Real-Time Probabilistic Flood Prediction: A Hybrid Bayesian-Hydrodynamic Approach

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Contents

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1 Document history

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Abstract

Flood risk assessment is a critical component of property valuation and management, particularly in the context of increasing urbanisation. The interaction between changing weather patterns and urban development creates compounding effects that can lead to negative feedback loops in property risk at individual and portfolio levels. Traditional approaches often fail to bridge the gap between insurance metrics (e.g., 100-year return periods) and property valuation frameworks that feed into mortgage models based on the probability of default and loss given default. This disconnect is particularly acute for long-term property holders, where annual insurance coverage must be reconciled with multi-decade investment horizons. We present a novel

approach to real-time flood prediction that combines Bayesian deep learning with computational fluid dynamics. Our framework integrates high-resolution weather data from HRRR with advanced hydrodynamic modelling to provide probabilistic forecasts of flood timing, location, and depth. The system continuously learns from new observations while maintaining physical consistency through neural operators and physics-informed constraints. We demonstrate significant improvements in prediction accuracy and computational efficiency compared to traditional methods. Our integrated pipeline spans four key domains:

1. weather pattern prediction using adaptive Fourier neural operators
2. precipitation modelling with specialised deep learning architectures,
3. terrain-aware flow dynamics leveraging manifold learning techniques
4. shallow water hydrodynamic modelling for flood propagation.

By maintaining physical consistency across these components while leveraging data-driven approaches, we achieve both computational efficiency and prediction accuracy suitable for operational deployment.

2 Introduction

2.1 Background

Flood risk assessment faces two significant evolving challenges in the modern context. First, climate change is altering traditional weather patterns, leading to shifts in the long-term assumptions about risks faced by properties. Second, increasing urbanisation places more significant pressure on infrastructure and often results in residential development in areas unsuitable for long-term occupation but approved due to short-term imperatives.

The intersection of these challenges creates potential negative feedback loops, where increases in either climate change impacts or urbanisation vulnerabilities can amplify property risk at both individual and portfolio levels. This dynamic is particularly problematic for long-term property holders, who must navigate between annual insurance coverage and multi-decade investment horizons.

A critical gap exists between the language of insurance (e.g., 100-year return periods) and the metrics needed for property valuation that feed into mortgage models. Properties that become uninsurable effectively become unsellable unless backed by special programs like Flood Re, creating a pressing need for effective risk transfer mechanisms between lenders with multi-decade investment horizons and insurers operating on annual cycles.

2.2 Related Work

Previous approaches to flood risk assessment have typically focused on either statistical methods based on historical data or deterministic physics-based models. Statistical methods often fail to capture the non-stationarity of climate patterns, while physics-based models can be computationally prohibitive for real-time applications. Recent work in machine learning for weather prediction [7] and computational fluid dynamics [9] has shown promise, but integration across these domains remains challenging.

The application of topological data analysis and manifold learning techniques to meteorological-hydrological modelling represents a significant advancement in handling high-dimensional data while preserving essential structures [8]. Uniform Manifold Approximation and Projection (UMAP) has shown promise in capturing both global weather patterns and local terrain-influenced dynamics [5].

Neural operators [**?**] and physics-informed neural networks [**?**] have emerged as powerful tools for maintaining physical consistency in machine learning models. These approaches enable data-driven methods to respect conservation laws and boundary conditions, critical for reliable flood prediction.

2.3 Contributions

Our work makes the following contributions:

- A unified framework that bridges weather forecasting, precipitation modelling, terrain analysis, and flood dynamics
- A hybrid approach combining deep learning with physical models to achieve both computational efficiency and physical consistency
- A manifold learning methodology for dimensionality reduction that preserves topological features critical for weather-to-flood mapping
- A real-time operational system for probabilistic flood prediction with quantified uncertainty
- An integrated risk assessment framework that connects physical flood predictions to financial impact metrics

2.4 Paper Organization

The remainder of this paper is organised as follows:

- Section 2 presents our methodology, including state space formulation and the integration of HRRR data.
- Section 3 details the weather prediction architecture and precipitation modelling.
- Section 4 explores the role of topology and manifold learning in connecting weather patterns to flood dynamics.
- Section 5 presents the shallow water equations and their numerical solution for flood modelling.
- Section 6 outlines the portfolio risk assessment methodology.
- Section 7 discusses the implementation architecture and real-time processing pipeline.
- Section 8 presents our results and case studies.
- Section 9 discusses the implications and limitations of our approach.
- Section 10 concludes with future research directions.

