Amplitude modulation

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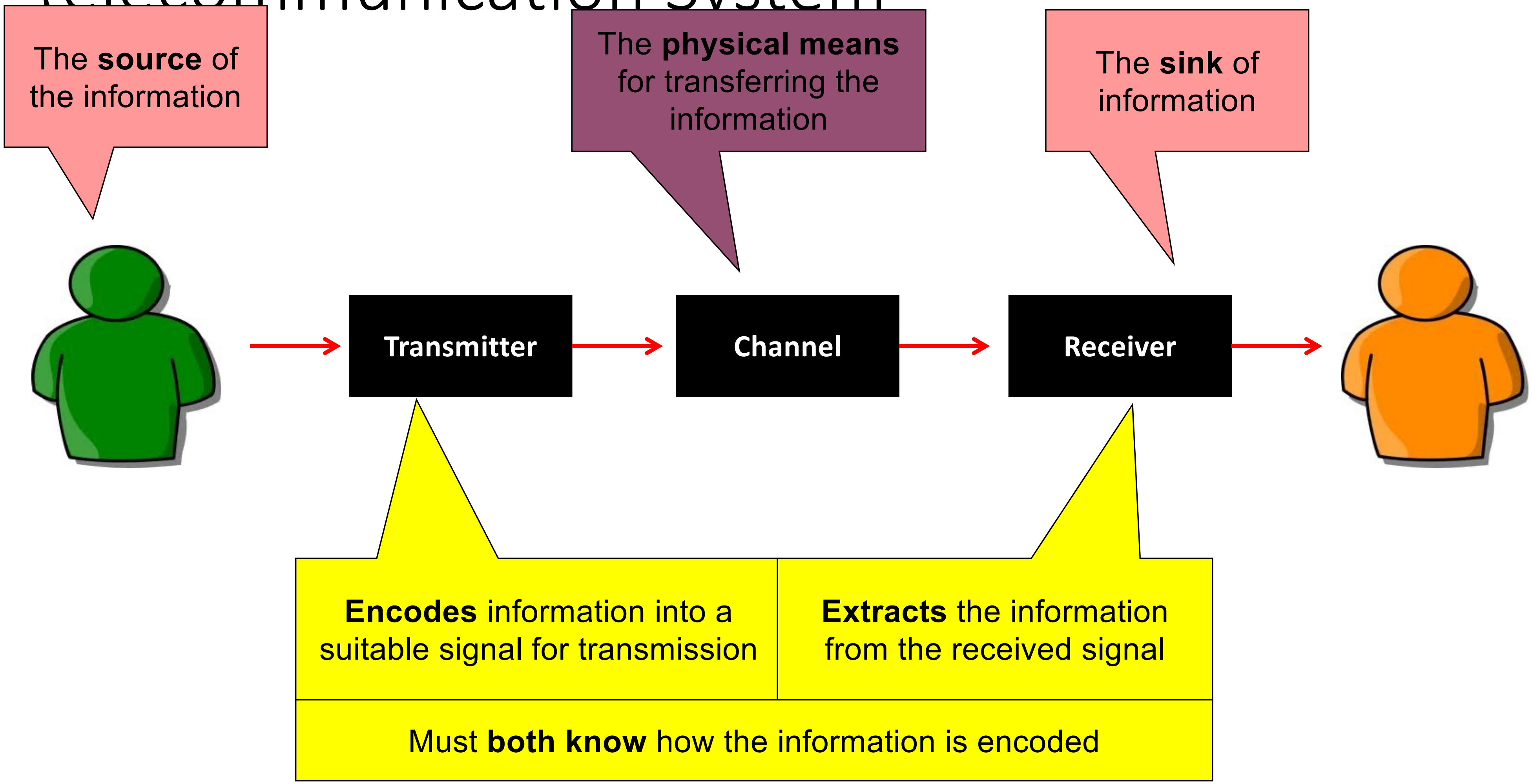
previous lecture

- Digital signal and PRBS
- Eye diagram
- Effect of filtering on data signals
- The impact of phase on a signal

