AI1110: Assignment-1

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12.13.6.16 Question: Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Answer: $(\frac{16}{31})$
Solution:
Let R and B be the events that the ball transferred from Box-1 to Box-2 is Red and Black respectively.

$$\Pr(R) = \frac{No.of Redballs}{TotalBalls} = \frac{3}{7} \tag{1}$$
$$\Pr(B) = \frac{No.of Blackballs}{TotalBalls} = \frac{4}{7} \tag{2}$$

Let $\Pr(X)$ be the Random variable for the outcomes of ball drawn from Bag II.

$$X = \begin{cases} 1, & \text{if ball drawn is red known that event R occurs} \\ 2, & \text{if ball drawn is red known that event B occur} \\ 3, & \text{if ball drawn is Black} \end{cases} \tag{3}$$

$$\Pr(X = 1) = \Pr(R) \times \frac{5}{10} = \frac{15}{70} \tag{4}$$
$$\Pr(X = 2) = \Pr(B) \times \frac{4}{10} = \frac{16}{70} \tag{5}$$

Let $\Pr(X = 3)$ denote the probabilty that ball drawn is Black. We know that the ball drawn will be Black or Red for sure. So,

$$\Pr(X = 3) = 1 - \Pr(X = 1) - \Pr(X = 2) \tag{6}$$
$$\Pr(X = 3) = 1 - \frac{15}{70} - \frac{16}{30} \tag{7}$$
$$\Pr(X = 3) = \frac{39}{70} \tag{8}$$

By Bayes Theorem,

$$\Pr(E_i|A) = \frac{\Pr(E_i)\Pr(A|E_i)}{\sum_{k=1}^{k=n}\Pr(E_k)\Pr(A|E_k)} \tag{9}$$
$$\Pr(B|E) = \frac{\Pr(E|B)\Pr(B)}{\Pr(E|B)\Pr(B) + \Pr(E|R)\Pr(R)} \tag{10}$$
$$\Pr(B|E) = \frac{\Pr(X = 1)}{\Pr(X = 1) + \Pr(X = 2)} \tag{11}$$

where $\Pr(B|E)$ is the conditional probability of transferred ball being Black, known that the final ball drawn is Red which is our desired final answer.

$$\Pr(B|E) = \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{4}{10} \times \frac{4}{7} + \frac{5}{10} \times \frac{3}{7}} \tag{12}$$
$$\Pr(B|E) = \frac{16}{31} \tag{13}$$