Adjoint Computational Electromagnetics Eigenvalue Problems

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The sensitivities of eigenvalues and eigenvectors due to perturbations in the deign variables are derived. The focus here is on linear, isotropic, nonmagnetic and transparent materials [1].

[1] In transparent materials, both ϵ and μ are real and positive.

Problem Formulation

Maxwell's equation for reads

$$\nabla \times \mathbf{E} = -j\mu_0\omega\mathbf{H} \tag{1a}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon_0\epsilon(r)\mathbf{E} \tag{1b}$$

which can be combined into a single PDE [2]

[2] We will refer to this equation as the master equation.

$$\nabla \times \frac{1}{\epsilon}\nabla \times \mathbf{H} = \left(\frac{\omega}{c}\right)^2 \mathbf{H} \tag{2}$$

Equation. 2 is a standard eigenvalue problem of the form,

$$Ax = \lambda x \tag{3}$$

where we have identified the operator $A = \nabla \times \nabla\times$ and the eigenvector $x = \mathbf{E}$ with associated eigenvalue $\lambda = \omega^2$. The operator A can be shown to be hermitian and positive-semidefinite. The eigenmodes can be normalized

$$x^T x = 1 \tag{4}$$

Sensitivity Derivatives (Nondegenerate)