Geometric models sum-up and nerves segmentation

Vedrana Dahl, Anders Dahl
DTU Compute
02506 Advanced Image Analysis
March 2020

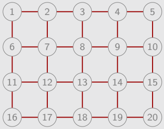
Todays lesson

Parts (videos)

- Part I Sum-up on geometric models: a parallel between MRF and deformable models
- Part II A few variants and extensions of MRF and deformable models, examples of using geometric models in research conducted at DTU Compute (optional)
- Part III A sneak-peak to layered surfaces, to be proposed as a mini-project later in the course (optional)
- Part IV Introduction to nerves segmentation problem and deliverable

Markov random fields and s-t grap cut

The set-up



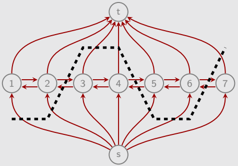
- Set of image pixels i with the pixel intensities d_i
- We know mean intensities μ_1 and μ_2
- We want to find a labeling which separates the two classes and is smooth

Modelling: Markov random fields

Find a a configuration f where $f_i \in \{1, 2\}$ which minimizes

$$E(f) = \sum_{\substack{i \\ f_i=1}} (\mu_1 - d_i)^2 + \sum_{\substack{i \\ f_i=2}} (\mu_2 - d_i)^2 + \beta \, \# \left\{ f_i \neq f_{i'} \,|\, i \text{ neighbouring } i' \right\}$$

Optimization: s-t graph cut



- An s-t graph constructed from to the segmentation energy
- An s-t cut corresponds to energy of the configuration: min-cut corresponds to the optimal configuration

Mumford-Shah functional and Chan-Vese algorithm

The set-up

- A foreground-background image (with unknown foreground intensity m_F and background intensity m_B) defined on $\Omega \subset \mathbb{R}^2$
- We want to find a curve which separates the two regions and is smooth

Modeling: piecewise constant Mumford-Shah

Find a curve Γ separatingΩ into Ω_{in} and Ω_{out} which minimizes the energy

$$E(\Gamma) = \int_{\Omega_{\mathrm{in}}} (m_{\mathrm{in}} - I)^2 \mathrm{d}x + \int_{\Omega_{\mathrm{out}}} (m_{\mathrm{out}} - I)^2 \mathrm{d}x + \lambda \mathrm{lenght}(\Gamma)$$

where m_{in} and m_{out} are mean intensities of regions

Solution: Chan-Vese algorithm

- Initialize a curve
- Until convergence alternate between
  - For given Γ find mean intensities m_{in} and m_{out}
  - For given m_{in} and m_{out} deform the curve to minimize the energy

Geometrical priors sum-up

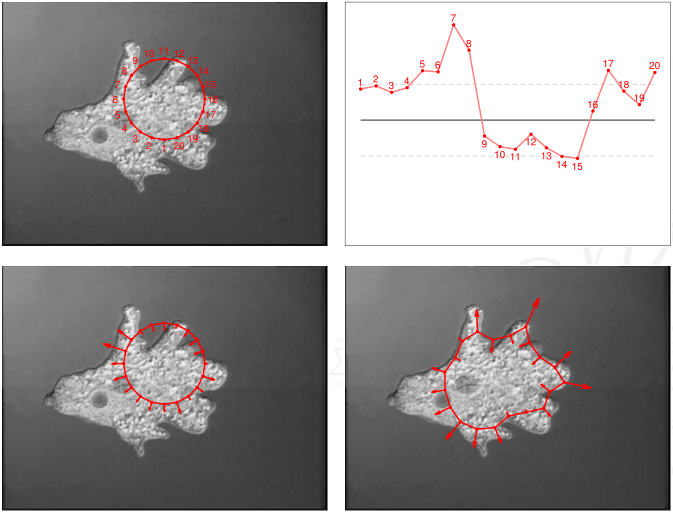
Similarities

- Optimization-based approaches to segmentation: modeling defines an objective function to be minimized
- Weighted combination of two parts: image-dependent part (data-term, likelihood, fidelity, external forces) and segmentation-dependent part (smoothness, prior, regularization, internal forces)
- For general cases the optimization is challenging, special cases are solvable and have been extensively researched

