Machine Learning Compendium

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Abstract

Looking up things take time. Let's just look into one single document

1 Reinforcement Learning

1.1 Bellman Equations

TODO backup diagramms

State Value Function

$$v_\pi(s) = \sum_{a \in A} \pi(s|a) Q_\pi(s,a) \tag{1}$$

Action Value Function

$$Q_\pi(s,a) = r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a v_\pi(s') \tag{2}$$

State Value Function recursive

$$v_\pi(s) = \sum_{a \in A} \pi(s|a)(r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a v_\pi(s')) \tag{3}$$

Action Value Function recursive

$$Q_\pi(s,a) = r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a \sum_{a \in A} \pi(a'|s') Q_\pi(s',a') \tag{4}$$

Optimal State Value Function

$$v_*(s) = \max_a Q_*(s,a) \tag{5}$$

Optimal Action State Value Function

$$Q_*(s,a) = r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a v_*(s') \tag{6}$$

Optimal State Value Function recursive

$$v_*(s) = \max_a r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a v_*(s') \tag{7}$$

Optimal Action State Value Function recursive

$$Q_*(a,s) = r_s^a + \gamma \sum_{s' \in S} P_{ss'}^a \max_{a'} Q_*(s',a') \tag{8}$$

1.2 Advantage Function

TODO

1.3 Policy, Policy Gradient

Policy: Distribution over actions given states

$$\pi_\theta(a|s) = P(a|s) \tag{9}$$

Policy Gradient

$$\nabla_\theta \pi_\theta(s|a) = \pi_\theta(s|a) \nabla_\theta \log \pi_\theta(s|a) \tag{10}$$

Note: this is valid for all probability distributions (the policy is a distribution over actions given states). The gradient term on the right hand side is called score function. The derivation basically uses the "log-trick".

2 Statistics

2.1 Probability

TODO: Probability, CDF, Variance, L_p-Space for Random-Variables, Jensens-Inequality for Random Variables etc.

2.2 Distributions

TODO: All the usual suspects

2.3 Estimation

TODO: ML, Score-Function, biased/unbiased, Cramér–Rao bound

2.4 Divergences

Definition 2.1. Divergence A divergence is a "sense" of distance between two probability distributions. It's not a metric, but a pre-metric. From wikipedia: Suppose S is a space of all probability distributions with common support. Then a divergence on S is a function $D(\Delta \,||\, \Delta) : S \to R$ satisfying [1]

1. $D(p \,||\, q) \geq 0 \forall p, q \in S$,
2. $D(p \,||\, q) = 0 \Leftrightarrow p = q$

Definition 2.2. f-Divergence

1. Generalization of whole family of divergences
2. From wikipedia: Let P and Q be two probability distributions over a space Ω such that P is absolutely continuous with respect to Q. Then, for a convex function f such that $f(1) = 0$, the f-divergence of P from Q is defined as: $D_f(P \parallel Q) \equiv \int_\Omega f\left(\frac{dP}{dQ}\right) dQ$

Definition 2.3. KL-Divergence

1. From wikipedia:P and Q are probability measures over a set X, and P is absolutely continuous with respect to Q, then the Kullback–Leibler divergence from Q to P is defined as $D_{\mathrm{KL}}(P \| Q) = \int_X \log \frac{dP}{dQ} \, dP = D_{t \log}$.
2. maxmizing likelihood is equivalent to minimizing $D_{KL}[P(.|\theta^*) \parallel P(.|\theta)]$, where $P(.|\theta^*)$ is the true distribution and $P(.|\theta)$ is our estimate

Definition 2.4. Jensen–Shannon divergence From wikipedia:$\mathrm{JSD}(P \parallel Q) = \frac{1}{2} D(P \parallel M) + \frac{1}{2} D(Q \parallel M)$, where $M = \frac{1}{2}(P + Q)$

TODO: Wasserstein & Wasserstein Dual