```
In[971]:= Clear["Global`*"]

In[972]:= (*Funktionen*)
GiveX[l_, y_] := Extract[l, Position[l, {xs_, ys_} /; ys == y]][[1, 1]]
GiveX[l_, Range_List] :=
 Extract[l, Position[l, {xs_, ys_} /; ys >= Range[[1]] && ys <= Range[[2]]]][[1, 1]]
GiveY[l_, x_] := Extract[l, Position[l, {xs_, ys_} /; xs == x]][[1, 2]]
GiveY[l_, Range_List] :=
 Extract[l, Position[l, {xs_, ys_} /; xs >= Range[[1]] && xs <= Range[[2]]]][[1, 2]]
GiveXY[l_, z_] := Extract[l, Position[l, {xs_, ys_, zs_} /; zs == z]][[1]][[{1, 2}]]
FindMaximumList[l_] := {GiveX[l, Max[l]], Max[l]}(*Annahme: y>x*)
CheckEq[l1_List, l2_List, n_] := Sequence[]
CheckEq[l1_List, l2_List, n_] :=
 l1[[n]] /; (l1[[n, 2]] ≥ l2[[n, 2]] && l1[[n, 2]] ≤ l2[[n + 5, 2]]) ||
   (l1[[n, 2]] ≤ l2[[n, 2]] && l1[[n, 2]] ≥ l2[[n + 5, 2]])
CheckEqPoints[l1_List, l2_List, start_: 1] :=
 Flatten[Table[{CheckEq[l1, l2, n]}, {n, start, Length[l1] - 5}] /. {} → Sequence[], 1]

In[981]:= (*Parameter*)
M = 50.; (*Masse des Gegengewichts*)
m = 0.5; (*Masse des Geschosses*)
d = 0.1; (*Dicke des Wurfarms*)
Tm = 0.;(*Plot und Integrations Range*)
Tp = 1.3;
θ0 = -Pi/4; (*Startwinkel des Wurfarms,
wenn keine Einschränkung aus Geometrie*)
n = 2000; (*Anzahl der Auswertungspunkte*)
(*Konstanten*)
g = 9.81;
(*Dichte Holz*)
ρ = 670.;

In[987]:= (*Startwerte*)
L0 = 3.5 (*Länge des Wurfarms*)
r0 = 2.86(*Länge der Schlinge*)
h0 = 1.0(*Länge der Aufhängung des Gegengewichts*)
x0 = 0.4(*Abstand Aufhängung des Wurfarms zum SP, guter Start: L0/4*)

Out[987]= 3.5

Out[988]= 2.86

Out[989]= 1.

Out[990]= 0.4
```

In[991]:=
```
(*abgeleitete Parameter*)
L1[L_, x_] := L / 2 - x
L2[L_, x_] := L / 2 + x
μ[L_] := L d^2 ρ
V[L_, x_] := M g L1[L, x] - m g L2[L, x] - μ[L] g x
i[L_, x_] := μ[L] (1 / 12 L^2 + x^2) + m L2[L, x]^2 + M L1[L, x]^2
```

In[996]:=
```
(*Einschränkung an den Startwinkel aus Geometrie*)
(*Achtung: θstart muss größer als Pi/4=0.785 sein,
sonst wird der optimale Abwurfwinkel nie erreicht*)
(*θstart[L_,h_,x_]:=Re[-ArcCos[(L1[L,x] Sin[Pi/4]+h)/L2[L,x]]
    HeavisideTheta[L2[L,x]-L1[L,x] Sin[Pi/4]-h]+
   θ0 HeavisideTheta[L1[L,x] Sin[Pi/4]+h-L2[L,x]]]*)
θstart[L_, h_, x_] := θ0
θstart[L0, h0, x0]
```

