wave for data transmission

Can be classified as bounded or guided:

- Copper wire (electrical)
- Coaxial cable (electromagnetic)
- Optical fibre (optical)

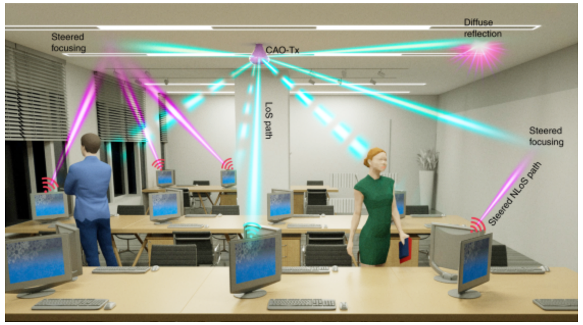




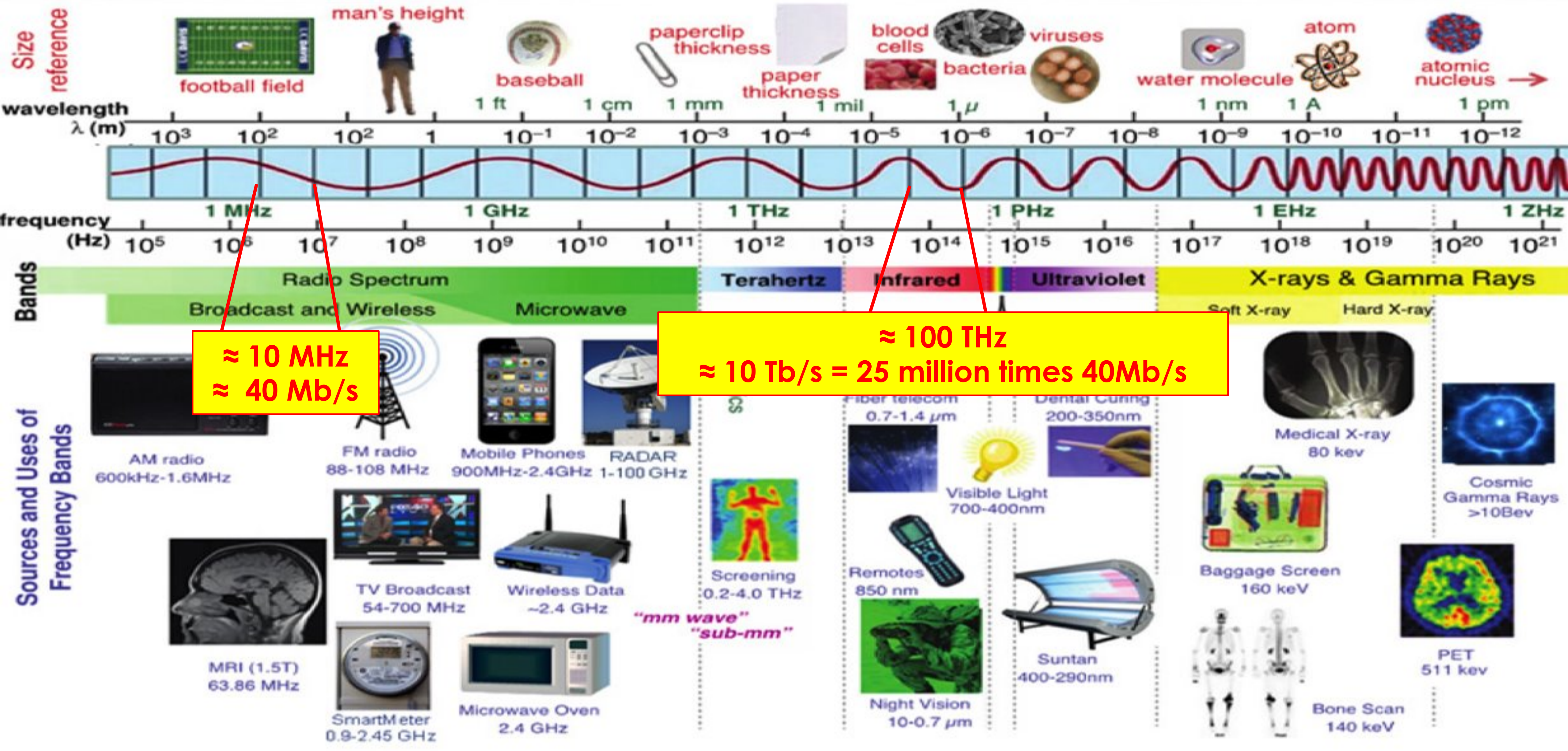


Unguided transmission (unbounded):

- Radio and microwave (electromagnetic)
- Optical (visual displays, LiFi etc.)

