PRACTICAL 6

Testing hypotheses about the population proportion

When the parameter of interest is a proportion, the methodology from Practical 5 still applies. The test statistic now becomes

$$z = \frac{\hat{p} - p}{SE}$$

where p is the proportion under the null hypothesis. One proceeds in the same manner, calculating the critical value(s) and/or p-value.

Consider this hypothetical example: A random sample of children who visit McDonalds was asked what flavour milkshake they prefer. Of 300 children asked, 172 indicated a preference for bubblegum flavour. Test

$$H_0\colon\ p = 0.5$$

against the alternative

$$H_a\colon\ p > 0.5$$

at the 1% level of significance.

The value for z is calculated to be 2.54, which is greater than the critical value of 2.326. The value for z lies in the rejection region, and the null hypothesis is rejected. The calculated p-value is 0.005, which is much smaller than 0.01. Again, this indicates the null hypothesis should be rejected.

Exercise

1. For the variable *X* in the tab *Mu66*, test the null hypothesis that the proportion of scores higher than 60 is equal to a half, against a two-sided alternative. Use a 10% level of significance.

2. In a medical trial, people received an experimental treatment. The participants represent a random sample of people who have the disease. In a group of 54 people, 22 made a complete recovery. Test the null hypothesis that the proportion of people who make a complete recovery is 1/2, against the one-sided alternative the proportion is less than 1/2. Use a 5% level of significance.

Hypothesis testing in small samples

When testing hypotheses about population means, the normal approximation only holds for “large" samples. When the sample size is small, the *t*-distribution gives a better representation of reality. The *t*-distribution is characterised by its *degrees of freedom* (*df*), which is directly related to sample size. When *df* ≥ 30, there is very little difference between the *t*-distribution and the normal approximation.

The *Excel* command *t.dist(x,deg_freedom,cumulative)* returns the tail probability associated with a negative *x* value. The *Excel* command *t.dist.2t(x,deg_freedom)* returns the two tail probabilities associated with a two-tailed test. The *Excel* command *t.dist.rt(x,deg_freedom)* returns the tail probability associated with a positive *x* value.

The command *t.inv(probability,deg_freedom)* returns the one-tailed negative critical value for the *t*-distribution with the specified degrees of freedom.
The command *t.inv.2t(probability,deg_freedom)* returns the two-tailed critical value for the *t*-distribution with the specified degrees of freedom.

Consider the following example:
We observe values 12*;* 15*;* 21*;* 14 and 16 from some population and wish to test the hypothesis

$$H_0: \mu = 12.5$$

The test statistic is calculated as usual,

$$t = \frac{\bar{x} - \mu}{{}^{s}/_{\sqrt{n}}}$$

and a value of 2.062 is obtained. If the normal approximation were used, and the alternative was two-sided, we would obtain a *p*-value of 0.039 (make sure you can find this value), leading us to reject the null hypothesis at the 5% level of significance. On the other hand, the *p*-value obtained from a *t*-distribution with *df* = 4 is 0.108, i.e. the null hypothesis is not rejected. The critical value for α = 0.05 is 2.776. The observed value of *t* is less than this, again confirming that the null hypothesis should not be rejected.
In general, the degrees of freedom for the one-sample *t*-test is *n* - 1 where *n* is the sample size.

Exercise

3. The carbon monoxide diffusing capacity (DL) of 20 randomly sampled smokers is given in the tab *Smoke*. The average for non-smokers is 100 DL. Researchers believe that the DL of current smokers may be significantly less. Test whether the mean DL reading of smokers is equal to 100 for α = 0.01.