Extensions to the `ebp` function in the **R** package emdi: additional data-driven transformations and empirical best prediction under informative sampling

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Abstract

The empirical best prediction (EBP) approach proposed by Molina and Rao (2010), and generalized in Guadarrama *et al.* (2016) as the census EBP, is implemented in the `ebp` function of the R package **emdi**. A first version of the function allowed for the estimation of point and MSE estimates under non-informative sampling. To ensure the normality of the error terms, the log and Box-Cox transformation were provided. For the latter, the transformation parameter can be estimated from the data as suggested in Rojas-Perilla *et al.* (2020). Their evaluation study further shows that the transformations log-shift and Dual transformation perform well. Furthermore, Guadarrama *et al.* (2018) reveal the benefits of considering the sampling design in the EBP under informative sampling. Therefore, the second version of the `ebp` comprises two new functionalities: (a) additional data-driven transformations for the EBP under non-informative sampling, the log-shift and Dual transformation, (b) the inclusion of sampling weights to consider informative sampling. The functionality of these extensions is demonstrated by examples based on synthetic data included in the package.

Keywords: Official statistics, survey statistics, small area estimation.

1. Introduction

The empirical best prediction (EBP) by Molina and Rao (2010) is one of the most popular unit-level models in the field of small area estimation (SAE) along with e.g., the World Bank method also known as ELL by Elbers *et al.* (2003). Therefore, it is one of the implemented small area estimation methods implemented in the R package **emdi** (Kreutzmann *et al.* 2019). However, the approach provided by function `ebp` relies, among others, on following assumptions: 1. normality of the error terms, and 2. a non-informative sampling design.
Several studies address the first aspect. While Diallo and Rao (2014) relax the normality assumption by allowing for skew-normal error terms, Graf *et al.* (2019) propose the usage of the more flexible distribution generalized beta of the second kind (GB2). Marino *et al.* (2019) investigate a semi-parametric empirical best predictor that estimates the distribution of the random effects from the data. Another, straighforward approach is transforming the dependent variable. Extensive evaluation studies in Rojas-Perilla *et al.* (2020) find that data-driven transformations help to achieve at least symmetric distributions. The transformations

compared are the Box-Cox (Box and Cox 1964), Dual (Yang 2006) and log-shift (Feng *et al.* 2016) transformation. Therefore, the Dual and log-shift transformations are added to the `ebp` function complementing the log and Box-Cox transformation that were already implemented in the first version of the package.
The second point affects the estimation when the sampling design of the survey used in the EBP is informative. Guadarrama *et al.* (2018) propose the incorporation of sampling weights by using the Pseudo empirical best linear unbiased predictor (PEBLUP) (You and Rao 2002) for the estimation of the model parameters. They show that the weighted EBP estimator has a lower bias than the unweighted EBP estimator under informative sampling and comparable results under non-informative sampling. The new version of the `ebp` function comprises both options.
The first version of the `ebp` function is explained in detail in Kreutzmann *et al.* (2019). Therefore, this vignette will focus only on short introductions to the newly implemented methodology and will further show how to use the functionality. Throughout the vignette, it is assumed that a finite population of size N is partitioned into D domains of sizes $N_1, \dots, N_D$. The index $i = 1, \dots, D$ refers to an ith domain and $j = 1, \dots, N_i$ to the jth household/individual. From the population, a random sample is drawn of size n with $n_1, \dots, n_D$ observations in each domain.

2. Additional data-driven transformations

The underlying model in the EBP is the nested error linear regression model that is a unit-level mixed model with a random intercept (Battese *et al.* 1988). One way to achieve the model assumptions in linear and linear mixed models is transforming the dependent variable. The transformation may help to achieve linearity, homoscedasticity and normality. The EBP crucially depends on the latter since the random domain-specific effect and the unit-level error term are drawn from a normal distribution in the Monte Carlo simulation. Therefore, we will focus on normality in this vignette even though the transformations may also improve the model in other aspects (Rojas-Perilla 2018, pp. 9-45).

2.1. Methodology

The most common transformation to achieve normality in the error terms is the log transformation, especially when the distribution of the dependent variable is right-skewed which is often observed for e.g., income. Since the logarithm cannot be applied for negative values, a deterministic shift can be added as follows:

$$y^*_{ij} = log(y_{ij} + s),$$

where y_{ij} is the variable of interest of domain i and unit j and s is a deterministic shift chosen such that $y_{ij} + s > 0$.
While the log transformation is especially beneficial for practitioners that are interested in the interpretation of parameters, it does not need to be the best option in a predictive model due to the missing ability to adapt to the data. In contrast, transformations with a transformation parameter λ can be fitted to the data. Rojas-Perilla *et al.* (2020) compare the log-shift (Feng *et al.* 2016), Box-Cox (Box and Cox 1964), and Dual transformation (Yang 2006) in the EBP context.

