Features and Scale Space

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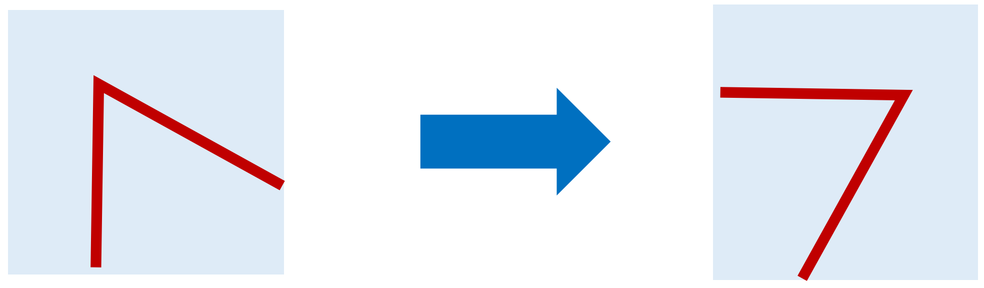
Harris Corner Detector: Invariances

- Translation Invariant?
- Rotation Invariant?
- Scale Invariant?
- Illumination Invariant?



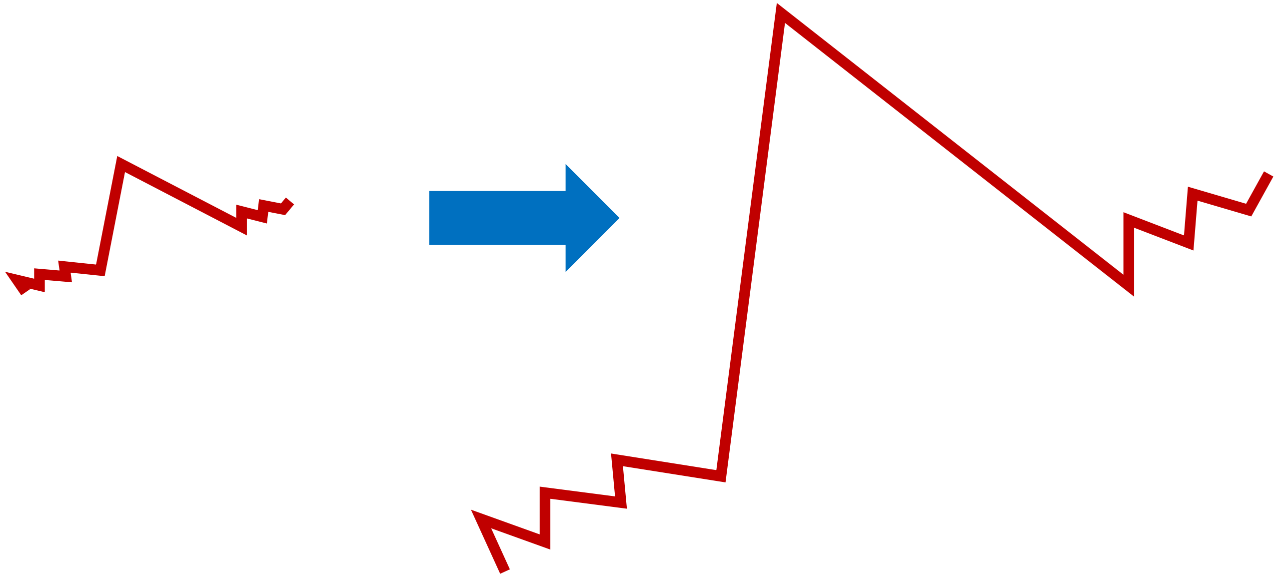


Image Pyramids

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Recall: Cascaded Convolutions

$$[1 \quad 1]^n$$

$$[1 \quad 1]$$

$$[1 \quad 2 \quad 1]$$

$$[1 \quad 3 \quad 3 \quad 1]$$

$$[1 \quad 4 \quad 6 \quad 4 \quad 1]$$

Interesting Aside

$$[1 \quad 1]^n$$

$$[1 \quad 1]$$

$$[1 \quad 2 \quad 1]$$

$$[1 \quad 3 \quad 3 \quad 1]$$

$$[1 \quad 4 \quad 6 \quad 4 \quad 1]$$

$$a_{nr} = \frac{n!}{r!\,(n-r)!} = \binom{n}{r}$$

n = number of elements in the filter minus 1

r = position of element in the filters (starting at 0)

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Interesting Aside

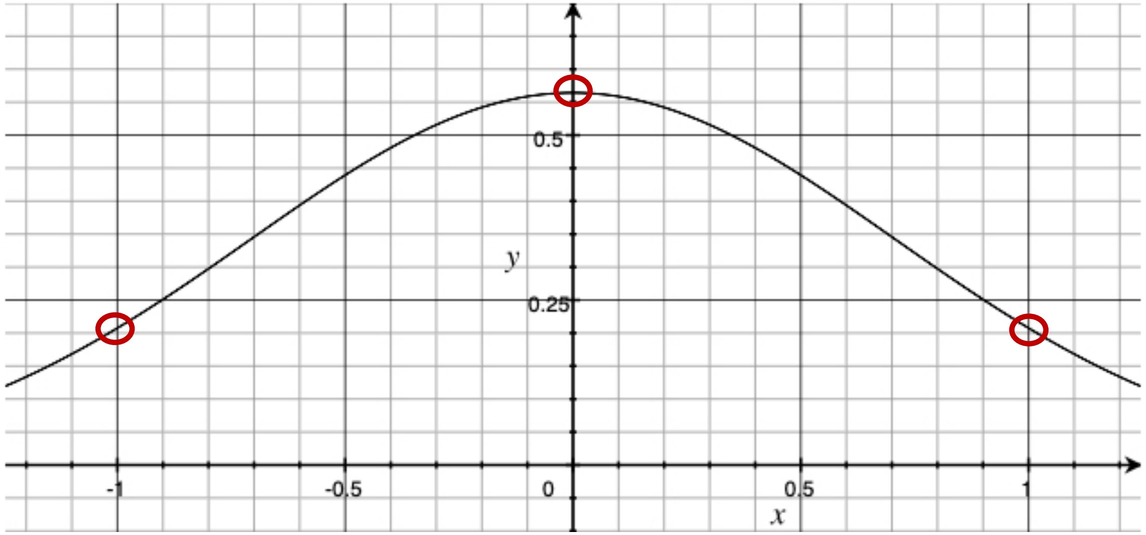
$$\frac{1}{2}[1 \quad 1]^n$$

$$\frac{1}{2}[1 \quad 1]$$

$$\frac{1}{4}[1 \quad 2 \quad 1]$$

$$\frac{1}{8}[1 \quad 3 \quad 3 \quad 1]$$

$$\frac{1}{16}[1 \quad 4 \quad 6 \quad 4 \quad 1]$$



$$\frac{1}{4}[1 \quad 2 \quad 1]$$

approximates Gaussian with $\sigma = \frac{1}{\sqrt{2}}$

$$\frac{1}{16}[1 \quad 4 \quad 6 \quad 4 \quad 1]$$

approximates Gaussian with $\sigma = 1$

Why?

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Interesting Aside

Fun facts:

- The distribution of the sum of two random variables $X + Y$ is the convolution of their two distributions. **Why?**

- Recall Convolution: $(f * g)(z) = \int_{-\infty}^{\infty} f(x)g(z-x)dx$

- $z = x + y \Rightarrow y = z - x$

- $p_{X+Y}(z) = \int_{-\infty}^{\infty} p_y(z-x)\, p_x(x)\, dx$

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Interesting Aside

$X\colon\ G_{\sigma_1}(x)$ and $Y\colon\ G_{\sigma_2}(y)$

$$G_\sigma(x+y) = (G_{\sigma_1} * G_{\sigma_2})(\mathrm{z})$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2$$

$$\frac{1}{4}[1 \quad 2 \quad 1]$$

approximates Gaussian with $\sigma = \frac{1}{\sqrt{2}}$

$$\frac{1}{16}[1 \quad 4 \quad 6 \quad 4 \quad 1]$$

approximates Gaussian with $\sigma = 1$

Gaussian Smoothing at Different Scales

$\sigma = 3$

$\sigma = 10$

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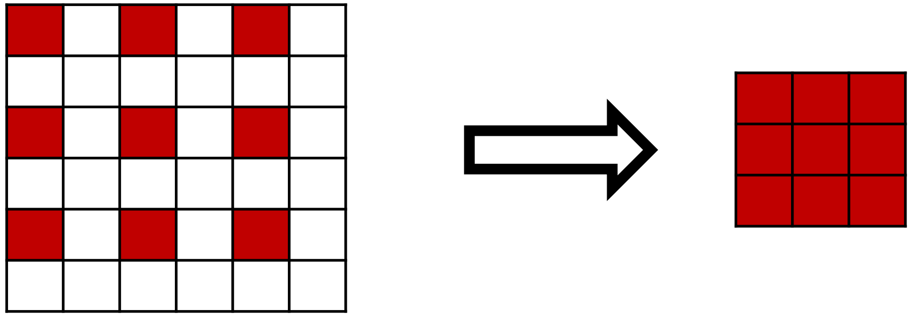
Subsampling

- A large amount of smoothing reduces the rate of change of neighboring pixels (frequency of features) in the image.

- We do not need to keep all the pixels around. Can progressively reduce the number of pixels as we smooth more and more.

- Leads to a “pyramid” representation if we ‘subsample’ at each level.



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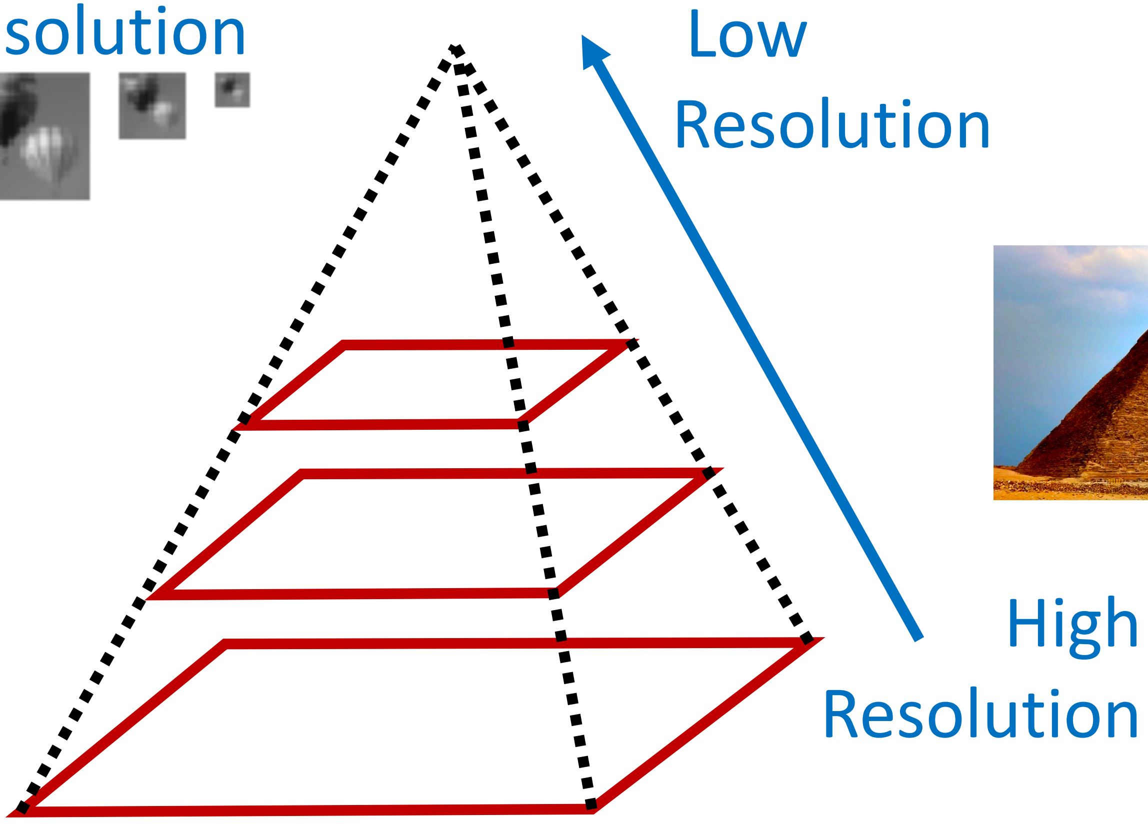
Gaussian Pyramid

- Synthesis: Smooth image with a Gaussian. Downsample. Repeat.

