Introduction to the Method of Moments

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This is an introduction the basic aspects of the Method of Moments (MOM).

Review

Given $g(x)$, find $f(x)$ in the interval $\Omega = [0, 1]$ satisfying

$$\begin{aligned} -\frac{d^2 f}{dx^2} &= g(x), \quad \Omega \\ f &= 0 \quad \partial\Omega \end{aligned} \tag{1}$$

This is a boundary value problem of the form $Lf = g$ for which

$$L = -\frac{d^2 f}{dx^2} \tag{2}$$

The operator L is hermitian and positive-definite

$$\langle Lf \mid g \rangle = \langle f \mid Lg \rangle \tag{3}$$

$$\langle Lf \mid f \rangle \geq 0 \tag{4}$$

The inverse of operator L can be obtained with the help of standard Green's function techniques

$$f(x) = L^{-1}(g) = \int_0^1 G(x, x')g(x')dx' \tag{5}$$

where G is the Green's function

$$G(x, x) = \begin{cases} x(1 - x') & x < x' \\ (1 - x)x' & x > x' \end{cases} \tag{6}$$

The operator L^{-1} is also Hermitian and positive-definite. Note that the boundary conditions must be specified for the domain of L, however, they are not required for L^{-1} (Green functions already accounts for them).

Method of Moments

Consider the steady state equation

$$Lf = g \tag{7}$$

where L is a linear operator, g is a known excitation and the response f is to be found. Let f be expanded as a superposition of a set of basis functions $|v_i\rangle$ with unknown expansion coefficients c_i

$$f = \sum_i c_i |v_i\rangle \tag{8}$$

For approximate solutions, (7) is truncated to a finite summation. Substituting (8) in (7), and using the linearity of L, we have

$$\sum_i c_i L |v_i\rangle = g \tag{9}$$

Taking the inner product of (9) with a set of test functions w_m, where $m = 1, 2, \ldots$

$$\sum_i c_i \langle w_m | L v_i\rangle = \langle w_m \mid g\rangle \tag{10}$$

Once the basis and test functions are chosen, Equation (10) becomes a standard linear system of algebraic equations. The procedure is similar to the method of wighted residual commonly used for finite element methods (but instead of a differential operator we have an integral operator) and the particular choice $w_i = v_i$ is known as Galerkin method.

The Collocation Method

The idea of the collocation method is to demand the satisfaction of (9) at a number of discrete points in the region of interest. The collocation method can be also seen as taking Dirac delta functions as testing functions.