Edge and Line Detection

Chetan Arora

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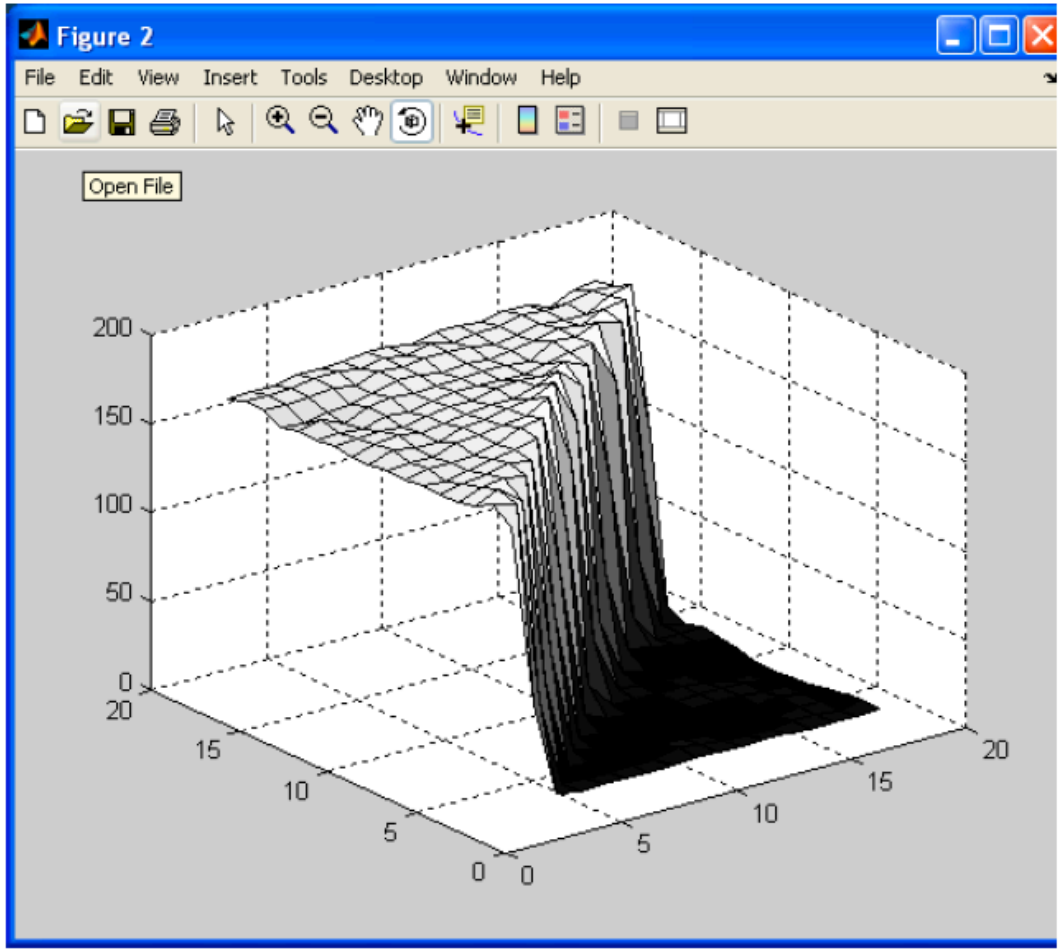
What are Edges?

- Mathematically: discontinuity in intensity or color.



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What are Edges?

- Mathematically: discontinuity in intensity or color.

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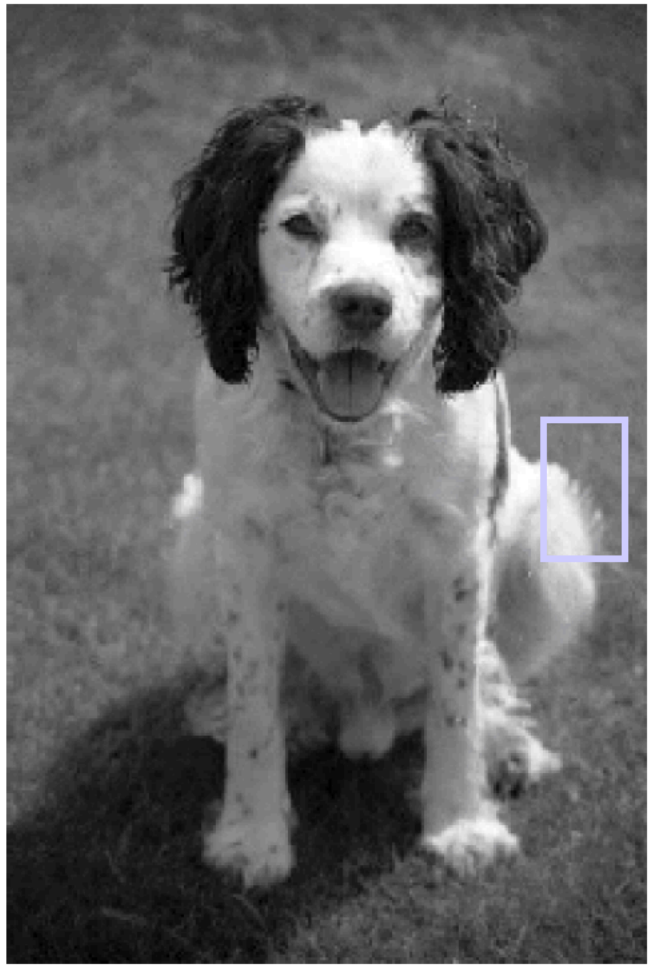
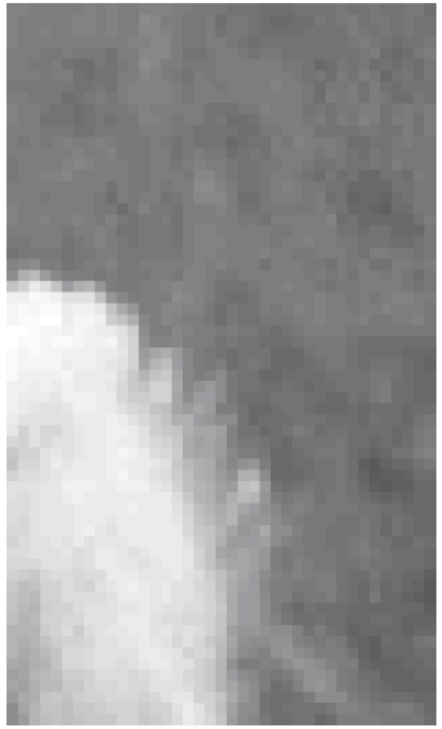
What are Edges?

- Semantically: object boundaries





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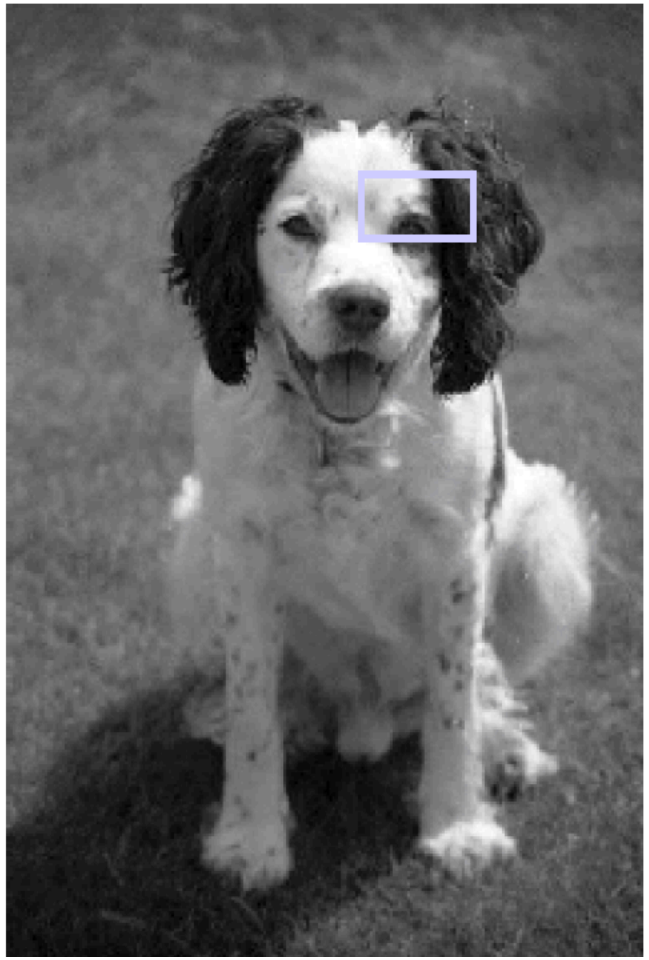
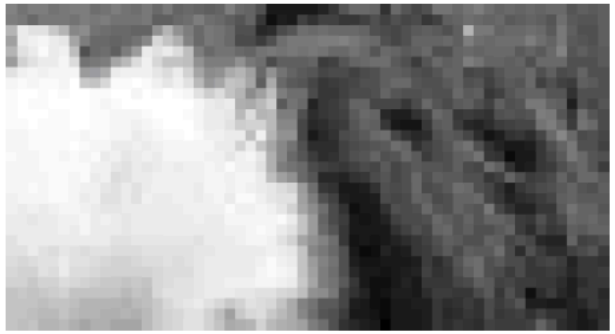
What are Edges?

- Semantically: boundaries of material properties





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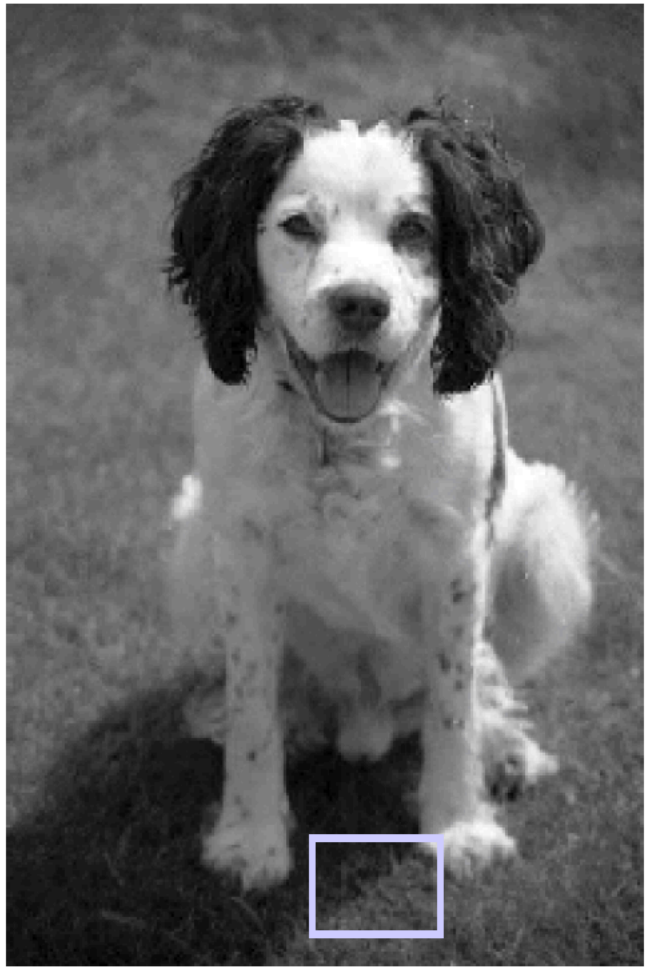
What are Edges?