Out[997]= $-\frac{\pi}{4}$

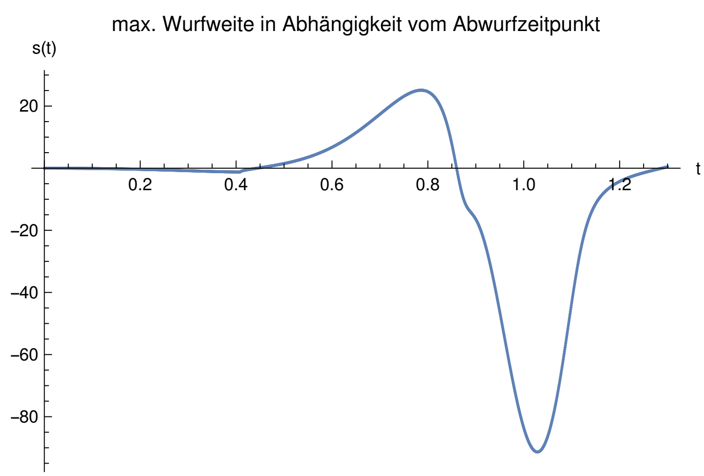
In[998]:=
```
(*Lösung der DGL*)
s[L_, r_, h_, x_] := NDSolve[{i[L, x] θ''[t] == M L1[L, x] h φ''[t] Sin[φ[t] + θ[t]] +
      M L1[L, x] h φ'[t]^2 Cos[φ[t] + θ[t]] - m L2[L, x] r ψ''[t] Cos[ψ[t] - θ[t]] +
      m L2[L, x] r ψ'[t]^2 Sin[ψ[t] - θ[t]] + V[L, x] Cos[θ[t]], h φ''[t] ==
    L1[L, x] θ''[t] Sin[φ[t] + θ[t]] + L1[L, x] θ'[t]^2 Cos[φ[t] + θ[t]] - g Sin[φ[t]],
   r ψ''[t] == -L2[L, x] θ''[t] Cos[ψ[t] - θ[t]] - L2[L, x] θ'[t]^2 Sin[ψ[t] - θ[t]] -
     g Cos[ψ[t]], θ[0] == θstart[L, h, x], θ'[0] == 0, φ[0] == 0,
   φ'[0] == 0, ψ[0] == -Pi + 0.1, ψ'[0] == 0}, {θ, φ, ψ}, {t, Tm, Tp}]
```

In[999]:=
```
Pv[L_, r_, h_, x_, t1_: 0, t2_: 0] := Block[{S}, S = s[L, r, h, x];
  Plot[Evaluate[
    Sqrt[(L2[L, x] θ'[t])^2 + (r ψ'[t])^2 + 2 L2[L, x] ψ'[t] θ'[t] Cos[ψ[t] - θ[t]]] /. S],
   {t, Tm, Tp}, PlotRange → All, GridLines → {{{t1, Red}, {t2, Green}}, None},
   PlotLabel → "Geschossgeschwindigkeit", AxesLabel → {"t", "v(t)"}]]
Ppsi[L_, r_, h_, x_, t1_: 0, t2_: 0] := Block[{S}, S = s[L, r, h, x];
  Plot[Evaluate[ψ[t] /. S], {t, Tm, Tp}, PlotRange → All, GridLines →
    {{{t1, Red}, {t2, Green}}, None}, PlotLabel → "Psi", AxesLabel → {"t", "ψ(t)"}]]
Pphi[L_, r_, h_, x_, t1_: 0, t2_: 0] := Block[{S}, S = s[L, r, h, x];
  Plot[Evaluate[φ[t] /. S], {t, Tm, Tp}, PlotRange → All, GridLines →
    {{{t1, Red}, {t2, Green}}, None}, PlotLabel → "Phi", AxesLabel → {"t", "ψ(t)"}]]
Pth[L_, r_, h_, x_, t1_: 0, t2_: 0] := Block[{S}, S = s[L, r, h, x];
  Plot[Evaluate[θ[t] /. S], {t, Tm, Tp}, PlotRange → All, GridLines →
    {{{t1, Red}, {t2, Green}}, None}, PlotLabel → "Theta", AxesLabel → {"t", "θ(t)"}]]
```

