In [1]:
```
import findspark
findspark.init()
findspark.find()

import pyspark
from pyspark import SparkContext, SparkConf, SQLContext
from pyspark.sql import SparkSession

from pyspark.sql import SparkSession

spark = SparkSession.builder \
    .master("local[*]") \
    .appName("GenericAppName") \
    .getOrCreate()
sc=spark.sparkContext
sqlContext = SQLContext(sc)
```

```
C:\temp\Hadoop\spark-3.3.0-bin-hadoop3\python\pyspark\sql\context.py:112: FutureWarning: Deprecated in 3.0.0. Use SparkSession.builder.getO
rCreate() instead.
  warnings.warn(
```

Task I

In [2]:
```
# Ingest data 2015-2022
from pyspark import SparkFiles

players_15 = spark.read.csv('FIFA_DATA/players_15.csv',header=True, inferSchema = True)
players_16 = spark.read.csv('FIFA_DATA/players_16.csv',header=True, inferSchema = True)
players_17 = spark.read.csv('FIFA_DATA/players_17.csv',header=True, inferSchema = True)
players_18 = spark.read.csv('FIFA_DATA/players_18.csv',header=True, inferSchema = True)
players_19 = spark.read.csv('FIFA_DATA/players_19.csv',header=True, inferSchema = True)
players_20 = spark.read.csv('FIFA_DATA/players_20.csv',header=True, inferSchema = True)
players_21 = spark.read.csv('FIFA_DATA/players_21.csv',header=True, inferSchema = True)
players_22 = spark.read.csv('FIFA_DATA/players_22.csv',header=True, inferSchema = True)
```

In [3]:
```
print(players_15.columns[55])
```

```
movement_agility
```

In [4]:
```
#Add new column for the year
from pyspark.sql.functions import lit

players_15 = players_15.withColumn('Year', lit(2015))
players_16 = players_16.withColumn('Year', lit(2016))
```

```
players_17 = players_17.withColumn('Year', lit(2017))
players_18 = players_18.withColumn('Year', lit(2018))
players_19 = players_19.withColumn('Year', lit(2019))
players_20 = players_20.withColumn('Year', lit(2020))
players_21 = players_21.withColumn('Year', lit(2021))
players_22 = players_22.withColumn('Year', lit(2022))
```

In [5]:
```
#Ensure every record can be uniquely identified
#combine fifa id and year as index
from pyspark.sql import functions as sf

players_15 = players_15.withColumn('data_ID', sf.concat(sf.col('sofifa_id'),sf.lit('_'), sf.col('Year')))
players_16 = players_16.withColumn('data_ID', sf.concat(sf.col('sofifa_id'),sf.lit('_'), sf.col('Year')))
players_17 = players_17.withColumn('data_ID', sf.concat(sf.col('sofifa_id'),sf.lit('_'), sf.col('Year')))
players_18 = players_18.withColumn('data_ID', sf.concat(sf.col('sofifa_id'),sf.lit('_'), sf.col('Year')))
players_19 = players_19.withColumn('data_ID', sf.concat(sf.col('sofifa_id'),sf.lit('_'), sf.col('Year')))
players_20 = players_20.withColumn('data_ID', sf.concat(sf.col('sofifa_id'),sf.lit('_'), sf.col('Year')))
players_21 = players_21.withColumn('data_ID', sf.concat(sf.col('sofifa_id'),sf.lit('_'), sf.col('Year')))
players_22 = players_22.withColumn('data_ID', sf.concat(sf.col('sofifa_id'),sf.lit('_'), sf.col('Year')))
```

In [6]:
```
# union the data to one dataset
data = players_15.union(players_16)
data = data.union(players_17)
data = data.union(players_18)
data = data.union(players_19)
data = data.union(players_20)
data = data.union(players_21)
data = data.union(players_22)
```

In [7]:
```
db_properties={}
#update your db username
db_properties['username']="postgres"
#update your db password
db_properties['password']="psql"
#make sure you got the right port number here
db_properties['url']= "jdbc:postgresql://localhost:5432/postgres"
#make sure you had the Postgres JAR file in the right location
db_properties['driver']="org.postgresql.Driver"
db_properties['table']= "fifa.data"



data.write.format("jdbc")\
.mode("overwrite")\
.option("url", db_properties['url'])\
.option("dbtable", db_properties['table'])\
.option("user", db_properties['username'])\
```

```
.option("password", db_properties['password'])\
.option("Driver", db_properties['driver'])\
.save()
```

In [8]:
```
data.printSchema()
```

```
root
 |-- sofifa_id: integer (nullable = true)
 |-- player_url: string (nullable = true)
 |-- short_name: string (nullable = true)
 |-- long_name: string (nullable = true)
 |-- player_positions: string (nullable = true)
 |-- overall: integer (nullable = true)
 |-- potential: integer (nullable = true)
 |-- value_eur: double (nullable = true)
 |-- wage_eur: double (nullable = true)
 |-- age: integer (nullable = true)
 |-- dob: timestamp (nullable = true)
 |-- height_cm: integer (nullable = true)
 |-- weight_kg: integer (nullable = true)
 |-- club_team_id: double (nullable = true)
 |-- club_name: string (nullable = true)
 |-- league_name: string (nullable = true)
 |-- league_level: integer (nullable = true)
 |-- club_position: string (nullable = true)
 |-- club_jersey_number: integer (nullable = true)
 |-- club_loaned_from: string (nullable = true)
 |-- club_joined: timestamp (nullable = true)
 |-- club_contract_valid_until: integer (nullable = true)
 |-- nationality_id: integer (nullable = true)
 |-- nationality_name: string (nullable = true)
 |-- nation_team_id: double (nullable = true)
 |-- nation_position: string (nullable = true)
 |-- nation_jersey_number: integer (nullable = true)
 |-- preferred_foot: string (nullable = true)
 |-- weak_foot: integer (nullable = true)
 |-- skill_moves: integer (nullable = true)
 |-- international_reputation: integer (nullable = true)
 |-- work_rate: string (nullable = true)
 |-- body_type: string (nullable = true)
 |-- real_face: string (nullable = true)
 |-- release_clause_eur: string (nullable = true)
 |-- player_tags: string (nullable = true)
 |-- player_traits: string (nullable = true)
 |-- pace: integer (nullable = true)
 |-- shooting: integer (nullable = true)
 |-- passing: integer (nullable = true)
 |-- dribbling: integer (nullable = true)
 |-- defending: integer (nullable = true)
 |-- physic: integer (nullable = true)
 |-- attacking_crossing: integer (nullable = true)
 |-- attacking_finishing: integer (nullable = true)
 |-- attacking_heading_accuracy: integer (nullable = true)
 |-- attacking_short_passing: integer (nullable = true)
```

```
|-- attacking_volleys: integer (nullable = true)
|-- skill_dribbling: integer (nullable = true)
|-- skill_curve: integer (nullable = true)
|-- skill_fk_accuracy: integer (nullable = true)
|-- skill_long_passing: integer (nullable = true)
|-- skill_ball_control: integer (nullable = true)
|-- movement_acceleration: integer (nullable = true)
|-- movement_sprint_speed: integer (nullable = true)
|-- movement_agility: integer (nullable = true)
|-- movement_reactions: integer (nullable = true)
|-- movement_balance: integer (nullable = true)
|-- power_shot_power: integer (nullable = true)
|-- power_jumping: integer (nullable = true)
|-- power_stamina: integer (nullable = true)
|-- power_strength: integer (nullable = true)
|-- power_long_shots: integer (nullable = true)
|-- mentality_aggression: integer (nullable = true)
|-- mentality_interceptions: integer (nullable = true)
|-- mentality_positioning: integer (nullable = true)
|-- mentality_vision: integer (nullable = true)
|-- mentality_penalties: integer (nullable = true)
|-- mentality_composure: string (nullable = true)
|-- defending_marking_awareness: integer (nullable = true)
|-- defending_standing_tackle: integer (nullable = true)
|-- defending_sliding_tackle: integer (nullable = true)
|-- goalkeeping_diving: integer (nullable = true)
|-- goalkeeping_handling: integer (nullable = true)
|-- goalkeeping_kicking: integer (nullable = true)
|-- goalkeeping_positioning: integer (nullable = true)
|-- goalkeeping_reflexes: integer (nullable = true)
|-- goalkeeping_speed: integer (nullable = true)
|-- ls: string (nullable = true)
|-- st: string (nullable = true)
|-- rs: string (nullable = true)
|-- lw: string (nullable = true)
|-- lf: string (nullable = true)
|-- cf: string (nullable = true)
|-- rf: string (nullable = true)
|-- rw: string (nullable = true)
|-- lam: string (nullable = true)
|-- cam: string (nullable = true)
|-- ram: string (nullable = true)
|-- lm: string (nullable = true)
|-- lcm: string (nullable = true)
|-- cm: string (nullable = true)
|-- rcm: string (nullable = true)
|-- rm: string (nullable = true)
|-- lwb: string (nullable = true)
```

```
|-- ldm: string (nullable = true)
|-- cdm: string (nullable = true)
|-- rdm: string (nullable = true)
|-- rwb: string (nullable = true)
|-- lb: string (nullable = true)
|-- lcb: string (nullable = true)
|-- cb: string (nullable = true)
|-- rcb: string (nullable = true)
|-- rb: string (nullable = true)
|-- gk: string (nullable = true)
|-- player_face_url: string (nullable = true)
|-- club_logo_url: string (nullable = true)
|-- club_flag_url: string (nullable = true)
|-- nation_logo_url: string (nullable = true)
|-- nation_flag_url: string (nullable = true)
|-- Year: integer (nullable = false)
|-- data_ID: string (nullable = true)
```

In [9]:

```
data_read = sqlContext.read.format("jdbc")\
    .option("url", db_properties['url'])\
    .option("dbtable", db_properties['table'])\
    .option("user", db_properties['username'])\
    .option("password", db_properties['password'])\
    .option("Driver", db_properties['driver'])\
    .load()

data_read.show(1, vertical=True)
```

```
-RECORD 0-----------------------------------------
 sofifa_id                   | 190745
 player_url                  | https://sofifa.co...
 short_name                  | M. Silvestri
 long_name                   | Marco Silvestri
 player_positions            | GK
 overall                     | 69
 potential                   | 74
 value_eur                   | 1100000.0
 wage_eur                    | 10000.0
 age                         | 25
 dob                         | 1991-03-02 00:00:00
 height_cm                   | 191
 weight_kg                   | 80
 club_team_id                | 8.0
 club_name                   | Leeds United
 league_name                 | English League Ch...
 league_level                | 2
 club_position               | RES
 club_jersey_number          | 12
 club_loaned_from            | null
 club_joined                 | 2014-07-09 00:00:00
 club_contract_valid_until   | 2018
 nationality_id              | 27
 nationality_name            | Italy
 nation_team_id              | null
 nation_position             | null
 nation_jersey_number        | null
 preferred_foot              | Right
 weak_foot                   | 2
 skill_moves                 | 1
 international_reputation    | 1
 work_rate                   | Medium/Medium
 body_type                   | Lean (185+)
 real_face                   | No
 release_clause_eur          | null
 player_tags                 | null
 player_traits               | Puncher
 pace                        | null
 shooting                    | null
 passing                     | null
 dribbling                   | null
 defending                   | null
 physic                      | null
 attacking_crossing          | 12
 attacking_finishing         | 20
 attacking_heading_accuracy  | 12
 attacking_short_passing     | 22
```

attacking_volleys	20
skill_dribbling	18
skill_curve	16
skill_fk_accuracy	11
skill_long_passing	20
skill_ball_control	22
movement_acceleration	58
movement_sprint_speed	61
movement_agility	65
movement_reactions	62
movement_balance	34
power_shot_power	16
power_jumping	74
power_stamina	34
power_strength	60
power_long_shots	16
mentality_aggression	31
mentality_interceptions	11
mentality_positioning	14
mentality_vision	49
mentality_penalties	16
mentality_composure	39
defending_marking_awareness	18
defending_standing_tackle	18
defending_sliding_tackle	16
goalkeeping_diving	78
goalkeeping_handling	59
goalkeeping_kicking	53
goalkeeping_positioning	67
goalkeeping_reflexes	76
goalkeeping_speed	60
ls	27+1
st	27+1
rs	27+1
lw	30
lf	29
cf	29
rf	29
rw	30
lam	31+1
cam	31+1
ram	31+1
lm	30+1
lcm	27+1
cm	27+1
rcm	27+1
rm	30+1
lwb	26+1

```
 ldm                         | 26+1
 cdm                         | 26+1
 rdm                         | 26+1
 rwb                         | 26+1
 lb                          | 26+1
 lcb                         | 27+1
 cb                          | 27+1
 rcb                         | 27+1
 rb                          | 26+1
 gk                          | 68+1
 player_face_url             | https://cdn.sofif...
 club_logo_url               | https://cdn.sofif...
 club_flag_url               | https://cdn.sofif...
 nation_logo_url             | null
 nation_flag_url             | https://cdn.sofif...
 Year                        | 2017
 data_ID                     | 190745_2017
only showing top 1 row
```

Task II

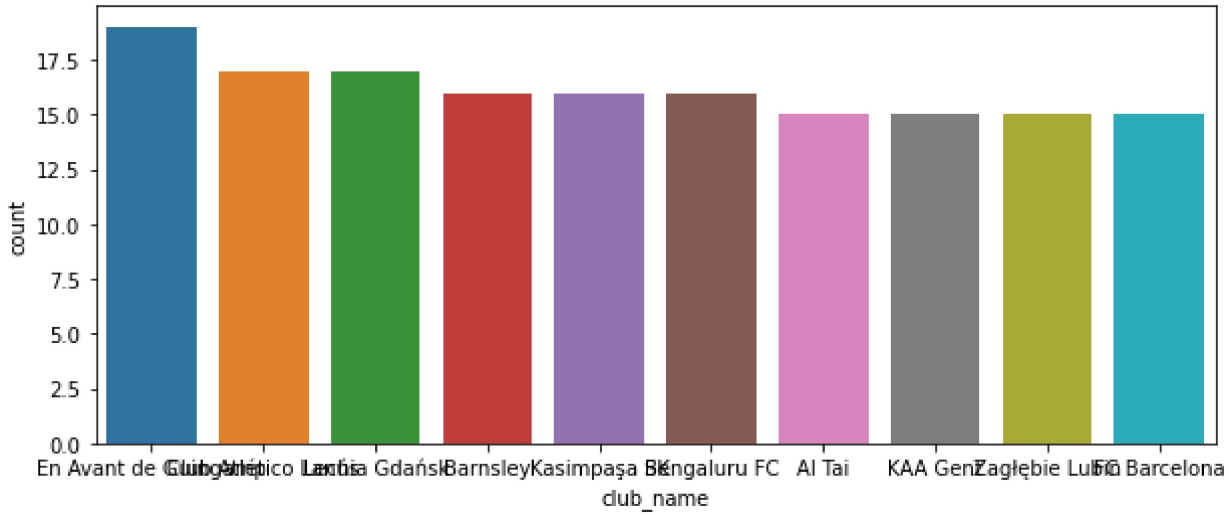
Find X

In [84]:

```python
#In order to find the club with highest numbers of players
#use where to find the data in 2022
#use .agg to Compute aggregates when(data_read["club_contract_valid_until"]=='2023' and returns the result as a DataFrame
#group the data by club name and sort them by the count of data
def findX(x):
    SORT_club = data_read.where(data_read['Year']=='2022').groupBy("club_name")
    highest_number_club = SORT_club.agg(F.count(F.when(data_read["club_contract_valid_until"]=='2023', "club_name")).alias("count")).sort("
    highest_number_club.show(x,vertical = True)
findX(5)
```

```
-RECORD 0-------------------------
 club_name | En Avant de Guingamp
 count     | 19
-RECORD 1-------------------------
 club_name | Club Atlético Lanús
 count     | 17
-RECORD 2-------------------------
 club_name | Lechia Gdańsk
 count     | 17
-RECORD 3-------------------------
 club_name | Barnsley
 count     | 16
-RECORD 4-------------------------
 club_name | Kasimpaşa SK
 count     | 16
only showing top 5 rows
```

In [90]:
```
# we drew the barplot of the count to give a direct interpretation
import seaborn as sns
from IPython import display
import matplotlib.pyplot as plt

SORT_club = data_read.where(data_read['Year']=='2022').groupBy("club_name")
highest_number_club = SORT_club.agg(F.count(F.when(data_read["club_contract_valid_until"]=='2023', "club_name")).alias("count")).sort("coun
plt.figure( figsize = ( 10, 4 ) )
sns.barplot( x="club_name", y="count", data=highest_number_club[:10])
plt.show()
```