3 Methodology

3.1 State Space Formulation

The model integrates physical flood dynamics and financial risk metrics in a unified state space. The complete state at time t is represented as:

$$S(t) = \{W(t), F(t), P(t)\} \tag{1}$$

- $W(t)$ is the state of the weather
- $F(t)$ is the state of the flood
- $V(t)$ is the vulnerability of the asset, its mortgage and insurance

Where each component captures distinct aspects of the flood risk system:

3.1.1 Weather State $W(t)$

The weather state $W(t)$ comprises:

$$W(t) = \{T(x, y, z), P(x, y, z), V(x, y, z), H(x, y, z), R(x, y), S(x, y), G(x, y)\} \tag{2}$$

Each component represents:

- $T(x, y, z)$: Temperature field [K]
  - Influences precipitation type (rain/snow)
  - Affects evaporation rates
  - Impacts soil moisture dynamics
- $P(x, y, z)$: Pressure field [hPa]
  - Drives atmospheric water vapor transport
  - Indicates storm system development

- Correlates with precipitation intensity

- $V(x, y, z)$: Wind velocity field [m/s]
  - Vector quantity (u, v, w) components
  - Affects precipitation distribution
  - Influences surface water dynamics
- $H(x, y, z)$: Humidity field [kg/kg]
  - Specific humidity ratio
  - Key for precipitation formation
  - Indicates atmospheric moisture content
- $R(x, y)$: Surface precipitation rate [mm/hr]
  - Direct HRRR measurement
  - Primary input for flood dynamics
  - Includes both stratiform and convective precipitation
- $S(x, y)$: Soil saturation level [dimensionless]
  - Ranges from 0 (dry) to 1 (saturated)
  - Affects infiltration rates
  - Key for runoff generation
- $G(x, y)$: Ground elevation/topology [m]
  - Digital elevation model
  - Determines flow direction
  - Critical for flood routing

3.1.2 Flood State $F(t)$

The flood state $F(t)$ describes the physical water dynamics:

$$F(t) = \{T(Q(x, y), D(x, y), A(x, y)\} \tag{3}$$

Components represent:

- $Q(x, y)$: Flow rate [$\mathbf{m^3}$/s]
  - Vector quantity of water flux
  - Flow velocity and acceleration
  - Impact of infrastructure
- $D(x, y)$: Water depth [m]
  - Flood creation due to water deceleration
  - Water rise over Gauge trigger levels (Alert etc)
  - Break of river banks
- $A(x, y)$: Affected area extent [$\mathbf{m^2}$]
  - Propagation of flood over ground area
  - Impact of local resilience and local terrain
  - Catchment sink areas

3.1.3 Vulnerability State $V(t)$

The vulnerability state $V(t)$ captures exposure and sensitivity:

$$V(t) = \{E(x,y), V(x,y), F(x,y)\} \tag{4}$$

Where:

- $E(x,y)$: Exposure [currency units]
  - Property value at risk
  - Infrastructure exposure
  - Local area price premium
- $R(x,y)$: Vulnerability
  - Property SOP/resilience for actual flood
  - Vulnerability at property level
  - Impact adjustment for other factors (e.g. density)
- $M(x,y)$: Mitigation measures [dimensionless]
  - Insurance Coverage
  - Impact on mortgage affordability
  - Revaluation of loan

Each state variable is discretised on a regular grid with spatial resolution Δx, Δy for horizontal coordinates and Δz for vertical coordinates where applicable. Temporal evolution follows the time step Δt determined by the CFL condition for numerical stability.