The log-shift transformation is the simplest way to make the log transformation more flexible and adpatable to the data. Instead of a deterministic shift, the data is shifted by an optimal shift before the logarithm is applied:

$$y_{ij}^{*} = log(y_{ij} + \lambda),$$

where y_{ij} is the variable of interest of domain i and unit j and $\lambda >= s$ is an estimated shift. When $\lambda = s$, this transformation equals the implemented log transformation with deterministic shift.
The Box-Cox transformation is a famous transformation in the family of power transformations. Since the Box-Cox transformation is not suitable for negative values, a deterministic shift can be added as for the log transformation. Its shifted version is defined by:

$$y_{ij}^{*}(\lambda) = \begin{cases} \frac{(y_{ij}+s)^{\lambda}-1}{\lambda} & \text{if } \lambda \neq 0; \\ \log(y_{ij} + s) & \text{if } \lambda = 0, \end{cases}$$

where y_{ij} is the variable of interest of domain i and unit j and s is a deterministic shift chosen such that $y_{ij} + s > 0$. One characteristic of the Box-Cox transformation is that the cases of no transformation when $\lambda = 1$ (the data is only shifted) and applying the log transformation with a deterministic shift when $\lambda = 0$ are covered. A known drawback is the truncation of y_{ij}^{*}. The transformed variable y_{ij}^{*} is bounded from below by $\frac{1}{\lambda}$ when $\lambda > 0$, and bounded from above by $\frac{-1}{\lambda}$ when $\lambda < 0$.
The Dual transformation overcomes this issue. It is originally only defined for strictly positive values, but Rojas-Perilla *et al.* (2020) include a deterministic shift as follows:

$$y_{ij}^{*}(\lambda) = \begin{cases} ((y_{ij} + s)^{\lambda} - (y_{ij} + s)^{-\lambda})/2\lambda & \lambda > 0; \\ \log(y_{ij} + s) & \lambda = 0, \end{cases}$$

where y_{ij} is the variable of interest of domain i and unit j and s is a deterministic shift chosen such that $y_{ij} + s > 0$. The transformation parameter λ cannot be negative.
For the estimation of the transformation parameter in linear mixed regression models, Gurka *et al.* (2006) propose maximum likelihood and residual maximum likelihood (REML) methods. Rojas-Perilla *et al.* (2020) investigate the maximum likelihood based methods as well as alternative approaches. So far, the package **emdi** only provides the REML approach for the model fitting and the estimation of the transformation parameter. For the explanation of how the transformations are included in the EBP, we refer to Kreutzmann *et al.* (2019, Section 2.2) and Rojas-Perilla *et al.* (2020).

2.2. Functionality

In the following, we will show how the new transformations can be used in the `ebp` function of package **emdi**. The argument `transformation` is determining the chosen transformation. In the new version of function `ebp` following options will be available:

- `no`: No transformation
- `log`: Log transformation with a deterministic shift
- `box.cox`: Box-Cox transformation with a deterministic shift

Transformation	Default interval
`box.cox`	c(-1, 2)
`dual`	c(0, 2)
`log.shift`	c(a, b) with $a = max(0, (min(y)) + 1)$, $b = \frac{(max(y)-min(y))}{2}$

Table 1: Default values for the estimation of the transformation parameter λ.

- `dual`: Dual transformation with a deterministic shift
- `log.shift`: Log transformation with an optimized shift

The Box-Cox transformation is chosen to be the default transformation since it covers the options of no transformation and the logarithm. For the REML estimation of the transformation parameter λ, an interval needs to be specified. To simplify the usage, a default interval is defined for all data-driven transformations, Box-Cox, Dual and log-shift, if no specific values are chosen. Table 1 shows the default intervals implemented for function `ebp`. In the following, the different data-driven transformations are applied with the data of Austrian districts provided in the package (Kreutzmann *et al.* 2019).

```
R> library("emdi")
R> # Load sample data set
R> data("eusilcA_smp")
R> data('eusilcA_pop')
```

Box-Cox transformation

The Box-Cox transformation remains the default transformation. The following code produces the same results that are shown in Kreutzmann *et al.* (2019). The estimated transformation parameter equals to 0.6046901. The summary also shows residual diagnostics that suggest a normally distributed random effect, while the Shapiro-Wilk test for the unit-level error rejects normality. A look at the kurtosis and the QQ-plots reveals that the problem lies in the tails. Outlying observations could be one driving factor for these observations.

```
R> ebp_bc <- ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl +
+                  unempl_ben + age_ben + surv_ben + sick_ben + dis_ben +
+                  rent + fam_allow + house_allow + cap_inv + tax_adj,
+                pop_data = eusilcA_pop, pop_domains = "district",
+                smp_data = eusilcA_smp, smp_domains = "district",
+                threshold = 10885.33)
R> summary(ebp_bc)

Empirical Best Prediction

Call:
 ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl + unempl_ben +
    age_ben + surv_ben + sick_ben + dis_ben + rent + fam_allow +
    house_allow + cap_inv + tax_adj, pop_data = eusilcA_pop,
```

```
    pop_domains = "district", smp_data = eusilcA_smp, smp_domains = "district",
    threshold = 10885.33)

Out-of-sample domains:  24
In-sample domains:  70

Sample sizes:
Units in sample:  1945
Units in population:  25000
                   Min. 1st Qu. Median      Mean 3rd Qu. Max.
Sample_domains       14    17.0   22.5  27.78571   29.00  200
Population_domains    5   126.5  181.5 265.95745  265.75 5857

Explanatory measures:
 Marginal_R2 Conditional_R2
   0.6325942       0.709266

Residual diagnostics:
               Skewness Kurtosis Shapiro_W    Shapiro_p
Error         0.7523871 9.646993 0.9619824 3.492626e-22
Random_effect 0.4655324 2.837176 0.9760574 1.995328e-01

ICC:  0.2086841

Transformation:
 Transformation Method Optimal_lambda Shift_parameter
        box.cox   reml      0.6046901               0

R> qqnorm(ebp_bc)
```

