Buck converter regulator design

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Goal of this project is to develop a regulator for controlling the output voltage of a DC/DC step down converter (buck converter). Firstly, a general overview of the project is presented. Full process with equations and derivation is described at the end of this document.

3 regulators are designed:

-PID

-LQR

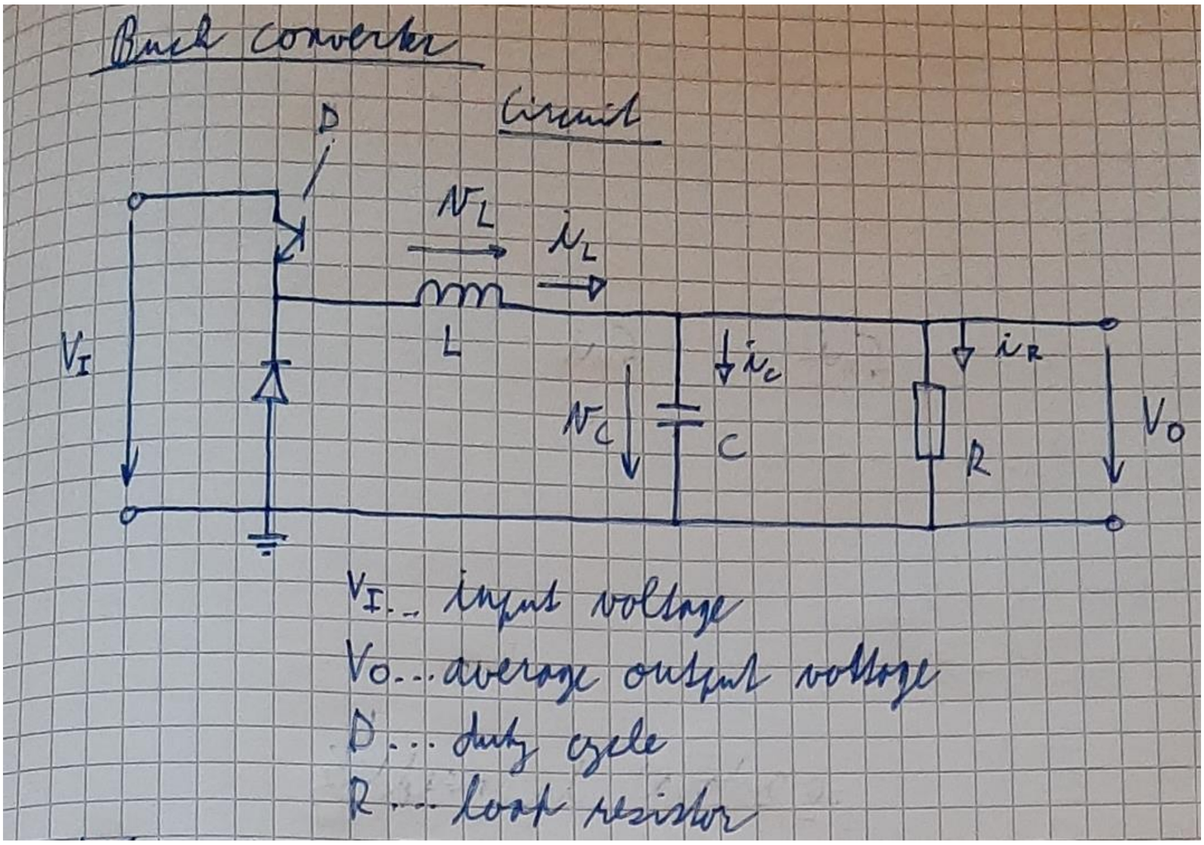
-Pole placement

MATLAB code and Simulink model of the PID regulator are provided as well.