High Resolution → Low Resolution



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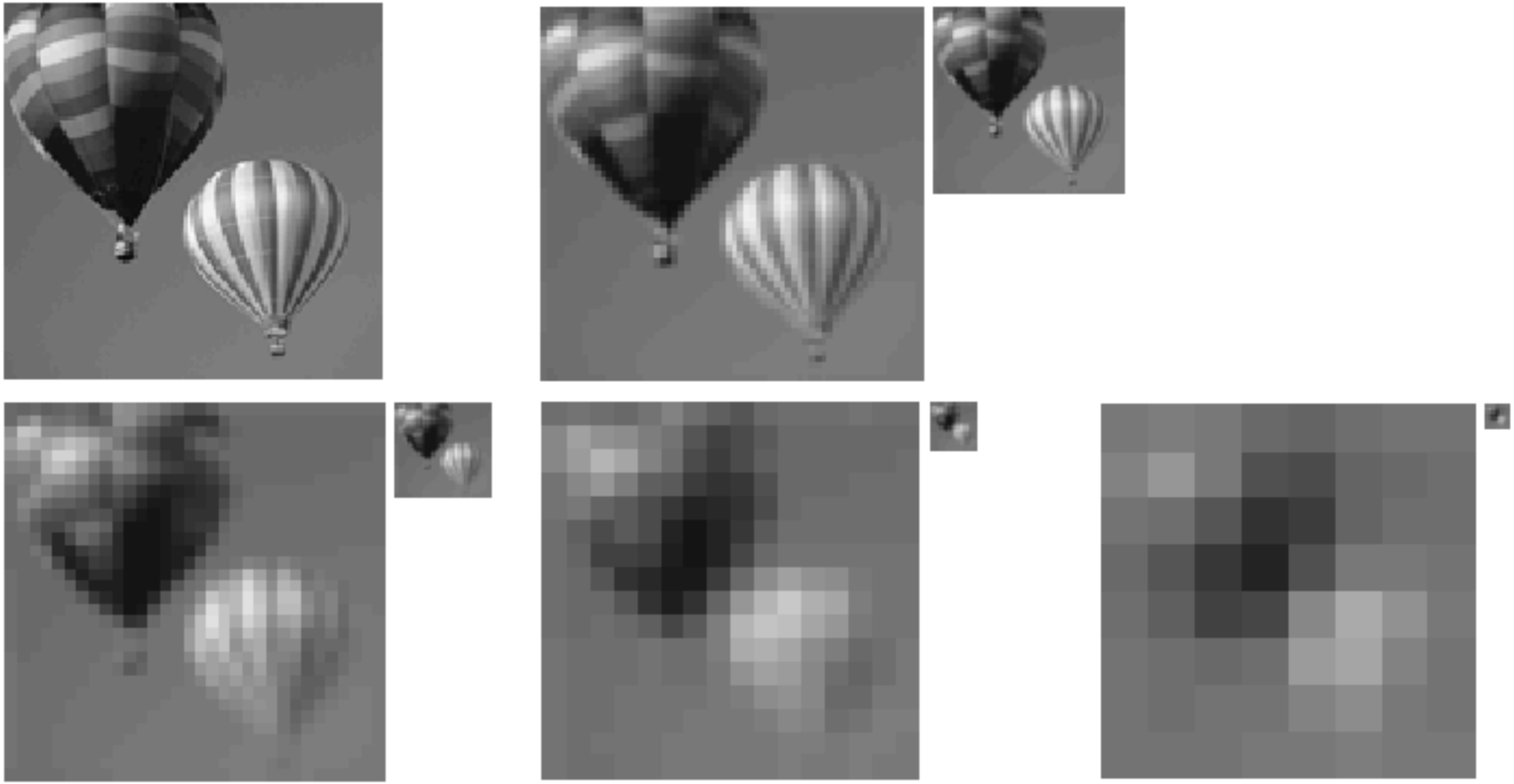
Smaller Image → Lower Resolution



Interesting Aside

- Subsampling without prior blurring is a bad idea (introduces aliasing)!

Downsampled

Blurred and Downsampled

Interesting Aside

- Subsampling without prior blurring is a bad idea (introduces aliasing)!

Downsampled

Blurred and Downsampled

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Interesting Aside

- Subsampling without prior blurring is a bad idea (introduces aliasing)!

Downsampled

Blurred and Downsampled

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What is Aliasing?

- A digital photograph of a striped shirt, or a brick wall (shown above) with high frequencies, with small distance between the stripes/bricks, can cause aliasing of the shirt/wall when it is sampled by the camera's image sensor. The aliasing appears as a moiré pattern.

- The “solution” to higher sampling in the spatial domain for this case would be to move closer to the shirt, use a higher resolution sensor, or to optically blur the image before acquiring it with the sensor.

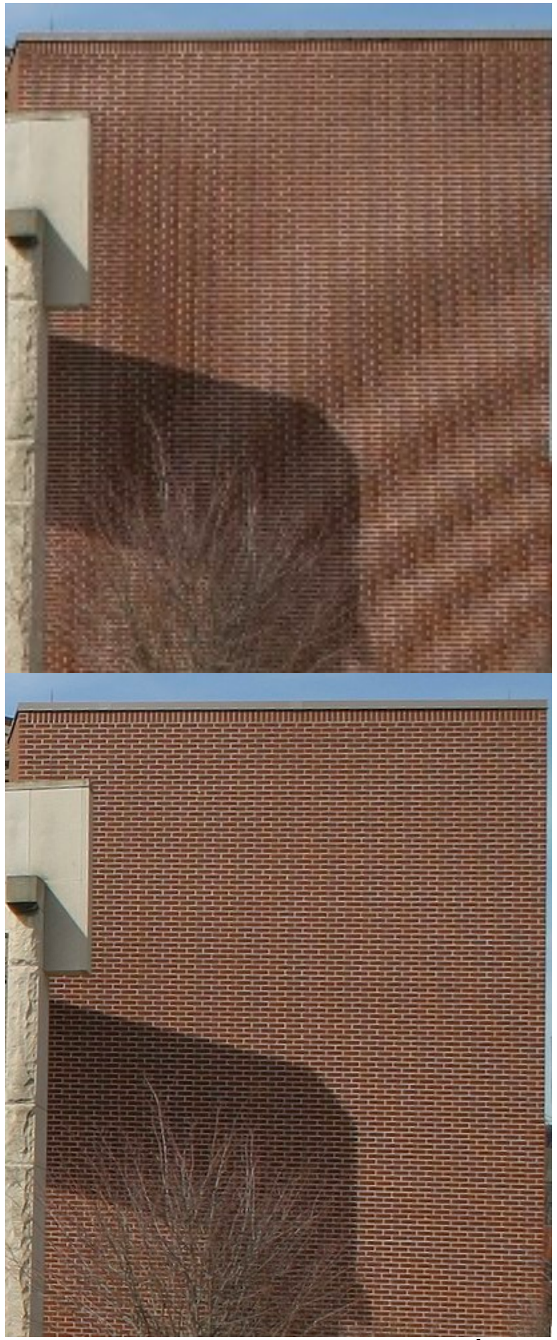
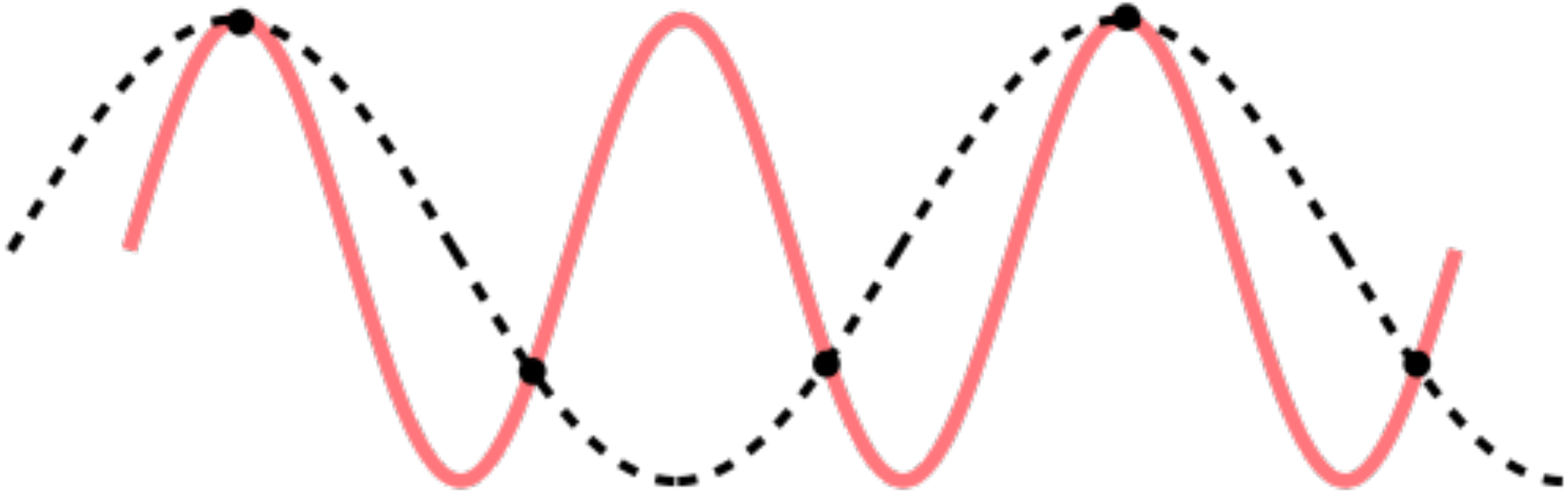


Image source: Wikipedia

What is Aliasing?



The samples of two sine waves can be identical when at least one of them is at a frequency above half the sample rate.

Image source: Wikipedia

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Constructing Gaussian Pyramid

Input Image $*\ g(\sigma = 1) *\ g(\sigma = 1) *\ g(\sigma = 1) *\ g(\sigma = 1)$ Subsample Image $*\ g(\sigma = 1) *\ g(\sigma = 1) *\ g(\sigma = 1) *\ g(\sigma = 1)$ Subsample Image

Effective σ $\quad \sqrt{2} \quad 2 \quad 2\sqrt{2} \quad 4$

$\mathbf{1}$

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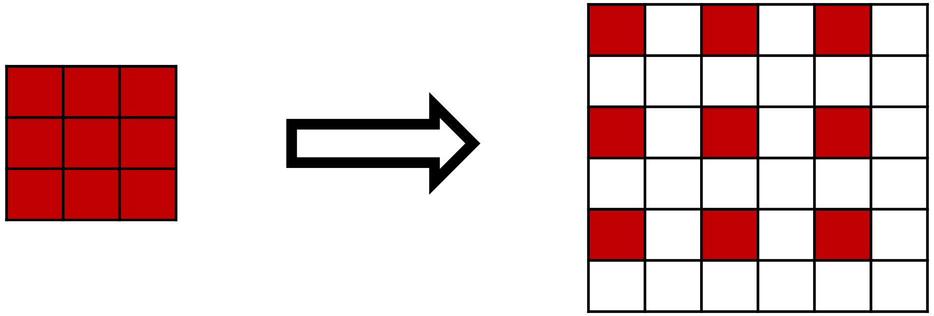
Upsampling



How to fill in the empty values?

- Interpolation:
  - Initially set empty pixels to zero
  - Convolve upsampled image with a Gaussian filter.
  - E.g. 5x5 kernel with sigma = 1.
  - Must also multiply by 4. **Why?**

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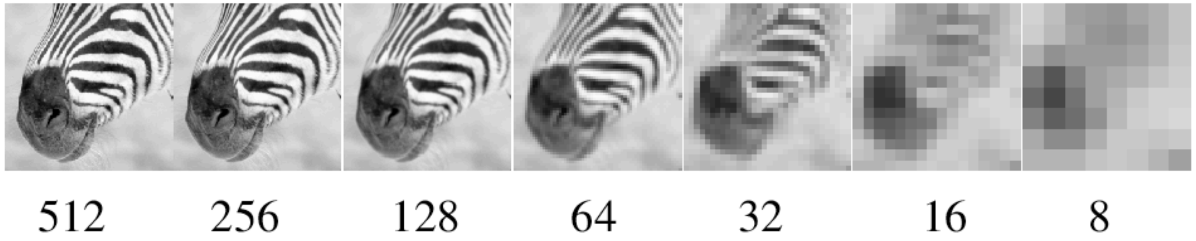
Scale Space

- Different scales are appropriate for describing different objects in the image

- We do not know the correct scale/size ahead of time.

- Image Pyramids are a useful representation for multi-resolution analysis of images.



