Wärme und Elektrizität, PHB2

$R = 8{,}31\ \frac{J}{mol*K}$; $N_A = 6{,}022\ \times 10^{23} mol^{-1}$; $k = 1{,}38 \times 10^{-23}\ \frac{J}{K}$; Wasser: $C_w =\ 4{,}18\ \ J\,g^{-1}K^{-1}$, $\lambda_{v,m} = h_{v,m} = 2257$ kJ/kg, $h_{s,m} = 334$ kJ/kg

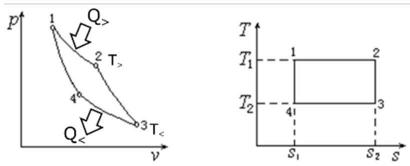
$\sigma = 5.67 \times 10^{-8}\, W/(m^2K^4)$, $1e = 1{,}602 * 10^{-19}$ C, $e/m_{el} = 1.7588 \cdot 10^{11}$ C/kg, $\varepsilon_0 = 8{,}854 \times 10^{-12}\, C^2N^{-1}m^{-2}$, $\mu_0 = 4\pi \times 10^{-7} VsA^{-1}m^{-1}$

Thermische Ausdehnung:	$\Delta l = l_0 \cdot \alpha \cdot \Delta T$;		$V \approx V_0(1 + 3\alpha\Delta T_C)$;	$\gamma = \frac{1}{V_0}\frac{\partial V}{\partial T}\Big\|_{T_0} = 3\alpha$
Verteilungsdichte f(x): $\int_{-\infty}^{\infty} f(x)dx = 1$ Gerichtete Verteilung, ($R = N_A \cdot k$)	$f_{MB} = \sqrt{\frac{m}{2\pi \cdot k \cdot T}}\, e^{-\frac{m \cdot v^2}{2 \cdot k \cdot T}}$		$\overline{v_x} = \int_{-\infty}^{\infty} v_x\, f(v_x)\, dv_x$	$\overline{v_x^2} = \int_{-\infty}^{\infty} v_x^2\, f(v_x)\, dv_x$
Ungerichtete 3D-Verteilung	$f_M = 4\pi v^2 \left(\frac{m}{2\pi \cdot k \cdot T}\right)^{\frac{3}{2}} e^{-\frac{m \cdot v^2}{2 \cdot k \cdot T}}$		$\overline{v} = \int_0^{\infty} v\, f_M(v) dv = \sqrt{\frac{8}{3\pi}} v_{rms}$	$\overline{v^2} = v_{rms}^2 = \int_0^{\infty} v^2 f_M(v) dv = \frac{3kT}{m}$
Boltzmann-Verteilung $\frac{n(E)}{n(E_0)} = e^{-\frac{\Delta E}{k*T}}$	$f_B = \frac{e^{-\frac{E}{kT}}}{\int_0^{\infty} e^{-\frac{E}{kT}} dE}$		$\Lambda = 1/(\sqrt{2}\sigma n)$ $\sigma = \pi(r_1 + r_2)^2$	$n = \frac{N}{V} = \frac{p}{kT}$
Transporteffekte, Bilanzgleichung: $V\,\frac{dn}{dt} = j_0\,A - j_1\,A$	Diffusion; $j_m = -D\frac{d\varphi}{dz}$ $D = \frac{1}{3}\bar{v}\Lambda$		Innere Reibung; $\tau = -\eta\frac{du}{dz}$ $\eta = \frac{1}{3}\rho\bar{v}\Lambda$	Wärmeleitung; $\dot{q} = -\lambda\frac{dT}{dz}$ $\lambda = \frac{1}{3}\overline{v_z} nmc_V\Lambda$
Wärmetransport, Wärmebilanz $\frac{dU}{dt} = \dot{Q}_1 - \dot{Q}_2$	Wärmeleitung: $\dot{\vec{q}} = -\lambda\vec{\nabla}T$	$\dot{Q} = -\frac{\Delta T}{R_{th}}$	Konvektion $\dot{q}_K = k_K\,\Delta T$, $R_k = \frac{1}{k_k A}$ Wärmestrom [J/m²] $\dot{Q}_m = vAu = vA\rho c_m \Delta T$ $\dot{q}_m = vu = v\rho c_m \Delta T$	Strahlung, Planck: $\frac{dI}{df} = \frac{2\pi h f^3}{c^2}\frac{1}{e^{\frac{hf}{kT}}-1}$ oder $\frac{dI}{d\lambda} = \frac{2\pi h c^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$, $I_S = \varepsilon\sigma T_S^4$ [W/m²] $1 = \alpha + r + \tau$ $\lambda_{max} = 2897.8\,\mu m\,K/T$ $\Phi = EA\sigma(T_1^4 - T_2^4)$ $E = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$
	Wärme- widerstand	$R_{th} = \frac{\Delta x}{\lambda\, A}$		
	Kontakt- widerstand	$R_C = \frac{\|\Delta T\|}{\dot{Q}_K}$		
Ideales Gas: $PV = \nu RT$	$C_V = \left(\frac{dU}{dT}\right)_V$	$C_p = \left(\frac{dH}{dT}\right)_p$	$c_V = \frac{f}{2}R,\ c_P = c_V + R, \varkappa = \frac{c_P}{c_V}$;	$\Delta Q = c \cdot m \cdot \Delta T = c_{mol} \cdot \nu \cdot \Delta T$
Der erste Hauptsatz: $\Delta U = \Delta Q + \Delta W$, reversible: $\Delta Q = \Delta U + \int_1^2 PdV$, $E_{kin} = U = \frac{f}{2}\nu RT = \nu c_V T$				
Reale Gase: $\left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT$	$\frac{dp}{dT} = \frac{\lambda_V}{T(v_{gas} - v_{fl})}$, $p(T) = p_0 \exp\left(-\left(\frac{\lambda_{V,mol}}{RT} - \frac{\lambda_{V,mol}}{RT_0}\right)\right)$, $T_0 = 373$ K, $P_0 = 10^5$ Pa			

