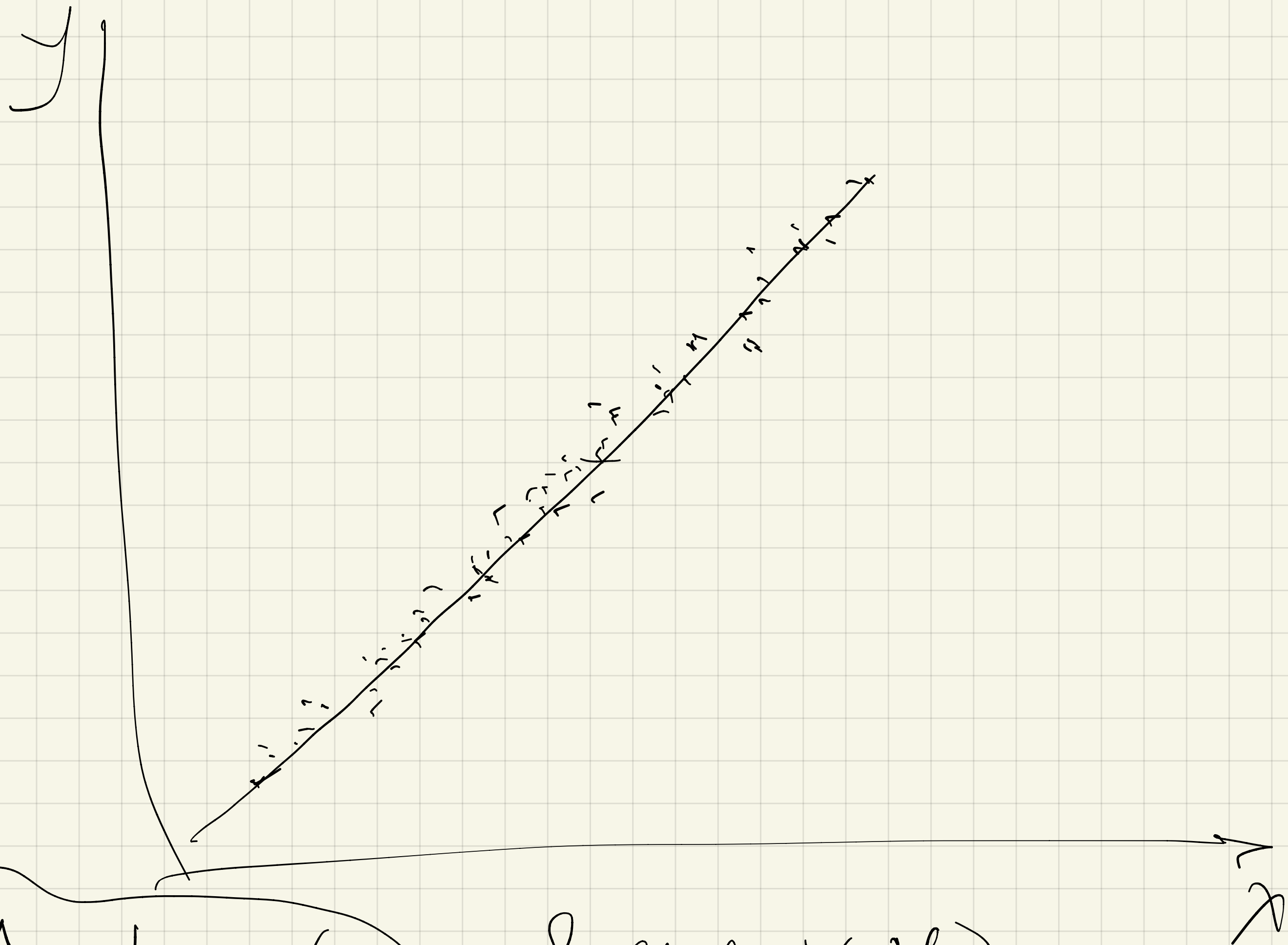
Linear regression

$$p(y|x,\theta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

LINE



$$\hat{y} = kx + b$$

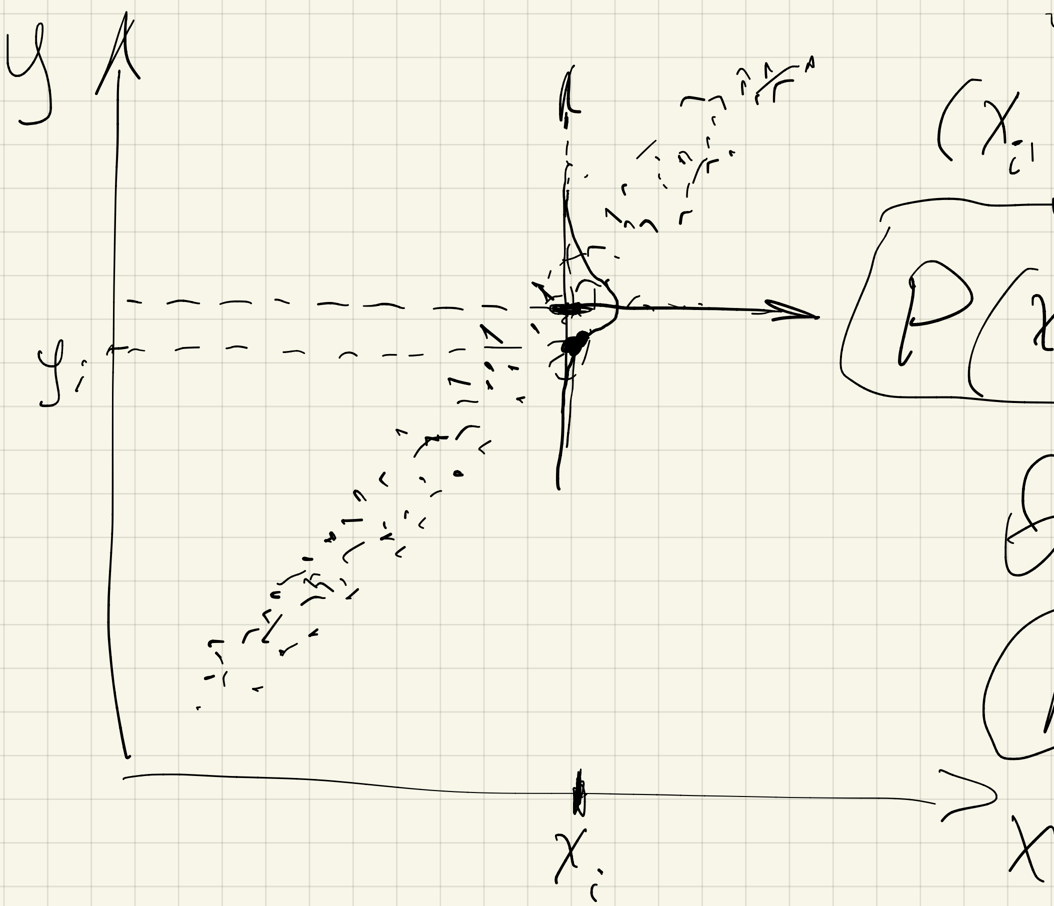
$$\mathcal{L}(k, b, X, Y) = \frac{1}{N} \sum_{i=1}^{N} \left((k x_i + b) - y_i \right)^2$$

$$k, b = \underset{\mathbb{K}, \mathbb{B}}{\arg\min}\ \mathcal{L}(k, b, X, Y)$$

LINE

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$P(A|B)\,P(B) = P(B|A)\,P(A)$$



