$$C_{(p,N)} + C_{(p,N-1)} = C_{(p+1,N)}$$

$$2\sum_{k=0}^{N-1}\binom{p-1}{k} + 2\sum_{k=0}^{N-1-1}\binom{p-1}{k} = 2\sum_{k=0}^{N-1}\binom{p}{k}$$

$$\sum_{k=0}^{N-1}\binom{p-1}{k} + \sum_{k=0}^{N-2}\binom{p-1}{k} = \sum_{k=0}^{N-1}\binom{p}{k}$$

$$1 + \sum_{k=1}^{N-1}\binom{p-1}{k} + \sum_{k=0}^{N-2}\binom{p-1}{k} = 1 + \sum_{k=1}^{N-1}\binom{p}{k}$$

$$\sum_{k=1}^{N-1}\binom{p-1}{k} + \sum_{k=0}^{N-2}\binom{p-1}{k} = \sum_{k=1}^{N-1}\binom{p}{k}$$

$$\sum_{k=1}^{N-1}\binom{p-1}{k} + \sum_{k=1}^{N-1}\binom{p-1}{k-1} = \sum_{k=1}^{N-1}\binom{p}{k}$$

$$\sum_{k=1}^{N-1}\left(\binom{p-1}{k} + \binom{p-1}{k-1}\right) = \sum_{k=1}^{N-1}\binom{p}{k}$$

$$\binom{p-1}{k} + \binom{p-1}{k-1} = \binom{p}{k}$$