Independent Set Lab Report

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Correctness

For the produced set to be maximal every node must either be
marked or have a marked neighbour. Algorithm R1 correctly com-
putes $\alpha(G)$ because if a node only has one neighbour either it or its
neighbour must be marked and since marking the node itself doesn't
add any constraints to the rest of the graph (while marking its neigh-
bour does) this is better.
Algorithm R2 correctly computes $\alpha(G)$ because if all three nodes are
connected the case is the same as in R1 that one must be marked and
it's better to mark the node itself. If the neighbours aren't connected
at least one of them will get marked (for similar reasons as R1 works)
and at most two will be marked (both children). Therefore one node
is kept to symbolize the node that may or may not be marked.

Empirical Running time
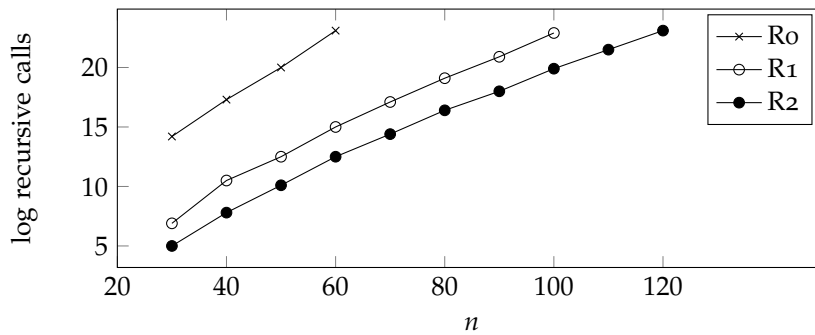
Experiments.
    The running times of algorithm R_0, R_1, and R_2 appear to be
$O(1.22^n)$, $O(1.17^n)$ and $O(1.15^n)$, respectively.

Theoretical Upper Bound

Denote be $T_i(n)$ the worst runtime of algorithm Ri on *any* graph on n
vertices. Note that $T_i(n)$ is a non-decreasing function of n. For R_0 we
can conclude that

$$T_0(n) \leq \max(T_0(n-1), T_0(n-1) + T_0(n-1-d_{\max}))$$
$$\leq T_0(n-1) + T_0(n-2)$$

with $d_{\max}$ the degree of the vertex we branch on. The hard part is
the one when there are no isolated vertices, in which case the vertex
u we are branching on has at least one neighbor.

For R1 we have that[1]

$$T_1(n) = [\ldots]$$

For R2 we have that

$$T_2(n) = [\ldots]$$

Worst Case Upper Bound   The running times of algorithm R_0, R_1, and R_2 are in $[\ldots], [\ldots]$, and $[\ldots]$, respectively.[2]

[1] Derive similar recursive bounds on $T_1(n)$ and $T_2(n)$.

[2] Replace the $[\ldots]$ by a function of n on the form $O(c^n)$. Use the recursive bounds you've derived above. *Hint: A recurrence of the form $T(n) \leq \sum_{i=1}^{k} a_i T(n-i)$ is called a linear homogeneous recurrence relation with constant coefficients. To solve it, you can set $T(n) \leq c^n$ where c is the largest real root to the characteristic polynomial $x^k - \sum_{i=1}^{k} a_i x^{k-i}$.*