PHYS 3142 HW 5

Due date: 11:59 PM 20$^{\text{th}}$ Mar. 2022

- Submit a report that includes your results and your python scripts
- Make sure your code can run
- Write comments in your code
- If you submit the assignment after the deadline or the report is missing, you can only get at most 80% of the full marks.
- If there is any kind of plagiarism, all students involved will get zero marks.

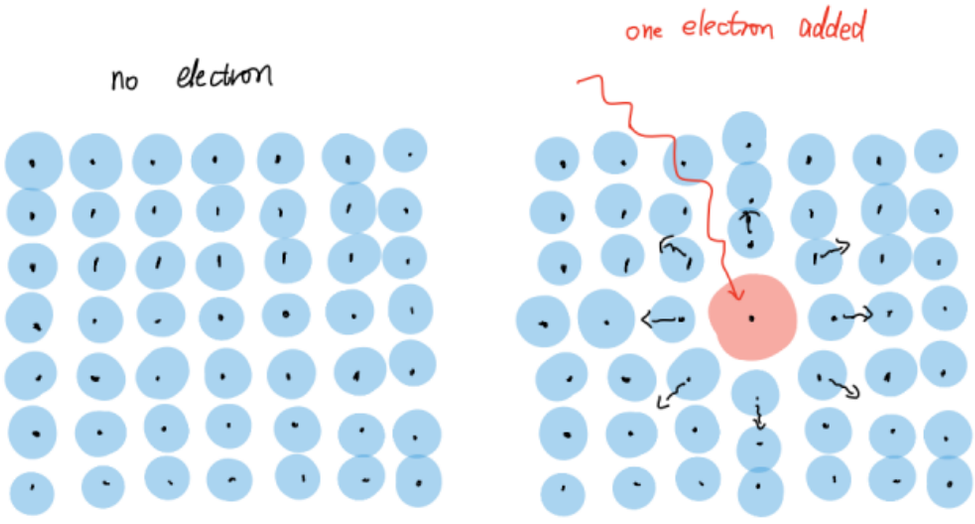
1 Impurity phonon problem (40 points)

When it comes to microscopic systems, the rules obeyed by the particles are far from the physics of our everyday life. It's impossible to observe or manipulate the system directly using eyes or hands. However, we can use particles like electrons to probe the systems! Imagine that we have an extremely sharp conducting tip, like the one used by the scanning tunneling microscope (STM)[1], and put the tip very close to the material's surface so that electrons could tunnel between the two systems. Near the zero point, suppose there's one electron escaping from the sample surface to the tip at $t = 0$, and tunnel back to the sample at $t = t_0$. Because of the lattice vibrations, there will also be phonons, which are likely to be coupled with the electrons(impurity phonon problem [2]). The probability of the system having the tunneling-in electron in each energy level is given by:

$$P(\omega) = \sum_{l=0}^{\infty} \frac{e^{-g} g^l}{l!} \delta(\omega - (\omega_0 + l\omega_E)) \tag{1}$$

l is an integer, and g is the coupling strength between electrons and phonons, ω_0 and ω_E are some constants.

We can see that the probability of getting the electron in each energy level l obeys the Poisson distribution! To simplify the problem, we ignore the delta function part.



1.1 Visualize the Poisson distribution

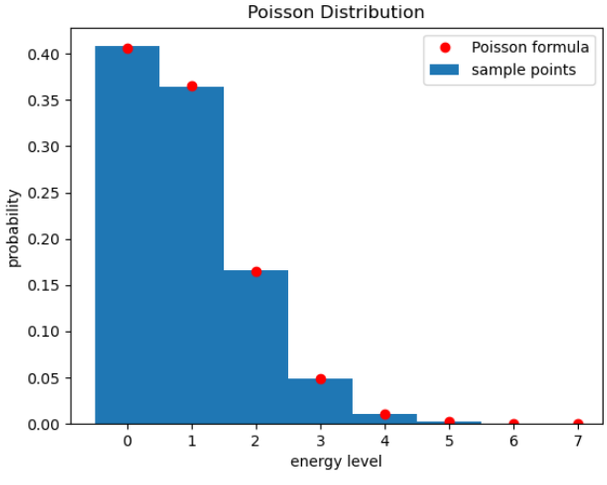
Now suppose that the experiment described above has been repeated for 30000 times, and the energy level l where the electron appears each time is recorded. The coupling strength g is found to be $g = 0.9$. The following code could be used to simulate this process:

```
from scipy.stats import poisson
array = poisson.rvs(mu= #g value above#, size= #array size#)
```

Please plot the histogram using sample points generated above (set density = True, also adjust the number of bins to make the graph look nicer). Compare the results with the scatter plot using the Poison distribution formula directly:

$$P(X = l) = \frac{e^{-g} g^l}{l!} \tag{2}$$

The result should look like the picture below:



Also please calculate the mean value from the sample points. Is it as expected? (For Poisson distribution the mean value should equal to g).