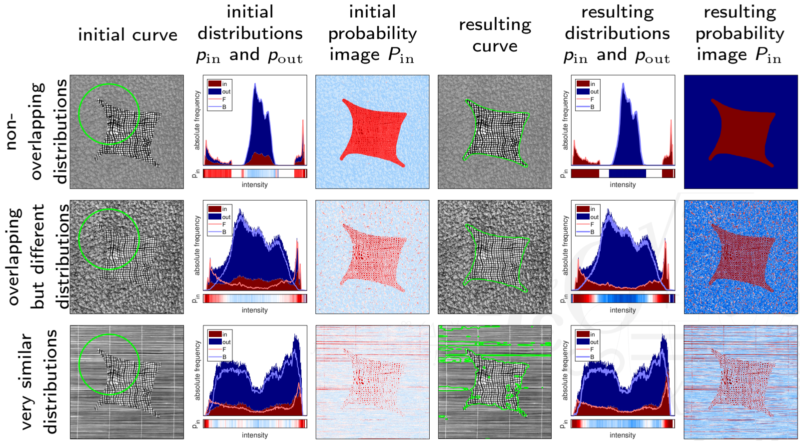
Differences

- The mathematical foundations: discrete pixels represented as graphs with neighborhood structure, and images as continuous functions in a variational framework
- Treatment of mean values μ_1 and μ_2, m_{in} and m_{out}. Note however the possiblity of formulating a "Chan-Vese-like" MRF algorithm
- In our implementation of Chan-Vese, the topology is constrained. Note that this is not a property of Chan-Vese model in general

Generalizing Chan-Vese model

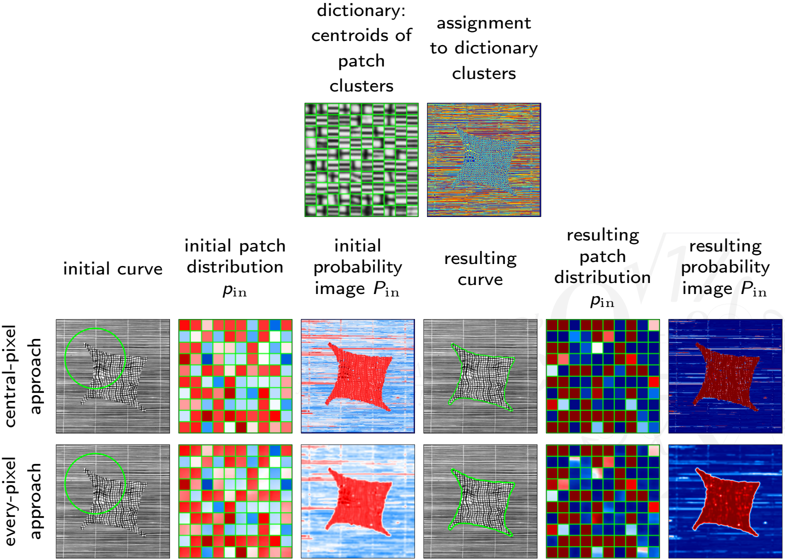


Generalizing Chan-Vese model[1]

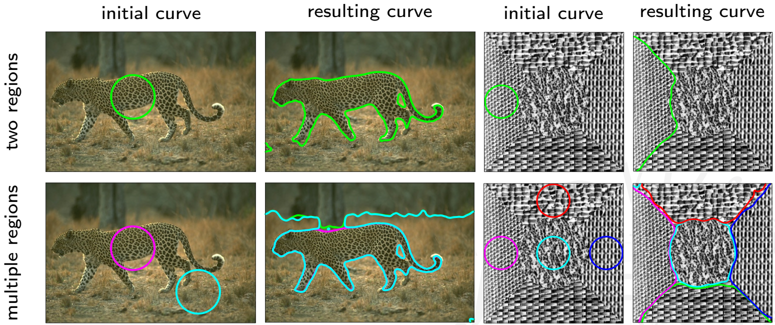


[1] SSVM 2017

Generalizing Chan-Vese model



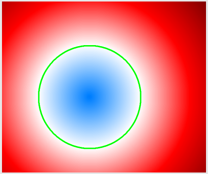
Generalizing Chan-Vese model



Generalizing curve representation

Deformable implicit curve

(Implicit equation for a circle:
$x^2 + y^2 = r^2$)