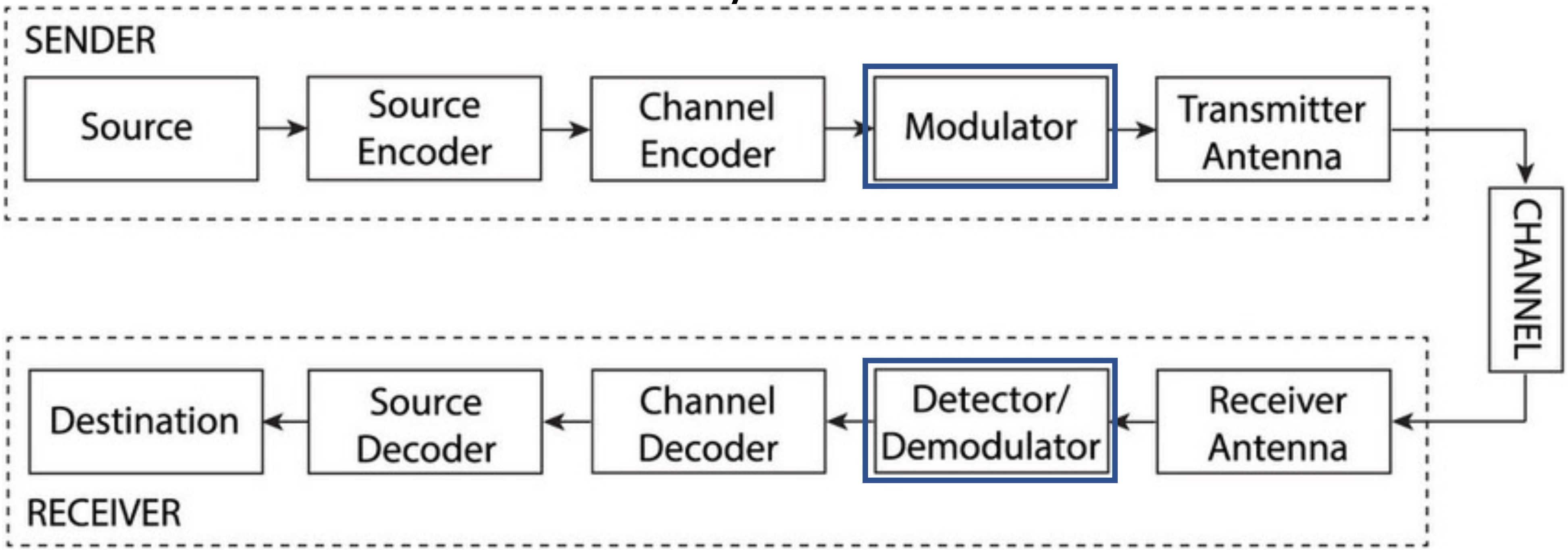
Today's lecture

- Modulation and modulation types
- Amplitude modulation – basics
- Spectrum of AM signals
- Bandwidth requirements
- Power of AM signal

telecommunication System



telecommunication System



- Baseband signal – signal generated by the source
- Modulation – shift in frequency in order to match the properties of the channel

Modulation

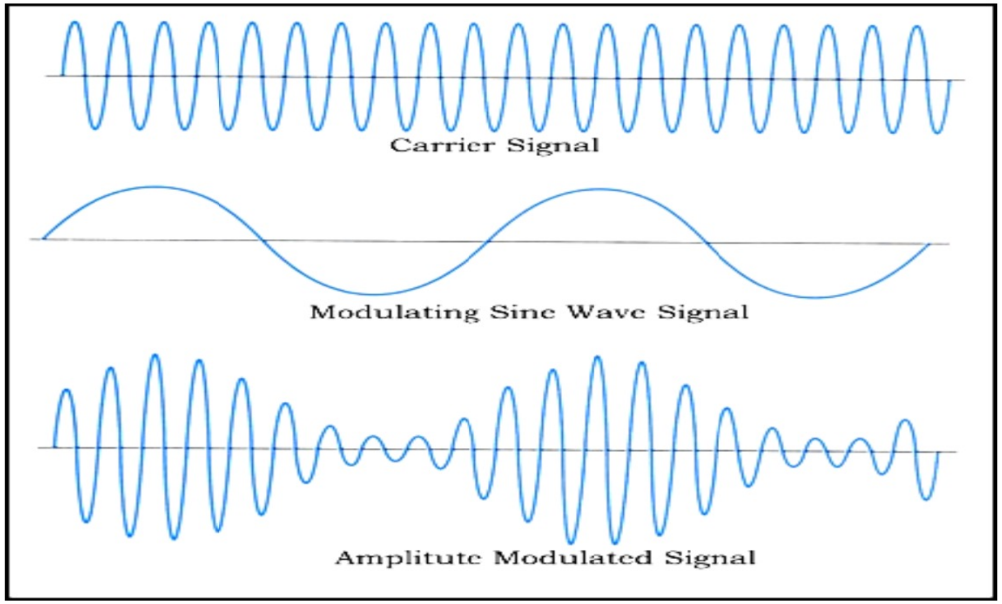
- Data may be encoded in a digital or analog form
- Modulation is a process of imprinting data on to a carrier
- Achieved by changing some characteristic of a carrier in accordance to the data
- If carrier is a sinusoidal wave it is continuous wave modulation
- Digital modulation - data to be transmitted is digital
- Analog modulation - data to be transmitted is analog
- The implementation of the transmitter and receiver is defined by the modulation scheme
- The basic modulation schemes are amplitude & angle (frequency and phase)

