Interactive Computer Graphics: Lecture 10

Ray tracing


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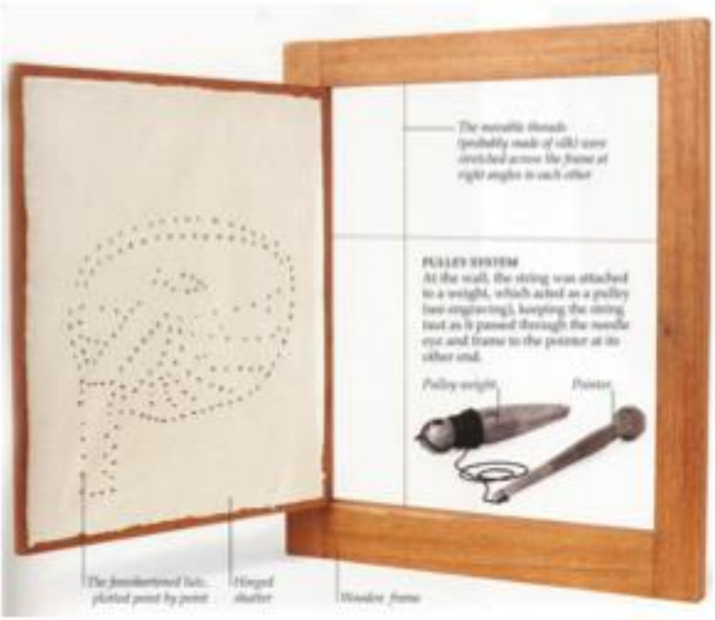
Direct and Global Illumination

- **Direct illumination**: A surface point receives light directly from all light sources in the scene.
  - Computed by the direct illumination model.
- **Global illumination**: A surface point receives light after the light rays interact with other objects in the scene.
  - Points may be in shadow.
  - Rays may refract through transparent material.
  - Computed by reflection and transmission rays.

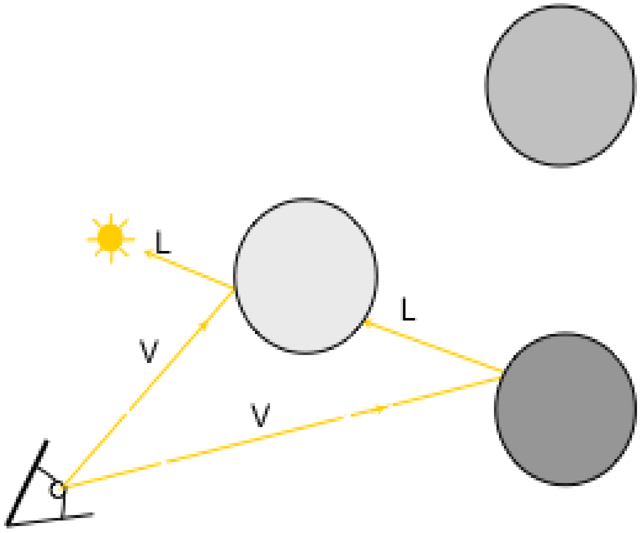
Albrecht Dürer's Ray Casting Machine

- Albrecht Dürer, 16th century



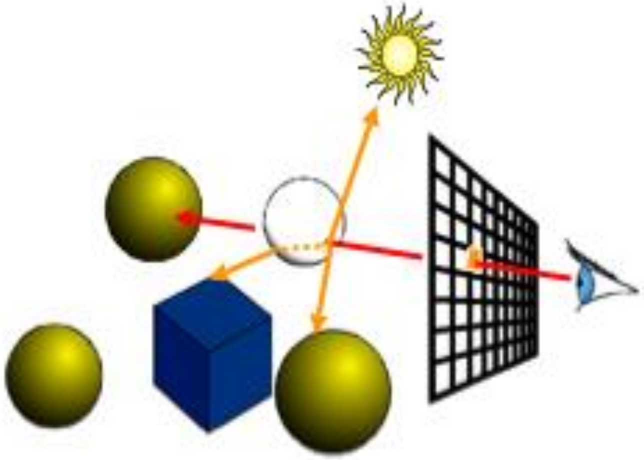
Arthur Appel, 1968

- On calculating the illusion of reality, 1968
- Cast one ray per pixel (ray casting).
  - For each intersection, trace one ray to the light to check for shadows
  - Only a local illumination model
- Developed for pen-plotters

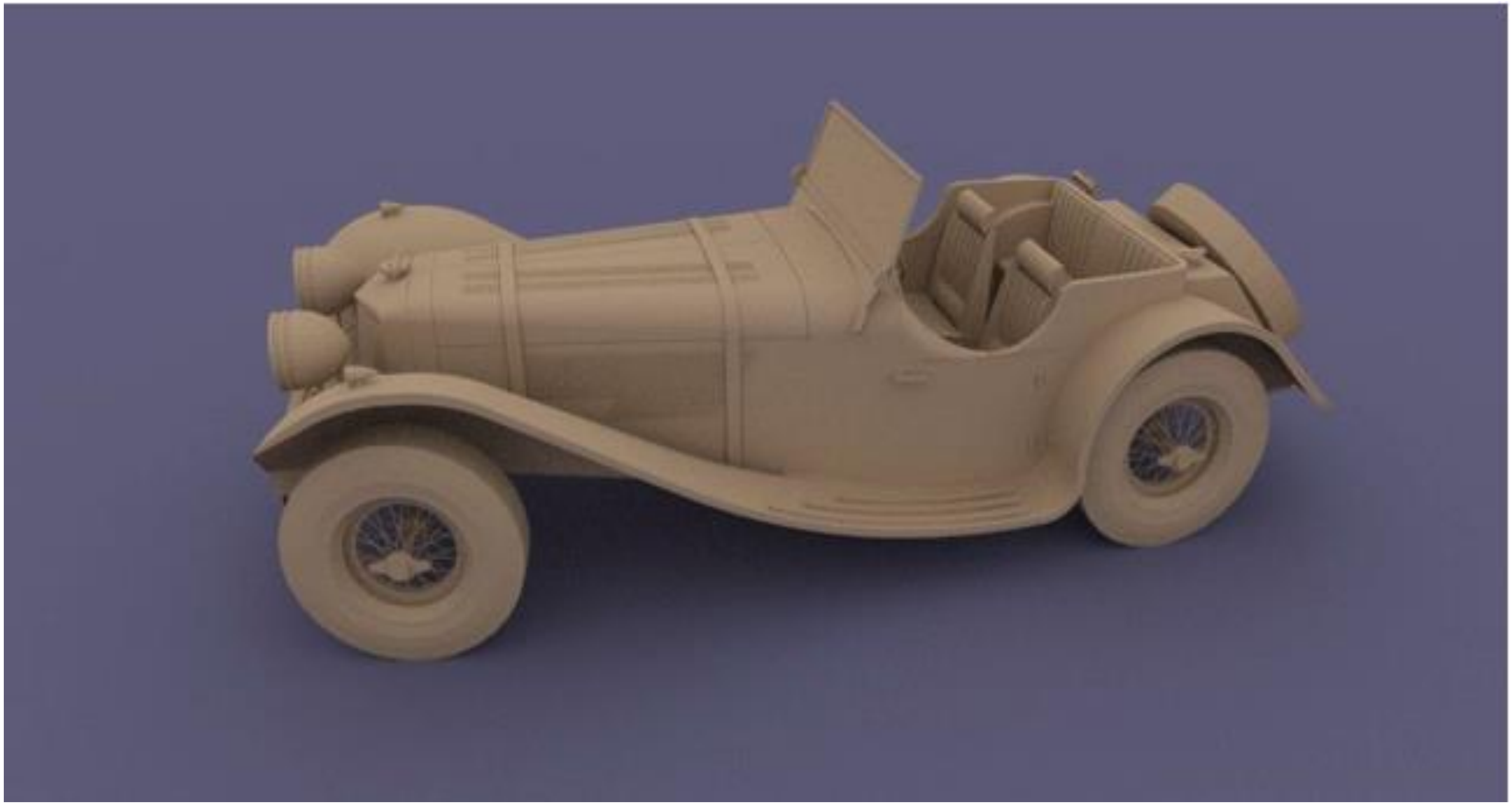


Ray casting

```
cast ray
      Intersect all objects
      color = ambient term
      For every light cast shadow ray
            col += local shading term
```

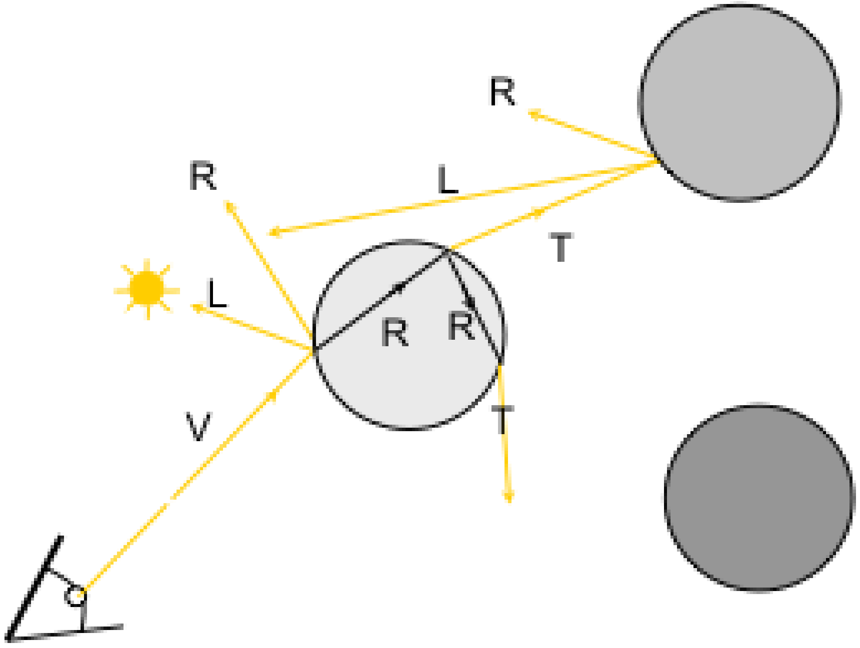


Ray casting

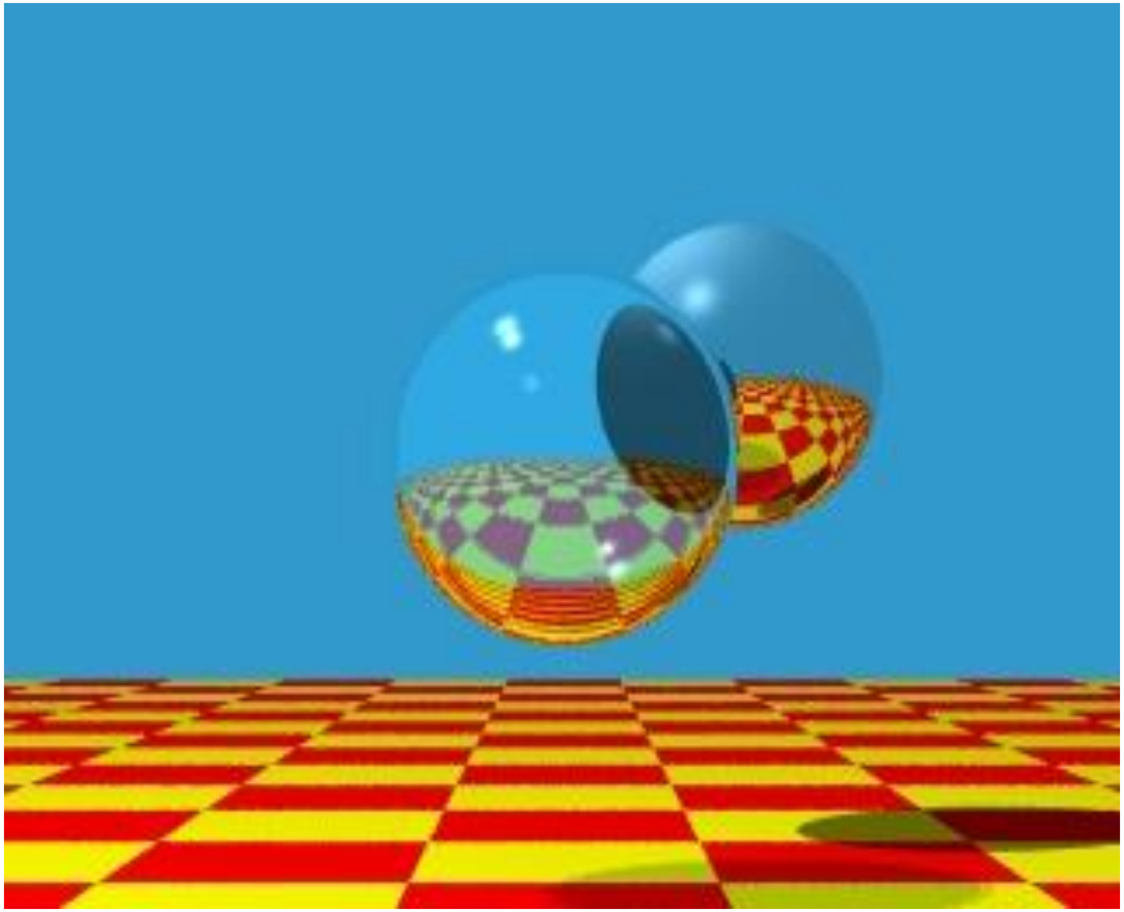


Turner Whitted, 1980

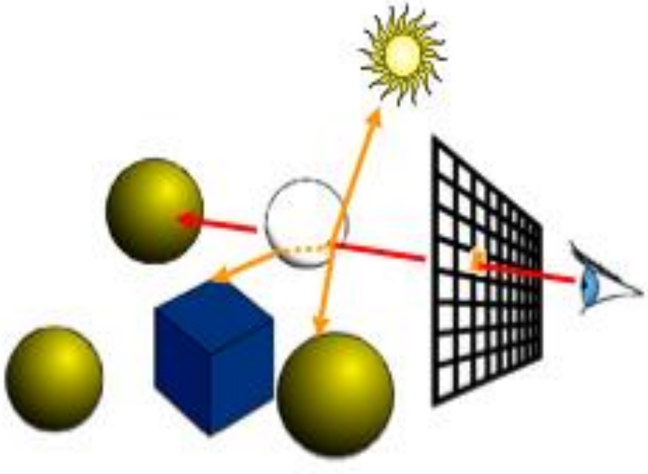
- An Improved Illumination Model for Shaded Display, 1980
- First global illumination model:
  - An object's color is influenced by lights and other objects in the scene
  - Simulates specular reflection and refractive transmission



Turner Whitted, 1980



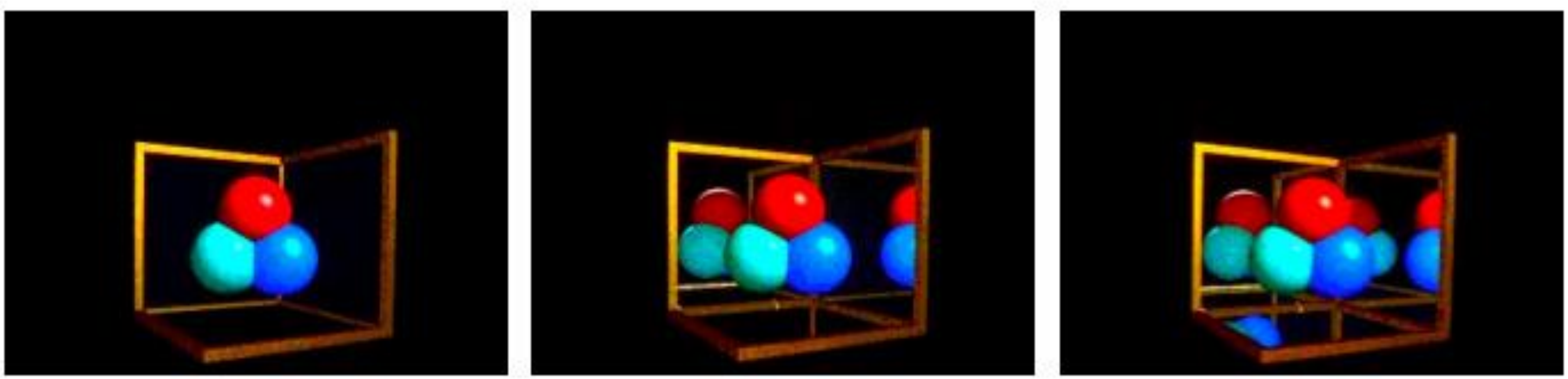
Recursive ray casting



```
trace ray
      Intersect all objects
      color = ambient term
      For every light
            cast shadow ray
            col += local shading term
      If mirror
            col += k_refl * trace reflected ray
      If transparent
            col += k_trans * trace transmitted ray
```

Does it ever end?