FindY

In [12]:
```
# the number of players older than 27 years oldfor each club
# we used where to find the players who are older than 27 and group the data by club_name, then we sort the data
# to find the club who has the highest number
# we sum the age for each club

import numpy as np
highest_number_over27_count = data_read.where(data_read['age']>27).groupBy("club_name").count().sort("count",ascending=False).toPandas()
highest_number_over27_sum = data_read.where(data_read['age']>27).groupBy("club_name").sum('age').toPandas()
print(highest_number_over27_count)
print(highest_number_over27_sum)
average_over27 = highest_number_over27_sum['sum(age)'].values/highest_number_over27_count['count'].values
print(type(average_over27),average_over27)
```

```
                  club_name  count
0                      None    874
1     İstanbul Başakşehir FK    133
2     Jeonbuk Hyundai Motors    118
3        FC Lokomotiv Moscow    108
4             Crystal Palace    106
...                      ...    ...
1007     Caracas Fútbol Club      3
1008          SC Freiburg II      2
1009            FC Helsingør      2
1010            FC Dordrecht      1
1011    Borussia Dortmund II      1

[1012 rows x 2 columns]
                      club_name  sum(age)
0                       Palermo      1105
1     CD Everton de Viña del Mar      1014
2                 Shonan Bellmare      1676
3                     Yeovil Town       847
4                      Göztepe SK      1553
...                           ...       ...
1007                         Bury      1731
1008            Rotherham United      1945
1009                 Bohemian FC      1269
1010               Middlesbrough      2064
1011                        Como       219

[1012 rows x 2 columns]
<class 'numpy.ndarray'> [1.26430206e+00 7.62406015e+00 1.42033898e+01 ... 6.34500000e+02
 2.06400000e+03 2.19000000e+02]
```

In [92]:
```
# we calculated the average and found the highest average
def findY(y):
    highest_number_over27_avg = data_read.where(data_read['age']>27).groupBy("club_name").avg('age').sort("avg(age)",ascending=False)
    highest_number_over27_avg.show(y,vertical =True)
findY(5)
```

```
-RECORD 0-------------------------
 club_name | Yokohama FC
 avg(age)  | 34.7037037037037
-RECORD 1-------------------------
 club_name | Wexford Youths
 avg(age)  | 34.0
-RECORD 2-------------------------
 club_name | Zamora Fútbol Club
 avg(age)  | 33.857142857142854
-RECORD 3-------------------------
 club_name | Centro Atlético F...
 avg(age)  | 33.6
-RECORD 4-------------------------
 club_name | CF Fuenlabrada
 avg(age)  | 33.54545454545455
only showing top 5 rows
```

In [93]:

```
top1_highest_number_over27_avg = data_read.where(data_read['age']>27).groupBy("club_name").avg('age').sort("avg(age)",ascending=False).limi
print("club with highest numbers over 27:",top1_highest_number_over27_avg['club_name'].values)
findY(1)
```

```
club with highest numbers over 27: ['Yokohama FC']
-RECORD 0--------------------
 club_name | Yokohama FC
 avg(age)  | 34.7037037037037
only showing top 1 row
```

In [15]:

```
# most frequent nation_positions for each year
# we also used where to filter the year, and group them by nation_position, count the number of each position and sort
# them to search the highest one.
most_frequent_nation_2015 = data_read.where(data_read['Year']==2015).groupBy("nation_position").count().sort("count",ascending=False).limit
print("Most frequent nation position 2015:",most_frequent_nation_2015['nation_position'].values)
most_frequent_nation_2016 = data_read.where(data_read['Year']==2016).groupBy("nation_position").count().sort("count",ascending=False).limit
print("Most frequent nation position 2016:",most_frequent_nation_2016['nation_position'].values)
most_frequent_nation_2017 = data_read.where(data_read['Year']==2017).groupBy("nation_position").count().sort("count",ascending=False).limit
print("Most frequent nation position 2017:",most_frequent_nation_2017['nation_position'].values)
most_frequent_nation_2018 = data_read.where(data_read['Year']==2018).groupBy("nation_position").count().sort("count",ascending=False).limit
print("Most frequent nation position 2018:",most_frequent_nation_2018['nation_position'].values)
most_frequent_nation_2019 = data_read.where(data_read['Year']==2019).groupBy("nation_position").count().sort("count",ascending=False).limit
print("Most frequent nation position 2019:",most_frequent_nation_2019['nation_position'].values)
most_frequent_nation_2020 = data_read.where(data_read['Year']==2020).groupBy("nation_position").count().sort("count",ascending=False).limit
print("Most frequent nation position 2020:",most_frequent_nation_2020['nation_position'].values)
most_frequent_nation_2021 = data_read.where(data_read['Year']==2021).groupBy("nation_position").count().sort("count",ascending=False).limit
print("Most frequent nation position 2021:",most_frequent_nation_2021['nation_position'].values)
most_frequent_nation_2022 = data_read.where(data_read['Year']==2022).groupBy("nation_position").count().sort("count",ascending=False).limit
print("Most frequent nation position 2022:",most_frequent_nation_2022['nation_position'].values)
```

```
Most frequent nation position 2015: ['SUB']
Most frequent nation position 2016: ['SUB']
Most frequent nation position 2017: ['SUB']
Most frequent nation position 2018: ['SUB']
Most frequent nation position 2019: ['SUB']
Most frequent nation position 2020: ['SUB']
Most frequent nation position 2021: ['SUB']
Most frequent nation position 2022: ['SUB']
```

In [16]:
```
# most_frequent_nation_2015 = data_read.where(data_read['Year']==2015).groupBy("nation_position").count().sort("count",ascending=False).lim
# print(most_frequent_nation_2015)
```

Task III

Data Engineering and cleaning

In [17]:
```
# Since we only use the skillsets of the players, we dropped all the unneccessary columns
drop_list = ['player_position',' nationality_name','work_rate','body_type','real_face','data_ID','short_name','long_name','dob','club_team_
                            'club_loaned_from','club_joined','club_contract_valid_until',
                            'nationality_id','nation_team_id','nation_position','nation_jersey_number',
                            'release_clause_eur','player_tags','player_traits','player_face_url',
                            'club_logo_url','club_flag_url','nation_logo_url','nation_flag_url',
                            'player_url','mentality_composure','goalkeeping_speed','club_position','league_level','league_name','club_name'
```

In [18]:
```
print(len(drop_list))
```

```
33
```

In [19]:
```
df_read = data_read.drop('player_positions','nationality_name','work_rate','body_type','real_face','data_ID','short_name','long_name','dob'
                            'club_loaned_from','club_joined','club_contract_valid_until',
                            'nationality_id','nation_team_id','nation_position','nation_jersey_number',
                            'release_clause_eur','player_tags','player_traits','player_face_url',
                            'club_logo_url','club_flag_url','nation_logo_url','nation_flag_url',
                            'player_url','mentality_composure','goalkeeping_speed','club_position','league_level','league_name','club_name'
```

In [20]:
```
print(len(data_read.columns))
print(len(df_read.columns))
```

```
112
79
```

In [21]:
```
df_read.printSchema()
```

```
root
 |-- sofifa_id: integer (nullable = true)
 |-- overall: integer (nullable = true)
 |-- potential: integer (nullable = true)
 |-- value_eur: double (nullable = true)
 |-- wage_eur: double (nullable = true)
 |-- age: integer (nullable = true)
 |-- height_cm: integer (nullable = true)
 |-- weight_kg: integer (nullable = true)
 |-- preferred_foot: string (nullable = true)
 |-- weak_foot: integer (nullable = true)
 |-- skill_moves: integer (nullable = true)
 |-- international_reputation: integer (nullable = true)
 |-- pace: integer (nullable = true)
 |-- shooting: integer (nullable = true)
 |-- passing: integer (nullable = true)
 |-- dribbling: integer (nullable = true)
 |-- defending: integer (nullable = true)
 |-- physic: integer (nullable = true)
 |-- attacking_crossing: integer (nullable = true)
 |-- attacking_finishing: integer (nullable = true)
 |-- attacking_heading_accuracy: integer (nullable = true)
 |-- attacking_short_passing: integer (nullable = true)
 |-- attacking_volleys: integer (nullable = true)
 |-- skill_dribbling: integer (nullable = true)
 |-- skill_curve: integer (nullable = true)
 |-- skill_fk_accuracy: integer (nullable = true)
 |-- skill_long_passing: integer (nullable = true)
 |-- skill_ball_control: integer (nullable = true)
 |-- movement_acceleration: integer (nullable = true)
 |-- movement_sprint_speed: integer (nullable = true)
 |-- movement_agility: integer (nullable = true)
 |-- movement_reactions: integer (nullable = true)
 |-- movement_balance: integer (nullable = true)
 |-- power_shot_power: integer (nullable = true)
 |-- power_jumping: integer (nullable = true)
 |-- power_stamina: integer (nullable = true)
 |-- power_strength: integer (nullable = true)
 |-- power_long_shots: integer (nullable = true)
 |-- mentality_aggression: integer (nullable = true)
 |-- mentality_interceptions: integer (nullable = true)
 |-- mentality_positioning: integer (nullable = true)
 |-- mentality_vision: integer (nullable = true)
 |-- mentality_penalties: integer (nullable = true)
 |-- defending_marking_awareness: integer (nullable = true)
 |-- defending_standing_tackle: integer (nullable = true)
 |-- defending_sliding_tackle: integer (nullable = true)
 |-- goalkeeping_diving: integer (nullable = true)
```

```
|-- goalkeeping_handling: integer (nullable = true)
|-- goalkeeping_kicking: integer (nullable = true)
|-- goalkeeping_positioning: integer (nullable = true)
|-- goalkeeping_reflexes: integer (nullable = true)
|-- ls: string (nullable = true)
|-- st: string (nullable = true)
|-- rs: string (nullable = true)
|-- lw: string (nullable = true)
|-- lf: string (nullable = true)
|-- cf: string (nullable = true)
|-- rf: string (nullable = true)
|-- rw: string (nullable = true)
|-- lam: string (nullable = true)
|-- cam: string (nullable = true)
|-- ram: string (nullable = true)
|-- lm: string (nullable = true)
|-- lcm: string (nullable = true)
|-- cm: string (nullable = true)
|-- rcm: string (nullable = true)
|-- rm: string (nullable = true)
|-- lwb: string (nullable = true)
|-- ldm: string (nullable = true)
|-- cdm: string (nullable = true)
|-- rdm: string (nullable = true)
|-- rwb: string (nullable = true)
|-- lb: string (nullable = true)
|-- lcb: string (nullable = true)
|-- cb: string (nullable = true)
|-- rcb: string (nullable = true)
|-- rb: string (nullable = true)
|-- gk: string (nullable = true)
|-- Year: integer (nullable = true)
```

In [22]:
```
# from pyspark.sql.functions import *

# # Notice I dropped isPenalty and isSTPlay
# null_counts_plays_df = df_read.select([count(when(isnan(c) | col(c).isNull(), c)).alias(c) \
#                         for c in df_read.columns])

# null_counts_plays_df.show(truncate=False, vertical=True)
```

impute 'pace','shooting','passing','dribbling','defending','physic'

In [23]:
```
# we checked the missing values of the whole dataset and we found out that the columns with
# larger amoungt of missing value: 'pace','shooting','passing','dribbling','defending','physic'
# Because there were many missing value in the data, we want to impute the missing value with the median value
```

In [24]:
```
from pyspark.ml.feature import Imputer
columns_to_be_imputed = ['pace']
value_not_in_dataset = -200

# Replace None/Missing Value with a value that can't be present in the dataset.
df_with_filled_na = df_read.fillna(-200, columns_to_be_imputed)

#Create new columns with imputed values. New columns will be suffixed with "_imputed"
imputer = Imputer (
            inputCols=columns_to_be_imputed,
            outputCols=["{}_imputed".format(c) for c in columns_to_be_imputed])\
            .setStrategy("median").setMissingValue(value_not_in_dataset)

df_imputed = imputer.fit(df_with_filled_na).transform(df_with_filled_na)
# we will drop the old column without imputation. We have only one column to be imputed
df_imputed_enhanced = df_imputed.drop(columns_to_be_imputed[0])

# We will rename our newly imputed column with the correct name
df_fully_imputed = df_imputed_enhanced.withColumnRenamed("pace_imputed","pace")
```

In [25]:
```
from pyspark.ml.feature import Imputer
columns_to_be_imputed = ['shooting']
value_not_in_dataset = -200

# Replace None/Missing Value with a value that can't be present in the dataset.
df_with_filled_na2 = df_fully_imputed.fillna(-200, columns_to_be_imputed)

#Create new columns with imputed values. New columns will be suffixed with "_imputed"
imputer = Imputer (
            inputCols=columns_to_be_imputed,
            outputCols=["{}_imputed".format(c) for c in columns_to_be_imputed])\
            .setStrategy("median").setMissingValue(value_not_in_dataset)

df_imputed2 = imputer.fit(df_with_filled_na2).transform(df_with_filled_na2)
# we will drop the old column without imputation. We have only one column to be imputed
df_imputed_enhanced2 = df_imputed2.drop(columns_to_be_imputed[0])

# We will rename our newly imputed column with the correct name
df_fully_imputed2 = df_imputed_enhanced2.withColumnRenamed("shooting_imputed","shooting")
```

In [26]:
```
from pyspark.ml.feature import Imputer
columns_to_be_imputed = ['passing']
value_not_in_dataset = -200

# Replace None/Missing Value with a value that can't be present in the dataset.
df_with_filled_na3 = df_fully_imputed2.fillna(-200, columns_to_be_imputed)
```

```
#Create new columns with imputed values. New columns will be suffixed with "_imputed"
imputer = Imputer (
            inputCols=columns_to_be_imputed,
            outputCols=["{}_imputed".format(c) for c in columns_to_be_imputed])\
            .setStrategy("median").setMissingValue(value_not_in_dataset)

df_imputed3 = imputer.fit(df_with_filled_na3).transform(df_with_filled_na3)
# we will drop the old column without imputation. We have only one column to be imputed
df_imputed_enhanced3 = df_imputed3.drop(columns_to_be_imputed[0])

# We will rename our newly imputed column with the correct name
df_fully_imputed3 = df_imputed_enhanced3.withColumnRenamed("passing_imputed","passing")
```

In [27]:
```
from pyspark.ml.feature import Imputer
columns_to_be_imputed = ['dribbling']
value_not_in_dataset = -200

# Replace None/Missing Value with a value that can't be present in the dataset.
df_with_filled_na4 = df_fully_imputed3.fillna(-200, columns_to_be_imputed)

#Create new columns with imputed values. New columns will be suffixed with "_imputed"
imputer = Imputer (
            inputCols=columns_to_be_imputed,
            outputCols=["{}_imputed".format(c) for c in columns_to_be_imputed])\
            .setStrategy("median").setMissingValue(value_not_in_dataset)

df_imputed4 = imputer.fit(df_with_filled_na4).transform(df_with_filled_na4)
# we will drop the old column without imputation. We have only one column to be imputed
df_imputed_enhanced4 = df_imputed4.drop(columns_to_be_imputed[0])

# We will rename our newly imputed column with the correct name
df_fully_imputed4 = df_imputed_enhanced4.withColumnRenamed("dribbling_imputed","dribbling")
```

In [28]:
```
from pyspark.ml.feature import Imputer
columns_to_be_imputed = ['defending']
value_not_in_dataset = -200

# Replace None/Missing Value with a value that can't be present in the dataset.
df_with_filled_na5 = df_fully_imputed4.fillna(-200, columns_to_be_imputed)

#Create new columns with imputed values. New columns will be suffixed with "_imputed"
imputer = Imputer (
            inputCols=columns_to_be_imputed,
            outputCols=["{}_imputed".format(c) for c in columns_to_be_imputed])\
            .setStrategy("median").setMissingValue(value_not_in_dataset)

df_imputed5 = imputer.fit(df_with_filled_na5).transform(df_with_filled_na5)
```

```
# we will drop the old column without imputation. We have only one column to be imputed
df_imputed_enhanced5 = df_imputed5.drop(columns_to_be_imputed[0])

# We will rename our newly imputed column with the correct name
df_fully_imputed5 = df_imputed_enhanced5.withColumnRenamed("defending_imputed","defending")
```

In [29]:
```
from pyspark.ml.feature import Imputer
columns_to_be_imputed = ['physic']
value_not_in_dataset = -200

# Replace None/Missing Value with a value that can't be present in the dataset.
df_with_filled_na6 = df_fully_imputed5.fillna(-200, columns_to_be_imputed)

#Create new columns with imputed values. New columns will be suffixed with "_imputed"
imputer = Imputer (
            inputCols=columns_to_be_imputed,
            outputCols=["{}_imputed".format(c) for c in columns_to_be_imputed])\
            .setStrategy("median").setMissingValue(value_not_in_dataset)

df_imputed6 = imputer.fit(df_with_filled_na6).transform(df_with_filled_na6)
# we will drop the old column without imputation. We have only one column to be imputed
df_imputed_enhanced6 = df_imputed6.drop(columns_to_be_imputed[0])

# We will rename our newly imputed column with the correct name
df_fully_imputed6 = df_imputed_enhanced6.withColumnRenamed("physic_imputed","physic")
```

In [30]:
```
from pyspark.sql.functions import *

# # Notice I dropped isPenalty and isSTPlay
# null_counts_plays_df = df_fully_imputed6.select([count(when(isnan(c) | col(c).isNull(), c)).alias(c) \
#                         for c in df_read.columns])

# null_counts_plays_df.show(truncate=False, vertical=True)
```

drop null

In [31]:
```
# we chaecked the missing value again and found out the rows with missing value
# Then we drop the rows
# Reason: The number of row was small compared to the entire dataset, so we just drop them
```