Modulating the Signal

- Data is usually modulated onto a sinusoidal signal/carrier:

$$s(t) = A cos(2\pi f t + \varphi)$$

- Each of the parameters of the sinewave can be modified to carry data
- Amplitude modulation changes the amplitude A of the carrier to represent data



Amplitude Modulation

- For the information signal m(t) and the carrier:

$$c(t) = A_c \cos(2\pi f_c t),$$

the AM modulated signal has the form:

AM signal

$$y(t) = A_c\,[1 + \mu \cdot m(t)]\cos(2\pi f_c t)$$

where:

$\mu = \frac{A_m}{A_c}$ - modulation index $\qquad A_m = max|m(t)|$

Amplitude Modulation

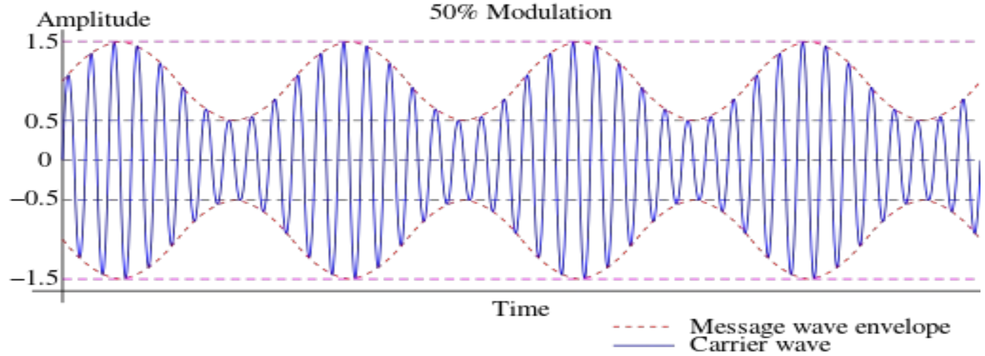
- The **<u>modulation depth or index</u> is often** expressed in %
- Assuming $A_m = max|\mathrm{m(t)}|$

