PHYS 3142 HW 2

Due date: 11:59 PM 27th Feb. 2022

- Submit a report that includes your results and your python scripts
- Make sure your code can run
- Write comments in your code
- If you submit the assignment after the deadline or the report is missing, you can only get at most 80% of the full marks.
- If there is any kind of plagiarism, all students involved will get zero marks.

1 Arc length of a cycloid

1. (a) Using a uniform sampling of time, track the motion of a point attached to a circle that rolls without slipping. (The position of the center of mass is $(t, 1)$. The circle is rotating with angular velocity $\frac{d\theta}{dt} = -1$ (clockwise).) In other words, reproduce only the blue scatter plot of the following figure. You must show that the dots do not distribute uniformly. Use $r = 1$. Attach the figure in png format in your report.

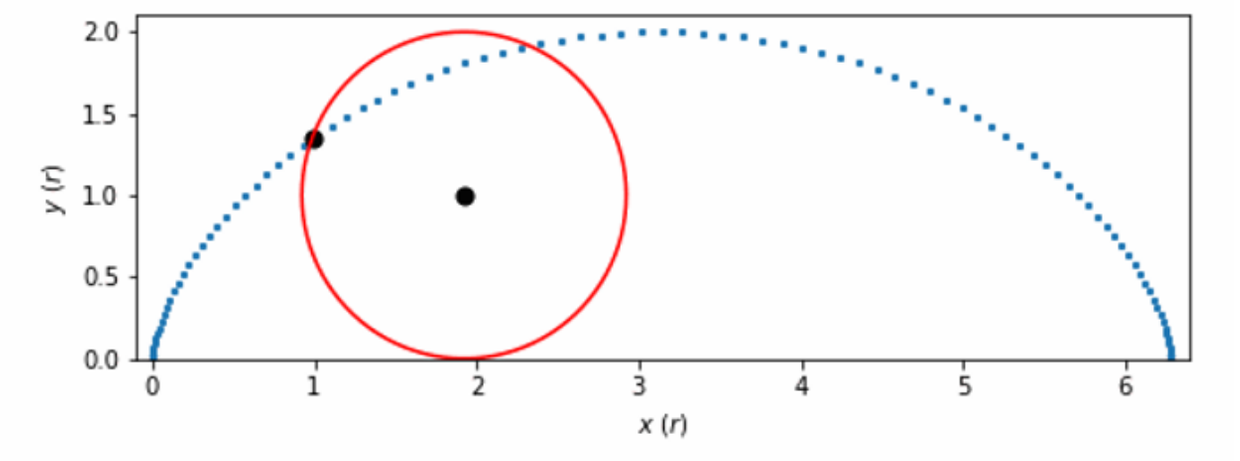


FIG. 1 A cycloid formed by tracing a point of a rotating circle that does not slip.

1. (b) Add up the distance between each two consecutive points, find the arc length of a cycloid.

2 Cross section of atomic orbitals

The eigenfunction of an electron in a hydrogen atom is $\psi_{nlm}(r,\theta,\phi) = R_{nl}(r)Y_l^m(\theta,\phi)$ which is complex. In many applications, real wavefunctions are used instead, they can be formed by taking linear combinations of the eigenfunctions.

2. (a) Following ref. [1] and other references on atomic orbitals, write down p_x as linear combinations of spherical harmonics. (For example, $d_{x,y} = i\sqrt{\frac{1}{2}}\left(Y_2^{-2} + Y_2^2\right)$.) What is the quantum number l for p_x and d_{xy} orbitals, respectively?

2. (b) After finding out s, p, d, f correspond to which values of l, and how to form real wavefunctions with different m. Plot the atomic orbitals $2p_x$ and $3d_{x,y}$. The numbers 2 and 3 are the principal quantum number n. See ref. 2. Plot the intersection of the atomic wavefunction with the xy plane ($z = 0$). Use pseudocolor plot with colormap "bwr". Choose your own range and figure size. Use appropriate units. The constant terms can be ignored and the magnitude of the wavefunction is not important here. Your plots should look like fig. 2 below.