Laplacian of Gaussian (LoG)

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Recall: Second Derivative

$$\frac{\partial}{\partial x} f(i,j) \cong f(i,j) - f(i-1,j)$$

$$\frac{\partial}{\partial y} f(i,j) \cong f(i,j) - f(i,j-1)$$

$$\begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\frac{\partial^2}{\partial x^2} f(i,j) \cong \frac{\partial}{\partial x} f(i+1,j) - \frac{\partial}{\partial x} f(i,j)$$

$$= f(i-1,j) + f(i+1,j) - 2f(i,j)$$

$$\begin{bmatrix} 1 & -1 \end{bmatrix} * \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$$

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Recall: Laplacian

Equation:

$$\nabla^2 f = \frac{\partial}{\partial x^2} f + \frac{\partial}{\partial y^2} f$$

Convolution:

$$\begin{bmatrix} 1 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

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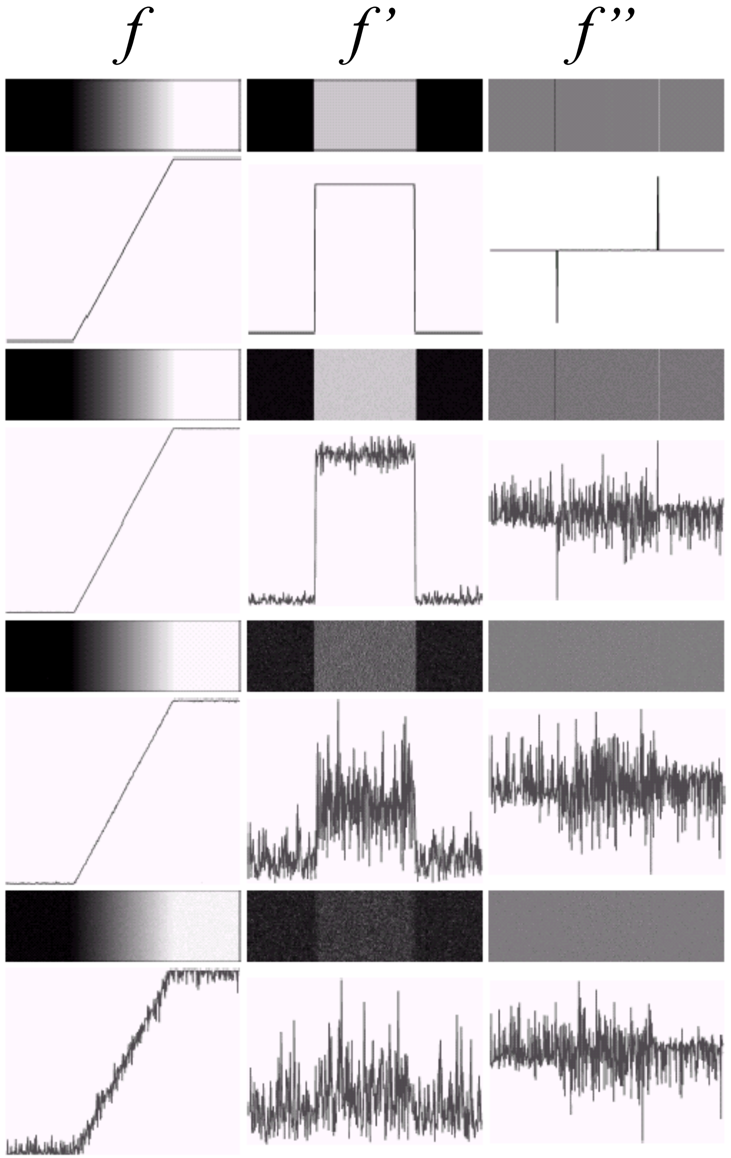
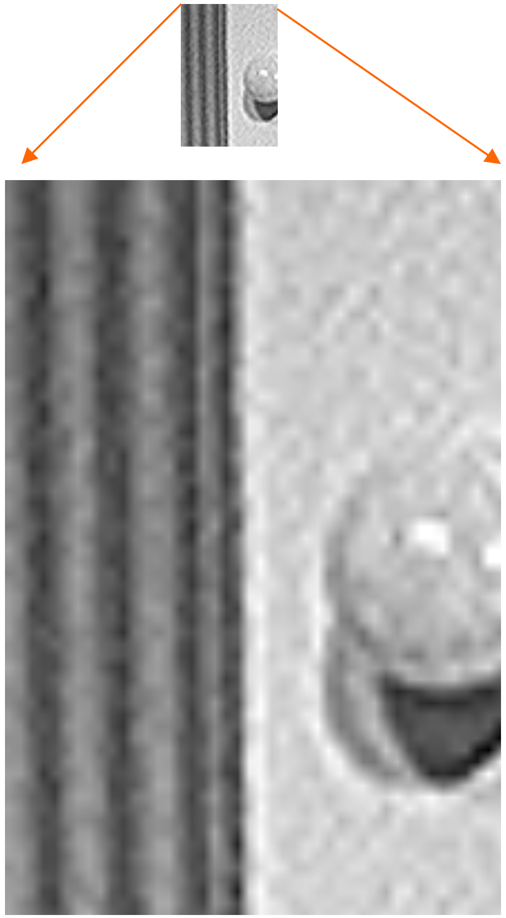
Recall: Effect of Noise on Derivatives





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Recall: Zero Crossing After Smoothing

Input

Laplacian after increasingly strong smoothing

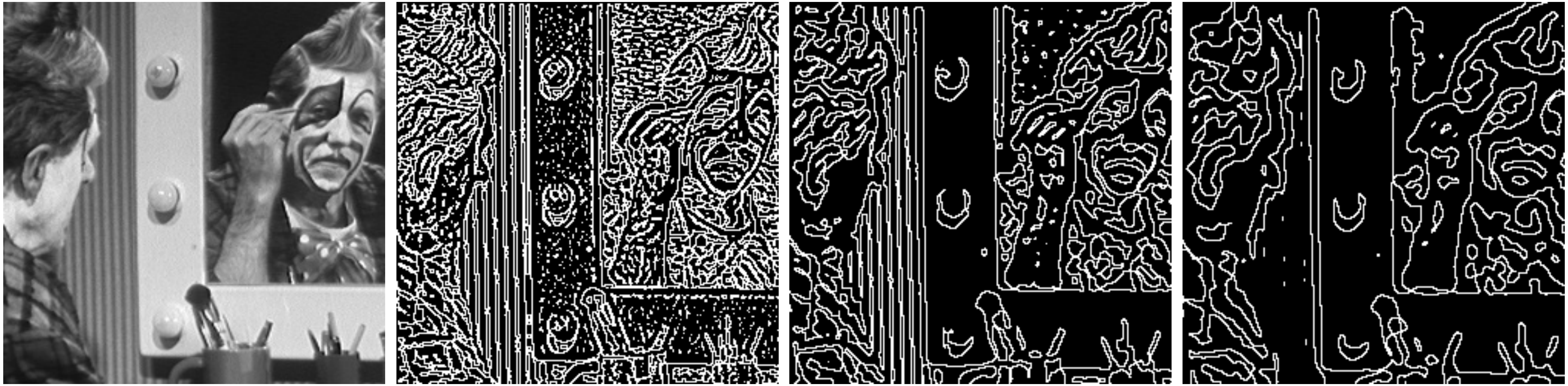

Image ⇨ Gaussian ⇨ Laplacian ⇨ Output

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LoG Filter

Image $\Rightarrow$ Gaussian $\Rightarrow$ Laplacian $\Rightarrow$ Output

$$\nabla^2(f * G)(x) \equiv (\nabla^2 G * f)(x)$$

Laplacian of Gaussian
(LoG) Filter

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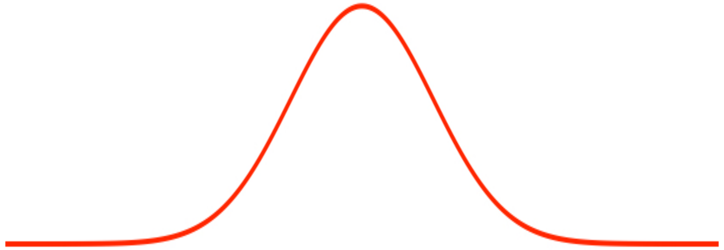
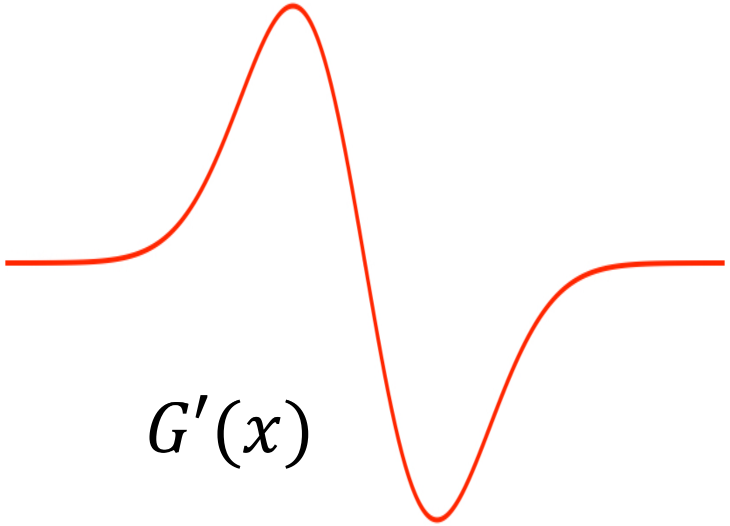
LoG Filter

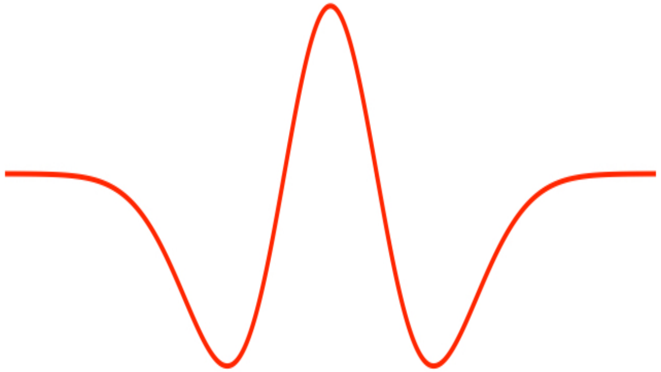


$G(x)$



$G'(x)$

Mexican Hat



$-G''(x)$

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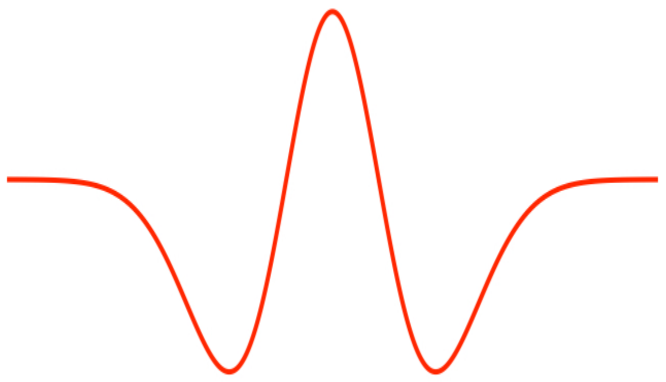
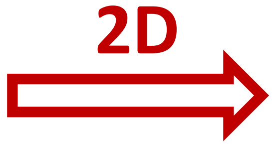
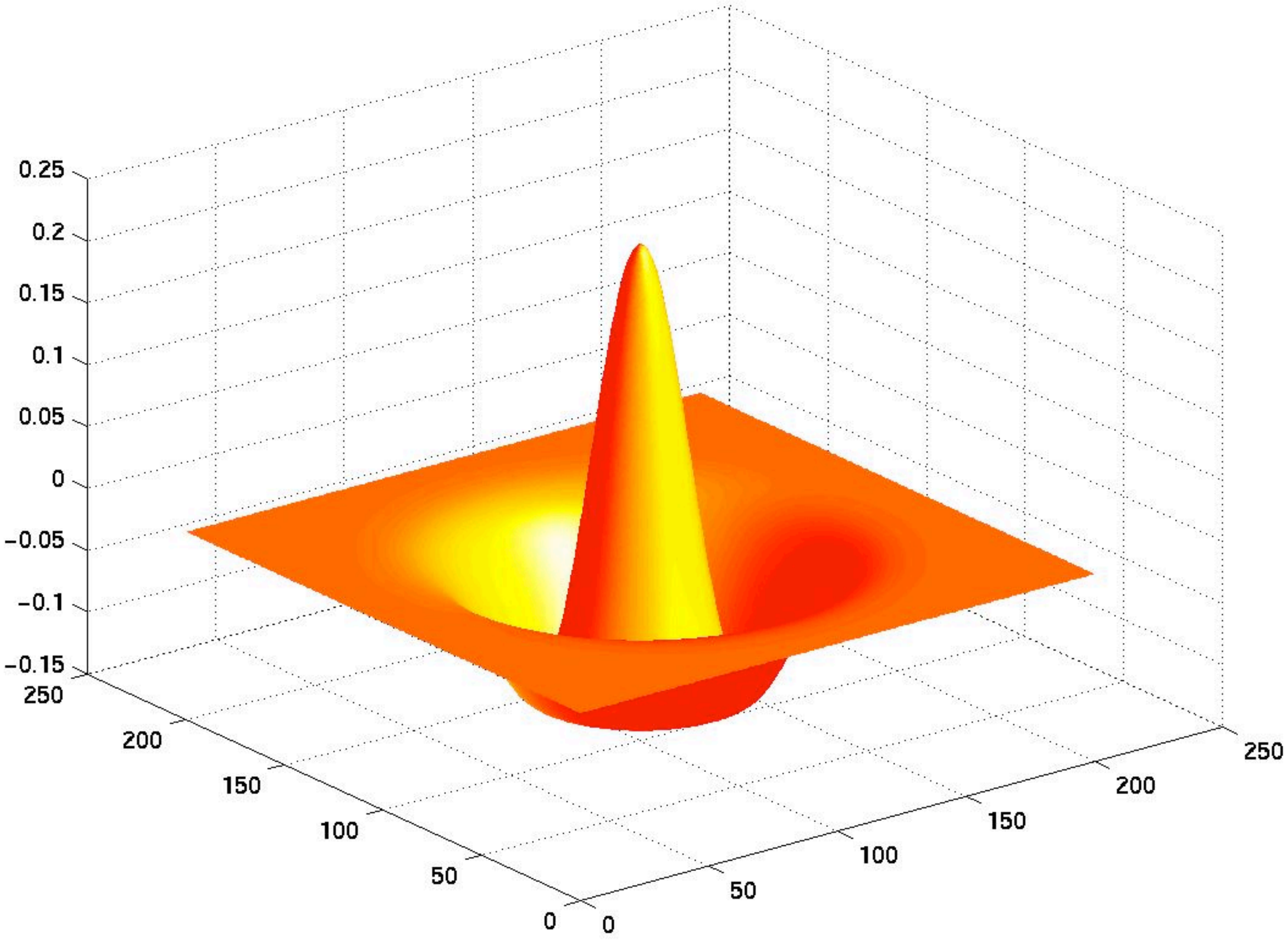
LoG Filter