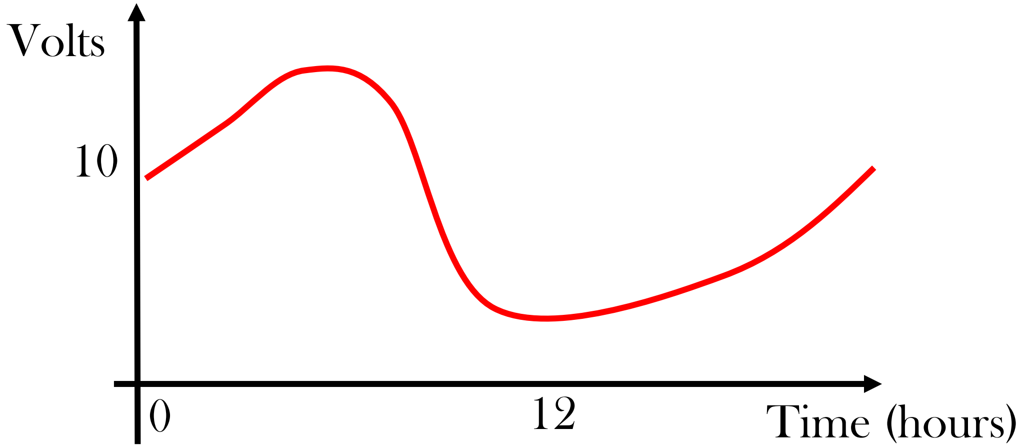


ELECTROMAGNETIC SPECTRUM

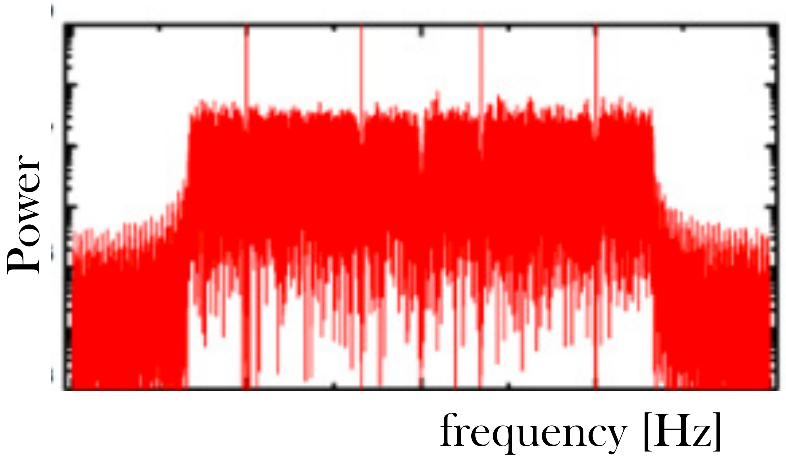


SIGNALS: TIME & FREQUENCY DOMAIN

- When we express how the signal changes with time, we define the signal in the time domain



- We can also look at what frequencies the signal consist of i.e. define it in the frequency domain



- The faster the signal changes – the higher the frequency of a signal
- The more complicated the shape, the higher the no. of frequency components

FOURIER COEFFICIENTS

- Any periodic signal can be generated by summing up sine waves with different amplitudes, frequencies and phases

$$s(t) = A_0 + \sum_{n=1}^{\infty} A_n \cdot \sin(2\pi nft) + \sum_{n=1}^{\infty} B_n \cdot \cos(2\pi nft)$$

$$A_n = \frac{2}{T}\int_0^T s(t)\cos(\frac{2n\pi t}{T})dt, \quad n \geq 0$$

$$B_n = \frac{2}{T}\int_0^T s(t)\sin(\frac{2n\pi t}{T})dt, \quad n \geq 1$$

Fourier series:

$$s(t) = \sum_{n=-N}^{N} c_n e^{-i\frac{2\pi n}{T}t}$$

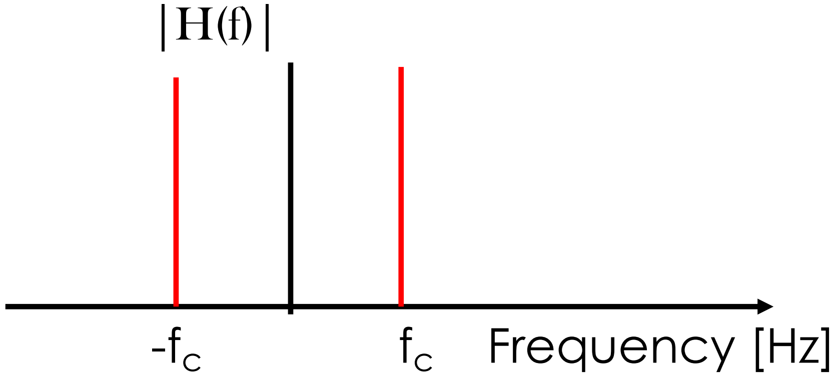
$$c_0 = \frac{a_0}{2}, \quad c_n = \frac{a_n - ib_n}{2}, \quad c_{-n} = \frac{a_n + ib_n}{2}.$$

- The coefficients A_n and B_n tell us what is the contribution of each frequency component to the overall signal