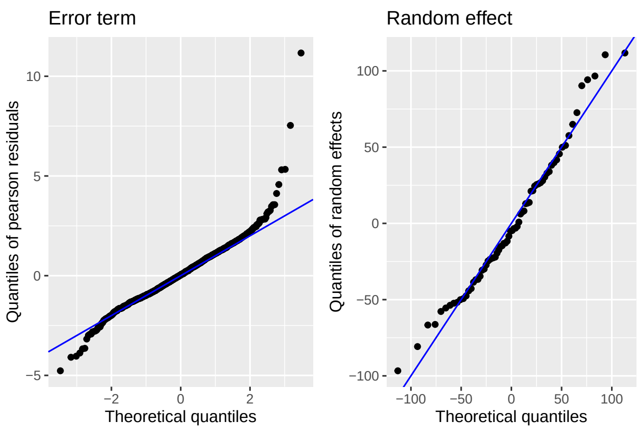


Figure 1: QQ-plots of the error term and random effect using the Box-Cox transformation.

Dual transformation

The Dual transformation is similar to the Box-Cox transformation. Therefore, the estimated transformation parameter differs only slightly with a value of 0.6047161 and also the diagnostics are comparable, with the conclusion of a normally distributed random effect and a distribution of the unit-level error that shows fat tails.

```
R> ebp_dual <- ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl +
+                    unempl_ben + age_ben + surv_ben + sick_ben + dis_ben +
+                    rent + fam_allow + house_allow + cap_inv + tax_adj,
+                  pop_data = eusilcA_pop, pop_domains = "district",
+                  smp_data = eusilcA_smp, smp_domains = "district",
+                  threshold = 10885.33, transformation = 'dual')
R> summary(ebp_dual)

Empirical Best Prediction

Call:
 ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl + unempl_ben +
    age_ben + surv_ben + sick_ben + dis_ben + rent + fam_allow +
    house_allow + cap_inv + tax_adj, pop_data = eusilcA_pop,
    pop_domains = "district", smp_data = eusilcA_smp, smp_domains = "district",
    threshold = 10885.33, transformation = "dual")

Out-of-sample domains:  24
In-sample domains:  70

Sample sizes:
Units in sample:  1945
Units in population:  25000
                   Min. 1st Qu. Median      Mean 3rd Qu. Max.
Sample_domains       14    17.0   22.5  27.78571   29.00  200
Population_domains    5   126.5  181.5 265.95745  265.75 5857

Explanatory measures:
 Marginal_R2 Conditional_R2
   0.6325965      0.7092674

Residual diagnostics:
              Skewness Kurtosis Shapiro_W    Shapiro_p
Error         0.752435 9.647438 0.9619800 3.487023e-22
Random_effect 0.465552 2.837214 0.9760562 1.995026e-01

ICC:  0.2086831

Transformation:
 Transformation Method Optimal_lambda Shift_parameter
           dual   reml      0.6047161               0
```

```
R> qqnorm(ebp_dual)
```

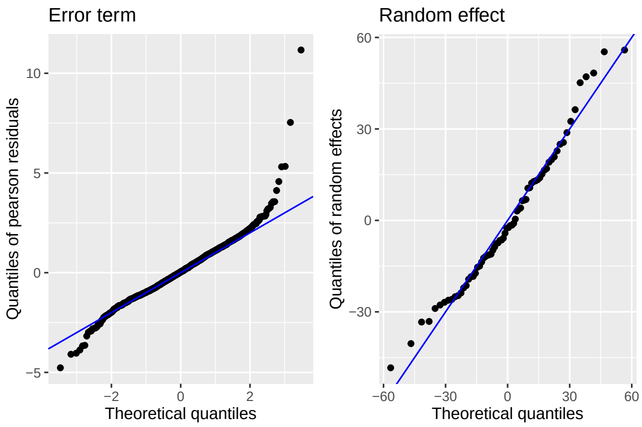


Figure 2: QQ-plots of the error term and random effect using the Dual transformation.

Log-shift transformation

In contrast to the Box-Cox and Dual transformation, the transformation parameter of the log-shift transformation is on the scale of the dependent variable. In this example, λ equals 27907.57. The diagnostics in the summary show that the log-shift transformation slightly improves the kurtosis. From the QQ-plots, it can be concluded that the normality of the unit-level error is most likely rejected because of two outliers.

```
R> ebp_ls <- ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl +
+                    unempl_ben + age_ben + surv_ben + sick_ben + dis_ben +
+                    rent + fam_allow + house_allow + cap_inv + tax_adj,
+                  pop_data = eusilcA_pop, pop_domains = "district",
+                  smp_data = eusilcA_smp, smp_domains = "district",
+                  threshold = 10885.33, transformation = 'log.shift')
R> summary(ebp_ls)

Empirical Best Prediction

Call:
 ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl + unempl_ben +
    age_ben + surv_ben + sick_ben + dis_ben + rent + fam_allow +
    house_allow + cap_inv + tax_adj, pop_data = eusilcA_pop,
    pop_domains = "district", smp_data = eusilcA_smp, smp_domains = "district",
    threshold = 10885.33, transformation = "log.shift")

Out-of-sample domains:  24
In-sample domains:  70

Sample sizes:
```

```
Units in sample:  1945
Units in population:  25000
                   Min. 1st Qu. Median      Mean 3rd Qu. Max.
Sample_domains       14    17.0   22.5  27.78571   29.00  200
Population_domains    5   126.5  181.5 265.95745  265.75 5857

Explanatory measures:
 Marginal_R2 Conditional_R2
   0.6233538      0.7054886

Residual diagnostics:
              Skewness Kurtosis Shapiro_W    Shapiro_p
Error        0.6222910 7.607189 0.9706711 1.705890e-19
Random_effect 0.4788713 2.726898 0.9737695 1.487627e-01

ICC:  0.2180689

Transformation:
 Transformation Method Optimal_lambda
      log.shift   reml       27907.57

R> qqnorm(ebp_ls)
```

