Lab Report: Marking Trees

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Results

For $i \in \{1, 2, 3\}$, the number of rounds R_i spent until the tree is completely marked in process i is given in the following table. The table shows the result of 1000 repeated trails.

N	R_1	R_2	R_3
3	2.5 ± 0.9	2	2
7	7 ± 2	4.5 ± 0.6	4
15	$1.7 \pm 7.0 \times 10^1$	$1.0 \pm 0.2 \times 10^1$	8
31	$4 \pm 2 \times 10^1$	$2.3 \pm 0.3 \times 10^1$	2×10^1
63	$1.1 \pm 0.4 \times 10^2$	$5.1 \pm 0.6 \times 10^1$	3×10^1
127	$2.7 \pm 0.8 \times 10^2$	$1.08 \pm 0.09 \times 10^2$	6×10^1
255	$6 \pm 2 \times 10^2$	$2.3 \pm 0.1 \times 10^2$	1×10^2
511	$1.4 \pm 0.3 \times 10^3$	$4.7 \pm 0.2 \times 10^2$	3×10^2
1023	$3.2 \pm 0.7 \times 10^3$	$9.7 \pm 0.3 \times 10^2$	5×10^2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
524 287	$3.2 \pm 0.3 \times 10^6$	$5.230 \pm 0.006 \times 10^5$	3×10^5
1 048 575	$6.8 \pm 0.7 \times 10^6$	$1.0468 \pm 0.0009 \times 10^6$	5×10^5

Analysis

Our experimental data indicates that $\mathbf{E}[R_1]$ is linearithmic, while $\mathbf{E}[R_2]$ and $\mathbf{E}[R_3]$ are linear.

Theoretically, the behaviour of R_1 can be explained as follows:

The problem is an "easier" version of the coupon collecting problem, which therefore can be used as an upper bound. An easier version of this problem would be if we, when we choose a node to mark, also mark its sibling and its parent. This would be equal to a version of the coupon collecting problem where you get three cards each round instead of one. Since both the upper and lower bound can be reduced to coupon collecting, which is linearithmic, the problem itself is linearithmic too.

Upper bound: cupon collecting Lower bound: Mark both sibling and parent of marked node -> a version of cupon collecting where you get three cards per pack