- Semantically: lighting boundary



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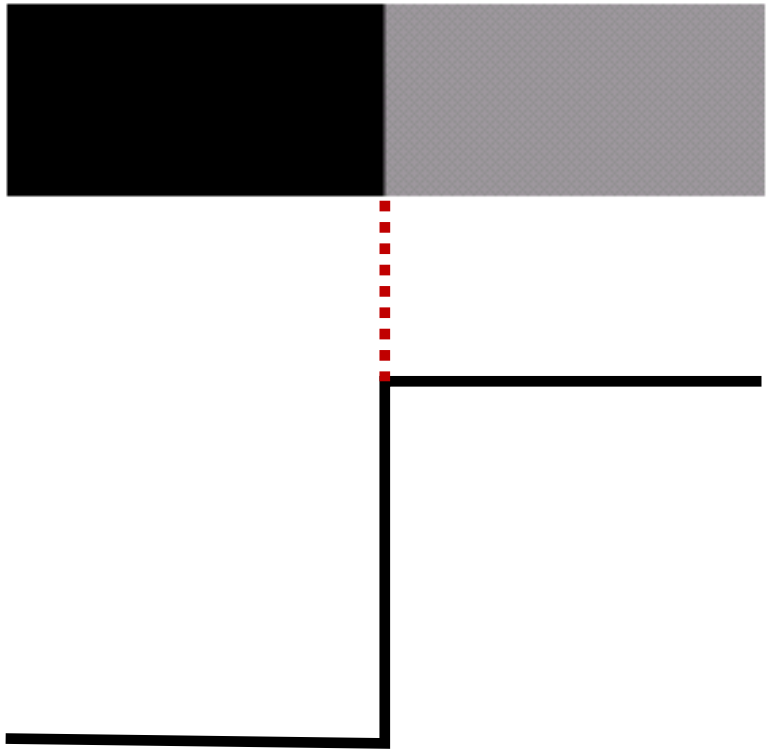
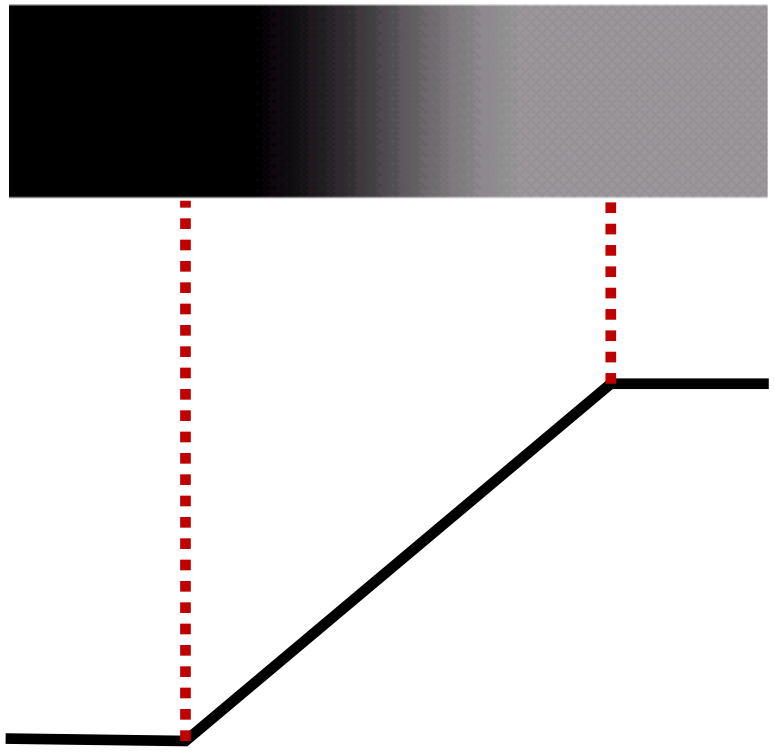
Types of Edges

Step edge



Ramp edge



Gray-level profile of the edge

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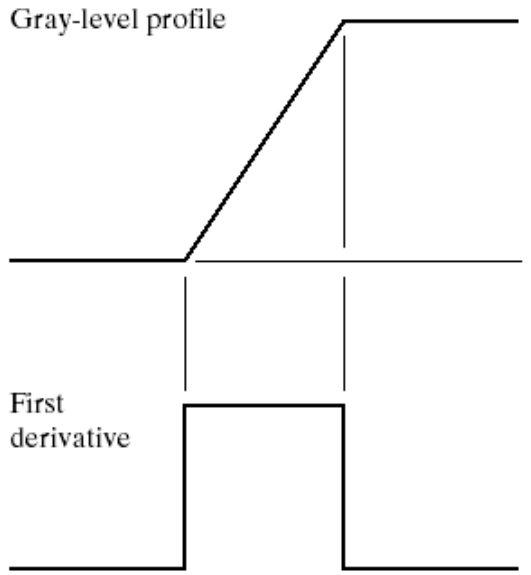
Derivate Response to a Ramp Edge



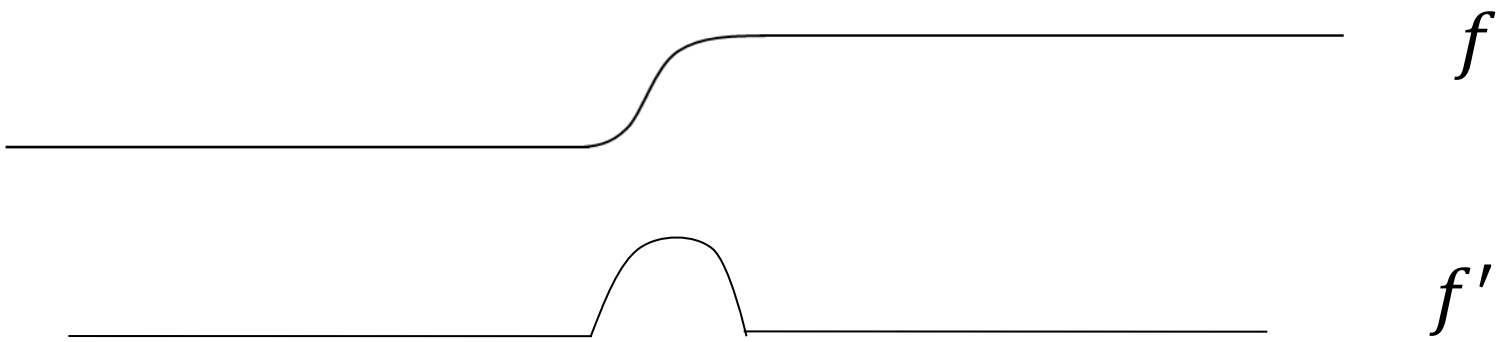


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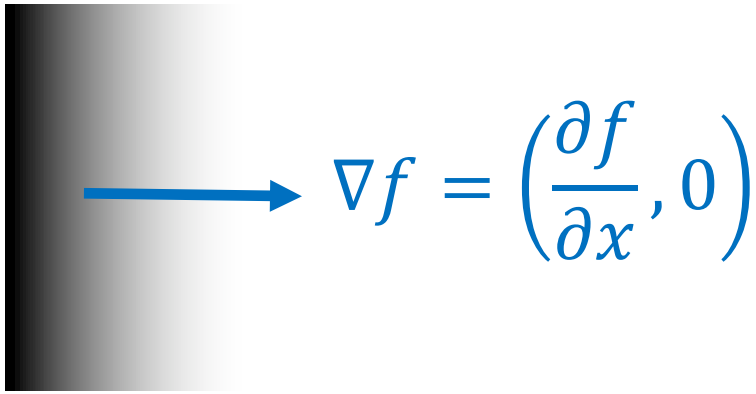
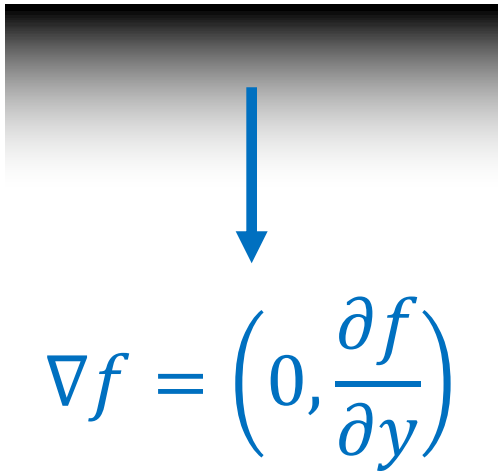
Image Gradient

- Gradient: The vector of derivatives $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (I_x, I_y)$

- Gradient: The direction of most rapid change in intensity





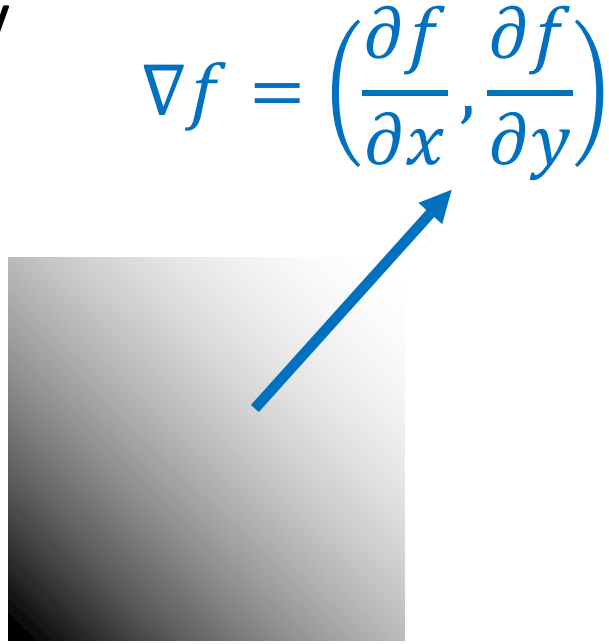


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Image Gradient

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

- Edge (Gradient) Magnitude: $|\nabla f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$
- Edge (Gradient) Direction: $\alpha = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$
- Directional Derivative in direction α: $\cos(\alpha)\frac{\partial f}{\partial x} + \sin(\alpha)\frac{\partial f}{\partial y}$

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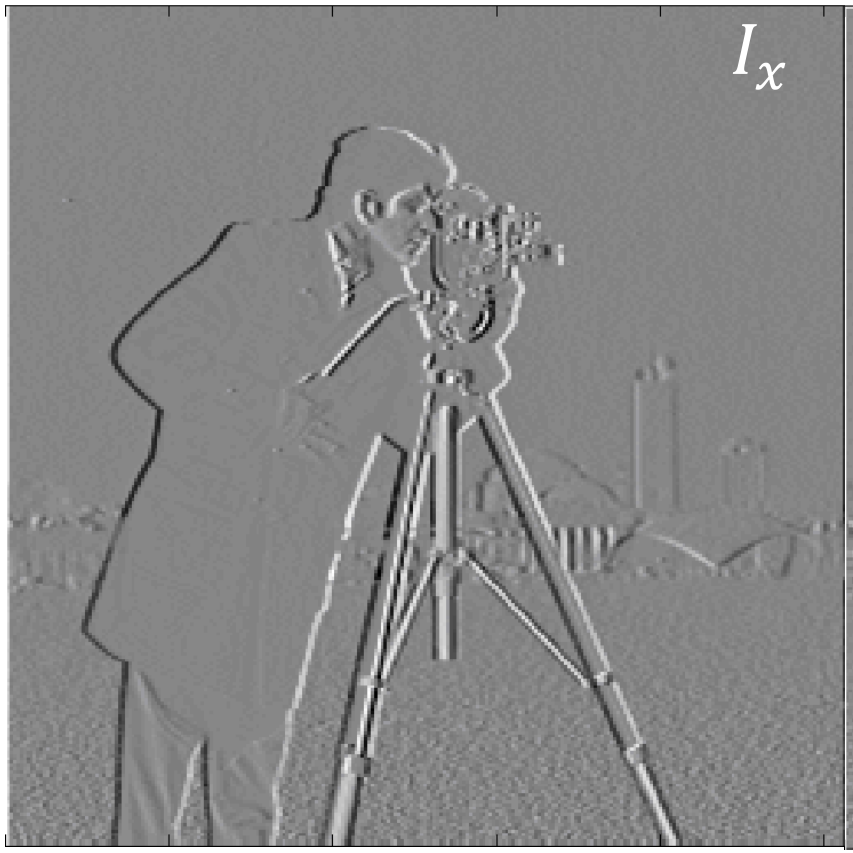
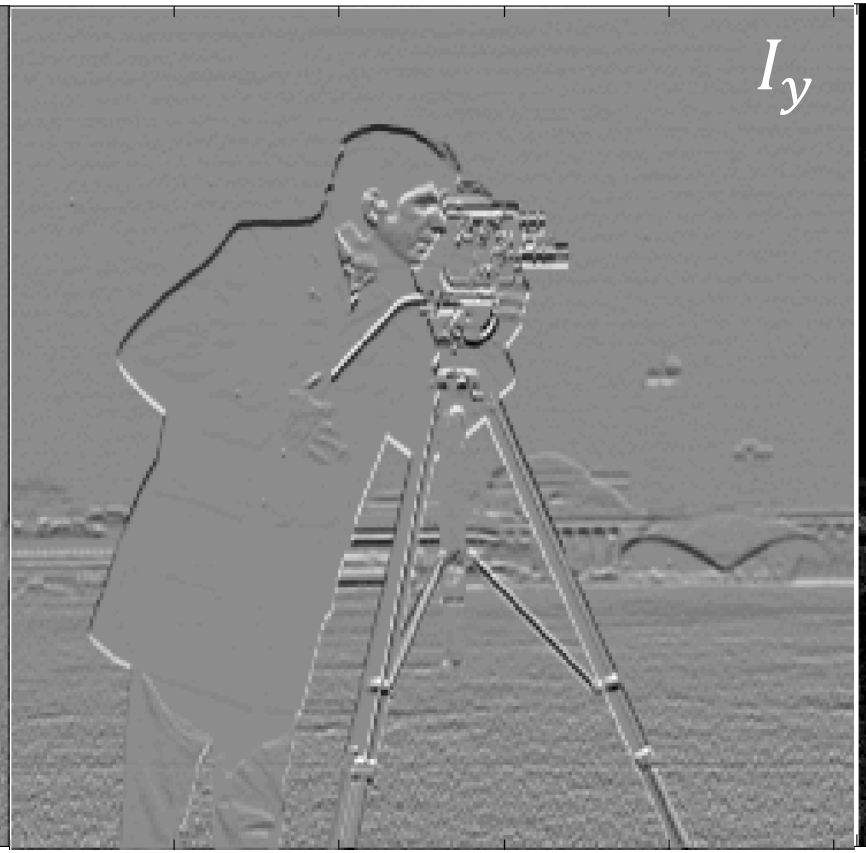
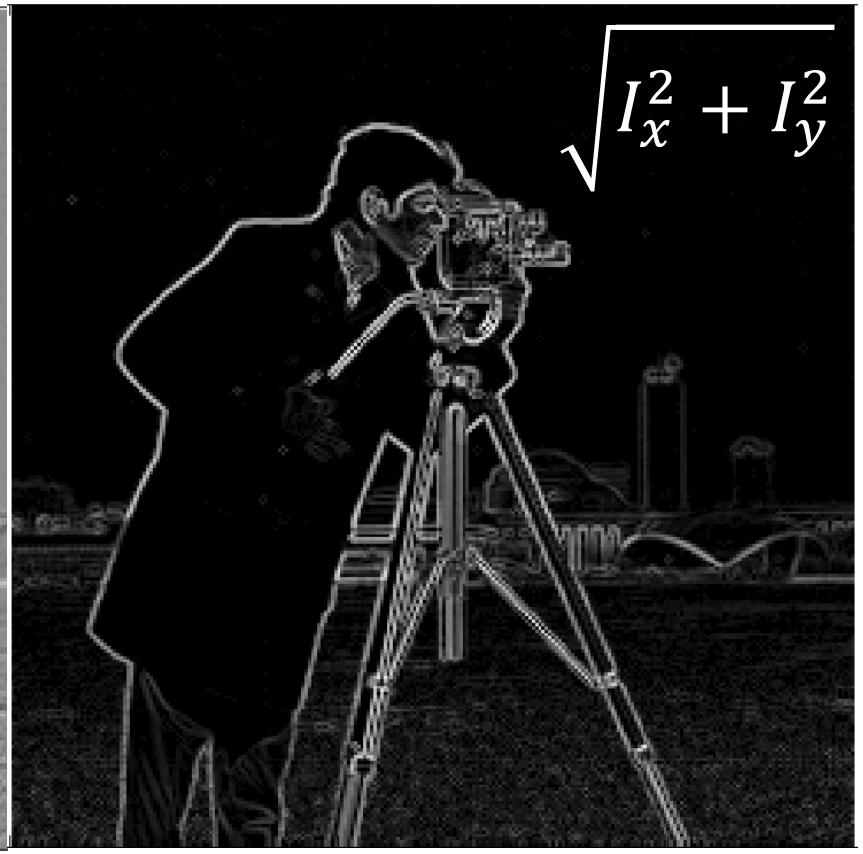
Image Gradient

