Introduction to the Method of Moments

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This is an introduction the basic aspects of the Method of Moments (MOM).

Review

Given $g(x)$, find $f(x)$ in the interval $\Omega = [0,1]$ satisfying

$$\begin{aligned} -\frac{d^2 f}{dx^2} &= g(x), \quad \Omega \\ f &= 0 \quad \partial\Omega \end{aligned} \tag{1}$$

This is a boundary value problem of the form $Lf = g$ for which

$$L = -\frac{d^2 f}{dx^2} \tag{2}$$

The operator L is hermitian and positive-definite

$$\langle Lf \,|\, g \rangle = \langle f \,|\, Lg \rangle \tag{3}$$

$$\langle Lf \,|\, f \rangle \geq 0 \tag{4}$$

The inverse of operator L can be obtained with the help of standard Green's function techniques

$$f(x) = L^{-1}(g) = \int_0^1 G(x,x')g(x')dx' \tag{5}$$

where G is the Green's function

$$G(x,x) = \begin{cases} x(1-x') & x < x' \\ (1-x)x' & x > x' \end{cases} \tag{6}$$

The operator L^{-1} is also Hermitian and positive-definite. Note that the boundary conditions must be specified for the domain of L, however, they are not required for L^{-1} (Green functions already accounts for them).