In [32]:
```
df_read = df_fully_imputed6.dropna()
```

In [33]:
```
null_counts_plays_df = df_read.select([count(when(isnan(c) | col(c).isNull(), c)).alias(c) \
                        for c in df_read.columns])

null_counts_plays_df.show(truncate=False, vertical=True)
```

```
-RECORD 0-------------------------
 sofifa_id                    | 0
 overall                      | 0
 potential                    | 0
 value_eur                    | 0
 wage_eur                     | 0
 age                          | 0
 height_cm                    | 0
 weight_kg                    | 0
 preferred_foot               | 0
 weak_foot                    | 0
 skill_moves                  | 0
 international_reputation     | 0
 attacking_crossing           | 0
 attacking_finishing          | 0
 attacking_heading_accuracy   | 0
 attacking_short_passing      | 0
 attacking_volleys            | 0
 skill_dribbling              | 0
 skill_curve                  | 0
 skill_fk_accuracy            | 0
 skill_long_passing           | 0
 skill_ball_control           | 0
 movement_acceleration        | 0
 movement_sprint_speed        | 0
 movement_agility             | 0
 movement_reactions           | 0
 movement_balance             | 0
 power_shot_power             | 0
 power_jumping                | 0
 power_stamina                | 0
 power_strength               | 0
 power_long_shots             | 0
 mentality_aggression         | 0
 mentality_interceptions      | 0
 mentality_positioning        | 0
 mentality_vision             | 0
 mentality_penalties          | 0
 defending_marking_awareness  | 0
 defending_standing_tackle    | 0
 defending_sliding_tackle     | 0
 goalkeeping_diving           | 0
 goalkeeping_handling         | 0
 goalkeeping_kicking          | 0
 goalkeeping_positioning      | 0
 goalkeeping_reflexes         | 0
 ls                           | 0
 st                           | 0
```

```
rs                  | 0
lw                  | 0
lf                  | 0
cf                  | 0
rf                  | 0
rw                  | 0
lam                 | 0
cam                 | 0
ram                 | 0
lm                  | 0
lcm                 | 0
cm                  | 0
rcm                 | 0
rm                  | 0
lwb                 | 0
ldm                 | 0
cdm                 | 0
rdm                 | 0
rwb                 | 0
lb                  | 0
lcb                 | 0
cb                  | 0
rcb                 | 0
rb                  | 0
gk                  | 0
Year                | 0
pace                | 0
shooting            | 0
passing             | 0
dribbling           | 0
defending           | 0
physic              | 0
```

Data preprocessing

label to binary

In [34]:
```
# Because there are only two values in the 'preferred_foot' column, we convert the value to 0 and 1
label_to_binary = udf(lambda name: 0.0 if name == 'Left' else 1.0)
df_read_bi = df_read.withColumn('preferred_foot', label_to_binary(col('preferred_foot')))
```

In [35]:
```
# df_read_bi.show(vertical = True)
```

string to int

In [36]:
```
# We found the format "XX+X" in some columns and we calculated them and converted them to int
def addall(string):
    if len(string)== 4:
        string = int(string[0:2])+int(string[-1])
    elif len(string) ==2:
        string = int(string)
    elif len(string)== 6:
        string = int(string[-2:])
    return string
```

In [37]:
```
from pyspark.sql.functions import udf
from pyspark.sql.types import IntegerType
```

In [38]:
```
addallUDF = udf(lambda i:addall(i), IntegerType())
```

In [39]:
```
str_to_int_list = ['ls','st','rs','lw','lf','cf','rf','rw','lam','cam','ram','lm','lcm','cm','rcm','rm','lwb',
                   'ldm','cdm','rdm','rwb','lb','lcb','cb','rcb','rb','gk']
```

In [40]:
```
df_read_int = df_read_bi.withColumn('ls', addallUDF(col('ls')))
df_read_int = df_read_int.withColumn('st', addallUDF(col('st')))
df_read_int = df_read_int.withColumn('rs', addallUDF(col('rs')))
df_read_int = df_read_int.withColumn('lw', addallUDF(col('lw')))
df_read_int = df_read_int.withColumn('lf', addallUDF(col('lf')))
df_read_int = df_read_int.withColumn('cf', addallUDF(col('cf')))
df_read_int = df_read_int.withColumn('rf', addallUDF(col('rf')))
df_read_int = df_read_int.withColumn('rw', addallUDF(col('rw')))
df_read_int = df_read_int.withColumn('lam', addallUDF(col('lam')))
df_read_int = df_read_int.withColumn('cam', addallUDF(col('cam')))
df_read_int = df_read_int.withColumn('ram', addallUDF(col('ram')))
df_read_int = df_read_int.withColumn('lm', addallUDF(col('lm')))
df_read_int = df_read_int.withColumn('lcm', addallUDF(col('lcm')))
df_read_int = df_read_int.withColumn('cm', addallUDF(col('cm')))
df_read_int = df_read_int.withColumn('rcm', addallUDF(col('rcm')))
df_read_int = df_read_int.withColumn('rm', addallUDF(col('rm')))
df_read_int = df_read_int.withColumn('lwb', addallUDF(col('lwb')))
df_read_int = df_read_int.withColumn('ldm', addallUDF(col('ldm')))
df_read_int = df_read_int.withColumn('cdm', addallUDF(col('cdm')))
df_read_int = df_read_int.withColumn('rdm', addallUDF(col('rdm')))
df_read_int = df_read_int.withColumn('rwb', addallUDF(col('rwb')))
df_read_int = df_read_int.withColumn('lb', addallUDF(col('lb')))
df_read_int = df_read_int.withColumn('lcb', addallUDF(col('lcb')))
df_read_int = df_read_int.withColumn('cb', addallUDF(col('cb')))
df_read_int = df_read_int.withColumn('rcb', addallUDF(col('rcb')))
df_read_int = df_read_int.withColumn('rb', addallUDF(col('rb')))
df_read_int = df_read_int.withColumn('gk', addallUDF(col('gk')))
```

cast data type

In [41]:
```
print(df_read_int.columns)
```

```
['sofifa_id', 'overall', 'potential', 'value_eur', 'wage_eur', 'age', 'height_cm', 'weight_kg', 'preferred_foot', 'weak_foot', 'skill_move
s', 'international_reputation', 'attacking_crossing', 'attacking_finishing', 'attacking_heading_accuracy', 'attacking_short_passing', 'atta
cking_volleys', 'skill_dribbling', 'skill_curve', 'skill_fk_accuracy', 'skill_long_passing', 'skill_ball_control', 'movement_acceleration',
'movement_sprint_speed', 'movement_agility', 'movement_reactions', 'movement_balance', 'power_shot_power', 'power_jumping', 'power_stamin
a', 'power_strength', 'power_long_shots', 'mentality_aggression', 'mentality_interceptions', 'mentality_positioning', 'mentality_vision',
'mentality_penalties', 'defending_marking_awareness', 'defending_standing_tackle', 'defending_sliding_tackle', 'goalkeeping_diving', 'goalk
eeping_handling', 'goalkeeping_kicking', 'goalkeeping_positioning', 'goalkeeping_reflexes', 'ls', 'st', 'rs', 'lw', 'lf', 'cf', 'rf', 'rw',
'lam', 'cam', 'ram', 'lm', 'lcm', 'cm', 'rcm', 'rm', 'lwb', 'ldm', 'cdm', 'rdm', 'rwb', 'lb', 'lcb', 'cb', 'rcb', 'rb', 'gk', 'Year', 'pac
e', 'shooting', 'passing', 'dribbling', 'defending', 'physic']
```

In [42]:
```
# we changed the type of data to DoubleType
```

In [43]:
```
from pyspark.sql.types import DoubleType
for i in df_read_int.columns:
    df_read_int = df_read_int.withColumn(i, df_read_int[i].cast(DoubleType()))
```

In [44]:
```
df_read_int.printSchema()
```

```
root
 |-- sofifa_id: double (nullable = true)
 |-- overall: double (nullable = true)
 |-- potential: double (nullable = true)
 |-- value_eur: double (nullable = true)
 |-- wage_eur: double (nullable = true)
 |-- age: double (nullable = true)
 |-- height_cm: double (nullable = true)
 |-- weight_kg: double (nullable = true)
 |-- preferred_foot: double (nullable = true)
 |-- weak_foot: double (nullable = true)
 |-- skill_moves: double (nullable = true)
 |-- international_reputation: double (nullable = true)
 |-- attacking_crossing: double (nullable = true)
 |-- attacking_finishing: double (nullable = true)
 |-- attacking_heading_accuracy: double (nullable = true)
 |-- attacking_short_passing: double (nullable = true)
 |-- attacking_volleys: double (nullable = true)
 |-- skill_dribbling: double (nullable = true)
 |-- skill_curve: double (nullable = true)
 |-- skill_fk_accuracy: double (nullable = true)
 |-- skill_long_passing: double (nullable = true)
 |-- skill_ball_control: double (nullable = true)
 |-- movement_acceleration: double (nullable = true)
 |-- movement_sprint_speed: double (nullable = true)
 |-- movement_agility: double (nullable = true)
 |-- movement_reactions: double (nullable = true)
 |-- movement_balance: double (nullable = true)
 |-- power_shot_power: double (nullable = true)
 |-- power_jumping: double (nullable = true)
 |-- power_stamina: double (nullable = true)
 |-- power_strength: double (nullable = true)
 |-- power_long_shots: double (nullable = true)
 |-- mentality_aggression: double (nullable = true)
 |-- mentality_interceptions: double (nullable = true)
 |-- mentality_positioning: double (nullable = true)
 |-- mentality_vision: double (nullable = true)
 |-- mentality_penalties: double (nullable = true)
 |-- defending_marking_awareness: double (nullable = true)
 |-- defending_standing_tackle: double (nullable = true)
 |-- defending_sliding_tackle: double (nullable = true)
 |-- goalkeeping_diving: double (nullable = true)
 |-- goalkeeping_handling: double (nullable = true)
 |-- goalkeeping_kicking: double (nullable = true)
 |-- goalkeeping_positioning: double (nullable = true)
 |-- goalkeeping_reflexes: double (nullable = true)
 |-- ls: double (nullable = true)
 |-- st: double (nullable = true)
```

```
|-- rs: double (nullable = true)
|-- lw: double (nullable = true)
|-- lf: double (nullable = true)
|-- cf: double (nullable = true)
|-- rf: double (nullable = true)
|-- rw: double (nullable = true)
|-- lam: double (nullable = true)
|-- cam: double (nullable = true)
|-- ram: double (nullable = true)
|-- lm: double (nullable = true)
|-- lcm: double (nullable = true)
|-- cm: double (nullable = true)
|-- rcm: double (nullable = true)
|-- rm: double (nullable = true)
|-- lwb: double (nullable = true)
|-- ldm: double (nullable = true)
|-- cdm: double (nullable = true)
|-- rdm: double (nullable = true)
|-- rwb: double (nullable = true)
|-- lb: double (nullable = true)
|-- lcb: double (nullable = true)
|-- cb: double (nullable = true)
|-- rcb: double (nullable = true)
|-- rb: double (nullable = true)
|-- gk: double (nullable = true)
|-- Year: double (nullable = true)
|-- pace: double (nullable = true)
|-- shooting: double (nullable = true)
|-- passing: double (nullable = true)
|-- dribbling: double (nullable = true)
|-- defending: double (nullable = true)
|-- physic: double (nullable = true)
```

In [45]:
```
df_read_features = df_read_int.withColumn('overall_new',col('overall')).drop('overall')
```

In [46]:
```
df_read_int.show(5,vertical = True)
```

```
-RECORD 0------------------------------
 sofifa_id                    | 190745.0
 overall                      | 69.0
 potential                    | 74.0
 value_eur                    | 1100000.0
 wage_eur                     | 10000.0
 age                          | 25.0
 height_cm                    | 191.0
 weight_kg                    | 80.0
 preferred_foot               | 1.0
 weak_foot                    | 2.0
 skill_moves                  | 1.0
 international_reputation     | 1.0
 attacking_crossing           | 12.0
 attacking_finishing          | 20.0
 attacking_heading_accuracy   | 12.0
 attacking_short_passing      | 22.0
 attacking_volleys            | 20.0
 skill_dribbling              | 18.0
 skill_curve                  | 16.0
 skill_fk_accuracy            | 11.0
 skill_long_passing           | 20.0
 skill_ball_control           | 22.0
 movement_acceleration        | 58.0
 movement_sprint_speed        | 61.0
 movement_agility             | 65.0
 movement_reactions           | 62.0
 movement_balance             | 34.0
 power_shot_power             | 16.0
 power_jumping                | 74.0
 power_stamina                | 34.0
 power_strength               | 60.0
 power_long_shots             | 16.0
 mentality_aggression         | 31.0
 mentality_interceptions      | 11.0
 mentality_positioning        | 14.0
 mentality_vision             | 49.0
 mentality_penalties          | 16.0
 defending_marking_awareness  | 18.0
 defending_standing_tackle    | 18.0
 defending_sliding_tackle     | 16.0
 goalkeeping_diving           | 78.0
 goalkeeping_handling         | 59.0
 goalkeeping_kicking          | 53.0
 goalkeeping_positioning      | 67.0
 goalkeeping_reflexes         | 76.0
 ls                           | 28.0
 st                           | 28.0
```

```
 rs                          | 28.0
 lw                          | 30.0
 lf                          | 29.0
 cf                          | 29.0
 rf                          | 29.0
 rw                          | 30.0
 lam                         | 32.0
 cam                         | 32.0
 ram                         | 32.0
 lm                          | 31.0
 lcm                         | 28.0
 cm                          | 28.0
 rcm                         | 28.0
 rm                          | 31.0
 lwb                         | 27.0
 ldm                         | 27.0
 cdm                         | 27.0
 rdm                         | 27.0
 rwb                         | 27.0
 lb                          | 27.0
 lcb                         | 28.0
 cb                          | 28.0
 rcb                         | 28.0
 rb                          | 27.0
 gk                          | 69.0
 Year                        | 2017.0
 pace                        | 69.0
 shooting                    | 54.0
 passing                     | 58.0
 dribbling                   | 63.0
 defending                   | 55.0
 physic                      | 66.0
-RECORD 1-----------------------------
 sofifa_id                   | 190780.0
 overall                     | 69.0
 potential                   | 73.0
 value_eur                   | 1000000.0
 wage_eur                    | 4000.0
 age                         | 27.0
 height_cm                   | 190.0
 weight_kg                   | 98.0
 preferred_foot              | 1.0
 weak_foot                   | 2.0
 skill_moves                 | 1.0
 international_reputation    | 1.0
 attacking_crossing          | 13.0
 attacking_finishing         | 12.0
 attacking_heading_accuracy  | 14.0
```

attacking_short_passing	34.0
attacking_volleys	17.0
skill_dribbling	19.0
skill_curve	13.0
skill_fk_accuracy	19.0
skill_long_passing	32.0
skill_ball_control	20.0
movement_acceleration	60.0
movement_sprint_speed	60.0
movement_agility	64.0
movement_reactions	67.0
movement_balance	52.0
power_shot_power	23.0
power_jumping	75.0
power_stamina	45.0
power_strength	86.0
power_long_shots	20.0
mentality_aggression	40.0
mentality_interceptions	28.0
mentality_positioning	13.0
mentality_vision	14.0
mentality_penalties	16.0
defending_marking_awareness	13.0
defending_standing_tackle	15.0
defending_sliding_tackle	15.0
goalkeeping_diving	72.0
goalkeeping_handling	62.0
goalkeeping_kicking	60.0
goalkeeping_positioning	64.0
goalkeeping_reflexes	75.0
ls	30.0
st	30.0
rs	30.0
lw	28.0
lf	27.0
cf	27.0
rf	27.0
rw	28.0
lam	29.0
cam	29.0
ram	29.0
lm	31.0
lcm	29.0
cm	29.0
rcm	29.0
rm	31.0
lwb	31.0
ldm	32.0

```
 cdm                          | 32.0
 rdm                          | 32.0
 rwb                          | 31.0
 lb                           | 31.0
 lcb                          | 33.0
 cb                           | 33.0
 rcb                          | 33.0
 rb                           | 31.0
 gk                           | 69.0
 Year                         | 2017.0
 pace                         | 69.0
 shooting                     | 54.0
 passing                      | 58.0
 dribbling                    | 63.0
 defending                    | 55.0
 physic                       | 66.0
-RECORD 2-------------------------------
 sofifa_id                    | 190783.0
 overall                      | 69.0
 potential                    | 69.0
 value_eur                    | 650000.0
 wage_eur                     | 9000.0
 age                          | 31.0
 height_cm                    | 190.0
 weight_kg                    | 79.0
 preferred_foot               | 1.0
 weak_foot                    | 2.0
 skill_moves                  | 2.0
 international_reputation     | 1.0
 attacking_crossing           | 57.0
 attacking_finishing          | 33.0
 attacking_heading_accuracy   | 64.0
 attacking_short_passing      | 68.0
 attacking_volleys            | 34.0
 skill_dribbling              | 50.0
 skill_curve                  | 60.0
 skill_fk_accuracy            | 28.0
 skill_long_passing           | 60.0
 skill_ball_control           | 58.0
 movement_acceleration        | 46.0
 movement_sprint_speed        | 34.0
 movement_agility             | 44.0
 movement_reactions           | 64.0
 movement_balance             | 34.0
 power_shot_power             | 31.0
 power_jumping                | 63.0
 power_stamina                | 80.0
 power_strength               | 80.0
```