- As a result, they give us the frequency composition of the signal

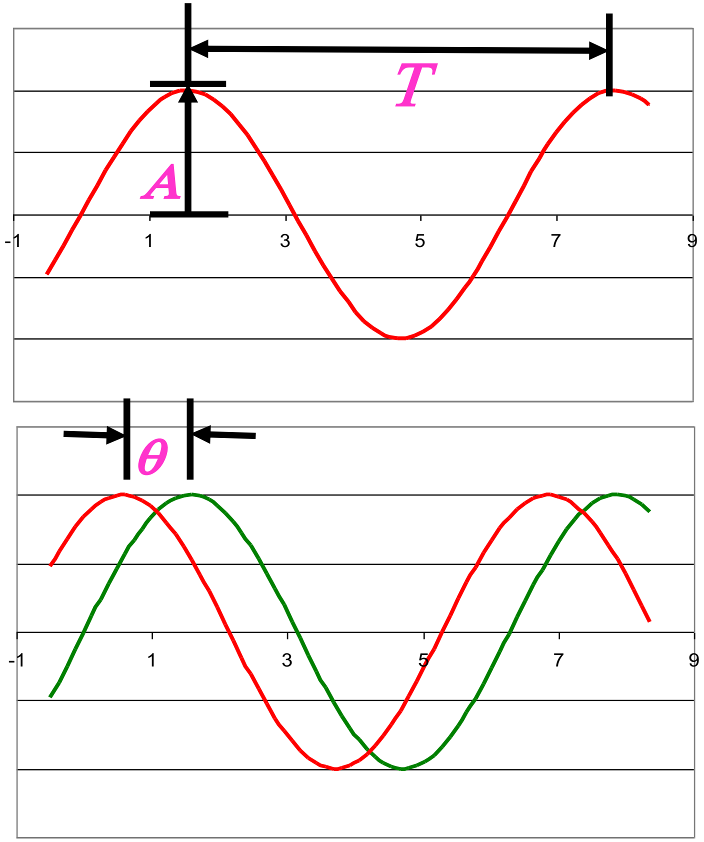
TIME AND FREQUENCY DOMAIN

- The simplest signal (at non-zero frequency) is a sine wave
- It consist of only 1 frequency component