- Stopping criteria:
  - Recursion depth: Stop after a number of bounces
  - Ray contribution: Stop if reflected / transmitted contribution becomes too small



0 recursion | 1 recursion | 2 recursions

Ray tracing: Primary rays

- For each ray we need to test which objects are intersecting the ray:
  - If the object has an intersection with the ray we calculate the distance between viewpoint and intersection
  - If the ray has more than one intersection, the smallest distance identifies the visible surface.
- Primary rays are rays from the view point to the nearest intersection point
- Local illumination is computed as before:

$$L = k_a + (k_d(\mathbf{n} \cdot \mathbf{l}) + k_s(\mathbf{v} \cdot \mathbf{r})^q) I_s$$

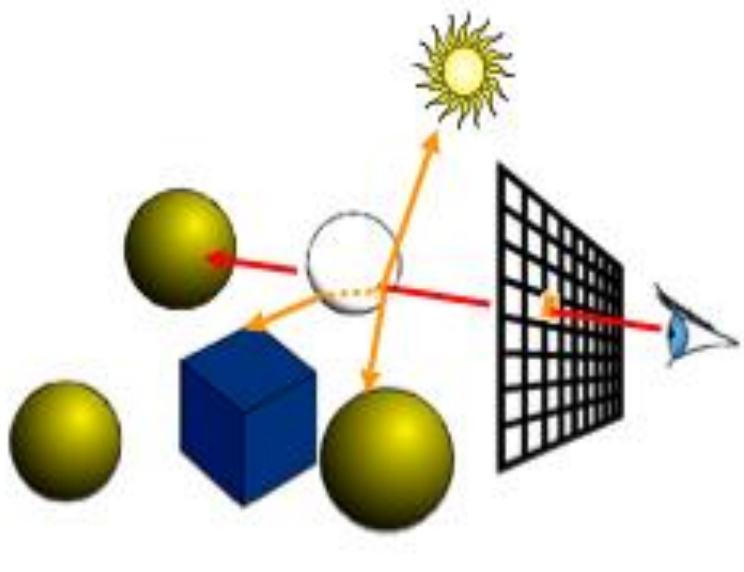
Ray tracing: Secondary rays

- Secondary rays are rays originating at the intersection points
- Secondary rays are caused by
  - rays reflected off the intersection point in the direction of reflection
  - rays transmitted through transparent materials in the direction of refraction
  - shadow rays

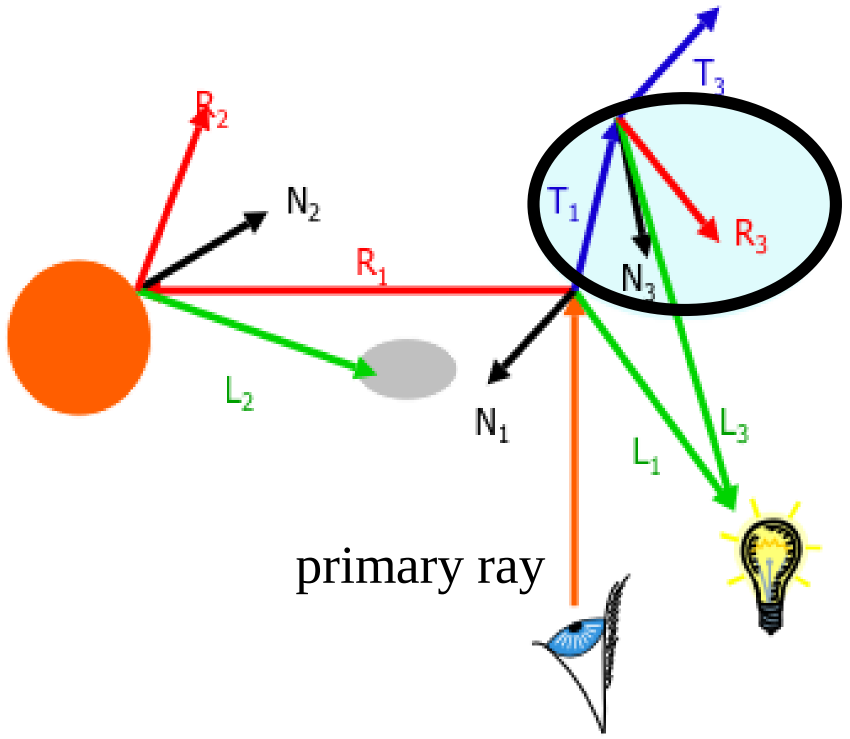
Recursive ray tracing: Putting it all together

- Illumination can be expressed as

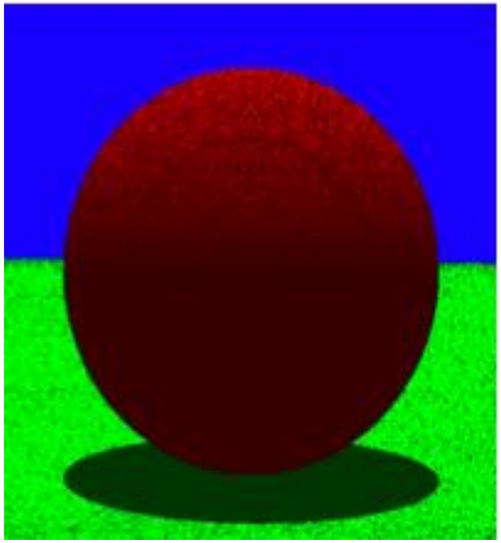
$$L = k_a + (k_d(\mathbf{n} \times \mathbf{l}) + k_s(\mathbf{v} \times \mathbf{r})^q)I_s \boxed{+ k_{reflected} L_{reflected} + k_{refracted} L_{refracted}}$$



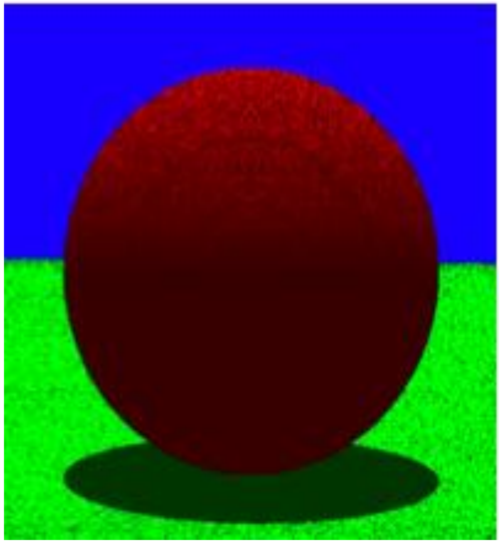
Recursive Ray Tracing: Ray Tree



Precision Problems



Precision Problems



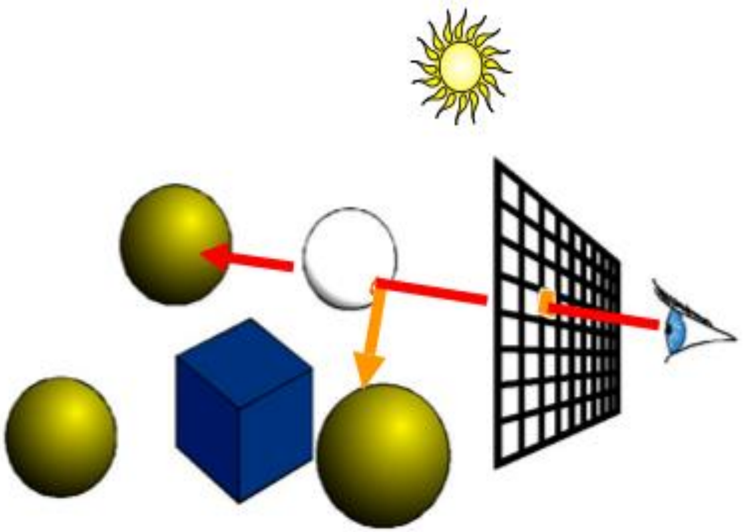
- In ray tracing, the origin of (secondary) rays is often below the surface of objects
  - Theoretically, the intersection point should be on the surface
  - Practically, calculation imprecision creeps in, and the origin of the new ray is slightly beneath the surface
- Result: the surface area is shadowing itself or the ray continues on the wrong side

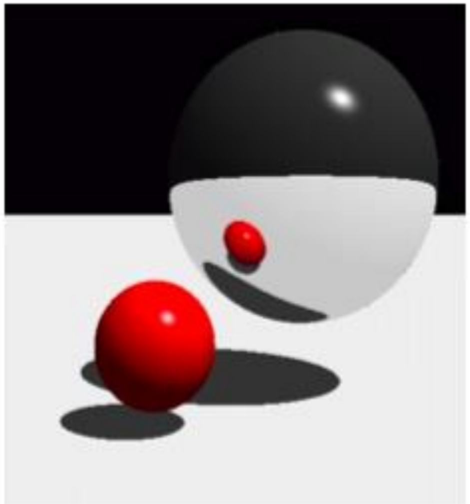
ε to the rescue ...

- Check if t is within some epsilon tolerance:
  - if abs(μ) < ε
    - point is on the surface
  - else
    - point is inside/outside
  - Choose the ε tolerance empirically
- Move the intersection point by epsilon along the surface normal so it is outside of the object
- Check if point is inside/outside surface by checking the sign of the implicit (sphere etc.) equation

Mirror reflection

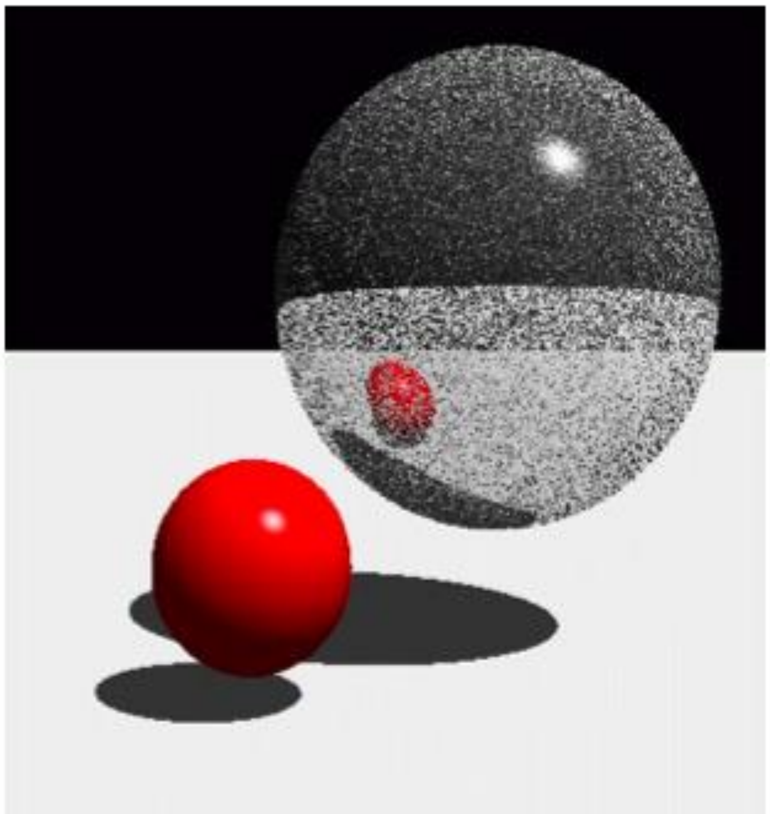
- Compute mirror contribution
- Cast ray in direction symmetric wrt. normal
- Multiply by reflection coefficient (color)