power_long_shots	32.0
mentality_aggression	76.0
mentality_interceptions	73.0
mentality_positioning	47.0
mentality_vision	55.0
mentality_penalties	25.0
defending_marking_awareness	65.0
defending_standing_tackle	71.0
defending_sliding_tackle	62.0
goalkeeping_diving	6.0
goalkeeping_handling	16.0
goalkeeping_kicking	7.0
goalkeeping_positioning	6.0
goalkeeping_reflexes	16.0
ls	50.0
st	50.0
rs	50.0
lw	51.0
lf	50.0
cf	50.0
rf	50.0
rw	51.0
lam	54.0
cam	54.0
ram	54.0
lm	56.0
lcm	61.0
cm	61.0
rcm	61.0
rm	56.0
lwb	64.0
ldm	68.0
cdm	68.0
rdm	68.0
rwb	64.0
lb	64.0
lcb	69.0
cb	69.0
rcb	69.0
rb	64.0
gk	18.0
Year	2017.0
pace	39.0
shooting	33.0
passing	60.0
dribbling	52.0
defending	68.0
physic	78.0

```
-RECORD 3------------------------------
 sofifa_id                    | 190792.0
 overall                      | 69.0
 potential                    | 72.0
 value_eur                    | 1000000.0
 wage_eur                     | 7000.0
 age                          | 26.0
 height_cm                    | 182.0
 weight_kg                    | 78.0
 preferred_foot               | 1.0
 weak_foot                    | 2.0
 skill_moves                  | 1.0
 international_reputation     | 1.0
 attacking_crossing           | 11.0
 attacking_finishing          | 10.0
 attacking_heading_accuracy   | 14.0
 attacking_short_passing      | 23.0
 attacking_volleys            | 13.0
 skill_dribbling              | 11.0
 skill_curve                  | 15.0
 skill_fk_accuracy            | 14.0
 skill_long_passing           | 20.0
 skill_ball_control           | 22.0
 movement_acceleration        | 53.0
 movement_sprint_speed        | 38.0
 movement_agility             | 56.0
 movement_reactions           | 65.0
 movement_balance             | 42.0
 power_shot_power             | 22.0
 power_jumping                | 75.0
 power_stamina                | 18.0
 power_strength               | 73.0
 power_long_shots             | 18.0
 mentality_aggression         | 17.0
 mentality_interceptions      | 17.0
 mentality_positioning        | 11.0
 mentality_vision             | 32.0
 mentality_penalties          | 14.0
 defending_marking_awareness  | 14.0
 defending_standing_tackle    | 18.0
 defending_sliding_tackle     | 16.0
 goalkeeping_diving           | 72.0
 goalkeeping_handling         | 66.0
 goalkeeping_kicking          | 67.0
 goalkeeping_positioning      | 65.0
 goalkeeping_reflexes         | 73.0
 ls                           | 26.0
 st                           | 26.0
```

```
 rs                          | 26.0
 lw                          | 25.0
 lf                          | 25.0
 cf                          | 25.0
 rf                          | 25.0
 rw                          | 25.0
 lam                         | 27.0
 cam                         | 27.0
 ram                         | 27.0
 lm                          | 25.0
 lcm                         | 25.0
 cm                          | 25.0
 rcm                         | 25.0
 rm                          | 25.0
 lwb                         | 24.0
 ldm                         | 26.0
 cdm                         | 26.0
 rdm                         | 26.0
 rwb                         | 24.0
 lb                          | 25.0
 lcb                         | 28.0
 cb                          | 28.0
 rcb                         | 28.0
 rb                          | 25.0
 gk                          | 69.0
 Year                        | 2017.0
 pace                        | 69.0
 shooting                    | 54.0
 passing                     | 58.0
 dribbling                   | 63.0
 defending                   | 55.0
 physic                      | 66.0
-RECORD 4-----------------------------
 sofifa_id                   | 190883.0
 overall                     | 69.0
 potential                   | 69.0
 value_eur                   | 775000.0
 wage_eur                    | 6000.0
 age                         | 28.0
 height_cm                   | 175.0
 weight_kg                   | 68.0
 preferred_foot              | 1.0
 weak_foot                   | 3.0
 skill_moves                 | 2.0
 international_reputation    | 1.0
 attacking_crossing          | 59.0
 attacking_finishing         | 49.0
 attacking_heading_accuracy  | 60.0
```

attacking_short_passing	67.0
attacking_volleys	48.0
skill_dribbling	57.0
skill_curve	61.0
skill_fk_accuracy	55.0
skill_long_passing	58.0
skill_ball_control	63.0
movement_acceleration	73.0
movement_sprint_speed	72.0
movement_agility	76.0
movement_reactions	71.0
movement_balance	83.0
power_shot_power	53.0
power_jumping	81.0
power_stamina	78.0
power_strength	66.0
power_long_shots	36.0
mentality_aggression	76.0
mentality_interceptions	69.0
mentality_positioning	55.0
mentality_vision	56.0
mentality_penalties	52.0
defending_marking_awareness	71.0
defending_standing_tackle	64.0
defending_sliding_tackle	68.0
goalkeeping_diving	13.0
goalkeeping_handling	9.0
goalkeeping_kicking	16.0
goalkeeping_positioning	12.0
goalkeeping_reflexes	11.0
ls	59.0
st	59.0
rs	59.0
lw	61.0
lf	59.0
cf	59.0
rf	59.0
rw	61.0
lam	61.0
cam	61.0
ram	61.0
lm	63.0
lcm	63.0
cm	63.0
rcm	63.0
rm	63.0
lwb	69.0
ldm	68.0

```
 cdm                       | 68.0
 rdm                       | 68.0
 rwb                       | 69.0
 lb                        | 69.0
 lcb                       | 69.0
 cb                        | 69.0
 rcb                       | 69.0
 rb                        | 69.0
 gk                        | 19.0
 Year                      | 2017.0
 pace                      | 72.0
 shooting                  | 48.0
 passing                   | 61.0
 dribbling                 | 63.0
 defending                 | 67.0
 physic                    | 72.0
only showing top 5 rows
```

In [47]:
```
feature_cols = df_read_int.columns
del feature_cols[2]
```

In [48]:
```
from pyspark.sql import Row
from pyspark.ml.linalg import Vectors
```

In [49]:
```
len(df_read_features.columns)
```

Out[49]:
```
79
```

In [50]:
```
null_counts_plays_df_2 = df_read_features.select([count(when(isnan(c) | col(c).isNull(), c)).alias(c) \
                        for c in df_read_features.columns])

null_counts_plays_df_2.show(truncate=False, vertical=True)
```

```
-RECORD 0-------------------------
 sofifa_id                    | 0
 potential                    | 0
 value_eur                    | 0
 wage_eur                     | 0
 age                          | 0
 height_cm                    | 0
 weight_kg                    | 0
 preferred_foot               | 0
 weak_foot                    | 0
 skill_moves                  | 0
 international_reputation     | 0
 attacking_crossing           | 0
 attacking_finishing          | 0
 attacking_heading_accuracy   | 0
 attacking_short_passing      | 0
 attacking_volleys            | 0
 skill_dribbling              | 0
 skill_curve                  | 0
 skill_fk_accuracy            | 0
 skill_long_passing           | 0
 skill_ball_control           | 0
 movement_acceleration        | 0
 movement_sprint_speed        | 0
 movement_agility             | 0
 movement_reactions           | 0
 movement_balance             | 0
 power_shot_power             | 0
 power_jumping                | 0
 power_stamina                | 0
 power_strength               | 0
 power_long_shots             | 0
 mentality_aggression         | 0
 mentality_interceptions      | 0
 mentality_positioning        | 0
 mentality_vision             | 0
 mentality_penalties          | 0
 defending_marking_awareness  | 0
 defending_standing_tackle    | 0
 defending_sliding_tackle     | 0
 goalkeeping_diving           | 0
 goalkeeping_handling         | 0
 goalkeeping_kicking          | 0
 goalkeeping_positioning      | 0
 goalkeeping_reflexes         | 0
 ls                           | 0
 st                           | 0
 rs                           | 0
```

```
lw                  | 0
lf                  | 0
cf                  | 0
rf                  | 0
rw                  | 0
lam                 | 0
cam                 | 0
ram                 | 0
lm                  | 0
lcm                 | 0
cm                  | 0
rcm                 | 0
rm                  | 0
lwb                 | 0
ldm                 | 0
cdm                 | 0
rdm                 | 0
rwb                 | 0
lb                  | 0
lcb                 | 1
cb                  | 1
rcb                 | 1
rb                  | 0
gk                  | 451
Year                | 0
pace                | 0
shooting            | 0
passing             | 0
dribbling           | 0
defending           | 0
physic              | 0
overall_new         | 0
```

In [51]:
```
df_read_features = df_read_features.dropna()
```

In [52]:
```
# we checked the data again to ensure there is no missing value in the dataset
null_counts_plays_df_2 = df_read_features.select([count(when(isnan(c) | col(c).isNull(), c)).alias(c) \
                        for c in df_read_features.columns])

null_counts_plays_df_2.show(truncate=False, vertical=True)
```

```
-RECORD 0-------------------------
 sofifa_id                    | 0
 potential                    | 0
 value_eur                    | 0
 wage_eur                     | 0
 age                          | 0
 height_cm                    | 0
 weight_kg                    | 0
 preferred_foot               | 0
 weak_foot                    | 0
 skill_moves                  | 0
 international_reputation     | 0
 attacking_crossing           | 0
 attacking_finishing          | 0
 attacking_heading_accuracy   | 0
 attacking_short_passing      | 0
 attacking_volleys            | 0
 skill_dribbling              | 0
 skill_curve                  | 0
 skill_fk_accuracy            | 0
 skill_long_passing           | 0
 skill_ball_control           | 0
 movement_acceleration        | 0
 movement_sprint_speed        | 0
 movement_agility             | 0
 movement_reactions           | 0
 movement_balance             | 0
 power_shot_power             | 0
 power_jumping                | 0
 power_stamina                | 0
 power_strength               | 0
 power_long_shots             | 0
 mentality_aggression         | 0
 mentality_interceptions      | 0
 mentality_positioning        | 0
 mentality_vision             | 0
 mentality_penalties          | 0
 defending_marking_awareness  | 0
 defending_standing_tackle    | 0
 defending_sliding_tackle     | 0
 goalkeeping_diving           | 0
 goalkeeping_handling         | 0
 goalkeeping_kicking          | 0
 goalkeeping_positioning      | 0
 goalkeeping_reflexes         | 0
 ls                           | 0
 st                           | 0
 rs                           | 0
```

```
lw                  | 0
lf                  | 0
cf                  | 0
rf                  | 0
rw                  | 0
lam                 | 0
cam                 | 0
ram                 | 0
lm                  | 0
lcm                 | 0
cm                  | 0
rcm                 | 0
rm                  | 0
lwb                 | 0
ldm                 | 0
cdm                 | 0
rdm                 | 0
rwb                 | 0
lb                  | 0
lcb                 | 0
cb                  | 0
rcb                 | 0
rb                  | 0
gk                  | 0
Year                | 0
pace                | 0
shooting            | 0
passing             | 0
dribbling           | 0
defending           | 0
physic              | 0
overall_new         | 0
```

In [53]:
```
# we assembled the features in the data to a vector so that it can be recognized by pyspark models
```

In [54]:
```
def transData(data):
    return data.rdd.map(lambda r: [r[-1], Vectors.dense(r[:-1])]).\
           toDF(['output','features'])

data= transData(df_read_features)
data.show()
```

```
+------+--------------------+
|output|            features|
+------+--------------------+
|  69.0|[190745.0,74.0,11...|
|  69.0|[190780.0,73.0,10...|
|  69.0|[190783.0,69.0,65...|
|  69.0|[190792.0,72.0,10...|
|  69.0|[190883.0,69.0,77...|
|  69.0|[191079.0,73.0,11...|
|  69.0|[191096.0,70.0,95...|
|  69.0|[191135.0,72.0,12...|
|  69.0|[191222.0,71.0,10...|
|  69.0|[191252.0,69.0,10...|
|  69.0|[191443.0,69.0,32...|
|  69.0|[191548.0,69.0,65...|
|  69.0|[191679.0,69.0,72...|
|  69.0|[191834.0,69.0,82...|
|  69.0|[192009.0,76.0,12...|
|  69.0|[192179.0,69.0,87...|
|  69.0|[192297.0,69.0,87...|
|  69.0|[192449.0,72.0,12...|
|  69.0|[192457.0,70.0,10...|
|  69.0|[192473.0,69.0,11...|
+------+--------------------+
only showing top 20 rows
```

In [55]:
```
# we normalized the features by using StandardScaler
```

In [56]:
```
from pyspark.ml.feature import StandardScaler
Scalerizer=StandardScaler().setInputCol("features").setOutputCol("norm_features")
data_norm = Scalerizer.fit(data).transform(data)
```

In [57]:
```
data_norm = data_norm.drop('features')
data_norm = data_norm.withColumn('features',col('norm_features')).drop('norm_features')
```

In [58]:
```
# data_norm.select('features').show(vertical = False)
```

In [59]:
```
# (trainingData, testData) = data_norm.randomSplit([0.8, 0.2])

# then we splited the data in to training and test dataset

(trainingData, testData) = data_norm.randomSplit([0.8, 0.2])
```

In [60]:
```
trainingData.describe().show()
```

```
+-------+-----------------+
|summary|           output|
+-------+-----------------+
|  count|           111721|
|   mean|65.68281701739154|
| stddev| 7.07615617760127|
|    min|             40.0|
|    max|             94.0|
+-------+-----------------+
```

In [61]:
```
print("Training Dataset Count: " + str(trainingData.count()))
print("Test Dataset Count: " + str(testData.count()))
```

```
Training Dataset Count: 111721
Test Dataset Count: 28009
```

Pyspark

Random Forest

In [63]:
```
from pyspark.ml.regression import RandomForestRegressor
from pyspark.ml.feature import VectorIndexer
from pyspark.ml.evaluation import RegressionEvaluator
from pyspark.ml.tuning import ParamGridBuilder, CrossValidator

rf = RandomForestRegressor(featuresCol = 'features', labelCol = 'output')

# Create ParamGrid for Cross Validation
rf_paramGrid = (ParamGridBuilder()
             .addGrid(rf.numTrees, [50,100,150,200,300,])
             .addGrid(rf.maxDepth, [3,4,5])
             .build())

evaluator = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")

rf_cv = CrossValidator(estimator=rf, estimatorParamMaps=rf_paramGrid,
                    evaluator=evaluator, numFolds=5)

# Train model.  This also runs the indexer.
model_rf = rf_cv.fit(trainingData)

# Make predictions.
```

```
predictions_rf = model_rf.transform(testData)
predictions_rf.show(5)
```

```
+------+-------------------+------------------+
|output|           features|        prediction|
+------+-------------------+------------------+
|  41.0|[6.40057623302199...|53.669842725778224|
|  42.0|[6.12293559300367...| 53.71640172811433|
|  44.0|[6.45133304662317...| 53.65709730806286|
|  44.0|[6.51719225878871...| 53.86573062028856|
|  44.0|[6.53197579673080...| 53.65709730806286|
+------+-------------------+------------------+
only showing top 5 rows
```

In [65]:
```
evaluator = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")
rmse = evaluator.evaluate(predictions_rf)
print("Root Mean Squared Error (RMSE) on test data = %g" % rmse)
```

```
Root Mean Squared Error (RMSE) on test data = 1.58698
```

Tune the parameters one by one and see the impact of each parameter

In [60]:
```
from pyspark.ml.regression import RandomForestRegressor
from pyspark.ml.feature import VectorIndexer
from pyspark.ml.evaluation import RegressionEvaluator

rf3 = RandomForestRegressor(featuresCol = 'features', labelCol = 'output',numTrees=100, maxDepth=5)

# Train model.  This also runs the indexer.
model3 = rf3.fit(trainingData)

# Make predictions.
predictions3 = model3.transform(testData)

predictions3.show(5)

evaluator3 = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")
rmse3 = evaluator3.evaluate(predictions3)
print("Root Mean Squared Error (RMSE) on test data = %g" % rmse3)
```

```
+------+------------------+-----------------+
|output|          features|       prediction|
+------+------------------+-----------------+
|  41.0|[6.53690364271165...|53.512072880945354|
|  44.0|[6.41599749220878...|53.512072880945354|
|  44.0|[6.58719665904405...|53.512072880945354|
|  44.0|[6.60650222035666...| 53.76495696951452|
|  45.0|[6.17380835592218...| 53.60100459946284|
+------+------------------+-----------------+
only showing top 5 rows

Root Mean Squared Error (RMSE) on test data = 1.60538
```

In [61]:
```
trainingData.show(5)
```

```
+------+------------------+
|output|          features|
+------+------------------+
|  40.0|[6.30738196932836...|
|  40.0|[6.46263810504961...|
|  41.0|[6.40057623302213...|
|  42.0|[6.12293559300381...|
|  42.0|[6.22430428355967...|
+------+------------------+
only showing top 5 rows
```

In [62]:
```
from pyspark.ml.regression import RandomForestRegressor
from pyspark.ml.feature import VectorIndexer
from pyspark.ml.evaluation import RegressionEvaluator

rf_1 = RandomForestRegressor(featuresCol = 'features', labelCol = 'output',numTrees=100, maxDepth=6)

# Train model.  This also runs the indexer.
model_1 = rf_1.fit(trainingData)

# Make predictions.
predictions_1 = model_1.transform(testData)

predictions_1.show(5)

evaluator_1 = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")
rmse_1 = evaluator_1.evaluate(predictions_1)
print("Root Mean Squared Error (RMSE) on test data = %g" % rmse_1)
```

```
+------+-------------------+-----------------+
|output|           features|       prediction|
+------+-------------------+-----------------+
|  41.0|[6.53690364271165...|52.39016251553484|
|  44.0|[6.41599749220878...|52.39016251553484|
|  44.0|[6.58719665904405...|52.39016251553484|
|  44.0|[6.60650222035666...|52.77358279353466|
|  45.0|[6.17380835592218...|52.47513371165757|
+------+-------------------+-----------------+
only showing top 5 rows

Root Mean Squared Error (RMSE) on test data = 1.33804
```