Electrical Circuit

This circuit lowers the input voltage based on the duty cycle which switches on and off the transistor. The average output voltage can be calculated as shown:

$$V_O = V_I \cdot D$$



Dynamic model

Differential equations describing the dynamics of this circuit are derived from the circuit above (see full derivation at the end of this document):

$$\frac{di_L}{dt} = \frac{DV_I}{L} - \frac{v_c}{L}$$

$$\frac{dv_c}{dt} = \frac{i_L}{C} - \frac{v_c}{RC}$$

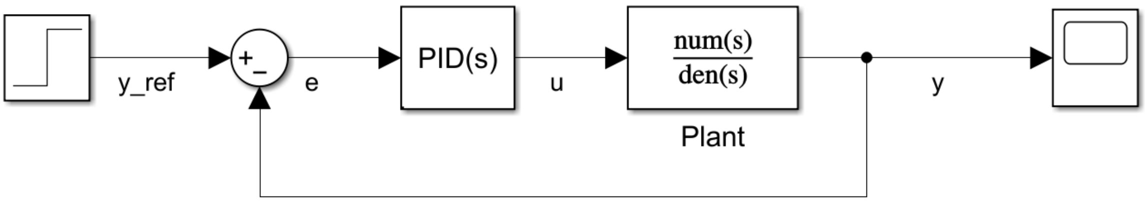
The system is linear so LTI control can be used.

PID regulator

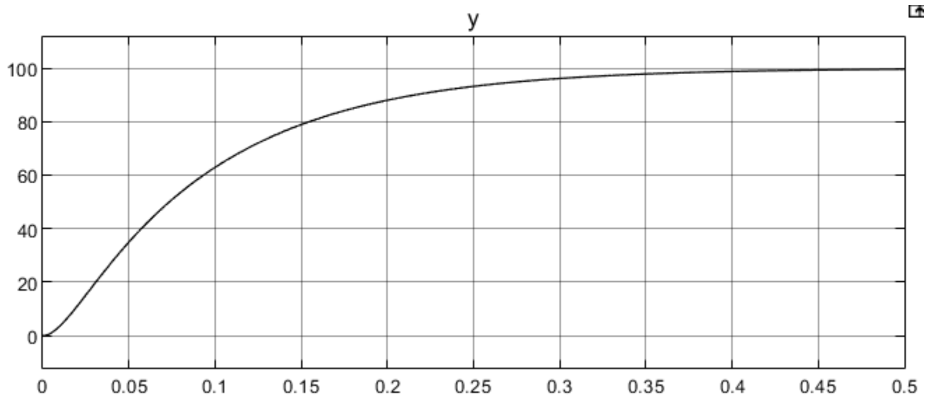
The transfer function of the buck converter (plant) is derived from the dynamic model:

$$G_P(s) = \frac{\frac{V_I}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Stability of the system can be assessed by computing the poles and checking if their real part is negative. With obtained transfer function a Simulink model of a PID regulator is created:



The constants of the PID regulator can be obtained by Ziegler-Nichols, or other methods. In this project, manual tuning was chosen because of it's simplicity. After defining the parameters of the buck converter (R, L, C, V_I and the desired output voltage). Step response of the whole system can be seen in the scope:



Regulator was tuned for little to no overshoot.

State space representation

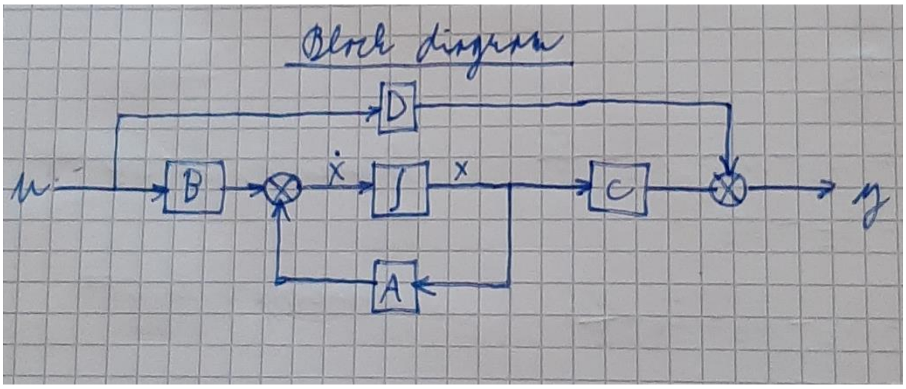
The dynamic model of the buck converter is represented with state space matricies and a block diagram (see full derivation at the end of this document):