A discrete implicit curve
represented on a 500×600 grid

- Curve deforms by updating the level set function
- Supports topology changes
- No multi-phase support

Deformable parametric curve

(Parametric equation for a circle:
$x = r \cos t,\ y = r \sin t,\ t = [0, 2\pi\rangle$)

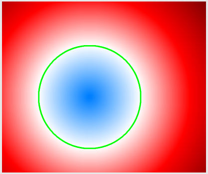
A discrete parametric curve
represented by 400 points

- Curve deforms by displacing curve points
- No support for topology changes
- No multi-phase support

Generalizing curve representation

Deformable implicit curve

(Implicit equation for a circle:
$x^2 + y^2 = r^2$)



A discrete implicit curve
represented on a 500×600 grid

- Curve deforms by updating the level set function
- Supports topology changes
- No multi-phase support

Deformable parametric curve

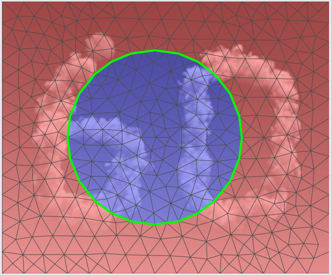
(Parametric equation for a circle:
$x = r\cos t,\ y = r\sin t,\ t = [0, 2\pi\rangle$)

A discrete parametric curve
represented by 400 points

- Curve deforms by displacing curve points
- No support for topology changes
- No multi-phase support

Something in between: Deformable mesh

Mesh-based curve representation [2]



A discrete curve represented on a triangle mesh
(triangles have phase labels, curve is where labels change) [3]

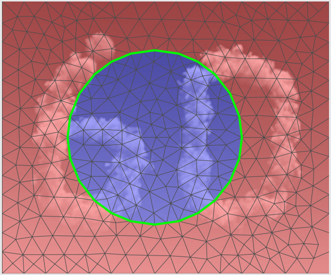
- Deformation by moving interface vertices (as with parametric curve)
- Supports topology changes (similar to level-set curve)
- Supports multi-phase

[2] Deformable Simplicial Complex framework developed at DTU Compute by Jakob Andreas Bærentzen, and others.

[3] Segmentation by Tuan Nguyen, DTU Compute

Something in between: Deformable mesh

Mesh-based curve representation [2]



A discrete curve represented on a triangle mesh
(triangles have phase labels, curve is where labels change) [3]

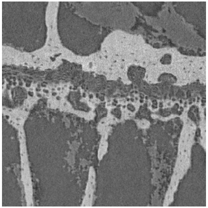
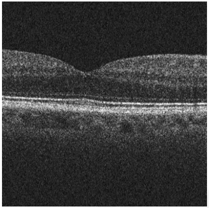
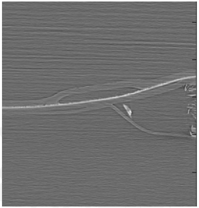
- Deformation by moving interface vertices (as with parametric curve)
- Supports topology changes (similar to level-set curve)
- Supports multi-phase

[2] Deformable Simplicial Complex framework developed at DTU Compute by Jakob Andreas Bærentzen, and others.

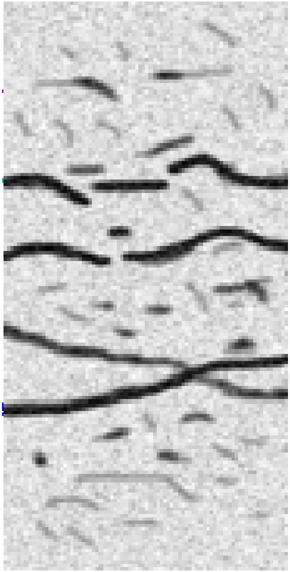
[3] Segmentation by Tuan Nguyen, DTU Compute

Optimal layered surface detection

- Layered terrain-like surfaces are commonly seen geometric segmentation model
- Can be generalized to tubular, closed and rolled surfaces