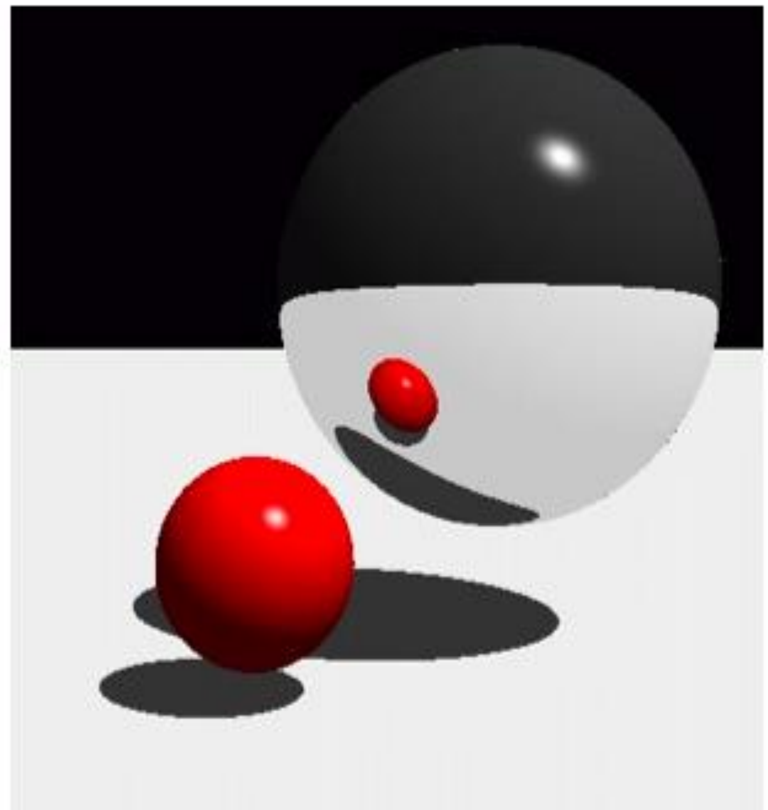


Mirror reflection

- Don’t forget to add epsilon to the ray

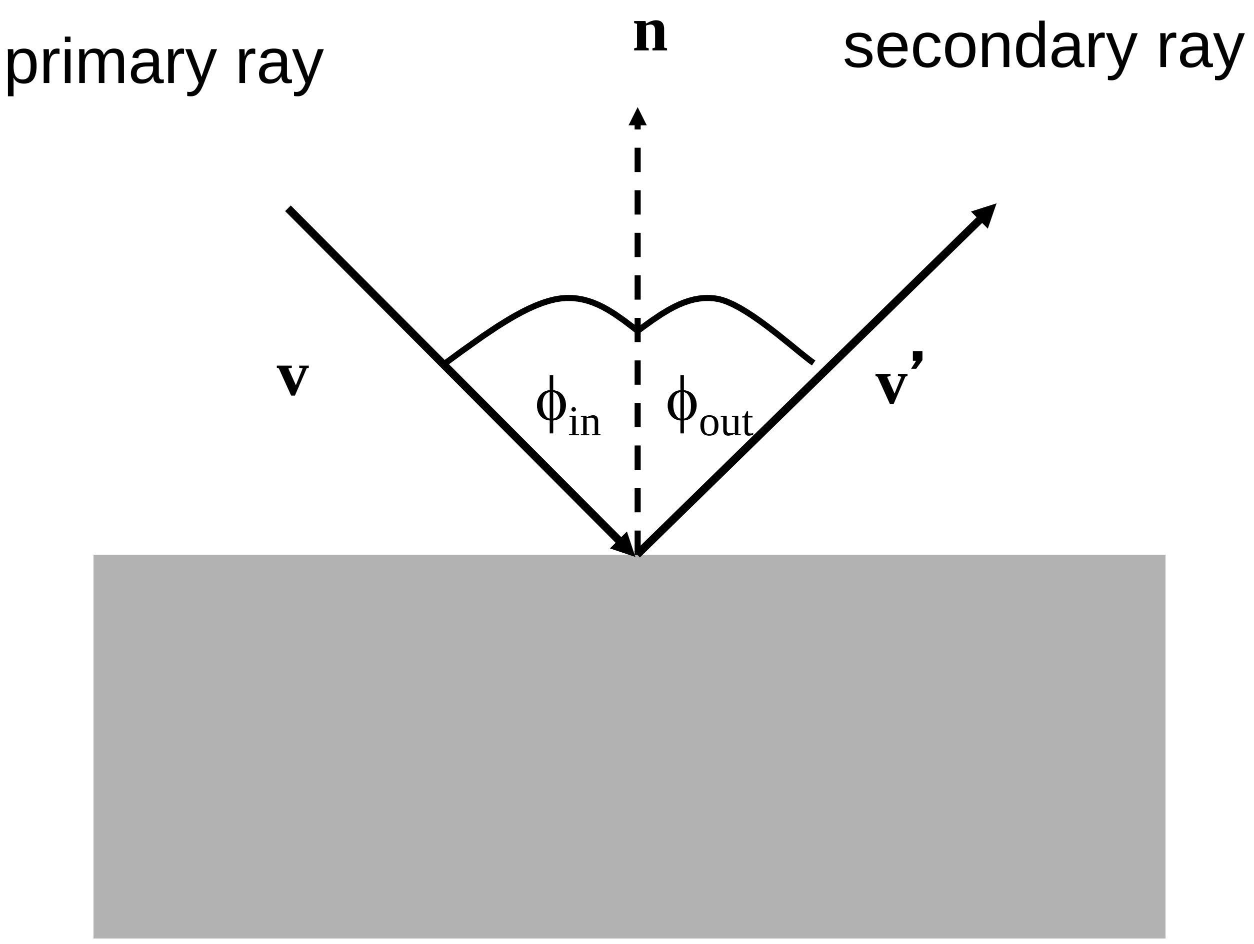


Without epsilon



With epsilon

Mirror reflection



Mirror reflection

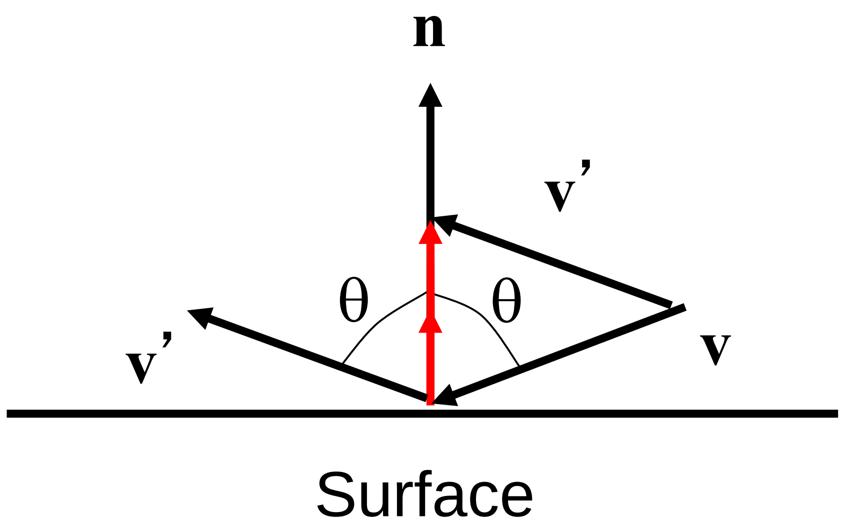
- To calculate illumination as a result of reflections
  - calculate the direction of the secondary ray at the intersection of the primary ray with the object.

given that

- **n** is the unit surface normal
- **v** is the direction of the primary ray
- **v'** is the direction of the secondary ray as a result of reflections

$$\mathbf{v}' = \mathbf{v} - (2\mathbf{v} \cdot \mathbf{n})\mathbf{n}$$

Mirror reflection



$$\mathbf{v}' = \mathbf{v} - (2\mathbf{v} \cdot \mathbf{n})\mathbf{n}$$

Mirror reflection

The $\mathbf{v}$, $\mathbf{v'}$ and $\mathbf{n}$ are unit vectors and coplanar so:

$$\mathbf{v'} = \alpha\, \mathbf{v} + \beta\, \mathbf{n}$$

Taking the dot product with $\mathbf{n}$ yields the eq.:

$$\mathbf{n}\cdot\mathbf{v'} = \alpha\, \mathbf{v}\cdot\mathbf{n} + \beta = \mathbf{v}\cdot\mathbf{n}$$

Requiring $\mathbf{v'}$ to be a unit vector yields the second eq.:

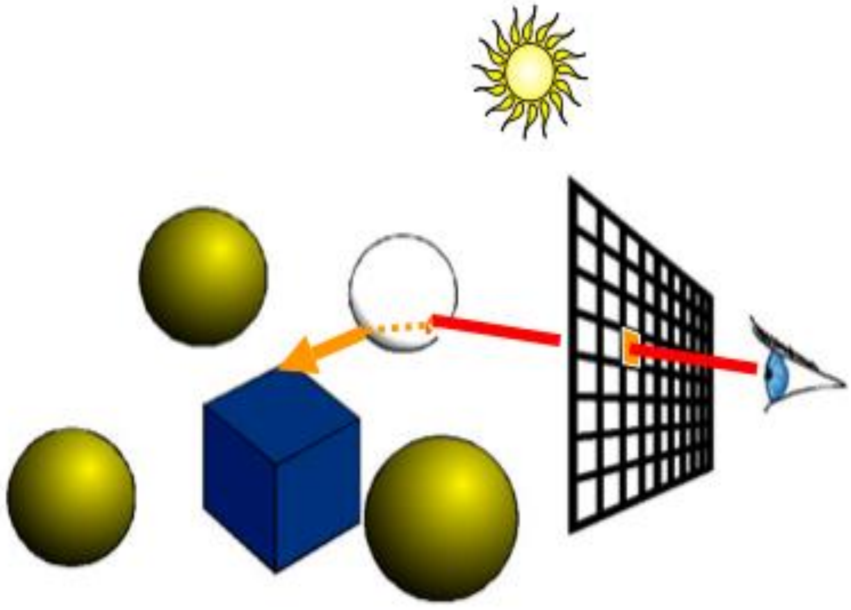
$$1 = \mathbf{v'}\cdot\mathbf{v'} = \alpha^2 + 2\,\alpha\,\beta\, \mathbf{v}\cdot\mathbf{n} + \beta^2$$

Solving both equations yields:

$$\mathbf{v'} = \mathbf{v} - (2\mathbf{v}\cdot\mathbf{n})\mathbf{n}$$

Transparency

- Compute transmitted contribution
- Cast ray in refracted direction
- Multiply by transparency coefficient

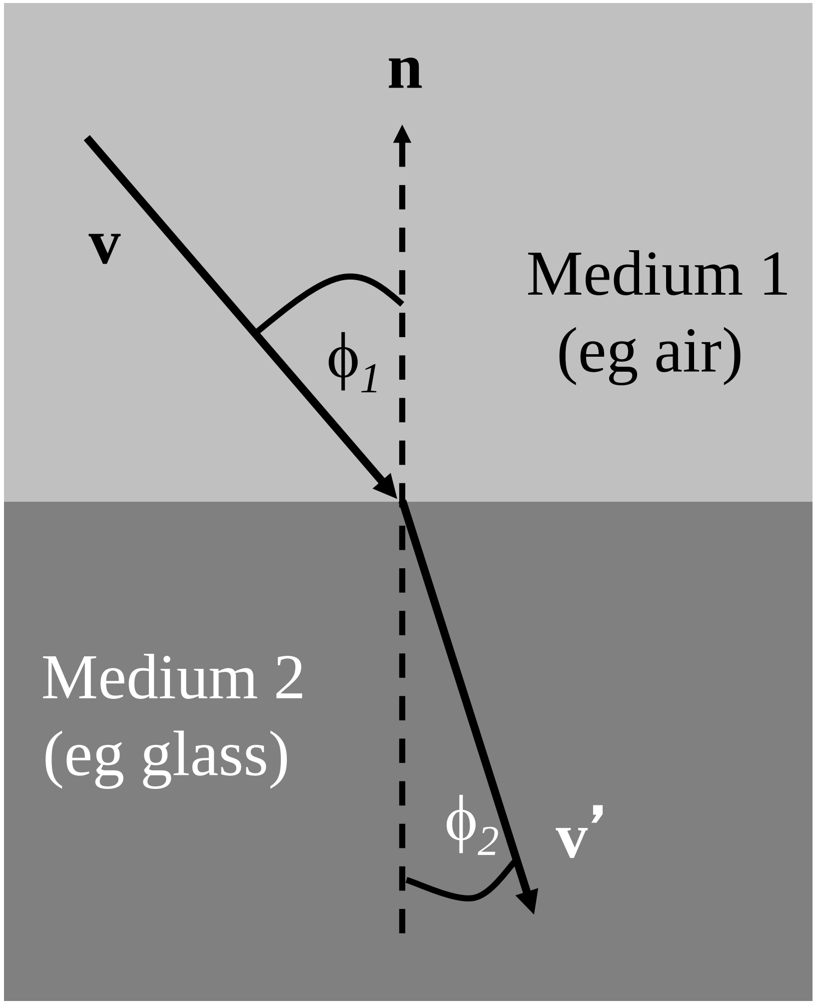


Refraction

- The angle of the refracted ray can be determined by Snell's law:

$$h_1 \sin(f_1) = h_2 \sin(f_2)$$

- η_1 is a constant for medium 1
- η_2 is a constant for medium 2
- ϕ_1 is the angle between the incident ray and the surface normal
- ϕ_2 is the angle between the refracted ray and the surface normal



Refraction

- In vector notation Snell's law can be written:

$$k_1(\mathbf{v}\times\mathbf{n}) = k_2(\mathbf{v}'\times\mathbf{n})$$

- The direction of the refracted ray is

$$\mathbf{v}' = \frac{\eta_1}{\eta_2}\left(\left[\sqrt{(\mathbf{n}\times\mathbf{v})^2 + \left(\frac{\eta_2}{\eta_1}\right)^2 - 1} - \mathbf{n}\times\mathbf{v}\right]\times\mathbf{n} + \mathbf{v}\right)$$

Refraction

- This equation only has a solution if

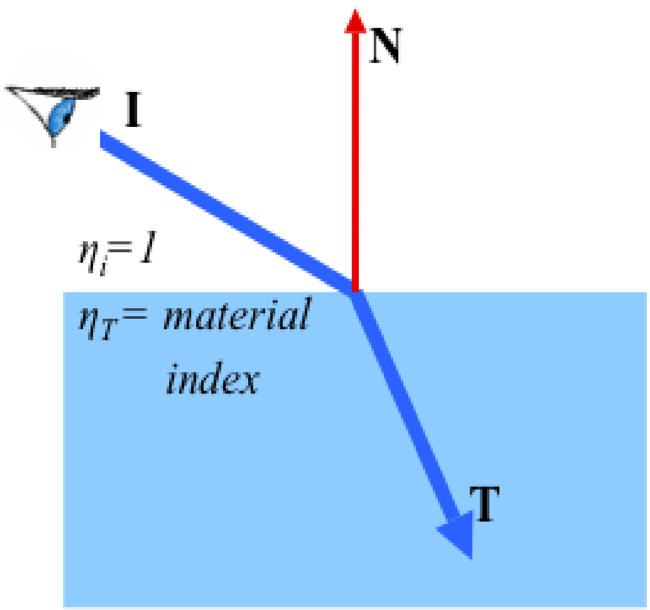
$$(\mathbf{n}\cdot\mathbf{v})^2 > 1 - \left(\frac{\eta_2}{\eta_1}\right)^2$$

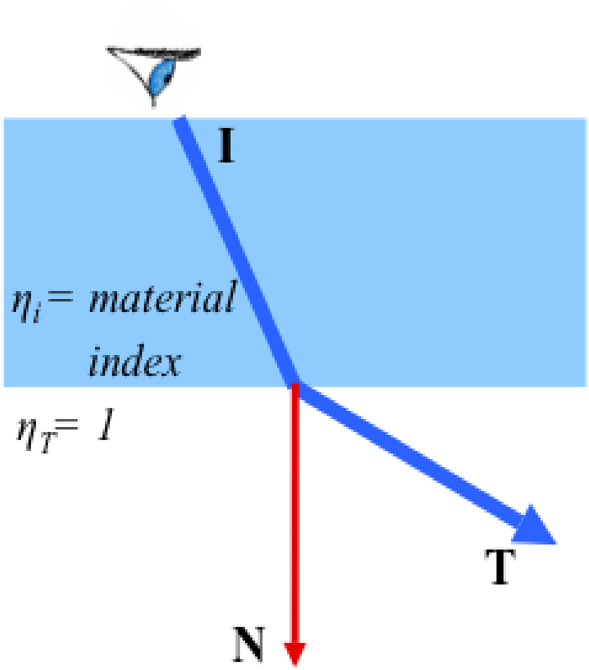
- This illustrates the physical phenomenon of the limiting angle:
  - if light passes from one medium to another medium whose index of refraction is low, the angle of the refracted ray is greater than the angle of the incident ray
  - if the angle of the incident ray is large, the angle of the refracted ray is larger than 90^{o}

  ➔ the ray is reflected rather than refracted

Refraction

- Make sure you know whether you are entering or leaving the transmissive material

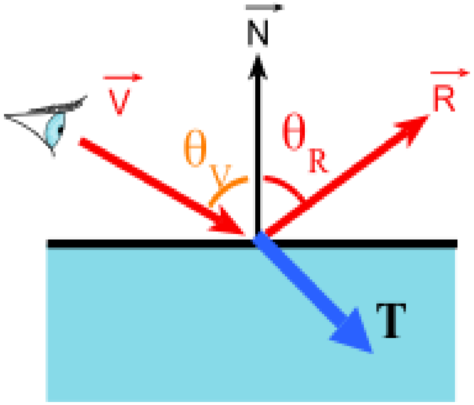




Amount of reflection and refraction

- Traditional (hacky) ray tracing
  - Constant coefficient reflection
  - Component per component multiplication
- Better: Mix reflected and refracted light according to the Fresnel factor.

