Geostrophic Speed

Contents

1 Key Characteristics

- **Force Balance:** Achieved when Coriolis force ($-f \times v$) balances pressure gradient force ($-\frac{1}{\rho}\nabla p$)
- **Flow Direction:** Parallel to isobars/isopycnals (anti-clockwise around lows in NH)
- **Mathematical Formulation:**
$$v_g = \frac{1}{f\rho}\frac{\partial p}{\partial n}$$
where:
  - $f = 2\Omega \sin\phi$ (Coriolis parameter)
  - ρ = fluid density
  - $\frac{\partial p}{\partial n}$ = pressure gradient normal to flow

2 Key Relationships

2.1 Atmospheric Version

$$v_g = \frac{g}{f}\frac{\Delta Z}{\Delta n}$$

where g = gravitational acceleration (9.81 m/s^2), ΔZ = geopotential height difference.

2.2 Oceanic Version

$$v_g = \frac{g}{f\rho_0}\frac{\partial \rho}{\partial x}\Delta z$$

Accounting for thermal wind adjustment in stratified flows.

3 Practical Calculation Example

Given parameters:

$$\begin{aligned}\Delta Z &= 60\,\mathrm{m}\ \text{over}\ 300\,\mathrm{km}\\ f_{45^\circ\mathrm{N}} &= 10^{-4}\,\mathrm{s}^{-1}\\ g &= 9.8\,\mathrm{m/s}^2\end{aligned}$$

Solution:

$$\begin{aligned}v_g &= \frac{9.8}{10^{-4}} \times \frac{60}{3\times 10^5}\\ &= 98\times 10^3 \times 2\times 10^{-4}\\ &= 19.6\,\mathrm{m/s} \quad (\sim 38\,\text{knots})\end{aligned}$$

4 Limitations and Extensions

- **Gradient Wind Approximation:** For curved flows:

$$\frac{v^2}{r} + fv = \frac{1}{\rho}\frac{\partial p}{\partial n}$$

- **Thermal Wind:** Vertical shear relationship:

$$\frac{\partial v_g}{\partial z} = \frac{g}{fT}\frac{\partial T}{\partial n}$$