4 Weather Prediction and Precipitation Modeling

4.1 Primitive-Equation Atmospheric Model

The foundation of our weather prediction system is based on the primitive equations that govern atmospheric motion and thermodynamics. These equations include:

- The momentum equations (equations of motion)
- The continuity equation
- The thermodynamic energy equation
- The equation of state (ideal gas law)
- The water vapor continuity equation

These coupled nonlinear partial differential equations form the fundamental basis for numerical weather prediction. The governing equations in Cartesian coordinates are expressed as:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{uv\tan\phi}{a} + \frac{uw}{a} + \frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega(w\cos\phi - v\sin\phi) = Fr_x \tag{5}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{u^2\tan\phi}{a} + \frac{uw}{a} + \frac{1}{\rho}\frac{\partial p}{\partial y} + 2\Omega u\sin\phi = Fr_y \tag{6}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{u^2+v^2}{a} + \frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega u\cos\phi + g = Fr_z \tag{7}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{8}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + (\gamma - \gamma_d)w + \frac{1}{c_p}\frac{dH}{dt} = 0 \tag{9}$$

$$\frac{\partial q_v}{\partial t} + u\frac{\partial q_v}{\partial x} + v\frac{\partial q_v}{\partial y} + w\frac{\partial q_v}{\partial z} + Q_v = 0 \tag{10}$$

$$P = \rho RT \tag{11}$$

These equations are typically simplified for computational efficiency using approximations such as the hydrostatic assumption, which replaces the vertical momentum equation with:

$$\frac{\partial p}{\partial z} = -\rho g \tag{12}$$

4.2 Reynolds' Equations for Turbulence

To account for unresolved turbulence, we employ Reynolds decomposition, splitting variables into mean and turbulent parts:

$$u = \bar{u} + u' \tag{13}$$

$$T = \bar{T} + T' \tag{14}$$

$$p = \bar{p} + p' \tag{15}$$

After applying Reynolds' postulates and averaging, the momentum equation becomes:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z} + \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x} - f\bar{v} - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z} + \frac{1}{\bar{\rho}}\left(\frac{\partial \bar{\tau}_{xx}}{\partial x} + \frac{\partial \bar{\tau}_{yx}}{\partial y} + \frac{\partial \bar{\tau}_{zx}}{\partial z}\right) = 0 \tag{16}$$

The turbulent stresses are defined as:

$$T_{xx} = -\bar{\rho}\overline{u'u'} \tag{17}$$

$$T_{yx} = -\bar{\rho}\overline{u'v'} \tag{18}$$

$$T_{zx} = -\bar{\rho}\overline{u'w'} \tag{19}$$

4.3 Weather Pattern Prediction Architecture

The core weather prediction system utilizes the FourCastNet architecture, comprising four key components:

4.3.1 Core AFNO Architecture

The Adaptive Fourier Neural Operator processes multi-channel weather data through:

$$\mathcal{F} = \mathcal{D} \circ \mathcal{M} \circ \mathcal{E} \tag{20}$$

Where the components are:

- $\mathcal{E}$: Patch embedding for input weather variables
- $\mathcal{M}$: Spatial/Channel mixing through AFNO layers
- $\mathcal{D}$: Linear decoder for prediction output

The AFNO layer operation is defined as:

$$\text{AFNO}(X) = \text{IFFT}(W \odot \text{FFT}(X)) \tag{21}$$

4.3.2 Two-Stage Prediction Process

Sequential predictions are generated through:

$$\begin{aligned} X_{k+1} &= \mathcal{F}(X_k) \\ X_{k+2} &= \mathcal{F}(X_{k+1}) \end{aligned} \tag{22}$$

With composite loss function:

$$\mathcal{L} = \alpha \|X_{k+1} - \hat{X}_{k+1}\|_2^2 + \beta \|X_{k+2} - \hat{X}_{k+2}\|_2^2 \tag{23}$$

4.3.3 Precipitation Model

Specialized precipitation prediction uses:

$$R_{k+1} = \mathcal{F}_R(\text{freeze}(\mathcal{F}(X_k))) \tag{24}$$

Where:

- $\mathcal{F}_R$: Dedicated AFNO for precipitation
- freeze($\mathcal{F}$): Pre-trained backbone with frozen parameters

4.3.4 Complete Inference Pipeline

The full prediction step combines both models:

$$[X_{k+1}, R_{k+1}] = \Phi(X_k) = [\mathcal{F}(X_k), \mathcal{F}_R(\mathcal{F}(X_k))] \tag{25}$$

Where X_k represents the multi-channel state vector:

$$X_k = [T_k, P_k, V_k, H_k, \ldots]^T \tag{26}$$

Including temperature (T), pressure (P), wind velocity (V), and humidity (H) fields.

4.4 AI-Enhanced Weather Prediction

While traditional numerical weather prediction (NWP) models rely on solving primitive equations using finite difference or spectral methods, recent advances in artificial intelligence offer promising alternatives that can dramatically improve computational efficiency while maintaining or enhancing forecast accuracy.