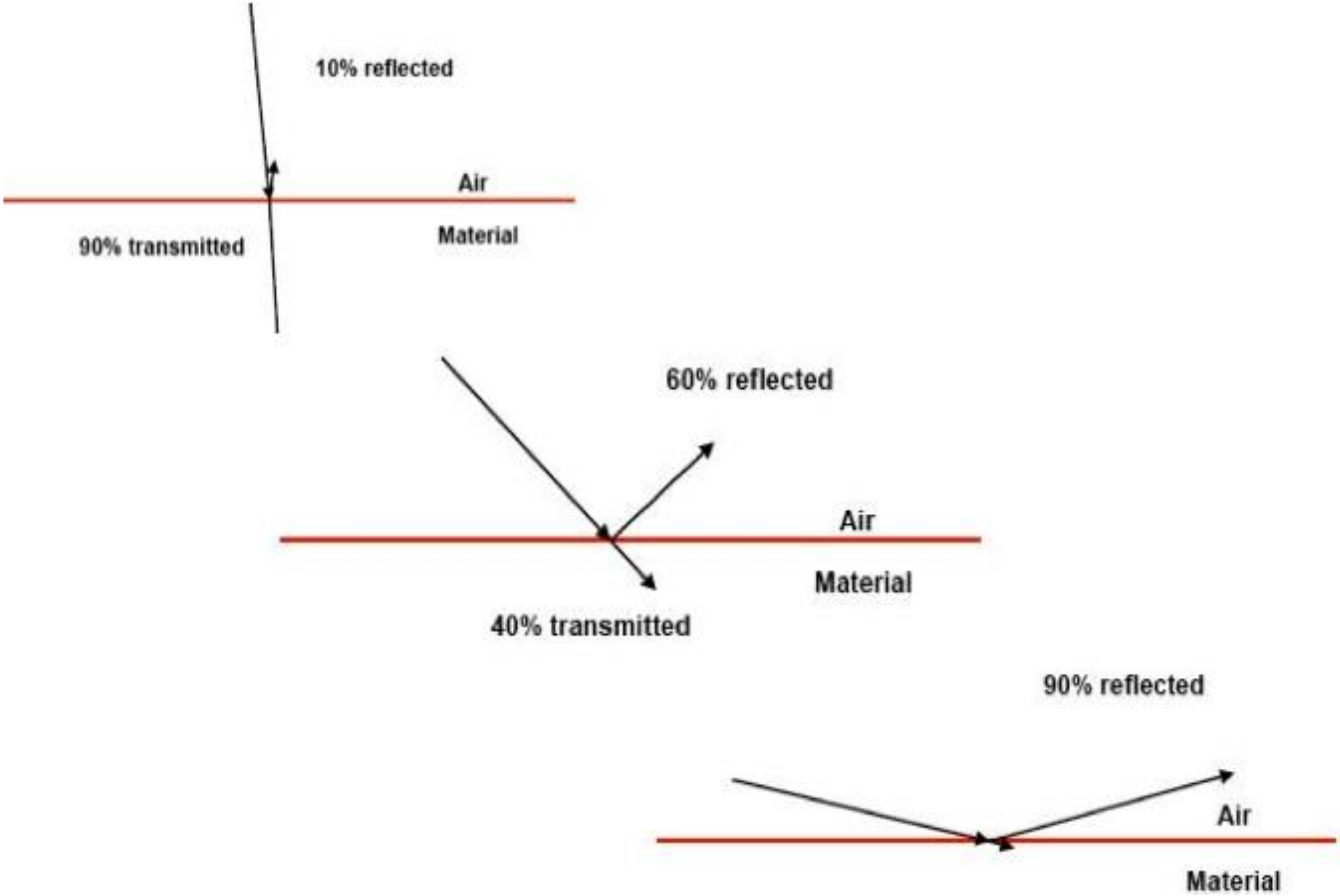
$$L = k_{fresnel} L_{reflected} + (1 - k_{fresnel}) L_{refracted}$$



Fresnel factor

- More reflection at grazing angle

Fresnel factor



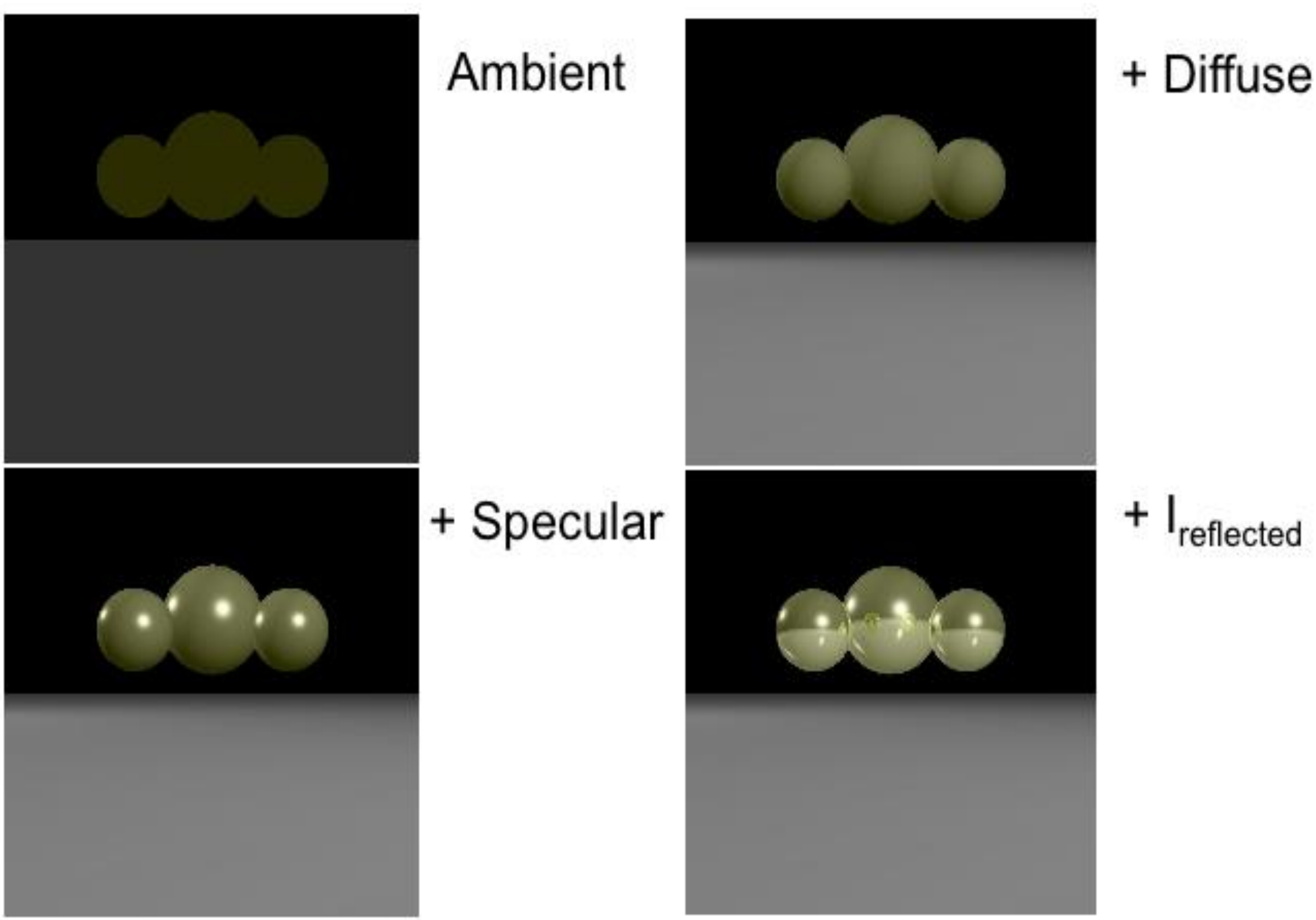
Schlick's Approximation

- Schlick's approximation

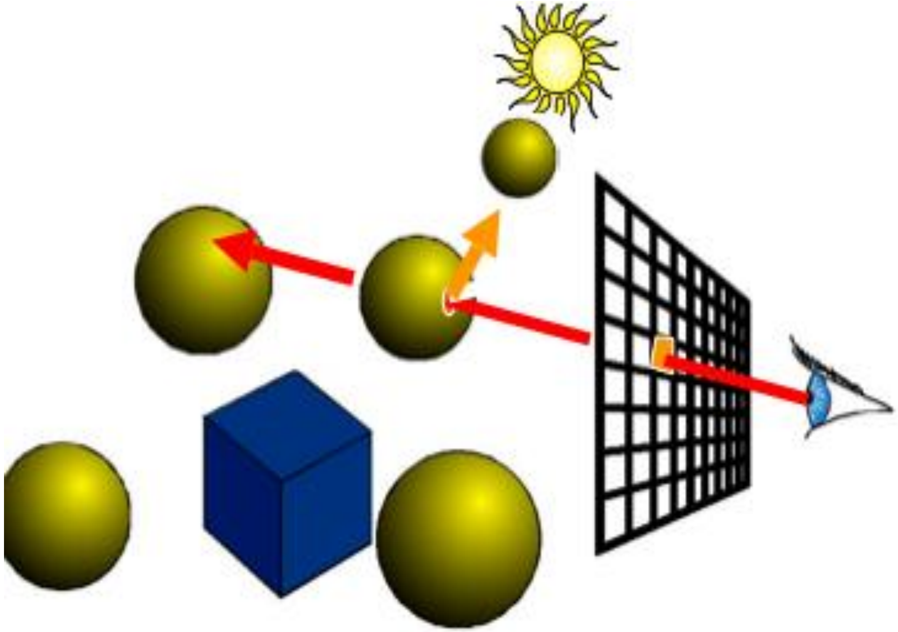
$$k_{fresnel}(\theta) = k_{fresnel}(0) + (1 - k_{fresnel}(0))(1 - (\mathbf{n} \cdot \mathbf{l}))^5$$

- $k_{fresnel}(0)$ = Fresnel factor at zero degrees
- Choose $k_{fresnel}(0) = 0.8$, this will look like stainless steel

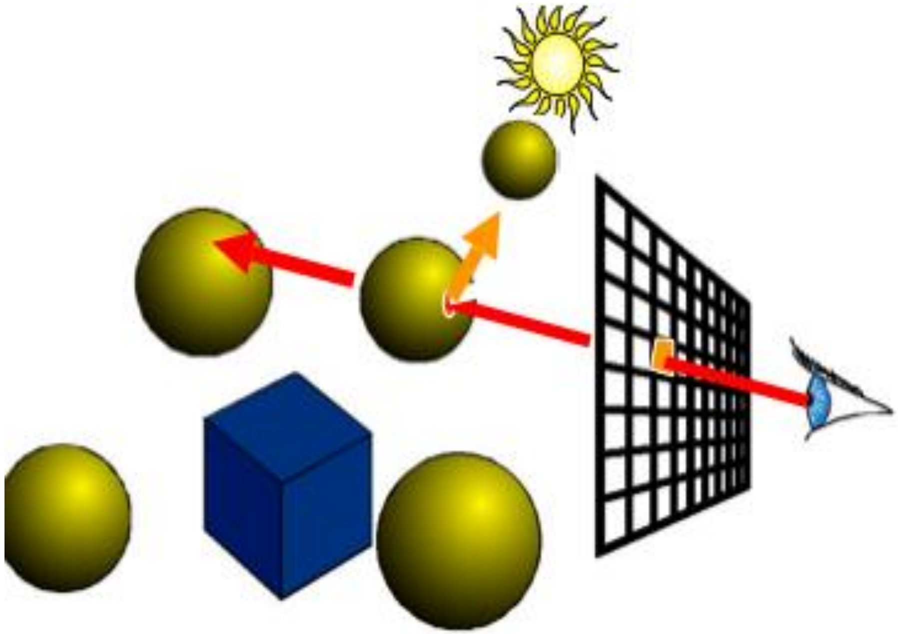
Example



How do we add shadows?



How do we add shadows?