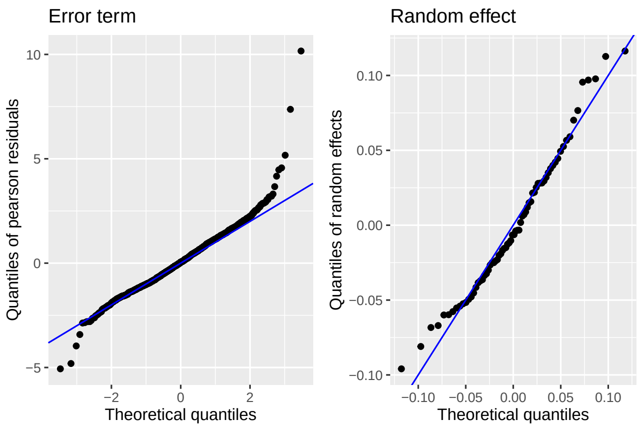


Figure 3: QQ-plots of the error term and random effect using the log-shift transformation.

Summary

While the transformations help to achieve normality for the random effect but not for the unit-level error term in this specific example, they all lead to almost symmetric distributions and show better results compared to an application without transformation.

3. Empirical Best Prediction under informative sampling

3.1. Methodology

Point Estimation

The EBP proposed by Molina and Rao (2010) assumes non-informative sampling, which means that the inclusion probability of the sample is not linked to the outcome variable of interest. The sampling design is said to be non-informative when

$$P(smp \mid y) = P(smp), \forall y \in \mathbb{R}^N, \forall smp,$$

where $P(smp \mid y)$ is the probability of sample smp.
In applications, where the sampling design is informative, the sampling weights should be included in the estimation of the model parameters in order to avoid biased results. Guadarrama *et al.* (2018) transfer the conditioning idea of the unweighted EBP to the EBP under informative sampling. Instead of conditioning on the unweighted sample mean $\bar{y}_{is}$ during prediction, they condition on the weighted sample mean $\bar{y}_{ij} = w_{i\cdot}^{-1} \sum_{j \in smp_i} w_{ij} y_{ij}$. w_{ij} is the sampling weight for the jth unit in domain i and $w_{i\cdot} = \sum_{j \in smp_i} w_{ij}$ is the sum of sampling weights within domain i.
The pseudo best (PB) estimator for $I_{ij} = i(y_{ij})$ is therefore $\tilde{I}_{ij}^{PB}(\theta) = E[i(y_{ij}) \mid \bar{y}_{iw}; \theta]$ with model parameters θ and the estimator of the additive domain parameter I_i is defined as

$$\tilde{I}_i^{PB}(\theta) = \frac{1}{N_i} \left[\sum_{j \in smp_i} i(y_{ij}) + \sum_{j \in nsmp_i} \tilde{I}_{ij}^{PB}(\theta) \right].$$

The abbreviation nsmp stands for the non-sampled observations in the census. In **emdi**, the PB is implemented as the census PB (CPB) and given by

$$\tilde{I}_i^{CPB}(\theta) = \frac{1}{N_i} \sum_{j \in i} \tilde{I}_{ij}^{PB}(\theta).$$

This means that the indicator is predicted for all observations in the census and not just for the out-of-sample elements. The reason behind the implementation is that sample observations can very rarely be identified in the census. This procedure is also mentioned in the methodology part of Guadarrama *et al.* (2018).

Analogue to the EBP, the PB estimator depends on the true values of the model parameters $\theta = (\beta, \sigma_u^2, \sigma_e^2)$, which are not known and therefore need to be estimated. θ in the pseudo EB predictor is replaced by a consistent estimator. The resulting predictor is called pseudo empirical best predictor (PEBP). The authors mention two ways of estimating the model parameters. One feasible approach of Pfeffermann and Sverchkov (2007) is based on the sample likelihood, The likelihood is used to find maximum likelihood (ML) estimates of the regression coefficients β and of the variances σ_u^2 and σ_e^2. In the second approach, β is estimated using the weighted method of moments of You and Rao (2002). The needed variance parameters σ_u^2 and σ_e^2 are estimated using ML (or REML). In **emdi**, the second approach is implemented and the variance parameters are estimated by REML.