Our AI approach to weather prediction leverages deep learning architectures specially designed for spatiotemporal forecasting. This section describes the methodology, but implementation details are reserved for future work.

4.4.1 Graph Neural Networks for Weather Forecasting

Graph Neural Networks (GNNs) provide a natural representation for weather data on irregular grids and can efficiently capture both local and long-range dependencies. Our approach uses:

- Mesh-based graph construction aligned with meteorological dynamics
- Message-passing layers that respect physical constraints
- Multi-scale aggregation to capture phenomena at different spatial scales
- Temporal attention mechanisms for sequence prediction

4.4.2 Physical Consistency Constraints

To ensure that AI predictions maintain physical consistency, we incorporate:

- Conservation law enforcers as regularisation terms
- Scale-aware loss functions for multi-resolution phenomena
- Uncertainty quantification layers to estimate prediction confidence
- Physics-guided attention mechanisms to prioritise physically relevant features

4.4.3 Data Integration Strategy

The model integrates multiple data sources:

- Historical reanalysis datasets for base training
- Satellite observations for real-time correction
- Ground station measurements for localised calibration
- Traditional NWP outputs for ensemble generation

This AI-based weather prediction component feeds directly into our precipitation modelling and the hydrological pipeline, seamlessly integrating meteorological forecasting to flood prediction.

5 Topological Analysis and Manifold Learning

5.1 UMAP in Meteorological-Hydrological Modeling

Applying Uniform Manifold Approximation and Projection (UMAP) to the complex domain of weather-to-flood modelling represents a significant advancement in our ability to handle high-dimensional meteorological data while preserving the essential topological structures that govern fluid dynamics over terrain.

5.1.1 UMAP Mathematical Framework

UMAP provides a topologically grounded approach to dimensionality reduction suited to meteorological-hydrological modelling. The mathematical foundation consists of several key components:

For each high-dimensional point x_i in the weather state space, UMAP constructs local neighbourhood relationships:

$$\rho_i = \min\{d(x_i, x_{ij}) \mid 1 \leq j \leq k,\ d(x_i, x_{ij}) > 0\} \tag{27}$$

The parameter σ_i is calibrated to ensure consistent local connectivity across the manifold:

$$\sum_{j=1}^{k} \exp\left(-\frac{\max(0, d(x_i, x_{ij}) - \rho_i)}{\sigma_i}\right) = \log_2(k) \tag{28}$$

This local distance calibration allows UMAP to adapt to the varying densities in atmospheric data, where specific weather patterns may be densely clustered while others are more sparsely distributed.

5.1.2 Fuzzy Topological Representation

UMAP constructs a weighted k-nearest neighbor graph where edge weights represent fuzzy set membership:

$$w_{ij} = \exp\left(-\frac{\max(0, d(x_i, x_j) - \rho_i)}{\sigma_i}\right) \tag{29}$$

The directed adjacency matrix A is symmetrized to form an undirected representation:

$$B = A + A^\top - A \circ A^\top \tag{30}$$

Where $\circ$ denotes the Hadamard product, this operation implements a probabilistic t-conorm representing the union of fuzzy simplicial sets, essential for preserving the topological structure of weather patterns across varying scales.

5.1.3 Multi-scale Weather Pattern Integration

The application of UMAP to integrate weather patterns with flood dynamics requires a multi-scale approach that respects the physics of fluid flow while reducing computational complexity.

We implement a hierarchical approach:

$$M = \{M_1, M_2, \ldots, M_m\} \tag{31}$$

Where each M_i represents a manifold at a specific scale, constructed using:

$$M_i = \text{UMAP}(\mathcal{F}_i(W(t))) \tag{32}$$

Here, $\mathcal{F}_i$ applies scale-specific filtering to the weather state $W(t)$, and the multi-scale representation is constructed through:

$$\mathcal{M}(W(t)) = \bigoplus_{i=1}^{m} \alpha_i M_i \tag{33}$$

Where α_i are scale-importance weights determined through variance analysis of historical weather-flood relationships.

5.1.4 Topological Feature Preservation

Critical topological features in the weather-to-flood mapping are preserved using persistent homology filtering:

$$PH(M) = \{(b_i, d_i) \mid i = 1, 2, \ldots, n\} \tag{34}$$

Where (b_i, d_i) represents topological features' birth and death times across the filtration. Features with high persistence (i.e., $d_i - b_i$ is large) correspond to stable atmospheric structures that significantly influence flood dynamics.

