## AI1110: Assignment-1

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12.13.6.16 Question: Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

**Answer:**  $(\frac{16}{31})$ **Solution:** 

Let Pr(X, Y) be the Random variable for the events of drawing balls from bags.

$$X = \begin{cases} 1, & \text{if ball is being drawn from Bag I} \\ 2, & \text{if ball is being drawn from Bag I} \end{cases}$$
 (1)

$$Y = \begin{cases} 1, & \text{if ball drawn is Red} \\ 2, & \text{if ball drawn is Black} \end{cases}$$
 (2)

$$Pr(X = 1, Y = 1) = \frac{No.ofRedballs}{TotalBalls} = \frac{3}{7}$$

$$Pr(X = 1, Y = 2) = \frac{No.ofBlackballs}{TotalBalls} = \frac{4}{7}$$
(4)

$$Pr(X = 1, Y = 2) = \frac{No.ofBlackballs}{TotalBalls} = \frac{4}{7}$$
(4)

$$Pr(X = 2, Y = 1) = Pr(X = 1, Y = 1) \times \frac{5}{10} + Pr(X = 1, Y = 2) \times \frac{4}{10}$$
 (5)

$$\Pr\left(X=2, Y=1\right) = \frac{15}{70} + \frac{16}{70} \tag{6}$$

$$\Pr(X=2, Y=1) = \frac{31}{70} \tag{7}$$

By total probability law, we know,

$$Pr(X = 2, Y = 2) = 1 - Pr(X = 2, Y = 1)$$
(8)

$$\Pr(X=2, Y=2) = \frac{39}{70} \tag{9}$$

Red ball from Bag I:	$Pr(X = 1, Y = 1) = \frac{3}{7}$	
Black ball from Bag I:	$Pr(X = 1, Y = 2) = \frac{4}{7}$	
Red ball from Bag II:	$Pr(X = 2, Y = 1) = \frac{31}{70}$	(
Black ball from Bag II:	$Pr(X = 2, Y = 2) = \frac{39}{70}$	

TABLE I FINAL PROBABILITIES OF THE EVENTS.

By Bayes Theorem,

$$\Pr(E_i|A) = \frac{\Pr(E_i)\Pr(A|E_i)}{\sum_{k=1}^{k=n}\Pr(E_k)\Pr(A|E_k)}$$
(11)

$$\Pr(B|E) = \frac{\Pr((X=2,Y=1)|(X=1,Y=1))\Pr(X=1,Y=1)}{\Pr(X=2,Y=1|X=1,Y=1)\Pr(X=1,Y=1) + \Pr(X=2,Y=1|X=1,Y=2)\Pr(X=1,Y=2)}$$

where Pr(B|E) is the conditional probability of transferred ball being Black, known that the final ball drawn is Red which is our desired final answer.

$$\Pr(X = 1 | Y = 2) = \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{4}{10} \times \frac{4}{7} + \frac{5}{10} \times \frac{3}{7}}$$
(13)

$$\Pr(X=1|Y=2) = \frac{16}{31} \tag{14}$$