For out-of-sample observations, the following relationships hold under the nested error population model:

$$
\begin{aligned}
y_{ij} \,|\bar{y}_{iw} &\overset{ind.}{\sim} \mathcal{N}(\mu^{w}_{ij|smp}, \sigma^{2w}_{ij|smp}),\\
\mu^{w}_{ij|smp} &= x_{ij}^{\top}\beta + \gamma_{iw}(\bar{y}_{iw} - \bar{x}_{iw}^{\top}\beta), \quad \sigma^{2w}_{ij|smp} = \sigma^2_u(1-\gamma_{iw}) + \sigma^2_e.
\end{aligned}
$$

The mean $\mu^{w}_{ij|smp}$ is obtained by replacing the unweighted best predictor of the domain effect u_i by its weighted version, given by $\tilde{u}_{iw} = \gamma_{iw}(\bar{y}_{iw} - \bar{x}_{iw}^{\top}\beta)$. This approach of conditioning on the weighted sample mean $\bar{y}_{iw}$ protects against informative sampling.
In **emdi**, a Monte Carlo procedure is applied to approximate the predictor for all indicators using the following algorithm:

1. The dependent variable is transformed according to chosen transformation ('no', 'log') to obtain $T(y_{ij}) = y^*_{ij}$.
2. The sample data is used to estimate the nested error linear regression model

   $$y^*_{ij} = x_{ij}^{\top}\beta + u_i + e_{ij},\; u_i \overset{iid}{\sim} \mathcal{N}(0,\sigma^2_u),\; e_{ij} \overset{iid}{\sim} \mathcal{N}(0,\sigma^2_e)$$

   with the `lme` function from the package Pinheiro *et al.* (2021). The shrinkage parameter $\hat{\gamma}_{iw} = \hat{\sigma}^2_u/(\hat{\sigma}^2_u + \hat{\sigma}^2_e\hat{\delta}^2_i)$, for $\hat{\delta}^2_i = w_{i\cdot}^{-2}\sum_{j\in smp_i} w^2_{ij}$ is also computed and the coefficients for the fixed effects (following You and Rao 2002) are then obtained as:

   $$\hat{\beta} = \left(\sum_{i=1}^{D}\sum_{j=1}^{n_i} w_{ij}x_{ij}(x_{ij} - \hat{\gamma}_{iw}\bar{x}_{iw})^{\top}\right)^{-1}\left(\sum_{i=1}^{D}\sum_{j=1}^{n_i} w_{ij}(x_{ij} - \hat{\gamma}_{iw}\bar{x}_{iw})y_{ij}\right),$$

   where $\bar{x}_{iw} = w_{i\cdot}^{-1}\sum_{j\in smp_i} w_{ij}x_{ij}$.
3. For $l = 1, ..., L$:
   (a) For in-sample domains (domains that are part of the sample data set), a synthetic population of the target variable is generated by $y^{*(l)}_{ij} = x_{ij}^{\top}\hat{\beta} + \hat{u}_i + \nu^{(l)}_{ij} + e^{(l)}_{ij}$, where $\nu^{(l)}_i \overset{iid}{\sim} \mathcal{N}(0, \hat{\sigma}^2_u(1-\hat{\gamma}_{iw}))$, $e_{ij} \overset{iid}{\sim} \mathcal{N}(0,\hat{\sigma}^2_e)$ and $\hat{u}_i = \hat{\gamma}_{iw}(\bar{y}_{iw} - \bar{x}_{iw}^{\top}\hat{\beta})$
   For out-of-sample domains (domains with no data in the sample), the conditional expectation of u_i cannot be computed, hence for these domains a synthetic population is generated by using $y^{*(l)}_{ij} = x_{ij}\top\hat{\beta} + \nu^{(l)}_{ij} + e^{(l)}_{ij}$, where $\nu^{(l)}_i \overset{iid}{\sim} \mathcal{N}(0,\hat{\sigma}^2_u))$ and $e_{ij} \overset{iid}{\sim} \mathcal{N}(0,\hat{\sigma}^2_e)$.
   (b) The predicted dependent variable is back-transformed to the original scale $y^{(l)}_{iy} = T^{-1}(y^{*(l)}_{ij})$ and the target indicator $I^{(l)}_i(y^{(l)}_{ij})$ is calculated in each domain.
4. Final estimates are computed by taking the mean over the L Monte Carlo simulations in each domain, $\hat{I}^{CPEBP}_i = \frac{1}{N_i}\sum_{j=1}^{N_i}\hat{I}^{PEBP}_{ij}$

Parametric bootstrap MSE estimator

Guadarrama *et al.* (2018) moreover propose a parametric bootstrap MSE estimator very close to the procedure in Molina and Rao (2010), which is based on the method developed by González-Manteiga *et al.* (2008). The bootstrap implemented in **emdi** takes into account, that in applications the sample can be identified within the census rarely:

1. The same nested error model is fit to the sample data (possibly under transformation) as for the point estimates and the model parameters are obtained $(\hat{\beta}, \hat{\sigma}_u^2, \hat{\sigma}_e^2)$.
2. For $b = 1, ..., B$, with large B, $u_i^{(b)} \sim \mathcal{N}(0, \hat{\sigma}_u^2)$ and $e_{ij}^{(b)} \sim \mathcal{N}(0, \hat{\sigma}_e^2)$, $j = 1, ..., N_i$, $i = 1, ..., D$ are generated independently.
3. B bootstrap populations are generated as

$$y_{ij}^{*(b)} = x_{ij}^\top \hat{\beta} + u_i^{(b)} + e_{ij}^{(b)},\ j = 1, ..., N_i,\ i = 1, ..., D.$$

4. From each bootstrap population the true value of the domain indicator $I_i^{(b)} = N_i^{-1} \sum_{j=1}^{N_i} i(y_{ij}^{(b)}),\ b = 1, ..., B$ is calculated.
5. Additionally a bootstrap sample is generated as

$$y_{ij}^{*(b)} = x_{ij}^\top \hat{\beta} + u_i^{(b)} + e_{ij}^{(b)},\ j = 1, ..., n_i,\ i = 1, ..., D.$$

   This sample is used in conjunction with the known population vectors $x_{ij}, j \in U_i$ to calculate the bootstrap PEBP of I_i, denoted $\hat{I}_i^{PEBP(b)}, b = 1, ..., B$.