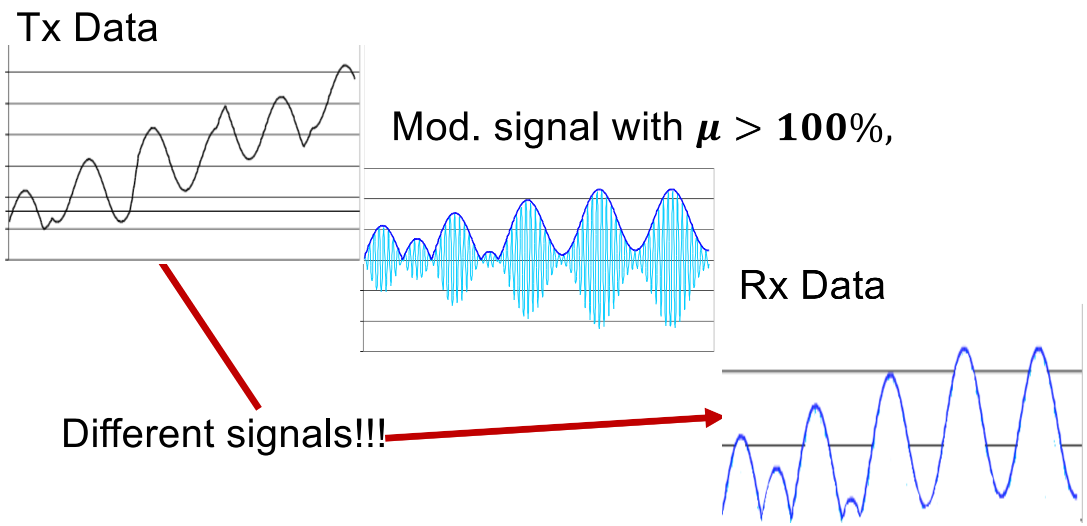
Modulation depth

$$\mu = \frac{max|\mathrm{m(t)}|}{A_c} \cdot 100\% = \frac{A_m}{A_c} \cdot 100\%$$

Amplitude Modulation

1. Carrier frequency >> max. f_m
2. Data signal constitutes the <u>envelop</u> of the modulated signal – red dashes
3. The above is true only for $\mu \leq 100\%$, $A_c[1 + \mu\, m(t)] \geq 0$





Amplitude Modulation

- If $\mu \geq 100\%$ we are dealing with overmodulated signal
- Such signal can no longer be detected using a simple envelop detector
- A more complex synchronous detection scheme needs to be used
- Thus to preserve the simplicity of the AM receiver the following needs to be met:

Modulation depth

$$\mu = \frac{\mathrm{A_m}}{\mathrm{A_c}} \leq 1 \text{ for all t}$$

spectrum of Am signal

- Let assume our modulating signal is $\mathrm{m(t)} = \mathrm{A}_m cos(2\pi \mathrm{f}_m \mathrm{t})$

- The modulated signal is then equal to:

$$y(t) = A_c[1 + \mu\, cos(2\pi f_m t)]cos(2\pi f_c t),$$

$$y(t) = A_c cos(2\pi f_c t) + A_c \mu[cos(2\pi f_m t) cos(2\pi f_c t)] =>$$

$$\boxed{\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}$$

$$y(t) = A_c cos(2\pi f_c t) + \frac{1}{2} A_c \mu[cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)]$$

spectrum of Am signal

$$y(t) = A_c cos(2\pi f_c t) + \frac{1}{2} A_c \mu[cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)]$$

$$G(f) = \Im\{\cos(2\pi At)\} = \int_{-\infty}^{\infty} \frac{e^{i2\pi At} + e^{-i2\pi At}}{2} e^{-i2\pi ft} dt$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{i2\pi At} e^{-i2\pi ft} dt + \int_{-\infty}^{\infty} e^{-i2\pi At} e^{-i2\pi ft} dt\right]$$

$$= \frac{1}{2}[\delta(f - A) + \delta(f + A)]$$

- The Fourier transform of y(t) is thus:

$$Y(f) = \frac{1}{2} A_c[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{4} \mu A_c[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]$$