Original







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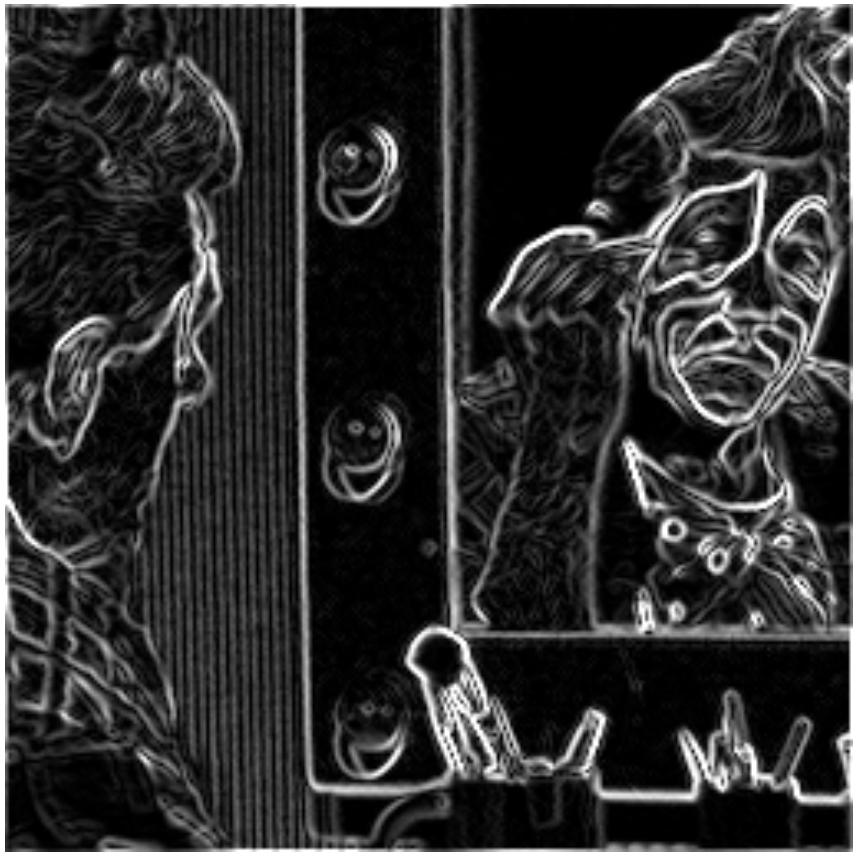
Edge Detection using Image Gradients

Original

|Gradient|



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Edge Detection using Image Gradients

Original

|Gradient|

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Image Gradients to Edges

|Gradient|

Cameraman

|Gradient| > 30

Binary Image

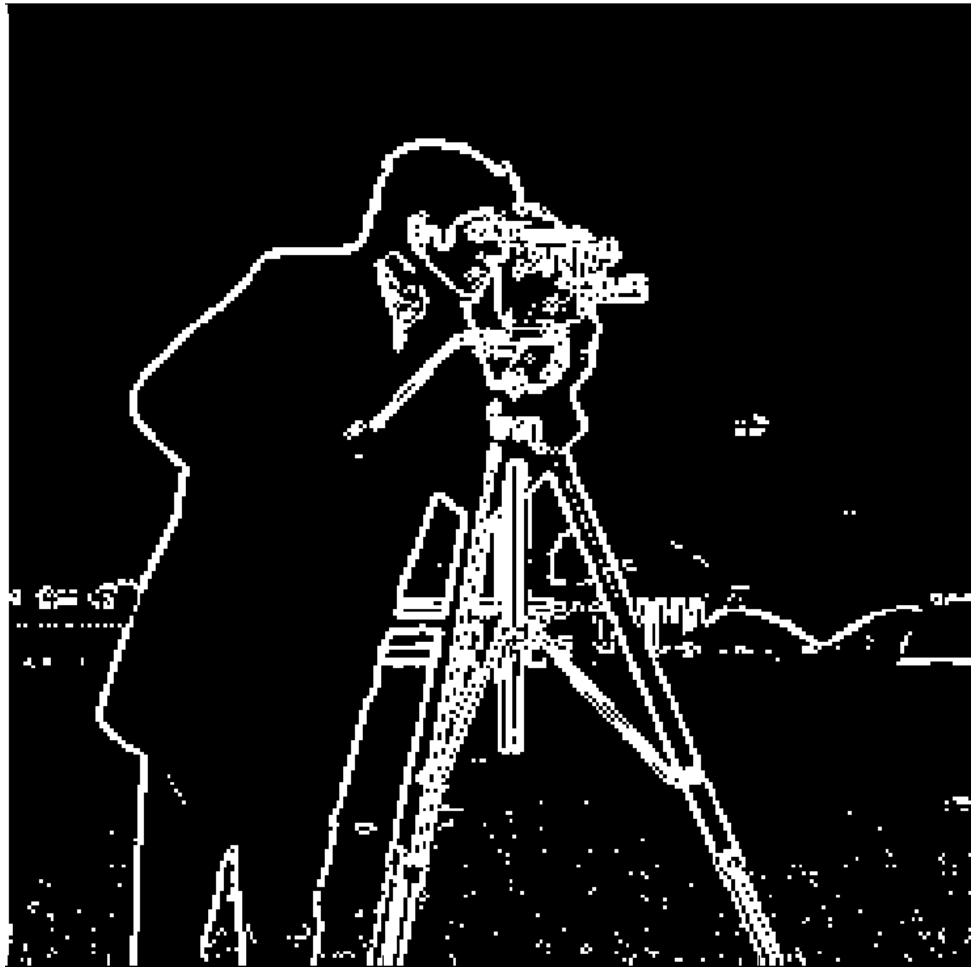

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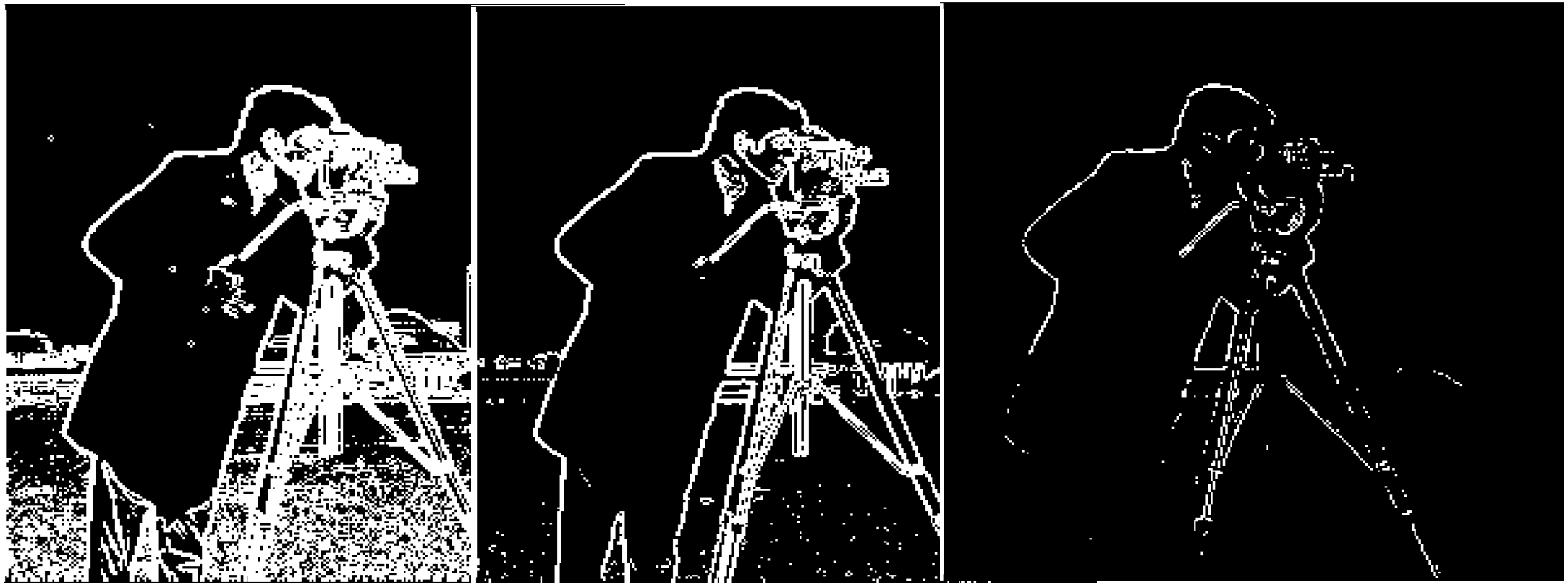
Image Gradients to Edges

Problem 1: How to choose the threshold



>10 >30 >80

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Image Gradients to Edges

- We smooth to reduce the effect noise on image gradients
- Smoothed derivative removes noise but blurs edges