Linear Regression

In [66]:
```
from pyspark.ml.classification import LogisticRegression
from pyspark.ml.tuning import ParamGridBuilder, CrossValidator
from pyspark.ml.evaluation import BinaryClassificationEvaluator

from sklearn.metrics import roc_curve
import pyspark.sql.functions as F
import pyspark.sql.types as T
import numpy
from matplotlib import pyplot as plt

from pyspark.ml.regression import RandomForestRegressor
from pyspark.ml.feature import VectorIndexer
from pyspark.ml.evaluation import RegressionEvaluator

from pyspark.sql.types import Row
from pyspark.ml.linalg import Vectors
from pyspark.ml.classification import MultilayerPerceptronClassifier
```

In [67]:
```
from pyspark.ml.regression import LinearRegression

lrg = LinearRegression(featuresCol = 'features', labelCol = 'output')

# Create ParamGrid for Cross Validation
lrg_paramGrid = ParamGridBuilder() \
    .addGrid(lrg.maxIter, [10, 30,200,10000]) \
    .addGrid(lrg.regParam, [0, 0.1, 2]) \
    .build()

evaluator = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")
```

```
lrg_cv = CrossValidator(estimator=lrg, estimatorParamMaps=lrg_paramGrid,
                    evaluator=evaluator, numFolds=5)

model_lrg = lrg_cv.fit(trainingData)

predictions_lrg = model_lrg.transform(testData)
predictions_lrg.show(5)
```

```
+------+-------------------+-----------------+
|output|           features|       prediction|
+------+-------------------+-----------------+
|  41.0|[6.40057623302199...|46.439497176678344|
|  42.0|[6.12293559300367...|  49.2242871853893|
|  44.0|[6.45133304662317...| 46.83448039704086|
|  44.0|[6.51719225878871...| 43.92618287833449|
|  44.0|[6.53197579673080...| 41.81840415146371|
+------+-------------------+-----------------+
only showing top 5 rows
```

In [69]:
```
evaluator = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")
rmse5 = evaluator.evaluate(predictions_lrg)
print("Root Mean Squared Error (RMSE) on test data = %g" % rmse5)
```

```
Root Mean Squared Error (RMSE) on test data = 1.82176
```

Tune one by one to find the impact for each hyperparameter

In [66]:
```
from pyspark.ml.regression import LinearRegression

# Define LinearRegression algorithm
lrg_1 = LinearRegression(featuresCol = 'features', labelCol = 'output',regParam=2.0)
# maxIter: int = 100
model_1 = lrg_1.fit(trainingData)
predictions_1 = model_1.transform(testData)

predictions_1.show(5)

evaluator_1 = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")
rmse_1 = evaluator_1.evaluate(predictions_1)
print("Root Mean Squared Error (RMSE) on test data = %g" % rmse_1)
```

```
+------+------------------+-----------------+
|output|          features|       prediction|
+------+------------------+-----------------+
|  41.0|[6.53690364271165...|44.227559545991866|
|  44.0|[6.41599749220878...| 47.81024214005528|
|  44.0|[6.58719665904405...|50.707025368422734|
|  44.0|[6.60650222035666...|48.697781658265555|
|  45.0|[6.17380835592218...|48.883788636232325|
+------+------------------+-----------------+
only showing top 5 rows

Root Mean Squared Error (RMSE) on test data = 2.14361
```

In [67]:

```
from pyspark.ml.regression import LinearRegression

# Define LinearRegression algorithm
lrg_2 = LinearRegression(featuresCol = 'features', labelCol = 'output',maxIter=10)
model_2 = lrg_2.fit(trainingData)
predictions_2 = model_2.transform(testData)

predictions_2.show(5)

evaluator_2 = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")
rmse_2 = evaluator_2.evaluate(predictions_2)
print("Root Mean Squared Error (RMSE) on test data = %g" % rmse_2)
```

```
+------+------------------+-----------------+
|output|          features|       prediction|
+------+------------------+-----------------+
|  41.0|[6.53690364271165...| 41.53811522459333|
|  44.0|[6.41599749220878...| 45.92256921728185|
|  44.0|[6.58719665904405...| 48.79175549801562|
|  44.0|[6.60650222035666...|44.533694117718795|
|  45.0|[6.17380835592218...| 45.96323644678648|
+------+------------------+-----------------+
only showing top 5 rows

Root Mean Squared Error (RMSE) on test data = 1.86664
```

In [68]:

```
from pyspark.ml.regression import LinearRegression

# Define LinearRegression algorithm
lrg_3 = LinearRegression(featuresCol = 'features', labelCol = 'output',maxIter=30)
model_3 = lrg_3.fit(trainingData)
predictions_3 = model_3.transform(testData)

predictions_3.show(5)
```

```
evaluator_3 = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")
rmse_3 = evaluator_3.evaluate(predictions_3)
print("Root Mean Squared Error (RMSE) on test data = %g" % rmse_3)
```

```
+------+-------------------+-----------------+
|output|           features|       prediction|
+------+-------------------+-----------------+
|  41.0|[6.53690364271165...| 40.53397568304155|
|  44.0|[6.41599749220878...| 45.15551922724518|
|  44.0|[6.58719665904405...|48.524183250218016|
|  44.0|[6.60650222035666...| 44.02133666015817|
|  45.0|[6.17380835592218...| 45.43579327618232|
+------+-------------------+-----------------+
only showing top 5 rows

Root Mean Squared Error (RMSE) on test data = 1.83285
```

In [69]:

```
from pyspark.ml.regression import LinearRegression

# Define LinearRegression algorithm
lrg_4 = LinearRegression(featuresCol = 'features', labelCol = 'output',maxIter=200)
model_4 = lrg_4.fit(trainingData)
predictions_4 = model_4.transform(testData)

predictions_4.show(5)

evaluator_4 = RegressionEvaluator(
    labelCol="output", predictionCol="prediction", metricName="rmse")
rmse_4 = evaluator_4.evaluate(predictions_4)
print("Root Mean Squared Error (RMSE) on test data = %g" % rmse_4)
```

```
+------+-------------------+-----------------+
|output|           features|       prediction|
+------+-------------------+-----------------+
|  41.0|[6.53690364271165...|40.73535289532248|
|  44.0|[6.41599749220878...|45.15357021804499|
|  44.0|[6.58719665904405...|48.66151100088803|
|  44.0|[6.60650222035666...|  44.129994473203|
|  45.0|[6.17380835592218...|45.64501921754493|
+------+-------------------+-----------------+
only showing top 5 rows

Root Mean Squared Error (RMSE) on test data = 1.82613
```

Tensorflow

In [70]:
```
from pyspark.sql.types import FloatType
```

In [71]:
```
to_array = udf(lambda v: v.toArray().tolist(), ArrayType(FloatType()))

df_test = testData
df_validate,df_train = trainingData.randomSplit([0.5,0.5])

df_train_pandas = df_train.withColumn('features', to_array('features')).toPandas()
df_validate_pandas = df_validate.withColumn('features', to_array('features')).toPandas()
df_test_pandas = df_test.withColumn('features', to_array('features')).toPandas()
```

In [72]:
```
import tensorflow as tf
from tensorflow import keras

x_train = tf.constant(np.array(df_train_pandas['features'].values.tolist()))
y_train = tf.constant(np.array(df_train_pandas['output'].values.tolist()))

x_validate = tf.constant(np.array(df_validate_pandas['features'].values.tolist()))
y_validate = tf.constant(np.array(df_validate_pandas['output'].values.tolist()))


x_test = tf.constant(np.array(df_test_pandas['features'].values.tolist()))
y_test = tf.constant(np.array(df_test_pandas['output'].values.tolist()))
```

In [73]:
```
print(x_train)
print(y_train)
```

```
tf.Tensor(
[[6.30738211e+00 7.97132349e+00 2.59242672e-03 ... 3.16481352e+00
  3.29432011e+00 7.40527344e+00]
 [6.22430420e+00 8.29017639e+00 3.45656904e-03 ... 6.43171787e+00
  3.48437715e+00 7.18747139e+00]
 [6.40443134e+00 8.60902977e+00 3.45656904e-03 ... 6.43171787e+00
  3.48437715e+00 7.18747139e+00]
 ...
 [6.02965415e-01 1.49860888e+01 1.50360746e+01 ... 9.29025936e+00
  2.09062624e+00 8.71208668e+00]
 [6.02965415e-01 1.49860888e+01 1.65051174e+01 ... 9.18816853e+00
  2.09062624e+00 8.71208668e+00]
 [4.58066463e+00 1.51455145e+01 1.91839581e+01 ... 9.69862270e+00
  1.52045548e+00 6.75186682e+00]], shape=(56176, 78), dtype=float64)
tf.Tensor([40. 42. 42. ... 94. 94. 94.], shape=(56176,), dtype=float64)
```

Neural networks

In [74]:
```
model_nn = keras.Sequential( [keras.layers.Dense(78,activation='relu'),
```

```
                        keras.layers.Dense(10,activation='relu'),
                        keras.layers.Dense(10,activation='relu'),
                        keras.layers.Dense(10,activation='relu') ,
                        keras.layers.Dense(1)] )
```

In [75]:
```
y_pred = model_nn(x_train)
model_nn.summary()
```

```
Model: "sequential"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense (Dense)               (56176, 78)               6162

 dense_1 (Dense)             (56176, 10)               790

 dense_2 (Dense)             (56176, 10)               110

 dense_3 (Dense)             (56176, 10)               110

 dense_4 (Dense)             (56176, 1)                11

=================================================================
Total params: 7,183
Trainable params: 7,183
Non-trainable params: 0
_________________________________________________________________
```

In [76]:
```
print(y_pred)
print(y_train)
```

```
tf.Tensor(
[[74.732086]
 [74.12477 ]
 [74.16112 ]
 ...
 [73.94921 ]
 [74.17415 ]
 [74.34247 ]], shape=(56176, 1), dtype=float32)
tf.Tensor([40. 42. 42. ... 94. 94. 94.], shape=(56176,), dtype=float64)
```

In [77]:
```
mse = keras.losses.MeanSquaredError()

model_nn.compile(optimizer = 'adam',
    loss=mse,
    metrics=[mse])
model_nn.fit(x_train,y_train, epochs = 20,validation_data=(x_validate,y_validate),verbose = 2)
```

```
loss = mse(y_train, y_pred).numpy()
print(loss)
```

```
Epoch 1/20
1756/1756 - 6s - loss: 14.1575 - mean_squared_error: 14.1562 - val_loss: 13.5213 - val_mean_squared_error: 13.6656 - 6s/epoch - 3ms/step
Epoch 2/20
1756/1756 - 5s - loss: 7.0432 - mean_squared_error: 7.0433 - val_loss: 6.2357 - val_mean_squared_error: 6.2705 - 5s/epoch - 3ms/step
Epoch 3/20
1756/1756 - 5s - loss: 5.6210 - mean_squared_error: 5.6212 - val_loss: 6.5882 - val_mean_squared_error: 6.6034 - 5s/epoch - 3ms/step
Epoch 4/20
1756/1756 - 5s - loss: 4.9807 - mean_squared_error: 4.9805 - val_loss: 4.2374 - val_mean_squared_error: 4.2553 - 5s/epoch - 3ms/step
Epoch 5/20
1756/1756 - 5s - loss: 4.7245 - mean_squared_error: 4.7238 - val_loss: 4.8499 - val_mean_squared_error: 4.8806 - 5s/epoch - 3ms/step
Epoch 6/20
1756/1756 - 6s - loss: 4.5663 - mean_squared_error: 4.5655 - val_loss: 4.4919 - val_mean_squared_error: 4.5089 - 6s/epoch - 3ms/step
Epoch 7/20
1756/1756 - 5s - loss: 4.3092 - mean_squared_error: 4.3104 - val_loss: 5.9678 - val_mean_squared_error: 5.9769 - 5s/epoch - 3ms/step
Epoch 8/20
1756/1756 - 5s - loss: 3.9615 - mean_squared_error: 3.9619 - val_loss: 7.5420 - val_mean_squared_error: 7.5455 - 5s/epoch - 3ms/step
Epoch 9/20
1756/1756 - 4s - loss: 3.6190 - mean_squared_error: 3.6186 - val_loss: 4.1830 - val_mean_squared_error: 4.1947 - 4s/epoch - 2ms/step
Epoch 10/20
1756/1756 - 5s - loss: 3.0706 - mean_squared_error: 3.0711 - val_loss: 2.4483 - val_mean_squared_error: 2.4569 - 5s/epoch - 3ms/step
Epoch 11/20
1756/1756 - 5s - loss: 2.8021 - mean_squared_error: 2.8017 - val_loss: 3.2102 - val_mean_squared_error: 3.2180 - 5s/epoch - 3ms/step
Epoch 12/20
1756/1756 - 5s - loss: 2.6239 - mean_squared_error: 2.6245 - val_loss: 2.4021 - val_mean_squared_error: 2.4094 - 5s/epoch - 3ms/step
Epoch 13/20
1756/1756 - 5s - loss: 2.5039 - mean_squared_error: 2.5045 - val_loss: 4.0864 - val_mean_squared_error: 4.0895 - 5s/epoch - 3ms/step
Epoch 14/20
1756/1756 - 5s - loss: 2.3757 - mean_squared_error: 2.3755 - val_loss: 2.2760 - val_mean_squared_error: 2.2826 - 5s/epoch - 3ms/step
Epoch 15/20
1756/1756 - 4s - loss: 2.2276 - mean_squared_error: 2.2274 - val_loss: 2.0466 - val_mean_squared_error: 2.0521 - 4s/epoch - 3ms/step
Epoch 16/20
1756/1756 - 5s - loss: 2.1436 - mean_squared_error: 2.1435 - val_loss: 1.8553 - val_mean_squared_error: 1.8600 - 5s/epoch - 3ms/step
Epoch 17/20
1756/1756 - 5s - loss: 1.9760 - mean_squared_error: 1.9758 - val_loss: 1.9688 - val_mean_squared_error: 1.9746 - 5s/epoch - 3ms/step
Epoch 18/20
1756/1756 - 5s - loss: 1.9285 - mean_squared_error: 1.9283 - val_loss: 1.8612 - val_mean_squared_error: 1.8662 - 5s/epoch - 3ms/step
Epoch 19/20
1756/1756 - 4s - loss: 1.8439 - mean_squared_error: 1.8442 - val_loss: 1.6813 - val_mean_squared_error: 1.6880 - 4s/epoch - 3ms/step
Epoch 20/20
1756/1756 - 5s - loss: 1.8171 - mean_squared_error: 1.8171 - val_loss: 1.6386 - val_mean_squared_error: 1.6437 - 5s/epoch - 3ms/step
132.08322
```

In [78]:
```
model_nn.evaluate(x_test,y_test, verbose = 2)
```

```
869/869 - 1s - loss: 1.6491 - mean_squared_error: 1.6550 - 985ms/epoch - 1ms/step
```

Out[78]:
```
[1.6490752696990967, 1.6549885272979736]
```

In [79]:
```
def cross_valiation(hyper,k,s_r,x,y,logdir):

    def data_split():
        for i in range(k):
            idx=tf.range(df_train_pandas.shape[0])
            splt_idx = int(s_r*df_train_pandas.shape[0])
            idx = tf.random.shuffle(idx)
            x_train, y_train = tf.gather(x, idx[:splt_idx]), tf.gather(y, idx[:splt_idx])
            x_valid, y_valid = tf.gather(x, idx[splt_idx:]), tf.gather(y, idx[splt_idx:])
            return x_train,y_train,x_valid,y_valid

    model = keras.Sequential()
    for _ in range(hparams[HP_DEPTH]):
        model.add(keras.layers.Dense(hparams[HP_WIDTH],activation='relu'))
        model.add(keras.layers.Dense(1))
        model.compile(optimizer = 'adam',
            loss=keras.losses.MeanSquaredError(),
            metrics=[keras.losses.MeanSquaredError(name = 'MSE')])
        x_train,y_train,x_valid,y_valid = data_split()
        history = model.fit(x_train, y_train, epochs=5, verbose = 2,validation_data = (x_valid, y_valid),
                  callbacks=[tf.keras.callbacks.TensorBoard(log_dir=logdir, histogram_freq=1)])
        accuracy = np.mean(history.history["MSE"])
        model.summary()
    return accuracy
```

In [80]:
```
from tensorboard.plugins.hparams import api as hp

HP_WIDTH = hp.HParam('NN_width', hp.Discrete([10,20,30]))
HP_DEPTH = hp.HParam('NN_depth', hp.Discrete([3,4,5]))


with tf.summary.create_file_writer('logs14813/hparam_tuning').as_default():
    hp.hparams_config(
    hparams=[HP_WIDTH, HP_DEPTH],
    metrics=[hp.Metric('MSE')],
  )
```

