

Multi-objective Model-Predictive Control for Dielectric Elastomer Wave Harvesters.

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Abstract:

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1. INTRODUCTION

2. MODEL AND PROBLEM STATEMENT

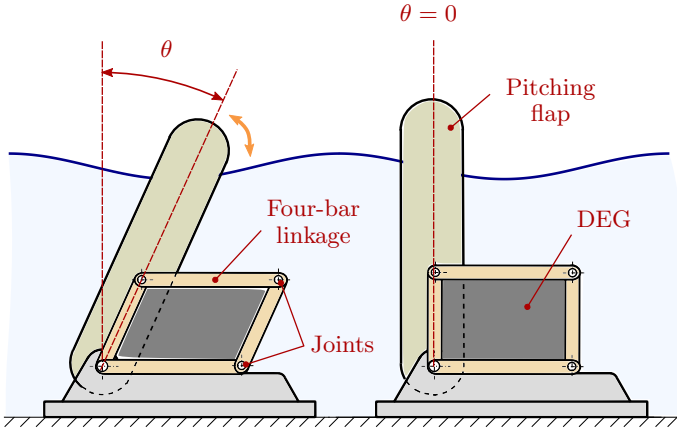


Fig. 1. Wave surge converter: A flap mounted to the sea floor is tilted by the wave motion. It is displayed in a generic (left) and the vertical equilibrium position (right).

2.1 Background

In this work, we investigate the Model-predictive Control (MPC) of the wave surge converter (see, e.g. Whittaker and Folley (2012)), displayed in Fig. 1. The device is pivoted to the sea, such that incoming waves excite an oscillatory motion. This motion distorts a dielectric elastomer generator (DEG) mounted to a deformable parallelogram Moretti et al. (2014). Through this parallelogram, the DEG applies a torque to the flap towards the equilibrium position $\theta = 0$. Applying a voltage to the DEG adds an electrostatically-induced torque in the same angular direction, making the system stiffer.

When a voltage is applied to the DEG, charge carrier accumulate in the DEG. By deforming the parallelogram

to a smaller area, the charge density increases, such that energy can be extracted. As we showed in our previous work Hoffmann et al. (2022), controlling the input to maximise the energy extracted from the system requires application of large voltages, damaging the DEG-material over time, and resulting in system failure after too much damage accumulated Chen et al. (2019). For that reason, we also took the damage into consideration as a second control goal, resulting in lower electric fields in the DEG.

2.2 Model

The dynamics of the wave surge

$$\begin{bmatrix} \dot{\theta} \\ \dot{\delta} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0^{1 \times n} \\ -I_h^{-1} K_h & -I_h^{-1} B_h & -I_h^{-1} C_r \\ 0^{n \times 1} & B_r & A_r \end{bmatrix} \begin{bmatrix} \theta \\ \delta \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ I_h^{-1} \\ 0^{n \times 1} \end{bmatrix} (d - C_0 \theta u), \quad (1)$$

contain the equation of angular motion and the wave loads generated by the interaction of the waves with the flap. Here, $\delta = \dot{\theta}$ describes the flap's angular velocity and $z \in \mathbb{R}^n$ the n -dimensional state vector for wave radiation (Yu and Falnes (1995)). The input u is the electrostatic force generated by the DEG when a voltage v is applied. Since the DEG's electrostatic force is proportional to the v^2 , u is restricted to be positive. **add u constraint explanation**

$$u \leq (E_{bd} h_1)^2 / \cos^2(\theta), \quad (2)$$

2.3 Cost functions

Our aim is the simultaneous minimisation of damage and maximisation of extracted energy, which we will employ in a multi-objective Optimal Control (MOOCP) setting. The extracted energy cost function is modelled as the negative generated energy

$$J_1 = \Psi(t_f) - \Psi(0) + \int_0^{t_f} \left(B_h \dot{\theta}^2 + z^T S_r z + \frac{u}{R_0} - d\dot{\theta} \right) dt$$

with $\Psi = \frac{1}{2} I_h \dot{\theta}^2 + \frac{1}{2} K_h \theta^2 + \frac{1}{2} z^T Q_r z + \frac{1}{2} C_0 (1 - \theta^2) u$,

(3)

with the storage function Ψ including kinematic, potential, electrostatic, and hydrodynamic energy contributions. Dissipations due to viscous, hydrodynamic, and electrical losses, and the power input by the incident wave are considered via the integral terms. [more explanations?](#), see [mathmod paper](#)

Under the assumption, that damage only starts accumulating after the applied electric field exceeds a threshold E_{th} , the cost function can be formulated as

$$J_2 = \alpha \int_0^{t_f} \left(\max\{\cos^2(\theta)u - E_{th}^2 h_l^2, 0\} \right)^{n_d} dt, \quad (4)$$

with a normalisation factor α rendering J_2 dimensionless, and an experimental parameter n_d . [too close to Mathmod paper?](#)

Equations (1), (2), (3), and (4) formulate the MOOCP *Problem 1*.

$$\begin{aligned} & \underset{u(t)}{\text{minimize}} \quad (J_1, J_2) \\ & \text{subject to dynamics (1)} \\ & \quad 0 \leq \cos^2(\theta)u \leq (E_{bd} h_l)^2. \end{aligned} \quad (5)$$

that is solved inside the MPC.

3. METHODS

3.1 Model-predictive Control

MPC arose from optimal control as one answer on how to “close the loop” (Rawlings et al. (2017)). In optimal control, a system’s behaviour is predicted into the future, while optimising the inputs to the system, such that a cost function is minimised. The working principle of MPC is repeatedly measuring the system’s state, solving an Optimal Control Problem (OCP), and applying the first few of the calculated inputs. [move to background or introduction](#)

3.2 Wave generation

3.3 Adaptive weight selection

When applying MPC, we do not know exactly how the system will perform over the deployment of the control. In the case of the WEC-DEG when driving the system with a set weighting, different sea states will result in a different damage accumulation over time. Since the DEG has to be replaced once it breaks down, an operator of multiple DEGs might be interested replacing all of the devices at once to save monetary costs. This means, that the breakdown of all the devices has to be synced, e.g. by changing the weighting of the damage cost function in a way, such that the accumulated damage cost at the designated break-down time does not exceed a fixed value J_2^d .

One easy way of deciding, if this damage goal is achievable with the current weighting is by evaluating the MPC performance over a short time into the past, estimating the average rate of damage accumulation, and predicting the damage at the break-down time. [can be shortened to just reflect the change of weighting](#)

Input: J_2^d , t_d , N , t_f , $k \geq 2$,
solve Problem 1;

4. NUMERICAL RESULTS

The following simulations were done using MATLAB. The optimisation problems were formulated and solved using the CasADi package by Andersson et al. (2019) and the Ipopt solver by Wächter and Biegler (2006).

4.1 Multi-objective Optimal Control

Methods from Hoffmann et al. (2022).

4.2 Model-predictive Control

5. CONCLUSION

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