Adjoint Computational Electromagnetics

Mohamed Kamal Abd Elrahman
6 October, Giza,
Egypt

October 30, 2019

Problem Formulation

Time harmonic coupled first order Maxwell's equation for a linear nonmagnetic and source-free medium could be written as

$$\nabla \times \mathbf{E} = -j\mu_0\omega\mathbf{H} \tag{1a}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E} \tag{1b}$$

Which could be combined into a single second order PDE

$$\nabla \times \nabla \times \mathbf{E} = \mu_0\epsilon_0\omega^2\epsilon_r\mathbf{E} = \left(\frac{\omega}{c}\right)^2 \epsilon_r\mathbf{E} \tag{2}$$

This can be written in the standard generalized eigenvalue problem form.

$$Ax = \lambda Bx \tag{3}$$

If the the following transformations have been made, $A = \nabla \times \nabla \times$, $\mathbf{B} = \left(\frac{1}{c}\right)^2 \epsilon_r$ and $x = \mathbf{E}$. The operators A and B can be shown to be positive-semidefinite and Hermitian. B is even a Hermitian operator. It can be shown that ω is real, and that two solutions x_i and x_j with different frequencies satisfy an orthonormality relation:

$$x_i^T \epsilon x_j = \delta_{ij} \tag{4}$$

Suppose we want to maximize the design objective $g(x, \lambda, \epsilon)$, the sensitivity of g is

$$\frac{dg}{d\epsilon} = \frac{\partial g}{\partial \epsilon} + \frac{\partial g}{\partial x}\frac{\partial x}{\partial \epsilon} + \frac{\partial g}{\partial \lambda}\frac{\partial \lambda}{\partial \epsilon} \tag{5}$$

Taking the derivative of the eigenvalue equation

$$(A - \lambda B)\frac{\partial x}{\partial \epsilon} = \frac{\partial \lambda}{\partial \epsilon}Bx + \lambda\frac{\partial B}{\partial \epsilon}x \tag{6}$$

Taking the derivative of the orthogonality equation