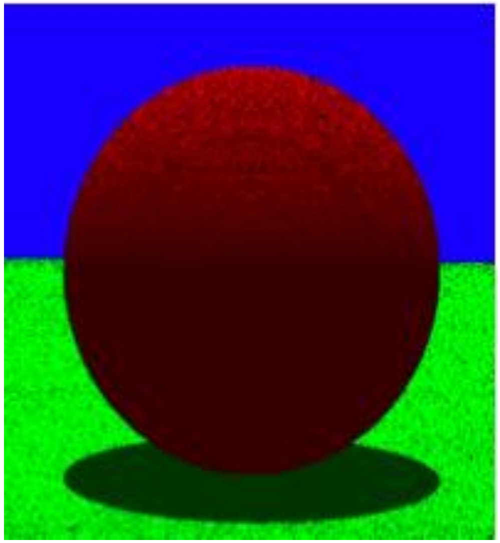


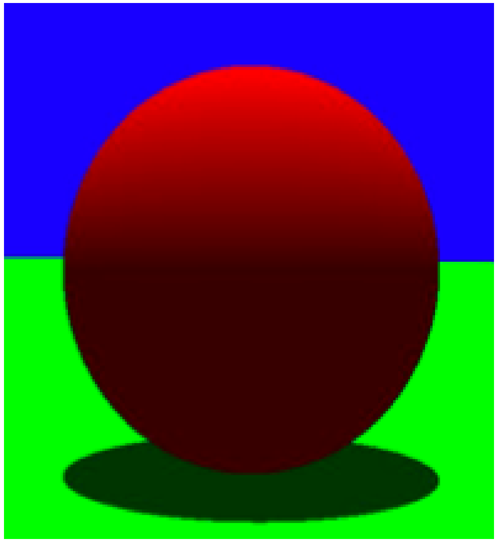
$$L = k_a + s(k_d(\mathbf{n} \cdot \mathbf{l}) + k_s(\mathbf{v} \cdot \mathbf{r})^q) I_s + k_{reflected} L_{reflected} + k_{refracted} L_{refracted}$$

$$s = \begin{cases} 0 & \text{if light source is obscured} \\ 1 & \text{if light source is not obscured} \end{cases}$$

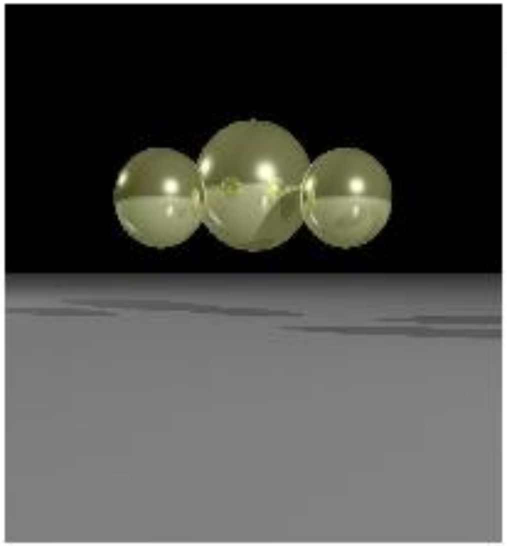
Shadows: Problems?

- Make sure to avoid self-shadowing

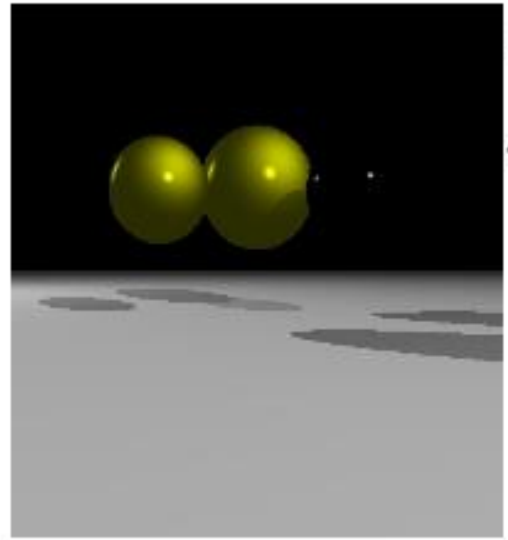




Example

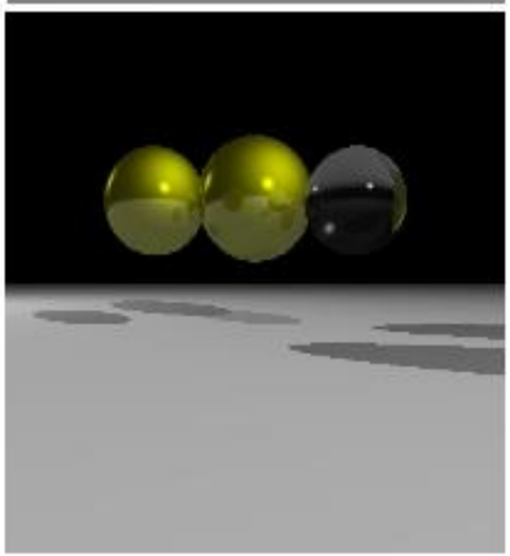


+ Shadows

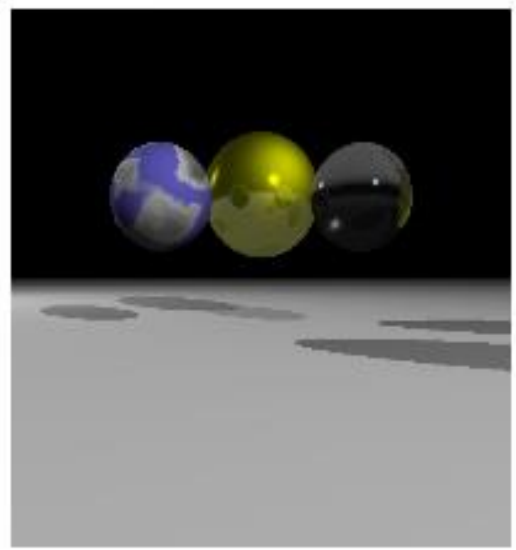


- $I_{reflected}$

+ transmitted



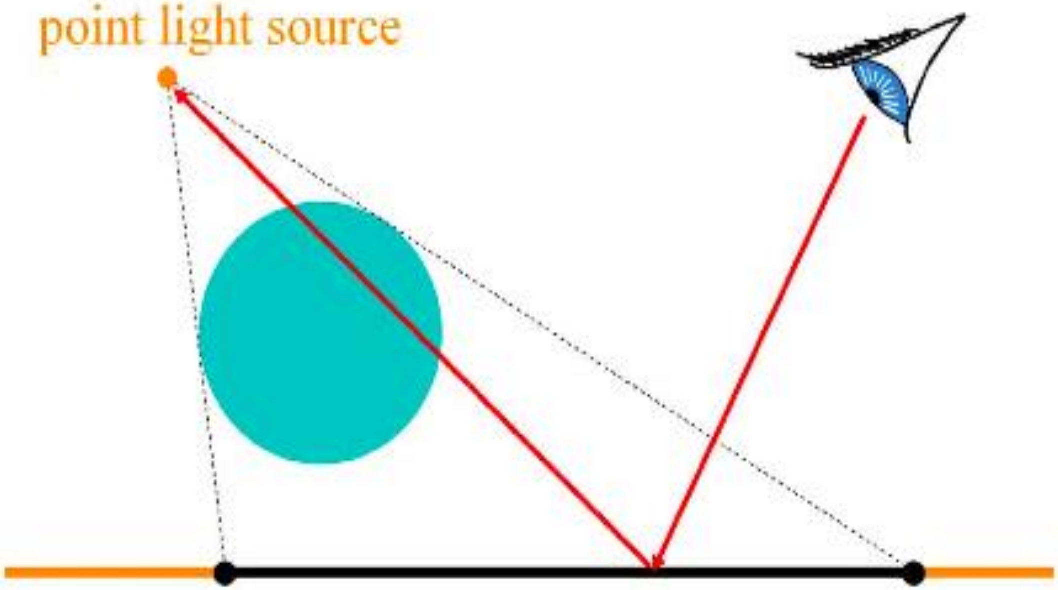
+ $I_{reflected}$

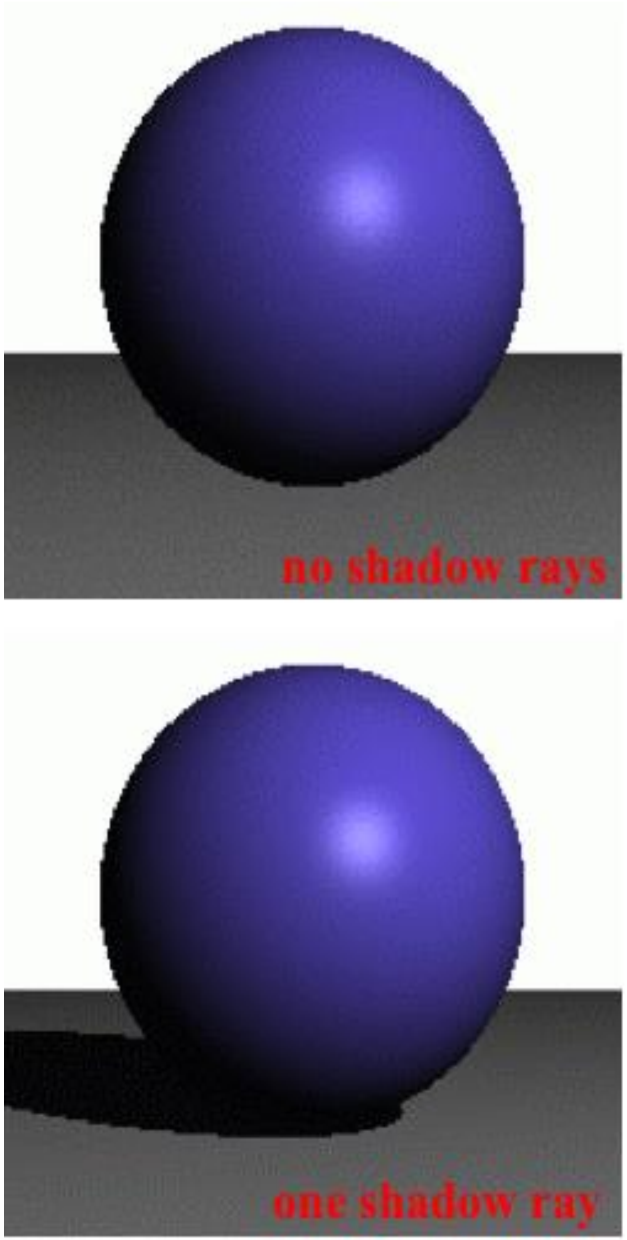


+ textures

Shadows

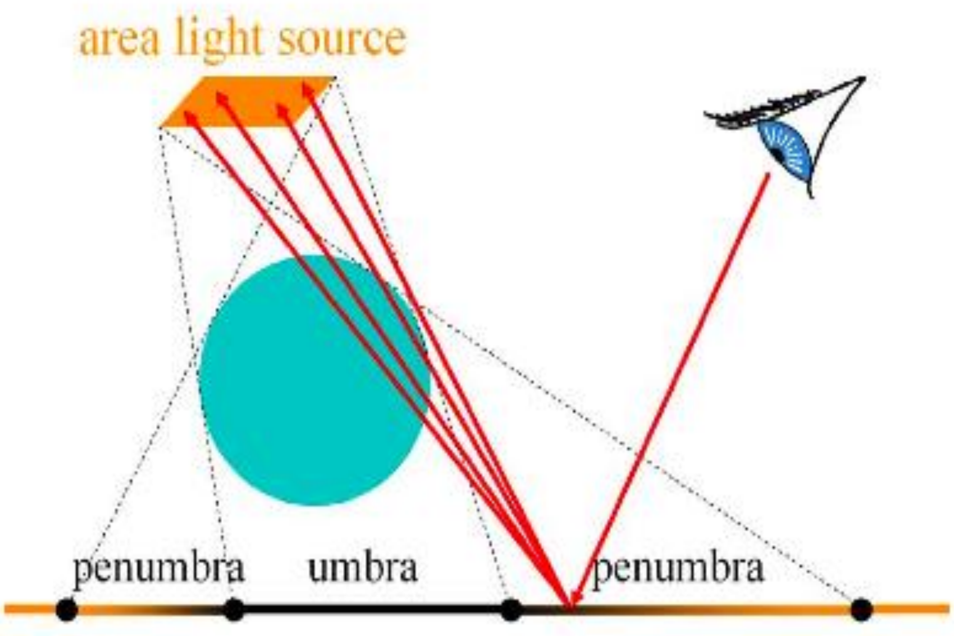
- One shadow ray per intersection per point light source