Image with Edge

Edge Location

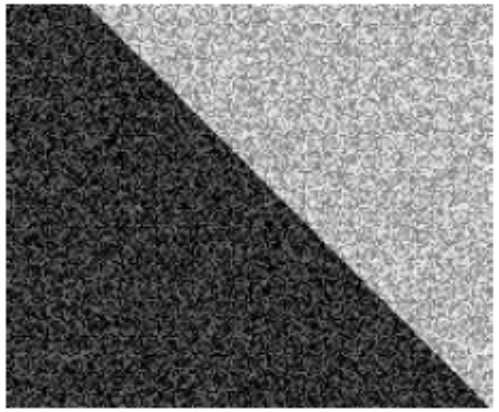
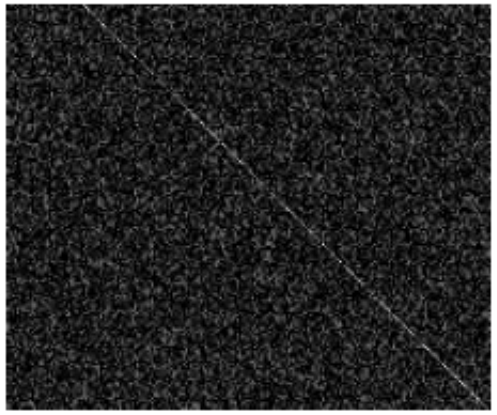


Image + Noise



Derivatives detect edge *and* noise

Smoothed derivative removes noise, but blurs edge

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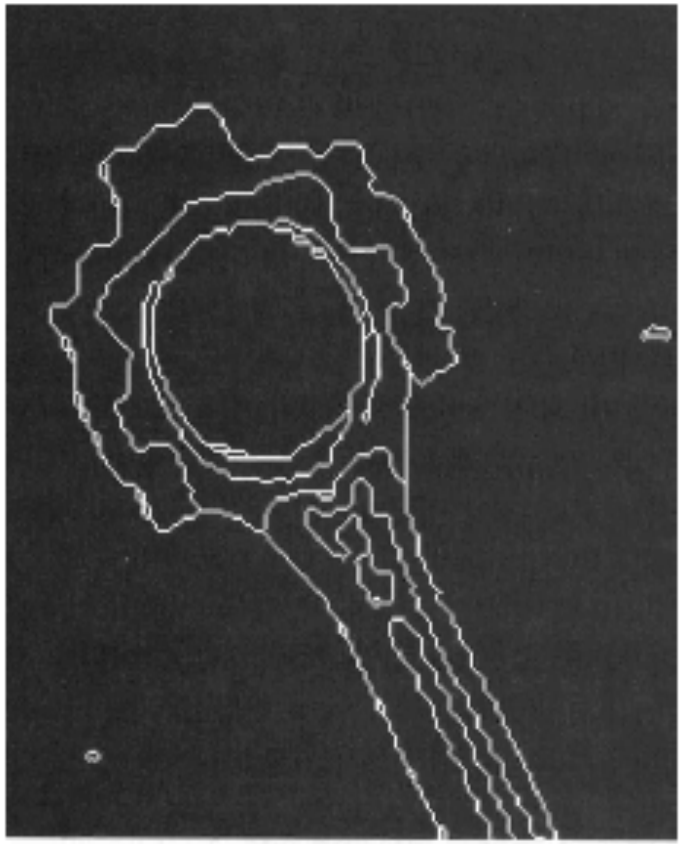
Image Gradients to Edges

Problem 2: How to get thin localized edges

Smoothing+Thresholding gives a binary mask with “thick” edges



We want thin, one-pixel wide, connected contours

Marr Hildreth Edge Detector

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Second Derivative

$$\frac{\partial}{\partial x} f(i,j) \cong f(i,j) - f(i-1,j)$$

$$\frac{\partial}{\partial y} f(i,j) \cong f(i,j) - f(i,j-1)$$

$$[1 \quad -1]$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\frac{\partial^2}{\partial x^2} f(i,j) \cong \frac{\partial}{\partial x} f'(i+1,j) - \frac{\partial}{\partial x} f'(i,j)$$

$$= f(i-1,j) + f(i+1,j) - 2f(i,j)$$

$$[1 \quad -1] * [1 \quad -1]$$

$$= [1 \quad -2 \quad 1]$$

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Laplacian

Equation: $$\nabla^2 f = \frac{\partial}{\partial x^2} f + \frac{\partial}{\partial y^2} f$$

Convolution: $$[1 \quad -2 \quad 1] + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

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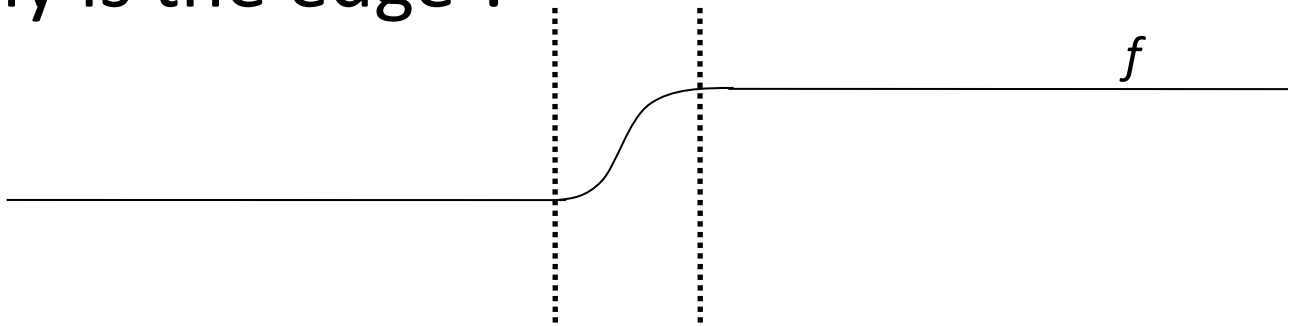
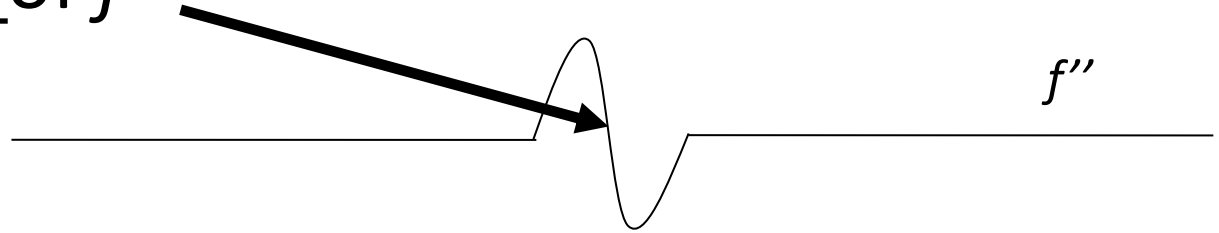
Edge Localization with Zero Crossing

Where exactly is the edge ?



Maximum of *f'*

Zero crossing of *f''*



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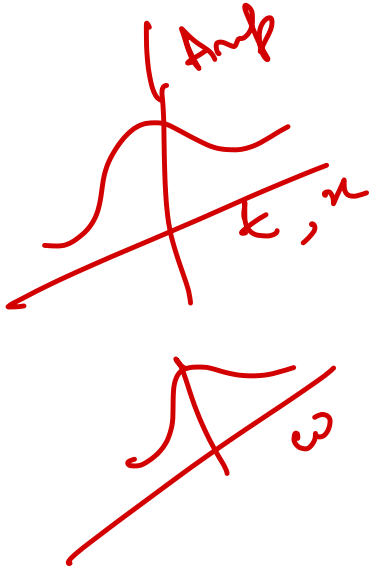
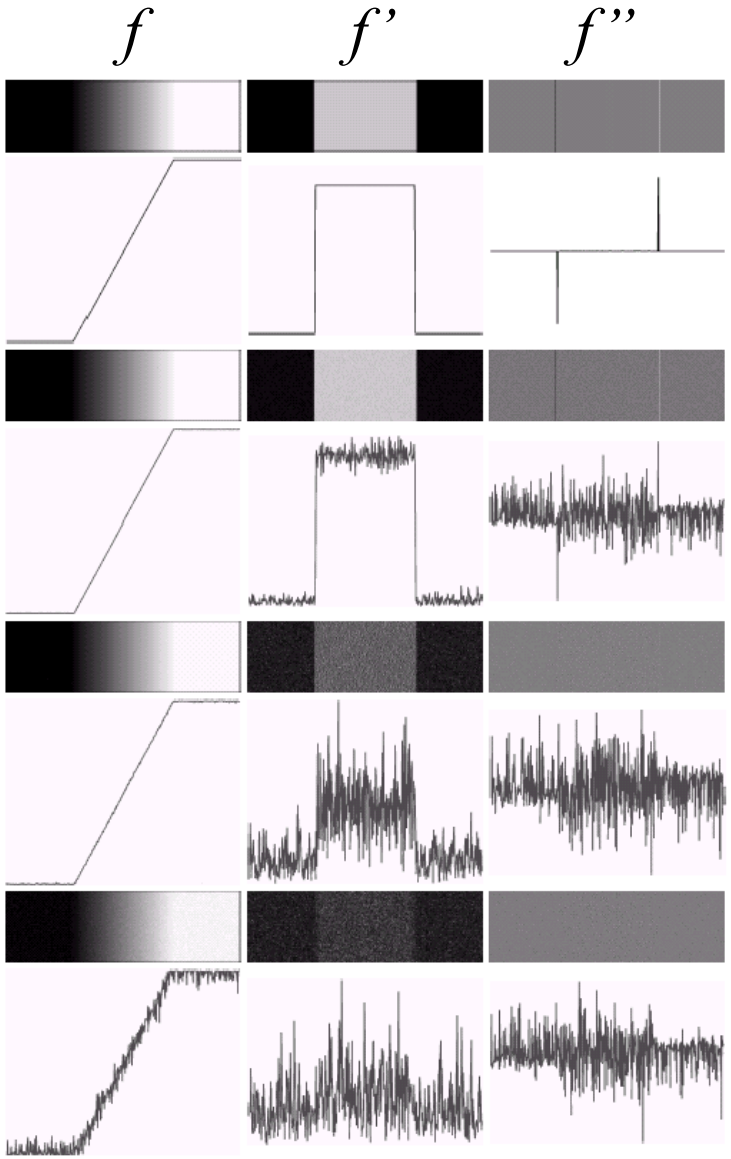
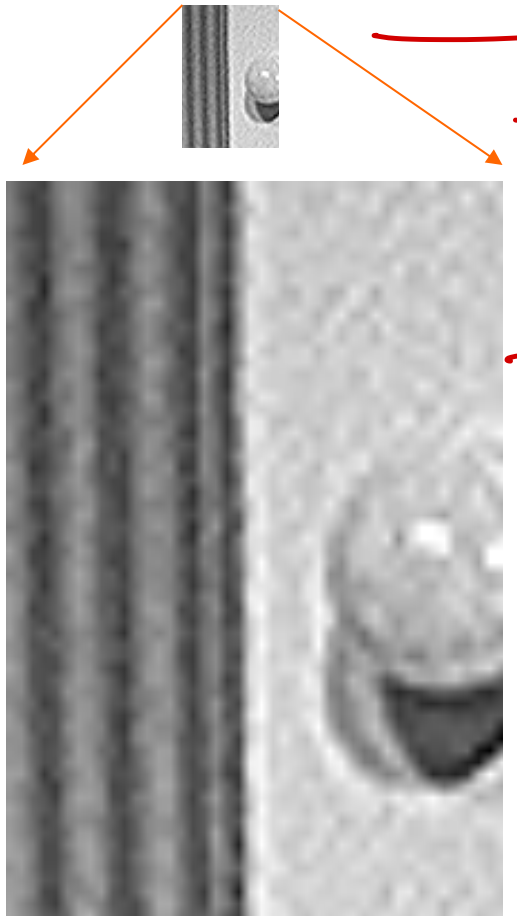
Effect of Noise on Derivatives





Gaussian filter
Low pass filter

Derivatives
High Pass filter



Noise
High frequency

$A \otimes B \equiv \left(A' B'\right)'$

A' = fourier transform

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Recall: Image Derivatives

- Good derivative filters are high-pass (edge) in one direction and low-pass (blur) in the orthogonal direction.

