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Exercise sheet 11 - Machine Intelligence I

11.1 - Cliques

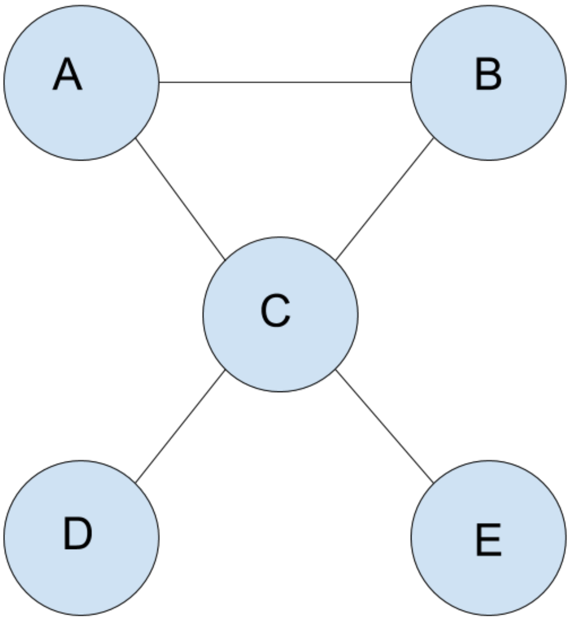
In the given graph, we have 10 vertices and 17 edges. These make up 10 1-vertex cliques and 17 2-vertex cliques. We also identify the 3-vertex cliques consisting of vertices
$\{A, C, H\}, \{A, C, G\}, \{B, C, D\}, \{B, C, G\}, \{C, H, I\}, \{G, H, I\}$. There is also one 4-vertex clique, among the vertices $\{C, G, H, I\}$.

11.2 - Cliques and Separators

(a)

The moralized graph of the DAG is an undirected graph where additional connections are added between nodes that share a child in the DAG.

Figure 1: Moralization of the DAG



(b)

Generally, the 1-vertex cliques are all the vertices of the moral graph, the 2-vertex cliques are formed by all the vertex pairs that are directly connected, and the only 3-vertex in the graph consists of $\{A, BC\}$. The separators of the graph are $\{C\}, \{A, C\}, \{B, C\}, \{C, D\}, \{C, E\}, \{A, B, C\}, \{A, C, E\}, \{A, C, D\}, \{B, C, D\}, \{B, C, E\}$.

The given formula works only for maximal cliques and minimal separators. We have $\{A, B, C\}, \{B, C\}, \{C, E\}$ as a maximal cliques and $\{C\}$ as minimal separator.

$$\begin{gathered} p(a,b,c,d,e) = p(e|c)p(d|c)p(c|a,b)p(a,b) = \\ = \frac{p(e,c)p(d,c)p(a,b,c)p(a,b)}{p(c)p(a,b)} = \frac{p(a,b,c)p(e,c)p(d,c)}{p(c)} \end{gathered} \tag{1}$$

We see that in equation above the product of the cliques is the numerator and the separator is the denominator.

11.3 Representation of the knowledge base

a)

```
'''This is the burglary example from Exercise 10.3'''
from bayesian.bbn import build_bbn

def f_burglary(burglary):
    if burglary is True:
        return 0.01
    return 0.99

def f_earthquake(earthquake):
    if earthquake is True:
        return 0.000001
    return 0.999999

def f_radioBroadcast(earthquake, radioBroadcast):
    table = dict()
    table['tt'] = 1.
    table['tf'] = 0.
    table['ft'] = 0.
    table['ff'] = 1.
    key = ''
    key = key + 't' if earthquake else key + 'f'
    key = key + 't' if radioBroadcast else key + 'f'
    return table[key]
```

```
def f_alarm(burglary, earthquake, alarm):
    table = dict()
    table['fft'] = 0.001
    table['fff'] = 0.999
    table['ftt'] = 0.41
    table['ftf'] = 0.59
    table['tft'] = 0.95
    table['tff'] = 0.05
    table['ttt'] = 0.98
    table['ttf'] = 0.02
    key = ''
    key = key + 't' if burglary else key + 'f'
    key = key + 't' if earthquake else key + 'f'
    key = key + 't' if alarm else key + 'f'
    return table[key]

if __name__ == '__main__':
    g = build_bbn(
        f_burglary,
        f_earthquake,
        f_radioBroadcast,
        f_alarm)
    g.q()
```

b)

The code presented above creates the following output when run:

```
+----------------+-------+----------+
| Node           | Value | Marginal |
+----------------+-------+----------+
| alarm          | False | 0.989510 |
| alarm          | True  | 0.010490 |
| burglary       | False | 0.990000 |
| burglary       | True  | 0.010000 |
| earthquake     | False | 0.999999 |
| earthquake     | True  | 0.000001 |
| radioBroadcast | False | 0.999999 |
| radioBroadcast | True  | 0.000001 |
+----------------+-------+----------+
```

Here we can easily see the marginal probabilities $p(A = t) = 0.989510$, $p(B = t) = 0.99, p(E = t) = 0.000001, p(R = t) = 0.000001$. This bayesian belief network uses a datastructure comprising four functions, one for each node in the DAG. For the independent events of a burglary and an earthquake occuring, we simply return the probability, whereas for the other nodes a probility is chosen from a table of probabilities depending on the input. When the *Pythonic Bayesian Belief Network Framework* is built, it computes the marginal probabilities.