Welcome to Advanced Image Analysis – 02506
Spring 2020

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DTU Compute
Advanced Image Analysis
February 2020

Reading Material

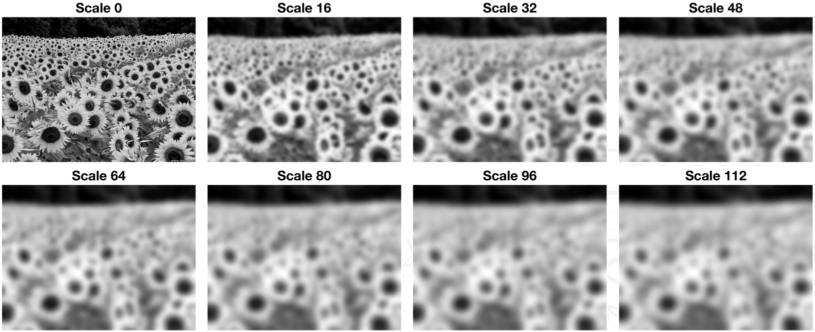
- Lindeberg, T. Scale-space theory: A framework for handling image structures at multiple scales. In: Proc. CERN School of Computing, Egmond aan Zee, The Netherlands, 8-21 September, 1996
- Lecture note Chapter 2 and 3

What to learn

- Scale space: To model an image at multiple scales
- Image geometry: To describe image features focusing blobs and Gaussian derivatives
  - Requires applying differential operators on images
- Combine scale space and image geometry

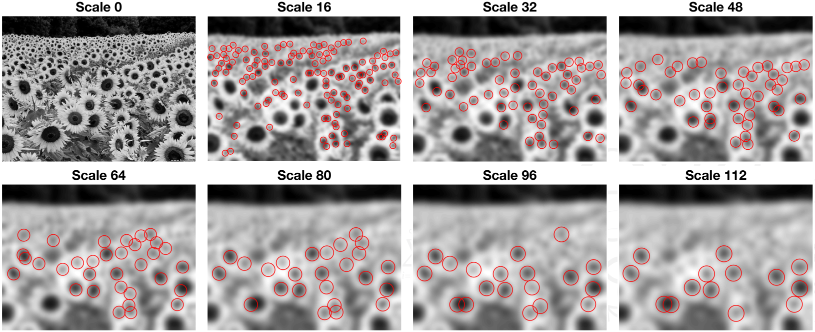
What to learn (Visualized)

- Scale space: To model an image at multiple scales



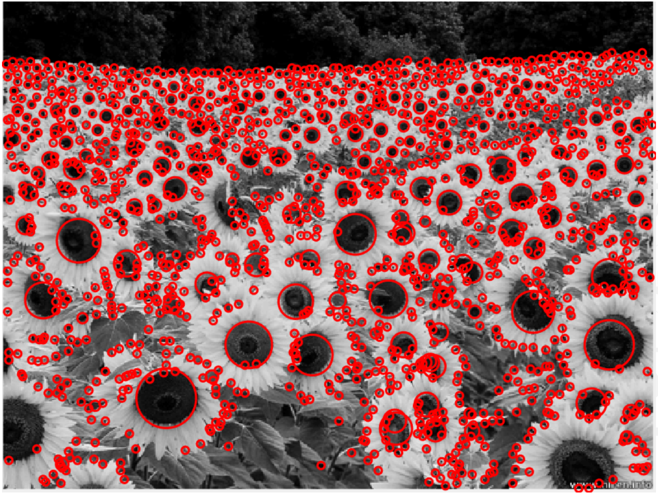
What to learn (Visualized)

- Image geometry: To describe texture as edges and **blobs**



What to learn (Visualized)

- Combine scale space and image geometry



Motivation: Why is scale space important?