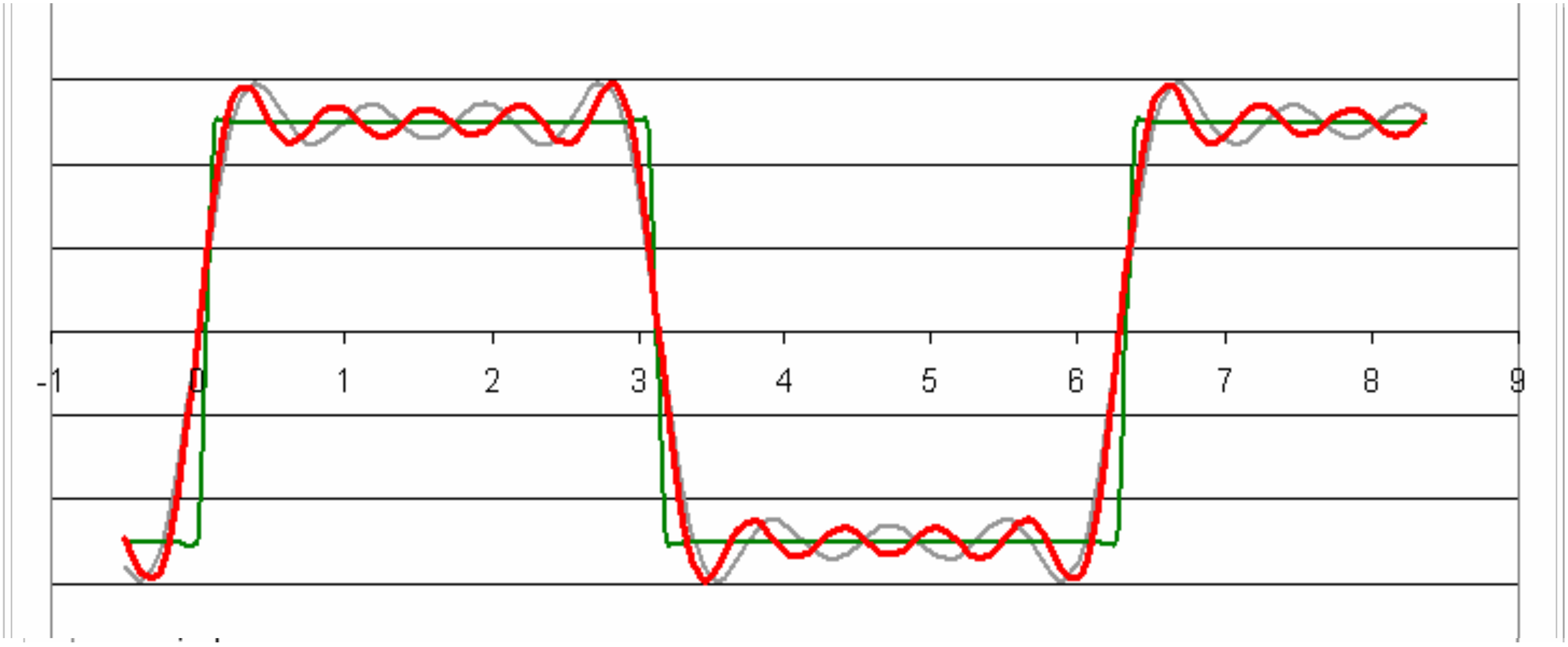
Spectrum of an ideal sinewave

T = period
A = Amplitude
f = frequency (1/T)
θ= phase shift



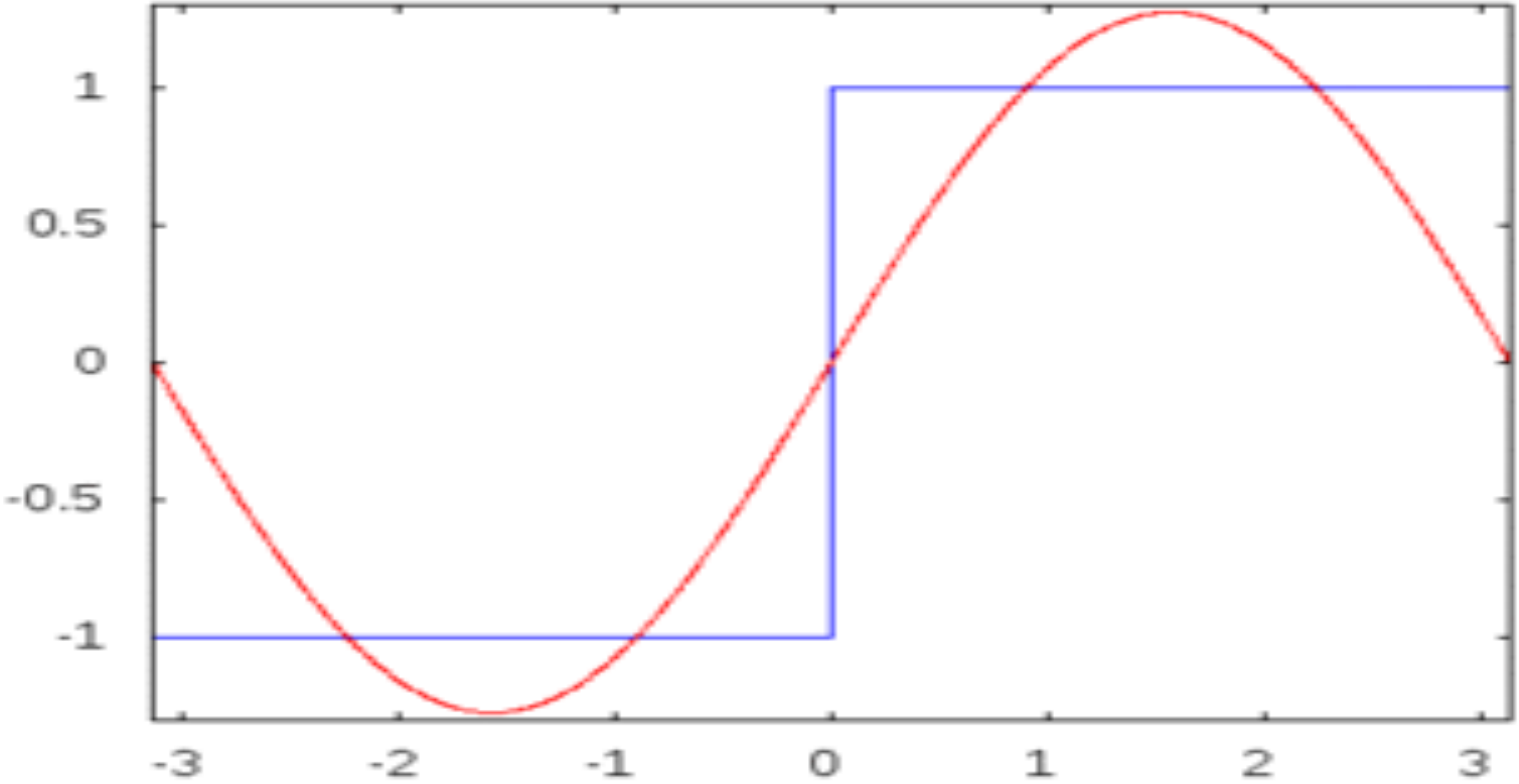
CONSTRUCTING A SQUARE WAVE

Square wave

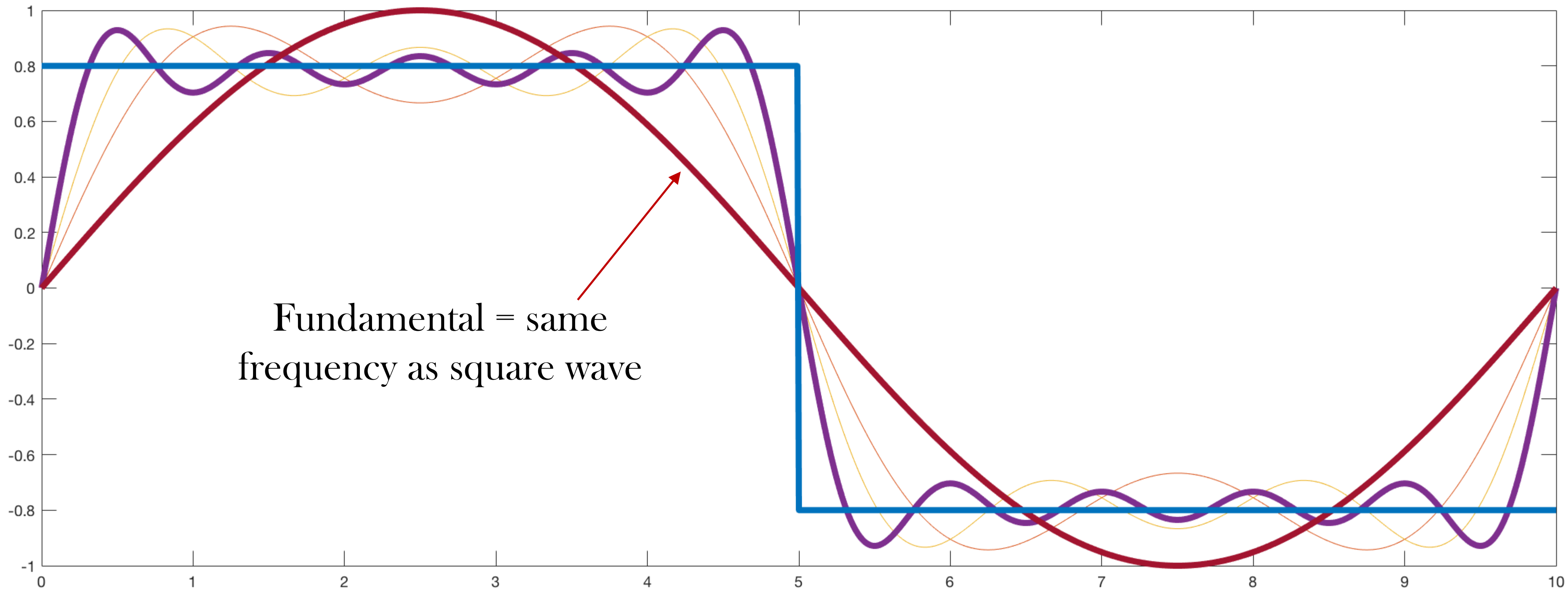


sin(x)+sin(3x)/3+sin(5x)/5+sin(7x)/7 sin(x)+sin(3x)/3+sin(5x)/5+sin(7x)/7+sin(9x)/9

SIGNALS: CONSTRUCTING A SQUARE WAVE



SIGNALS: CONSTRUCTING A SQUARE WAVE



- To achieve a vertical line you need an infinite no. of harmonics!
- Such transitions (0 s rise time) can't be realised! Can only be approximated!

SPECTRUM

