Sensitivity Calculations

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Sensitivities are used as measures of robustness for engineering systems. In many applications, one is interested about the system performance under small variations of a set of design parameters. In inverse device design, sensitivities guides the search within the space spanned by a set of design parameters.

Problem Formulation

Assuming $\mathbf{G}$ is a vector of design merits $(G_1[\mathbf{x}], G_2[\mathbf{x}], \ldots, G_n[\mathbf{x}])$, where each component is a scaler function of m design parameters $\mathbf{x}$ $(x_1, x_2, \ldots, x_m)$. The goal is to find the sensitivity of the design merit G_i with respect to the design parameter x_j:

$$S_{ij} = \frac{dG_i}{dx_j} \tag{1}$$

The entries S_{ij} form the elements of the $n \times m$ Jacobian matrix S which maps m input parameters to n output merits. The Jacobian could be seen as a generalization of the slope constant s in $g(x) = sx$, but now is used for multivariate vector functions. The entries of row S_i^T are the sensitivities of the merit function G_i with respect to all the design parameters. The entries of column S_j are the sensitivities of all the merit functions with respect to the single design parameter x_j

Numerical Differentiation

Finite Difference Method

Complex Step Method

Automatic Differentiation

Automatic Forward-Mode Differentiation

Automatic Reverse-Mode Differentiation