6. A bootstrap estimator of MSE$(\hat{I}_i^{PEBp})$ is then given by

$$mse(\tilde{I}_i^{PEBP}) = \frac{1}{B} \sum_{b=1}^{B} (\hat{I}_i^{PEBP(b)} - I_i^{(b)})^2.$$

3.2. Functionality

Overall, there are only slight changes for the user from the first version of the function `ebp`. Since the Pseudo EBP method uses survey weights in the estimation part, the `ebp` function has a new argument `weights`. The argument defaults to `NULL` and is to be used like the argument `smp_domains`, when the sampling design is informative. The function expects a character string as input for the argument that indicates the name of the weights variable in the sample dataset. The variable itself has to be numeric.

Since Rojas-Perilla *et al.* (2020) find that the usage of data-driven transformations can be favourable for the estimation of the EBP under non-informative sampling, the `transformation` argument of the `ebp` function in **emdi** is set to "box.cox" implying that the dependent variable is transformed with the Box-Cox transformation. Users of the weighted version of the EBP will have to choose "no" for no transformation or "log" for a logarithmic transformation, because data-driven transformations for the PEBP are still a topic for research. If the argument is not changed an informative message will be displayed and the estimation process will be halted.

While two options to estimate the MSE are provided for the unweighted EBP, the MSE estimation for PEBP allows only for a parametric bootstrap which is the default for both estimation approaches (`boot_type="parametric"`).

Model estimation

The original version of the EBP can still be used without any changes to the arguments of the function.The following function call almost equals the shown example in Kreutzmann *et al.* (2019):

```
R> ebp_noweights <- ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl +
+                          unempl_ben + age_ben + surv_ben + sick_ben + dis_ben +
+                          rent + fam_allow + house_allow + cap_inv + tax_adj,
+                        pop_data = eusilcA_pop, pop_domains = "district",
+                        smp_data = eusilcA_smp, smp_domains = "district",
+                        threshold = 10885.33, MSE = TRUE)
```

```
 Bootstrap started
 10  of  50  Bootstrap iterations completed        Approximately  00:00:02:06  remaining

 20  of  50  Bootstrap iterations completed        Approximately  00:00:01:33  remaining

 30  of  50  Bootstrap iterations completed        Approximately  00:00:01:02  remaining

 40  of  50  Bootstrap iterations completed        Approximately  00:00:00:31  remaining

 Bootstrap completed
```

When using the PEBP, the aforementioned changes to two arguments are necessary in order to run the model:

- `weights`: Adding the name of the variable that indicates the sampling weights
- `transformation`: Since the default transformation, Box-Cox, is not yet available for the weighted EBP, it needs to be changed to "no" or "log"

```
R> ebp_weights <- ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl +
+                        unempl_ben + age_ben + surv_ben + sick_ben + dis_ben +
+                        rent + fam_allow + house_allow + cap_inv + tax_adj,
+                      pop_data = eusilcA_pop, pop_domains = "district",
+                      smp_data = eusilcA_smp, smp_domains = "district",
+                      threshold = 10885.33, MSE = TRUE,
+                      weights = "weight", transformation = "log")
```

```
 Bootstrap started
 10  of  50  Bootstrap iterations completed        Approximately  00:00:01:42  remaining

 20  of  50  Bootstrap iterations completed        Approximately  00:00:01:15  remaining

 30  of  50  Bootstrap iterations completed        Approximately  00:00:00:49  remaining

 40  of  50  Bootstrap iterations completed        Approximately  00:00:00:24  remaining

 Bootstrap completed
```

The `model` component of an 'ebp' object that considers weights has a new list element with the weighted coefficients (You and Rao 2002). The other return components for the class are the same as for the unweighted EBP. After running the model, all the S3 methods that are available for the unweighted version of the EBP, can also be used to inspect the estimation results of the weighted EBP. An overview of all available methods can be found with `help(emdiObject)`.