- The spectrum of a signals shows the amplitude and phase of all the frequency components making up the signal
- Observing signals in time and frequency domains allows us to see different characteristics of the signal

APERIODIC SIGNALS

- Aperiodic signal can be considered a periodic signal with T→∞
- To find frequency components of an aperiodic signal we use the Fourier transform

The Fourier Transform .com

$$\mathscr{F}\{g(t)\} = G(f) = \int_{-\infty}^{\infty} g(t)e^{-i2\pi ft}\,dt$$

$$\mathscr{F}^{-1}\{G(f)\} = g(t) = \int_{-\infty}^{\infty} G(f)e^{i2\pi ft}\,df$$

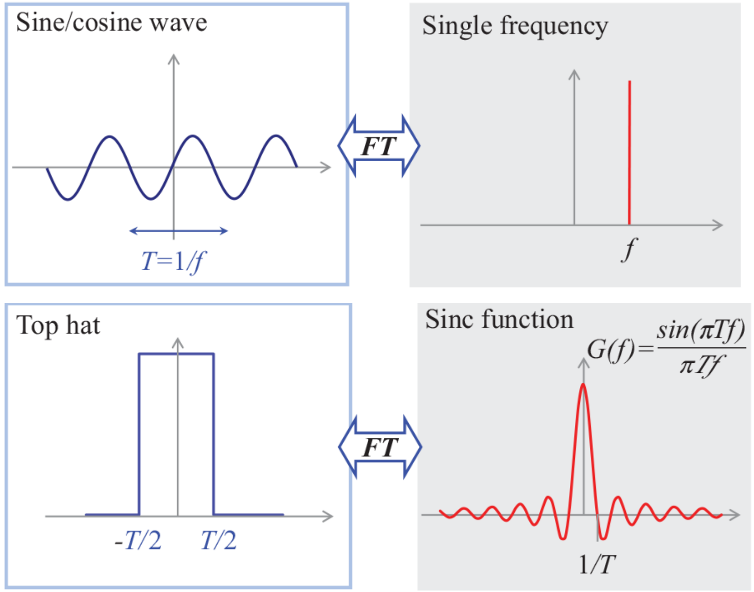
Fourier series:

