$$C \propto \frac{\int I^*_{pump} I_{probe}\, dx}{\int I_{pump} I_{probe}\, dx} = \frac{\overbrace{\int I_{probe}\, dx}^{1} \cdot c^*}{\int I_1 \cdot I_2\, dx} = \frac{c^*}{S_{12}}$$

$I^*_{pump} = \text{const}\ (\sigma_{pump} \gg \sigma_{probe})$

$$I_{12}(x) = \frac{1}{\sqrt{2\pi}\,\sigma_{12}} S_{12} \exp\left(-\frac{(x-\mu_n)^2}{2\sigma_n^2}\right)$$

$$c^* = \max(I_{pump}) = \frac{1}{\sqrt{2\pi}\,\sigma_{pump}}$$

$$S_{12} = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_1^2+\sigma_2^2}} \exp\left(-\frac{(\mu_1-\mu_2)^2}{2(\sigma_1^2+\sigma_2^2)}\right)$$

$$\Rightarrow C = \frac{\frac{\sqrt{2\pi}\,\sigma_{12}}{\sqrt{2\pi}\,\sigma_{pump}}}{\sigma_1} \exp\left(\frac{(\mu_1-\mu_2)^2}{2\sigma_{12}^2}\right) = \sqrt{\frac{\sigma_1^2+\sigma_2^2}{\sigma_1^2}} \exp\left(\frac{(\mu_1-\mu_2)^2}{2\sigma_{12}^2}\right)$$

$$I_1 \cdot I_2 = I_{12} \qquad \int I_{12}\, dx = 1 \cdot S_{12}$$

$$\sqrt{\sigma_1^2+\sigma_2^2} = \sigma_{12}$$

C for $\mu_1 = \mu_2$; $\sigma_1 \gg \sigma_2$: $C \to \underbrace{\sqrt{\frac{\sigma_1^2+\sigma_2^2}{\sigma_1^2}}}_{\sim 1} \cdot 1 = 1$ ✓

C for $\sigma_1 \ll \sigma_2$; $\mu_1 = \mu_2$

$$C \to \sqrt{\frac{\sigma_2^2}{\sigma_1^2}} = \frac{\sigma_2}{\sigma_1} \to \text{large}$$

C for $\sigma_1 = \sigma_2$, $\mu_1 = \mu_1$: $C \to \sqrt{2}$

C for $\sigma = \sigma_1 = \sigma_2$, $\mu_1 \neq \mu_1$: $C \to \sqrt{2} \exp\left(\frac{\Delta\mu^2}{4\sigma^2}\right)$

$$\sigma_1 \tfrac{1}{2} = \sigma_2$$

$$\sqrt{\frac{\frac{5}{4}}{\frac{1}{4}}} = \sqrt{\frac{5}{1}} = \sqrt{5} \sim 2{,}25$$

$\sigma^2 \gg \Delta\mu^2 \Rightarrow C \to \sqrt{2} \cdot 1$

$\sigma^2 \sim \Delta\mu^2 \to C \Rightarrow \sqrt{2} \exp\left(\frac{1}{4}\right)$

$\sigma^2 \ll \Delta\mu^2 \to C \Rightarrow$ very large $\propto \exp(\Delta\mu^2)$

$$G_{12} = G_1 \cdot G_2 \overset{?}{=} \frac{1}{2\pi\sigma_1\sigma_2} \exp\left(-\frac{1}{2}\left[\frac{(x-\mu_1)^2}{\sigma_1^2} + \frac{(x-\mu_2)^2}{\sigma_2^2}\right]\right)$$

$$= \frac{1}{2\pi\sigma_1\sigma_2} \exp\left(-\frac{1}{2\sigma_1^2\sigma_2^2}\left[\sigma_2^2(x^2-2\mu_1 x+\mu_1^2) + \sigma_1^2(x^2-2\mu_2 x+\mu_2^2)\right]\right)$$

$$= -\!\!\shortparallel\!\!- \exp\left(-\frac{1}{2\sigma_1^2\sigma_2^2}\left[x^2(\sigma_1^2+\sigma_2^2) - 2x(\mu_1\sigma_2^2+\mu_2\sigma_1^2) + (\sigma_1^2\mu_2^2+\sigma_2^2\mu_1^2)\right]\right)$$

