AI1110: Assignment-1

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12.13.6.16 Question: Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Answer: $(\frac{16}{31})$
Solution: *By Bayes's Theorem we have,for any i =1,2,3,..n , where* $E_1, E_2, \ldots, E_n$ *be a set of events associated with a sample space S, where all the events* $E_1, E_2, \ldots, E_n$ *form a partition of S.For an event A associated with S,*

$$\Pr(E_i|A) = \frac{\Pr(E_i)\Pr(A|E_i)}{\sum_{k=1}^{k=n}\Pr(E_k)\Pr(A|E_k)}$$

Let R and B be the events that the ball transferred from Box-1 to Box-2 is Red and Black respectively.

$$\Pr(R) = \frac{No.ofRedballs}{TotalBalls} = \frac{3}{7} \tag{1}$$

$$\Pr(B) = \frac{No.ofBlackballs}{TotalBalls} = \frac{4}{7} \tag{2}$$

Let's take the conditional probability for event E, which denotes the outcome of final ball drawn being Red.
When event R occurs, in Box-2 the count of red balls becomes 5 and black balls remains 5.

$$\Pr(E|R) = \frac{5}{10} \tag{3}$$

When event B occurs, in Box-2 the count of red balls remains 4 and black balls becomes 5.

$$\Pr(E|B) = \frac{4}{10} \tag{4}$$

By Bayes Theorem,

$$\Pr(E|B) = \frac{\Pr(E|B).\Pr(B)}{\Pr(E|B).\Pr(B) + \Pr(E|R).\Pr(R)} \tag{5}$$

where P(E/B) is the probability of final ball being red known that the transferred ball is Black, which is our desired final answer.

$$\Pr(E|B) = \frac{\frac{4}{10}\cdot\frac{4}{7}}{\frac{4}{10}\cdot\frac{4}{7} + \frac{5}{10}\cdot\frac{3}{7}} \tag{6}$$

$$\Pr(E|B) = \frac{16}{31} \tag{7}$$

The probability that the transferred ball is Black is $\frac{16}{31}$.