Optimal layered surface detection



- Terrain-like surfaces

$$z = f(x, y)\,.$$

- Smoothness

$$|f(x+n, y) - f(x, y)| < \Delta\,,$$

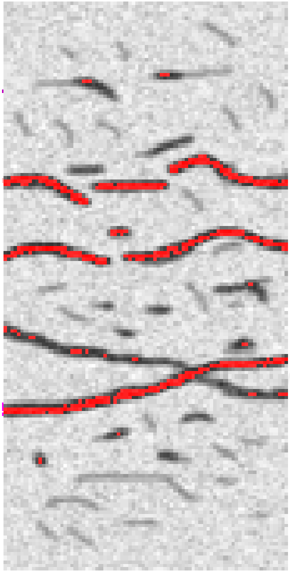
$$|f(x, y+n) - f(x, y)| < \Delta\,.$$

- Optimality (surface cost)

$$\min \sum_{x,y} c(x, y, f(x, y))\,.$$

- Geometric constraints reduce the number of acceptable outcomes.
- Optimal solution can be found using a graph-cut based search.
- Additional modelling options: layered surfaces, region based cost.

Optimal layered surface detection



- Terrain-like surfaces

$$z = f(x, y)\,.$$

- Smoothness

$$|f(x+n, y) - f(x, y)| < \Delta\,,$$

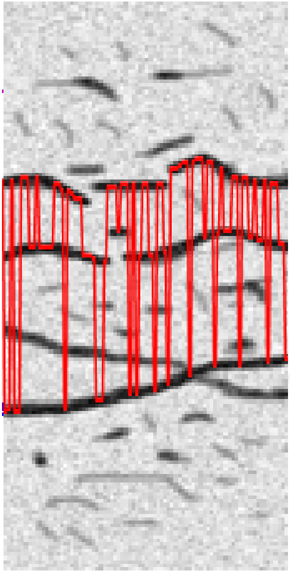
$$|f(x, y+n) - f(x, y)| < \Delta\,.$$

- Optimality (surface cost)

$$\min \sum_{x,y} c(x, y, f(x, y))\,.$$

- Geometric constraints reduce the number of acceptable outcomes.
- Optimal solution can be found using a graph-cut based search.
- Additional modelling options: layered surfaces, region based cost.

Optimal layered surface detection



- Terrain-like surfaces

$$z = f(x, y)\,.$$

- Smoothness

$$|f(x+n, y) - f(x, y)| < \Delta\,,$$

$$|f(x, y+n) - f(x, y)| < \Delta\,.$$

- Optimality (surface cost)

$$\min \sum_{x,y} c(x, y, f(x, y))\,.$$

- Geometric constraints reduce the number of acceptable outcomes.
- Optimal solution can be found using a graph-cut based search.
- Additional modelling options: layered surfaces, region based cost.

Optimal layered surface detection

- Terrain-like surfaces

$$z = f(x, y)\,.$$

- Smoothness

$$|f(x+n, y) - f(x, y)| < \Delta\,,$$

$$|f(x, y+n) - f(x, y)| < \Delta\,.$$

- Optimality (surface cost)

$$\min \sum_{x,y} c(x, y, f(x, y))\,.$$

- Geometric constraints reduce the number of acceptable outcomes.
- Optimal solution can be found using a graph-cut based search.
- Additional modelling options: layered surfaces, region based cost.

Optimal layered surface detection

- Terrain-like surfaces

$$z = f(x, y)\,.$$

- Smoothness

$$|f(x+n, y) - f(x, y)| < \Delta\,,$$

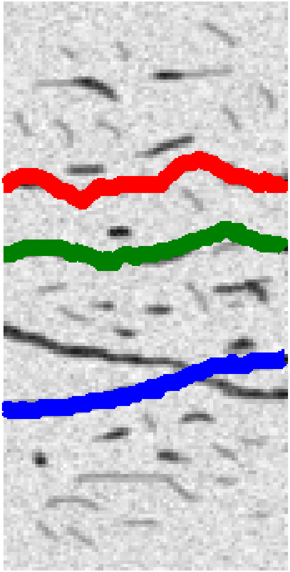
$$|f(x, y+n) - f(x, y)| < \Delta\,.$$

- Optimality (surface cost)