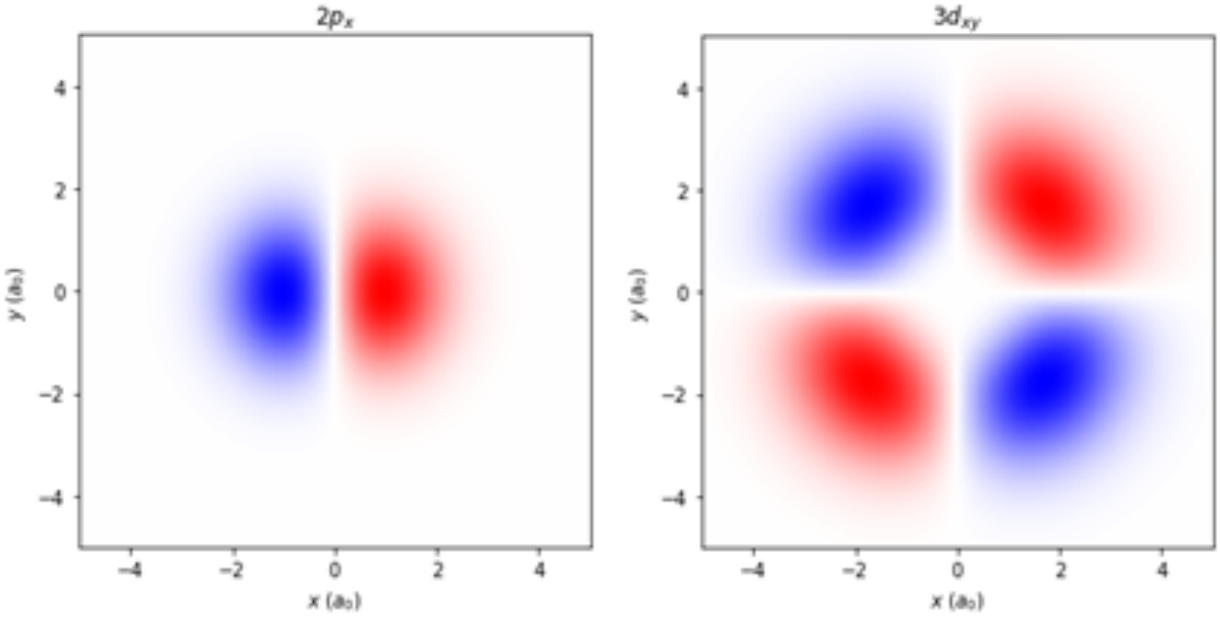


FIG. 2 Pseudocolorplots of the atomic orbitals $2p_x$ and $3d_{xy}$.

OPTIONAL

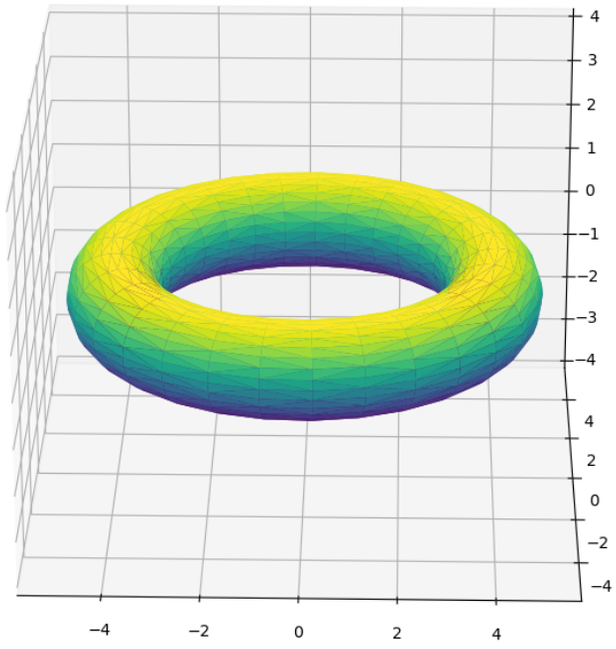
3 Plot the torus (10 points)

Torus is a basic and commonly used concept in physics and math, especially when topology is involved. The frequently referenced genus n surface in topology, which is also called g-torus or g-holed torus, uses torus as a ”counting unit”. the genus 0 orientable surface is just a sphere, and a genus g surface is a torus containing g holes.[3] The parametric function for a ring torus is[4]:

$$x(\theta, \phi) = (R + r\cos\theta)\cos\phi$$
$$y(\theta, \phi) = (R + r\cos\theta)\sin\phi$$
$$z(\theta, \phi) = r\sin\theta$$

The range of the angles is :$\theta, \phi = [0, 2\pi)$.

R is the radius from the center of torus to the center of the tube, and r is the radius of the tube. (R>r) Please use the plot_trisurf to plot a torus.



References

[1] Real spherical harmonics, in Table of spherical harmonics - Wikipedia. https://en.wikipedia.org/wiki/Table_of_spherical_harmonicsReal_spherical_harmonics.

[2] The Radial Wavefunction solutions. https://quantummechanics.ucsd.edu/ph130a/130_notes/node233.html.

[3] Genus g surface. https://en.wikipedia.org/wiki/Genus_g_surface#Genus_2.

[4] Torus. https://en.wikipedia.org/wiki/Torus.