$\sigma_{12} = \sqrt{\sigma_1^2+\sigma_2^2}$

$$= -\!\!\shortparallel\!\!- \exp\left(-\frac{(\sigma_1^2+\sigma_2^2)}{2\sigma_1^2\sigma_2^2}\left[x^2 - 2x\frac{\mu_1\sigma_2^2+\mu_2\sigma_1^2}{\sigma_{12}^2} + \frac{\sigma_1^2\mu_2^2+\sigma_2^2\mu_1^2}{\sigma_{12}^2} + \varepsilon - \varepsilon\right]\right)$$

$$\left(\frac{\mu_1\sigma_2^2+\mu_2\sigma_1^2}{\sigma_{12}^2}\right)^2 \overset{!}{=} \frac{\sigma_1^2\mu_2^2+\sigma_2^2\mu_1^2}{\sigma_{12}^2} + \varepsilon$$

$$\sigma_{12}^4\varepsilon = \cancel{\mu_1^2\sigma_2^4} + \cancel{\mu_2^2\sigma_1^4} + 2\mu_1\mu_2\sigma_1^2\sigma_2^2 - \sigma_1^2\mu_2^2(\sigma_1^2+\sigma_2^2) - \sigma_2^2\mu_1^2(\sigma_2^2+\sigma_1^2)$$

$$= -\left(\mu_1^2\sigma_1^2\sigma_2^2 + \mu_2^2\sigma_1^2\sigma_2^2 - 2\mu_1\mu_2\sigma_1^2\sigma_2^2\right) = -\sigma_1^2\sigma_2^2(\mu_1-\mu_2)^2$$

$$\Rightarrow \varepsilon = -\frac{\sigma_1^2\sigma_2^2}{\sigma_{12}^4}(\mu_1-\mu_2)^2$$

$$\boxed{\sigma_n = \frac{\sigma_1\sigma_2}{\sigma_{12}}}$$

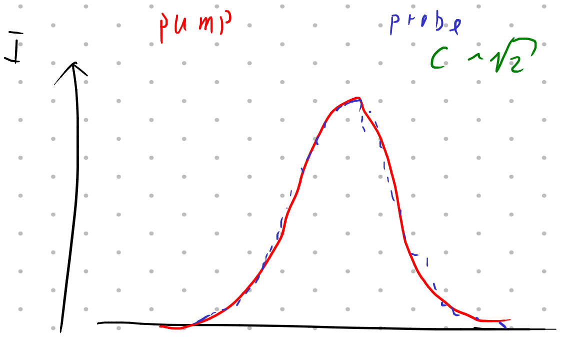
$$\frac{1}{2\sigma_n^2} \qquad \mu_n$$

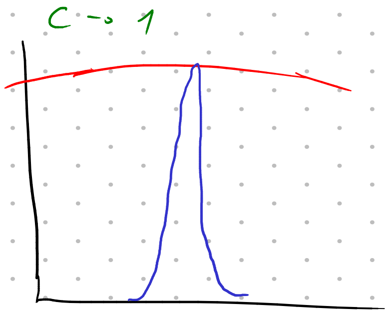
$$\Rightarrow G_{12} = \frac{1}{2\pi\sigma_1\sigma_2} \exp\left(-\frac{\cancel{\sigma_{12}^2}}{2\sigma_1^2\sigma_2^2\sigma_{12}^{\cancel{2}\,4}}\sigma_1^2\sigma_2^2(\mu_1-\mu_2)^2\right) \exp\left(-\frac{(\sigma_1^2+\sigma_2^2)}{2\sigma_1^2\sigma_2^2}\left[x - \frac{\mu_1\sigma_2^2+\mu_2\sigma_1^2}{\sigma_{12}^2}\right]^2\right)$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma_n} \underbrace{\frac{\sigma_n}{\sqrt{2\pi}\,\sigma_1\sigma_2} \exp\left(-\frac{(\mu_1-\mu_2)^2}{2\sigma_{12}^2}\right)}_{S_n} \exp\left(-\frac{(x-\mu_n)^2}{2\sigma_n^2}\right)$$