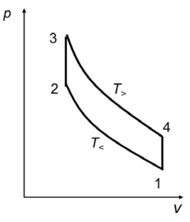
Prozess	Anfangs- und Endzustand	Arbeit Volumenarbeit **dW (-PdV)**	Wärme $\Delta U = \Delta Q + \Delta W$ $\Delta Q = \nu C_V\Delta T + PdV$
Isochor (V = const.)	$V_1 = V_2$ $\frac{T_2}{T_1} = \frac{p_2}{p_1}$	dW = 0	$\nu C_V(T_2 - T_1)$
Isobar (p = const.)	$P_1 = P_2$ $\frac{T_2}{T_1} = \frac{V_2}{V_1}$	$P(V_1 - V_2)$ oder $\nu R(T_1 - T_2)$	$\Delta Q = \nu C_p\Delta T + VdP$ $= \nu C_p(T_2 - T_1)$
Isotherm (**T** = const.)	$T_1 = T_2$ $P_1V_1 = P_2V_2$	$PV = \nu RT,\quad dW = -PdV$ $dW = -\nu RT\frac{dV}{V},\ \Delta W = -\nu RT \cdot \ln\frac{V_2}{V_1}$	$d(PV) = 0$ $PdV + VdP = 0$ $\Delta Q = -\Delta W$
Adiabatisch **(isentrop)** $PV^k = const.$	$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\kappa}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\kappa-1}$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\kappa-1}{\kappa}}$	$VdP + PdV = \nu RdT$ $PdV = \nu RdT + \kappa PdV$ $(\kappa - 1)PdV = -\nu RdT$ $-PdV = \frac{\nu R}{\kappa - 1}dT$ $\Delta W = \frac{\nu R}{\kappa - 1}(T_2 - T_1) = \nu C_V(T_2 - T_1)$	dQ = 0
Polytrop $pV^n = const.$	$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{n}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$	$PdV + VdP = \nu RdT$ $VdP = -nPdV$ $-(n-1)PdV = \nu RdT$ $\Delta W = \frac{\nu R}{n-1}(T_2 - T_1)$	$\Delta U = \Delta Q + \Delta W$ $\Delta Q = \nu C_V(T_2 - T_1) + \frac{-\nu R}{n-1}(T_2 - T_1)$ $\Delta Q = \nu C_V(T_2 - T_1)\left(\frac{n - \kappa}{n - 1}\right)$

