PHYS 3142 HW 10

Due date: 11:59 PM 1st May 2022

- Submit a report that includes your results and your python scripts
- Make sure your code can run
- Write comments in your code
- If you submit the assignment after the deadline or the report is missing, you can only get at most 80% of the full marks.
- If there is any kind of plagiarism, all students involved will get zero marks.

1 Nonlinear pendulum (50 points)

For a frictionless pendulum, the motion of the pendulum is

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta \tag{1}$$

For this problem, use $g = 4$, $L = 1$ and $m = 1$. Here g is the gravitational acceleration, L is the length of a massless cord that supports the mass m. And we have initial conditions $\theta(t = 0) = \theta_0$ which will be specified below and $\omega(t = 0) = 0$. $\omega = \frac{d\theta}{dt}$ is the angular velocity.

1. a For small initial angle $\theta_0 \ll 1$ (θ is in radians), the equation can be simplified to

$$\frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta \tag{2}$$

Then for this linear differential equation, the solution is well-known. It is $\theta \approx \theta_0 \cos(2\pi t/T_l)$. $T_l = 2\pi\sqrt{L/g}$ is the period in this linear approximation. For $\theta_0 = 0.7$, use the second-order Runge-Kutta method plot the solution θ and the linear approximation in the same graph. Use 100 time steps, from $t = 0$ to $t = 5\pi$.

1. b Because the total energy of this frictionless pendulum should be strictly zero, the error can be seen more clearly from the total energy. For the nonlinear pendulum, the total energy is the sum of the potential energy $mgl(\cos\theta_0 - \cos\theta)$ and the kinetic energy $\frac{1}{2}ml^2\omega^2$. For $\theta_0 = 1$, use 100 time steps, for time from $t = 0$ to $t = 5\pi$. Plot the total energy as a function of time for both the second-order Runge-Kutta method and the fourth-order Runge-Kutta method. Since their magnitudes are quite different, plot them in separate graphs.

2 Projectile motion (50 points)

The range of a cannon or a mortar is significantly affected by air resistance. Consider a cannon that shoots a projectile that weights 15 kg with initial velocity 200 m/s. The launch angle θ_0 will be specified or calculated. Including air resistance, the equations of motion are

$$\frac{d^2x}{dt^2} = -bv_x\sqrt{v_x^2 + v_y^2} \tag{3}$$

$$\frac{d^2y}{dt^2} = -g - bv_y\sqrt{v_x^2 + v_y^2} \tag{4}$$

where $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$ and $g = 10$.

2. a If the launch angle is $\theta_0 = \pi/4$. Plot the trajectories with and without including the air resistance

$b = 0.0003\ \mathrm{m}^{-1}$ on the same graph. Use the fourth-order Runge-Kutta method with a time step $h = 0.1$ s and stop the calculation when the cannon ball hits the ground.

2. b Including air resistance with $b = 0.0003\ \mathrm{m}^{-1}$ and also use the fourth-order Runge-Kutta method with a time step $h = 0.1$ s. Change your program to find the distance when the cannonball hits the ground, with the launch angle θ_0 as the input. Use Golden ratio search, Newton's method or Gradient descent method, find the launch angle that gives the maximum shooting range. Your result for θ_0 should be accurate to at least 2 significant figures. You can further reduce the time step h to check the convergence of your result but it is not required here.

Optional

3 Shooting method (10 points)

Using the same parameters (including the same initial velocity and the same b) given in Q2, find the launch angle that hits a target 1500 m away. The require angle should be smaller than $\pi/3$. The target is at the same height as the cannon.

This is a boundary value problem. This can be treated as an initial value problem by the shooting method, which is trying different initial values until the boundary conditions are satisfied. This can be turned into a root finding problem, you should use binary search to find the required launch angle θ_0. Your result for θ_0 should be accurate to at least 2 significant figures.