The filtered representation M' retains only features with persistence above a threshold τ:

$$M' = \text{Filter}_\tau(M) = \{x \in M \mid \exists (b_i, d_i) \in PH(M) : (d_i - b_i) > \tau, x \in \text{Support}(b_i, d_i)\} \tag{35}$$

5.2 Weather-to-Flood Mapping

The dimensionality-reduced weather representation serves as input to a non-linear mapping function Ψ that predicts flood dynamics:

$$F(t + \Delta t) = \Psi(\mathcal{M}(W(t)), F(t)) \tag{36}$$

This function Ψ is implemented as a neural operator that respects physical conservation laws:

$$\Psi = \mathcal{D} \circ \mathcal{N} \circ \mathcal{E} \tag{37}$$

Where:

- $\mathcal{E}$ is an encoder mapping the combined weather-flood state to a latent space
- $\mathcal{N}$ is a neural operator integrating the dynamics forward in time
- $\mathcal{D}$ is a decoder ensuring the output satisfies physical constraints

5.3 Uncertainty Quantification

The manifold learning approach enables rigorous uncertainty quantification by tracking how errors propagate through the dimensionality reduction.

5.3.1 Manifold Distortion Metrics

We quantify local distortion introduced by UMAP using the trustworthiness and continuity measures:

$$T(k) = 1 - \frac{2}{nk(2n - 3k - 1)} \sum_{i=1}^{n} \sum_{j \in U_i^k} (r(i, j) - k) \tag{38}$$

$$C(k) = 1 - \frac{2}{nk(2n - 3k - 1)} \sum_{i=1}^{n} \sum_{j \in V_i^k} (\hat{r}(i, j) - k) \tag{39}$$

Where U_i^k are the points in the k-neighborhood of i in the low-dimensional space but not in the high-dimensional space, and V_i^k are the points in the k-neighborhood of i in the high-dimensional space but not in the low-dimensional space.

5.3.2 Error Propagation Through the Manifold

The propagation of input uncertainties through the UMAP reduction is modelled using:

$$\Sigma_{\mathcal{M}} = J_{\mathcal{M}} \Sigma_W J_{\mathcal{M}}^T \tag{40}$$

Where $J_{\mathcal{M}}$ is the Jacobian of the manifold mapping and Σ_W is the covariance matrix of the weather state uncertainties.

5.3.3 Ensemble Approach to Uncertainty

To capture the full uncertainty in the weather-to-flood mapping, we implement an ensemble approach:

$$\{F^{(i)}(t + \Delta t)\}_{i=1}^{N_e} = \{\Psi(\mathcal{M}(W^{(i)}(t)), F^{(i)}(t))\}_{i=1}^{N_e} \tag{41}$$

Where $\{W^{(i)}(t)\}_{i=1}^{N_e}$ represents an ensemble of weather states capturing input uncertainty, and N_e is the ensemble size.

The probabilistic flood prediction is then characterised by:

$$P(F(t+\Delta t) \in A) = \frac{1}{N_e} \sum_{i=1}^{N_e} \mathbf{1}_A(F^{(i)}(t+\Delta t)) \tag{42}$$

For any region A in the flood state space.

6 Shallow Water Hydrodynamic Modeling

6.1 Shallow Water Equations

The shallow water equations (also known as Saint-Venant equations) provide the mathematical foundation for our flood propagation model. These equations are particularly suitable for modelling flood dynamics where the horizontal scale of the flow is much larger than the water depth.

The core shallow water equations, derived from the Navier-Stokes equations under the assumption of hydrostatic pressure distribution and depth-averaged velocity, are:

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (vh)}{\partial y} = R(x,y,t) \tag{43}$$

$$\frac{\partial (uh)}{\partial t} + \frac{\partial (u^2 h + gh^2/2)}{\partial x} + \frac{\partial (uvh)}{\partial y} = gh(S_{0x} - S_{fx}) \tag{44}$$

$$\frac{\partial (vh)}{\partial t} + \frac{\partial (uvh)}{\partial x} + \frac{\partial (v^2 h + gh^2/2)}{\partial y} = gh(S_{0y} - S_{fy}) \tag{45}$$

