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Exercise sheet 06 - Machine Intelligence I

6.4

Since the logarithm is a strictly convex function (at least for positive arguments), then *Jensen's inequality* implies:

$$D_{KL}(p||q) := \sum_{x\in X} p(x) log\frac{p(x)}{q(x)} \geq log \sum_{x\in X}(p(x)\frac{p(x)}{q(x)}) = log(1) = 0 \qquad (1)$$

If $\forall x : p(x) = q(x)$ this becomes:

$$D_{KL}(p||q) := \sum_{x\in X} p(x) log\frac{p(x)}{q(x)} = \sum_{x\in X} p(x) log(1) = 0 \qquad (2)$$

If $D_{KL}(p||q) = 0$, then we get:

$$\sum_{x\in X} p(x) log\frac{p(x)}{q(x)} = 0 \implies \sum_{x\in X}(p(x)) \sum_{x\in X} log\frac{p(x)}{q(x)} = 0 \qquad (3)$$

Since $p(x)$ and $q(x)$ are always positive, this implies that for all $x \in X$ we must have:

$$log\frac{p(x)}{q(x)} = 0 \implies \frac{p(x)}{q(x)} = 1 \implies p(x) = q(x)$$