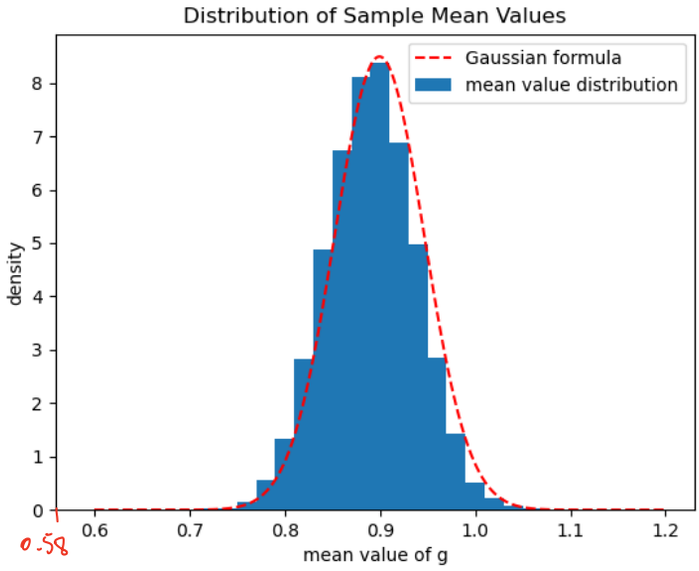
1.2 Check the central limit theorem

Now suppose 5000 undergraduate students are asked to reproduce this experiment and find out the coupling strength g. Unfortunately, each of them is only capable of completing 400 runs of the experiment because of the power failure. Furthermore, they don't have chance to go back to school and finish the following experiments since then due to the COVID-19.

They discussed online and figured out a solution: they could calculate the mean values individually and the distribution of all the mean values should obey the normal distribution with the same mean value as the original one! Please simulate this process, plot a histogram and find out the final mean value of all the mean values. Is the value as expected?

What's the standard error (Use formula from Lec 9, Page 10) and standard deviation of the mean value? Is the standard error as expected and why? Please use the standard deviation and mean value calculated to obtain a Gaussian distribution curve. Is the plot as expected? (You should obtain a similar graph as the following one)

(FYI: Here we assume that the coupling between the electrons and phonons are linear, also we only pick out one species of phonons that obey Einstein's model (There's no dispersion).)



2 Calculate the value of π (30 points)

During the lectures and labs, you have got familiar with calculating the value of π using Monte Carlo method. Please write codes to define a function of the Monte Carlo simulation for this process using 10000 points.(You could refer to the example codes on Canvas under the folder Lec10) Then

please repeat this procedure for 4000 times, calculating the standard deviation, mean value and plot a histogram for the results. With the standard deviation, what's the probability of obtaining π within the interval from $\pi+\sigma$ to $\pi-\sigma$ (σ is the standard deviation calculated out), if this procedure is repeated for another 4000 times? Is the result as expected?

3 Vaccine's efficiency (15 points)

It was reported that Pfizer and Moderna's vaccine's efficacy is more than 95%[3]. According to their data, among 3410 total cases of the suspected but unconfirmed COVID-19 in the overall study population, 1594 occurred in the vaccine group while 1816 in the placebo group. However, if we assume the PRC false negative rate is α, what will be the vaccine efficiency (VE) now? Please plot the VE versus the PRC false negative rate plot for α from 0 to 0.5. (You need to at least discretize the value of α from 0 to 0.5 for 100 points)

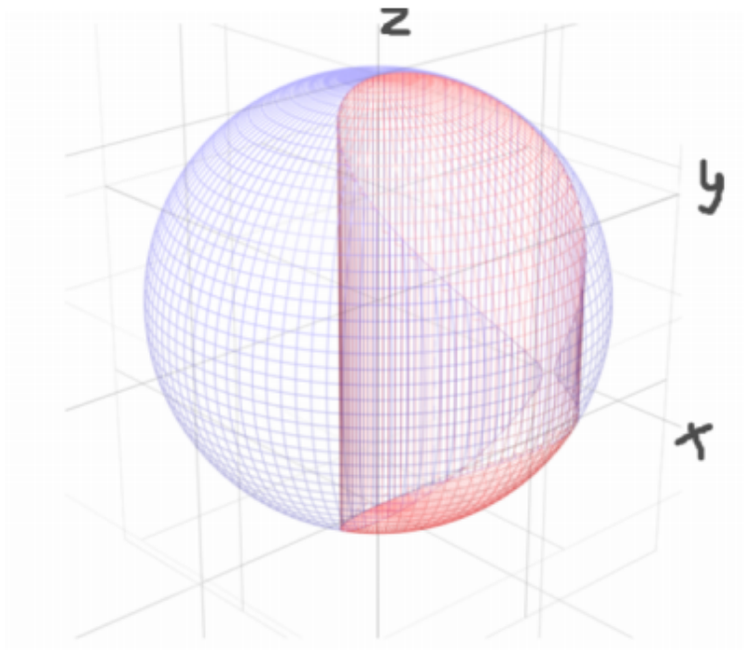
4 Infected or not? (15 points)

The current population of Hong Kong is 7.403 million [4], while there have been 538602 confirmed cases by nucleic acid tests on the fifth wave of COVID-19 [5] (updated on 2022/03/09). What's the probability for a random people of getting infected? If we assume that the PRC false positive rate is 0.03, what is the actual probability of getting infected, if a random person was tested positive using PCR?

Optional

5 The volume of intersection (10 points)

Find the volume of the intersection of a sphere and a cylinder, using Monte Carlo techniques. The sphere is centered at the origin with radius 1, while The cylinder is centered at (1/2, 0, 0). The radius of the base for this cylinder is 1/2. The vertical axis of this cylinder is parallel to z axis labelled below. (See the following figure) Report your uncertainty.



References

[1] Scanning Tunneling Microscope - Wikipedia. https://en.wikipedia.org/wiki/Scanning_tunneling_microscope.

[2] Adrian Po. Impurity Phonon Problem from PHYS5340.

[3] Vaccine's efficiency data. https://www.fda.gov/media/144416/download.

[4] The population of Hong Kong. https://www.censtatd.gov.hk/tc/.

[5] Statistics on the fifth Wave of COVID-19. https://www.covidvaccine.gov.hk/pdf/5th_wave_statistics.pdf.