Where:

- h is the water depth [m]
- u, v are the depth-averaged velocity components in the x and y directions [m/s]
- g is the gravitational acceleration [m/s$^{\mathbf{2}}$]
- $R(x,y,t)$ is the precipitation rate [m/s]
- S_{0x}, S_{0y} are the bed slopes in the x and y directions
- S_{fx}, S_{fy} are the friction slopes in the x and y directions

6.1.1 Physical Parameter Specification

The physical parameters in the hydrodynamic equations are defined as:

$$S_{fx} = \frac{n^2 u\sqrt{u^2+v^2}}{h^{4/3}}, \quad S_{fy} = \frac{n^2 v\sqrt{u^2+v^2}}{h^{4/3}} \tag{46}$$

Where n is Manning's roughness coefficient, typically ranging:

$$n = \begin{cases} 0.01 - 0.013 & \text{for smooth concrete} \\ 0.02 - 0.025 & \text{for gravel beds} \\ 0.03 - 0.05 & \text{for natural channels} \\ 0.05 - 0.08 & \text{for flood plains} \end{cases} \tag{47}$$

The bed slopes S_{0x}, S_{0y} are derived from topography G(x,y):

$$S_{0x} = -\frac{\partial G}{\partial x}, \quad S_{0y} = -\frac{\partial G}{\partial y} \tag{48}$$

6.2 Shallow-Fluid Approximations

For simplicity and computational efficiency, we employ additional approximations to the shallow water equations. For an incompressible, hydrostatic, and inviscid fluid, the governing equations can be expressed as:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv + g\frac{\partial h}{\partial x} = 0 \tag{49}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu + g\frac{\partial h}{\partial y} = 0 \tag{50}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} + h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{51}$$

Where f is the Coriolis parameter representing Earth's rotation effects, which becomes important for large-scale flood dynamics.

For one-dimensional flow scenarios, such as river channel flow, these equations simplify further to:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - fv + g\frac{\partial h}{\partial x} = 0 \tag{52}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + fu + g\frac{\partial H}{\partial y} = 0 \tag{53}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} = 0 \tag{54}$$

Where $\frac{\partial H}{\partial y} = -\frac{f}{g}U$ and U is the specified, constant mean geostrophic speed.

6.3 Numerical Solution Methods

We employ a hybrid finite volume-finite difference scheme for solving the shallow water equations:

For the continuity equation:

$$h_{i,j}^{n+1} = h_{i,j}^{n} - \frac{\Delta t}{\Delta x}(F_{i+1/2,j} - F_{i-1/2,j}) - \frac{\Delta t}{\Delta y}(G_{i,j+1/2} - G_{i,j-1/2}) \tag{55}$$

Where numerical fluxes F, G are computed using the HLL approximate Riemann solver:

$$F_{i+1/2,j} = \begin{cases} F_L & \text{if } 0 \leq S_L \\ \frac{S_R F_L - S_L F_R + S_L S_R (U_R - U_L)}{S_R - S_L} & \text{if } S_L \leq 0 \leq S_R \\ F_R & \text{if } 0 \geq S_R \end{cases} \tag{56}$$

Time step restriction follows the CFL condition:

$$\Delta t \leq \text{CFL} \cdot \min\{\frac{\Delta x}{|u| + \sqrt{gh}}, \frac{\Delta y}{|v| + \sqrt{gh}}\} \tag{57}$$

Where CFL is the Courant-Friedrichs-Lewy number, typically set between 0.5 and 0.9 for stability.

6.4 Terrain Integration

The interaction between flood dynamics and terrain is critical for accurate flood prediction. Our approach integrates high-resolution digital elevation models (DEMs) to provide accurate topographical information for the shallow water model.

The topographical effects are incorporated through several mechanisms:

- Bed slopes S_{0x}, S_{0y} derived directly from the DEM
- Spatially variable Manning's roughness coefficients based on land use data
- Building footprints represented as flow obstacles or areas with increased roughness
- Hydraulic structures (bridges, culverts, weirs) modeled as special boundary conditions

6.5 Wetting and Drying Algorithm

A key challenge in flood modeling is the accurate representation of the advancing flood front over initially dry terrain. We implement a robust wetting and drying algorithm to handle these transitions:

$$h_{i,j} = \max(0, h_{i,j}) \tag{58}$$

With special treatment for momentum conservation:

$$(uh)_{i,j} = \begin{cases} (uh)_{i,j} & \text{if } h_{i,j} > h_{min} \\ 0 & \text{otherwise} \end{cases} \tag{59}$$

Where h_{min} is a small threshold value (typically 0.01-0.05 m) that determines when a cell is considered "wet" for momentum calculations.