Model diagnostics

The function call returned in the summary indicates if sampling weights are used for the estimation. Furthermore, it is made clear in the PEBP output that the returned explanatory measures and residual diagnostics belong to the mixed model used for the estimation of the variance components. While the data information as the in- and out-of-sample domains or sample sizes do not differ between the models, differences can be seen in the model diagnostics which is due to the different transformation used.

```
R> # without weights
R> summary(ebp_noweights)

Empirical Best Prediction

Call:
 ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl + unempl_ben +
    age_ben + surv_ben + sick_ben + dis_ben + rent + fam_allow +
    house_allow + cap_inv + tax_adj, pop_data = eusilcA_pop,
    pop_domains = "district", smp_data = eusilcA_smp, smp_domains = "district",
    threshold = 10885.33, MSE = TRUE)

Out-of-sample domains:  24
In-sample domains:  70

Sample sizes:
Units in sample:  1945
Units in population:  25000
```

```
                   Min. 1st Qu. Median      Mean 3rd Qu. Max.
Sample_domains       14    17.0   22.5  27.78571   29.00  200
Population_domains    5   126.5  181.5 265.95745  265.75 5857

Explanatory measures:
 Marginal_R2 Conditional_R2
   0.6325942       0.709266

Residual diagnostics:
               Skewness Kurtosis Shapiro_W    Shapiro_p
Error         0.7523871 9.646993 0.9619824 3.492626e-22
Random_effect 0.4655324 2.837176 0.9760574 1.995328e-01

ICC:  0.2086841

Transformation:
 Transformation Method Optimal_lambda Shift_parameter
        box.cox   reml      0.6046901               0

R> # with weights
R> summary(ebp_weights)

Empirical Best Prediction

Call:
 ebp(fixed = eqIncome ~ gender + eqsize + cash + self_empl + unempl_ben +
    age_ben + surv_ben + sick_ben + dis_ben + rent + fam_allow +
    house_allow + cap_inv + tax_adj, pop_data = eusilcA_pop,
    pop_domains = "district", smp_data = eusilcA_smp, smp_domains = "district",
    threshold = 10885.33, transformation = "log", MSE = TRUE,
    weights = "weight")

Out-of-sample domains:  24
In-sample domains:  70

Sample sizes:
Units in sample:  1945
Units in population:  25000
                   Min. 1st Qu. Median      Mean 3rd Qu. Max.
Sample_domains       14    17.0   22.5  27.78571   29.00  200
Population_domains    5   126.5  181.5 265.95745  265.75 5857

Explanatory measures for the mixed model:
 Marginal_R2 Conditional_R2
   0.5022296      0.5909727

Residual diagnostics for the mixed model:
```

```
                Skewness  Kurtosis Shapiro_W    Shapiro_p
Error          -2.1828119 17.863231 0.8670156 8.641339e-38
Random_effect -0.6609709  3.361441 0.9682563 7.261244e-02

ICC:  0.1782811

Transformation:
 Transformation Shift_parameter
            log               0
```

The method `plot` works independent of the usage of weights and plots residual diagnostics for the mixed model used in the estimation of the variance components.

```
R> plot(ebp_weights)
```

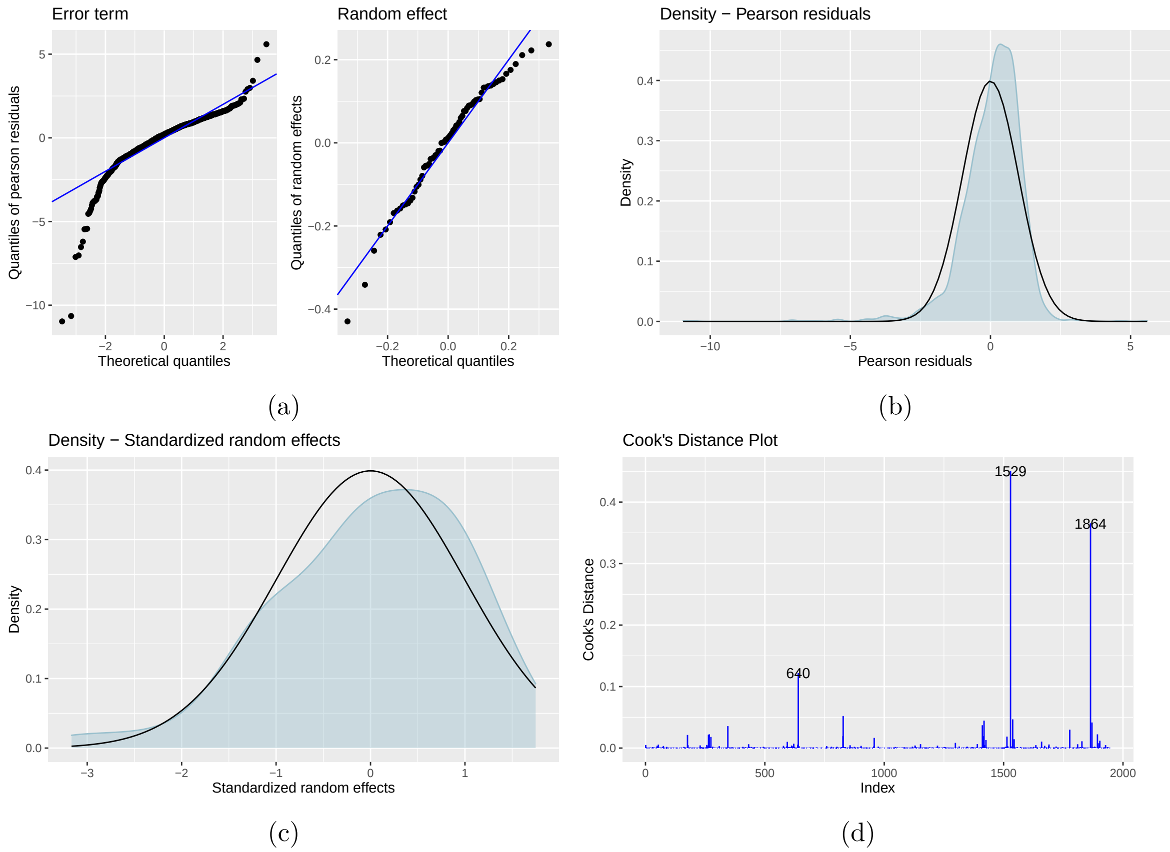


Figure 4: Output of `plot(ebp_weights)`: (a) normal quantile-quantile (Q-Q) plots of the error term and random effects, (b) and (c): kernel densities of the distribution of the error term and random effects (blue) in comparison to a standard normal distribution (black), (d): Cooks distance plot. All results refer to the error terms from the mixed model with log transformation.

