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Exercise sheet 10 - Machine Intelligence I

10.1 - Directed Acylic Graphs and Graphical Models

(a)

Nodes represent the random variables, edges the correlative relationship between those nodes, and the edge direction symbolizes causation.

(b)

Two nodes are conditionally independent if their combined probability, given a parent, is equal to the product of individual probabilities of the nodes given the parent. This enables a decomposition of the graph. Conditional Independence is shown in the graph structure as a lack of edges between nodes.

(c)

Here is a step-by-step visualization of the algorithm.

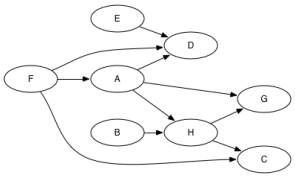


Figure 1: initial state

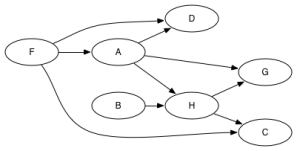


Figure 2: i = 1

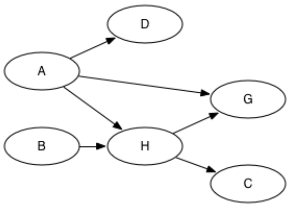
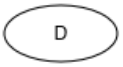


Figure 3: i = 2



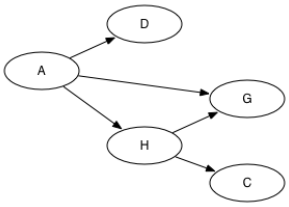


Figure 4: i = 3

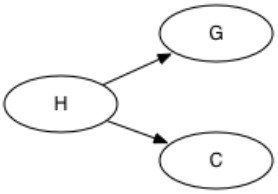


Figure 5: i = 4

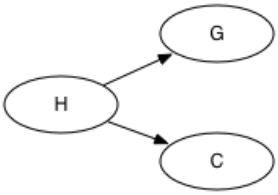


Figure 6: i = 5

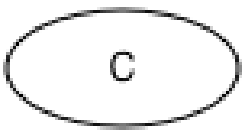


Figure 7: i = 6

Figure 8: i = 7

This results in the topological sorting: E,F,B,A,D,H,C,G

(d)

The factorization of join distribution for the DAG is:
$P(F)*P(E)*P(B)*P(A|F)*P(D|E,A,F)*P(H|B,A,F)*P(G|H,B,A,F)*$
$P(C|H,B,A,F)$
Given conditional independence, this can be reduced to:
$P(F)*P(E)*P(B)*P(A|F)*P(D|E,A)*P(H|B,A)*P(G|H,A)*P(C|H,F)$

(e)

The Markov Blanket of the node, A, is: $\{F,B,E,D,H\}$
Where:
$\{F\}$ is the parent node
$\{D,H\}$ are children nodes
$\{B,E\}$ are the children's parents nodes

(f)

A naive Bayes Classifier assigns a class label to a node x based on the class's probability and the probability of the node belonging to that class.

$$\bar{y} = \underset{k}{\text{argmax}} P(C_k) \prod_i P(x_i|C_k) \tag{1}$$