Introduction to the Adjoint Method

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Outline

Adjoint Sensitivity Analysis of Linear Systems

- Consider the Ordinary Differential Equation (ODE)

$$a(t,z)\frac{d^2\psi}{dt^2} + b(t,z)\frac{d\psi}{dt} + c(t,z)\psi = s(t,z) \tag{1.1}$$

- z is a design parameter that we want to change in order to minimize the objective function G

$$G = \int_0^T g(\psi, z)\, dt \tag{1.2}$$

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- To minimize G, we need the gradient with respect to the design parameter z.
- The design parameter z can then be updated with the steepest descent
$$z = z - \alpha \frac{dG}{dz} \tag{1.3}$$
- Local descent involves iteratively determining a descent direction ($\frac{dG}{dz}$) and then taking a step α in that direction and repeating that process until convergence or some termination condition is met

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- Lets modify the objective function G

$$G = \int_0^T g(\psi, z)\, dt + \int_0^T \lambda(t) R\, dt \tag{1.4}$$

- Where the residual
  $R = 0 = a(t,z)\frac{d^2\psi}{dt^2} + b(t,z)\frac{d\psi}{dt} + c(t,z)\psi - s(t,z)$
- The equation is still valid and for any λ, since we are multiplying λ by zero and just added a zero to the whole equation.