$$\boxed{\mu_n = \frac{\mu_1\sigma_2^2+\mu_2\sigma_1^2}{\sigma_{12}^2}}$$

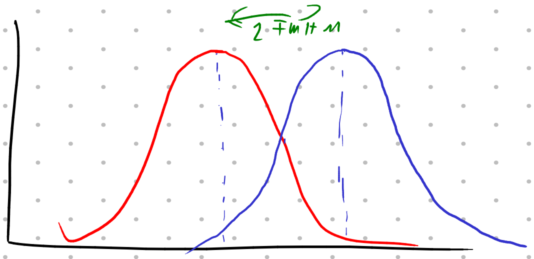
$$S_n = \frac{1}{\sqrt{2\pi}\,\sigma_{12}} \exp\left(-\frac{(\mu_1-\mu_2)^2}{2\sigma_{12}^2}\right)$$

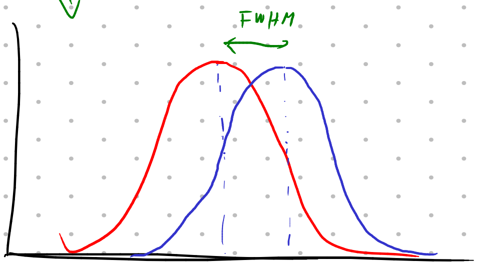




$\mathrm{FWHM} = \sqrt{2 \ln 2}\, \sigma$ for $\Delta\mu = 2\sqrt{2\ln 2}\,\sigma \Rightarrow \Delta\mu^2 = 8\ln 2\, \sigma^2$

$$C = \sqrt{2}\, \exp\left(\frac{1}{2} \cdot \frac{8\ln 2\, \sigma^2}{2\sigma^2}\right) = \sqrt{2}\, \exp(2\ln 2) = 4\sqrt{2} \approx 5{,}6$$





$\mu_1 = \mu_2 \qquad \sigma_1 = 3\,\sigma_2$

$$\cancel{C = \sqrt{\frac{10}{1}} = \sqrt{2}\sqrt{5} \approx 3{,}2}$$

$$\sqrt{\frac{\frac{10}{9}}{1}} = \frac{\sqrt{10}}{3} \approx 1{,}1$$

$$A = \left(\sqrt{2\pi}\,\sigma\right)^{-\frac{1}{2}}$$

$$\int A \cdot \exp\left(-\frac{(x-\mu_n)^2}{4\sigma^2}\right) dx \;\rightarrow\; \tilde{\sigma} = \sqrt{2}\,\sigma \;\Rightarrow\; \int \tilde{A}\, \exp\left(-\frac{(x-\mu_n)^2}{2\tilde{\sigma}^2}\right) dx$$

as in tutorial 1.2

$$\mathrm{Overlap} = \int \sqrt{I_1 \cdot I_2}\, dx$$

$$= \int \sqrt{I_{12}}\, dx = \sqrt{S_{12}} \int \sqrt{\mathrm{gauss}}\, dx$$

$$\tilde{A} = A \cdot c \overset{!}{=} \frac{1}{\sqrt{2\pi}\,\tilde{\sigma}}$$

$$c^{-1} = \underbrace{\sqrt{2\pi}\,\tilde{\sigma}}_{\sqrt{\pi}\,\tilde{\sigma}}\, A = \frac{\sqrt{2\pi}\,\sigma}{(2\pi)^{\frac{1}{2}\cdot\frac{1}{2}}\,\sigma^{\frac{1}{2}}} = \sqrt{\frac{\sigma}{2\pi}} \Rightarrow c = \sqrt{\frac{2\pi}{\sigma}}$$

