PHYS 3142 HW 1

Due date: 11:59 PM 20^{th} Feb. 2022

- Submit a report that includes your results and your python scripts
- Make sure your code can run
- Write comments in your code
- If you submit the assignment after the deadline or the report is missing, you can only get at most 80% of the full marks.
- If there is any kind of plagiarism, all students involved will get zero marks.

1 Title Case Capitalization (50 points)

Capitalize all words in a title entered, except ”a”, ”an”, ”the”,”and”, ”at”, ”by”, ”for”, ”in”, ”of”, ”on”, ”to”, ”up”, ”and”, ”as”, ”but”, ”or”, and ”nor”. Including the second part of hyphenated words [1]. You can use python methods for strings to split a sentence and capitalize a word or a letter. Take screen capture and attach it to your report (submit it along with your code). Your output should look like the following.

Enter a title: welcome to the world of data-drive modeling for master students offered by HKUST in a great and green campus on the hill.

Capitalized: Welcome to the World of Data-Driven Modeling for Master Students Offered by HKUST in a Great and Green Campus on the Hill.

2 Encryption and Decryption Using Modular Arithmetic (50 points)

Use brute force method to decipher a message encrypted using the affine cipher [2]. A program is given to you to convert a letter in the alphabet to an integer in $[0, 25]$ and vice versa.

2. (a) For the set $\mathbb{Z}/26 = \{0, 1, 2, ..., 24, 25\}$. Find the multiplicative inverses by trying all combinations for every element. Let a be an element in that set, a^{-1} is another element in that set such that $(a^{-1} \times a) \pmod{26} = 1$. For example, the multiplicative inverse for 7 is 15. Out of the 26 integers, how many of them have a multiplicative inverse? (hint: between 10 and 20) And what is the condition for its existence? (hint: related to the integer 26).

2. (b) Confirm the affine cipher can be deciphered. The affine cipher transforms x to y by

$$y = (a \times x) + b \pmod{26}. \tag{1}$$

If a^{-1} exists, this can be deciphered using

$$x = a^{-1} \times (y - b) \pmod{26}, \tag{2}$$

where x, y, a, b and a^{-1} are integers in the set $\mathbb{Z}/26$.

Using $a = 7$ and $b = 18$, check that $13 = (a \times 3) + b \pmod{26}$ and $3 = a^{-1} \times (13 - b) \pmod{26}$.

2. (c) Use the given program to convert between letters and numbers. Try all possible combinations of a and b to decipher the following message: $gkstmdodikbojsydzkpuibtzwuigu$. Given that the first two letters are co. What does the message say? What values of a and b did you use to decrypt it?

OPTIONAL

3 Print the Pascal's triangle (10 points)

Pascal's triangle is a triangular array of the binomial coefficients.[3] Please define a function which could print out the similar pyramid as shown below (An example when the number of lines is 8), given the number of lines of the pyramid.

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
    1   6   15  20  15  6   1
  1   7   21  35  35  21  7   1
```

4 Insertion sort (10 points)

Insertion sort is one of the simplest sorting algorithms, which is widely used. For each pass, the first unsorted values is inserted in the right place of the sorted partition. The process could be viewed from the YouTube video[4]. Please sort the list below in the increasing order using insertion sort:

$[14, 46, 43, 27, 57, 41, 45, 21, 70]$

References

[1] Title case capitalization - american psychological association. https://apastyle.apa.org/style-grammar-guidelines/capitalization/title-case.

[2] Affine cipher - wikipedia. https://en.wikipedia.org/wiki/Affine_cipher.

[3] Pascal's triangle - wikipedia. https://en.wikipedia.org/wiki/Pascal%27s$_triangle$.

[4] Insertion sort - toutube video. https://www.youtube.com/watch?v=JU767SDMDvA.