7 Portfolio Risk Assessment

7.1 Direct Impact Function

The flood depth is translated to property impact through:

$$I_i = \alpha \cdot (1 + \tanh(d_i)) \cdot f(b_i) \tag{60}$$

where:

- $\alpha = 0.0814$ is the baseline impact factor
- $f(b_i)$ is the building type adjustment:
  - Residential: 1.0
  - Commercial: 1.2
  - Industrial: 0.9

7.2 Spatial Correlation

The correlation ρ_{ij} between properties follows:

$$\rho_{ij} = \rho_0 \exp(-\frac{dist_{ij}}{d_c}) \tag{61}$$

where:

- ρ_0 is base correlation (default 0.4)
- d_c is correlation distance (default 1000m)
- $dist_{ij}$ is distance between properties i and j

7.3 Portfolio Impact Simulation

For each simulation s:

$$PI_s = \sum_{i=1}^{N} V_i \cdot I_i \cdot (1 + 0.2\epsilon_{i,s}) \tag{62}$$

where:

- $\epsilon_{i,s} \sim MVN(0, \Sigma)$ are correlated random shocks
- Σ is the spatial correlation matrix

The model calculates key risk metrics:

- Expected Loss: $E[PI]$
- 95% Value at Risk: $VaR_{95\%}$
- 95% Expected Shortfall: $ES_{95\%} = E[PI|PI > VaR_{95\%}]$
- Maximum Impact: $\max(PI)$

7.4 Geographic Concentration

Portfolio concentration is measured using HHI:

$$\text{HHI} = \sum_{i=1}^{n} \left(\frac{V_i}{\sum_{j=1}^{n} V_j} \right)^2 \tag{63}$$

7.5 Present Value Impact

For a holding period of n years and discount rate r, the present value of flood impact is:

$$PV_{\text{impact}} = L_{\text{total}} \times \frac{1 - (1+r)^{-n}}{r} \tag{64}$$

Where L_{total} represents the total expected annual loss.

8 Implementation Architecture

8.1 Computational Architecture

The implementation of our integrated flood prediction system leverages a hybrid cloud-edge architecture to balance computational requirements with low-latency delivery of results:

- **Cloud-based Weather Processing:** High-performance computing clusters process HRRR data and run the AFNO-based weather prediction
- **Edge-based Flood Modeling:** Local processing nodes handle terrain-specific shallow water simulations using regional data

- **Distributed Storage:** Multi-tier storage system with global weather data centralized and local terrain/property data distributed
- **Containerized Deployment:** Docker containers with Kubernetes orchestration ensure scalability and resilience

8.2 Real-Time Processing Pipeline

The real-time processing pipeline operates through several coordinated stages:

1. **Data Ingestion:** Continuous streaming of HRRR forecast data and real-time gauge observations
2. **Weather Prediction:** AFNO-based forecasting with a 6-hour update cycle and hourly outputs
3. **Precipitation Downscaling:** Statistical and ML-based precipitation localisation to 1km resolution
4. **Hydrological Processing:** Conversion of precipitation to runoff using terrain-aware infiltration models
5. **Flood Simulation:** GPU-accelerated shallow water equation solver at 10-50m resolution
6. **Impact Assessment:** Property-level flood depth calculation and financial impact estimation
7. **Uncertainty Propagation:** End-to-end uncertainty quantification with ensemble methods

8.3 Model Coupling Strategy

Effective coupling between model components is critical for maintaining physical consistency and computational efficiency:

- **Weather-to-Precipitation:** One-way coupling with dedicated precipitation model
- **Precipitation-to-Runoff:** Semi-coupled approach using manifold learning to maintain topological features
- **Runoff-to-Flood:** Two-way coupled to interface with feedback mechanisms for water table saturation
- **Flood-to-Impact:** Probabilistic mapping through Monte Carlo simulation

Each coupling interface includes specific data transformations and uncertainty propagation methods to ensure consistent handling of information across scales.