$$I_{12} = G_{12} \cdot S_{12} \Rightarrow \int \sqrt{I_{12}}\, dx = \int \sqrt{G_{12}(x) \cdot S_{12}}\, dx = \sqrt{S_{12}} \cdot \int \sqrt{G_{12}}\, dx$$

$$\int \sqrt{G_{12}}\, dx = \int \left(\frac{1}{\sqrt{2\pi}\,\sigma_n}\right)^{\frac{2}{2}} \exp\left(-\frac{(x-\mu_n)^2}{2 \cdot 2\,\sigma_n^2}\right) \stackrel{!}{=} \int \frac{c}{\sqrt{2\pi}\,\tilde{\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\tilde{\sigma}}\right) dx = c \cdot 1$$

$$\frac{c}{\sqrt{2\pi}\,\tilde{\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\,\tilde{\sigma}^2}\right) \stackrel{!}{=} \sqrt{\frac{1}{\sqrt{2\pi}\,\sigma_n}} \exp\left(\frac{(x-\mu)^2}{4\,\sigma_n^2}\right) \quad / \cdot \exp\left(\frac{(x-\mu)^2}{2\tilde{\sigma}^2}\right) \quad / \cdot \sqrt{\sqrt{2\pi}\,\sigma}$$

$$\tilde{\sigma}^2 = 2\sigma_n$$

$$\Rightarrow c\, \frac{\sqrt[4]{2\pi}\,\sqrt{\sigma_n}}{\sqrt{2\pi}\,\tilde{\sigma}} = \exp\left((x-\mu)^2 \left(\frac{4\sigma_n^2 - 2\tilde{\sigma}^2}{8\sigma^2\,\tilde{\sigma}^2}\right)\right) = 1$$

correct according to internet

$$\Rightarrow c = \frac{\sqrt{2\pi}\;\tilde{\sigma}}{\sqrt[4]{2\pi}\sqrt{\sigma_n}} = \frac{\sqrt[4]{2\pi}\,\sqrt{2}\,\sigma_{12}}{\sqrt{\sigma_n}} = \underline{\sqrt[4]{8\pi}\,\sqrt{\sigma_n}} = \sqrt[4]{8\pi}\,\sqrt{\frac{\sigma_1\sigma_2}{\sigma_{12}}}$$

$$c\sqrt{S_n} = \sqrt{\frac{\cancel{\sqrt{8\pi}}^{\,2}\ \frac{\sigma_1\sigma_2}{\sigma_{12}}}{\cancel{\sqrt{2\pi}}_{\,1}\ \sigma_{12}}}\ \exp\left(-\frac{(\mu_1-\mu_2)^2}{4\,\sigma_{12}}\right) = \underline{\sqrt{\frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}}\ \exp\left(-\frac{(\mu_1-\mu_2)^2}{4\,\sigma_{12}}\right)}$$

if $\sigma_1 = \sigma_2 = \sigma$, $\mu_1 = \mu_2$: $\quad c\sqrt{S_n} = \sqrt{\frac{2\sigma^2}{\sigma^2+\sigma^2}} \cdot 1 = \sqrt{\frac{2}{2}} = 1$ ✓

$$\int \sqrt{I_1 \cdot I_2}\, dx = \int \sqrt{I_{12}}\, dx = \int \sqrt{S_{12}\, G_{12}}\, dx =$$

$$= \sqrt{S_{12}} \int \sqrt{G_{12}}\, dx = \sqrt{S_{12}} \int \sqrt{c} \cdot \tilde{G}_{22}\, dx = \sqrt{S_{12}\, c}$$