In [81]:
```
import datetime
for hp_width in HP_WIDTH.domain.values:
    for hp_depth in (HP_DEPTH.domain.values):
        hparams = {
            HP_WIDTH: hp_width,
            HP_DEPTH: hp_depth,
        }
        run_name = f"run-WIDTH{int(hparams[HP_WIDTH])}-DEPTH{hparams[HP_DEPTH]}"
```

```
        print('--- Starting trial: %s' % run_name)
        print({h.name: hparams[h] for h in hparams})

        run_dir = 'logs14813/hparam_tuning/' + datetime.datetime.now().strftime("%Y%m%d-%H%M%S") + run_name
        accuracy = cross_valiation(hparams,10,0.7,x_train, y_train,run_dir)

        with tf.summary.create_file_writer(run_dir).as_default():
          hp.hparams(hparams)  # record the values used in this trial
          tf.summary.scalar("MSE", accuracy, step=1)
```

```
--- Starting trial: run-WIDTH10-DEPTH3
{'NN_width': 10, 'NN_depth': 3}
Epoch 1/5
1229/1229 - 3s - loss: 69.7070 - MSE: 69.7002 - val_loss: 17.4015 - val_MSE: 17.4038 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 12.9095 - MSE: 12.9097 - val_loss: 11.1434 - val_MSE: 11.1429 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 9.6867 - MSE: 9.6868 - val_loss: 9.2053 - val_MSE: 9.2047 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 8.1534 - MSE: 8.1532 - val_loss: 7.4019 - val_MSE: 7.4012 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 7.0980 - MSE: 7.0975 - val_loss: 6.2104 - val_MSE: 6.2098 - 2s/epoch - 2ms/step
Model: "sequential_1"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_5 (Dense)             (None, 10)                790

 dense_6 (Dense)             (None, 1)                 11

=================================================================
Total params: 801
Trainable params: 801
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 109.6528 - MSE: 109.6406 - val_loss: 14.2646 - val_MSE: 14.2666 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 10.5917 - MSE: 10.5912 - val_loss: 8.3117 - val_MSE: 8.3121 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 7.8114 - MSE: 7.8115 - val_loss: 7.5303 - val_MSE: 7.5294 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 6.2548 - MSE: 6.2548 - val_loss: 5.9674 - val_MSE: 5.9678 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 5.4212 - MSE: 5.4209 - val_loss: 5.0603 - val_MSE: 5.0598 - 2s/epoch - 2ms/step
Model: "sequential_1"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_5 (Dense)             (None, 10)                790

 dense_6 (Dense)             (None, 1)                 11

 dense_7 (Dense)             (None, 10)                20

 dense_8 (Dense)             (None, 1)                 11

=================================================================
```

```
Total params: 832
Trainable params: 832
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 190.4884 - MSE: 190.4648 - val_loss: 6.1861 - val_MSE: 6.1857 - 3s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 2s - loss: 5.3184 - MSE: 5.3182 - val_loss: 5.0793 - val_MSE: 5.0785 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 4.6593 - MSE: 4.6594 - val_loss: 4.5452 - val_MSE: 4.5443 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 4.2538 - MSE: 4.2538 - val_loss: 4.2055 - val_MSE: 4.2049 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 3s - loss: 3.9812 - MSE: 3.9811 - val_loss: 3.8692 - val_MSE: 3.8684 - 3s/epoch - 2ms/step
Model: "sequential_1"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_5 (Dense)             (None, 10)                790

 dense_6 (Dense)             (None, 1)                 11

 dense_7 (Dense)             (None, 10)                20

 dense_8 (Dense)             (None, 1)                 11

 dense_9 (Dense)             (None, 10)                20

 dense_10 (Dense)            (None, 1)                 11

=================================================================
Total params: 863
Trainable params: 863
Non-trainable params: 0
_________________________________________________________________
--- Starting trial: run-WIDTH10-DEPTH4
{'NN_width': 10, 'NN_depth': 4}
Epoch 1/5
1229/1229 - 3s - loss: 522.6877 - MSE: 522.6264 - val_loss: 27.7947 - val_MSE: 27.8055 - 3s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 3s - loss: 24.4647 - MSE: 24.4640 - val_loss: 20.5609 - val_MSE: 20.5685 - 3s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 17.4486 - MSE: 17.4476 - val_loss: 14.3351 - val_MSE: 14.3391 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 12.9500 - MSE: 12.9502 - val_loss: 11.2775 - val_MSE: 11.2796 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 11.0090 - MSE: 11.0100 - val_loss: 10.6013 - val_MSE: 10.6031 - 2s/epoch - 2ms/step
Model: "sequential_2"
```

```
 Layer (type)                Output Shape              Param #
=================================================================
 dense_11 (Dense)            (None, 10)                790

 dense_12 (Dense)            (None, 1)                 11

=================================================================
Total params: 801
Trainable params: 801
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 458.4666 - MSE: 458.4126 - val_loss: 63.8939 - val_MSE: 63.9003 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 53.6201 - MSE: 53.6201 - val_loss: 44.1140 - val_MSE: 44.1183 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 37.9260 - MSE: 37.9247 - val_loss: 31.1998 - val_MSE: 31.2029 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 24.7849 - MSE: 24.7828 - val_loss: 17.4751 - val_MSE: 17.4755 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 13.5879 - MSE: 13.5877 - val_loss: 10.6515 - val_MSE: 10.6500 - 2s/epoch - 2ms/step
Model: "sequential_2"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_11 (Dense)            (None, 10)                790

 dense_12 (Dense)            (None, 1)                 11

 dense_13 (Dense)            (None, 10)                20

 dense_14 (Dense)            (None, 1)                 11

=================================================================
Total params: 832
Trainable params: 832
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 695.4990 - MSE: 695.4178 - val_loss: 57.3379 - val_MSE: 57.3541 - 3s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 2s - loss: 46.6103 - MSE: 46.6133 - val_loss: 40.3770 - val_MSE: 40.3885 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 33.7918 - MSE: 33.7906 - val_loss: 29.0054 - val_MSE: 29.0139 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 21.5343 - MSE: 21.5333 - val_loss: 15.7405 - val_MSE: 15.7458 - 2s/epoch - 2ms/step
Epoch 5/5
```

```
1229/1229 - 2s - loss: 11.1094 - MSE: 11.1087 - val_loss: 10.1750 - val_MSE: 10.1787 - 2s/epoch - 2ms/step
Model: "sequential_2"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_11 (Dense)            (None, 10)                790

 dense_12 (Dense)            (None, 1)                 11

 dense_13 (Dense)            (None, 10)                20

 dense_14 (Dense)            (None, 1)                 11

 dense_15 (Dense)            (None, 10)                20

 dense_16 (Dense)            (None, 1)                 11

=================================================================
Total params: 863
Trainable params: 863
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 4s - loss: 1248.1918 - MSE: 1248.0376 - val_loss: 45.5892 - val_MSE: 45.5894 - 4s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 2s - loss: 39.1724 - MSE: 39.1737 - val_loss: 31.6969 - val_MSE: 31.6968 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 26.6132 - MSE: 26.6122 - val_loss: 19.4637 - val_MSE: 19.4625 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 14.2774 - MSE: 14.2768 - val_loss: 12.0076 - val_MSE: 12.0094 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 10.2604 - MSE: 10.2603 - val_loss: 10.2550 - val_MSE: 10.2551 - 2s/epoch - 2ms/step
Model: "sequential_2"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_11 (Dense)            (None, 10)                790

 dense_12 (Dense)            (None, 1)                 11

 dense_13 (Dense)            (None, 10)                20

 dense_14 (Dense)            (None, 1)                 11

 dense_15 (Dense)            (None, 10)                20

 dense_16 (Dense)            (None, 1)                 11
```

```
 dense_17 (Dense)            (None, 10)                20

 dense_18 (Dense)            (None, 1)                 11

=================================================================
Total params: 894
Trainable params: 894
Non-trainable params: 0
_________________________________________________________________
--- Starting trial: run-WIDTH10-DEPTH5
{'NN_width': 10, 'NN_depth': 5}
Epoch 1/5
1229/1229 - 3s - loss: 476.5839 - MSE: 476.5289 - val_loss: 29.2964 - val_MSE: 29.3007 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 25.9606 - MSE: 25.9594 - val_loss: 21.6556 - val_MSE: 21.6594 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 18.3905 - MSE: 18.3901 - val_loss: 15.0761 - val_MSE: 15.0795 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 13.1669 - MSE: 13.1660 - val_loss: 10.7960 - val_MSE: 10.8009 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 10.2869 - MSE: 10.2864 - val_loss: 9.2446 - val_MSE: 9.2490 - 2s/epoch - 2ms/step
Model: "sequential_3"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_19 (Dense)            (None, 10)                790

 dense_20 (Dense)            (None, 1)                 11

=================================================================
Total params: 801
Trainable params: 801
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 318.6873 - MSE: 318.6536 - val_loss: 46.5412 - val_MSE: 46.5286 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 38.1630 - MSE: 38.1622 - val_loss: 29.2737 - val_MSE: 29.2646 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 21.8978 - MSE: 21.8968 - val_loss: 13.8629 - val_MSE: 13.8567 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 11.5420 - MSE: 11.5418 - val_loss: 10.3700 - val_MSE: 10.3655 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 9.5435 - MSE: 9.5431 - val_loss: 10.1343 - val_MSE: 10.1304 - 2s/epoch - 2ms/step
Model: "sequential_3"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
```

```
 dense_19 (Dense)            (None, 10)                790

 dense_20 (Dense)            (None, 1)                 11

 dense_21 (Dense)            (None, 10)                20

 dense_22 (Dense)            (None, 1)                 11

=================================================================
Total params: 832
Trainable params: 832
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 416.6308 - MSE: 416.5794 - val_loss: 11.6535 - val_MSE: 11.6511 - 3s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 2s - loss: 9.9114 - MSE: 9.9145 - val_loss: 9.2920 - val_MSE: 9.2899 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 3s - loss: 8.6710 - MSE: 8.6710 - val_loss: 8.3207 - val_MSE: 8.3186 - 3s/epoch - 3ms/step
Epoch 4/5
1229/1229 - 3s - loss: 8.0196 - MSE: 8.0195 - val_loss: 7.8673 - val_MSE: 7.8651 - 3s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 3s - loss: 7.5011 - MSE: 7.5012 - val_loss: 7.2152 - val_MSE: 7.2132 - 3s/epoch - 2ms/step
Model: "sequential_3"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_19 (Dense)            (None, 10)                790

 dense_20 (Dense)            (None, 1)                 11

 dense_21 (Dense)            (None, 10)                20

 dense_22 (Dense)            (None, 1)                 11

 dense_23 (Dense)            (None, 10)                20

 dense_24 (Dense)            (None, 1)                 11

=================================================================
Total params: 863
Trainable params: 863
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 2305.4561 - MSE: 2305.1672 - val_loss: 48.7877 - val_MSE: 48.8102 - 3s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 2s - loss: 41.6964 - MSE: 41.6969 - val_loss: 37.7130 - val_MSE: 37.7367 - 2s/epoch - 2ms/step
```

```
Epoch 3/5
1229/1229 - 2s - loss: 31.7350 - MSE: 31.7346 - val_loss: 27.1738 - val_MSE: 27.1860 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 3s - loss: 21.3779 - MSE: 21.3773 - val_loss: 15.5924 - val_MSE: 15.5978 - 3s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 3s - loss: 13.5231 - MSE: 13.5230 - val_loss: 11.8494 - val_MSE: 11.8506 - 3s/epoch - 2ms/step
Model: "sequential_3"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_19 (Dense)            (None, 10)                790

 dense_20 (Dense)            (None, 1)                 11

 dense_21 (Dense)            (None, 10)                20

 dense_22 (Dense)            (None, 1)                 11

 dense_23 (Dense)            (None, 10)                20

 dense_24 (Dense)            (None, 1)                 11

 dense_25 (Dense)            (None, 10)                20

 dense_26 (Dense)            (None, 1)                 11

=================================================================
Total params: 894
Trainable params: 894
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 513.7387 - MSE: 513.6749 - val_loss: 12.0590 - val_MSE: 12.0553 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 11.2359 - MSE: 11.2354 - val_loss: 9.9632 - val_MSE: 9.9608 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 9.6040 - MSE: 9.6044 - val_loss: 9.6938 - val_MSE: 9.6929 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 8.2819 - MSE: 8.2816 - val_loss: 7.9046 - val_MSE: 7.9032 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 7.0416 - MSE: 7.0414 - val_loss: 6.7960 - val_MSE: 6.7959 - 2s/epoch - 2ms/step
Model: "sequential_3"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_19 (Dense)            (None, 10)                790

 dense_20 (Dense)            (None, 1)                 11
```

```
 dense_21 (Dense)            (None, 10)                20

 dense_22 (Dense)            (None, 1)                 11

 dense_23 (Dense)            (None, 10)                20

 dense_24 (Dense)            (None, 1)                 11

 dense_25 (Dense)            (None, 10)                20

 dense_26 (Dense)            (None, 1)                 11

 dense_27 (Dense)            (None, 10)                20

 dense_28 (Dense)            (None, 1)                 11

=================================================================
Total params: 925
Trainable params: 925
Non-trainable params: 0
_________________________________________________________________
--- Starting trial: run-WIDTH20-DEPTH3
{'NN_width': 20, 'NN_depth': 3}
Epoch 1/5
1229/1229 - 3s - loss: 31.6602 - MSE: 31.6572 - val_loss: 12.7759 - val_MSE: 12.7738 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 10.3628 - MSE: 10.3627 - val_loss: 8.5658 - val_MSE: 8.5664 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 8.1465 - MSE: 8.1467 - val_loss: 7.4850 - val_MSE: 7.4860 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 7.0048 - MSE: 7.0048 - val_loss: 6.7486 - val_MSE: 6.7499 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 6.2607 - MSE: 6.2607 - val_loss: 5.9642 - val_MSE: 5.9654 - 2s/epoch - 2ms/step
Model: "sequential_4"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_29 (Dense)            (None, 20)                1580

 dense_30 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,601
Trainable params: 1,601
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
```

```
1229/1229 - 3s - loss: 214.7521 - MSE: 214.7276 - val_loss: 31.8031 - val_MSE: 31.8015 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 19.2511 - MSE: 19.2499 - val_loss: 11.7134 - val_MSE: 11.7114 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 10.1315 - MSE: 10.1310 - val_loss: 10.1297 - val_MSE: 10.1281 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 8.0900 - MSE: 8.0897 - val_loss: 6.9480 - val_MSE: 6.9468 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 6.5768 - MSE: 6.5765 - val_loss: 6.3803 - val_MSE: 6.3799 - 2s/epoch - 2ms/step
Model: "sequential_4"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_29 (Dense)            (None, 20)                1580

 dense_30 (Dense)            (None, 1)                 21

 dense_31 (Dense)            (None, 20)                40

 dense_32 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,662
Trainable params: 1,662
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 76.4574 - MSE: 76.4488 - val_loss: 10.1545 - val_MSE: 10.1556 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 6.9969 - MSE: 6.9966 - val_loss: 6.2184 - val_MSE: 6.2194 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 3s - loss: 5.7206 - MSE: 5.7206 - val_loss: 5.0210 - val_MSE: 5.0218 - 3s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 3s - loss: 5.1095 - MSE: 5.1093 - val_loss: 4.5184 - val_MSE: 4.5189 - 3s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 4.6499 - MSE: 4.6496 - val_loss: 5.3976 - val_MSE: 5.3975 - 2s/epoch - 2ms/step
Model: "sequential_4"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_29 (Dense)            (None, 20)                1580

 dense_30 (Dense)            (None, 1)                 21

 dense_31 (Dense)            (None, 20)                40

 dense_32 (Dense)            (None, 1)                 21
```

```
 dense_33 (Dense)            (None, 20)                40

 dense_34 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,723
Trainable params: 1,723
Non-trainable params: 0
_________________________________________________________________
--- Starting trial: run-WIDTH20-DEPTH4
{'NN_width': 20, 'NN_depth': 4}
Epoch 1/5
1229/1229 - 3s - loss: 87.3124 - MSE: 87.3030 - val_loss: 19.4831 - val_MSE: 19.4836 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 14.1766 - MSE: 14.1761 - val_loss: 11.4002 - val_MSE: 11.4033 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 9.6470 - MSE: 9.6472 - val_loss: 8.1581 - val_MSE: 8.1596 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 7.2578 - MSE: 7.2576 - val_loss: 6.0401 - val_MSE: 6.0404 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 5.7713 - MSE: 5.7717 - val_loss: 4.7100 - val_MSE: 4.7096 - 2s/epoch - 1ms/step
Model: "sequential_5"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_35 (Dense)            (None, 20)                1580

 dense_36 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,601
Trainable params: 1,601
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 251.6135 - MSE: 251.5826 - val_loss: 8.0732 - val_MSE: 8.0725 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 6.4188 - MSE: 6.4189 - val_loss: 5.5131 - val_MSE: 5.5129 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 5.1755 - MSE: 5.1755 - val_loss: 4.5808 - val_MSE: 4.5806 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 4.4637 - MSE: 4.4635 - val_loss: 4.1960 - val_MSE: 4.1957 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 4.0078 - MSE: 4.0076 - val_loss: 3.7216 - val_MSE: 3.7214 - 2s/epoch - 2ms/step
Model: "sequential_5"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
```

```
 dense_35 (Dense)            (None, 20)                1580

 dense_36 (Dense)            (None, 1)                 21

 dense_37 (Dense)            (None, 20)                40

 dense_38 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,662
Trainable params: 1,662
Non-trainable params: 0

_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 143.1483 - MSE: 143.1321 - val_loss: 16.5469 - val_MSE: 16.5474 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 10.9631 - MSE: 10.9624 - val_loss: 8.3676 - val_MSE: 8.3688 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 7.8332 - MSE: 7.8334 - val_loss: 7.0570 - val_MSE: 7.0579 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 6.8916 - MSE: 6.8923 - val_loss: 6.3456 - val_MSE: 6.3465 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 6.1439 - MSE: 6.1437 - val_loss: 5.7334 - val_MSE: 5.7343 - 2s/epoch - 2ms/step
Model: "sequential_5"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_35 (Dense)            (None, 20)                1580

 dense_36 (Dense)            (None, 1)                 21

 dense_37 (Dense)            (None, 20)                40

 dense_38 (Dense)            (None, 1)                 21