- Objects have different physical size
- The distance to the camera may be unknown

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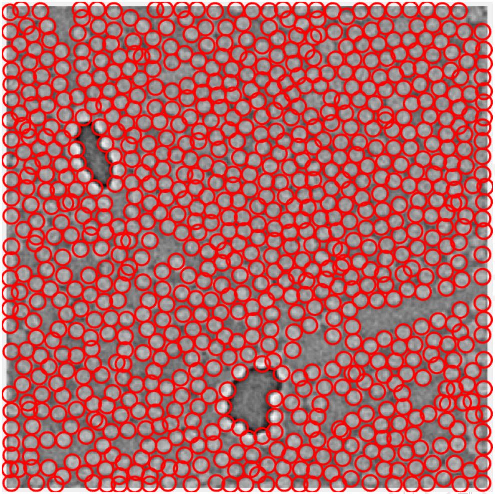
- Scale-space is the foundation for interest feature based analysis

Motivation: Problem of counting fibers

- Determine the position of carbon fibers
- Number of fibers: 861 (some double and some missing)



Scale space

- 2D Convolution/filtering
- Gaussian filter

Scale space properties

- Core idea: Represent an image at multiple scales
- Fine scale features are suppressed at coarser scale
- New features do not appear from smoothing, (edge at coarser scale is also there at fine scale)
- Increased coarseness achieved by changing filter extent

Scale space formulation

For an image $f : \mathbb{R}^2 \to \mathbb{R}$ the scale space $L : \mathbb{R}^2 \times R_+ \to \mathbb{R}$ is

$$L(x; t) = \int_{\xi \in \mathbb{R}^2} f(x - \xi) g(\xi; t) d\xi, \quad (1)$$

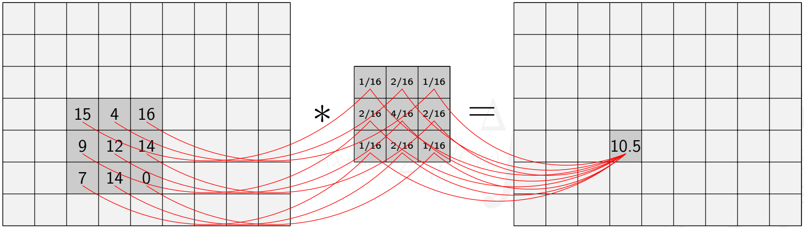
where $g : \mathbb{R}^2 \times R_+ \to \mathbb{R}$ is the 2D Gaussian.

$$g(x; t) = \frac{1}{(2\pi t)} \exp\left(\frac{-(x_1^2 + x_2^2)}{2t}\right). \quad (2)$$

with scale parameter (variance) $t = \sigma^2$.

Filtering/convolution: Small reminder

$$L(x;t) = \int_{\xi \in \mathbb{R}^2} f(x-\xi) g(\xi;t) d\xi \approx \sum_{j=y-k}^{y+k} \sum_{i=x-k}^{x+k} f(i,j) g(x-i, y-j)$$



Alternate scale space formulation

$$L = g * f = f * g$$

where * denotes convolution

Useful property: The (isotropic) Gaussian is separable

$$g(x;t) = \frac{1}{\sqrt{2\pi t}} \exp\left(\frac{-x_1^2}{2t}\right) \frac{1}{\sqrt{2\pi t}} \exp\left(\frac{-x_2^2}{2t}\right)$$

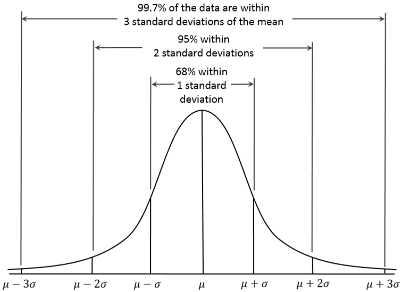
This seperability means that

$$L = g_1 * g_2^T * f$$

where $g_1 = g_2$

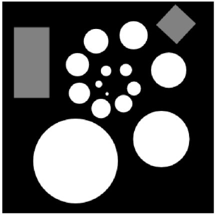
The size of a Gaussian filter

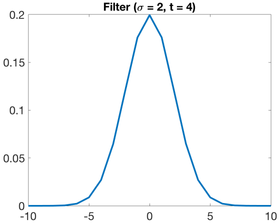
- Ideally: Infinitly big – but impractible
- Emperic rule: 3σ, 4σ or 5σ rule
- Using $5\sigma = 5\sqrt{t}$ rule
  - if $\sigma = 2$, filter size of Gaussian is $2 * 5 * 2 + 1 = 21$
  - if $\sigma = 20$, filter size of Gaussian is $2 * 5 * 20 + 1 = 201$