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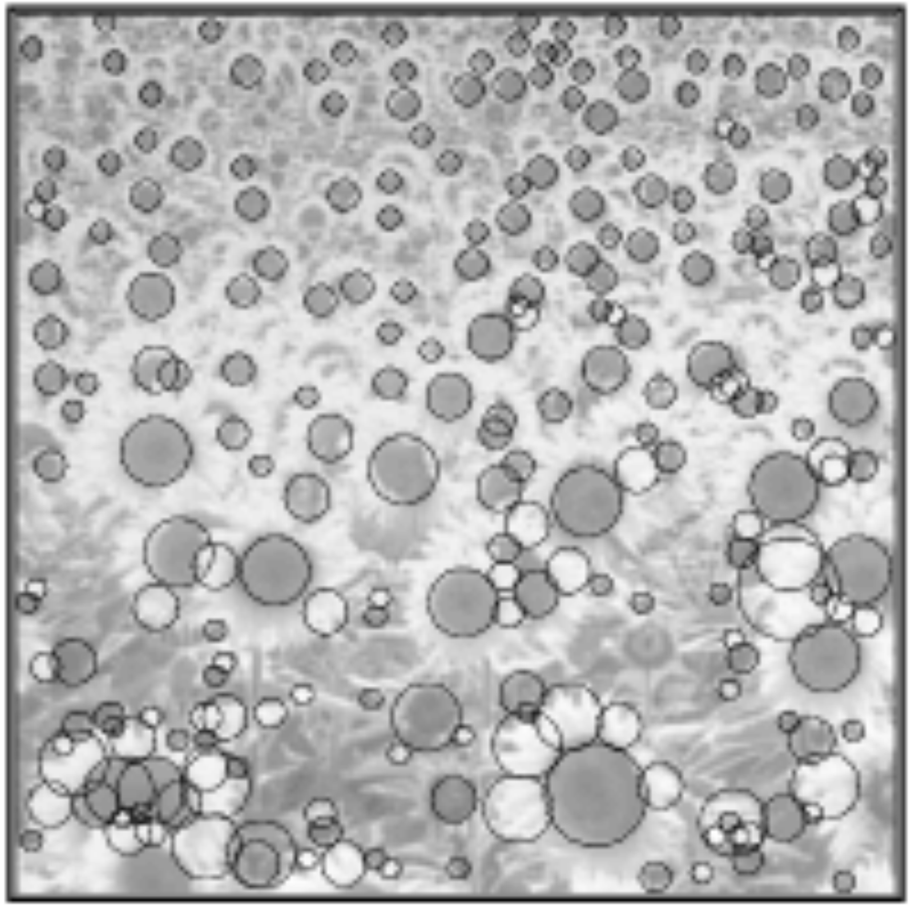
Blob Detection using LoG

- How can an edge finder be used to find blobs in an image?

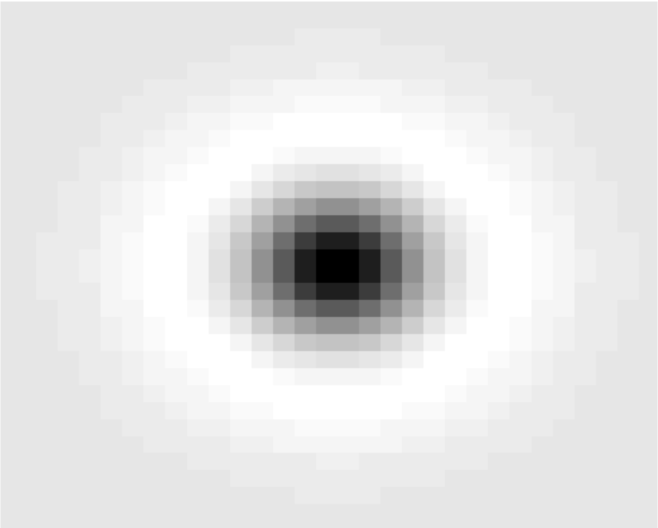


Lindeberg. IJCV 1998.
Feature detection with automatic scale selection

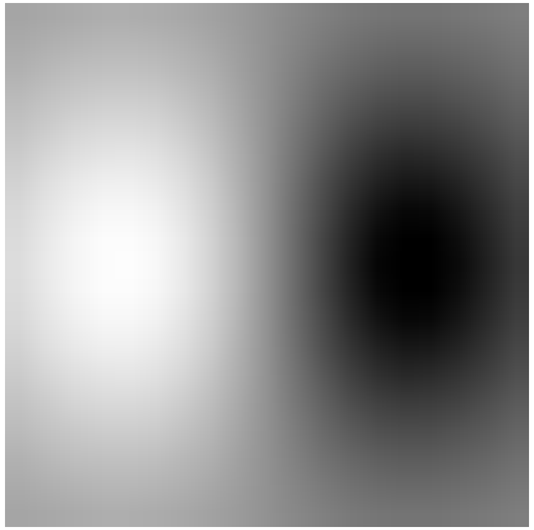
Blob Detection using LoG

Key Idea

- Cross correlation with a flipped filter (in a convolution) can be viewed as comparing a little “picture” of what you want to find against all local regions in the image.



Maximum response
Dark blob on light background



Maximum response
Vertical edge; lighter on left

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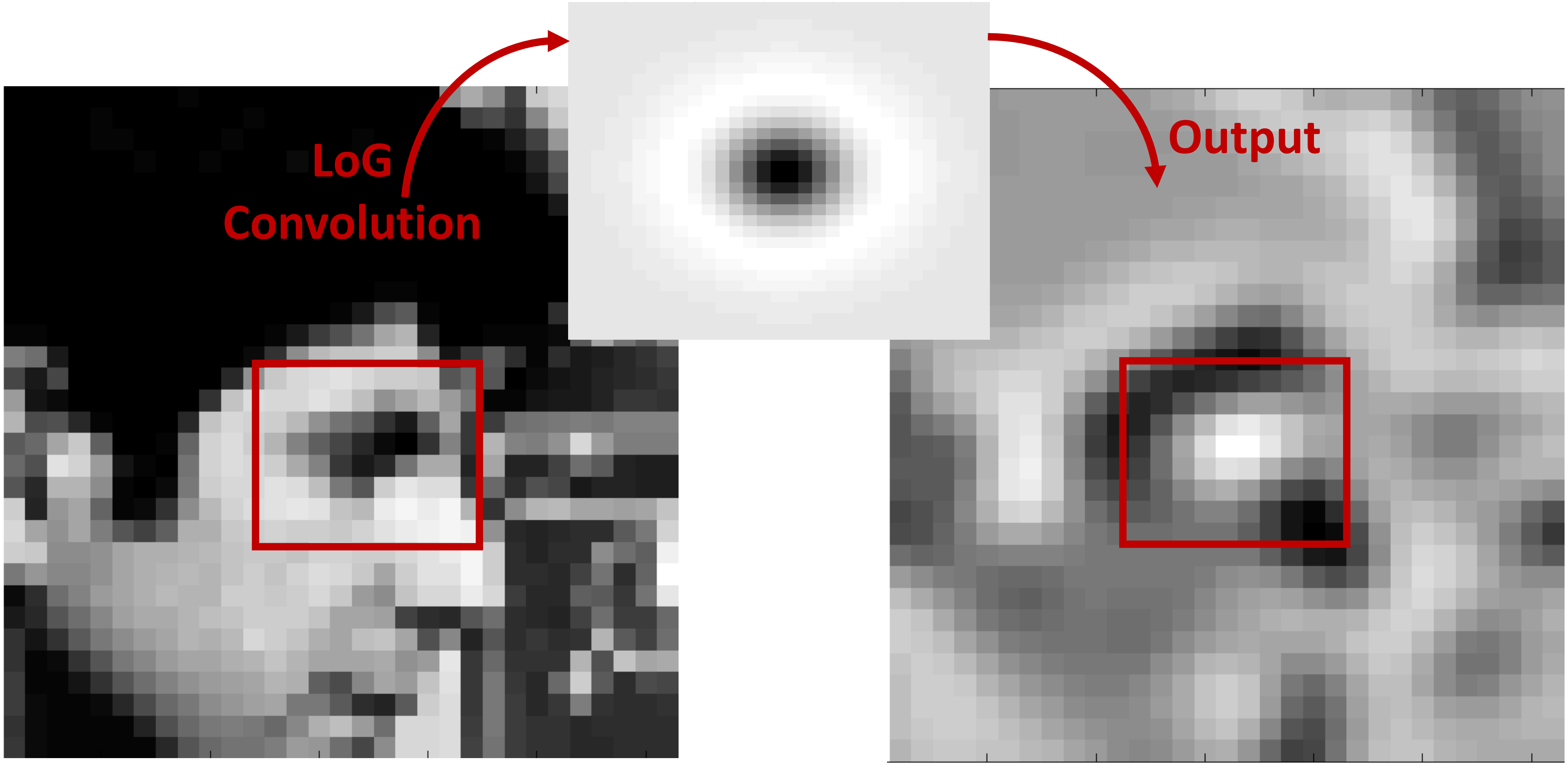
Blob Detection using LoG



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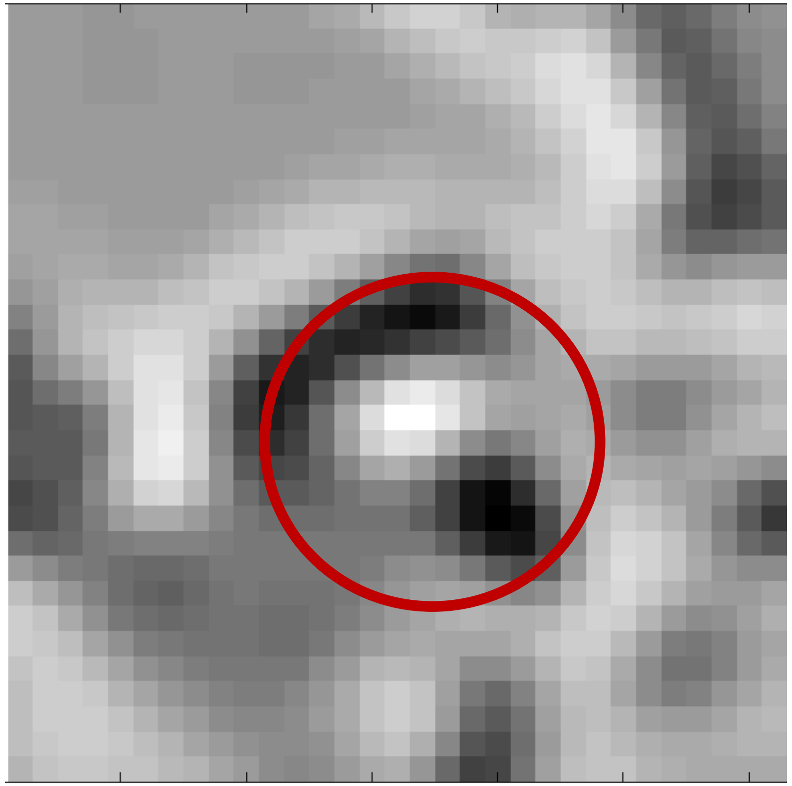
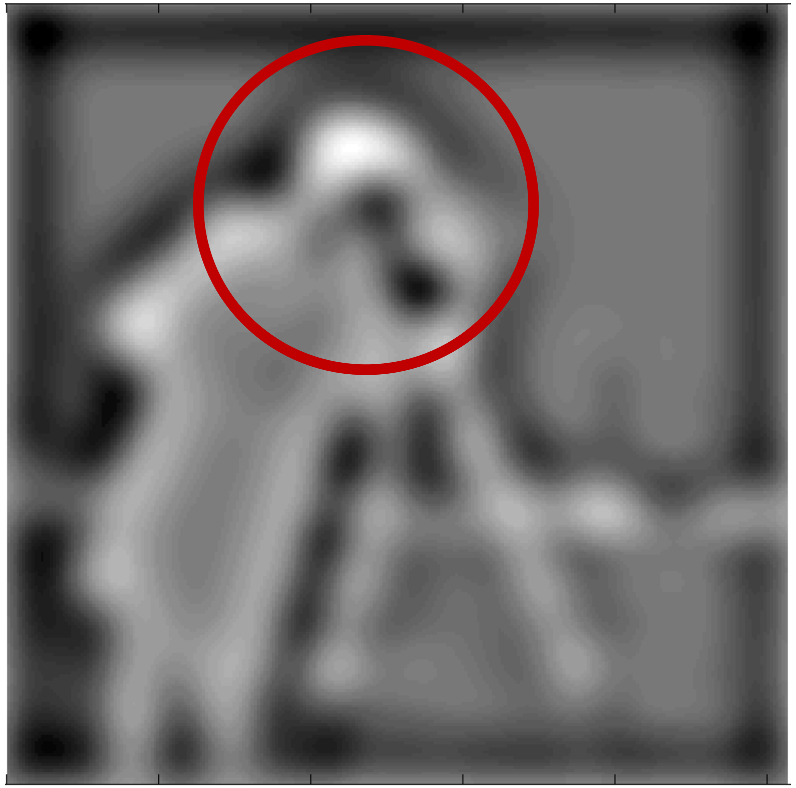
Blob Detection using LoG

- Scale of blob (size ; radius in pixels) is determined by the sigma parameter of the LoG filter.