 dense_39 (Dense)            (None, 20)                40

 dense_40 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,723
Trainable params: 1,723
Non-trainable params: 0

_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 16.6831 - MSE: 16.6816 - val_loss: 5.7838 - val_MSE: 5.7839 - 3s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 3s - loss: 5.7062 - MSE: 5.7060 - val_loss: 5.3774 - val_MSE: 5.3773 - 3s/epoch - 2ms/step
```

```
Epoch 3/5
1229/1229 - 2s - loss: 5.3697 - MSE: 5.3693 - val_loss: 5.0290 - val_MSE: 5.0291 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 3s - loss: 5.0689 - MSE: 5.0688 - val_loss: 4.6185 - val_MSE: 4.6184 - 3s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 3s - loss: 4.7122 - MSE: 4.7122 - val_loss: 4.4155 - val_MSE: 4.4156 - 3s/epoch - 2ms/step
Model: "sequential_5"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_35 (Dense)            (None, 20)                1580

 dense_36 (Dense)            (None, 1)                 21

 dense_37 (Dense)            (None, 20)                40

 dense_38 (Dense)            (None, 1)                 21

 dense_39 (Dense)            (None, 20)                40

 dense_40 (Dense)            (None, 1)                 21

 dense_41 (Dense)            (None, 20)                40

 dense_42 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,784
Trainable params: 1,784
Non-trainable params: 0
_________________________________________________________________
--- Starting trial: run-WIDTH20-DEPTH5
{'NN_width': 20, 'NN_depth': 5}
Epoch 1/5
1229/1229 - 3s - loss: 18.4341 - MSE: 18.4346 - val_loss: 10.3422 - val_MSE: 10.3414 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 8.4925 - MSE: 8.4922 - val_loss: 7.1017 - val_MSE: 7.1017 - 2s/epoch - 1ms/step
Epoch 3/5
1229/1229 - 2s - loss: 6.9772 - MSE: 6.9775 - val_loss: 7.1272 - val_MSE: 7.1281 - 2s/epoch - 1ms/step
Epoch 4/5
1229/1229 - 2s - loss: 6.0618 - MSE: 6.0617 - val_loss: 5.2572 - val_MSE: 5.2573 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 5.4154 - MSE: 5.4151 - val_loss: 5.0872 - val_MSE: 5.0878 - 2s/epoch - 2ms/step
Model: "sequential_6"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_43 (Dense)            (None, 20)                1580
```

```
 dense_44 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,601
Trainable params: 1,601
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 222.9749 - MSE: 222.9533 - val_loss: 35.5399 - val_MSE: 35.5379 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 26.8101 - MSE: 26.8090 - val_loss: 17.7155 - val_MSE: 17.7133 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 14.9769 - MSE: 14.9766 - val_loss: 11.9409 - val_MSE: 11.9392 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 10.9279 - MSE: 10.9278 - val_loss: 8.9933 - val_MSE: 8.9924 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 3s - loss: 8.5932 - MSE: 8.5933 - val_loss: 7.6981 - val_MSE: 7.6979 - 3s/epoch - 2ms/step
Model: "sequential_6"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_43 (Dense)            (None, 20)                1580

 dense_44 (Dense)            (None, 1)                 21

 dense_45 (Dense)            (None, 20)                40

 dense_46 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,662
Trainable params: 1,662
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 4s - loss: 20.7785 - MSE: 20.7770 - val_loss: 6.7311 - val_MSE: 6.7292 - 4s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 2s - loss: 6.2699 - MSE: 6.2699 - val_loss: 5.6614 - val_MSE: 5.6597 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 5.1215 - MSE: 5.1213 - val_loss: 4.7349 - val_MSE: 4.7345 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 4.2270 - MSE: 4.2271 - val_loss: 3.6799 - val_MSE: 3.6792 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 3.6098 - MSE: 3.6100 - val_loss: 3.3832 - val_MSE: 3.3830 - 2s/epoch - 2ms/step
Model: "sequential_6"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
```

```
=================================================================
 dense_43 (Dense)            (None, 20)                1580

 dense_44 (Dense)            (None, 1)                 21

 dense_45 (Dense)            (None, 20)                40

 dense_46 (Dense)            (None, 1)                 21

 dense_47 (Dense)            (None, 20)                40

 dense_48 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,723
Trainable params: 1,723
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 185.4881 - MSE: 185.4695 - val_loss: 33.1431 - val_MSE: 33.1431 - 3s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 3s - loss: 18.1768 - MSE: 18.1763 - val_loss: 11.8693 - val_MSE: 11.8703 - 3s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 10.3248 - MSE: 10.3243 - val_loss: 9.1549 - val_MSE: 9.1548 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 8.7906 - MSE: 8.7904 - val_loss: 8.1745 - val_MSE: 8.1740 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 7.7071 - MSE: 7.7068 - val_loss: 6.8180 - val_MSE: 6.8179 - 2s/epoch - 2ms/step
Model: "sequential_6"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_43 (Dense)            (None, 20)                1580

 dense_44 (Dense)            (None, 1)                 21

 dense_45 (Dense)            (None, 20)                40

 dense_46 (Dense)            (None, 1)                 21

 dense_47 (Dense)            (None, 20)                40

 dense_48 (Dense)            (None, 1)                 21

 dense_49 (Dense)            (None, 20)                40

 dense_50 (Dense)            (None, 1)                 21
```

```
=================================================================
Total params: 1,784
Trainable params: 1,784
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 370.6485 - MSE: 370.6024 - val_loss: 10.5139 - val_MSE: 10.5112 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 3s - loss: 7.9271 - MSE: 7.9268 - val_loss: 5.7239 - val_MSE: 5.7227 - 3s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 5.2404 - MSE: 5.2406 - val_loss: 4.5698 - val_MSE: 4.5689 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 3s - loss: 4.1068 - MSE: 4.1066 - val_loss: 3.5299 - val_MSE: 3.5292 - 3s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 3s - loss: 3.3665 - MSE: 3.3664 - val_loss: 2.9228 - val_MSE: 2.9223 - 3s/epoch - 2ms/step
Model: "sequential_6"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_43 (Dense)            (None, 20)                1580

 dense_44 (Dense)            (None, 1)                 21

 dense_45 (Dense)            (None, 20)                40

 dense_46 (Dense)            (None, 1)                 21

 dense_47 (Dense)            (None, 20)                40

 dense_48 (Dense)            (None, 1)                 21

 dense_49 (Dense)            (None, 20)                40

 dense_50 (Dense)            (None, 1)                 21

 dense_51 (Dense)            (None, 20)                40

 dense_52 (Dense)            (None, 1)                 21

=================================================================
Total params: 1,845
Trainable params: 1,845
Non-trainable params: 0
_________________________________________________________________
--- Starting trial: run-WIDTH30-DEPTH3
{'NN_width': 30, 'NN_depth': 3}
Epoch 1/5
1229/1229 - 2s - loss: 22.9941 - MSE: 22.9922 - val_loss: 10.3455 - val_MSE: 10.3508 - 2s/epoch - 2ms/step
```

```
Epoch 2/5
1229/1229 - 2s - loss: 8.4658 - MSE: 8.4654 - val_loss: 7.4150 - val_MSE: 7.4177 - 2s/epoch - 1ms/step
Epoch 3/5
1229/1229 - 2s - loss: 6.1264 - MSE: 6.1260 - val_loss: 4.5978 - val_MSE: 4.5987 - 2s/epoch - 1ms/step
Epoch 4/5
1229/1229 - 2s - loss: 4.8197 - MSE: 4.8196 - val_loss: 4.0424 - val_MSE: 4.0435 - 2s/epoch - 1ms/step
Epoch 5/5
1229/1229 - 2s - loss: 4.1383 - MSE: 4.1386 - val_loss: 3.5956 - val_MSE: 3.5964 - 2s/epoch - 2ms/step
Model: "sequential_7"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_53 (Dense)            (None, 30)                2370

 dense_54 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,401
Trainable params: 2,401
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 287.0855 - MSE: 287.0576 - val_loss: 41.3187 - val_MSE: 41.3190 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 28.4010 - MSE: 28.3999 - val_loss: 18.0117 - val_MSE: 18.0154 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 14.1309 - MSE: 14.1302 - val_loss: 11.5198 - val_MSE: 11.5223 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 10.5202 - MSE: 10.5198 - val_loss: 9.4835 - val_MSE: 9.4843 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 8.0337 - MSE: 8.0334 - val_loss: 6.7466 - val_MSE: 6.7476 - 2s/epoch - 2ms/step
Model: "sequential_7"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_53 (Dense)            (None, 30)                2370

 dense_54 (Dense)            (None, 1)                 31

 dense_55 (Dense)            (None, 30)                60

 dense_56 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,492
Trainable params: 2,492
Non-trainable params: 0
_________________________________________________________________
```

```
Epoch 1/5
1229/1229 - 3s - loss: 248.5283 - MSE: 248.5043 - val_loss: 34.6186 - val_MSE: 34.6245 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 21.0667 - MSE: 21.0650 - val_loss: 10.3852 - val_MSE: 10.3848 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 8.8700 - MSE: 8.8694 - val_loss: 7.0444 - val_MSE: 7.0451 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 5.8768 - MSE: 5.8766 - val_loss: 4.3580 - val_MSE: 4.3597 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 4.2889 - MSE: 4.2886 - val_loss: 3.6039 - val_MSE: 3.6045 - 2s/epoch - 2ms/step
Model: "sequential_7"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_53 (Dense)            (None, 30)                2370

 dense_54 (Dense)            (None, 1)                 31

 dense_55 (Dense)            (None, 30)                60

 dense_56 (Dense)            (None, 1)                 31

 dense_57 (Dense)            (None, 30)                60

 dense_58 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,583
Trainable params: 2,583
Non-trainable params: 0
_________________________________________________________________
--- Starting trial: run-WIDTH30-DEPTH4
{'NN_width': 30, 'NN_depth': 4}
Epoch 1/5
1229/1229 - 2s - loss: 24.3315 - MSE: 24.3302 - val_loss: 9.7458 - val_MSE: 9.7468 - 2s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 8.8406 - MSE: 8.8401 - val_loss: 7.9723 - val_MSE: 7.9725 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 7.0977 - MSE: 7.0977 - val_loss: 5.9458 - val_MSE: 5.9461 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 6.1402 - MSE: 6.1399 - val_loss: 5.1559 - val_MSE: 5.1562 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 5.4999 - MSE: 5.4999 - val_loss: 5.7218 - val_MSE: 5.7225 - 2s/epoch - 2ms/step
Model: "sequential_8"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_59 (Dense)            (None, 30)                2370
```

```
 dense_60 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,401
Trainable params: 2,401
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 442.1064 - MSE: 442.0554 - val_loss: 38.1442 - val_MSE: 38.1435 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 28.7432 - MSE: 28.7421 - val_loss: 20.1864 - val_MSE: 20.1875 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 16.0373 - MSE: 16.0374 - val_loss: 12.7755 - val_MSE: 12.7773 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 11.6391 - MSE: 11.6388 - val_loss: 11.2070 - val_MSE: 11.2079 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 9.1475 - MSE: 9.1474 - val_loss: 7.7600 - val_MSE: 7.7619 - 2s/epoch - 2ms/step
Model: "sequential_8"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_59 (Dense)            (None, 30)                2370

 dense_60 (Dense)            (None, 1)                 31

 dense_61 (Dense)            (None, 30)                60

 dense_62 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,492
Trainable params: 2,492
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 26.9687 - MSE: 26.9660 - val_loss: 7.7166 - val_MSE: 7.7179 - 3s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 2s - loss: 7.1220 - MSE: 7.1216 - val_loss: 6.1439 - val_MSE: 6.1454 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 6.0351 - MSE: 6.0350 - val_loss: 5.6020 - val_MSE: 5.6023 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 5.1482 - MSE: 5.1481 - val_loss: 4.4884 - val_MSE: 4.4893 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 4.5792 - MSE: 4.5789 - val_loss: 4.0759 - val_MSE: 4.0766 - 2s/epoch - 2ms/step
Model: "sequential_8"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
```

```
=================================================================
 dense_59 (Dense)            (None, 30)                2370

 dense_60 (Dense)            (None, 1)                 31

 dense_61 (Dense)            (None, 30)                60

 dense_62 (Dense)            (None, 1)                 31

 dense_63 (Dense)            (None, 30)                60

 dense_64 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,583
Trainable params: 2,583
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 15.1015 - MSE: 15.1002 - val_loss: 4.2403 - val_MSE: 4.2402 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 4.2167 - MSE: 4.2165 - val_loss: 3.8036 - val_MSE: 3.8035 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 3.9636 - MSE: 3.9636 - val_loss: 3.6645 - val_MSE: 3.6640 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 3.7364 - MSE: 3.7366 - val_loss: 5.5376 - val_MSE: 5.5368 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 3.5470 - MSE: 3.5474 - val_loss: 3.1517 - val_MSE: 3.1514 - 2s/epoch - 2ms/step
Model: "sequential_8"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_59 (Dense)            (None, 30)                2370

 dense_60 (Dense)            (None, 1)                 31

 dense_61 (Dense)            (None, 30)                60

 dense_62 (Dense)            (None, 1)                 31

 dense_63 (Dense)            (None, 30)                60

 dense_64 (Dense)            (None, 1)                 31

 dense_65 (Dense)            (None, 30)                60

 dense_66 (Dense)            (None, 1)                 31
```

```
=================================================================
Total params: 2,674
Trainable params: 2,674
Non-trainable params: 0
_________________________________________________________________
--- Starting trial: run-WIDTH30-DEPTH5
{'NN_width': 30, 'NN_depth': 5}
Epoch 1/5
1229/1229 - 2s - loss: 1510.8827 - MSE: 1510.6924 - val_loss: 17.6088 - val_MSE: 17.6064 - 2s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 14.2162 - MSE: 14.2162 - val_loss: 12.1156 - val_MSE: 12.1127 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 10.7347 - MSE: 10.7351 - val_loss: 9.9981 - val_MSE: 9.9953 - 2s/epoch - 1ms/step
Epoch 4/5
1229/1229 - 2s - loss: 9.5285 - MSE: 9.5280 - val_loss: 9.0421 - val_MSE: 9.0392 - 2s/epoch - 1ms/step
Epoch 5/5
1229/1229 - 2s - loss: 8.6157 - MSE: 8.6157 - val_loss: 7.9695 - val_MSE: 7.9668 - 2s/epoch - 1ms/step
Model: "sequential_9"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_67 (Dense)            (None, 30)                2370

 dense_68 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,401
Trainable params: 2,401
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 2s - loss: 221.7756 - MSE: 221.7551 - val_loss: 42.8513 - val_MSE: 42.8545 - 2s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 31.8899 - MSE: 31.8886 - val_loss: 22.0609 - val_MSE: 22.0665 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 16.2120 - MSE: 16.2108 - val_loss: 13.9488 - val_MSE: 13.9515 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 11.5623 - MSE: 11.5625 - val_loss: 10.2698 - val_MSE: 10.2709 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 9.4035 - MSE: 9.4035 - val_loss: 8.7993 - val_MSE: 8.8001 - 2s/epoch - 2ms/step
Model: "sequential_9"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_67 (Dense)            (None, 30)                2370

 dense_68 (Dense)            (None, 1)                 31
```

```
 dense_69 (Dense)            (None, 30)                60

 dense_70 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,492
Trainable params: 2,492
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 178.0667 - MSE: 178.0448 - val_loss: 9.3654 - val_MSE: 9.3620 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 8.7637 - MSE: 8.7635 - val_loss: 8.7139 - val_MSE: 8.7109 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 2s - loss: 7.6368 - MSE: 7.6366 - val_loss: 7.4373 - val_MSE: 7.4349 - 2s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 3s - loss: 6.7849 - MSE: 6.7848 - val_loss: 6.4158 - val_MSE: 6.4148 - 3s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 2s - loss: 5.9909 - MSE: 5.9908 - val_loss: 5.0049 - val_MSE: 5.0038 - 2s/epoch - 2ms/step
Model: "sequential_9"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_67 (Dense)            (None, 30)                2370

 dense_68 (Dense)            (None, 1)                 31

 dense_69 (Dense)            (None, 30)                60

 dense_70 (Dense)            (None, 1)                 31

 dense_71 (Dense)            (None, 30)                60

 dense_72 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,583
Trainable params: 2,583
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 3s - loss: 220.3110 - MSE: 220.2840 - val_loss: 5.8001 - val_MSE: 5.8056 - 3s/epoch - 2ms/step
Epoch 2/5
1229/1229 - 2s - loss: 5.8431 - MSE: 5.8433 - val_loss: 6.1834 - val_MSE: 6.1870 - 2s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 3s - loss: 5.5993 - MSE: 5.5992 - val_loss: 5.4977 - val_MSE: 5.5005 - 3s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 3s - loss: 5.1799 - MSE: 5.1798 - val_loss: 4.4188 - val_MSE: 4.4218 - 3s/epoch - 2ms/step
```

```
Epoch 5/5
1229/1229 - 4s - loss: 4.8030 - MSE: 4.8029 - val_loss: 4.9690 - val_MSE: 4.9708 - 4s/epoch - 3ms/step
Model: "sequential_9"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_67 (Dense)            (None, 30)                2370

 dense_68 (Dense)            (None, 1)                 31

 dense_69 (Dense)            (None, 30)                60

 dense_70 (Dense)            (None, 1)                 31

 dense_71 (Dense)            (None, 30)                60

 dense_72 (Dense)            (None, 1)                 31

 dense_73 (Dense)            (None, 30)                60

 dense_74 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,674
Trainable params: 2,674
Non-trainable params: 0
_________________________________________________________________
Epoch 1/5
1229/1229 - 4s - loss: 22.2586 - MSE: 22.2565 - val_loss: 4.8362 - val_MSE: 4.8354 - 4s/epoch - 3ms/step
Epoch 2/5
1229/1229 - 3s - loss: 4.7347 - MSE: 4.7344 - val_loss: 4.1874 - val_MSE: 4.1866 - 3s/epoch - 2ms/step
Epoch 3/5
1229/1229 - 3s - loss: 4.6565 - MSE: 4.6563 - val_loss: 5.5949 - val_MSE: 5.5945 - 3s/epoch - 2ms/step
Epoch 4/5
1229/1229 - 2s - loss: 4.5016 - MSE: 4.5014 - val_loss: 3.9336 - val_MSE: 3.9329 - 2s/epoch - 2ms/step
Epoch 5/5
1229/1229 - 3s - loss: 4.5062 - MSE: 4.5065 - val_loss: 4.5744 - val_MSE: 4.5736 - 3s/epoch - 2ms/step
Model: "sequential_9"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 dense_67 (Dense)            (None, 30)                2370

 dense_68 (Dense)            (None, 1)                 31

 dense_69 (Dense)            (None, 30)                60

 dense_70 (Dense)            (None, 1)                 31
```

```
 dense_71 (Dense)            (None, 30)                60

 dense_72 (Dense)            (None, 1)                 31

 dense_73 (Dense)            (None, 30)                60

 dense_74 (Dense)            (None, 1)                 31

 dense_75 (Dense)            (None, 30)                60

 dense_76 (Dense)            (None, 1)                 31

=================================================================
Total params: 2,765
Trainable params: 2,765
Non-trainable params: 0
_________________________________________________________________
```