Scale space filtering

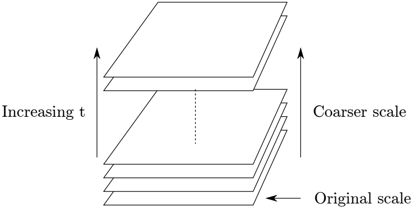
Original image (σ = 0, t = 0)





Scale space image (σ = 2, t = 4)

Implementation –The linear scale space

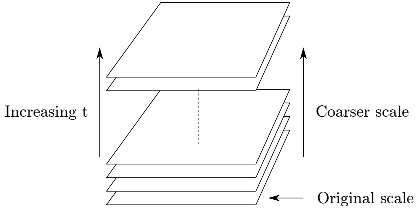


- Use repeated semi-group property

$$
\begin{aligned}
L(x, t_1) &= h(x, t_1) * f(x) \\
L(x, t_2) &= L(x, t_1 + t_1) = h(x, t_1) * L(x, t_1) \\
L(x, t_3) &= L(x, t_2 + t_1) = h(x, t_1) * L(x, t_2)
\end{aligned}
$$

- if $t_1 = 1$ than $t_2 = 2, t_3 = 3, t_4 = 4$ and so on.

Implementation –The linear scale space



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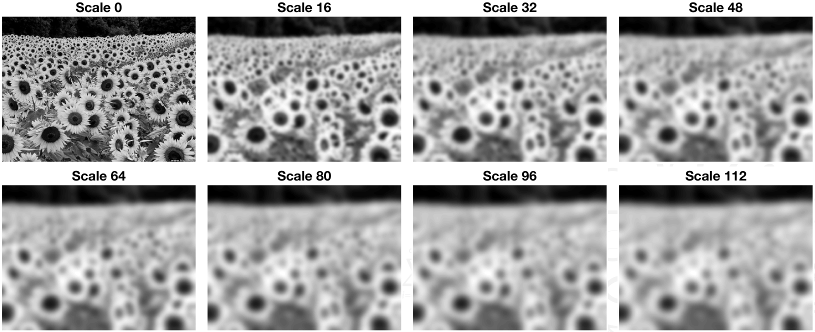
\end{aligned}
$$

- if $t_1 = 2$ than $t_2 = 4, t_3 = 6, t_4 = 8$ and so on.

Scale space example

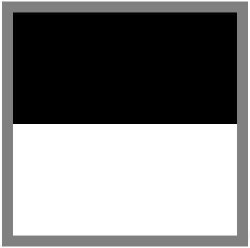


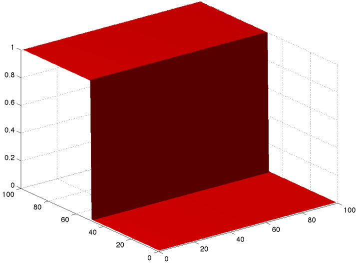
You will experiment with t steps and more scales during the exercise

Fundamental image features

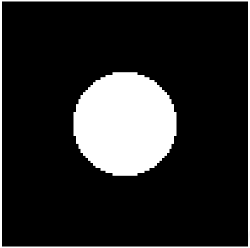
- Introducing edges and blobs
- Image derivatives (useful properties)
- Image geometry (edge and blob formulas)