LoG $\sigma = 2$



LoG $\sigma = 10$

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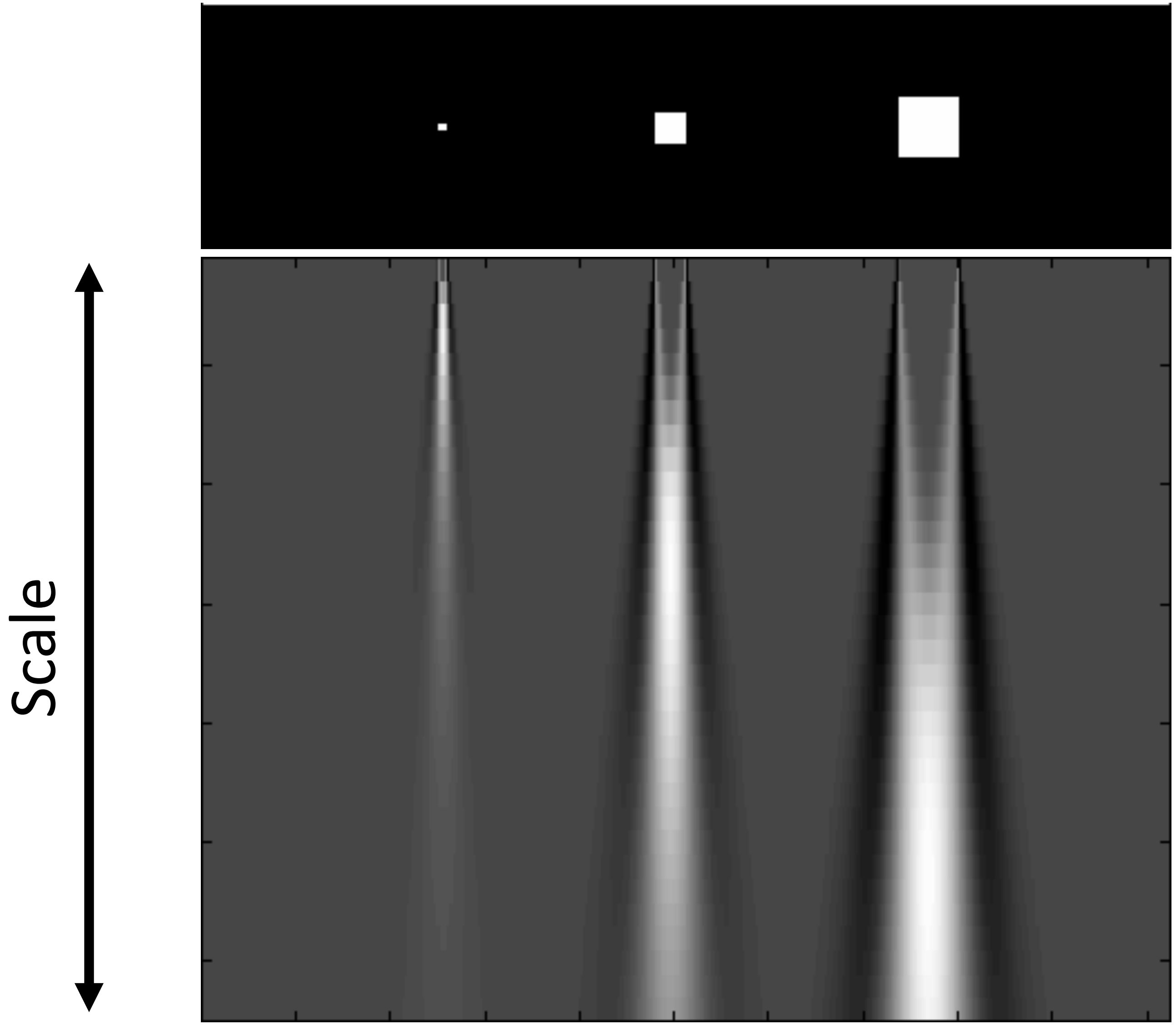
Local Scale Space Maxima



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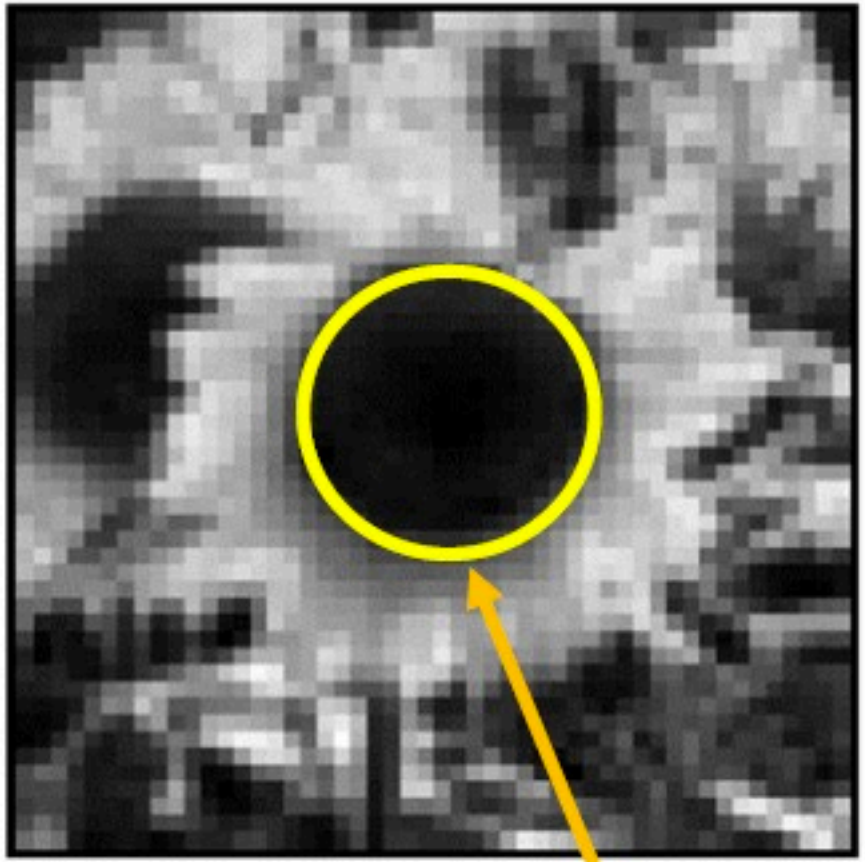
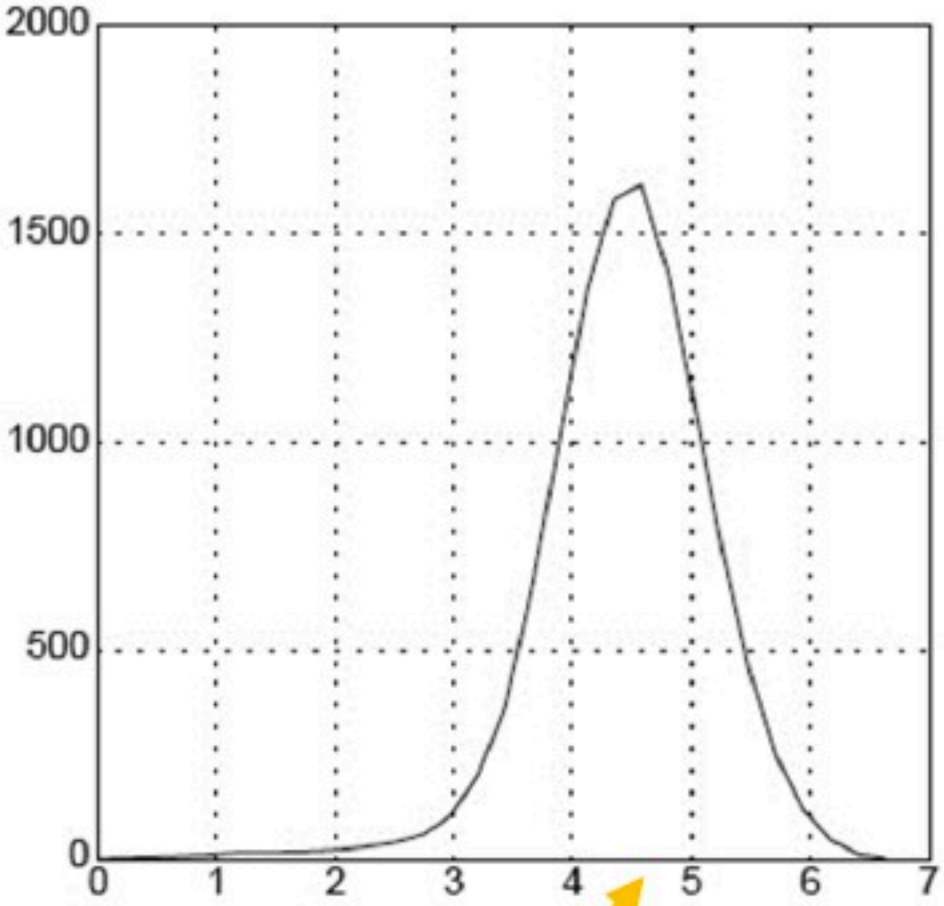
Local Scale Space Maxima





Characteristic scale

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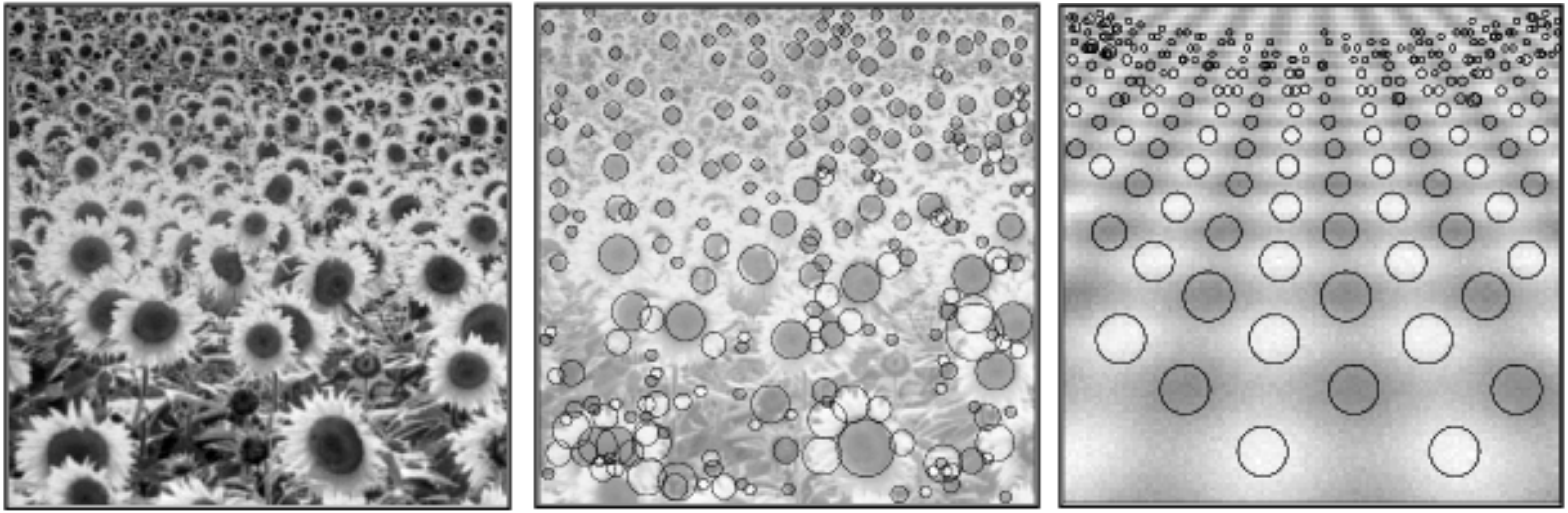
Blob Detection using LoG

- Blobs are detected as local extrema in space and scale, within the LoG scale-space volume.