Sobel (X)

$$\frac{1}{8}\begin{bmatrix}1 & 0 & -1\\ 2 & 0 & -2\\ 1 & 0 & -1\end{bmatrix}$$

Sobel (Y)

$$\frac{1}{8}\begin{bmatrix}1 & 2 & 1\\ 0 & 0 & 0\\ -1 & -2 & -1\end{bmatrix}$$

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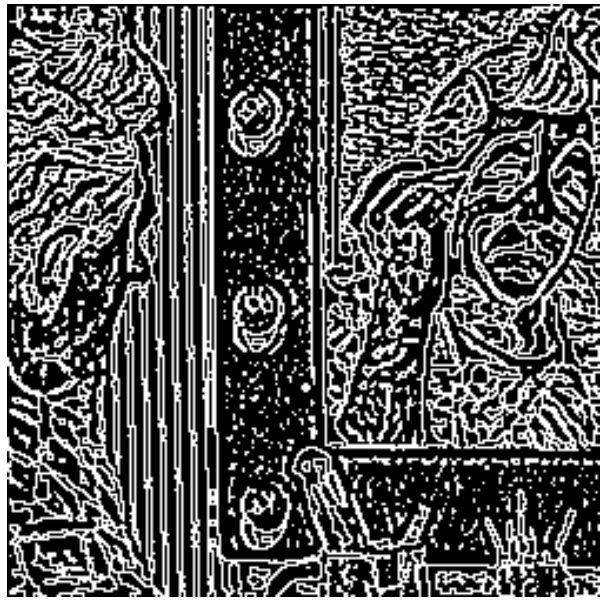
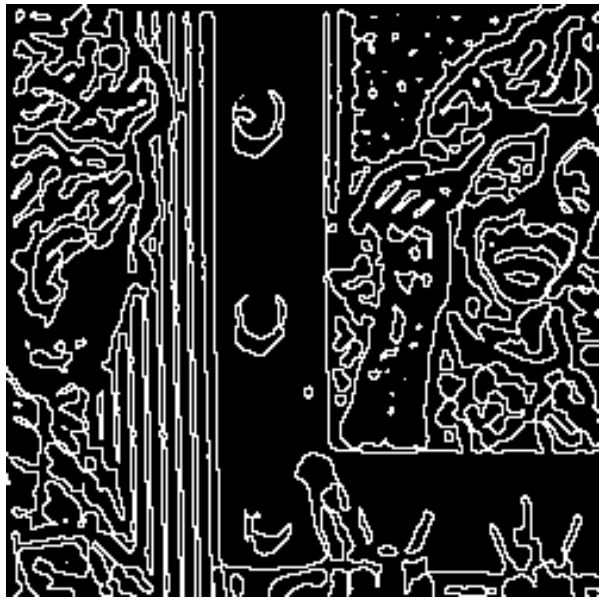
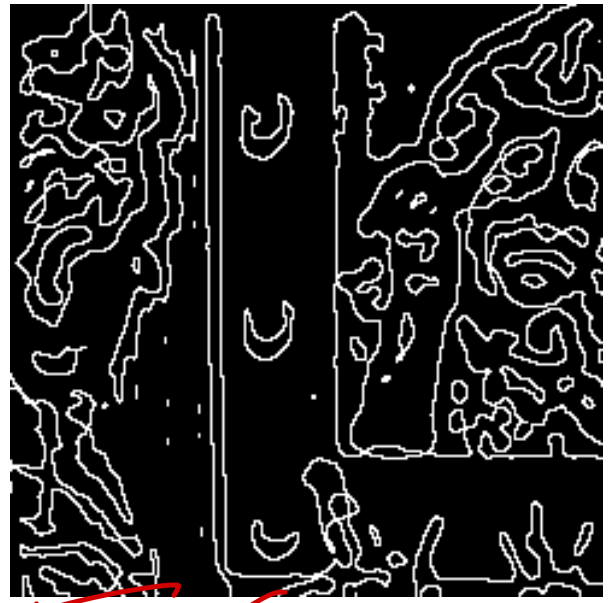
Zero Crossing After Smoothing

Input

Laplacian after increasingly strong smoothing







Canny Edge Detector

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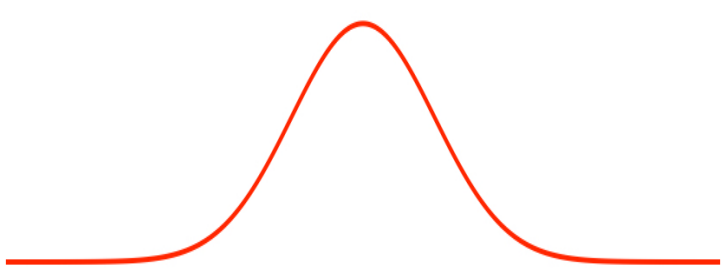
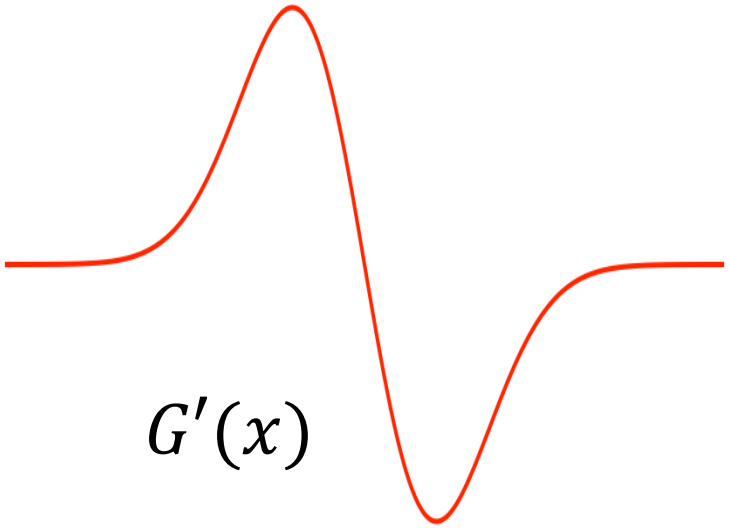
Canny Edge Detector

Step 1: Compute Edge Points

- Compute image derivatives G_x, G_y using derivative of Gaussian
  - Smooth with a Gaussian.
  - Use simple derivative kernels $[1 \quad -1]$.



$G(x)$



$G'(x)$

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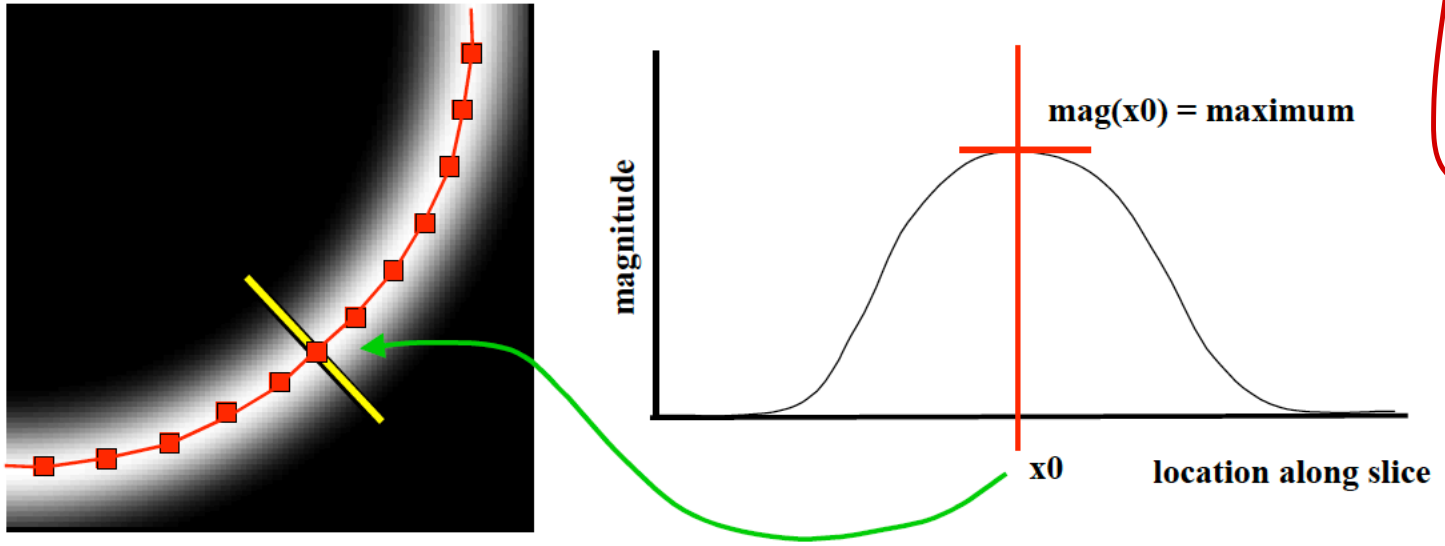
Canny Edge Detector

Step 2: Thin Edges

- We want to mark points along curve where the magnitude is largest.
- We can do this by looking for a maximum along a 1D intensity slice normal to the curve (**non-maximum suppression**).
- These points should form a one-pixel wide curve.



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Canny Edge Detector

Step 2: Thin Edges

- Compute edge direction: $\tan(\alpha) = {}^{G_y}/_{G_x}$.
- Edge point is the local maxima in edge direction. Find using non maximal suppression

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Canny Edge Detector

Step 3: Threshold and find connected edges

- How to pick threshold?
- How to connect edges?

Hysteresis

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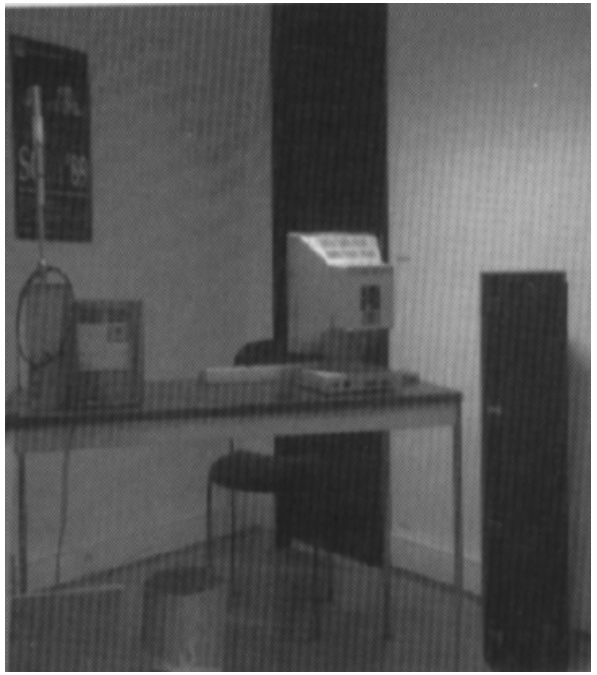
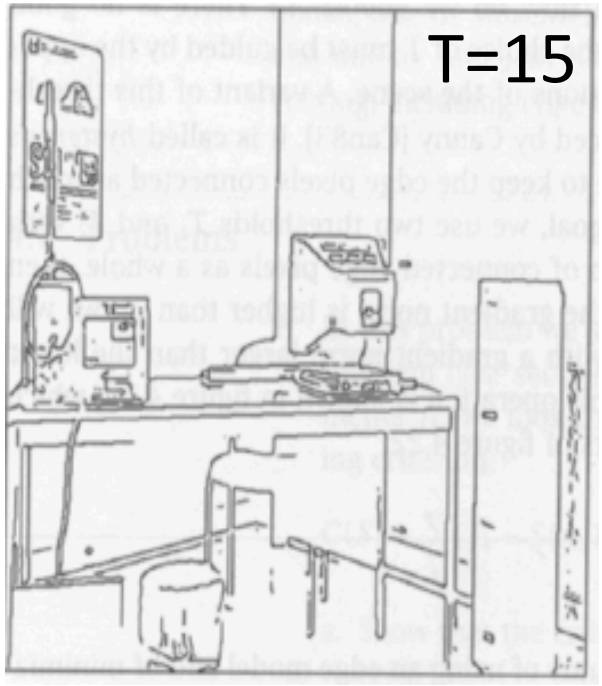
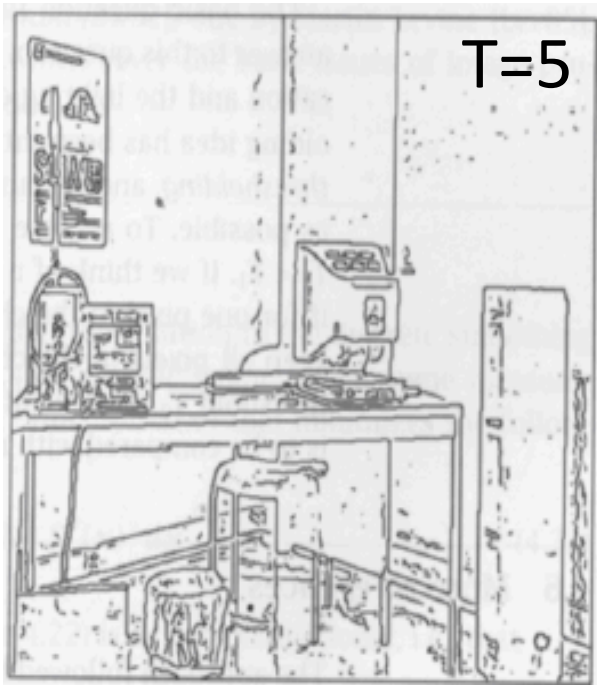
How to Choose the Gradient Threshold

- If the threshold is too high:
  - Very few (none) edges, High MISDETECTIONS, Many gaps
- If the threshold is too low:
  - Too many (all pixels) edges, High FALSE POSITIVES, many extra edges







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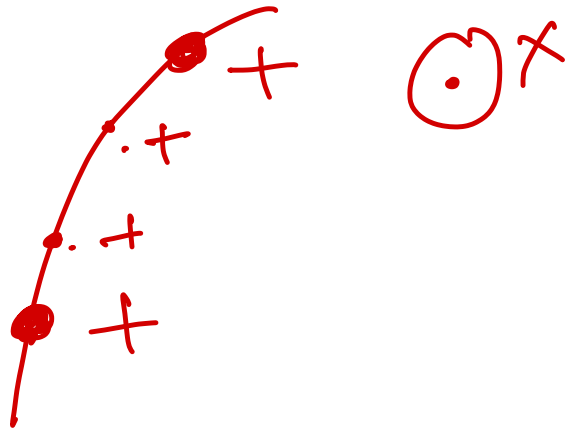
Hysteresis Thresholding and Edge Linking

- Keep both a high threshold H and a low threshold L.
  - Any edges with strength < L are discarded.
  - Any edge with strength > H are kept.