	Innere Energie $U = \frac{f}{2}\nu RT$	Enthalpie H = U + PV	Entropie S: reduzierte Wärme $\Delta S = \Delta Q/T$, $S = k\ln\Omega$
Differential	$dU = \nu C_v dT$	$dH = \nu C_p dT$	$dS = \nu C_p\frac{dT}{T} - \nu R\frac{dP}{P} = \nu C_v\frac{dT}{T} + \nu R\frac{dV}{V} = \nu C_v\frac{dP}{P} + \nu C_p\frac{dV}{V}$
Änderung	$\Delta U = \nu C_v\Delta T$	$\Delta H = \nu C_p\Delta T$	$\Delta S = \nu C_p ln\frac{T_2}{T_1} - \nu R ln\frac{P_2}{P_1} = \nu C_v ln\frac{T_2}{T_1} + \nu R ln\frac{V_2}{V_1} = \nu C_v ln\frac{P_2}{P_1} + \nu C_p ln\frac{V_2}{V_1}$

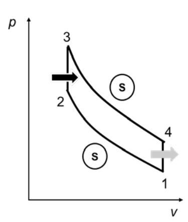
Carnot-Prozess: $\eta = \frac{|\Delta W|}{Q_>} \overset{Carnot}{\hat{=}} \frac{T_> - T_<}{T_>}$; $\varepsilon_{WP} = \frac{Q_>}{|\Delta W|} = \frac{T_>}{T_> - T_<}$; $\varepsilon_{KM} = \frac{Q_<}{|\Delta W|} = \frac{T_<}{T_> - T_<}$, $Q_> - Q_< = \Delta W, \eta = \frac{|\Delta W|}{Q_>}$



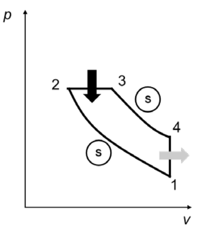
Carnot-Prozess



Stirling-Prozess



Otto-Prozess



Diesel-Prozess

Stirlingmotor:

$W_{ges} = W_{12} + W_{34}$

$= \nu RT_> \ln\frac{V_2}{V_1} + \nu RT_< \ln\frac{V_4}{V_3}$

$= \nu RT_> \ln\frac{V_2}{V_1} + \nu RT_< \ln\frac{V_1}{V_2}$

$Q_{34} = \nu RT_> \ln\frac{V_1}{V_2}$

$\eta = \frac{|W_{ges}|}{Q_{34}} = \frac{\nu R \ln\frac{V_1}{V_2}(T_> - T_<)}{\nu RT_> \ln\frac{V_1}{V_2}}$

$= \frac{T_> - T_<}{T_>}$

Ottomotor

$Q_> = C_v(T_3 - T_2)$

$Q_< = C_v(T_4 - T_1)$

$\eta = \frac{\Delta W}{Q_>} = \frac{Q_> - Q_<}{Q_>}$

$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$

$\eta = 1 - \frac{1}{\left(\frac{V_>}{V_<}\right)^{\kappa - 1}}$

Dieselmotor

$Q_> = C_p(T_3 - T_2)$

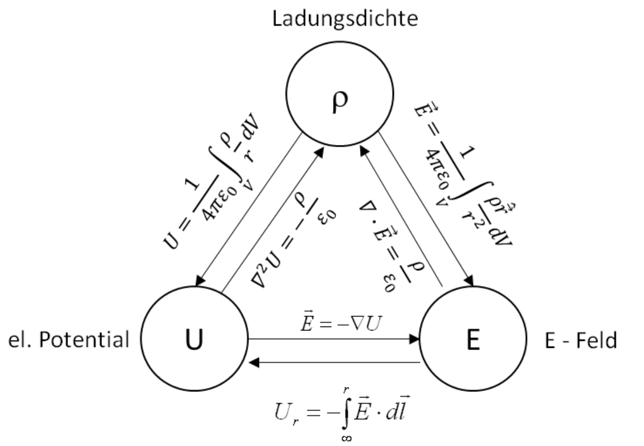
$Q_< = C_v(T_4 - T_1)$

$\eta = \frac{\Delta W}{Q_>} = 1 - \frac{Q_<}{Q_>}$

$= 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$

$\eta = 1 - \frac{(V_3/V_2)^\kappa - 1}{\kappa(V_3/V_2 - 1)(V_1/V_2)^{\kappa - 1}}$

Elektrostatik



Gauss'sches Gesetz $\oint\!\!\oint_A \vec{E} \cdot d\vec{A} = \frac{\sum q_i}{\varepsilon_0}$; $\oint\!\!\oint_A \vec{D} \cdot d\vec{A} = \sum q_f$

