

中山大学本科生期末考试

考试科目：《高等代数》(B 卷)

学年学期：2019 年第 1 学期

姓 名：_____

学 院/系：数据院

学 号：_____

考试方式：闭卷

年级专业：_____

考试时长：120 分钟

班 别：_____

警示 《中山大学授予学士学位工作细则》第八条：“考试作弊者，不授予学士学位。”

-----以下为试题区域，共三道大题，总分 100 分，考生请在答题纸上作答-----

一、问答题（共 3 小题，共 20 分）

- 1) (5 分) For a matrix equation $A\vec{x} = \vec{b}$, where \vec{x} is the solution vector, please tell (1) when there is no solution, (2) when there is a unique solution, and when there are infinitely many solutions.
- 2) (9 分) Detail the following matrix factorization: 1) LU factorization; 2) Diagonalization; 3) QR decomposition.
- 3) (6 分) What is the difference between orthogonal set and orthonormal set? What is orthogonal matrix?

二、计算题（共 5 小题，共 60 分）

- 1) (15 分) Compute the reduced echelon form for the following matrix

$$\begin{pmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{pmatrix}$$

Please point out the pivot positions. Is the above matrix invertible? If it is invertible, please compute its inverse.

- 2) (12 分) Compute the determinant of the following matrices (6% for each):

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -1 & 1 \end{pmatrix}, \begin{pmatrix} x & -1 & 0 & 0 \\ 0 & x & -1 & 0 \\ 0 & 0 & x & -1 \\ a_4 & a_3 & a_2 & x+a_1 \end{pmatrix}$$

- 3) (8 分) Are the following vectors linearly independent? Why?

$$\vec{a}_1 = \begin{pmatrix} 1 \\ -2 \\ 4 \\ -8 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} 1 \\ 3 \\ 9 \\ 27 \end{pmatrix}, \vec{a}_3 = \begin{pmatrix} 1 \\ 4 \\ 16 \\ 64 \end{pmatrix}, \vec{a}_4 = \begin{pmatrix} 3 \\ -3 \\ 3 \\ -3 \end{pmatrix}$$

4) (12 分) Given the following matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & -5 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{pmatrix}$$

Please

- Describe its column space and a basis for it. (3%)
- Compute the rank of matrix A (3%)
- Find an orthogonal basis for the column space of matrix A . (6%)

5) (13 分) Given the following matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

Please

- Compute its eigenvalues and eigenvectors. (6%)
- Is matrix A diagonalizable? If it is, please diagonalize it? (5%)
- What is the determinant value of A ? (2%)

三、证明题 (共 3 小题, 共 20 分)

1) (8 分) Suppose $\{\vec{w}_1, \dots, \vec{w}_m\}$ is an orthogonal basis of a subspace W of R^n . Then

(a) (2 分) For any \vec{y} in R^n , what is its orthogonal projection onto W ?

(b) (6 分) Denote the orthogonal projection of \vec{y} onto W by $\hat{\vec{y}}$. Please prove that there exist $\vec{z} \in W^\perp$ such that $\vec{y} = \hat{\vec{y}} + \vec{z}$. Please also prove this kind of decomposition, namely reconstructing any vector by addition of two vectors in W and the complementary W^\perp respectively, is unique.

2) (6 分) Let $W = \{(x_1 \ x_2 \ \dots \ x_n)^T \mid x_i \in R, i=1,2,\dots,n; x_1 + x_2 + \dots + x_n = 0\}$. Please prove that W is a subspace of R^n .

3) (6 分) Let $\vec{\eta}$ be a solution of matrix equation $A\vec{x} = \vec{b}$, where A is a matrix. Let $\{\vec{\xi}_1, \vec{\xi}_2, \dots, \vec{\xi}_r\}$ be a basis of the null space of matrix A , and \vec{b} is not zero. Please prove:

(a) (3 分) $\vec{\eta}, \vec{\xi}_1, \vec{\xi}_2, \dots, \vec{\xi}_r$ are linearly independent

(b) (3 分) $\vec{\eta}, \vec{\eta} + \vec{\xi}_1, \vec{\eta} + \vec{\xi}_2, \dots, \vec{\eta} + \vec{\xi}_r$ are linearly independent