- An edge P with strength between L and H is kept only if there is a path of edges with strength > L connecting P to an edge of strength > H.

Edge Linking

- In practice, hysteresis thresholding is combined with edge linking, by choosing the neighbors only along the edge direction. This links the edge pixels along the edge direction.

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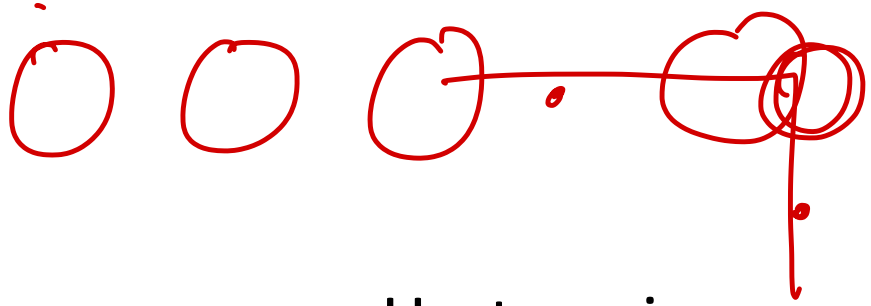
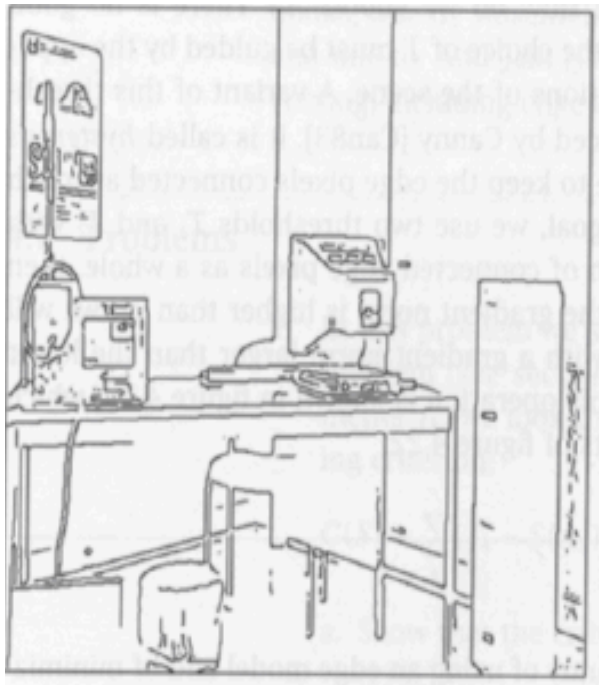
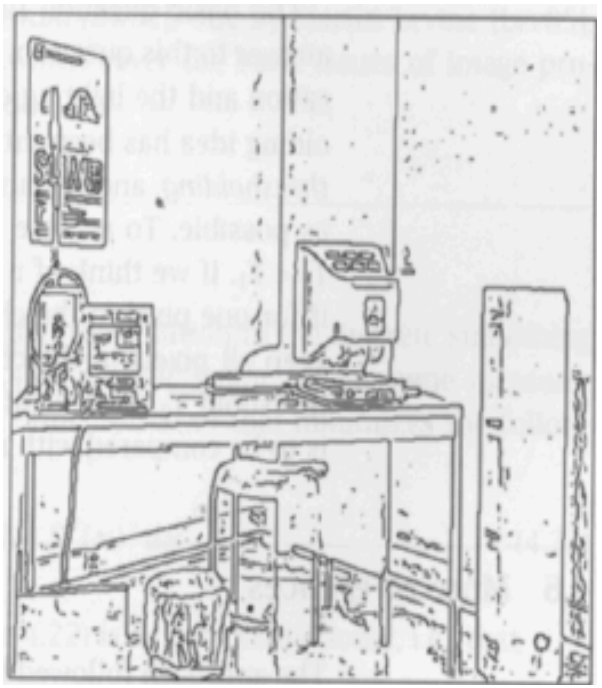
Hysteresis



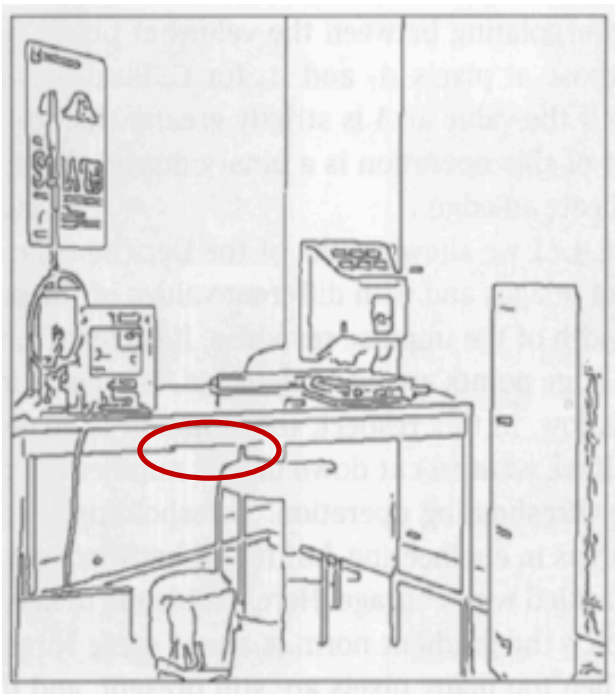
T=15



T=5



Hysteresis
H=15, L=5

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Canny Edge Detector

- Compute image derivatives G_x, G_y using derivative of Gaussian
  - Smooth with a Gaussian.
  - Use simple derivative kernels $[1 \quad -1]$.

- Compute edge direction: $\tan(\alpha) = {}^{G_y}/_{G_x}$. Edge point is the local maxima in edge direction.

- Use only edge points with gradient above threshold. Use hysteresis based thresholding and edge linking with two thresholds. Low threshold accepted only if neighbor to high threshold

Image Sharpening

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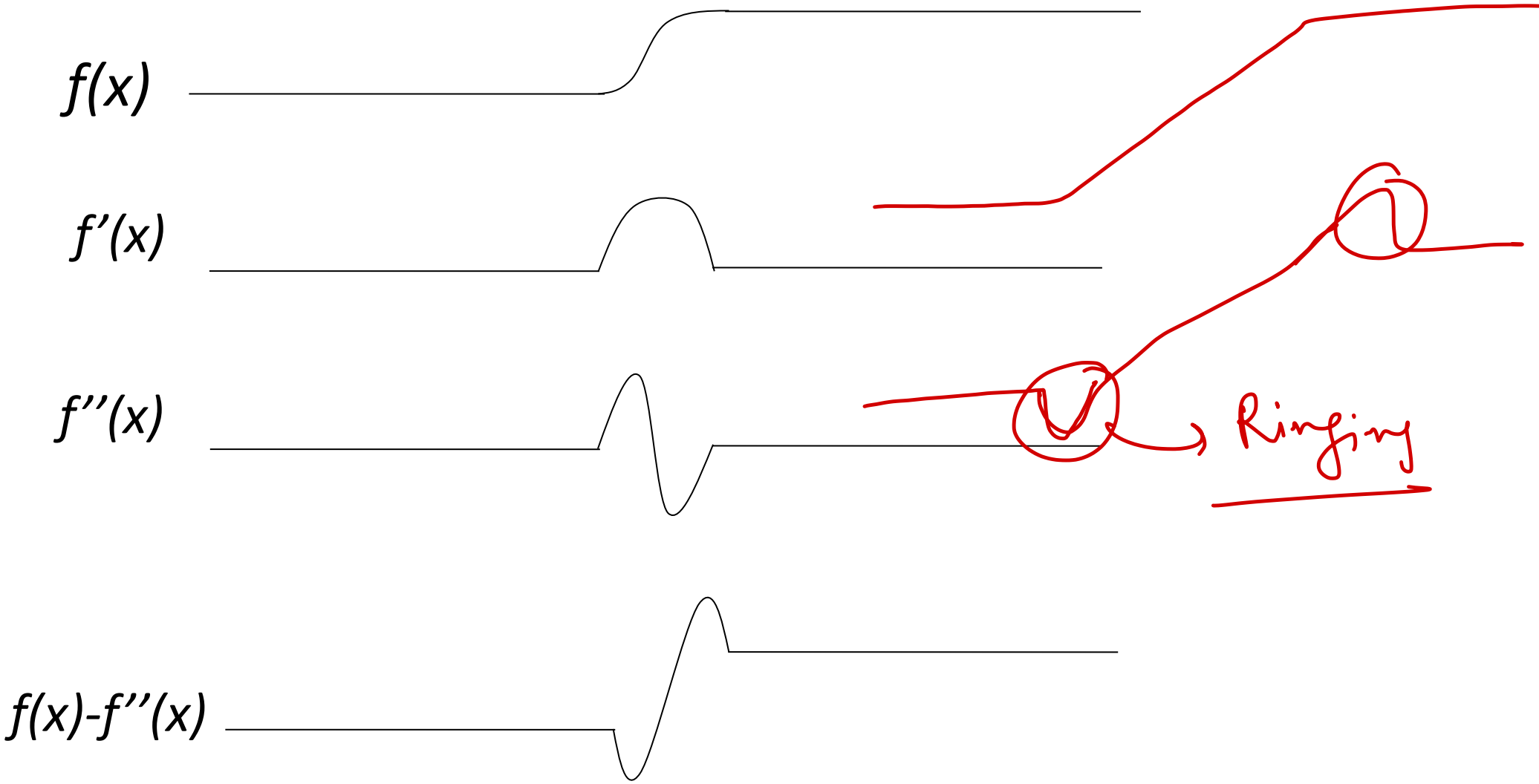
Sharpening by Subtracting the Laplacian



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Sharpening by Subtracting the Laplacian

Equation: $$\nabla^2 f = \frac{\partial}{\partial x^2} f + \frac{\partial}{\partial y^2} f$$

Convolution: $$[1 \quad -2 \quad 1] + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Subtracting the Laplacian from the image:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

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Sharpening Example

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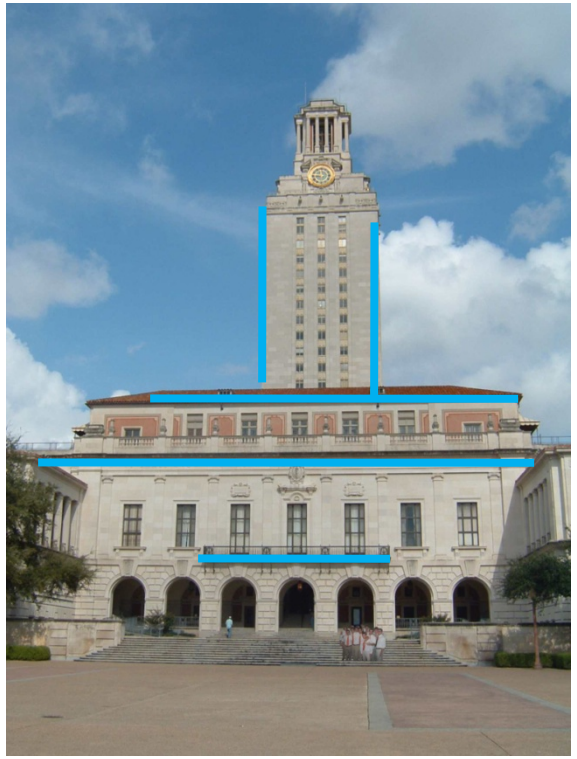
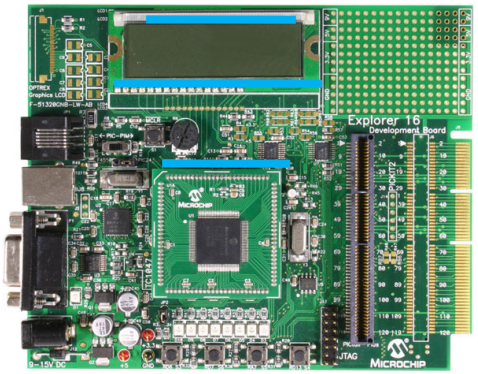
Line Detection

- Why detect lines? Many objects characterized by presence of straight lines





- Why aren’t we done just by running edge detection?