$(x_i, y_i) \sim \mathcal{N}(\mu_i, \sigma_i^2)$

$P(x, y, \theta)$

$\theta \Leftarrow \mu_i = \theta x_i$

$P(y_i, x_i, \theta)$ LINE

$$P(\theta | X, Y)\, P(X, Y) = P(X, Y | \theta)\, P(\theta)$$

$$P(\theta | X, Y) = \frac{P(X, Y | \theta) \cdot P(\theta)}{P(X, Y)}$$

$$P(X, Y | \theta) = \prod_{i=1}^{N} P(x_i, y_i | \theta)$$

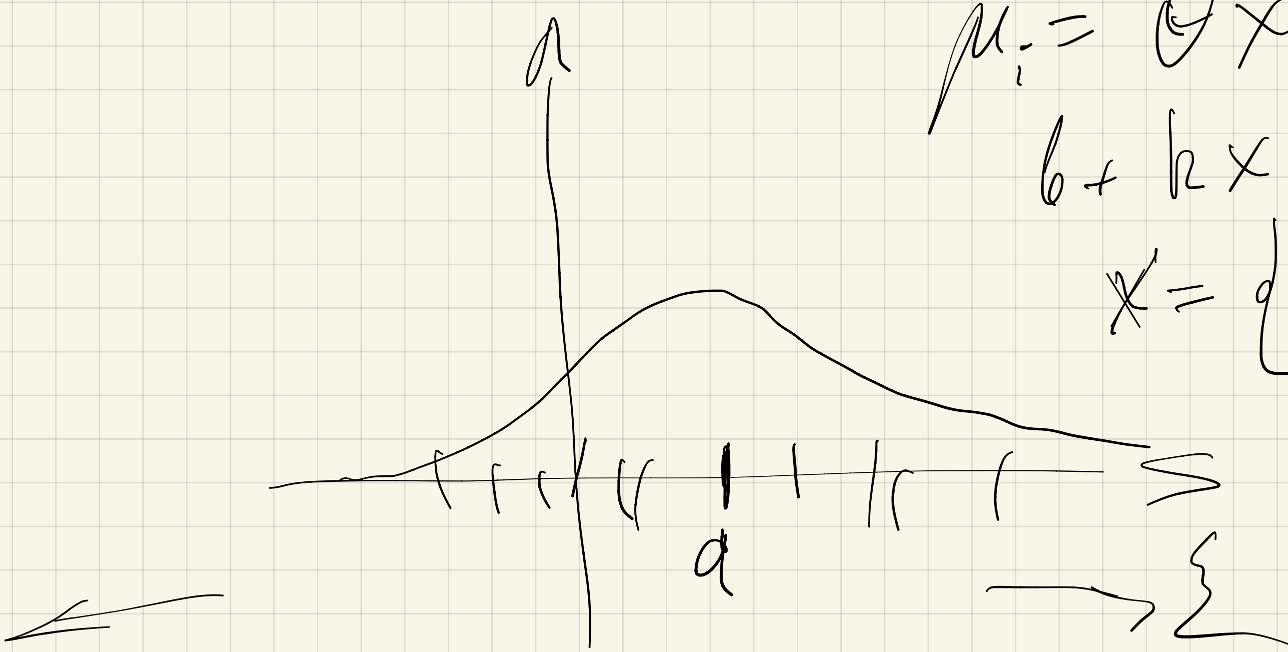
(independence)

$$P(x_i, y_i | \theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(y_i - \mu_i)^2}{2\sigma^2}}$$

$$\mu_i = \theta x_i$$

$$b + kx$$

$$x = [1; x]$$



$X = \{1; x_1; x_2; x_3 \dots x_d\}$

$\theta X = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \dots \theta_d x_d$

L $\quad \mu_i = \theta x_i$

I $\quad P(X, y|\theta) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{(y_i-\mu_i)^2}{2\sigma_i^2}}$

N

E - equivariance $\quad \sigma_i^2 = \sigma^2$

$P(X, y|\theta)$ — Правдоподобие выборки
likelihood

$$\ln P(X, y|\theta) = \ln \prod_{i=1}^{N} (\dots) =$$

$$= \sum_{i=1}^{N} \ln\left(\frac{1}{\sqrt{2\pi\sigma_i^2}} \cdot e^{-\frac{(\;)^2}{2\sigma_i^2}}\right)$$

$$= \sum_{i=1}^{N} \ln \frac{1}{\sqrt{2\pi\sigma_i^2}} \cdot e^{-\frac{(y_i - \theta x_i)^2}{2\sigma_i^2}} =$$

$$= \sum_{i=1}^{N} \ln \frac{1}{\sqrt{2\pi\sigma^2}} + \sum_{i=1}^{N} \ln e^{-\frac{(y_i - \theta x_i)^2}{2\sigma^2}}$$