In[1003]:=
```
thList[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{t, (θ[t] /. S)[[1]]}, {t, Tm, Tp, (Tp - Tm) / n}]]
psiList[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{t, (ψ[t] /. S)[[1]]}, {t, Tm, Tp, (Tp - Tm) / n}]]
phiList[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{t, (φ[t] /. S)[[1]]}, {t, Tm, Tp, (Tp - Tm) / n}]]
(*Betrag der Geschossgeschwindigkeit*)
vList[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{t, Re[(Sqrt[(L2[L, x] θ'[t])^2 + (r ψ'[t])^2 + 2 L2[L, x] ψ'[t]
          θ'[t] Cos[ψ[t] - θ[t]]] /. S)[[1]]]}, {t, Tm, Tp, (Tp - Tm) / n}]]
(*x-Komponente der Geschossgeschwindigkeit*)
vxList[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{t, (-L2[L, x] Sin[θ[t]] θ'[t] - r Sin[ψ[t]] ψ'[t] /. S)[[1]]},
   {t, Tm, Tp, (Tp - Tm) / n}]]
(*y-Komponente der Geschossgeschwindigkeit*)
vyList[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{t, (L2[L, x] Cos[θ[t]] θ'[t] + r Cos[ψ[t]] ψ'[t] /. S)[[1]]},
   {t, Tm, Tp, (Tp - Tm) / n}]]
(*Integrierte Wurfweite*)
sWurf[vx_, vy_, h_] := vx (vy + Sqrt[2 g h + vy^2]) / g
sWurfList[L_, r_, h_, x_] := Block[{S, vx, vy, H},
  S = s[L, r, h, x];
  vx = ((-L2[L, x] Sin[θ[t]] θ'[t] - r Sin[ψ[t]] ψ'[t]) /. S)[[1]];
  vy = ((L2[L, x] Cos[θ[t]] θ'[t] + r Cos[ψ[t]] ψ'[t]) /. S)[[1]];
  H = ((L Sin[θ[t]] + h) /. S)[[1]];
  Table[{t, Re[sWurf[vx, vy, H]]}, {t, Tm, Tp, (Tp - Tm) / n}]]
tmax[L_, r_, h_, x_] := FindMaximumList[Abs[sWurfList[L, r, h, x]]][[1]]
smax[L_, r_, h_, x_] := GiveY[sWurfList[L, r, h, x], tmax[L, r, h, x]]
vmax[L_, r_, h_, x_] := GiveY[vList[L, r, h, x], tmax[L, r, h, x]]
```

In[1014]:=
```
ListPlot[sWurfList[L0, r0, h0, x0], PlotRange → All,
 PlotLabel → "max. Wurfweite in Abhängigkeit vom Abwurfzeitpunkt",
 AxesLabel → {"t", "s(t)"}]
```

Out[1014]=

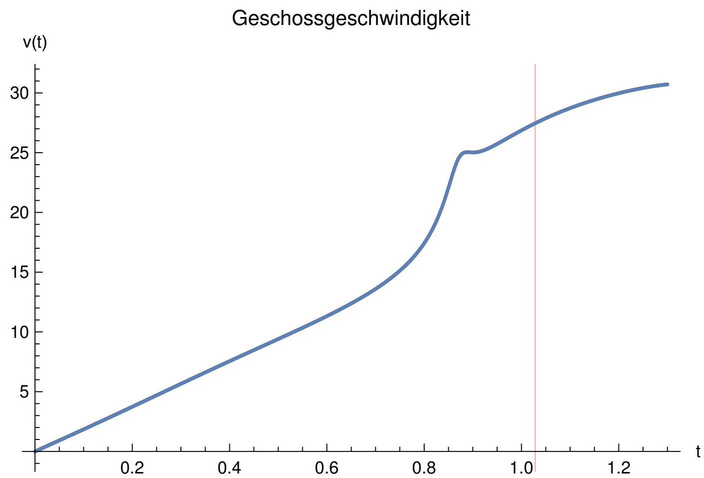


In[1015]:=
```
t0 = tmax[L0, r0, h0, x0]
s0 = smax[L0, r0, h0, x0]
Show[Pv[L0, r0, h0, x0, t0], ListPlot[vList[L0, r0, h0, x0]]]
Show[Pth[L0, r0, h0, x0, t0], ListPlot[thList[L0, r0, h0, x0]]]
Show[Ppsi[L0, r0, h0, x0, t0], ListPlot[psiList[L0, r0, h0, x0]]]
```