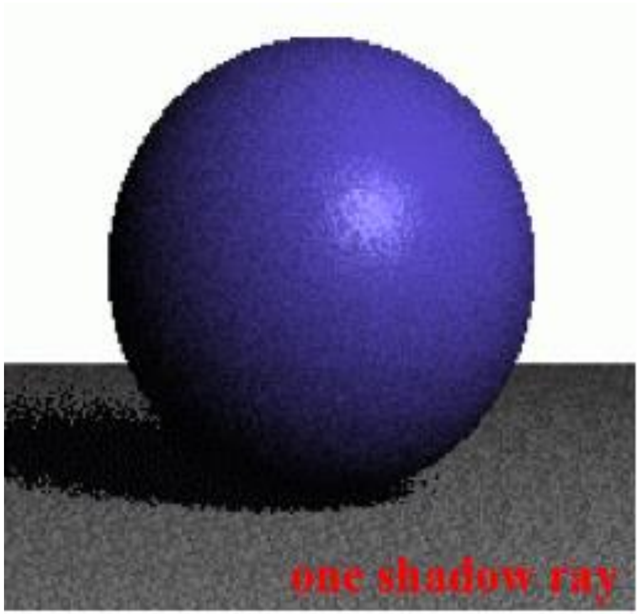


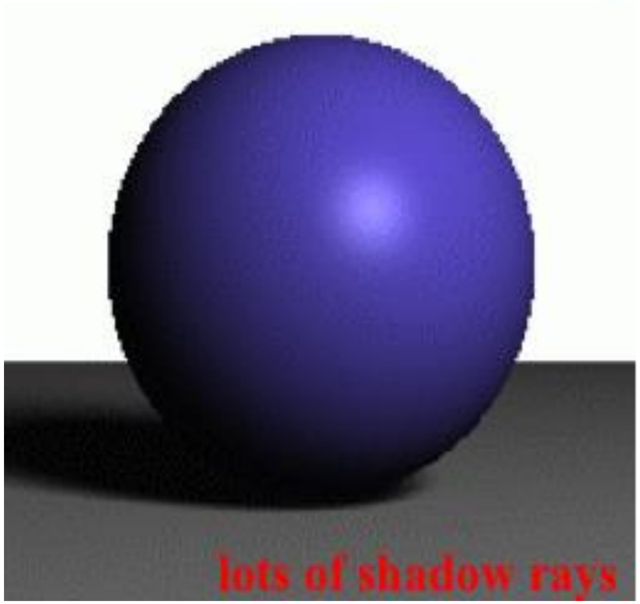


Soft shadows

- Multiple shadow rays to sample area light source

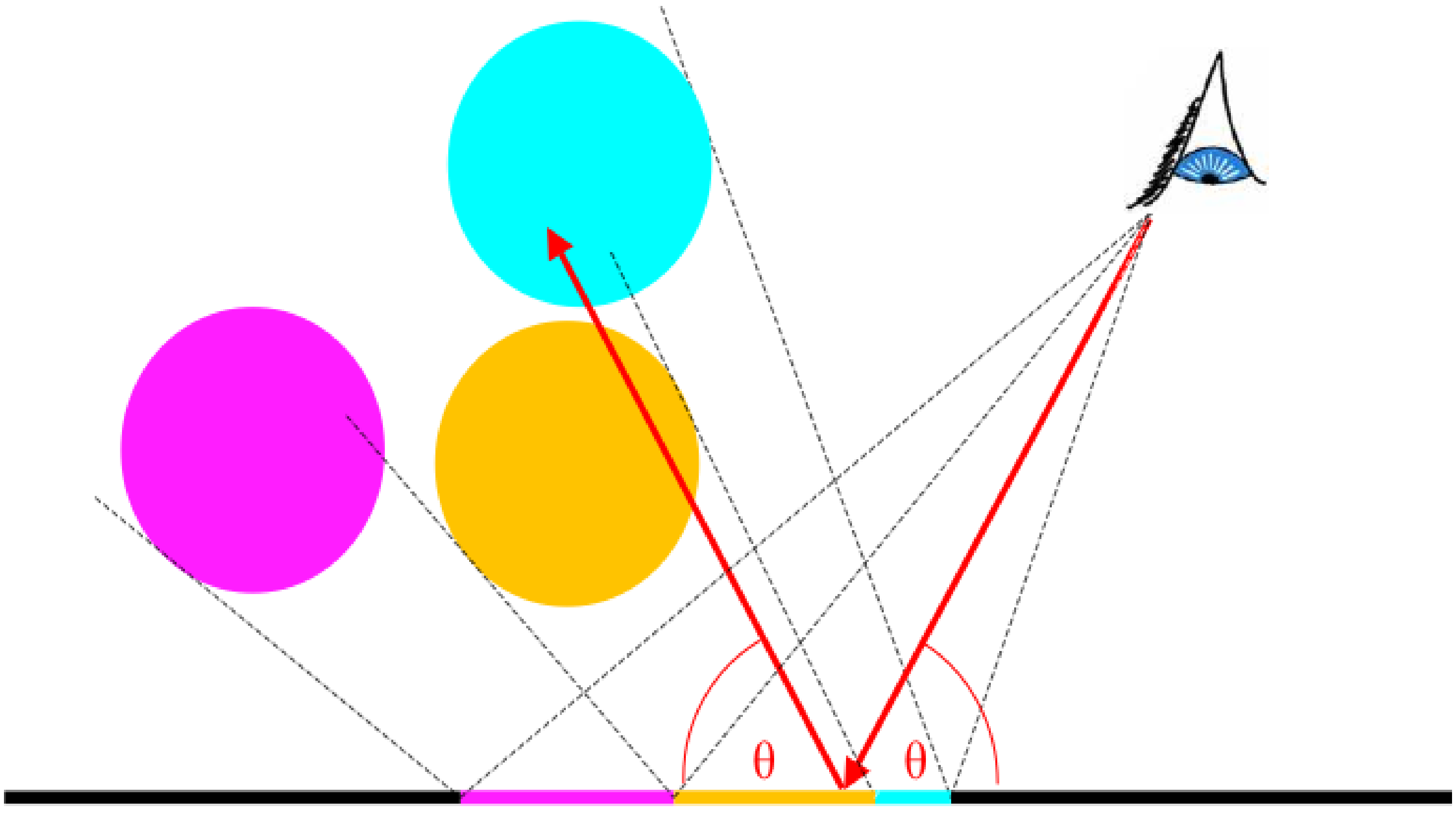






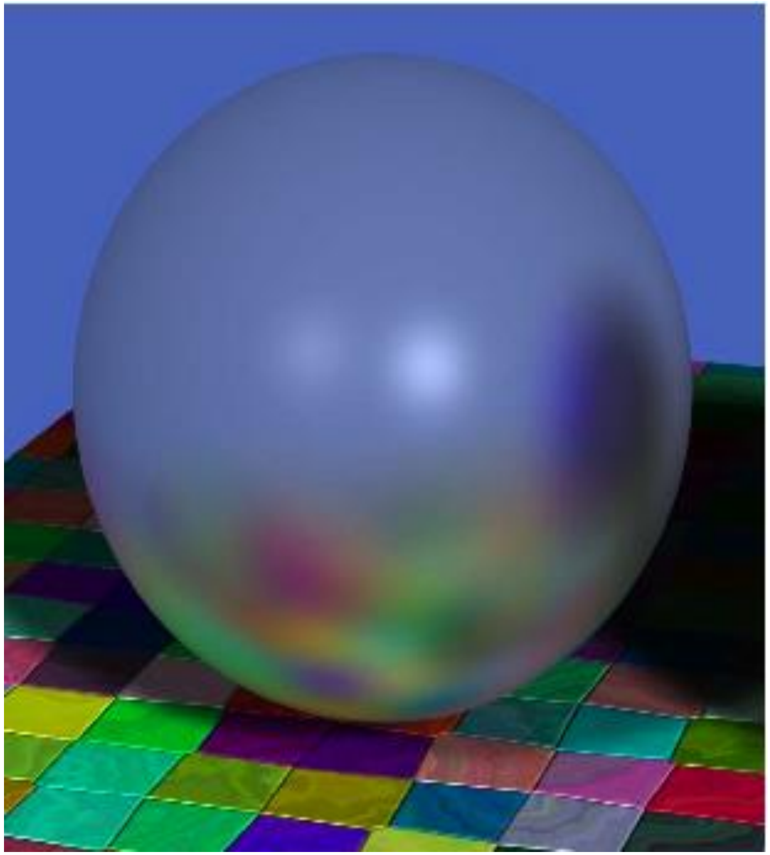
Reflection: Conventional ray tracing

- One reflection per intersection



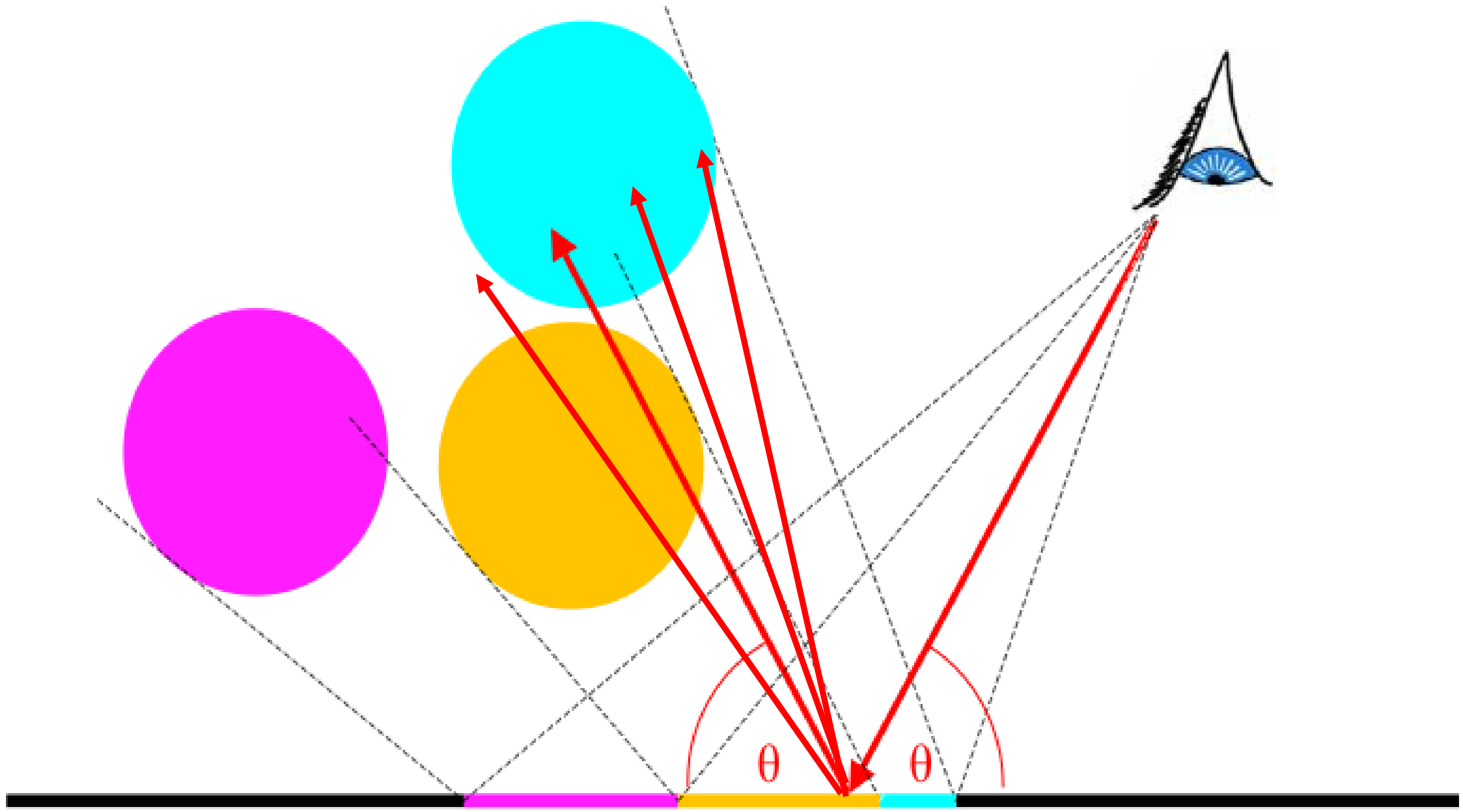
Reflection: Conventional ray tracing

- How can we create effects like this?

