

# PHYS 3142 HW 4

Due date: 11:59 PM 14<sup>th</sup> Mar. 2022

- Submit a report that includes your results and your python scripts
- Make sure your code can run
- Write comments in your code
- If you submit the assignment after the deadline or the report is missing, you can only get at most 80% of the full marks.
- If there is any kind of plagiarism, all students involved will get zero marks.

## 1 Romberg integration (20 points)

Use Romberg integration, calculate

$$\int_0^1 e^{-x^2} dx. \quad (1)$$

Print all  $R_{i,m}$  (up to  $i = 5$  and  $m = 5$ , see fig. 1) in the console and attach the results in your report (round them to 6 decimal places). The number of intervals  $N$  is doubled for each iteration, starting from  $N = 1$ . Your results can be checked by using `scipy.integrate.romberg` with the argument `show = True`.

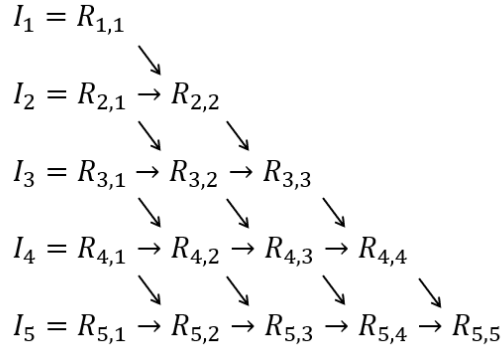


FIG. 1 Graphical representation of the iterative processes in Romberg integration.  $I_i$  is the trapezoidal rule with  $N_i$  many intervals. The number of intervals double for the next  $i$ . And  $R_{i,m}$  denotes the  $m^{\text{th}}$  extrapolation.

## 2 Comparisons of numerical integration methods (80 points)

Compute the following integration.

$$\int_0^1 4x^3 e^{x^4} dx. \quad (2)$$

Use 3 different methods, including (1) adaptive Simpson's rule, (2) Romberg's method and (3) Gaussian quadrature. Compare with the analytical results to plot the errors versus the number of intervals  $N$  in a log-log plot. Use the following values of  $N$  for all 3 methods, (4, 8, 16, 32, 64, 128). (Use `gaussxw.py` in the folder for lecture 7 on Canvas to calculate the weights and integration points for Gauss quadrature.)

## Optional

### 3 Variable transformation (10 points)

Make the variable transformation  $x = \frac{1-z}{z}$ . Use the analytical result

$$\int_0^\infty \frac{1}{1+x^2} dx = \frac{\pi}{2} \quad (3)$$

to find the error. Use adaptive trapezoidal rule for the above integral starting with  $N = 1$ , double  $N$  in each iteration. Repeat the iteration until  $|error| < 10^{-8}$  ( $N$  is the number of intervals). How many iterations do you need (how large is  $N$ )?

## 4 Singular integrand (10 points)

Please use two ways to do the integral below:

$$I = \int_0^1 f(x)dx = \int_0^1 \frac{\cos x + \sin^2 \sqrt{x}}{\sqrt{x}} dx \quad (4)$$

### 4.1 subtraction

Please think of a function  $g(x)$ , which makes there no singular point over the integration interval for the integrand  $f(x) - g(x)$ , while  $g(x)$  could be integrated numerically or analytically. Please use the  $g(x)$  you found to compute the value of  $I$  to the accuracy smaller or equal to  $10^{-5}$ .

### 4.2 truncation

Please truncate the integration at a small value  $\epsilon$ :

$$I = \int_{\epsilon}^1 \frac{\cos x + \sin^2 \sqrt{x}}{\sqrt{x}} dx \quad (5)$$

show the value  $I$  versus number of slices  $N$  plot for  $\epsilon$  in the list  $[0.1, 0.01, 0.001, 0.0001, 0.00001]$ . What are the convergence values?