$$+ \frac{1}{4} \mu A_c[\delta(f - f_c + f_m) + \delta(f + f_c - f_{m_c})]$$

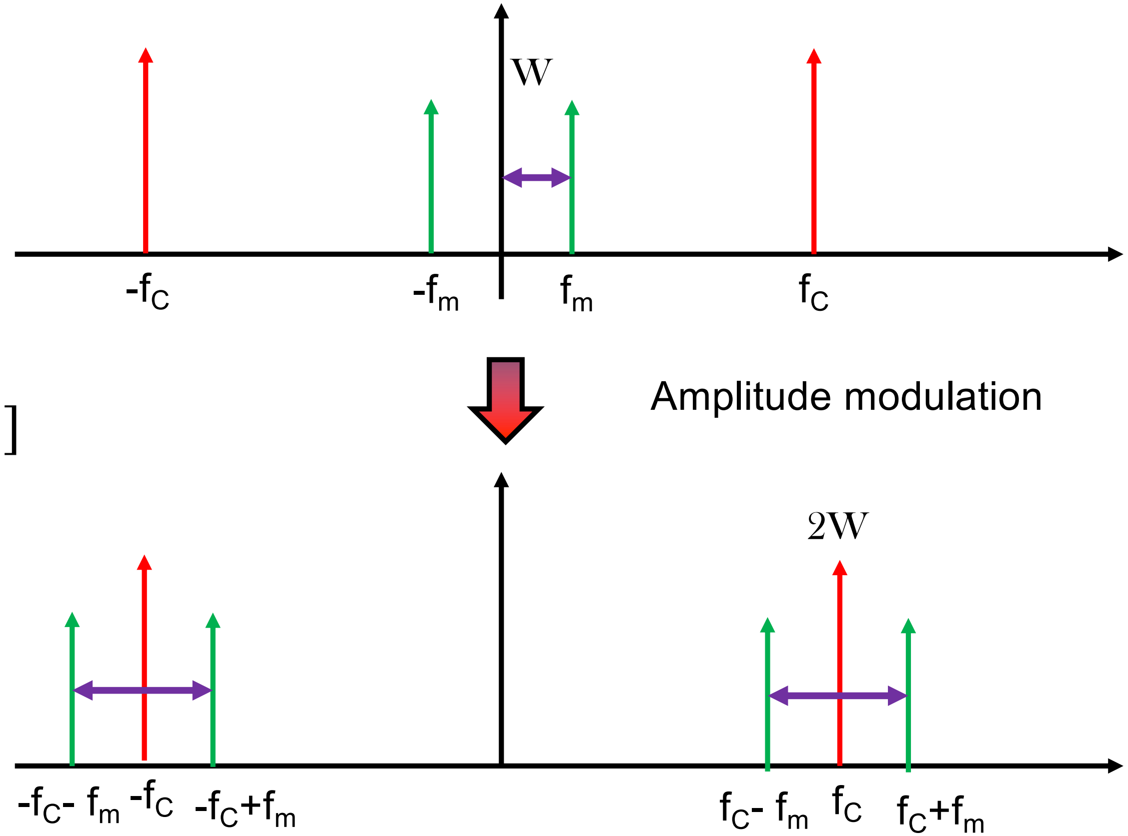
Bandwidth Requirement

$$Y(f) =$$

$$\frac{1}{2}A_c[\delta(f-f_c)+\delta(f+f_c)]$$

$$+\frac{1}{4}\mu A_c[\delta(f-f_c-f_m)+\ \delta(f+f_c+f_{m_c})]$$

$$+\frac{1}{4}\mu A_c[\delta(f-f_c+f_m)+\delta(f+f_c-f_{m_c})]$$

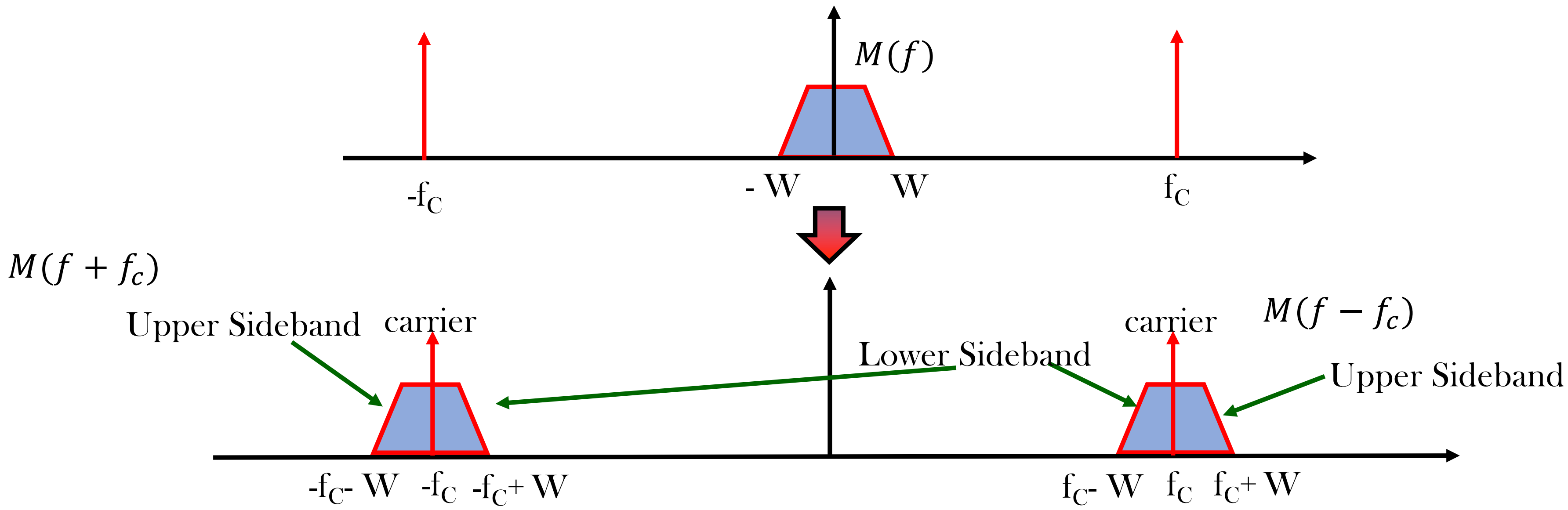


Faithful transmission of a signal at frequency f_m, requires a bandwidth of **2*f_m**

Bandwidth Requirement

- Previous result holds when the modulating signal is a sine wave
- More generally, if the modulating signal is m(t) with Fourier transform **M**(f):

$$Y(f) = \frac{1}{2}A_c[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{4}\mu A_c[M(f - f_c) + M(f + f_c)]$$



Conventional AM is also known as Double-Sideband AM (DSB-AM)

Transmitted Power

- To calculate power of an AM signal we can break it into its frequency components and calculate the power of each

$$P = VI = \frac{V^2}{R} = I^2 R\text{, assume R=1Ohm}$$

