## PHYS 3142 HW 7

Due date: 11:59 PM 3<sup>rd</sup> Apr. 2022

- Submit a report that includes your results and your python scripts
- Make sure your code can run
- Write comments in your code
- If you submit the assignment after the deadline or the report is missing, you can only get at most 80% of the full marks.
- If there is any kind of plagiarism, all students involved will get zero marks.

## 1 Metropolis algorithm for 2D Ising model (100 points)

Use the Metropolis algorithm to simulate the Ising model in a square lattice with  $10 \times 10$  sites. Use *periodic* boundary conditions. The energy is given by the Hamiltonian which is

$$H = -J \sum_{\langle i,j \rangle}^{N} S_i S_j \tag{1}$$

where  $S_i = \pm 1$  and  $\langle i, j \rangle$  denotes the nearest neighbors.

Use J=2 and  $\beta=\frac{1}{k_B}=1$ . Plot the energy  $E=\frac{H}{N}$ , magnetization M, heat capacity  $C_v$  and susceptibility  $\chi$  with temperature from T=1 to T=20 with  $\Delta T=0.2$ .

$$M = \frac{1}{N} \sum_{i}^{N} S_i \tag{2}$$

$$C_v = \frac{\langle E^2 \rangle - \langle E \rangle^2}{k_B T^2} \tag{3}$$

$$\chi = \frac{\langle M^2 \rangle - \langle M \rangle^2}{T} \tag{4}$$

## **Optional**

## 2 Considering antiferromagnetic interactions(10 points)

Please use the Metropolis algorithm to simulate the Ising model in square lattice with 10\*10 sites. The periodic boundary condition is used. The Hamiltonian is:

$$H = -\sum_{\langle i,j \rangle}^{N} J_{i,j} S_i S_j \tag{5}$$

Now we assume that  $J_x=2$  while  $J_y=-1$ . (i.e. now the coefficient J is different along x and y direction. The lattice has ferromagnetic interaction along x direction and antiferromagnetic interaction along y direction.) What's the Energy, Magnetization, Heat capacity and Susceptibility of the system then? Please plot the figure of these observables over Temperature from T=1 to T=10 with  $\Delta T=0.2$