$$A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{V_I}{L} \end{bmatrix}$$

$$c = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

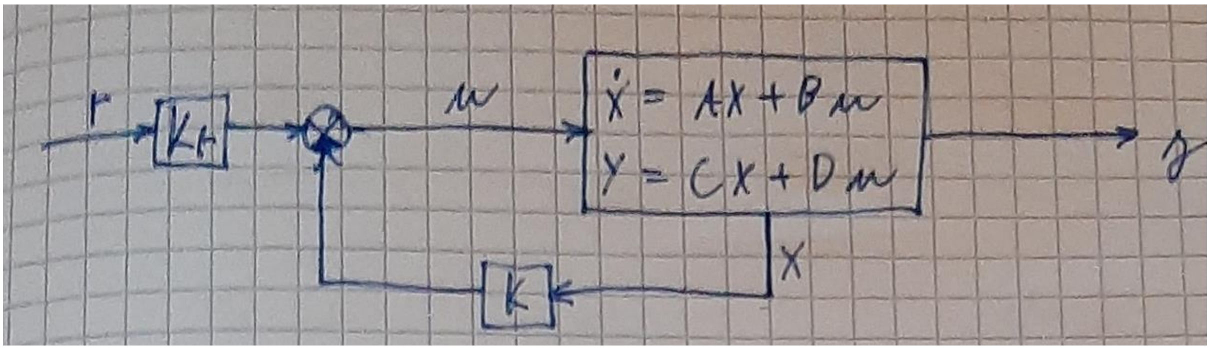
$$D = 0$$



With this representation, state space control methods can be used.

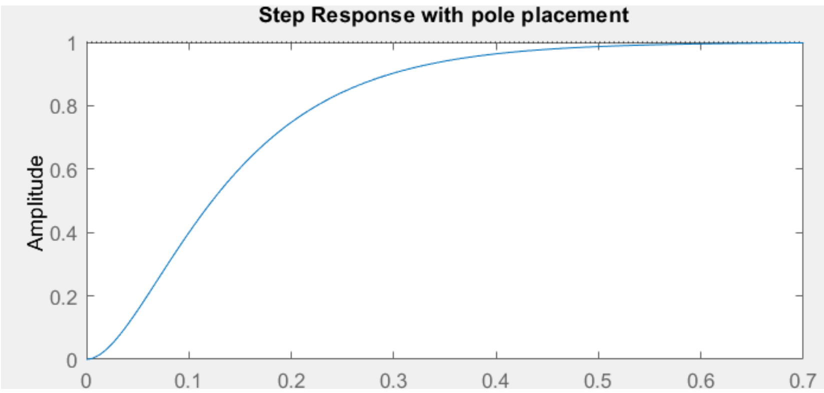
Pole placement

Pole placement block diagram is an extension of the state space block diagram:



By choosing poles of the system, we can insure stability and control parameters like overshoot, settle time etc. It is however not obvious how the placement of the poles impacts all of the parameters. This can be solved by using LQR.

The gain matrix K and scale factor K_r were calculated in matlab (see code). By placing poles at -10 + 0i and -20 + 0i, a step response was obtained:



