Линейная регрессия

$$P(A, B) = P(A|B)P(B) = P(B|A)P(A)$$

$$p(\theta, \mathcal{T}) \qquad \mathcal{T} = \{x_i, y_i\} \quad i = 1 \ldots N$$

$$P(\theta, \mathcal{T}) = P(\theta|\mathcal{T})P(\mathcal{T}) = P(\mathcal{T}|\theta)P(\theta)$$

$P(\theta)$

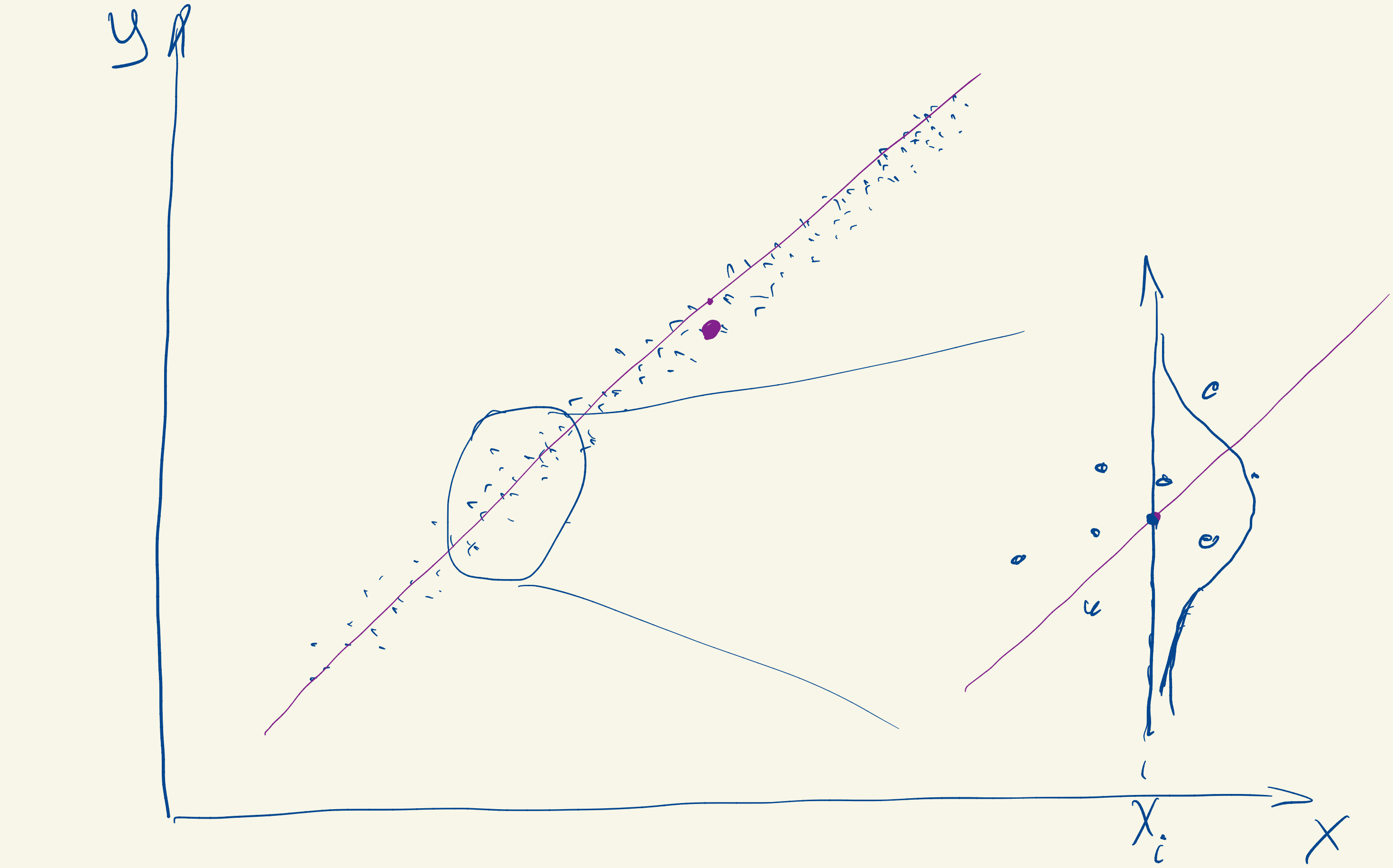
$P(\mathcal{T})$

$$\underbrace{P(\theta|\mathcal{T})}_{\text{апостериорная вер-ть } \theta \text{ при усл.-вии выборки } \mathcal{T}} = \frac{P(\mathcal{T}|\theta)\,P(\theta)}{P(\mathcal{T})}$$

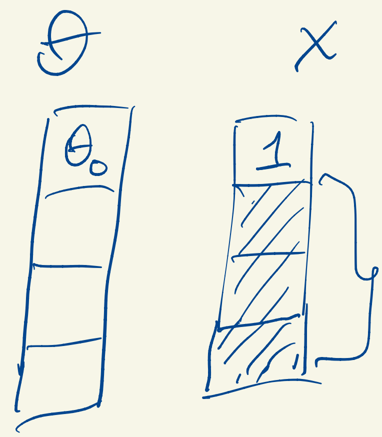
$P(\theta)$ — плотность априорного распределения θ

$P(\mathcal{T}|\theta)$ — <u>правдоподобие выборки</u>

$$\theta^* = \arg\max_{\Theta} P(\mathcal{T}|\theta)$$



LINE



① L: $\mu_i = \underline{\theta^T x_i}$
linear

③ N: $y_i \sim N(\mu_i, \sigma_i^2)$
normal distribution

④ $\sigma_i^2 = \sigma^2$ e-equivariance

② I I.I.d. Independent & identically distributed.

$$P(x_i, y_i, \theta) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{(y_i-\mu_i)^2}{2\sigma_i^2}} =$$

$$= \frac{1}{\sqrt{\dots}} e^{-\frac{(y_i - \theta^T x_i)^2}{2\sigma_i^2}}$$

$$P(A, B, C) = P(A) \cdot P(B) \cdot P(C)$$

$$P(\mathcal{J} | \theta) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{(y_i - \theta^T x_i)^2}{2\sigma_i^2}}$$

$$\theta^* = \underset{\Theta}{\operatorname{argmax}}\ P(\mathcal{J} | \theta)$$

$$\frac{\partial P(\mathcal{J}|\theta)}{\partial \theta} = 0$$

$$\underset{\Theta}{\operatorname{argmax}}\ \ln(P(\mathcal{J}|\theta)) = \underset{\Theta}{\operatorname{argmax}}\ P(\mathcal{J}|\theta)$$

$$\ln P(\mathcal{J}|\theta) = \sum_{i=1}^{N} \left(\ln \frac{1}{\sqrt{\dots}} + \ln e^{-\frac{(\dots)^2}{\dots}}\right) =$$

$$= N \cdot \ln \frac{1}{\sqrt{2\pi\sigma^2}} + \sum -\frac{1}{2\sigma^2} (y_i - \theta^T x_i)^2$$

$$\theta^* = \arg\max_{\Theta} \left[\cancel{N \cdot \ln \frac{1}{\sqrt{2\pi\sigma^2}}} + \sum -\frac{1}{2\sigma^2} (y_i - \theta^T x_i)^2 \right] =$$

$$= \arg\min_{\Theta} \boxed{\sum_{i=1}^{N} (y_i - \theta^T x_i)^2}$$

mean squared error MSE

$$p(x, y, \theta) = \alpha e^{-\frac{|y_i - \mu_i|}{\beta}} \Rightarrow$$

mean absolute error MAE