

Tutorial 1: 3D space, transformations and animations.

This tutorial uses the following notation:

- Position vectors are denoted by boldface capital letters: **P**, **Q**, **V** etc. Position vectors are the same as Cartesian coordinates, and represent position relative to the origin.
- Direction vectors are indicated by boldface lowercase letters **d**, **n** etc. Direction vectors are independent of any origin.
- Scalars are represented by italics: *a*, *b*, etc.

A plane is an object that is only defined in Cartesian space, however, each plane has a normal vector, whose size is non zero, and whose direction is at right angles to that plane. We can find a normal vector by taking the cross product of any two direction vectors which are parallel to the plane.

Analysis of 3D scenes

1. Given three points:

$$\begin{aligned}\mathbf{P}_1 &= (10, 20, 5) \\ \mathbf{P}_2 &= (15, 10, 10) \\ \mathbf{P}_3 &= (25, 20, 10)\end{aligned}$$

find two direction vectors which are parallel to the plane defined by \mathbf{P}_1 , \mathbf{P}_2 and \mathbf{P}_3 . Hence find a normal vector to the plane.

2. A plane is defined in vector terms by the equation:

$$\mathbf{n} \cdot (\mathbf{P} - \mathbf{P}_1) = 0$$

where $\mathbf{P} = (x, y, z)$ is the locus of a point on the plane, and \mathbf{P}_1 is any point known to be in the plane.

For the points given in part 1, expand the vector plane equation to find the Cartesian form of the plane equation, (i.e. $ax + by + cz + d = 0$).

Verify that you get the same result using either \mathbf{P}_1 or \mathbf{P}_2 .

3. Write a procedure, in any programming language you like, which takes as input three points and returns the coefficients of the Cartesian plane equation (a , b , c and d).
4. Starting from any point on a face of a polyhedron, an inner surface normal is a normal vector to the plane of the face whose direction points into the polyhedron.

A tetrahedron is defined by the three points of part 1, and a fourth point $\mathbf{P}_4 = (30, 20, 10)$. Determine whether the normal vector that you calculated in part 1 is an inner surface normal, and if not find the inner surface normal.

5. Two lines intersect at a point \mathbf{P}_1 , and are in the directions defined by \mathbf{d}_1 and \mathbf{d}_2 . Provided that \mathbf{d}_1 and \mathbf{d}_2 represent different directions, the two lines define a plane.

Any point on the plane can be reached by travelling from \mathbf{P}_1 in direction \mathbf{d}_1 by some distance μ and then in direction \mathbf{d}_2 by a distance ν . Using this fact construct the parametric equation of any point on the plane of part 1 in terms of $\mu, \nu, \mathbf{P}_1, \mathbf{P}_2$ and \mathbf{P}_3 . By taking the dot product with a normal vector to the plane \mathbf{n} , show that the parametric plane equation is equivalent to the vector plane equation of part 2.

Animations

6. In a computer graphics animation scene an object is defined as a planar polyhedron. The object centre is located at position $\mathbf{P} = (0, 0, 10)$, and the scene is drawn, as normal, in perspective projection with the viewpoint at the origin and the view direction along the z-axis. Calculate the transformation matrix that will shrink the object in size by a factor of 0.8 towards its centre point.
7. Use your matrix of part 1 to check what happens to the points $(0, 0, 10)$ and $(0, 0, 5)$. Is your result what you expect?
8. In a different animation, the object, defined above is required to rotate clockwise, looking from the origin, while shrinking. In each successive frame it is to rotate by 15° while shrinking to 0.8 of its original size. The rotation axis is to be the z axis, and the shrinkage is, as before, towards the object's centre. Given that $\cos(15^\circ) = .97$ and $\sin(15^\circ) = .26$, what is the transformation matrix that will achieve this animation?
9. The scene above is to be drawn in perspective projection with the plane of projection being $z = 2$. Find the combined transformation that will do animation of part 1 followed by the perspective projection. Is your matrix singular?
10. Use your matrix to find the transformation and perspective projection of the points $(0, 0, 10)$ and $(0, 0, 5)$ in homogenous coordinates and then in Cartesian coordinates.
11. The scene is to be viewed from a moving viewpoint specified by its position \mathbf{C} and a left-handed viewing coordinate system $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$. At one point in the animation the view direction is $\mathbf{w} = (-1, 0, 0)^T$, and the viewpoint is given by $\mathbf{C} = (50, 10, -10)$. Given that the view is in the horizontal plane ($\mathbf{v} = (0, 1, 0)^T$) find the value of \mathbf{u} .
12. Hence, or otherwise, find the viewing transformation matrix.