LQR

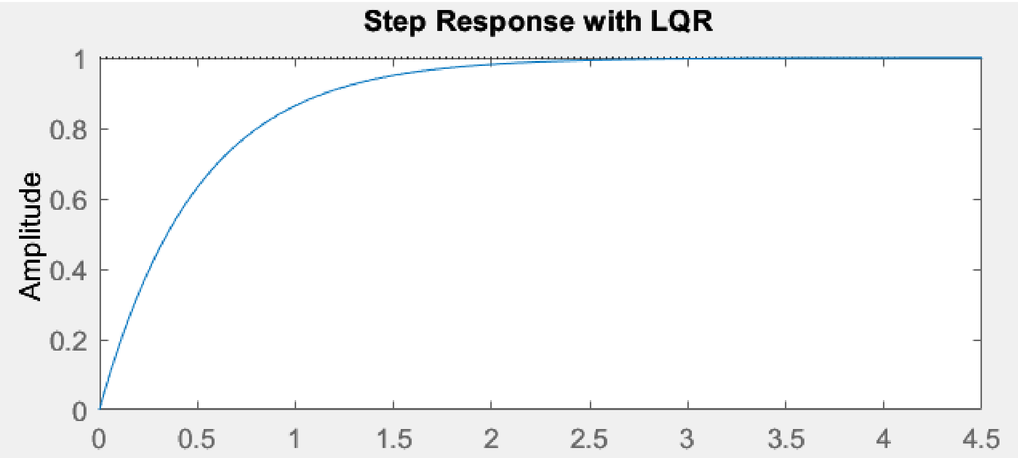
Linear quadratic regulator, or LQR is simular to pole placement. Instead of choosing the poles of the system, matricies Q and R are chosen. Matrix Q assigns a cost of the state variables and matrix R cost of the input. These matricies together define a cost function which is then minimazed by finding an optimal K matrix. After obtaining the gain matrix K, the process is the same as in pole placement.

In this project, matrix Q corresponds to the state variables i_L and v_c. The inductor current i_L is penalized heavily because of its potential to destroy the circuit by overcurrent. On the other hand, duty cycle is not penalized much, because it doesn't cause any problems.

$Q = \begin{bmatrix} 1000 & 0 \\ 0 & 10 \end{bmatrix}$ corresponds to… $\begin{bmatrix} i_L \\ v_c \end{bmatrix}$

$R = [0.001]$ corresponds to the duty cycle.

By setting the parameters of LQR in MATLAB, a step response is obtained:



By chosing different values for Q, there is a tradeoff for penalazing the inductor current and settle time.

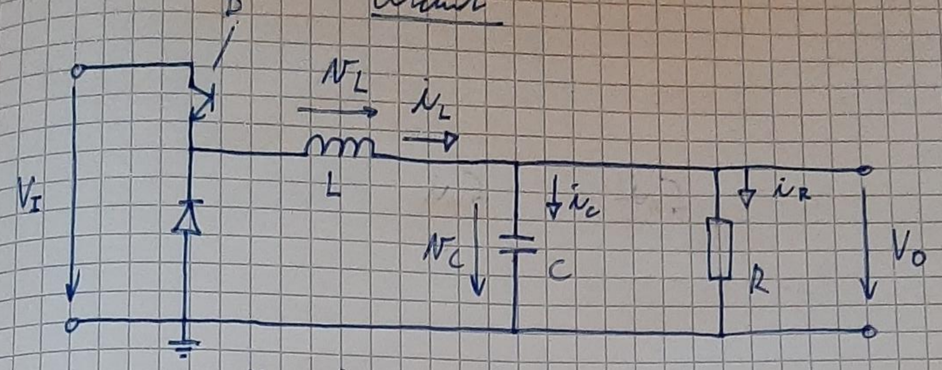
Conclusion

This project was done in one afternoon by myself. I am very interested in control systems and I hope this project shows it. Thank you for your time.

Full derivation is presented below:

Buck converter

Circuit



V_I ... input voltage
V_O ... average output voltage
D ... duty cycle
R ... load resistor

$$\frac{dv_C}{dt} = \frac{1}{C} i_C$$

$$\frac{di_L}{dt} = \frac{1}{L} v_L$$

T_{ON}: $i_C = i_L - i_R = i_L - \frac{v_C}{R}$ ... transistor is ON

$v_L = V_I - v_C$

T_{OFF}: $i_C = i_L - i_R = i_L - \frac{v_C}{R}$ ... transistor is OFF

$v_L = -v_C$

$$i_{C\,AVG} = i_L - \frac{v_C}{R}$$

$$v_{L\,AVG} = D(V_I - v_C) + (1-D)(-v_C) = D \cdot V_I - \cancel{Dv_C} - v_C + D\cancel{v_C}$$

$$v_{L\,AVG} = DV_I - v_C$$

$$\frac{dv_C}{dt} = \frac{i_L}{C} - \frac{v_C}{RC}$$

$$\frac{di_L}{dt} = \frac{DV_I}{L} - \frac{v_C}{L}$$

state space model

$i_L = x_1$ $\quad u = d$ ... input (duty cycle)