Lindeberg. IJCV 1998.
Feature detection with automatic scale selection

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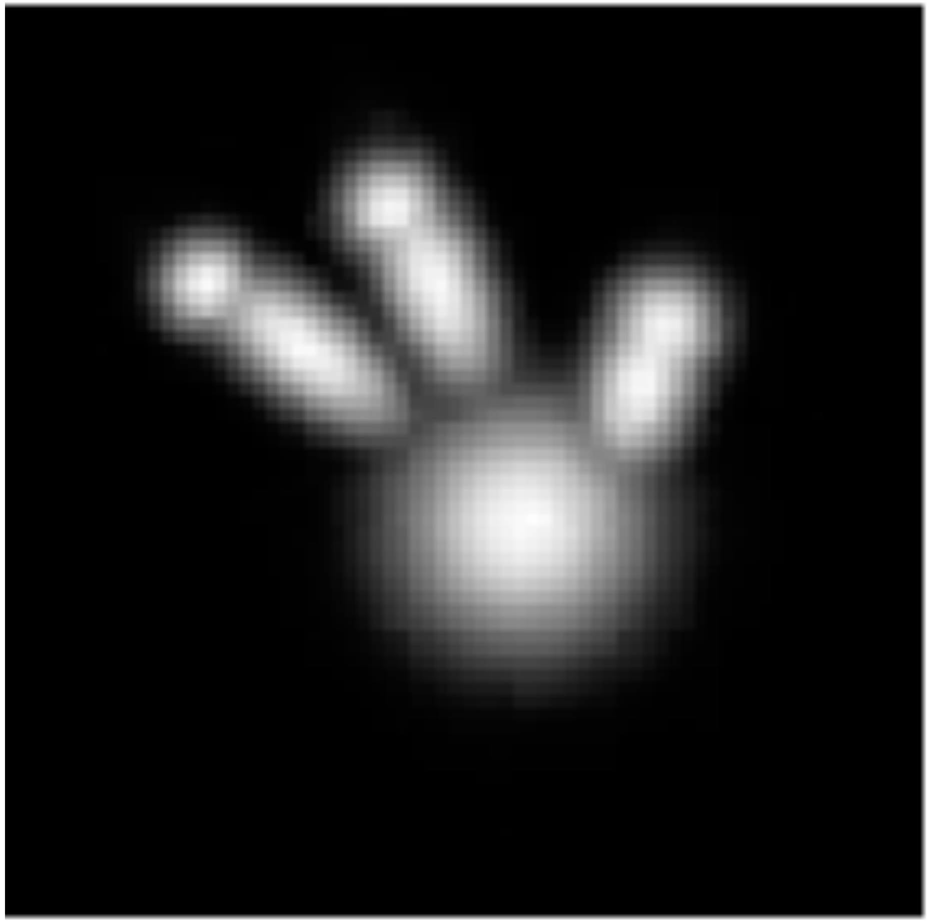
Gesture Recognition via Blob Detection



Lars Bretzner, Ivan Laptev, and Tony Lindeberg. Automatic Face and Gesture Recognition, 2002
Hand Gesture Recognition using Multi-Scale Colour Features, Hierarchical Models and Particle Filtering

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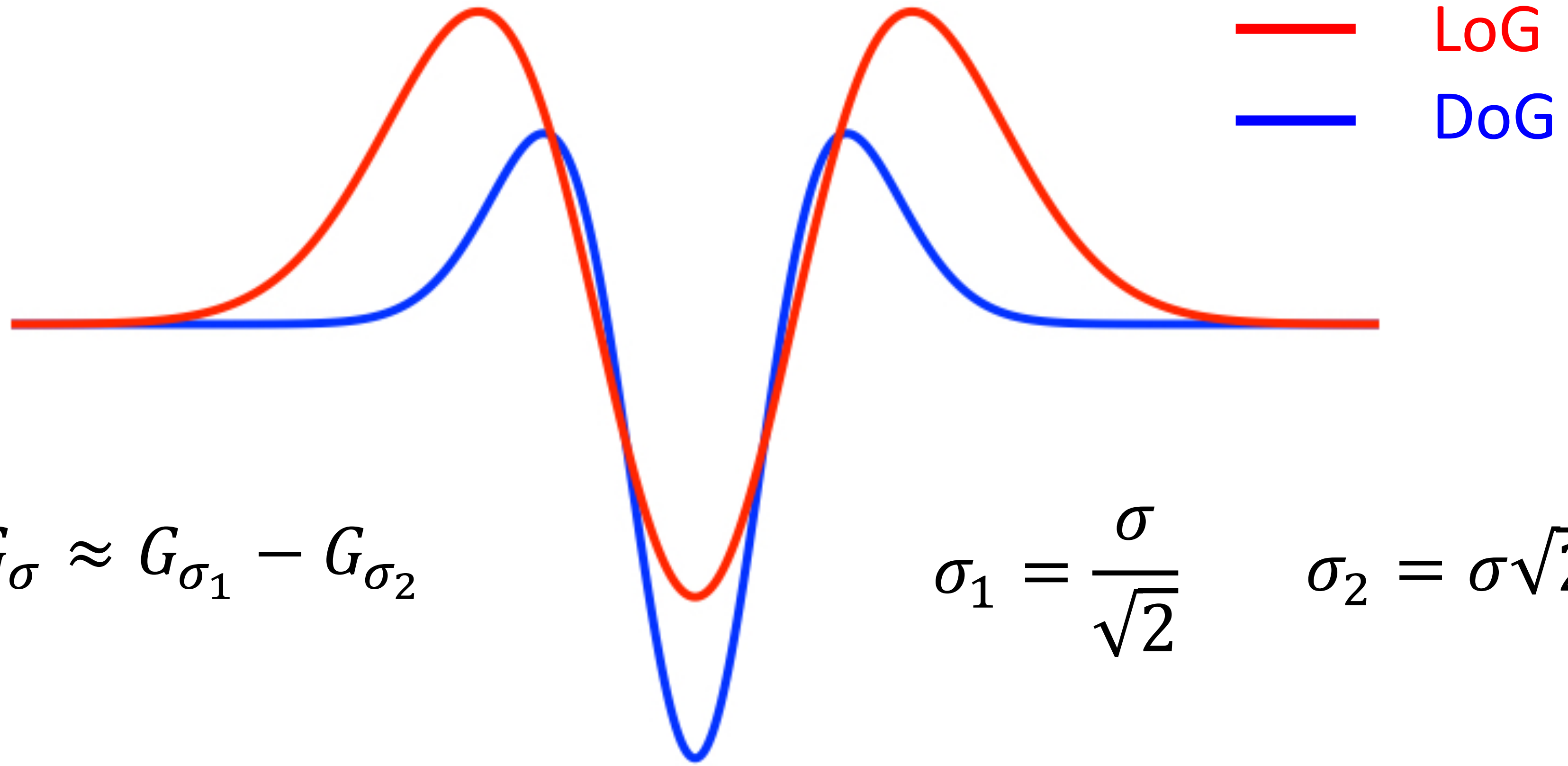
LoG → DoG

- LoG can be approximate by a Difference of two Gaussians (DoG) at different scales.



$$\nabla^2 G_\sigma \approx G_{\sigma_1} - G_{\sigma_2}$$

$$\sigma_1 = \frac{\sigma}{\sqrt{2}} \qquad \sigma_2 = \sigma\sqrt{2}$$

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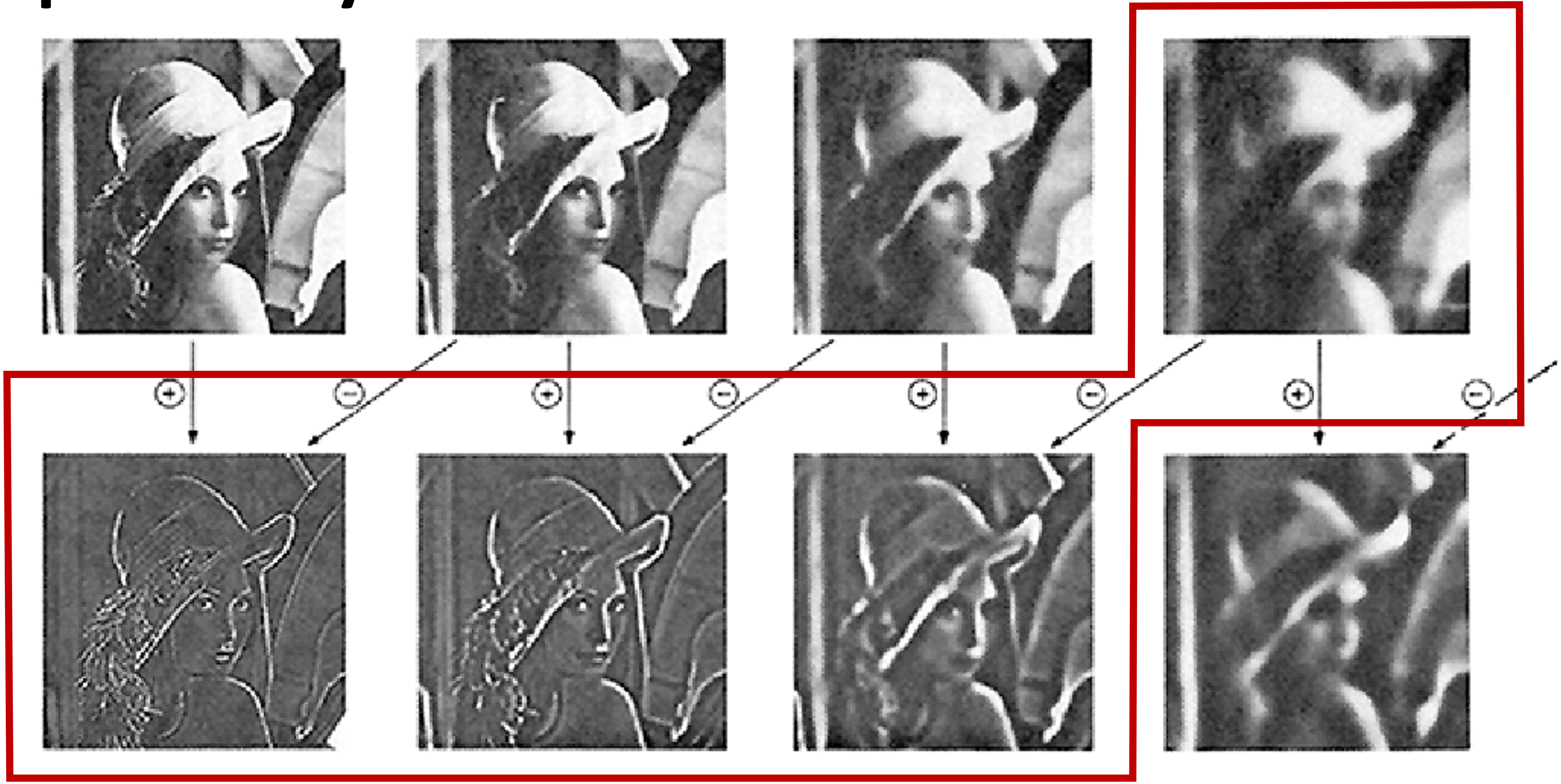
Laplacian Pyramids

Store only these



Scale Invariant Detectors

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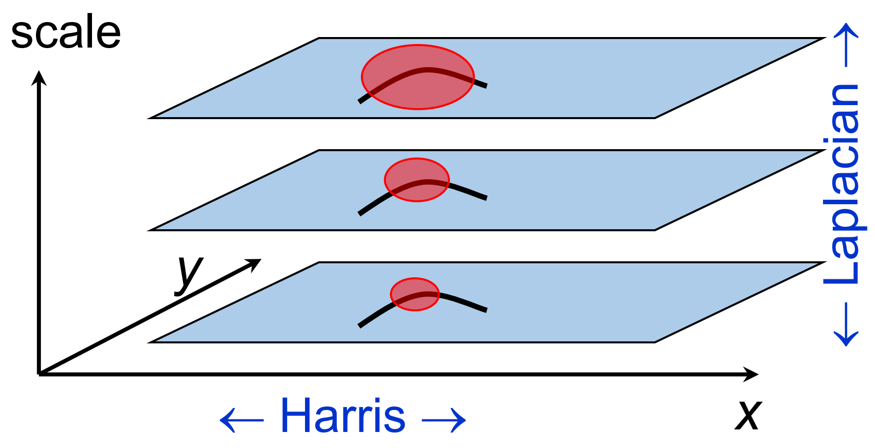
Scale Invariant Detectors

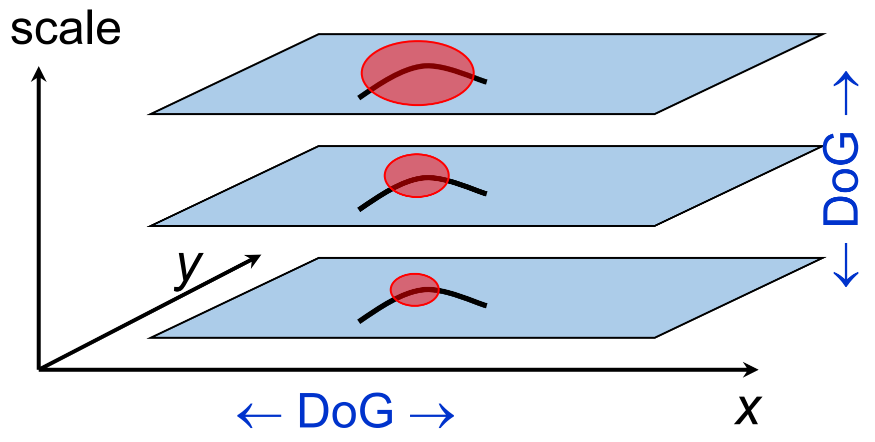
Harris-Laplacian

- Find local maximum of:
  - Harris corner detector in space (image coordinates)
  - Select points which are also maxima of Laplacian in scale