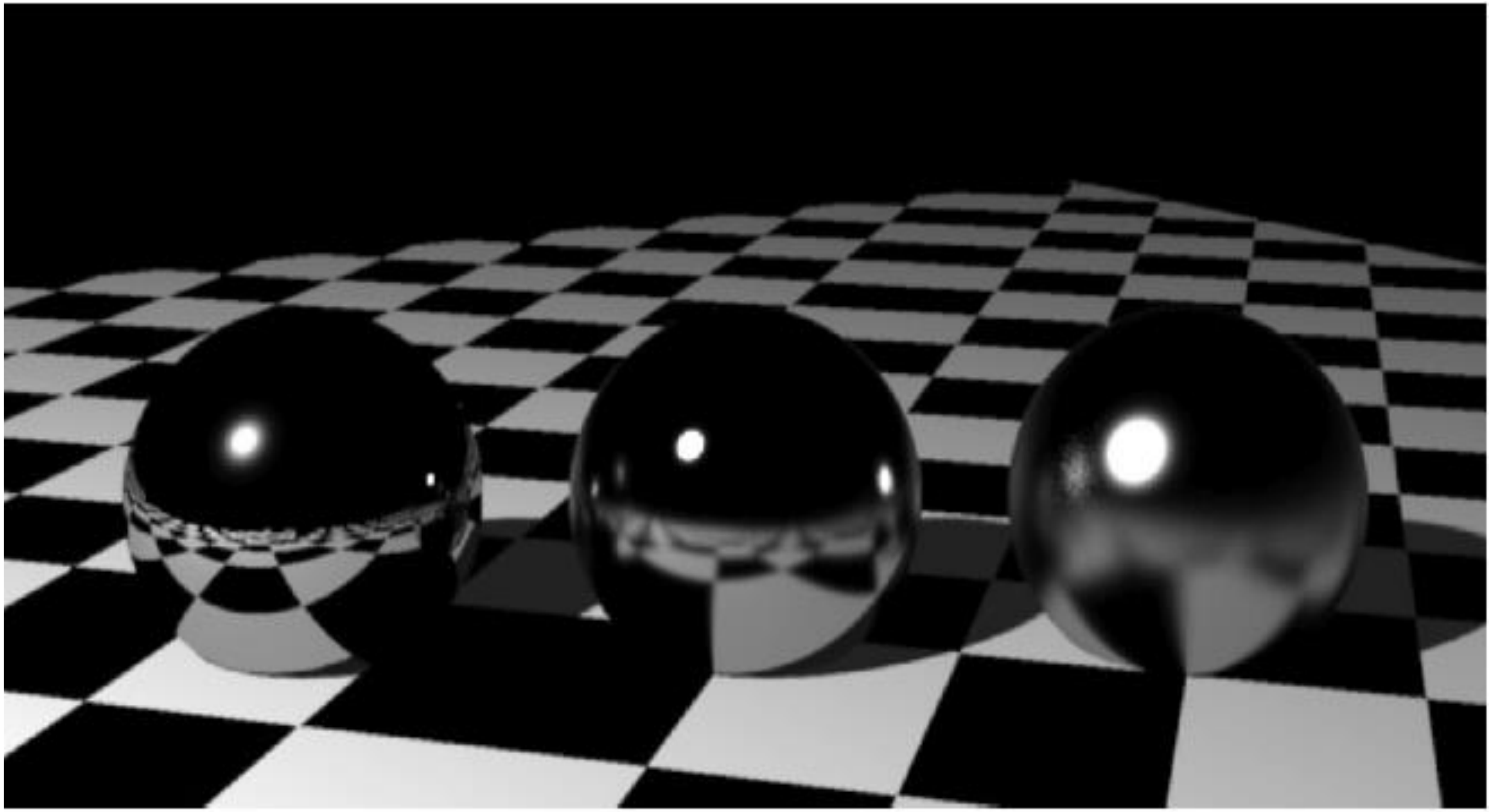


Reflection: Monte Carlo ray tracing

- Random reflection rays around mirror direction

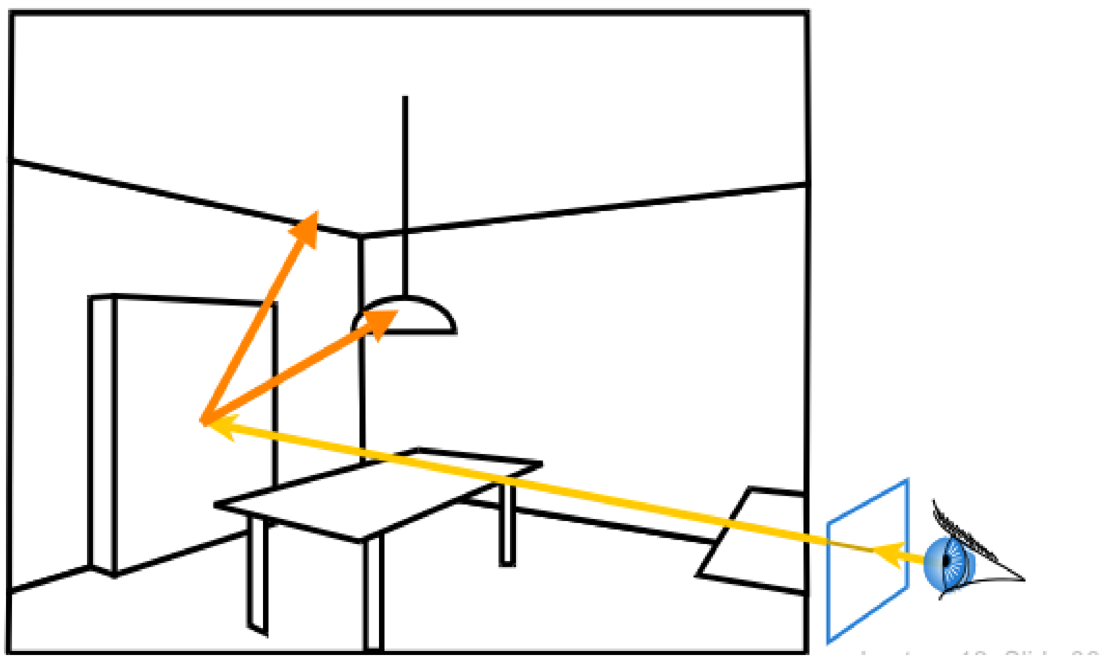


Glossy surfaces



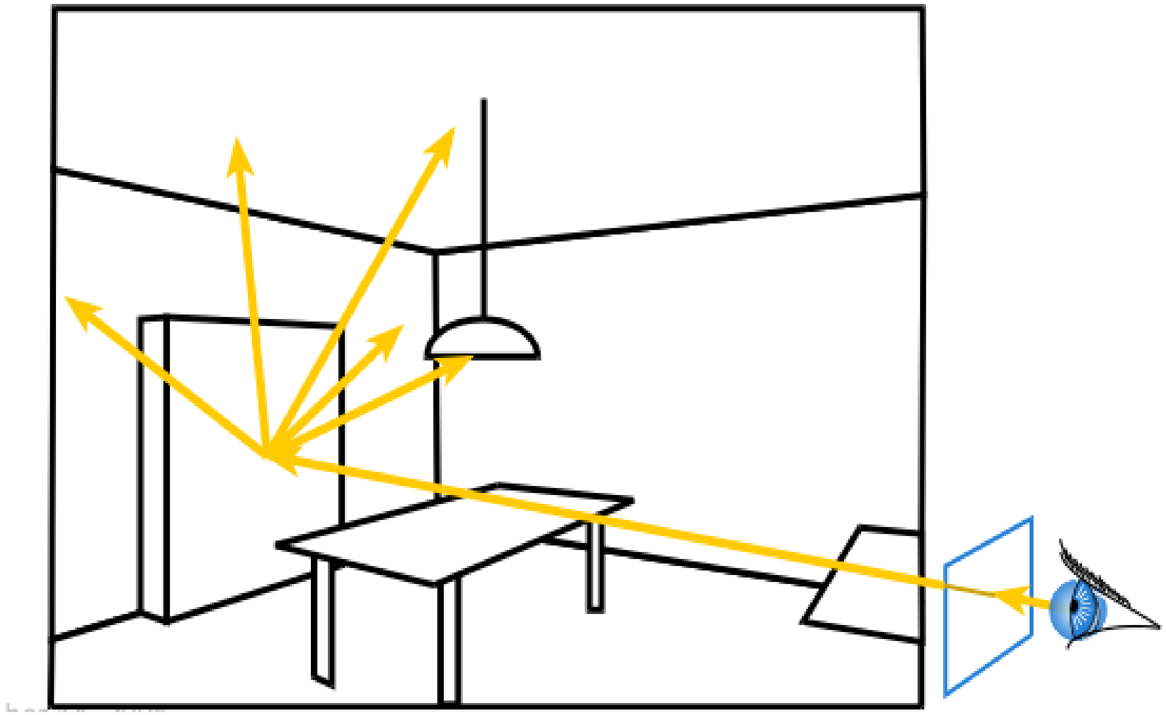
Ray tracing

- Cast a ray from the eye through each pixel
- Trace secondary rays (light, reflection, refraction)



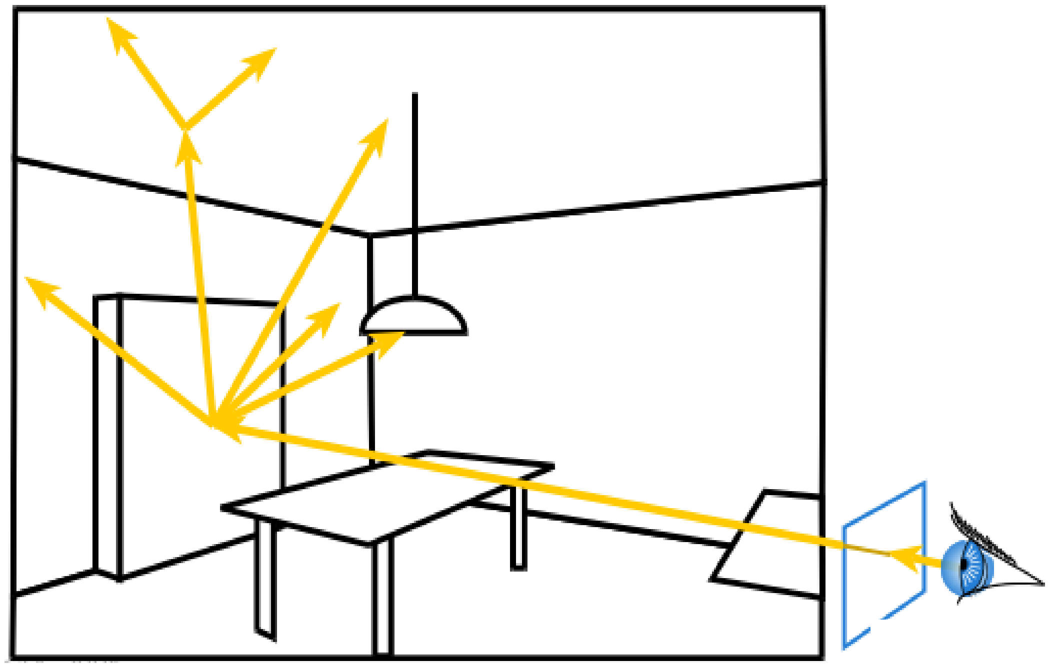
Monte-Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
  - Accumulate radiance contribution



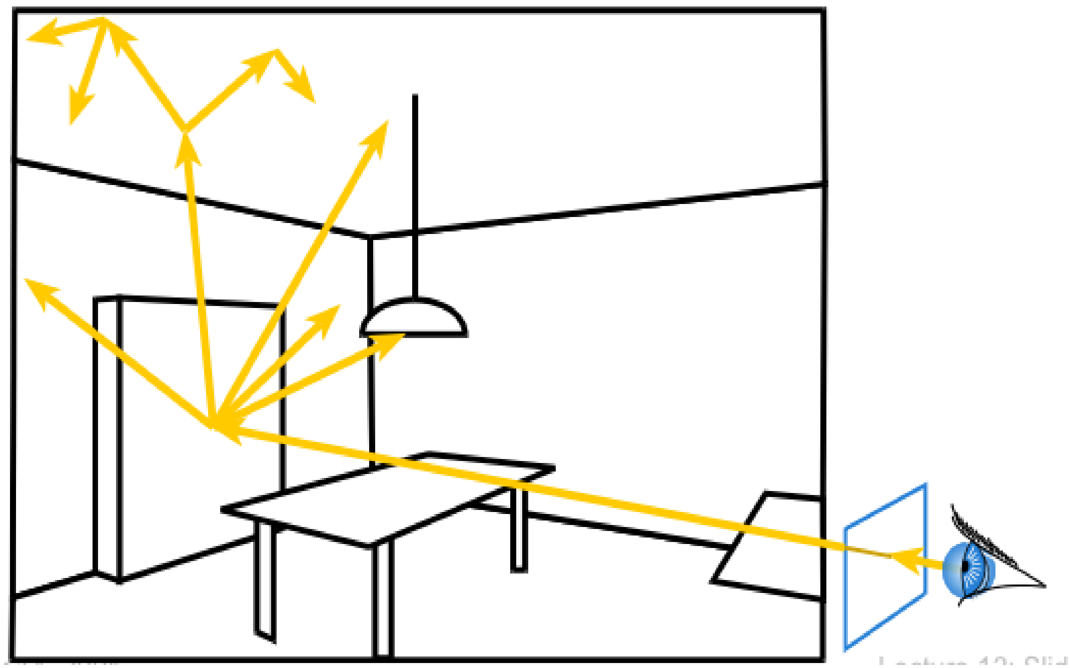
Monte-Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



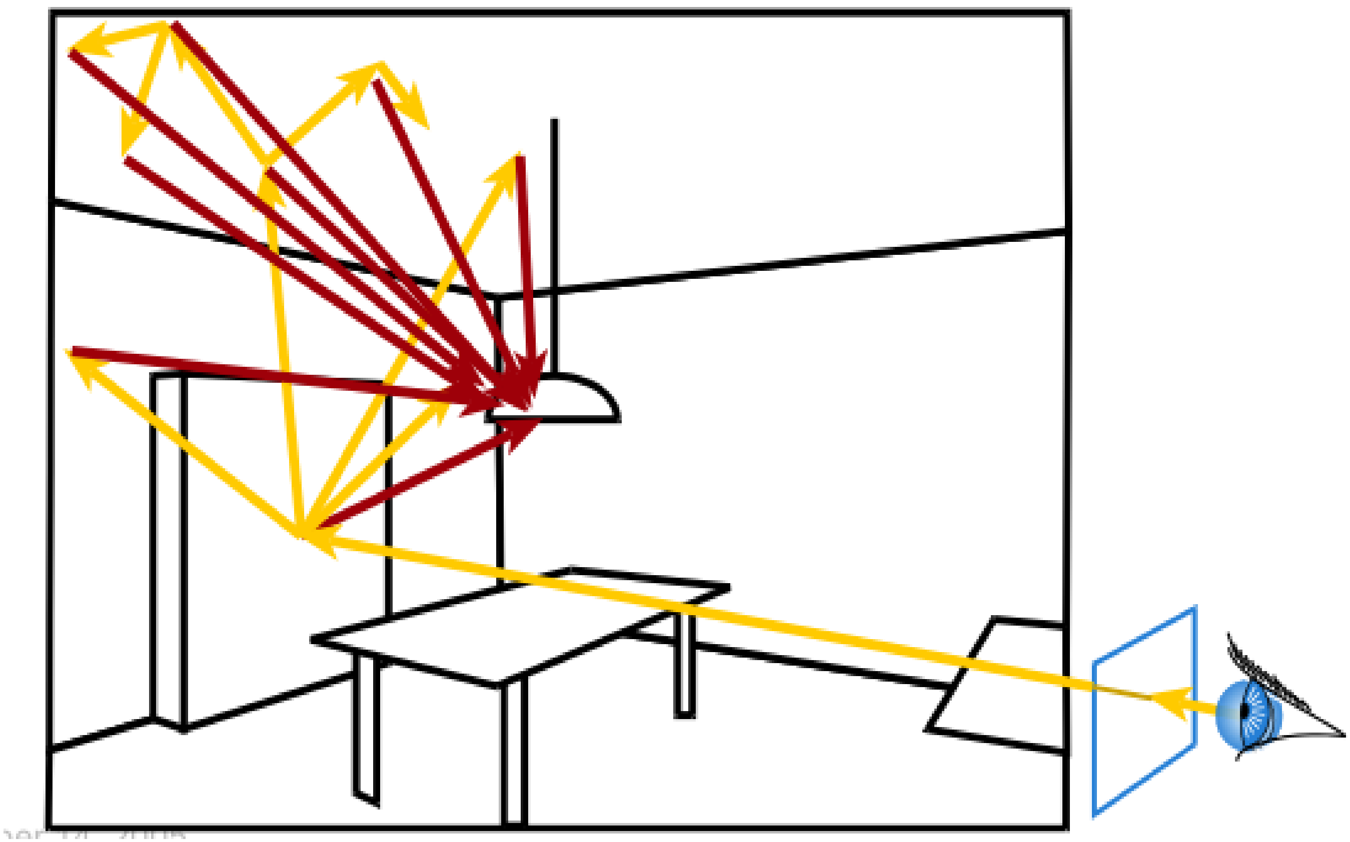
Monte-Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



