AI1110: Assignment-1

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12.13.6.16 Question: Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Answer: $(\frac{16}{31})$
Solution:
Let $\Pr(X, Y)$ be the Random variable for the events of drawing balls from bags.

$$X = \begin{cases} 1, & \text{if ball is being drawn from Bag I} \\ 2, & \text{if ball is being drawn from Bag I} \end{cases} \tag{1}$$

$$Y = \begin{cases} 1, & \text{if ball drawn is Red} \\ 2, & \text{if ball drawn is Black} \end{cases} \tag{2}$$

$$\Pr(X = 1, Y = 1) = \frac{No.of Redballs}{TotalBalls} = \frac{3}{7} \tag{3}$$

$$\Pr(X = 1, Y = 2) = \frac{No.of Blackballs}{TotalBalls} = \frac{4}{7} \tag{4}$$

$$\Pr(X = 2, Y = 1) = \Pr(X = 1, Y = 1) \times \frac{5}{10} + \Pr(X = 1, Y = 2) \times \frac{4}{10} \tag{5}$$

$$\Pr(X = 2, Y = 1) = \frac{15}{70} + \frac{16}{70} \tag{6}$$

$$\Pr(X = 2, Y = 1) = \frac{31}{70} \tag{7}$$

By total probability law, we know,

$$\Pr(X = 2, Y = 2) = 1 - \Pr(X = 2, Y = 1) \tag{8}$$

$$\Pr(X = 2, Y = 2) = \frac{39}{70} \tag{9}$$

Red ball from Bag I:	$\Pr(X = 1, Y = 1)=\frac{3}{7}$
Black ball from Bag I:	$\Pr(X = 1, Y = 2)=\frac{4}{7}$
Red ball from Bag II:	$\Pr(X = 2, Y = 1)=\frac{31}{70}$
Black ball from Bag II:	$\Pr(X = 2, Y = 2)=\frac{39}{70}$

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TABLE I
FINAL PROBABILITIES OF THE EVENTS.

By Bayes Theorem,

$$\Pr(E_i|A) = \frac{\Pr(E_i)\Pr(A|E_i)}{\sum_{k=1}^{k=n}\Pr(E_k)\Pr(A|E_k)} \tag{11}$$

$$\Pr(B|E) = \frac{\Pr((X=2, Y=1)|(X=1, Y=1))\Pr(X=1, Y=1)}{\Pr(X=2, Y=1|X=1, Y=1)\Pr(X=1, Y=1) + \Pr(X=2, Y=1|X=1, Y=2)\Pr(X=1, Y=2)} \tag{12}$$

where $\Pr(B|E)$ is the conditional probability of transferred ball being Black, known that the final ball drawn is Red which is our desired final answer.

$$\Pr(B|E) = \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{4}{10} \times \frac{4}{7} + \frac{5}{10} \times \frac{3}{7}} \tag{13}$$

$$\Pr(B|E) = \frac{16}{31} \tag{14}$$