To analyse the model coefficients, we added an additional argument `weights` to the `coef.ebp` method, which defaults to `FALSE`. When using the default, the coefficients for the mixed model are displayed. Setting the argument to `TRUE` returns the weighted regression coefficients as in You and Rao (2002).

```
R> # default
R> head(coef(ebp_weights), 2)
```

```
                (Intercept) genderfemale      eqsize         cash
Neusiedl am See    9.259754  -0.01087928 -0.06553293 2.984645e-05
Oberwart           9.079019  -0.01087928 -0.06553293 2.984645e-05
                   self_empl   unempl_ben      age_ben     surv_ben
Neusiedl am See 2.297232e-05 1.988227e-05 3.017273e-05 2.969203e-05
Oberwart        2.297232e-05 1.988227e-05 3.017273e-05 2.969203e-05
                    sick_ben     dis_ben         rent    fam_allow
Neusiedl am See 2.640426e-05 3.46888e-05 1.459455e-05 3.068898e-06
Oberwart        2.640426e-05 3.46888e-05 1.459455e-05 3.068898e-06
                 house_allow      cap_inv       tax_adj
Neusiedl am See 5.035249e-05 1.752919e-05 -1.194406e-05
Oberwart        5.035249e-05 1.752919e-05 -1.194406e-05
```

```
R> # weighted coefficients
R> coef(ebp_weights, weights = TRUE)
```

```
  (Intercept)  genderfemale        eqsize          cash     self_empl
 9.150540e+00  3.396113e-03 -6.104507e-02  3.272744e-05  2.486388e-05
   unempl_ben       age_ben      surv_ben      sick_ben       dis_ben
 2.182578e-05  3.355522e-05  3.122066e-05  2.847899e-05  3.778051e-05
         rent     fam_allow   house_allow       cap_inv       tax_adj
 1.514303e-05  3.655281e-07  4.582541e-05  1.807290e-05 -1.191243e-05
```

Estimation results

The analysis of estimation results does also not differ between the estimation of the EBP or the PEBP. Exemplarily, we show the first five rows of the head count ratio and the poverty gap for the Austrian districts for both estimation approaches.

```
R> # without weights
R> head(estimators(ebp_noweights, indicator = c("Head_Count", "Poverty_Gap")))
```

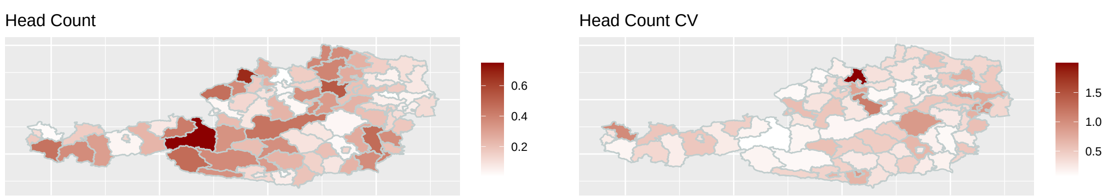
```
               Domain Head_Count Poverty_Gap
1 Eisenstadt-Umgebung 0.03252174 0.006666679
2  Eisenstadt (Stadt) 0.02918919 0.007225508
3             Güssing 0.15459459 0.034668361
4         Jennersdorf 0.35877551 0.099130465
5         Mattersburg 0.07908257 0.015634492
6     Neusiedl am See 0.09316129 0.017250369
```

```
R> # with weights
R> head(estimators(ebp_weights, indicator = c("Head_Count", "Poverty_Gap")))

               Domain Head_Count Poverty_Gap
1 Eisenstadt-Umgebung 0.04939130  0.00844820
2  Eisenstadt (Stadt) 0.03567568  0.00756965
3             Güssing 0.19216216  0.03653299
4         Jennersdorf 0.39306122  0.09112366
5         Mattersburg 0.10733945  0.01791921
6      Neusiedl am See 0.11122581  0.01800804
```

The point and uncertainty estimates can also be plotted on maps to analyse the spatial distribution of e.g., poverty incidence.

```
R> # load shape file
R> load_shapeaustria()
R> # plot for PEBP (weights) for head count ratio
R> map_plot(object = ebp_weights, CV = TRUE,
+           map_obj = shape_austria_dis, indicator = c("Head_Count"),
+           map_dom_id = "PB")
```



(a) (b)

Figure 5: Map of predictions (a) and CV (b) of the head count ratio from the EBP using sampling weights.

Summary

The usage of function `ebp` changes only slightly for the user when weights are added compared to the unweighted option. All diagnostic and analysis tools are available for both options.

3.3. Conclusion

This vignette shows the most recent changes of function `ebp` in the R package **emdi**: (a) additional data-driven transformations, (b) the inclusion of sampling weights into the estimation procedure of the EBP. A topic for further research is the inclusion of the above described data-driven transformations in the estimation of the PEBP.

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