

Exercise sheet 11 - Machine Intelligence I

11.1 - Cliques

In the given graph, we have 10 vertices and 17 edges. These make up 10 1-vertex cliques and 17 2-vertex cliques. We also identify the 3-vertex cliques consisting of vertices

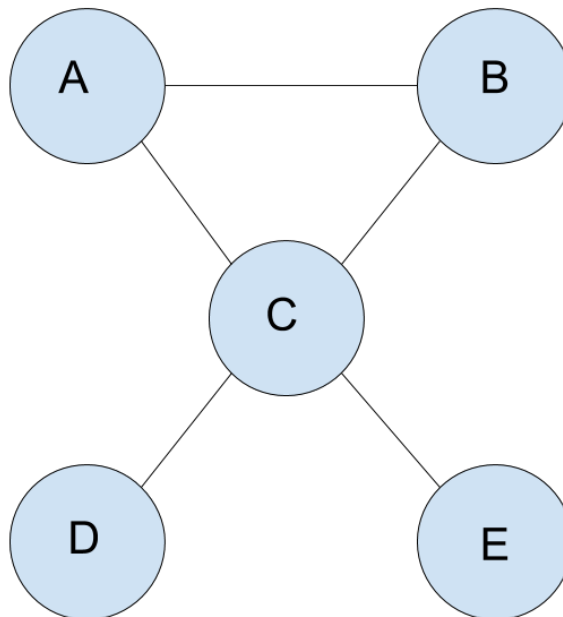
$\{A, C, H\}, \{A, C, G\}, \{B, C, D\}, \{B, C, G\}, \{C, H, I\}, \{G, H, I\}$. There is also one 4-vertex clique, among the vertices $\{C, G, H, I\}$.

11.2 - Cliques and Separators

(a)

The moralized graph of the DAG is an undirected graph where additional connections are added between nodes that share a child in the DAG.

Figure 1: Moralization of the DAG



(b)

Generally, the 1-vertex cliques are all the vertices of the moral graph, the 2-vertex cliques are formed by all the vertex pairs that are directly connected,

and the only 3-vertex in the graph consists of $\{A, BC\}$. The separators of the graph are $\{C\}, \{A, C\}, \{B, C\}, \{C, D\}, \{C, E\}, \{A, B, C\}, \{A, C, E\}, \{A, C, D\}, \{B, C, D\}, \{B, C, E\}$.

The given formula works only for maximal cliques and minimal separators. We have $\{A, B, C\}, \{B, C\}, \{C, E\}$ as a maximal cliques and $\{C\}$ as minimal separator.

$$\begin{aligned}
 p(a, b, c, d, e) &= p(e|c)p(d|c)p(c|a, b)p(a, b) = \\
 &= \frac{p(e, c)p(d, e)p(a, b, c)p(a, b)}{p(c)p(a, b)} = \frac{p(a, b, c)p(e, c)p(d, c)}{p(c)} \tag{1}
 \end{aligned}$$

11.3 Representation of the knowledge base