Martin Lundfall, Henri Bunting, Malte Siemers, Patrik Bey

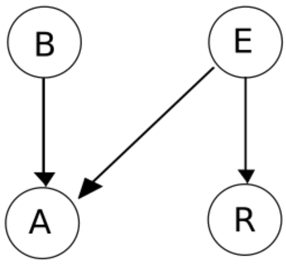
Exercise sheet 12 - Machine Intelligence I

12.1 - Implementation of belief propagation

a)

The directed acyclic graph of the network is the previously illustrated figure below.

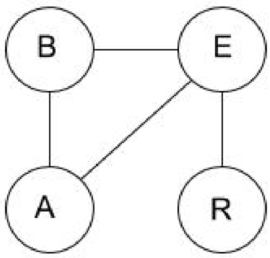
Figure 1: DAG illustrating joint probability distribution



b)

First, we construct the moral graph of DAG above.

Figure 2: Moral graph of the network

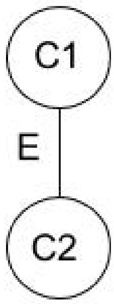


To construct the junction tree, we compress cliques to single nodes. The node C1 is clique of $\{A, B, E\}$ in the moral graph above and the node C2 is the clique $\{E, R\}$ and the link is the separator E. The clique potential at C1 is the product:

$$
\begin{aligned}
P(E)P(B)P(A|B,E) = \\
P(E)P(B) \sum_{e \in E, b \in B} (P(A|e,b) \\
= 0.000001 \cdot 0.01 \cdot \\
(0.001 \cdot 0.99 \cdot 0.99999 + 0.41 \cdot 0.99 \cdot 0.000001 + \\
0.95 \cdot 0.01 \cdot 0.99999 + 0.98 \cdot 0.01 \cdot 0.000001) = 1.049^{-10}
\end{aligned}
$$

The separator potential is initialized to 1. The clique potential of C2 is: $P(E)P(R|E) = P(E) = 0.000001$

Figure 3: Junction tree of the network



c)

The following marginals were calculated with the bayesian package for python we also used in assignment 11:

Node	Value	Marginal
alarm	False	0.989510
alarm	True	0.010490
burglary	False	0.990000
burglary	True	0.010000
earthquake	False	0.999999
earthquake	True	0.000001
radioBroadcast	False	0.999999
radioBroadcast	True	0.000001

d)

The probability that a burglary has happend when the alarm turns on is

$$P(B=t|A=t) = \frac{P(A=t|B=t)P(B=t)}{P(A=t)} =$$

$$\frac{(P(A=t|B=t,E=t)P(E=t)+P(A=t|B=t,E=f)P(E=f))P(B=t)}{P(A=t)} =$$

$$\frac{(0.98*10^{-6}+0.95*0.999999)*0.01}{0.010490} = 0.905624...$$

The influence of hearing the radio while an alarm goes off is the following:

$$P(B=t|A=t,E=t) = \frac{P(E=t,A=t|B=t)P(B=t)}{P(A=t,E=t)} =$$

$$= \frac{P(A=t|E=t,B=t)P(B=t)P(E=t)}{P(A=t|E=t)P(E=t)} =$$

$$= \frac{P(A=t|B=t,E=t)P(B=t)}{P(A=t|B=t,E=t)P(B=t)+P(A=t|B=f,E=t)P(B=f)}$$

$$= \frac{0.98*0.01}{0.98*0.01+0.41*0.99} = 0.02357...$$

The probability, that a burglary had happened when the alarm goes off while one heard about an earthquake in the radio is drastically smaller than without the information on the earthquake.

12.2 - Bayesian Inference

a

Generative model
Generative models diverge from strict assignment of labels to allow for probabilistic assignment. It represents a hypothesis about the causes.
We can say that some latent variable, for example y*, is the driving factor behind our label assignment function, y.
An example of a generative model is additive noise. We can model our y as:

$$y = y * +\eta$$

Here, y is modeled by the latent $y * +\eta$ for some noise function η

Prior distribution
The prior distribution represents some initial beliefs of values for our parameters. It is modeled by: $P(\theta)$

Likelihood function
$L(\theta)$
In Maximum Likelihood estimation, we take a subset of the observations and maximize this to give us a θ which we then use to make predictions for future features. It is a product of the probabilities.

Posterior distribution
After observing data, we update our beliefs.
$(P(x1, ..., xn|\theta) * P(\theta))/P(x1, ..., xn)$

Predictive distribution
How we can predict new data.
$P(x^{n+1}|x1, ..., xn) = \int(P(x^{n+1}|\theta) * P(\theta|x1, ..., xn)d\theta$
While this is a major advantage for Bayesians, it is also computationally expensive.

b

In the Bayesian approach, we update the knowledge base after every new piece of data, so we do not encounter the problem of over-/under-fitting.