Problem Formulation

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Current FYP Trajectory:
The student project allocation problem that will be investigated in this thesis involves 3 sets : students, lecturers and projects. The Model A variations will explore objective functions between the student and projects sets. The Model B variations will explore the objective functions between lecturers and projects and the Model C variations will combine Model A and B concepts to generate an objective function dependent on all 3 sets. Model C variations take a dynamic integer programming approach and explore matching techniques.

1 Student Project Allocation Optimization Problem Modelling

1.1 Model A1: Naive objective function: Allocate projects to students based solely on their preferences

This model simply allocates projects to students from a subset of project preferences the students shortlists from the entire list of project proposals. This model is naive as it's far from optimal but allows for an intuitive problem formulation which will then be improved for optimality.Model A1 is designed such there are T_S students. Each student can choose a subset of T_C projects from T_P proposed projects. In this model, a student can only be allocated to one project and a project can only be allocated to one student - although in distinct real-world applications such as group project allocations, a project can be allocated to multiple students.

Where,

- T_C is the total number of project preferences.
- T_P is the total number of projects proposed.
- T_S is the total number of students.

Problem Formulation

1. Decision Variables

- $X_{i,j}$: A student can only be allocated a project that has been proposed by a lecturer. This decision can be defined with the binary variable, $X_{i,j}$:

$$X_{i,j} = \begin{cases} 1, & \text{if student } i \text{ is allocated to project } j \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

  Note : The index, i represents the i^{th} student, $\{i|1 \leq i \leq T_S, i \in \mathbb{Z}\}$. The index, j represents the j^{th} project, $\{j|1 \leq j \leq T_P, j \in \mathbb{Z}\}$

- $C_{i,j}$: A student can only be allocated a project that has been chosen as one of their preferences.

$$C_{i,j} = \begin{cases} 1, & \text{if project j is chosen as a preference by student i} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

2. Constraints

- **Each student should only be allocated one project**
  Where student i has to be allocated to 1 of the total T_P proposed projects,

$$X_{i,1} + \cdots + X_{i,T_P} = 1 \tag{3}$$

  This is generalized as:

$$\sum_{j=1}^{T_P} X_{i,j} = 1 \tag{4}$$

- **Each project should only be allocated to one student**
  Generalized by:

$$\sum_{i=1}^{T_S} X_{i,j} = 1 \tag{5}$$

- **Lastly, each student can only be allocated a project that is part of their preferences subset**
  Where student i has to choose T_C project preferences,

$$C_{i,1} + \cdots + C_{i,T_P} = T_C \tag{6}$$

  Notice, if project j, is not chosen by student i then $C_{i,j} = 0$ and project j would not be applicable for allocation to student i.
  This is generalized as:

$$\sum_{j=1}^{T_P} C_{i,j} = T_C \tag{7}$$

Out of the 5 possible preferences, student i can only be allocated to 1 of their project preferences.

$$C_{i,1} \times X_{i,1} + \cdots + C_{i,T_P} \times X_{i,T_P} = 1 \tag{8}$$

This is generalized as:

$$\sum_{j=1}^{T_P} C_{i,j} \times X_{i,j} = 1 \tag{9}$$

3. Objective Function

$$Z_A = \sum_{i=1}^{T_S} \sum_{j=1}^{T_P} (C_{i,j} \times X_{i,j}) \tag{10}$$

1.2 Model A2: Improvement on Naive objective function: Allocate projects to students based on their preference rankings

This model has students rank their chosen project preferences instead of simply selecting a subset of project preferences as in Model A1. Although rankings can be defined by percentage weights, non-integer weights for a more precise ranking, Model A2 will explore integer weights that are linear to the total number of preferences, T_P. Therefore, given that $T_P = 5$, students should assign a weight of 5 to their least preferable project and a weight of 1 to their most preferable project. To favor lower rankings this optimization problem will be a **minimization** problem. If the weights were reversed then we would formulate a maximization problem.

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1. Decision variables
   - $X_{i,j}$ remains unchanged
   - $C_{i,j}$: Can take any value in set C where, $C \in [1, 2, 3, 4, 5]\}$. Since students are assigning integer values to their project preferences the variable $C_{i,j}$ is no longer a binary variable as in Model A1 but an integer variable i.e $C_{i,j} \in \mathbb{Z}$

2. Constraints
   The first two constraints in Model A1 remain unchanged however the last constraint changes as follows.
   - **Each student can only be allocated a project that is part of their preferences subset**
     Where student i ranks their 5 project preferences by assigning each of the project preferences with integer $C_{i,j}$,

$$\underbrace{C_{i,1} \times X_{i,1} + \cdots + C_{i,T_P} \times X_{i,T_P}}_{\text{Preference Equation}} \in C \tag{11}$$

where, $C \in [1, 2, 3, 4, 5]\}$

$$\text{Preference Equation} = \begin{cases} 1, & \text{if student } i \text{ allocated 1st preference} \\ 2, & \text{if student } i \text{ allocated 2nd preference} \\ 3, & \text{if student } i \text{ allocated 3rd preference} \\ 4, & \text{if student } i \text{ allocated 4th preference} \\ 5, & \text{if student } i \text{ allocated 5th preference} \end{cases} \tag{12}$$

Hence,

$$\underbrace{C_{i,1} \times X_{i,1} + \cdots + C_{i,T_P} \times X_{i,T_P}}_{\text{Preference Equation}} \geq 1 \tag{13}$$

This is generalized as:

$$\sum_{j=1}^{T_P} C_{i,j} \times X_{i,j} \geq 1 \tag{14}$$

3. Objective Function

$$\text{Minimize } Z_A = \sum_{i=1}^{T_S} \sum_{j=1}^{T_P} (C_{i,j} \times X_{i,j}) \tag{15}$$