PHYS 3142 HW 3

Due date: 11:59 PM 6th Mar. 2022

- Submit a report that includes your results and your python scripts
- Make sure your code can run
- Write comments in your code
- If you submit the assignment after the deadline or the report is missing, you can only get at most 80% of the full marks.
- If there is any kind of plagiarism, all students involved will get zero marks.

1 Speeding up the program

Please use broadcasting or other methods to speed up the following simulation codes. You need to roughly speed it up 5 times as the minimum requirement. (Generally speaking, there are no restrictions on how to speed up the program, as long as you are dealing with the same problem illustrated by the codes below). You may want to use `np.random.rand` to get an array of random numbers.

```
import numpy as np
import time


#only one bullet in the gun
```

```python
num_test=20000000
num_pos=20
pos_take=tuple(range(10))

start_time=time.time()
np.random.seed(10)
num_lose=0
for nt in range(num_test):
    A=np.zeros(num_pos)
    A[np.random.randint(0,num_pos)]=1

    for n in range(num_pos):
        if A[n]==1:
            if n in pos_take:
                num_lose += 1
            break

print("The probability of losing is:", num_lose/num_test)
print("Time:",time.time()-start_time)
print("Number of tests:",num_test)
```

2 Derive the equations BY HAND

During the lectures, several basic methods of integration are introduced. Now instead of directly using the formula, you should convince yourself by checking the results using Taylor expansions. Please derive the following results by hand!

2.1 Simpson's rule

First, approximate a function $f(x)$ with a quadratic polynomial.

$$P(x) = a_0 + a_1 x + a_2 x^2. \tag{1}$$

Find constants a_0, a_1 and a_2 such that $P(x') = f(x')$ at $x' = a, a+h$, and $a+2h$. Integrate this quadratic polynomial that approximates the original function to derive the Simpson's rule.

$$\int_a^{a+2h} P(x)dx = \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]. \tag{2}$$

Then, use $N+1$ many points to divide the interval (a, b) into N many intervals to derive the composite Simpson's rule.

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + f(b) + 4\sum_{k=1}^{\frac{N}{2}} f(a+(2k-1)h) + 2\sum_{k=1}^{\frac{N}{2}-1} f(a+2kh)\right], \tag{3}$$

where $h = \frac{b-a}{N}$.

2.2 The error of trapezoidal rule

Follow the slides for Lecture 6, derive

$$error = \frac{1}{12}h^2[f'(a) - f'(b)] + O(h^4) \tag{4}$$

OPTIONAL

3 Truncation error in Apéry's constant (10 points)

The Riemann zeta function is defined as

$$\zeta(z) = \lim_{N\to\infty}\sum_{n=1}^{N}\frac{1}{n^z} = 1 + \frac{1}{2^z} + \frac{1}{3^z} + \dots \tag{5}$$

where z is a complex number. And the series is convergent for $Re(z) > 1$. But in practice we only have the partial sums

$$S_N = \sum_{n=1}^{N}\frac{1}{n^z} = 1 + \frac{1}{2^z} + \frac{1}{3^z} + \dots + \frac{1}{N^z} \tag{6}$$

The truncation error can be approximated by comparing the partial sums of different orders, Error $\approx S_M - S_N$, where $M \gg N$. In fig. 3, I plotted the error versus N using $M = 10^5$ for $z = 2$. For $N = 10^4$, the error is about $10^{-4.05}$ (smaller than 0.0001), meaning that my result should be accurate to at least the 3$^{\text{rd}}$ decimal place. Then, instead of $S_{10^4} = 1.644834\dots$, I can write my result as $\zeta(2) \approx 1.645$.

Similarly, calculate $\zeta(z)$ and plot the error for $z = 3$. Write down your result (using $N = 10^4$) with the appropriate number of digits. Use `plt.semilogy(x,y,base=10)` for your plot. $\zeta(3)$ is called Apéry's constant.

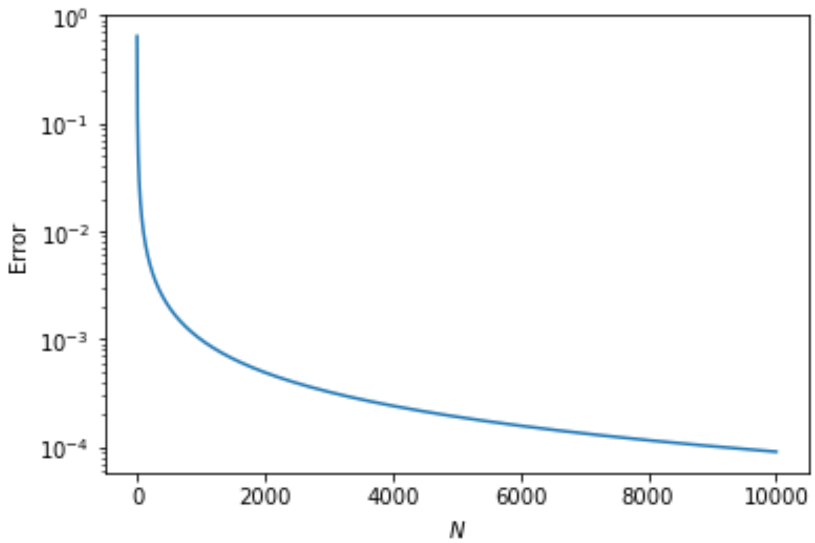

FIG. 3 Error of $\zeta(2)$ versus the number of terms in the summation.