$$\min \sum_{x,y} c(x, y, f(x, y))\,.$$

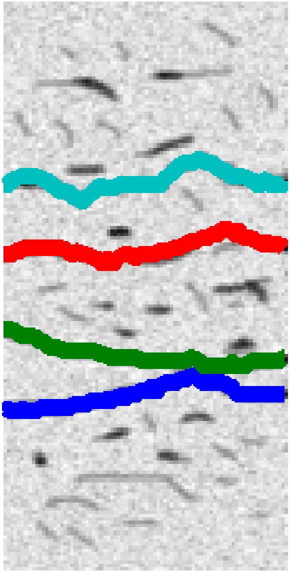
- Geometric constraints reduce the number of acceptable outcomes.
- Optimal solution can be found using a graph-cut based search.
- Additional modelling options: layered surfaces, region based cost.

Optimal layered surface detection



- Terrain-like surfaces
$$z = f(x, y)\,.$$
- Smoothness
$$|f(x+n, y) - f(x, y)| < \Delta\,,$$
$$|f(x, y+n) - f(x, y)| < \Delta\,.$$
- Optimality (surface cost)
$$\min \sum_{x,y} c(x, y, f(x, y))\,.$$
- Geometric constraints reduce the number of acceptable outcomes.
- Optimal solution can be found using a graph-cut based search.
- Additional modelling options: layered surfaces, region based cost.

Optimal layered surface detection



- Terrain-like surfaces

$$z = f(x, y)\,.$$

- Smoothness

$$|f(x+n, y) - f(x, y)| < \Delta\,,$$

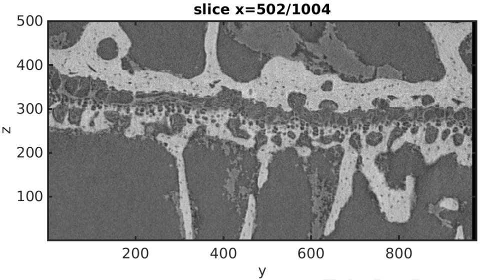
$$|f(x, y+n) - f(x, y)| < \Delta\,.$$

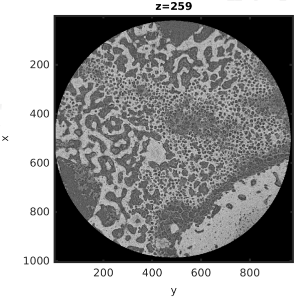
- Optimality (surface cost)

$$\min \sum_{x,y} c(x, y, f(x, y))\,.$$

- Geometric constraints reduce the number of acceptable outcomes.
- Optimal solution can be found using a graph-cut based search.
- Additional modelling options: layered surfaces, region based cost.

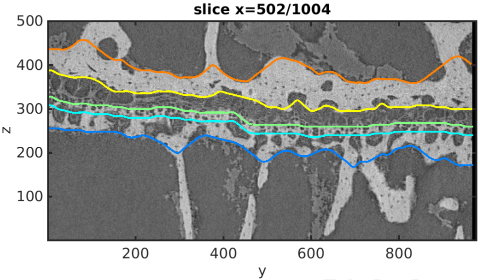
Growth plate in mice tibia[4]

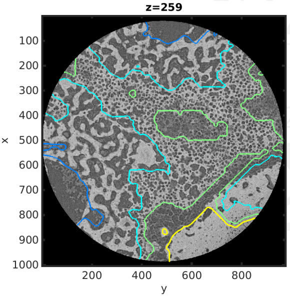




[4] With Maria Thomsen (DTU Physics and Novo Nordisk)

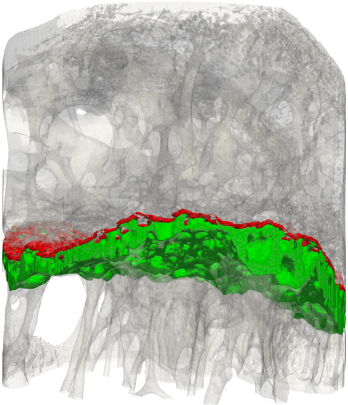
Growth plate in mice tibia[4]