SIFT

- Find local maximum of:
  - Difference of Gaussians in space and scale

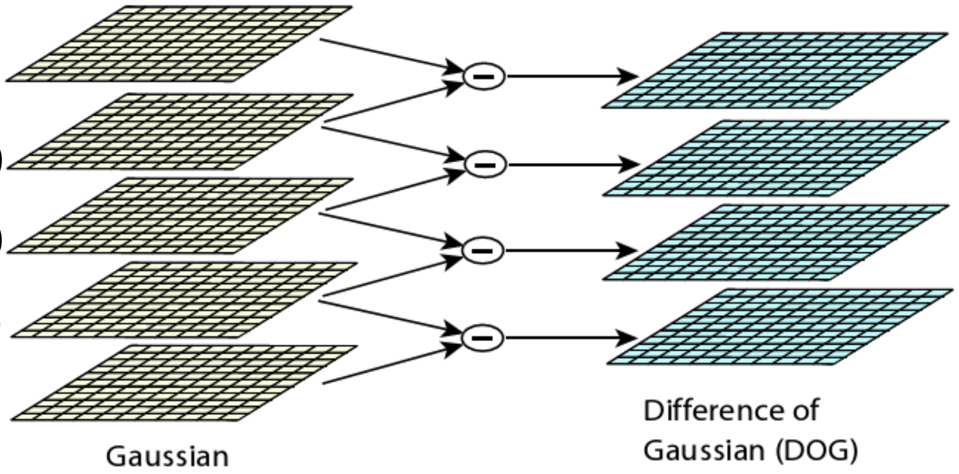


Difference of Gaussians

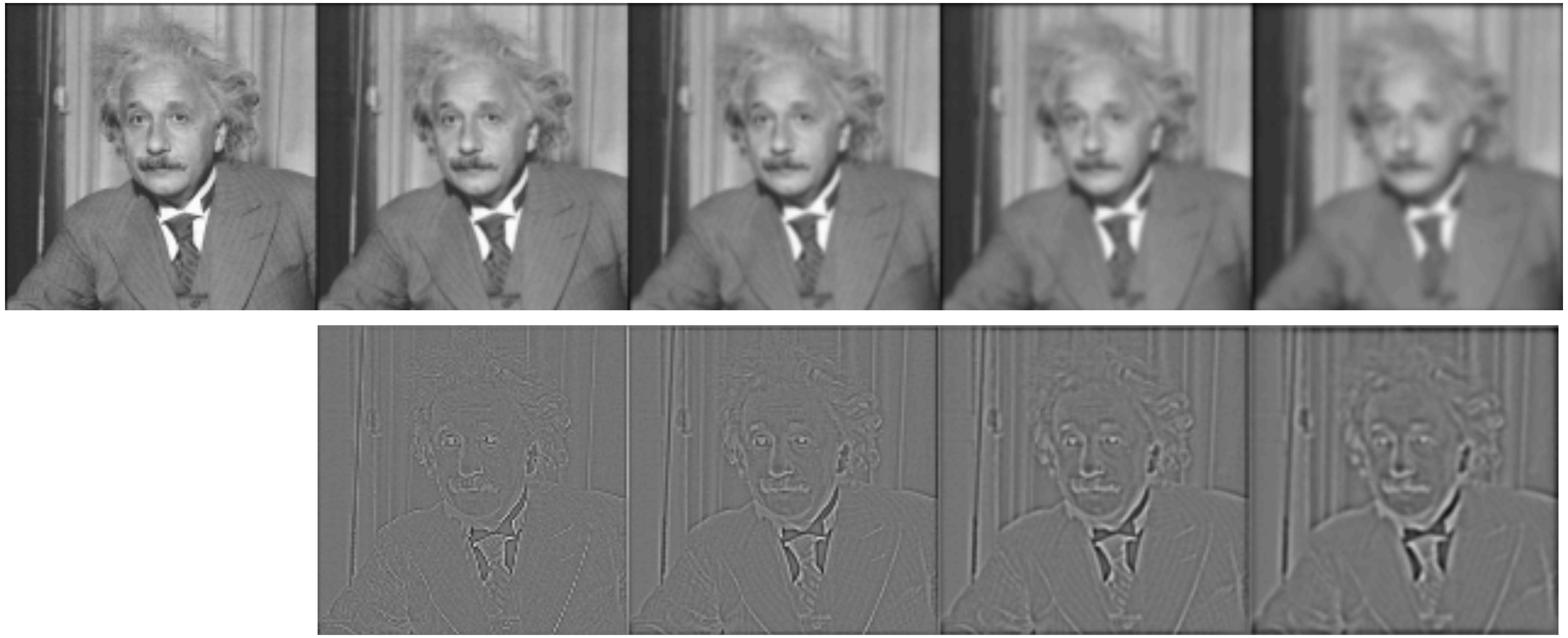
$I * G(k^2\sigma)$

$I * G(k\sigma)$

$I * G(\sigma)$



$$D(\sigma) \equiv I * (G(k\sigma) - G(\sigma))$$



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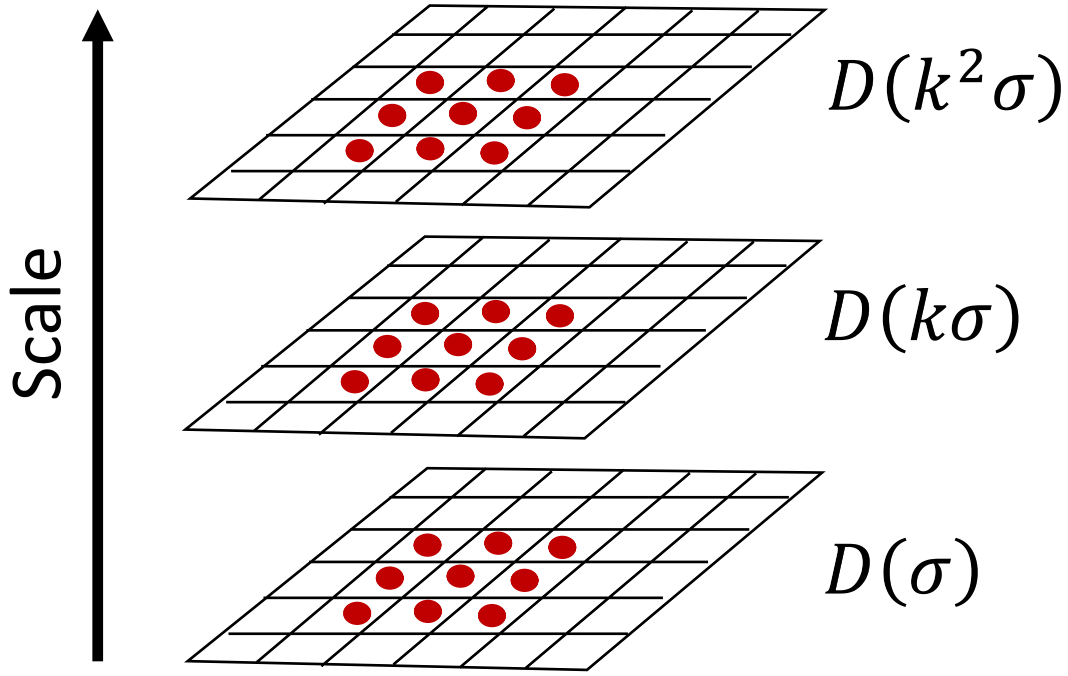
Scale Space Extrema

- Choose all extrema within 3×3×3 neighborhood.



X is selected if it's response is larger or smaller than all 26 neighbors

Rotation Invariant Matching (Descriptors)

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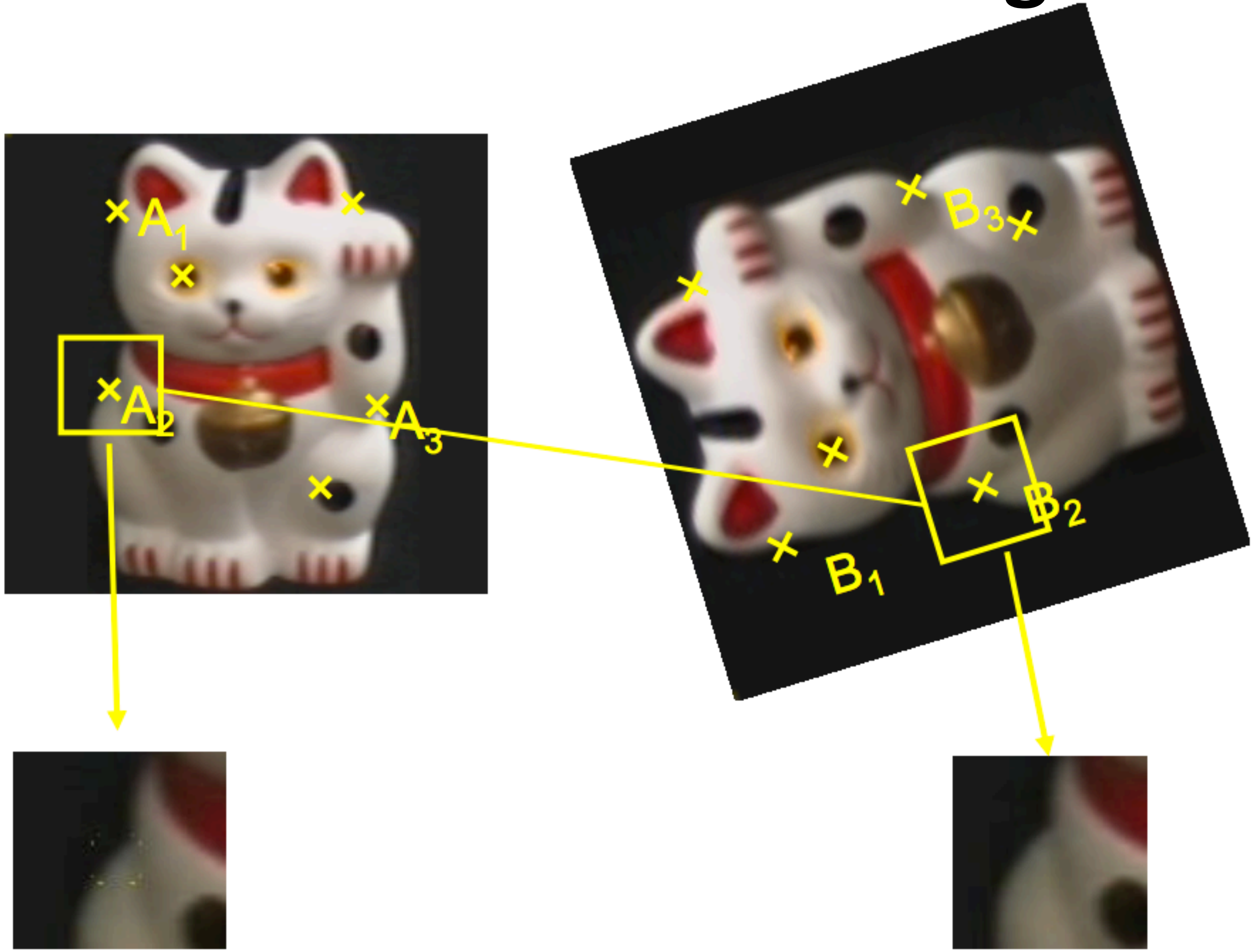
What is Rotation Invariant Matching?



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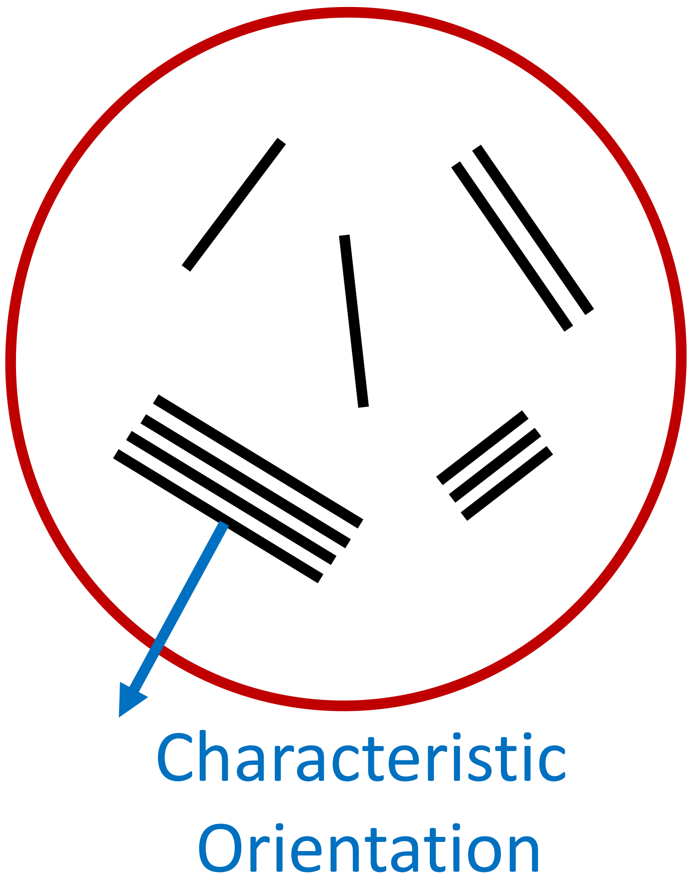
Rotation Invariance: Key Idea