$v_C = x_2$ $\quad y = x_2$ ... output (voltage)

$$\dot{x}_1 = -\frac{1}{L} x_2 + \frac{V_I}{L} u$$

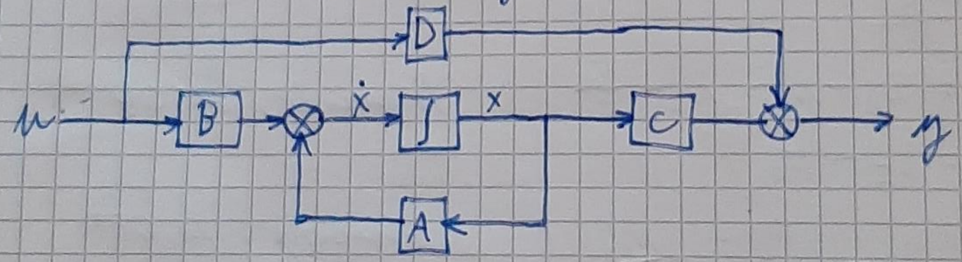
$$\dot{x}_2 = \frac{1}{C} x_1 - \frac{1}{RC} x_2$$

$$\dot{x} = Ax + Bu$$

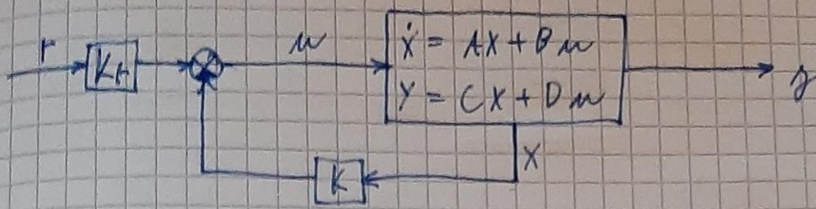
$$y = Cx + \overset{0}{D} u$$

$$A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{V_I}{L} \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Block diagram



Pole placement



$$u = rk_r - Kx$$

$$\dot{x} = Ax + Bu$$

$$\dot{x} = Ax + B(rk_r - Kx)$$

$$\dot{x} = \underbrace{(A - BK)}_{A_{cl}}x + \underbrace{Br\,k_r}_{\text{scaled input}}$$

A_{cl} ... new closed loop A matrix

LQR

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_L \\ v_C \end{bmatrix} \rightarrow Q = \begin{bmatrix} 1000 & 0 \\ 0 & 10 \end{bmatrix}$$

... penalise i_L

... penalise v_C

$u = d \rightarrow R = 0{,}001$ ...... penalise duty cycle

block diagram similar to pole placement ...

PID

SISO model

$$\dot{x}_1 = -\frac{1}{L}x_2 + \frac{V_I}{L}u \quad \xrightarrow{x_2 = v_c = y} \quad \dot{x}_1 = -\frac{1}{L}y + \frac{V_I}{L}u$$

$$\dot{x}_2 = \frac{1}{C}x_1 - \frac{1}{RC}x_2 \qquad\qquad \dot{x}_2 = \frac{1}{C}x_1 - \frac{1}{RC}y$$

$\mathcal{L}\{\}$

$$X_1 S = -\frac{1}{L}Y + \frac{V_I}{L}U \rightarrow X_1 = -\frac{1}{SL}Y + \frac{V_I}{L}U$$

$$YS = \frac{1}{C}X_1 - \frac{1}{RC}Y \rightarrow YS = -\frac{1}{SLC}Y + \frac{V_I}{LC}U - \frac{1}{RC}Y$$

$\cdot S$

$$Y\left(S^2 + \frac{1}{RC}s + \frac{1}{LC}\right) = \frac{V_I}{LC}U$$

$$\boxed{G_p(S) = \frac{\frac{V_I}{LC}}{S^2 + \frac{1}{RC}s + \frac{1}{LC}}}$$

transfer function of the buck converter (= plant)

Block diagram