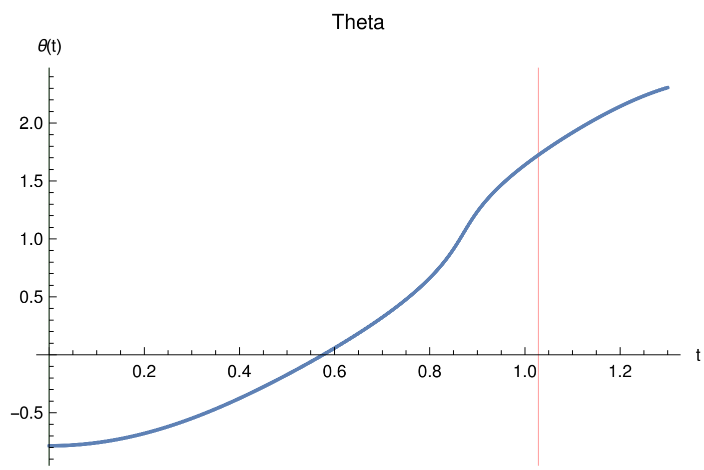
Out[1015]= 1.0283

Out[1016]= -91.3603

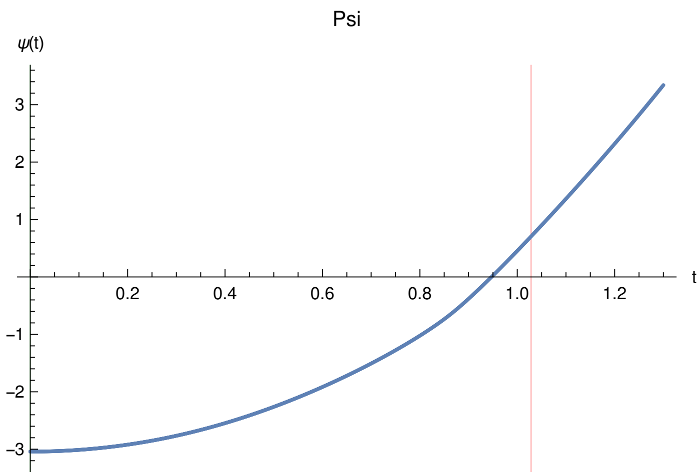
Out[1017]=



Out[1018]=



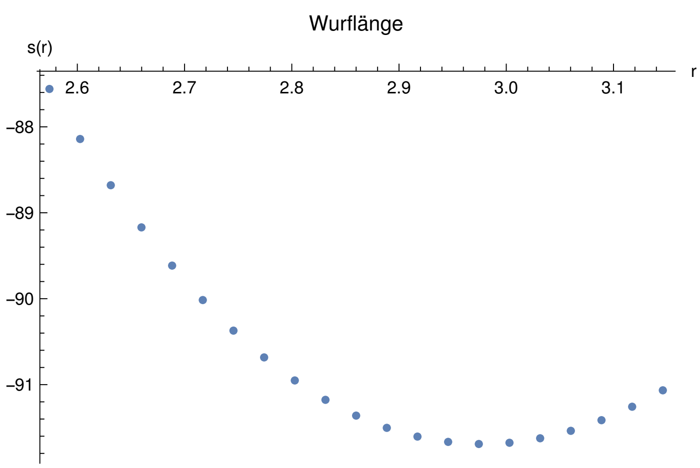
Out[1019]=



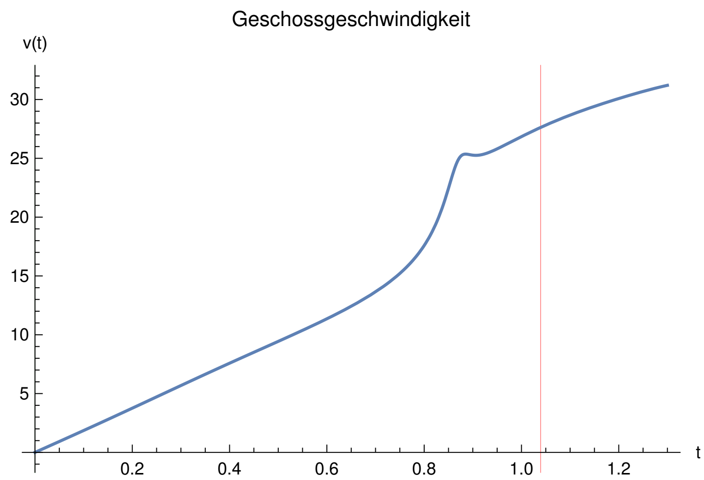
In[1020]:=

```
(*1D Analyse in r*)
smaxr = Table[{r, smax[L0, r, h0, x0]}, {r, 0.9 r0, 1.1 r0, 0.1 r0 / 10}];
ListPlot[smaxr, PlotLabel → "Wurflänge", AxesLabel → {"r", "s(r)"}]
rmax = FindMaximumList[Abs[smaxr]][[1]];
r0 = rmax;
t0 = tmax[L0, r0, h0, x0];
Pv[L0, r0, h0, x0, t0]