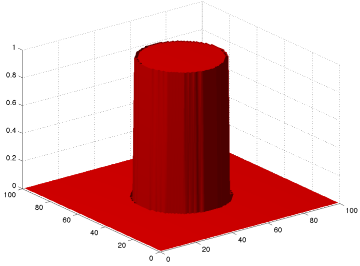
Scale space – edge over scales





Scale space – a blob over scales





Basic property

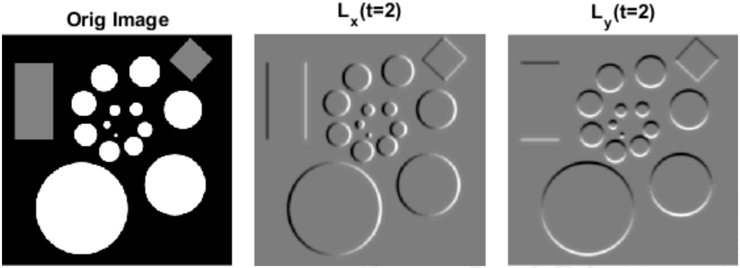
Differentiation of convolutions is commutative

$$\frac{\partial L}{\partial x_i} = g * \frac{\partial f}{\partial x_i} = \frac{\partial g}{\partial x_i} * f$$

Therefore scale space derivative by Gaussian derivatives

$$\begin{aligned}\partial_{x_1^\alpha x_2^\beta} L &= (\partial_{x_1^\alpha x_2^\beta} g) * f \\ &= (\partial_{x_1^\alpha} g_1 * \partial_{x_2^\beta} g_2^T) * f\end{aligned}$$

Derivative example



Orientation of derivatives

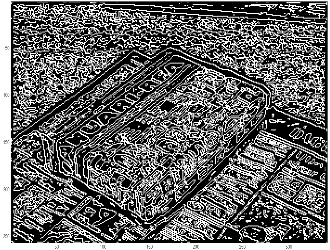
- L_x and L_y are row and column derivatives
- We may use another choice of coordinate system (i.e. rotation)
- Change derivative orientation by

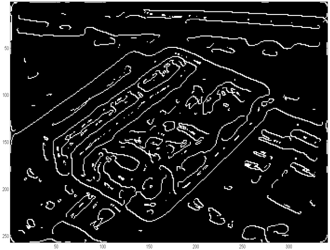
$$L_\phi = L_x \cos(\phi) + L_y \sin(\phi)$$

Oriented derivatives example

$$L_{45} = L_x \cos(45) + L_y \sin(45)$$

Edge detection





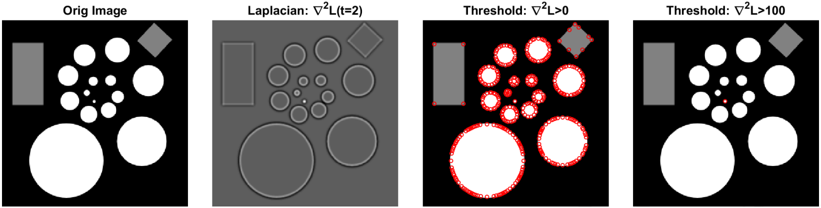
Blob detection

- Find the spatial maxima of the Laplacian

$$\nabla^2 L = L_{xx} + L_{yy} = L_{uu} + L_{vv} \tag{3}$$

- Dark blobs are found as local maxima (over 3×3 region)
- Bright blobs found as local image minima (over 3×3 region)

Blob detection example



- Note how thresholding the laplacian remove false positives
- Blobs ploted using $radii = \sqrt{2t} = 2.82$

Automatic scale selection

- Features should be found over scale to be scale invariant
- Done by calculating the normalized derivative (images "flatten"over scale)

$$\partial_{\xi_i} = t^{\gamma/2}\partial_{x_i} = t^{y/2}L_{x_j} \tag{4}$$

- γ should be tuned to the task at hand
- The feature scale, is where the scale normalize derivative is maximum over scales

