PHYS 3142 HW 4

Due date: 11:59 PM 14^{th} Mar. 2022

- Submit a report that includes your results and your python scripts
- Make sure your code can run
- Write comments in your code
- If you submit the assignment after the deadline or the report is missing, you can only get at most 80% of the full marks.
- If there is any kind of plagiarism, all students involved will get zero marks.

1 Romberg integration (20 points)

Use Romberg integration, calculate

$$\int_0^1 e^{-x^2} dx. \tag{1}$$

Print all $R_{i,m}$ (up to $i = 5$ and $m = 5$, see fig. 1) in the console and attach the results in your report (round them to 6 decimal places). The number of intervals N is doubled for each iteration, starting from $N = 1$. Your results can be checked by using `scipy.integrate.romberg` with the argument `show = True`.

$$
\begin{array}{lllll}
I_1 = R_{1,1} & & & & \\
& \searrow & & & \\
I_2 = R_{2,1} & \rightarrow R_{2,2} & & & \\
& \searrow & \searrow & & \\
I_3 = R_{3,1} & \rightarrow R_{3,2} & \rightarrow R_{3,3} & & \\
& \searrow & \searrow & \searrow & \\
I_4 = R_{4,1} & \rightarrow R_{4,2} & \rightarrow R_{4,3} & \rightarrow R_{4,4} & \\
& \searrow & \searrow & \searrow & \searrow \\
I_5 = R_{5,1} & \rightarrow R_{5,2} & \rightarrow R_{5,3} & \rightarrow R_{5,4} & \rightarrow R_{5,5}
\end{array}
$$

FIG. 1 Graphical representation of the iterative processes in Romberg integration. I_i is the trapezoidal rule with N_i many intervals. The number of intervals double for the next i. And $R_{i,m}$ denotes the m^{th} extrapolation.

2 Comparisons of numerical integration methods (80 points)

Compute the following integration.

$$\int_0^1 4x^3 e^{x^4} dx. \tag{2}$$

Use 3 different methods, including (1) adaptive Simpson's rule, (2) Romberg's method and (3) Gaussian quadrature. Compare with the analytical results to plot the errors versus the number of intervals N in a log-log plot. Use the following values of N for all 3 methods, $(4, 8, 16, 32, 64, 128)$. (Use `gaussxw.py` in the folder for lecture 7 on Canvas to calculate the weights and integration points for Gauss quadrature.)

Optional

3 Variable transformation (10 points)

Make the variable transformation $x = \frac{1-z}{z}$. Use the analytical result

$$\int_0^\infty \frac{1}{1+x^2} dx = \frac{\pi}{2} \tag{3}$$

to find the error. Use adaptive trapezoidal rule for the above integral starting with $N = 1$, double N in each iteration. Repeat the iteration until $|error| < 10^{-8}$ (N is the number of intervals). How many iterations do you need (how large is N)?

4 Singular integrand (10 points)

Please use two ways to do the integral below:

$$I = \int_0^1 f(x)dx = \int_0^1 \frac{\cos x + \sin^2\sqrt{x}}{\sqrt{x}}dx \tag{4}$$

4.1 subtraction

Please think of a function $g(x)$, which makes there no singular point over the integration interval for the integrand $f(x) - g(x)$, while $g(x)$ could be integrated numerically or analytically. Please use the $g(x)$ you found to compute the value of I to the accuracy smaller or equal to 10^{-5}.

4.2 truncation

Please truncate the integration at a small value ϵ:

$$I = \int_\epsilon^1 \frac{\cos x + \sin^2\sqrt{x}}{\sqrt{x}}dx \tag{5}$$

show the value I versus number of slices N plot for ϵ in the list [0.1,0.01,0.001,0.0001,0.00001]. What are the convergence values?