Print["Abwurfzeitpunkt t0 = ", t0]
Print["neuer Startwert für L0 = ", L0]
Print["neuer Startwert für r0 = ", r0]
Print["neuer Startwert für h0 = ", h0]
Print["neuer Startwert für x0 = ", x0]
Print["vmax = ", vmax[L0, r0, h0, x0]]
Print["smax = ", smax[L0, r0, h0, x0]]
```

Out[1021]=



Out[1025]=

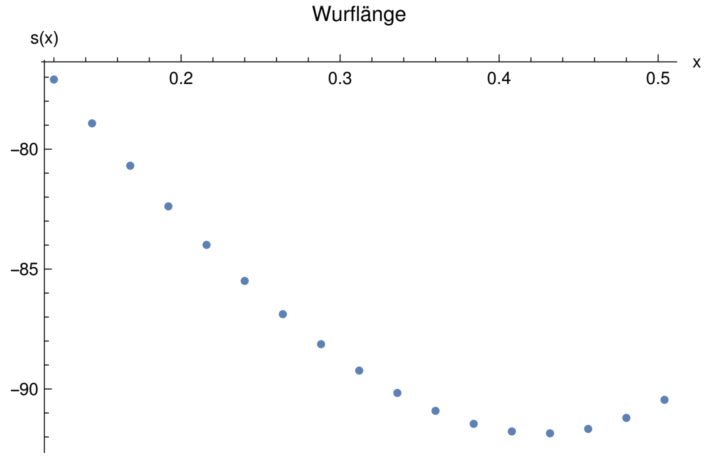



```
Abwurfzeitpunkt t0 = 1.03935

neuer Startwert für L0 = 3.5

neuer Startwert für r0 = 2.9744

neuer Startwert für h0 = 1.

neuer Startwert für x0 = 0.4

vmax = 27.6247

smax = -91.6904
```