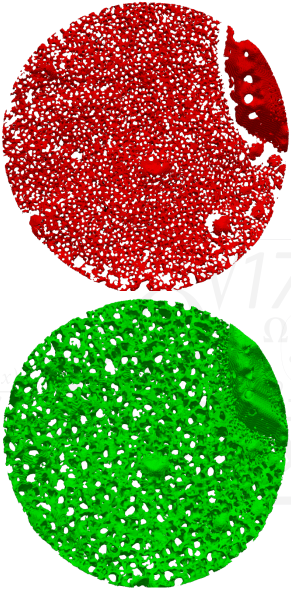




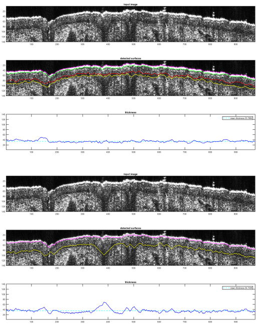
[4] With Maria Thomsen (DTU Physics and Novo Nordisk)

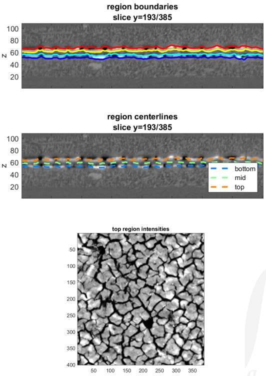
Growth plate in mice tibia

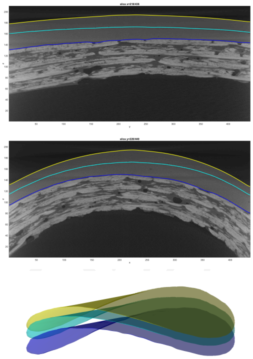




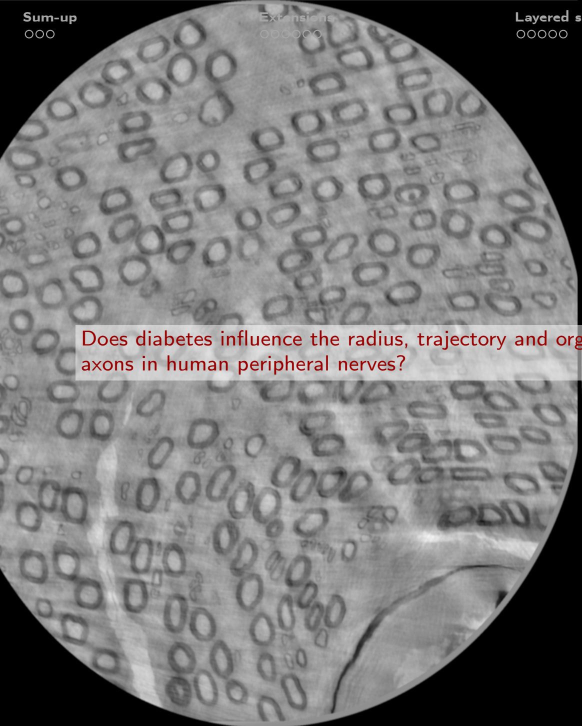
Other examples

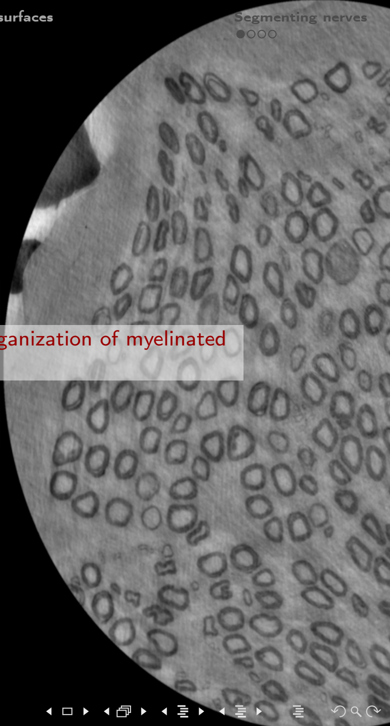




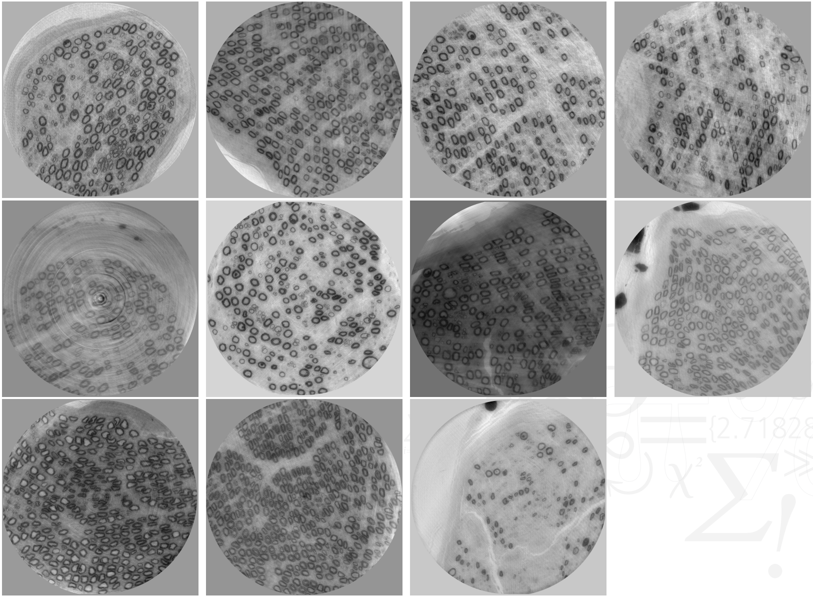


Layers of skin captured using optical coherence tomography, layers of high temperature polymer electrolyte membrane fuel cell captured using