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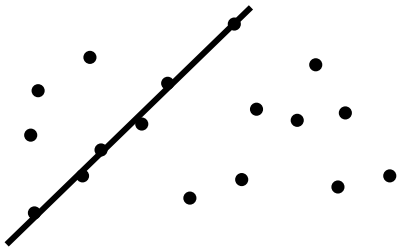
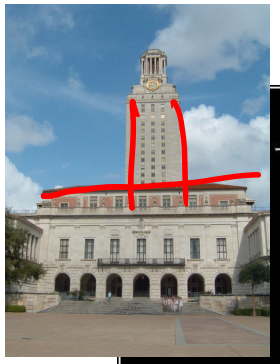
Difficulty of Line Detection

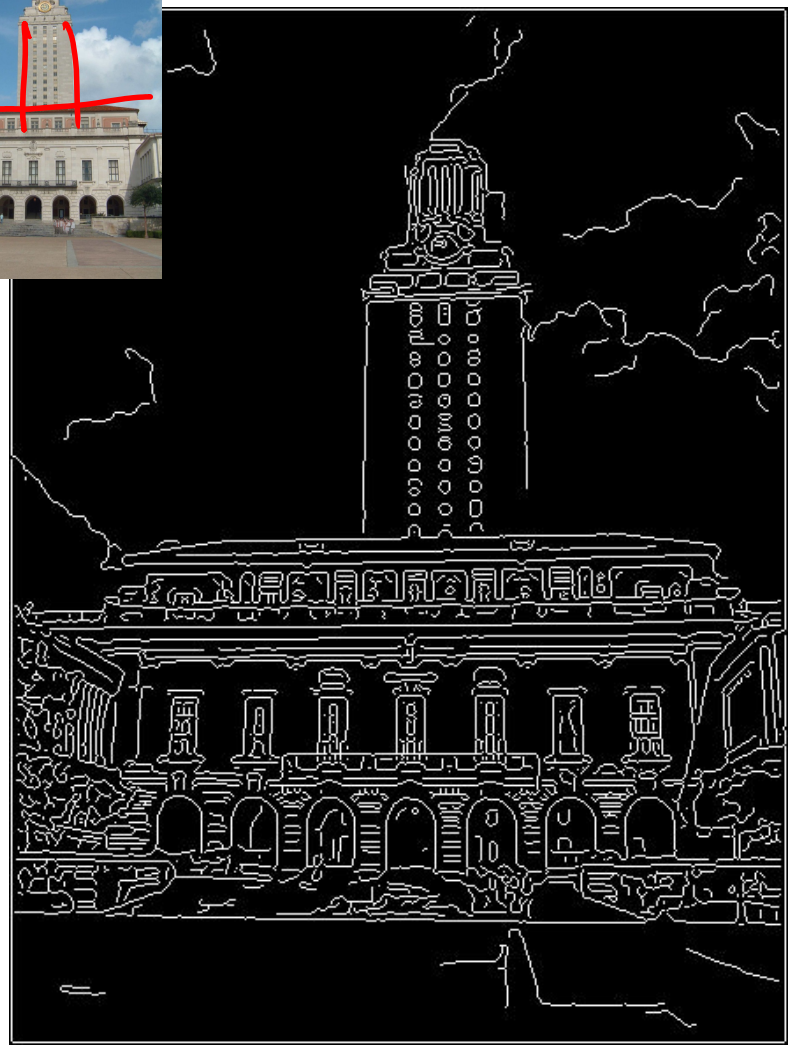


- Extra edge points (clutter), multiple models:
  - Which points go with which line, if any?

- Only some parts of each line detected, and some parts are missing:
  - How to find a line that bridges missing evidence?

- Noise in measured edge points, orientations:
  - How to detect true underlying parameters?





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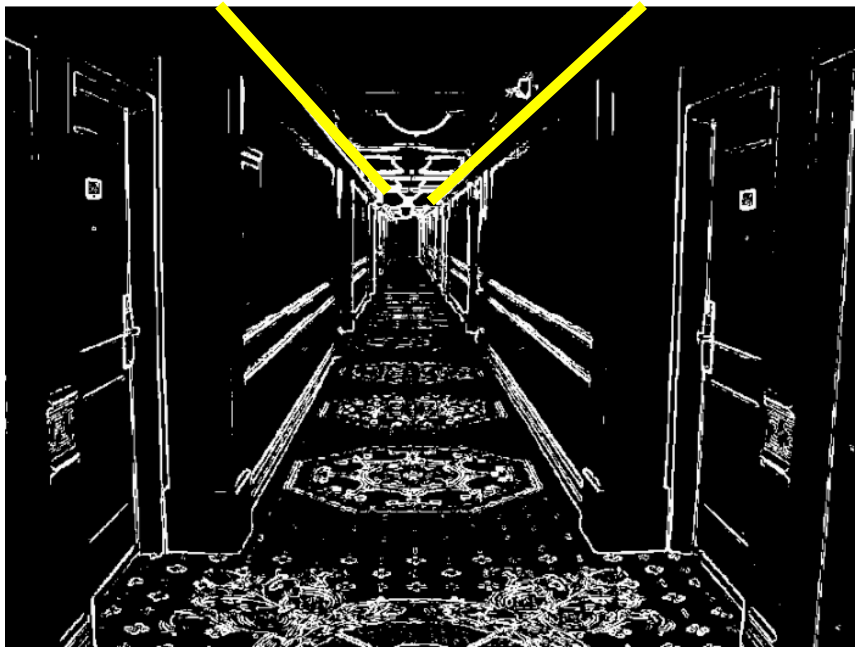
Edges to Lines: Problem Statement

- Assume that we have performed some edge detection, and a thresholding of the edge magnitude image.
- Thus, we have some pixels that may partially describe the boundary of some objects.
- We wish to find sets of pixels that make up straight lines.

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Hough Transform

- Can be used to detect lines, circles etc.
- Introduced in 1962 (Hough 1962) and first used to find lines in images a decade later (Duda 1972).
- The goal is to find the location of lines in images. It can give robust detection under noise and partial occlusion
- **Caveat**: Hough transform can detect structures **ONLY** if their parametric equation is known.


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Detecting Lines using Hough Transform

- Consider a point of known coordinates (x_i, y_i)
  - There are many lines passing through the point (x_i, y_i)

- Straight lines that pass that point have the form $y_i = \boldsymbol{a}x_i + \boldsymbol{b}$
  - Common to them is that they satisfy the above equation for some set of parameters $(\boldsymbol{a}, \boldsymbol{b})$

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Detecting Lines using Hough Transform

- This equation can obviously be rewritten as follows:
  - $\boldsymbol{b} = -\boldsymbol{a}x_i + y_i$
  - We can now consider x and y as parameters
  - $\boldsymbol{a}$ and $\boldsymbol{b}$ as variables.

- This is a line in $(\boldsymbol{a}, \boldsymbol{b})$ space parameterized by x and y.
  - A single point (x_i, y_i) gives a line in (a, b) space.
  - Another point (x_j, y_j) will give rise to another line in (a, b) space.

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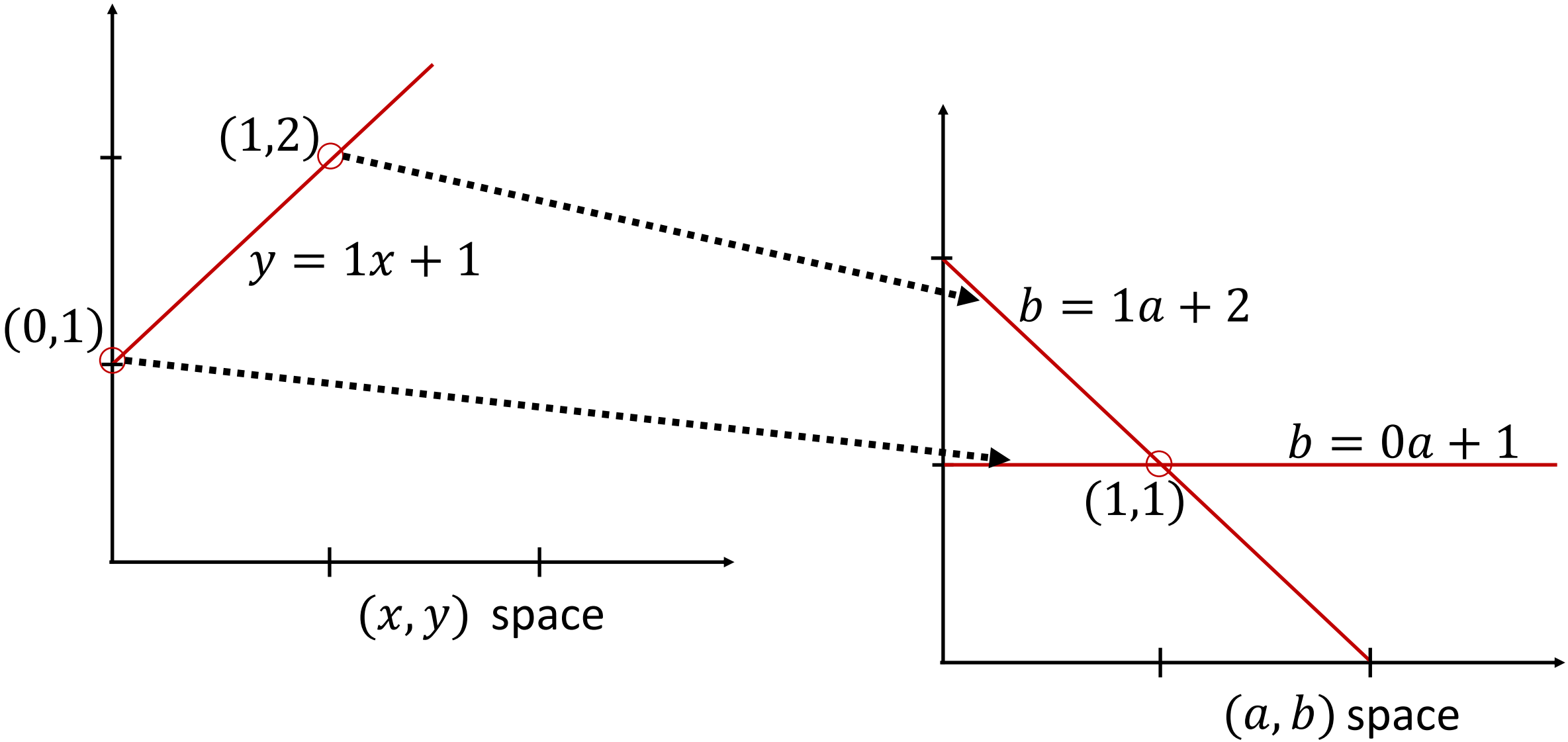
Detecting Lines using Hough Transform

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Detecting Lines using Hough Transform

- Two points (x_i, y_i) and (x_j, y_j) define a line in the (x, y) plane with parameters (a_k, b_k).
- These two points give rise to two different lines in (a, b) space.
- In (a, b) space these lines will intersect at a point (a_k, b_k)
- All points on the line (a_k, b_k) in (x, y) space will parameterize lines that intersect at point (a_k, b_k) in (a, b) space.


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Detecting Lines using Hough Transform

- Quantize the parameter space (a, b) by dividing it into **accumulator** cells
- For each pair of points (x_i, y_i) and (x_j, y_j) detected as an edge:
  - Find the intersection (a_k, b_k) in (a, b) space.
  - Increase the value of cell $[[a_{\min}, a_{\max}], [b_{\min}, b_{\max}]]$ containing (a_k, b_k)
- Cells receiving more than a certain number of counts (also called **votes**) are assumed to correspond to lines in (x, y) space.