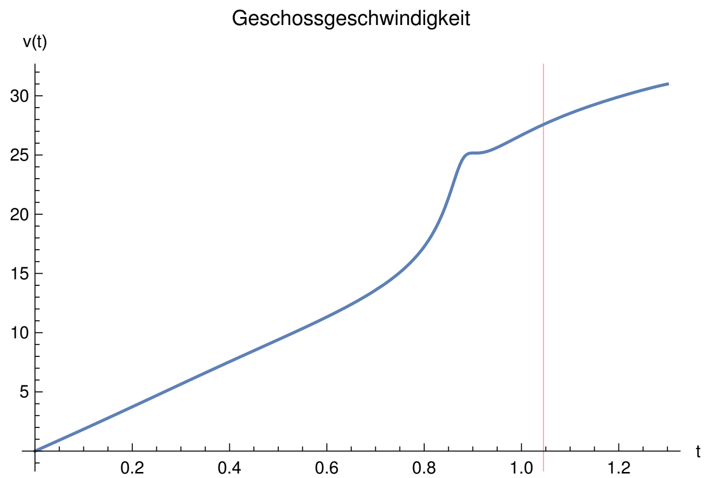
In[1033]:=

```
(*1D Analyse in x*)
smaxx = Table[{x, smax[L0, r0, h0, x]}, {x, 0.3 x0, 1.3 x0, 0.6 x0 / 10}];
ListPlot[smaxx, PlotLabel → "Wurflänge", AxesLabel → {"x", "s(x)"}]
xmax = FindMaximumList[Abs[smaxx]][[1]];
x0 = xmax;
t0 = tmax[L0, r0, h0, x0];
Pv[L0, r0, h0, x0, t0]
Print["Abwurfzeitpunkt t0 = ", t0]
Print["neuer Startwert für L0 = ", L0]
Print["neuer Startwert für r0 = ", r0]
Print["neuer Startwert für h0 = ", h0]
Print["neuer Startwert für x0 = ", x0]
Print["vmax = ", vmax[L0, r0, h0, x0]]
Print["smax = ", smax[L0, r0, h0, x0]]
```

Out[1034]=



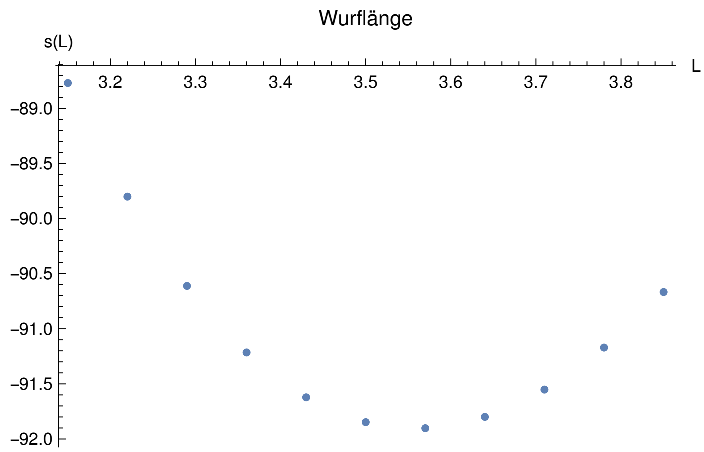
Out[1038]=



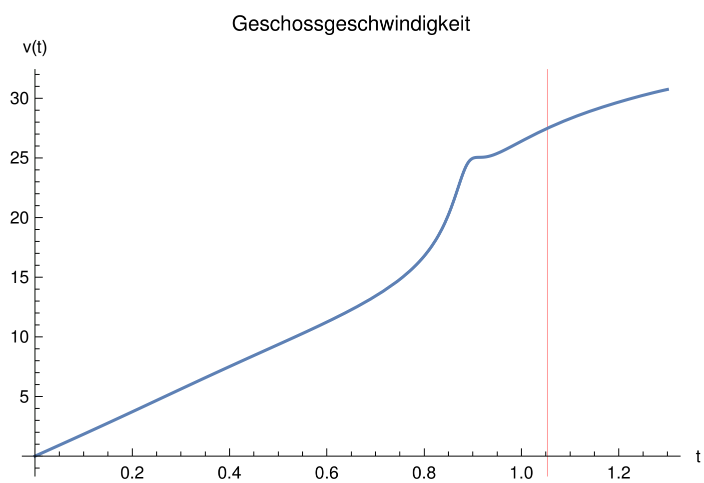

```
Abwurfzeitpunkt t0 = 1.0452
neuer Startwert für L0 = 3.5
neuer Startwert für r0 = 2.9744
neuer Startwert für h0 = 1.
neuer Startwert für x0 = 0.432
vmax = 27.5762
smax = -91.8473
```