Specifically for some feature types

Feature type	Strength measure	Value of γ
Edge	$t^{\gamma/2} L_v$	1/2
Ridge	$t^{2\gamma}(L_{pp} - L_{qq})^2$	3/4
Corner	$t^{2\gamma} L_v^2 L_{uu}$	1
Blob	$t^{\gamma} \nabla^2 L$	1

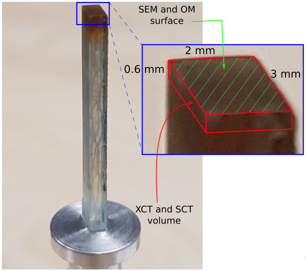
(5)

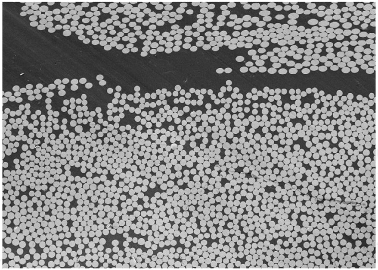
Formulas for ridges and more presented next week.

Exercise

- Compute Gaussian and its second order derivative
- Detect blobs at one scale
- Detect blobs with scale selection
- Detect blobs in real data
- Modified altgorithm

Data



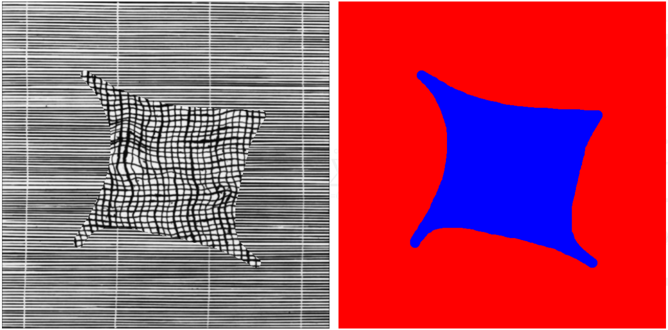


Segmentation

Map from image to label space

$$g(x, y) \to \ell \text{ , where } \ell \in 1, ..., L$$

Supervised segmentation



Features

Gaussian derivatives

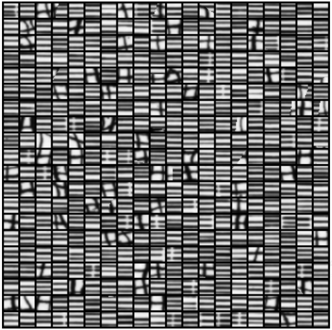
$$L_x xy = I * \frac{\partial^3 g}{\partial x^2 \partial y}$$

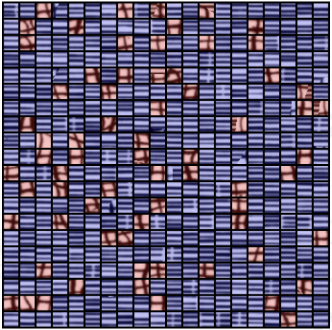
Image of feature vectors

$$\begin{aligned} F \quad = \quad & [L, L_x, L_y, L_{xx}, L_{xy}, L_{yy}, L_{xxx}, L_{xxy}, \\ & L_{xyy}, L_{yyy}, L_{xxxx}, L_{xxxy}, L_{xxyy}, L_{xyyy}, L_{yyyy}] \end{aligned}$$

Features

Image patches





Compute probabilities

Label for each class

$$p(\ell = \lambda) = \frac{1}{m} \sum_{f \in C} \delta(\ell(f) - \lambda) \ , \tag{6}$$

where C is a cluster with m elements f, $\ell(f)$ is the label of f, and

$$\delta(x) = \begin{cases} 1 & \text{if} \quad x = 0 \\ 0 & \text{else} \end{cases} . \tag{7}$$

What did we learn

Last exercise on scale-space

- Scale space – use Gaussian to obtain features at a range of scales
- Image derivatives – Gaussian derivatives in scale space
- Image geometric features (edges, blobs) – based on filtering with the Gaussian and its derivative

Todays exercise on feature-based segmentation

- Segmentation with features
- Compute image features
- Feature-based label probabilities