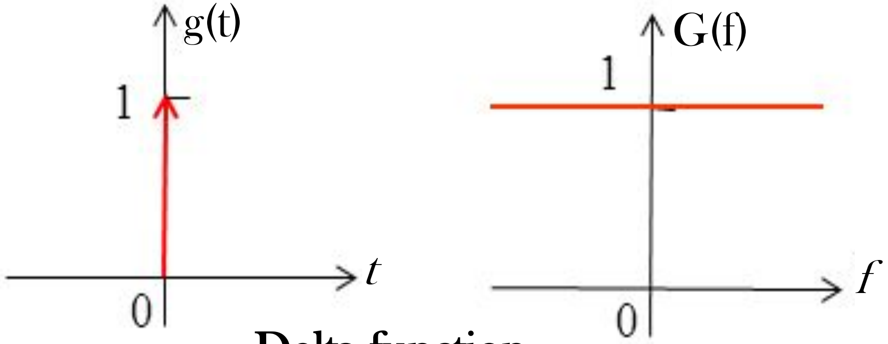
$$f(t) = \sum_{n=-N}^{N} c_n e^{-i\frac{2\pi n}{T}t}$$

- The frequency components of the periodic signal are separated by the fundamental frequency (1/T) i.e.: its spectrum is discrete
- For aperiodic signals, the T is infinite, so the spectrum is continuous

RELATIONSHIP BETWEEN TIME & FREQUENCY

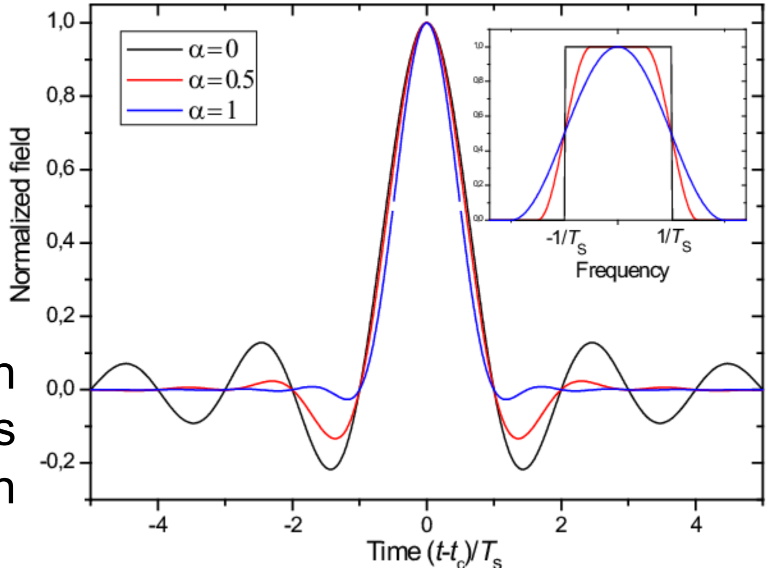
- There is an inverse relationship between the time and frequency domain representation of a signal
- We may specify an arbitrary function of time or frequency, but not both simultaneously
- Signals limited in time domain are infinite in frequency domain and vice versa





