Adjoint Computational Electromagnetics
Ray Optics

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The adjoint method is used to inverse design High frequency EM problems where light rays are governed by the Eikonal equation.

The Eikonal Equation

The Eikonal equation is $F = ma$ for ray optics!. It can be written as a differential equation in the phase.

$$\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 - \epsilon(x,y) = 0 \tag{1}$$

where $\epsilon(x,y)$ is the dielectric profile. Gradient of a wavefront at a position (x,y) defines the direction of the wave propagation. The contours of solution are the wavefronts.

Formulation

$$\begin{aligned} \min_{\epsilon} \quad & G(\epsilon) = \int g(\phi,\epsilon)dxdy \\ \text{s.t.} \quad & A(\phi) = \left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 - \epsilon(x,y) = 0 \\ & I(\phi(x_s),\epsilon) = 0 \end{aligned} \tag{2}$$

Sensitivity Analysis

$$L = G(\epsilon) + \int \lambda A(\phi)\, dxdy \tag{3}$$