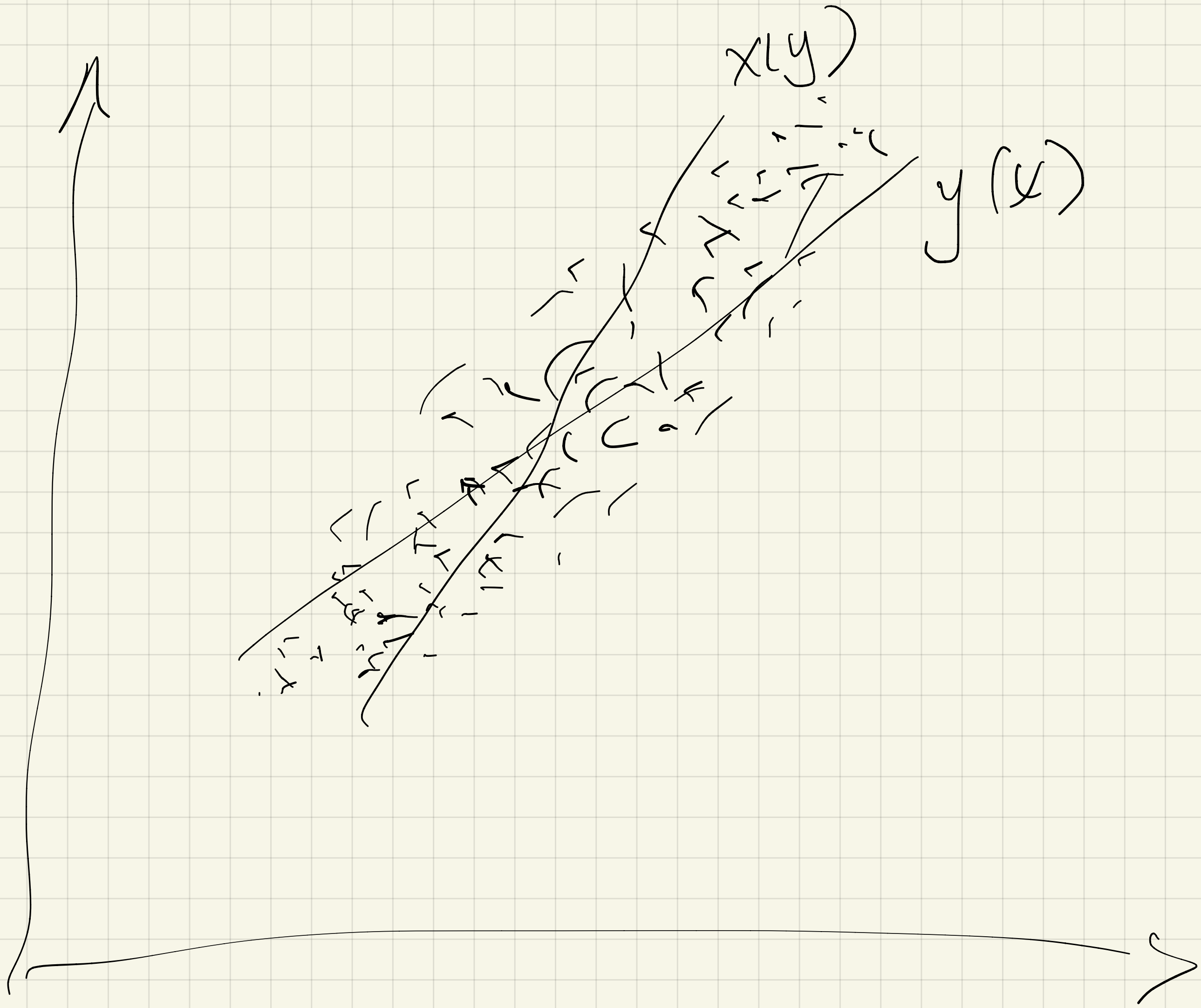
$$\hat{\theta} = \underset{\Theta}{\operatorname{argmax}}\, P(X, y \mid \theta) =$$

$$= \underset{\Theta}{\operatorname{argmax}} \left(\cancel{\sum_{i=1}^{N} \ln \frac{1}{\sqrt{2\pi\sigma^2}}} + \sum_{i=1}^{N} \left(- \frac{(y_i - \theta x_i)^2}{2\sigma^2} \right) \right.$$

$$= \underset{\Theta}{\operatorname{argmax}} \sum_{i=1}^{N} \left(- \frac{(y_i - \theta x_i)^2}{2\sigma^2} \right) =$$

$$= \underset{\Theta}{\operatorname{argmin}} \boxed{\sum_{i=1}^{N} (y_i - \theta x_i)^2}$$

$$\mu_i = \theta x_i = \hat{y}$$



$$\mathcal{L}(X, Y, \theta) = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

$$\hat{y}_i = \mu_i = \theta x_i \qquad x_i^{(0)} = 1$$

θ_0 - bias

$$(w) \rightarrow \boxed{\frac{\alpha}{2} e^{-\alpha |x - \mu|}} \qquad \mu_i = \theta x_i$$