Shape Optimization of Photonic Devices

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This is an introduction to inverse design of photonic devices through shape optimization.

Introduction

In topology optimization(TO), every pixel in the design domain is taken as a design variable. However, in shape optimization(SO), the device boundary interface takes the role. SO is very useful, we never face the gray region problems as in TO and we don't need an additional projection stage. Inverse design with TO is a very dangerous process!, you need to have a good initial guess from the beginning and you need further constraints to prevent very small features from appearing. In TO, the discovered devices looks random and its theory of operation is very hard to understand in most cases. In SO, the discovered devices inherits so mush from the initial design guesses. In practice SO and TO are used, sometimes we have no knowledge of a good initial device to start the optimization with, TO could help us discover some initial device ideas. Other times, we have an excellent device idea, and we enhance the device performance with shape optimization.

Formulation

Suppose we want to minimize the objective function G by varying the interface between two adjacent materials with dielectric constants ϵ_1 and ϵ_2. We assume G can be written in the following form:

$$F(\Omega(t)) = \int_{\Omega(t)} f(\psi)\, d\Omega \tag{1}$$

ψ is the field. Ω represents the volume of the design region and Γ is its interface. The goals is to change Γ in a way that minimizes F. The time rate of change of F is

$$\frac{dF}{dt} = \int_{\Omega(t)} \left(\frac{\partial f}{\partial t} + \nabla \cdot (f\boldsymbol{V}) \right) d\Omega \tag{2}$$

Using the divergence theorem, this can be written as:

$$\frac{dF}{dt} = \int_{\Omega(t)} \frac{\partial f}{\partial t}\, d\Omega + \int_{\Gamma(t)} f V_n\, d\Gamma \tag{3}$$

where V_n is normal perturbation velocity component at the interface. The time rate of change of f could be expanded with the chain rule:

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial \psi}\frac{\partial \psi}{\partial t} \tag{4}$$

Making use of the identity:

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \boldsymbol{V} \cdot \nabla \phi \tag{5}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial \psi}\left(\frac{d\psi}{dt} - \boldsymbol{V} \cdot \nabla \psi\right) \tag{6}$$

Substituting the last result in 3

$$\frac{dF}{dt} = \int_{\Omega(t)} \left(\frac{\partial f}{\partial \psi}\left(\frac{d\psi}{dt} - \boldsymbol{V} \cdot \nabla \psi\right)\right) d\Omega + \int_{\Gamma(t)} f V_n \, d\Gamma \tag{7}$$

Now, we need to git rid of $\frac{d\psi}{dt}$. The method of Lagrange multiplier can be used.

$$\frac{dF}{dt} = \int_{\Omega(t)} \left(\frac{\partial f}{\partial \psi}\left(\frac{d\psi}{dt} - \boldsymbol{V} \cdot \nabla \psi\right)\right) d\Omega + \int_{\Gamma(t)} f V_n \, d\Gamma + \int_{\Omega(t)} \lambda^T \left(\frac{\partial A}{\partial t}\psi + A\frac{\partial \psi}{\partial t}\right) \tag{8}$$

$$\frac{dF}{dt} = \int_{\Omega(t)} \left(\frac{\partial f}{\partial \psi}\left(\frac{d\psi}{dt} - \boldsymbol{V} \cdot \nabla \psi\right)\right) d\Omega + \int_{\Gamma(t)} f V_n \, d\Gamma + \int_{\Omega(t)} \lambda^T \left(\left(\frac{dA}{dt} - \boldsymbol{V} \cdot \nabla A\right)\psi + A\left(\frac{d\psi}{dt} - \boldsymbol{V} \cdot \nabla \psi\right)\right) \tag{9}$$

collecting terms in $\frac{d\psi}{dt}$ and $\frac{\partial A}{\partial t} = 0$

$$\left(\frac{\partial f}{\partial \psi} + \lambda^T A\right)\frac{d\psi}{dt} \tag{10}$$

$$\frac{dF}{dt} = \int_{\Omega(t)} \left(\left(\lambda^T A + \frac{\partial f}{\partial \psi}\right)(-\boldsymbol{V} \cdot \nabla \psi)\right) d\Omega + \int_{\Gamma(t)} f V_n \, d\Gamma \tag{11}$$

$$\frac{dF}{dt} = \int_{\Gamma(t)} f V_n \, d\Gamma \tag{12}$$