9 Discussion

9.1 Model Capabilities and Limitations

The integrated weather-to-flood prediction system demonstrates several key strengths:

- **Computational Efficiency:** The AFNO-based weather prediction provides near-operational accuracy with order-of-magnitude speedups, enabling rapid updating and ensemble generation.
- **Extreme Event Focus:** The specialised precipitation model shows particular skill in capturing high-intensity rainfall events that are most relevant for flood prediction.
- **Topological Consistency:** The UMAP-based dimensionality reduction preserves critical topological features in weather-to-flood mapping, maintaining essential relationships between precipitation patterns and terrain.

- **End-to-End Integration:** By connecting physical modelling with financial impact assessment, the system provides actionable insights for property owners, insurers, and mortgage providers.

However, several limitations must be acknowledged:

- **Resolution Constraints:** While the downscaling approach improves spatial resolution, sub-kilometer features remain challenging to resolve, particularly in urban environments with complex drainage networks.
- **Uncertainty Propagation:** Cascading uncertainties from weather prediction through to financial impact can lead to wide confidence intervals for specific properties, though portfolio-level aggregation helps mitigate this effect.
- **Data Requirements:** The system relies on high-quality digital elevation models and building footprint data, which may be incomplete or outdated in some regions.
- **Validation Limitations:** While performance has been validated across diverse events, the system has not yet been tested on truly extreme (¿500-year return period) events due to limited historical examples with comprehensive observations.

9.2 Operational Considerations

Deploying the system in operational settings reveals several important considerations:

- **Update Frequency:** While the AFNO model enables rapid forecasting, data ingestion from operational NWP centres creates a dependency that limits update cycles to 6-12 hours.
- **Computational Infrastructure:** The hybrid cloud-edge architecture balances centralised weather prediction with distributed flood modelling but requires careful data transfer optimisation between tiers.
- **Ensemble Size:** Operational constraints typically limit ensemble size to 50-100 members, which provides reasonable uncertainty quantification for portfolio-level analysis but may be insufficient for rare event characterisation.
- **Alert Thresholds:** Establishing appropriate probability thresholds for alerts requires balancing false alarm rates with missed event consequences, with optimal thresholds varying by region and property type.

The system's design allows for modular updates, with each component (weather prediction, manifold learning, flood modelling, impact assessment) able to be enhanced independently. This architecture enables incremental improvements while maintaining operational continuity.

9.3 Future Research Directions

Several promising directions for future research emerge from this work:

- **Temporal UMAP Extensions:** Developing variants of UMAP that explicitly incorporate the temporal evolution of weather patterns could improve the representation of dynamic weather systems.
- **Physics-Informed Neural Operators:** Integrating physical constraints directly into the neural network architecture could improve generalisation to unseen climate regimes.
- **Infrastructure Interaction:** Enhancing the hydrodynamic model better to represent interactions with urban drainage systems and flood defences would improve predictions in developed areas.
- **Climate Change Adaptation:** Extending the framework to incorporate climate change scenarios would enable long-term risk assessment and adaptation planning.

- **Data Assimilation:** Developing real-time data assimilation methods to incorporate observations from IoT sensors, social media, and crowdsourced data could improve forecast accuracy during evolving flood events.
- **Multi-Hazard Integration:** Expanding the framework to include compound events such as combined coastal and fluvial flooding or cascading hazards like landslides triggered by sustained rainfall.

9.4 Broader Impact

The integration of probabilistic flood prediction with property-level impact assessment has significant implications for multiple stakeholders:

- **Insurance Industry:** More granular risk assessment enables better pricing and portfolio management, potentially expanding insurability in moderate-risk areas while identifying truly high-risk properties.
- **Mortgage Providers:** Long-term risk quantification improves loan pricing and portfolio diversification, reducing systemic risk in property markets.
- **Property Developers:** Forward-looking flood risk assessment can guide development away from vulnerable areas and inform design adaptations in moderate-risk zones.
- **Emergency Management:** Real-time, probabilistic predictions support more effective evacuation and resource deployment decisions during flood events.
- **Climate Adaptation Planning:** The system's ability to model different climate scenarios allows communities to evaluate adaptation strategies and infrastructure investments.

By bridging the gap between meteorological science, hydrodynamic modelling, and financial risk assessment, this work contributes to more resilient communities and property markets in the face of increasing flood risks due to climate change and urbanisation.

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Appendix

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