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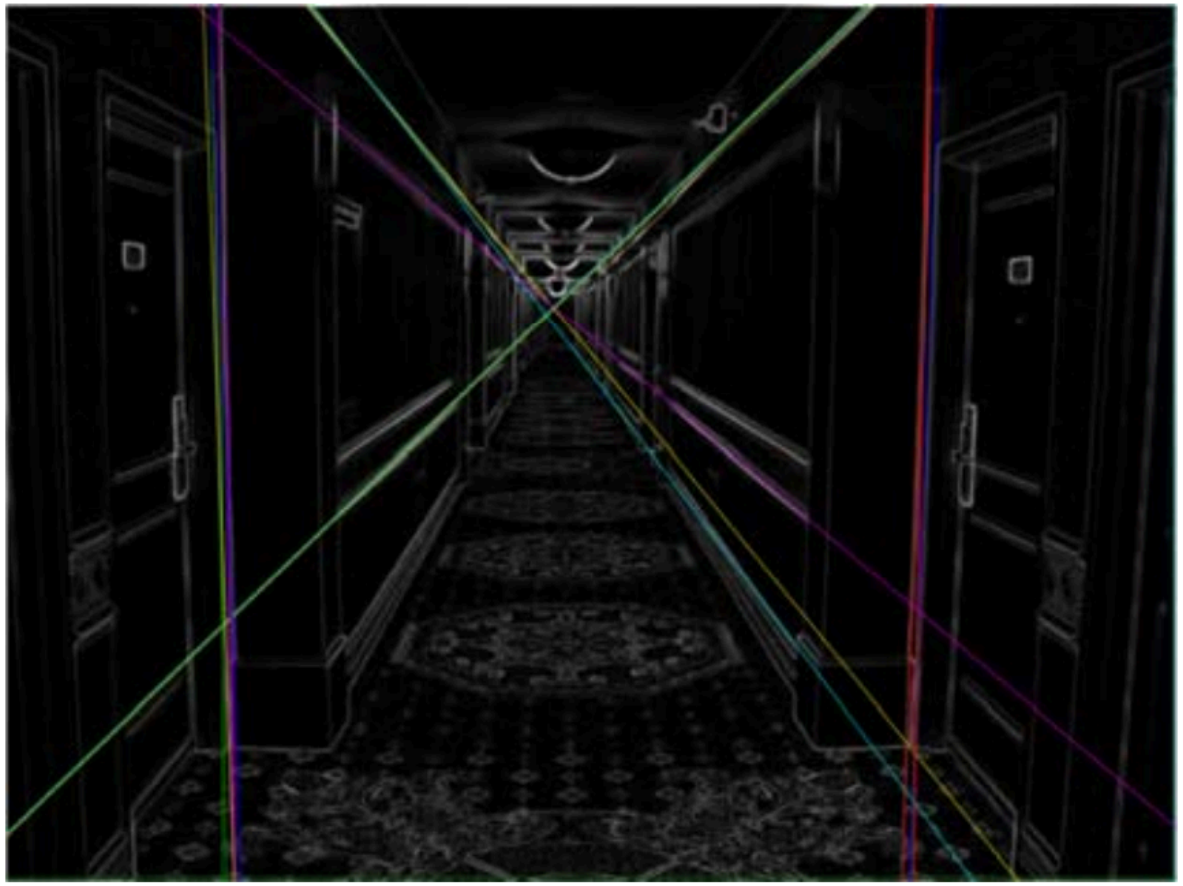
Hough Transform: Example



Top 20 most voted lines

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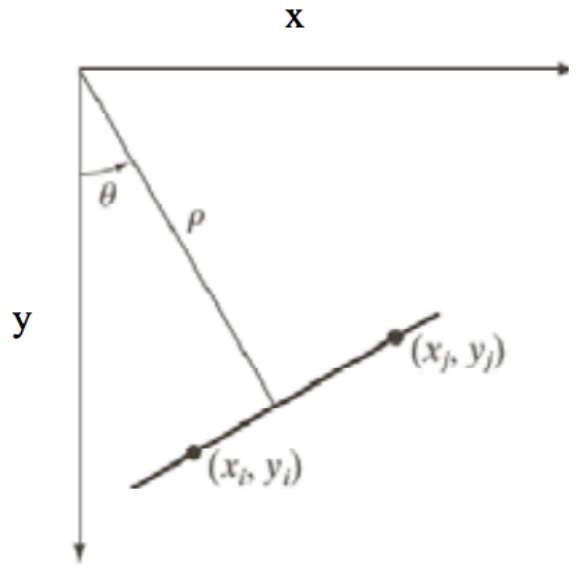
Other Hough Transformations

- We can represent lines as polar coordinates instead of $y = ax + b$

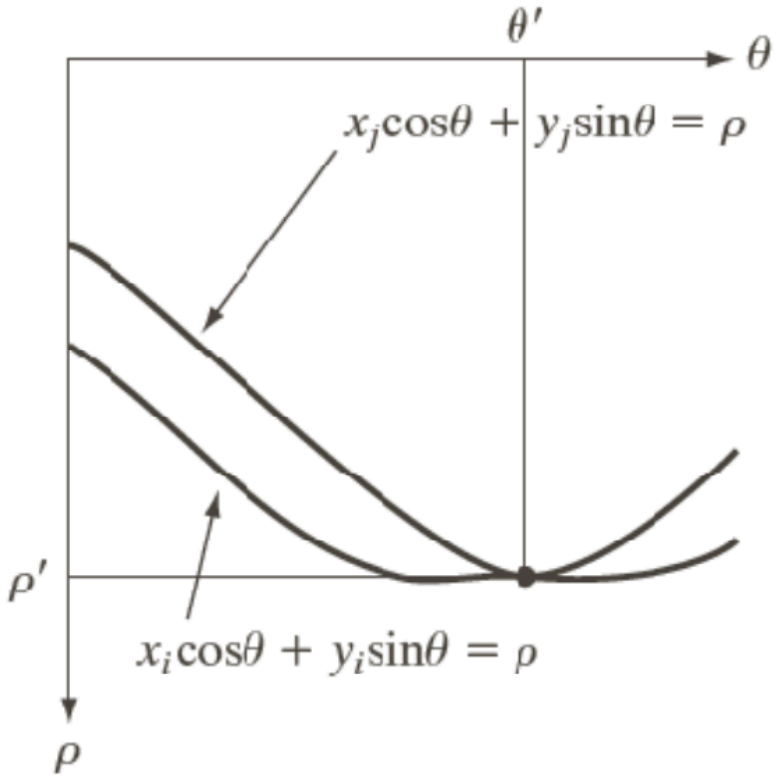
$$x * \cos(\theta) + y * \sin(\theta) = \rho$$



- Lines in (x, y) space are not lines in (ρ, θ) space

- Horizontal line: $\theta = 0$, $\rho =$ intercept with the y-axis.

- Vertical line: $\theta = 90$, $\rho =$ intercept with the x-axis.

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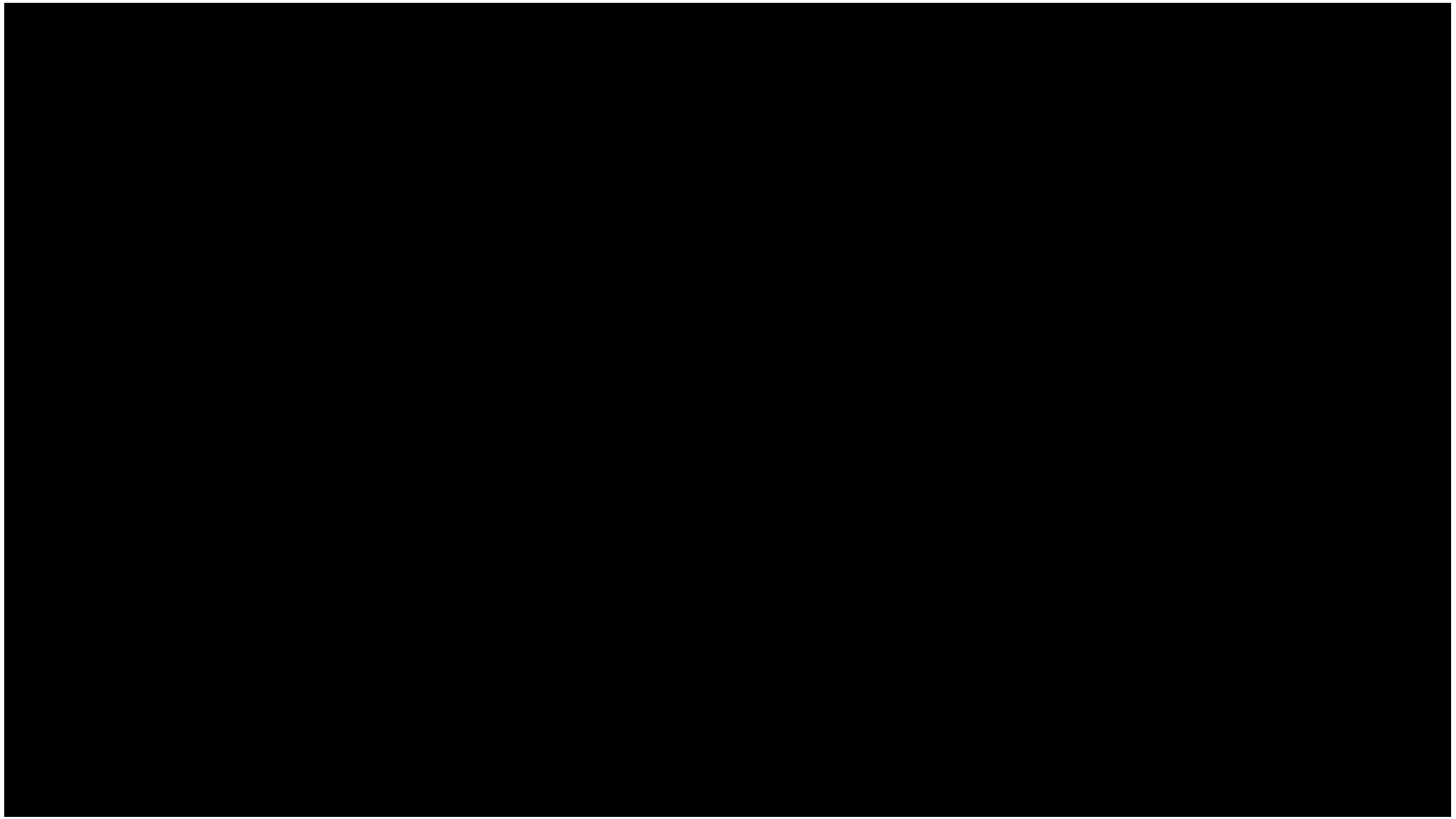
Other Hough Transformations



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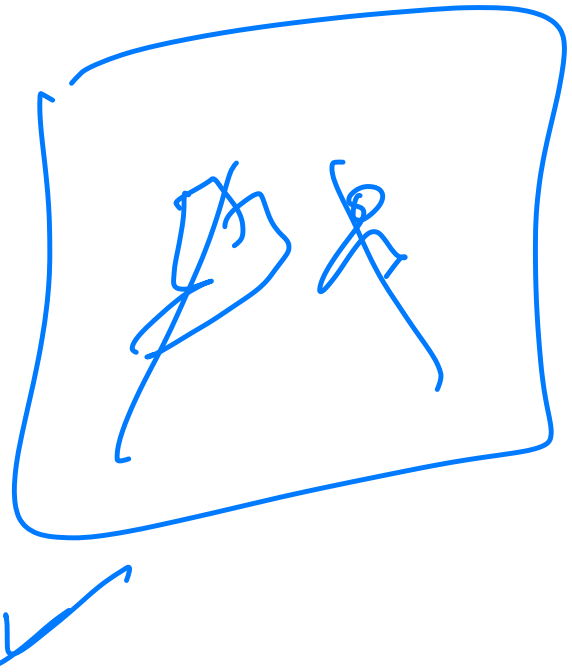
Hough Transform: Advantages



- Conceptually simple.
- Easy implementation
- Handles missing and occluded data very gracefully.
- Can be adapted to many types of forms, not just lines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Hough Transform: Disadvantages

- Computationally complex for objects with many parameters.
- Accuracy of estimation (accumulator cell size) needs to be traded off with complexity of estimation.
- Looks for only one single type of object

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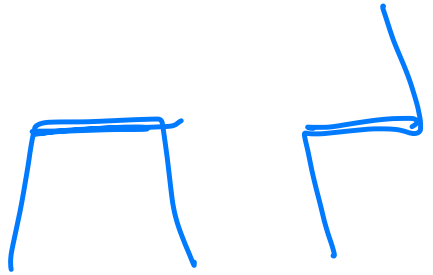
Hough Transform: Disadvantages



- Can be “fooled” by “apparent lines”.

- The length and the position of a line segment cannot be determined.

- Co-linear line segments cannot be separated.