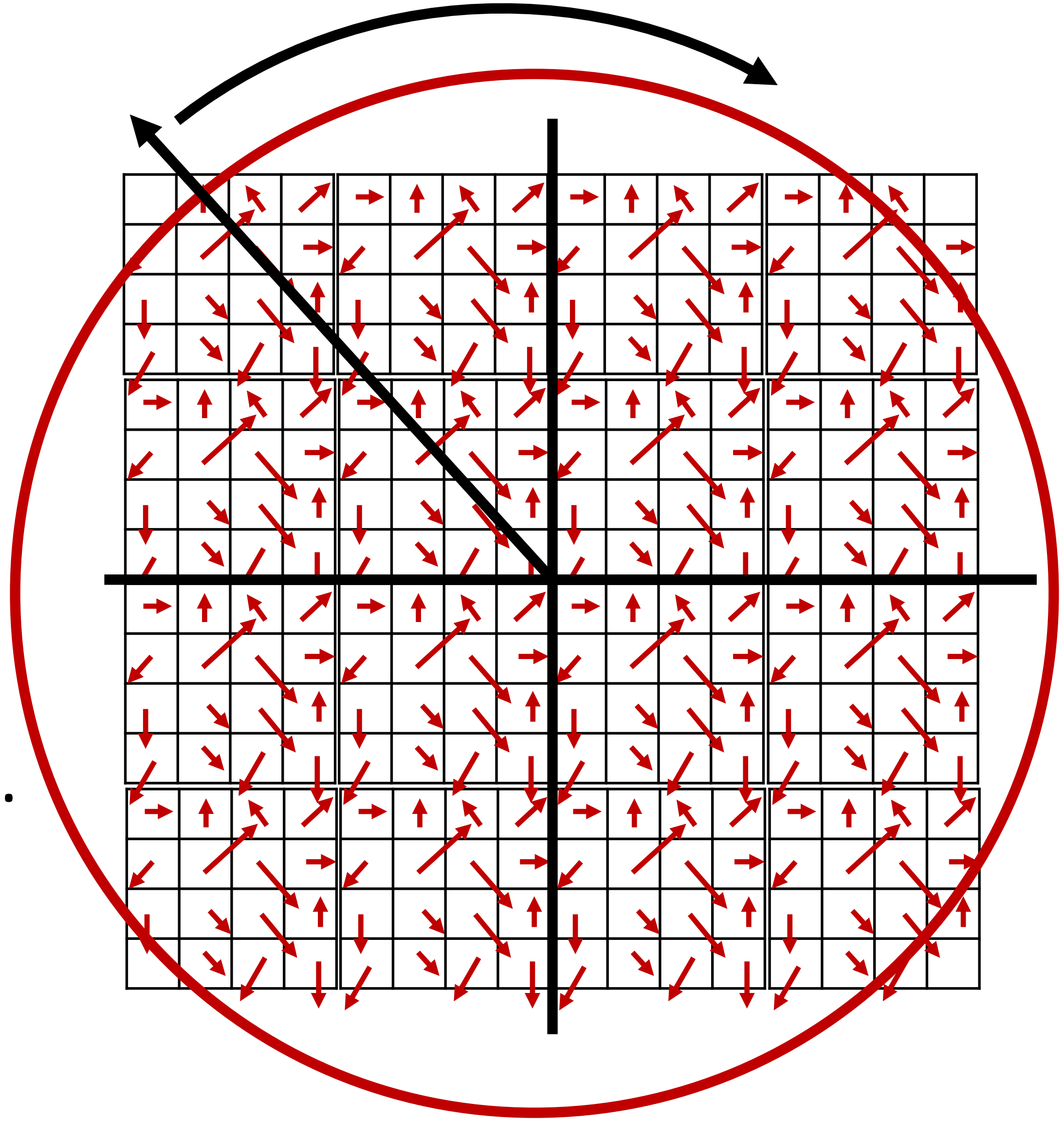
- We are given a keypoint and its scale from DoG
- Select a characteristic orientation for the keypoint (based on the most prominent gradient there)
- Describe all features **relative** to this orientation
- Causes descriptor to be rotation invariant!
  - If the keypoint is rotated in another image, the descriptor will be the same, since it is **relative** to the characteristic orientation



SIFT Descriptor

- Use the blurred image associated with the keypoint's scale
- Take image gradients over the keypoint neighborhood.
- To become rotation invariant, rotate the gradient directions by keypoint orientation.
- We could have rotated the whole image as well (slower)

Rotate Gradient Directions



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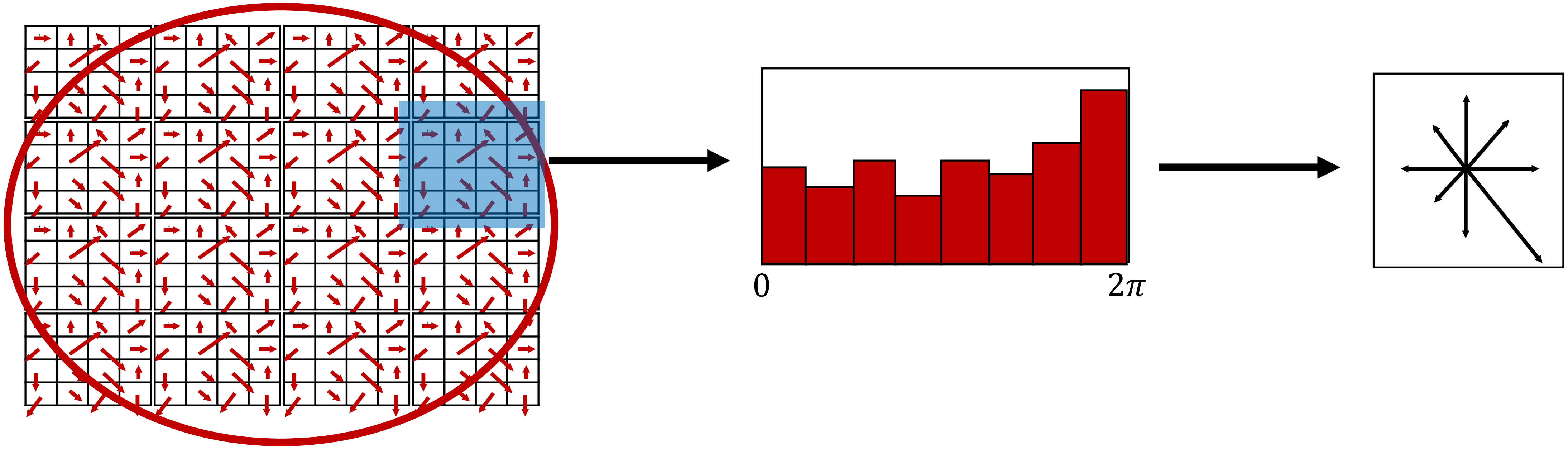
SIFT Descriptor



- Using precise gradient locations is fragile. Instead, create an orientation histogram of a 4x4 cells.
- A gradient's contribution is divided among the nearby histograms based on distance. Halfway between two histogram locations = Half contribution to both. Also, scales down gradient contributions far from the center.

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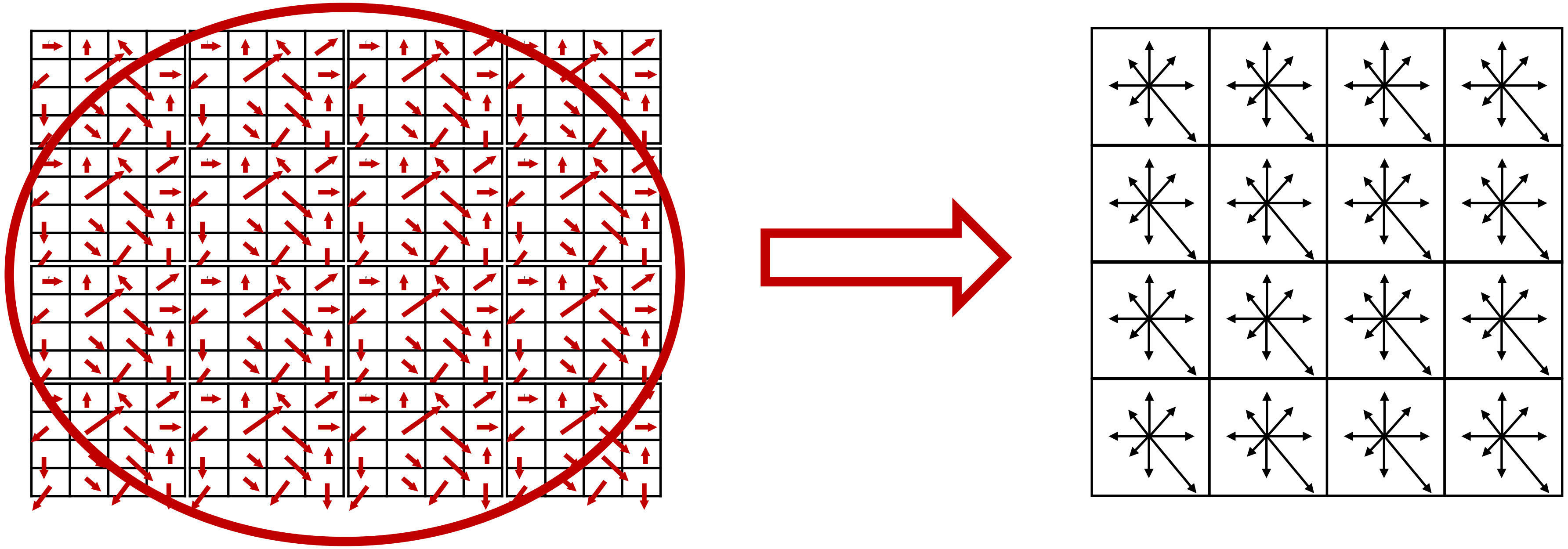
SIFT Descriptor



- SIFT uses 8 orientation bins per histogram, and a 4x4 histogram array, yielding 8 x 4x4 = 128 numbers, for a 16x16 window.
- A SIFT descriptor is a 128 dimensional vector, invariant to rotation (we rotated the gradients) and scale (worked with the scaled image from DoG).
- Use Euclidean distance between the two descriptor vectors to match.

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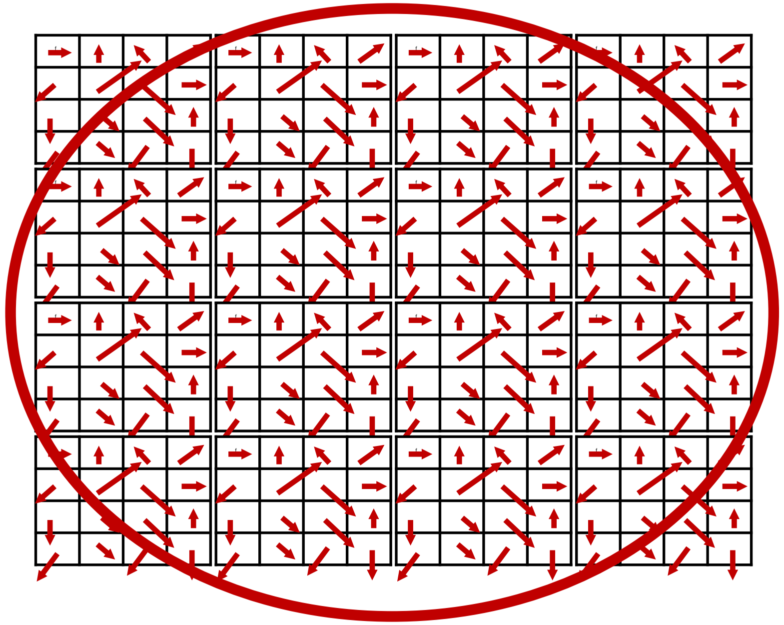
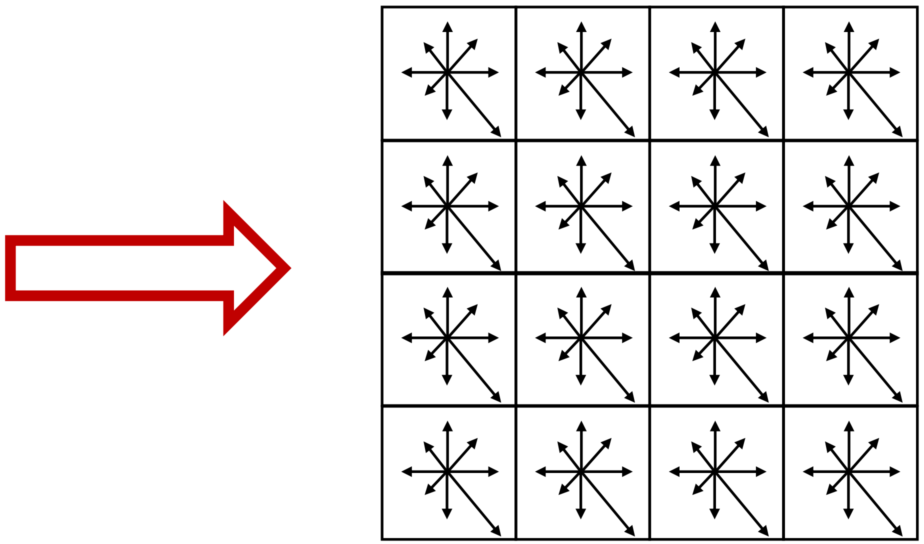
SIFT Descriptor: Illumination Robustness





- SIFT descriptor is made of gradients. Invariant to small changes in brightness
- A higher-contrast in an image increases the magnitude of gradients linearly. To correct for contrast changes, normalize the vector (unit magnitude)
- Large gradients from 3D illumination effects effect robustness. Clamp all values in the vector to be ≤ 0.2. Then normalize the vector again.

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Histogram of Oriented Gradients

- Local object appearance and shape can often be characterized rather well by the distribution of local intensity gradients or edge directions.