Dielektrikum, Polarisation

$\sigma' = \vec{P} \cdot \vec{n}$

$\vec{E} = \vec{E}_0 + \vec{E}'$

$\vec{P} = \chi_e \varepsilon_0 \vec{E}$

$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

$|D| = \sigma_0$

$\varepsilon_r = 1 + \chi_{el}$

Dipolmoment: $\vec{p} = q\vec{d}$

Elektrischer Dipol: $U = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}$

Kondensator

$C_0 = \frac{\varepsilon_0 A}{d} = \frac{Q_0}{U_0}$

$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \varepsilon_r C_0$

$\sigma_0 = \frac{Q_0}{A} = D = \frac{C_0 U_0}{A}$

$= \frac{\varepsilon_0 U_0}{d}$

$E_0 = \frac{\Delta U}{d} = \frac{\sigma_0}{\varepsilon_0}$

$W = \frac{1}{2} C U^2$

Energiedichte

$w_E = \frac{1}{2} ED$

elektrische Kraft

$F_E = -\frac{dW}{dx}$

Randbedingungen

$E_{t1} = E_{t2}$

$\varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}$

$D_{n1} = D_{n2}$

$\frac{tan\theta_1}{tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$

Magnetostatik

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \vec{r}}{r^3}$ $\quad d\vec{F} = Id\vec{l} \times \vec{B}$ $\quad \vec{f} = q\vec{E} + q\vec{v} \times \vec{B}$

Amper'sches Durchflutungsgesetz

$\oint_l \vec{H} \cdot d\vec{l} = \sum_i I_i$, $\quad \sum_i H_i l_i = NI$

Geradlinige Leitung $B = \frac{\mu_0 I}{2\pi r}$

Ringförmige Leitung $\vec{B} = \frac{\mu_0 \cdot I \cdot R^2}{2 \cdot (R^2 + x^2)^{\frac{3}{2}}}$

lange Spule: $\vec{B} = \mu_0 \cdot \frac{N}{L} \cdot I$

Magnetischer Fluß Geschlossene Fläche:

$\oint\!\!\oint_A \vec{B} \cdot d\vec{A} = 0$ $\qquad \Phi_m = \iint_{offene\ Fläche} \vec{B} \cdot d\vec{A}$

Magnetisierung

$I' = \oint \vec{M} \cdot \vec{dl}$

$\vec{B} = \vec{B_0} + \vec{B}'$

$\vec{B} = \mu_0 \mu_r \vec{H}$

$\mu_r = 1 + \chi_{mag}$

$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$

Das magnetische Moment

$m = IA$

Energiedichte

$w_M = \frac{1}{2} BH$

magnetischer Druck

$P_M = -\frac{B^2}{2\mu_0}$

Randbedingungen

$B_{n1} = B_{n2}$

$H_{t1} = H_{t2}$

$\frac{B_{t1}}{B_{t2}} = \frac{\mu_1}{\mu_2}$

$\frac{tan\theta_1}{tan\theta_2} = \frac{\mu_1}{\mu_2}$

Induktionsgesetz

$\varepsilon_{ind} = \oint_L \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt} = -\iint_A \frac{\partial \vec{B}}{\partial t} d\vec{A}$

Eigeninduktivität L

$L = \frac{\Phi}{I}$, $\varepsilon_L = -L\frac{di}{dt}$, $\quad$ Energie: $W = \frac{1}{2} L I^2$

Gegeninduktivität M

$\varepsilon_{12} = -\frac{d\psi_{12}}{dt} = -M\frac{di_1}{dt}$,

$\varepsilon_{21} = -\frac{d\psi_{21}}{dt} = -M\frac{di_2}{dt}$,

Transformator

$\frac{U_1}{U_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$

Satz von Gauß:

$\int_A \vec{E}(x,y) \cdot d\vec{A} = \int_V \nabla \cdot \vec{E}(x,y) dV$

Satz von Stokes:

$\int_{\partial A} \vec{E}(x,y,z) \cdot d\vec{r} = \int_A \vec{\nabla} \times \vec{E}(x,y,z) \cdot d\vec{A}$

Maxwell-Gleichungen

$\vec{\nabla} \cdot \vec{D} = \rho_0$

$\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = -\mu_0 \mu_r \frac{\partial \vec{H}}{\partial t}$

$\vec{\nabla} \times \vec{H} = j_0 + \frac{\partial \vec{D}}{\partial t}$