$$y(t) = A_c cos(2\pi f_c t) + \frac{\mu A_c}{2}\left[cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)\right]$$

$$\mathrm{P}(y(t)) = \left(\frac{A_c}{\sqrt{2}}\right)^2 + 2\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2 = \frac{A_c^2}{2} + \left(\frac{\mu A_c}{2}\right)^2$$

For sinewave $V_{RMS} = \frac{V}{\sqrt{2}}, I_{RMS} = \frac{I}{\sqrt{2}},\quad \mu = \frac{A_m}{A_c} \cdot 100\%$

Power of the carrier

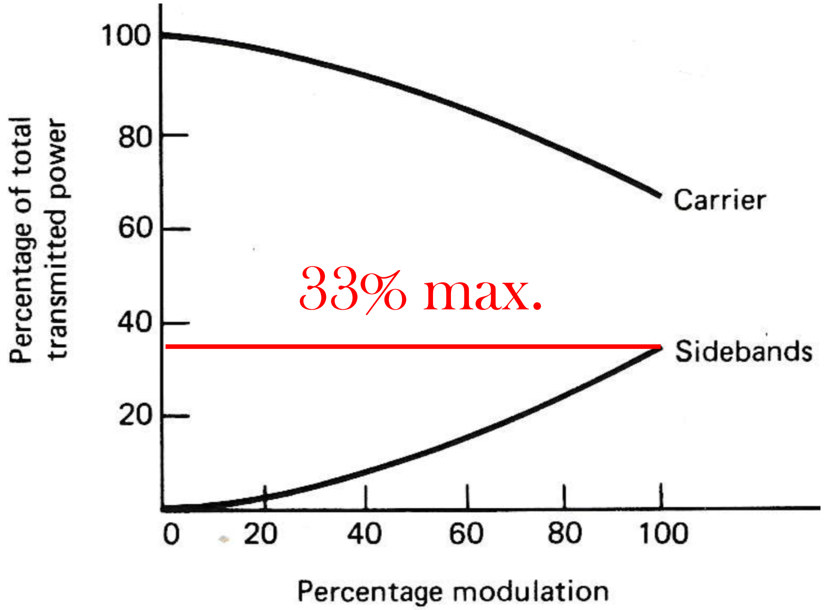
Power of the sidebands

Transmitted Power

$$P(y(t)) = \frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{4}$$

- For $P(y(t)) = const$, the power distribution between carrier & sidebands depends on μ
- The ratio of sideband power to total signal power is:

$$R = \frac{\frac{\mu^2 A_c^2}{4}}{\frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{4}} \Rightarrow \quad R = \frac{\mu^2}{2 + \mu^2}$$



➢ R is max. for $\mu = 1$ => R=33%

➢ $\frac{2}{3}$ of power wasted since carrier doesn't carry information

what have we learnt?

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