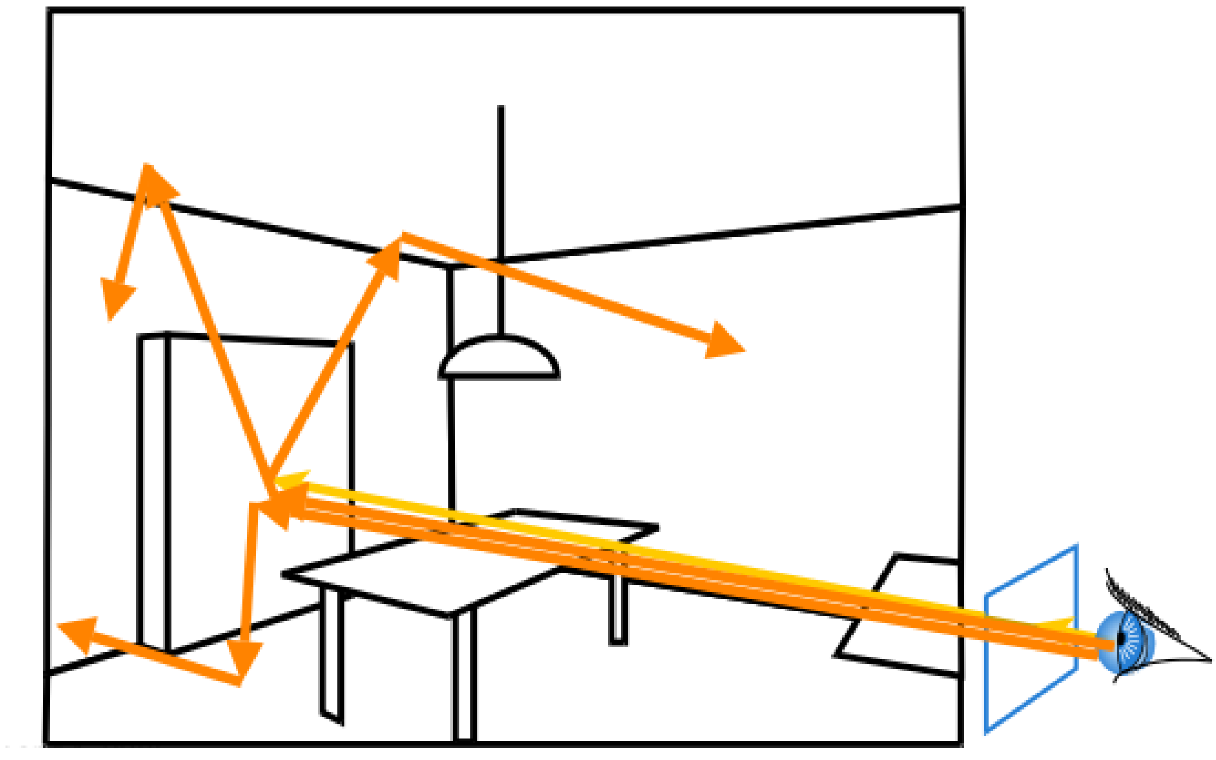
Monte-Carlo Ray Tracing

- Send rays to light

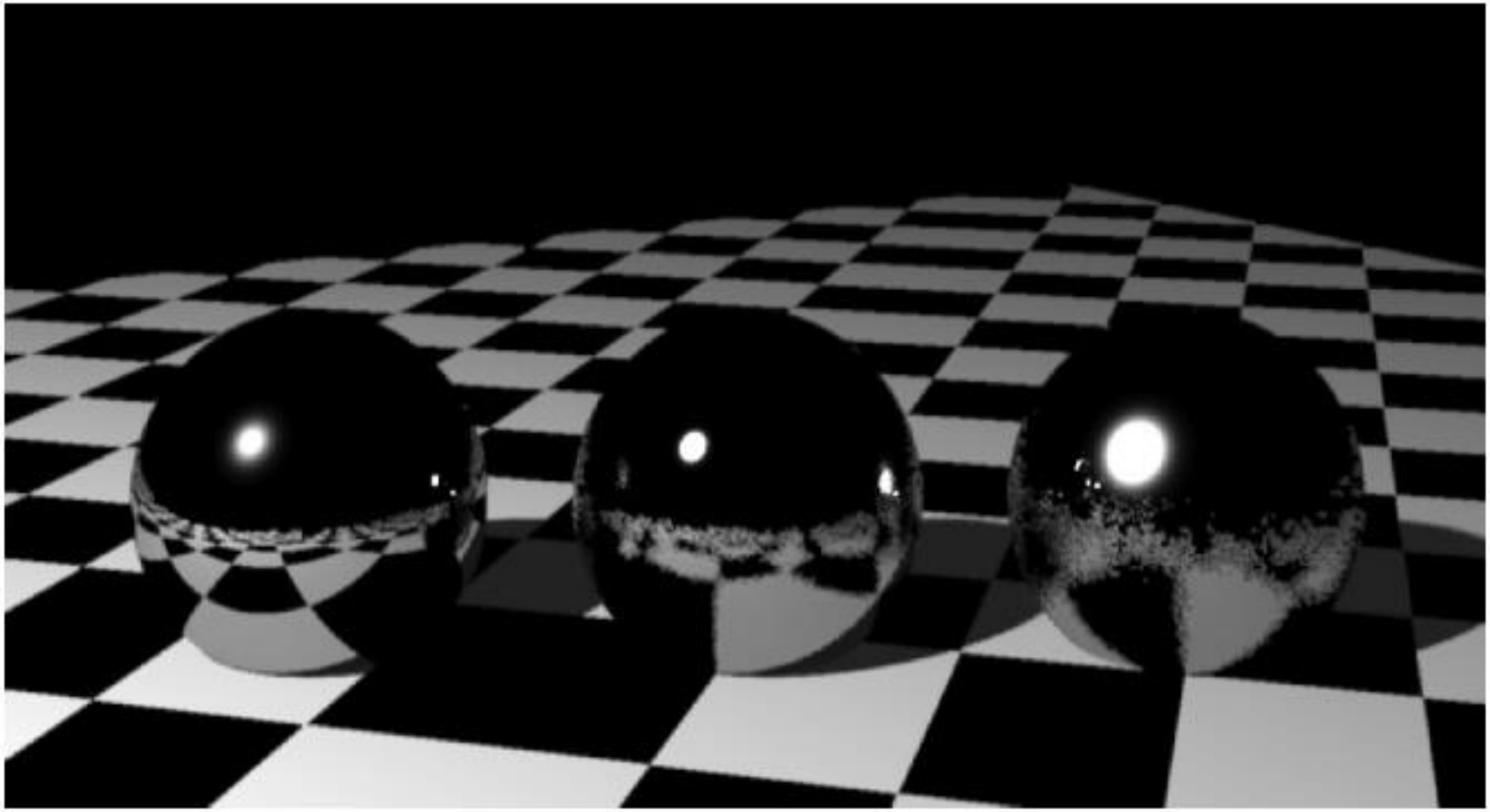


Monte-Carlo Path Tracing

- Trace only one secondary ray per recursion
- But send many primary rays per pixel

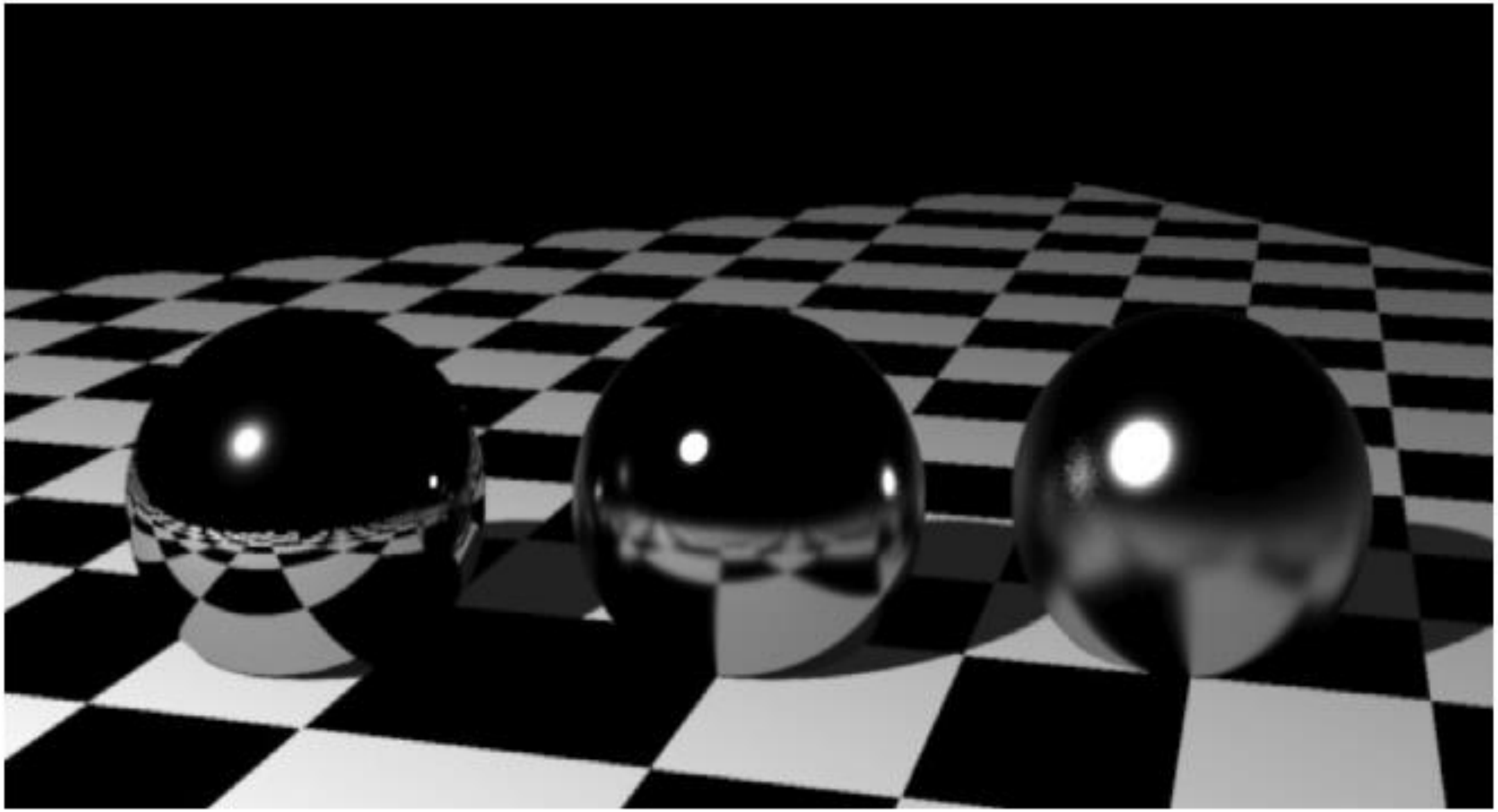


Example



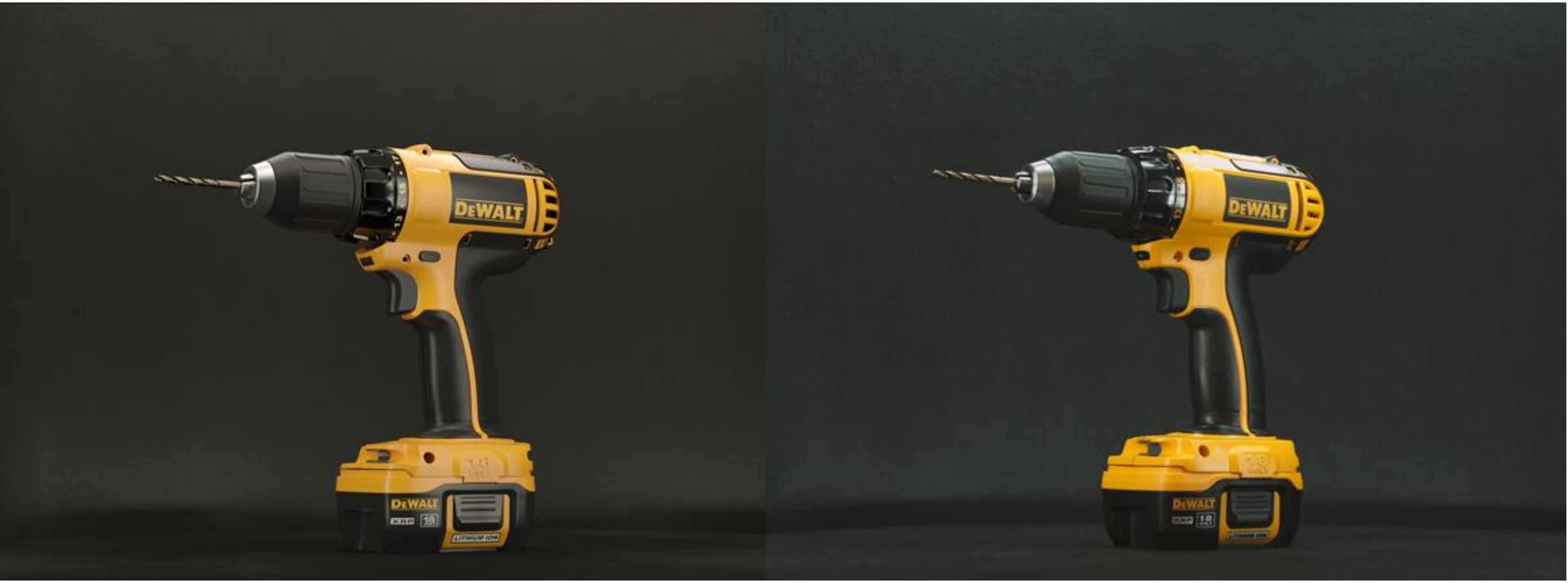
1 sample per ray

Example



256 samples per ray

Some cool pictures



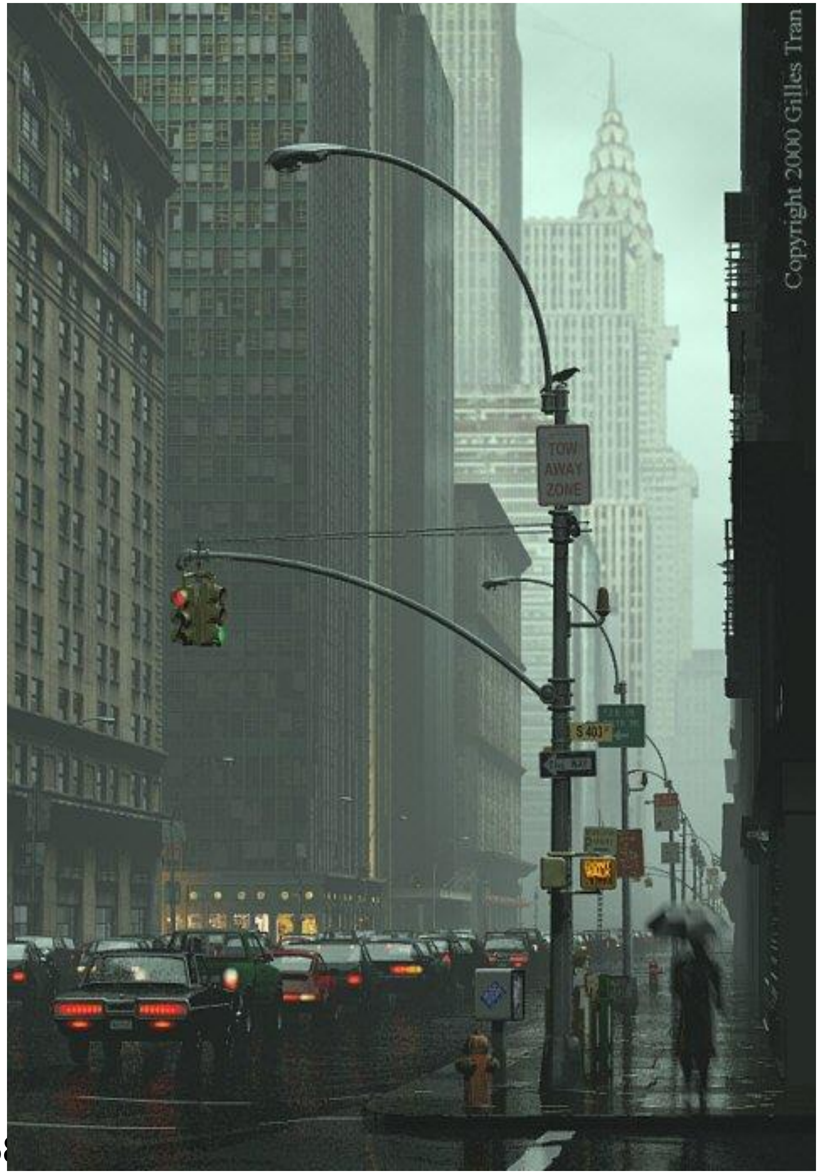
Some cool pictures

"Fukaya House" Waro Kishi

Some cool pictures

Some cool pictures

Some cool pictures



Some cool pictures

Some cool pictures

Some cool pictures

took 4.5 days to render! (10 years ago)