Linear Regression

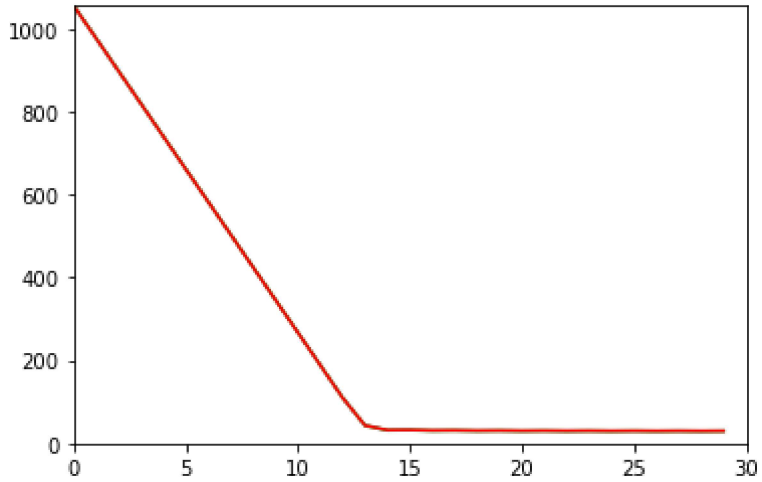
In [82]:

```
tf.compat.v1.disable_eager_execution()
# tf.compat.v1.enable_eager_execution()
import matplotlib.pyplot as plt
import tensorflow as tf
import numpy as np

from sklearn.datasets import load_boston

def append_bias_reshape(x_train,y_train):
    n_training_samples = x_train.shape[0]
    n_dim = x_train.shape[1]
    f = np.reshape(np.c_[np.ones(n_training_samples),x_train],[n_training_samples,n_dim+1])
    l = np.reshape(y_train,[n_training_samples,1])

    return f,l

if __name__ == '__main__':


#     x,y = read_boston_data()
#     norm_features = feature_normalize(x)
    f,l = append_bias_reshape(x_train,y_train)
    n_dim = f.shape[1]

    rnd_indices = np.random.rand(len(f)) < 0.80
```

```
x_train = f[rnd_indices]
y_train = l[rnd_indices]
x_test = f[~rnd_indices]
y_test = l[~rnd_indices]



learning_rate = 0.0001
training_epochs = 30
cost_history = []
test_history = []

X = tf.compat.v1.placeholder(tf.float32,[None,n_dim])
Y = tf.compat.v1.placeholder(tf.float32,[None,1])
W = tf.Variable(tf.ones([n_dim,1]))

init = tf.compat.v1.initialize_all_variables()

y_ = tf.matmul(X,W)
cost = tf.reduce_mean(tf.abs(y_-Y))
training_step = tf.compat.v1.train.GradientDescentOptimizer(learning_rate).minimize(cost)

sess = tf.compat.v1.Session()
sess.run(init)

for epoch in range(training_epochs):
    sess.run(training_step,feed_dict={X:x_train,Y:y_train})
    c = sess.run(cost,feed_dict={X:x_train,Y:y_train})
    print(c)
    t = sess.run(cost,feed_dict={X:x_test,Y:y_test})
    print(t)
    cost_history.append(c)
    test_history.append(t)

plt.plot(range(len(test_history)),test_history,color = 'green')
plt.plot(range(len(cost_history)),cost_history,color = 'red')

plt.axis([0,training_epochs,0,np.max(cost_history)])
plt.show()
```

```
WARNING:tensorflow:From C:\Users\Julie\AppData\Roaming\Python\Python39\site-packages\tensorflow\python\util\tf_should_use.py:243: initializ
e_all_variables (from tensorflow.python.ops.variables) is deprecated and will be removed after 2017-03-02.
Instructions for updating:
Use `tf.global_variables_initializer` instead.
1056.453
1057.1117
977.57007
978.2302
898.6871
899.349
819.8042
820.46765
740.9212
741.58624
662.0383
662.70483
583.1554
583.8235
504.27246
504.9421
425.38953
426.06076
346.5066
347.17938
267.62366
268.29797
188.7407
189.41664
109.858086
110.53527
43.021408
43.00537
32.117527
31.522545
32.228374
32.035816
31.210054
30.619781
31.336395
31.12399
30.76221
30.179024
30.885654
30.656467
30.473074
29.895824
30.635881
30.395695
```

```
30.296503
29.7233
30.465446
30.216637
30.162111
29.593527
30.315517
30.059908
30.061953
29.496569
30.223164
29.963722
30.006186
29.442467
30.151493
29.889187
```



In [86]:

```python
# tf.compat.v1.disable_eager_execution()
# tf.compat.v1.enable_eager_execution()
import matplotlib.pyplot as plt
import tensorflow as tf
import numpy as np

from sklearn.datasets import load_boston

def append_bias_reshape(x_train,y_train):
    n_training_samples = x_train.shape[0]
    n_dim = x_train.shape[1]
    f = np.reshape(np.c_[np.ones(n_training_samples),x_train],[n_training_samples,n_dim+1])
    l = np.reshape(y_train,[n_training_samples,1])

    return f,l
```

```
if __name__ == '__main__':


#     x,y = read_boston_data()
#     norm_features = feature_normalize(x)
    f,l = append_bias_reshape(x_train,y_train)
    n_dim = f.shape[1]

    rnd_indices = np.random.rand(len(f)) < 0.80

    x_train = f[rnd_indices]
    y_train = l[rnd_indices]
    x_test = f[~rnd_indices]
    y_test = l[~rnd_indices]



    learning_rate = 0.001
    training_epochs = 30
    cost_history = []
    test_history = []

    X = tf.compat.v1.placeholder(tf.float32,[None,n_dim])
    Y = tf.compat.v1.placeholder(tf.float32,[None,1])
    W = tf.Variable(tf.ones([n_dim,1]))

    init = tf.compat.v1.initialize_all_variables()

    y_ = tf.matmul(X,W)
    cost = tf.reduce_mean(tf.abs(y_-Y))
    training_step = tf.compat.v1.train.GradientDescentOptimizer(learning_rate).minimize(cost)

    sess = tf.compat.v1.Session()
    sess.run(init)

    for epoch in range(training_epochs):
        sess.run(training_step,feed_dict={X:x_train,Y:y_train})
        c = sess.run(cost,feed_dict={X:x_train,Y:y_train})
        print(c)
        t = sess.run(cost,feed_dict={X:x_test,Y:y_test})
        print(t)
        cost_history.append(c)
        test_history.append(t)

    plt.plot(range(len(test_history)),test_history,color = 'green')
    plt.plot(range(len(cost_history)),cost_history,color = 'red')
```

```
    plt.axis([0,training_epochs,0,np.max(cost_history)])
    plt.show()
```

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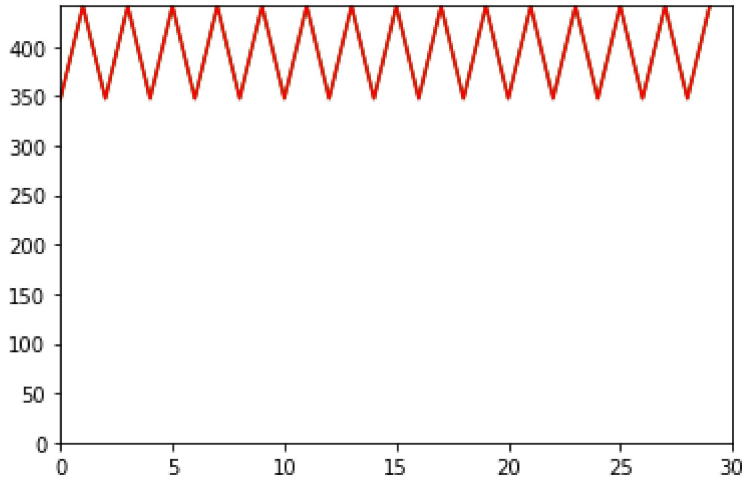
```



In [87]:

```python
# tf.compat.v1.disable_eager_execution()
# tf.compat.v1.enable_eager_execution()
import matplotlib.pyplot as plt
import tensorflow as tf
import numpy as np

from sklearn.datasets import load_boston

def append_bias_reshape(x_train,y_train):
    n_training_samples = x_train.shape[0]
    n_dim = x_train.shape[1]
    f = np.reshape(np.c_[np.ones(n_training_samples),x_train],[n_training_samples,n_dim+1])
    l = np.reshape(y_train,[n_training_samples,1])

    return f,l

if __name__ == '__main__':


#     x,y = read_boston_data()
```

```
#     norm_features = feature_normalize(x)
    f,l = append_bias_reshape(x_train,y_train)
    n_dim = f.shape[1]

    rnd_indices = np.random.rand(len(f)) < 0.80

    x_train = f[rnd_indices]
    y_train = l[rnd_indices]
    x_test = f[~rnd_indices]
    y_test = l[~rnd_indices]



    learning_rate = 0.00001
    training_epochs = 300
    cost_history = []
    test_history = []

    X = tf.compat.v1.placeholder(tf.float32,[None,n_dim])
    Y = tf.compat.v1.placeholder(tf.float32,[None,1])
    W = tf.Variable(tf.ones([n_dim,1]))

    init = tf.compat.v1.initialize_all_variables()

    y_ = tf.matmul(X,W)
    cost = tf.reduce_mean(tf.abs(y_-Y))
    training_step = tf.compat.v1.train.GradientDescentOptimizer(learning_rate).minimize(cost)

    sess = tf.compat.v1.Session()
    sess.run(init)

    for epoch in range(training_epochs):
        sess.run(training_step,feed_dict={X:x_train,Y:y_train})
        c = sess.run(cost,feed_dict={X:x_train,Y:y_train})
        print(c)
        t = sess.run(cost,feed_dict={X:x_test,Y:y_test})
        print(t)
        cost_history.append(c)
        test_history.append(t)

    plt.plot(range(len(test_history)),test_history,color = 'green')
    plt.plot(range(len(cost_history)),cost_history,color = 'red')

    plt.axis([0,training_epochs,0,np.max(cost_history)])
    plt.show()
```

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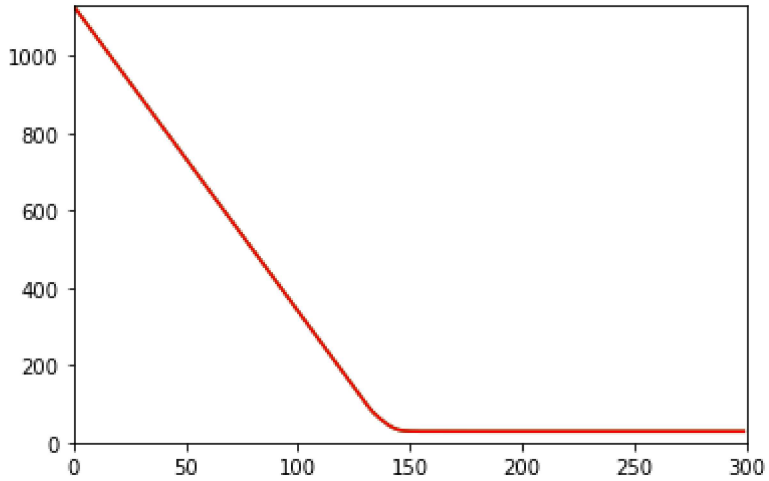

In [88]:

```python
# tf.compat.v1.disable_eager_execution()
# tf.compat.v1.enable_eager_execution()
import matplotlib.pyplot as plt
import tensorflow as tf
import numpy as np

from sklearn.datasets import load_boston
```

```
def append_bias_reshape(x_train,y_train):
    n_training_samples = x_train.shape[0]
    n_dim = x_train.shape[1]
    f = np.reshape(np.c_[np.ones(n_training_samples),x_train],[n_training_samples,n_dim+1])
    l = np.reshape(y_train,[n_training_samples,1])

    return f,l

if __name__ == '__main__':


#     x,y = read_boston_data()
#     norm_features = feature_normalize(x)
    f,l = append_bias_reshape(x_train,y_train)
    n_dim = f.shape[1]

    rnd_indices = np.random.rand(len(f)) < 0.80

    x_train = f[rnd_indices]
    y_train = l[rnd_indices]
    x_test = f[~rnd_indices]
    y_test = l[~rnd_indices]



    learning_rate = 0.000001
    training_epochs = 3000
    cost_history = []
    test_history = []

    X = tf.compat.v1.placeholder(tf.float32,[None,n_dim])
    Y = tf.compat.v1.placeholder(tf.float32,[None,1])
    W = tf.Variable(tf.ones([n_dim,1]))

    init = tf.compat.v1.initialize_all_variables()

    y_ = tf.matmul(X,W)
    cost = tf.reduce_mean(tf.abs(y_-Y))
    training_step = tf.compat.v1.train.GradientDescentOptimizer(learning_rate).minimize(cost)

    sess = tf.compat.v1.Session()
    sess.run(init)

    for epoch in range(training_epochs):
        sess.run(training_step,feed_dict={X:x_train,Y:y_train})
        c = sess.run(cost,feed_dict={X:x_train,Y:y_train})
        print(c)
        t = sess.run(cost,feed_dict={X:x_test,Y:y_test})
```

```
        print(t)
        cost_history.append(c)
        test_history.append(t)

    plt.plot(range(len(test_history)),test_history,color = 'green')
    plt.plot(range(len(cost_history)),cost_history,color = 'red')

    plt.axis([0,training_epochs,0,np.max(cost_history)])
    plt.show()
```

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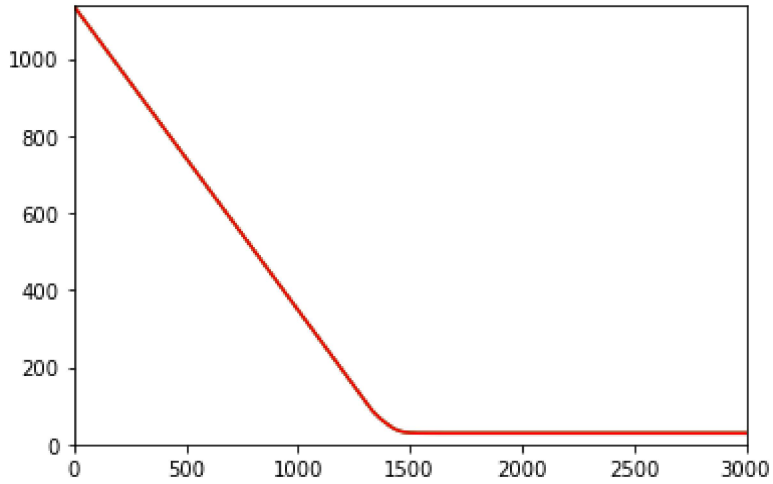

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