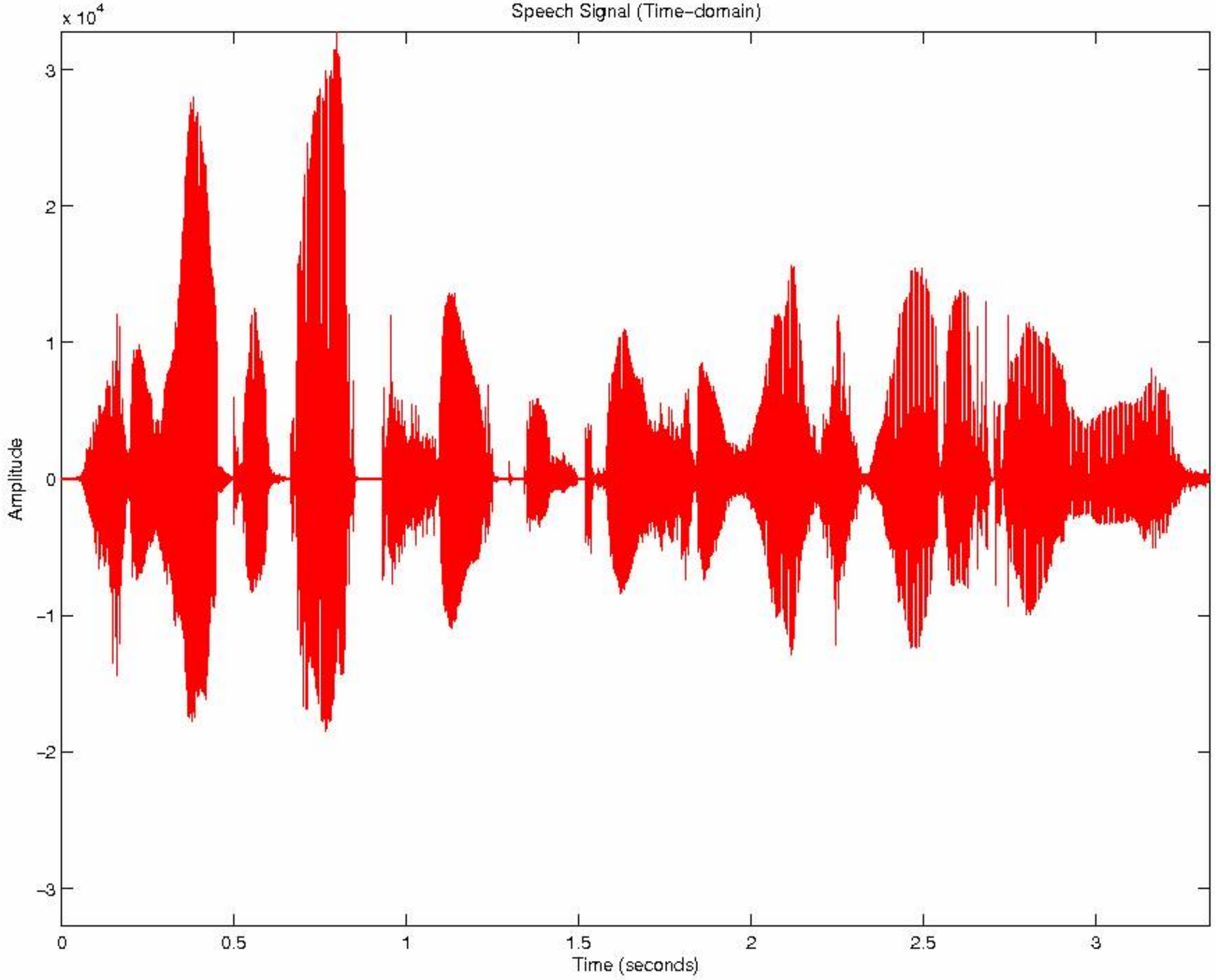
Delta function

Nyquist pulses with different roll-off factors in the time domain



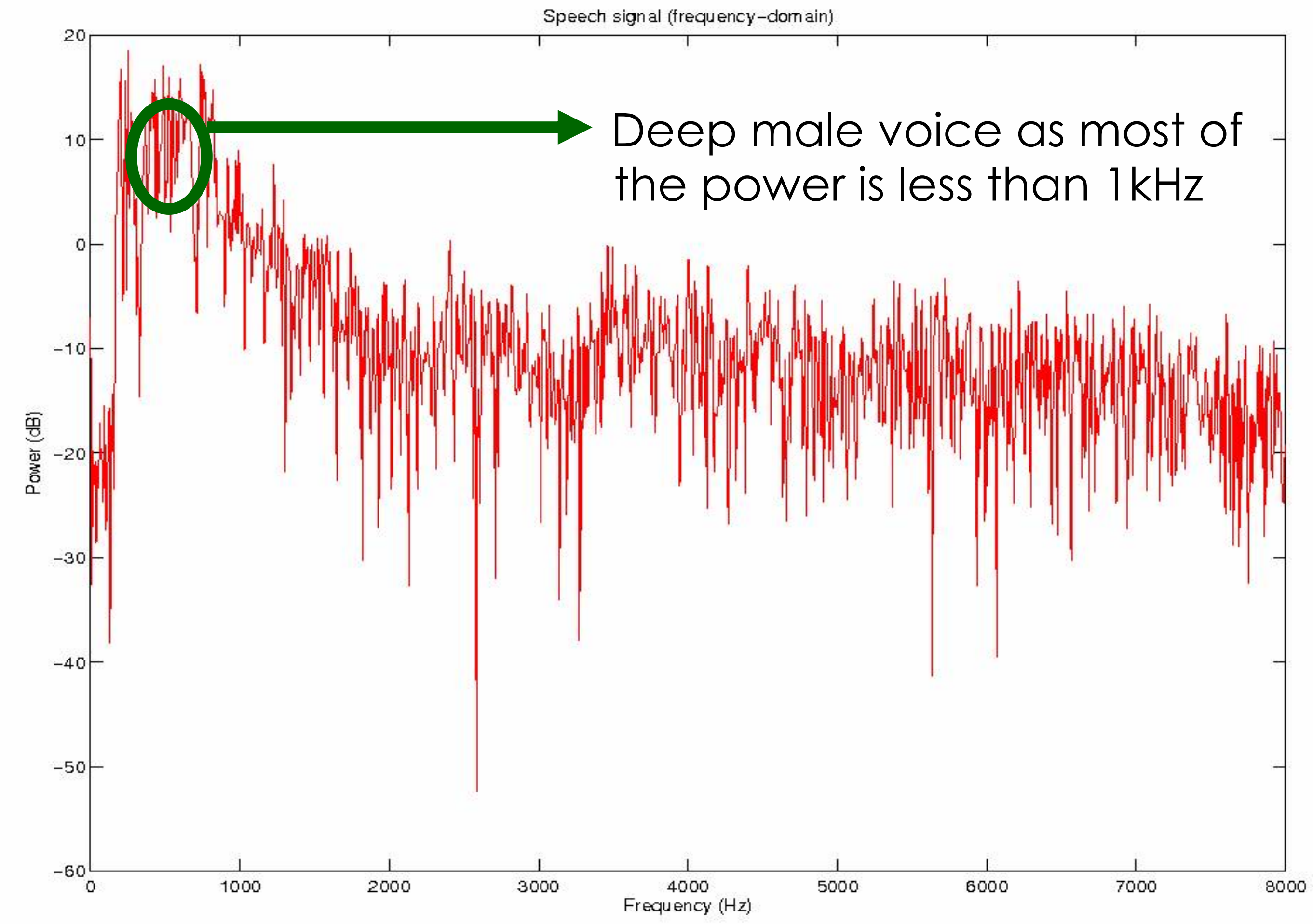
SPOKEN WORD: TIME DOMAIN

- Here we can clearly see the words spoken e.g.: how quickly, how loud etc.

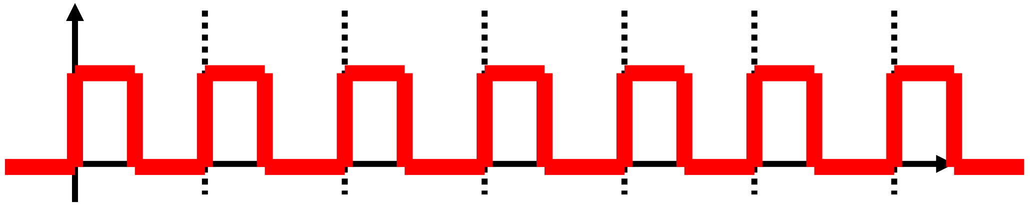


SPOKEN WORD: FREQUENCY DOMAIN

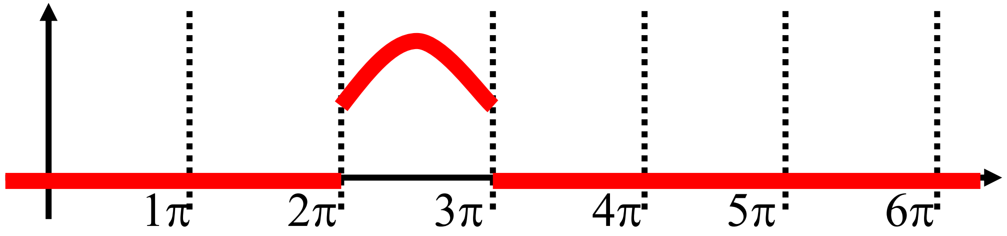
- Gives information about the pitch (frequency) of the voice



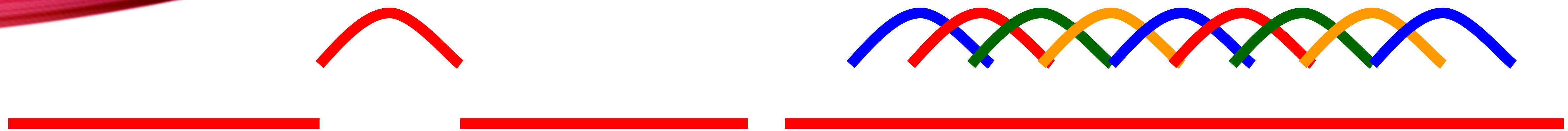
ENERGY AND POWER SIGNALS



- Periodic waveforms exist for infinite time
- Their energy keeps building up, thus theoretically, such signals have infinite energy
- Thus, we call them power waveforms and quote their power [W] (power = energy over time e.g. signal period)
- Other signals exist only for a limited time
- Such signals have 0 power (finite energy divided by infinite time)
- Such waveforms are defined as energy signals



POWER AND ENERGY



- In digital communication systems we produce a pulse to represent data e.g. 1 pulse = 1 bit
- Such pulse is of finite duration and can only be defined through its energy
- A system that sends out a continues stream of pulses may be to be a power signal
- Each waveform/pulse is energy-defined, but the combination of all the pulses corresponds to a power or at least an average power level.

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