In[1046]:=
```
(*1D Analyse in L*)
smaxL = Table[{L, smax[L, r0, h0, x0]}, {L, 0.9 L0, 1.1 L0, 0.2 L0 / 10}];
ListPlot[smaxL, PlotLabel → "Wurflänge", AxesLabel → {"L", "s(L)"}]
Lmax = FindMaximumList[Abs[smaxL]][[1]];
L0 = Lmax;
t0 = tmax[L0, r0, h0, x0];
Pv[L0, r0, h0, x0, t0]
Print["Abwurfzeitpunkt t0 = ", t0]
Print["neuer Startwert für L0 = ", L0]
Print["neuer Startwert für r0 = ", r0]
Print["neuer Startwert für h0 = ", h0]
Print["neuer Startwert für x0 = ", x0]
Print["vmax = ", vmax[L0, r0, h0, x0]]
Print["smax = ", smax[L0, r0, h0, x0]]
```

Out[1047]=



Out[1051]=



```
Abwurfzeitpunkt t0 = 1.05365

neuer Startwert für L0 = 3.57

neuer Startwert für r0 = 2.9744

neuer Startwert für h0 = 1.

neuer Startwert für x0 = 0.432

vmax = 27.4916

smax = -91.902
```

In[1059]:=
```
(*Notiz: Optimierung über h konvergiert nicht. h
  sollte deshalb so groß wie technisch möglich gewählt werden.*)
(*1D Analyse in h*)
(*smaxh=Table[{h,smax[L0,r0,h,x0]},{h,0.9h0,1.1 h0,0.2h0/10}];
ListPlot[smaxh,PlotLabel→"Wurflänge",AxesLabel→{"h","s(h)"}]
 hmax=FindMaximumList[Abs[smaxh]][[1]];
h0=hmax;
t0=tmax[L0,r0,h0,x0];
Pv[L0,r0,h0,x0,t0]
 Print["Abwurfzeitpunkt t0 = ",t0]
 Print["neuer Startwert für L0 = ",L0]
 Print["neuer Startwert für r0 = ",r0]
 Print["neuer Startwert für h0 = ",h0]
 Print["neuer Startwert für x0 = ",x0]
 Print["vmax = ",vmax[L0,r0,h0,x0]]
 Print["smax = ",smax[L0,r0,h0,x0]]*)
```

In[1060]:=
```
Tend = t0;
(*Vektoren*)
r1List[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{Evaluate[(L2[L, x] Cos[θ[t]]) /. S][[1]],
    Evaluate[(L2[L, x] Sin[θ[t]]) /. S][[1]]}, {t, Tm, Tend, 0.01}]]
r2List[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{Evaluate[(L2[L, x] Cos[θ[t]] + r Cos[ψ[t]]) /. S][[1]],
    Evaluate[(L2[L, x] Sin[θ[t]] + r Sin[ψ[t]]) /. S][[1]]}, {t, Tm, Tend, 0.01}]]
r3List[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{Evaluate[(-L1[L, x] Cos[θ[t]]) /. S][[1]],
    Evaluate[(-L1[L, x] Sin[θ[t]]) /. S][[1]]}, {t, Tm, Tend, 0.01}]]
r4List[L_, r_, h_, x_] := Block[{S}, S = s[L, r, h, x];
  Table[{Evaluate[(-L1[L, x] Cos[θ[t]] - h Sin[φ[t]]) /. S][[1]],
    Evaluate[(-L1[L, x] Sin[θ[t]] - h Cos[φ[t]]) /. S][[1]]}, {t, Tm, Tend, 0.01}]]
Manipulate[
 Show[ListPlot[{r2List[L0, r0, h0, x0], r1List[L0, r0, h0, x0]},
   PlotRange → {{-L2[L0, x0] - r0, L2[L0, x0] + r0}, {-L2[L0, x0] - r0, L2[L0, x0] + r0}},
   Frame → True, ImageSize → {600, 600}, AspectRatio → 1],
  Graphics[Line[{r4List[L0, r0, h0, x0][[Round[n]]],
     r3List[L0, r0, h0, x0][[Round[n]]], {0, 0}, r1List[L0, r0, h0, x0][[Round[n]]],
     r2List[L0, r0, h0, x0][[Round[n]]]}]]], {n, 1, Length[r1List[L0, r0, h0, x0]]}]
```

Out[1065]=