X-ray micro CT and layers of paint on composite material shown using X-ray micro CT.

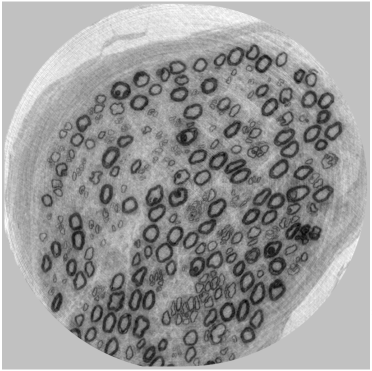


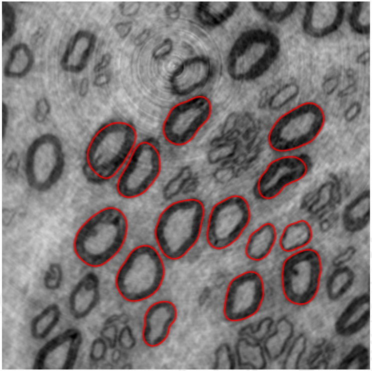


Does diabetes influence the radius, trajectory and organization of myelinated axons in human peripheral nerves?



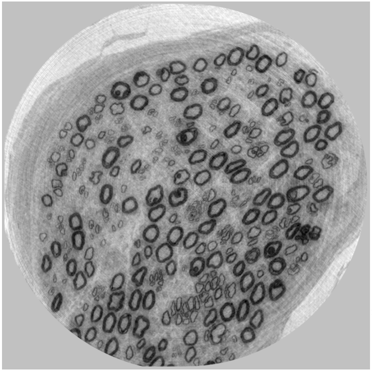
Segmentation of nerve fibers[5]

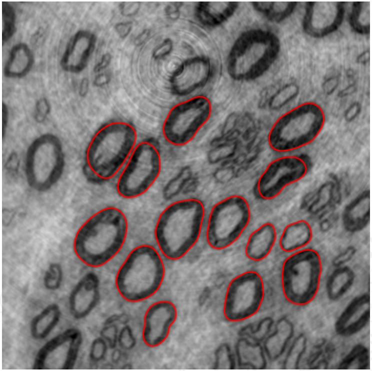




[5] with Martin Bech and Lars Dahlin

Segmentation of nerve fibers[5]





[5] with Martin Bech and Lars Dahlin

Segmentation of nerve fibers